

Make Illegal States Unrepresentable

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- Running example — interpreter for arithmetic & conditional expressions

Questions welcome throughout the talk!

Our language

$\langle \text{expr} \rangle ::= \mathbb{Z}$
| $\langle \text{expr} \rangle + \langle \text{expr} \rangle$
| $\langle \text{expr} \rangle - \langle \text{expr} \rangle$
| $\langle \text{expr} \rangle * \langle \text{expr} \rangle$
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For example if 5 + 6 == 11 then
 12 / 6 is a valid expression
 else 3 * 4

Interpreter in C

```
<expr> :=  $\mathbb{Z}$ 
        | <expr> + <expr>
        | <expr> - <expr>
        | <expr> * <expr>
        | <expr> / <expr>
        | <expr> == <expr>
        | if <expr> then <expr>
          else <expr>
```

```
typedef enum {
    Const, Add, Sub, Mul, Div, Eq, Ite
} expr_tag;

typedef struct expr {
    expr_tag tag;
    union {
        int int_value;
        struct expr** args;
    } info;
} expr;
```

Interpreter in C

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<expr> := ℤ  
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        | <expr> - <expr>  
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else 3 * 4
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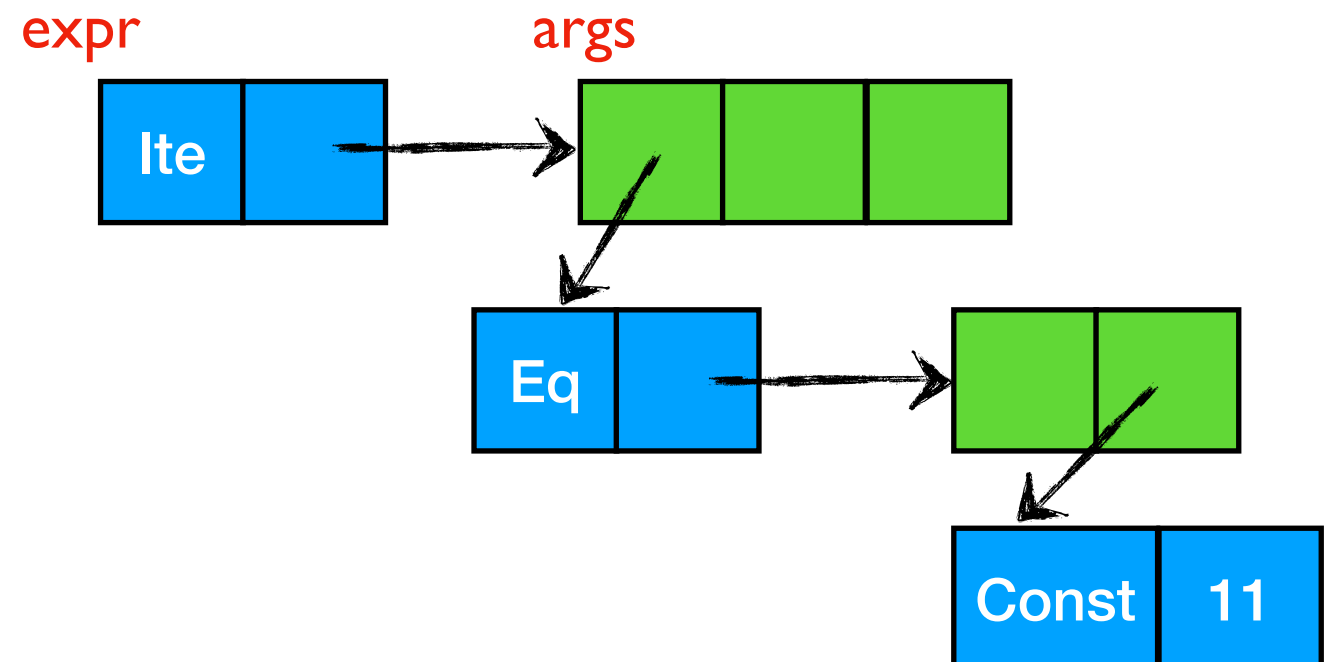
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```
if 5 + 6 == 11 then  
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else 3 * 4
```



Demo: interp.c

Invariants

- Although the code works for the example, there are several invariants not handled by the code
 - ✦ An expression may be NULL
 - ✦ The argument array of an expression may be NULL
 - ✦ An expression may not have the right number of arguments
 - ✦ The denominator of the division may be 0

Invariants

- Others invariants are less clear
 - ◆ The first argument of the if-then-else expression may not be a predicate (==)
 - ✿ Should `if 5 + 6 then 24 else 23` be allowed?
 - ◆ Integer overflow not handled
 - ◆ Memory safety not ensured
 - ✿ corrupted, non-NULL pointers for expressions
 - ✿ A not-NULL expression may not always be a well-formed expression

F* programming languages

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F* programming languages

- A solver-aided (Z3) general purpose programming language
 - ✦ Functional programming language
 - ✦ Advanced type system to express rich invariants
- Programs can be extracted to OCaml, F#, C, Wasm and ASM.
- Main use case is Project Everest at Microsoft — a drop in replacement for HTTPS stack
 - ✦ Verified implementations of TLS 1.2 and 1.3, and underlying cryptographic primitives
 - ✦ Also in, Mozilla Firefox, WireGuard etc.

Our language — vl

$\langle \text{expr} \rangle := \mathbb{Z}$
| $\langle \text{expr} \rangle + \langle \text{expr} \rangle$
| $\langle \text{expr} \rangle - \langle \text{expr} \rangle$
| $\langle \text{expr} \rangle * \langle \text{expr} \rangle$

Demo: interpvl.fst

Invariants

- ♦ ~~An expression may be NULL~~
- ♦ ~~The argument array of an expression may be NULL~~
 - ♦ *No NULL pointers in F*!*
- ♦ ~~An expression may not have the right number of arguments~~
 - ♦ *Constructors take arguments!*
- ♦ The denominator of the division may be 0
- ♦ The first argument of the if-then-else expression may not be a predicate (==)
- ♦ ~~Integer overflow not handled~~
 - ♦ *Infinite precision integers by default*
- ♦ ~~Memory safety not ensured~~
 - ♦ *Garbage collection*

