

Image and Video Processing

Morphological Image Processing

Morphological operations can be used to remove imperfections in the segmented image and provide information on the form and structure of a region shape

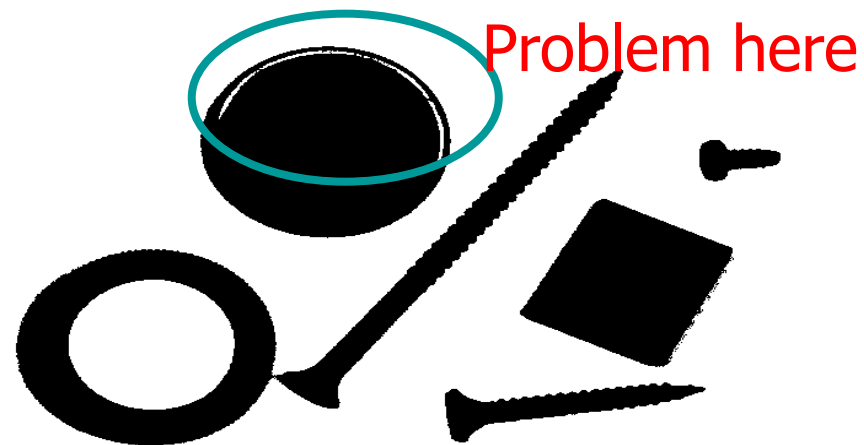
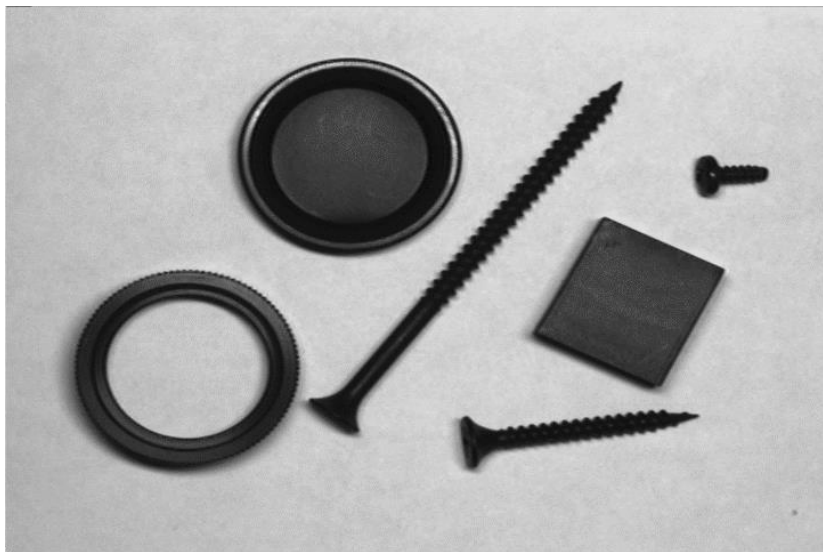
In this topic we will consider

- What is morphology?
- Simple morphological operations
- Compound operations
- Morphological algorithms

What Is Morphology?

- Morphological image processing (or *morphology*) describes a range techniques for extracting image components that are useful in the representation & description of region shape such as boundaries, skeletons etc.
- Some of the operations are frequently applied as post-processing to remove imperfections introduced during segmentation, and so typically operate on binary images.

Quick Example 1



How do we fill "missing pixels"?

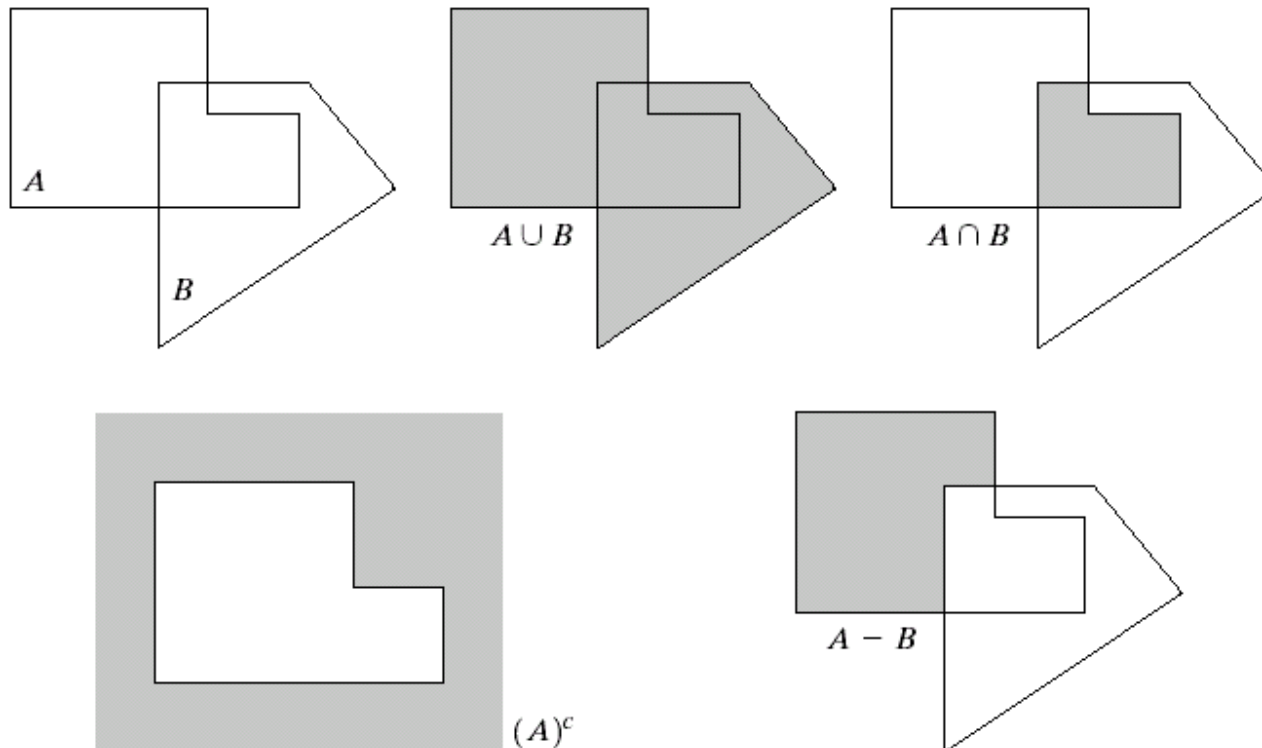


Image after segmentation



Image after segmentation and
morphological processing

- Language of mathematical morphology is set theory



a	b	c
d	e	

FIGURE 9.1

(a) Two sets A and B . (b) The union of A and B . (c) The intersection of A and B . (d) The complement of A . (e) The difference between A and B .