Our language — v2

$\langle \text{expr} \rangle := \mathbb{Z}$
| $\langle \text{expr} \rangle + \langle \text{expr} \rangle$
| $\langle \text{expr} \rangle - \langle \text{expr} \rangle$
| $\langle \text{expr} \rangle * \langle \text{expr} \rangle$
| $\langle \text{expr} \rangle / \langle \text{expr} \rangle$

Demo: interpv2.fst

Invariants

- ♦ ~~An expression may be NULL~~
- ♦ ~~The argument array of an expression may be NULL~~
- ♦ ~~An expression may not have the right number of arguments~~
- ♦ ~~The denominator of the division may be 0~~
 - ♦ *F* enforces denominator to be non-zero (subtyping)*
- ♦ The first argument of the if-then-else expression may not be a predicate (==)
- ♦ ~~Integer overflow not handled~~
- ♦ ~~Memory safety not ensured~~

Our language — v3

$\langle \text{expr} \rangle := \mathbb{N}$
| $\langle \text{expr} \rangle + \langle \text{expr} \rangle$
| $\langle \text{expr} \rangle - \langle \text{expr} \rangle$
| $\langle \text{expr} \rangle * \langle \text{expr} \rangle$
| $\langle \text{expr} \rangle / \langle \text{expr} \rangle$

Demo: `interp v3.fst`

Our language — v4

`<const> ::= \mathbb{N} | Bool`

`<expr> ::= <const>`

`| <expr> + <expr>`

`| <expr> - <expr>`

`| <expr> * <expr>`

`| <expr> / <expr>`

`| <expr> == <expr>`

`| if <expr> then <expr> else <expr>`

Our language — v4

$\langle \text{const} \rangle ::= \mathbb{N} \mid \text{Bool}$

$\langle \text{expr} \rangle ::= \langle \text{const} \rangle$

$\mid \langle \text{expr} \rangle + \langle \text{expr} \rangle$

$\mid \langle \text{expr} \rangle - \langle \text{expr} \rangle$

$\mid \langle \text{expr} \rangle * \langle \text{expr} \rangle$

$\mid \langle \text{expr} \rangle / \langle \text{expr} \rangle$

$\mid \langle \text{expr} \rangle == \langle \text{expr} \rangle$

$\mid \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{expr} \rangle \text{ else } \langle \text{expr} \rangle$

Must reject

if 5 + 6 then 23 else 24

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$\langle \text{const} \rangle ::= \mathbb{N} \mid \text{Bool}$

$\langle \text{expr} \rangle ::= \langle \text{const} \rangle$

| $\langle \text{expr} \rangle + \langle \text{expr} \rangle$

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Allow only boolean
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Add static type checking to our language!

Our language — v4

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`<expr> ::= <const>`

`| <expr> + <expr>`

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`| <expr> * <expr>`

`| <expr> / <expr>`

`| <expr> == <expr>`

`| if <expr> then <expr> else <expr>`

Must reject

`if 5 + 6 then 23 else 24`

Allow only boolean
expressions here

Add static type checking to our language!

Demo: `interp v4.fst`

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- ♦ ~~An expression may be NULL~~
- ♦ ~~The argument array of an expression may be NULL~~
- ♦ ~~An expression may not have the right number of arguments~~
- ♦ ~~The denominator of the division may be 0~~
- ♦ ~~The first argument of the if-then-else expression may not be a predicate
(==)~~
 - ♦ *Encode static types using type-level functions*
- ♦ ~~Integer overflow not handled~~
- ♦ ~~Memory safety not ensured~~

Summary

- Functional programming offers a concise way to write expressive programs
- Rich type systems help express and enforce program invariants
- Slides + programs available at

<https://github.com/prismlab/aicte-compilers-lecture-2021>