- Logic operations involving binary images

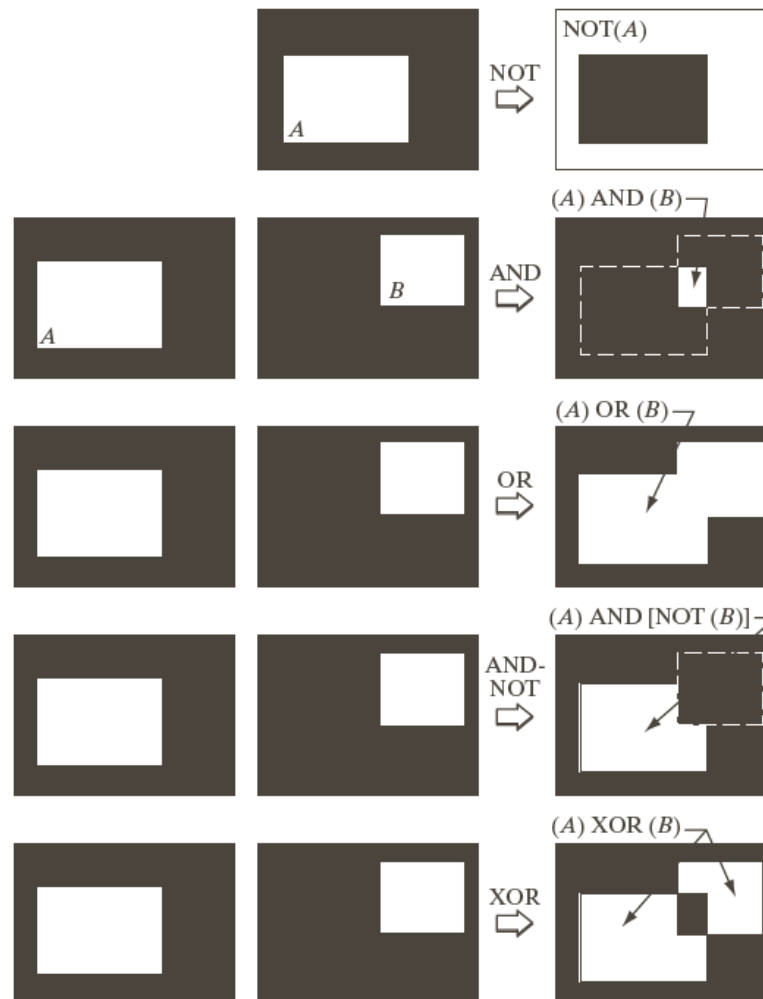


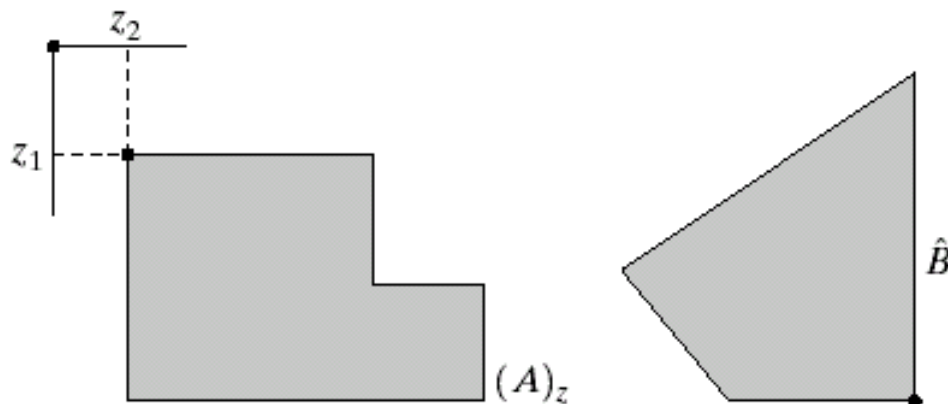
FIGURE 2.33

Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

- Reflection and Translation:

$$\hat{B} = \{w \mid w \in -b, \text{ for } b \in B\}$$

$$(A)_z = \{c \mid c \in a + z, \text{ for } a \in A\}$$



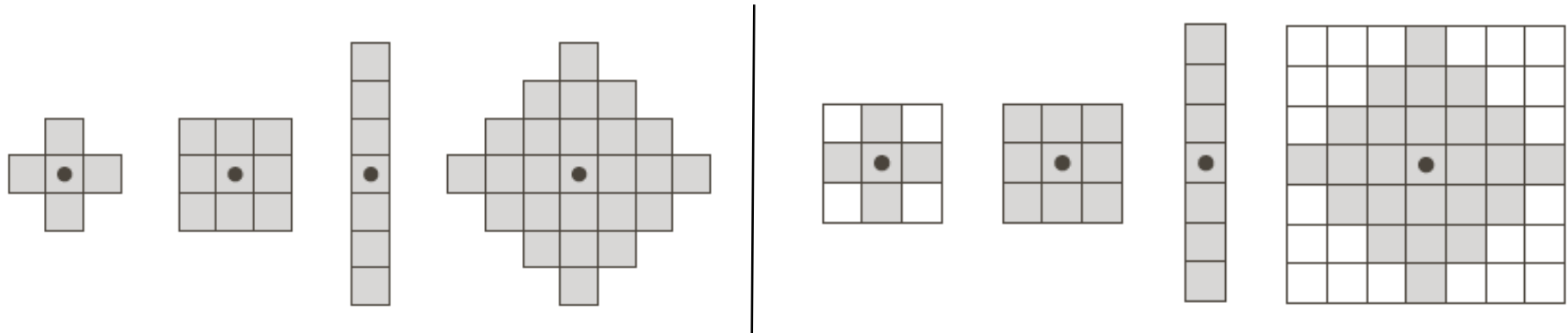
a b

FIGURE 9.2

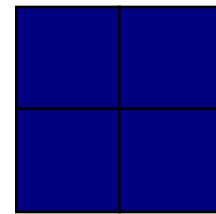
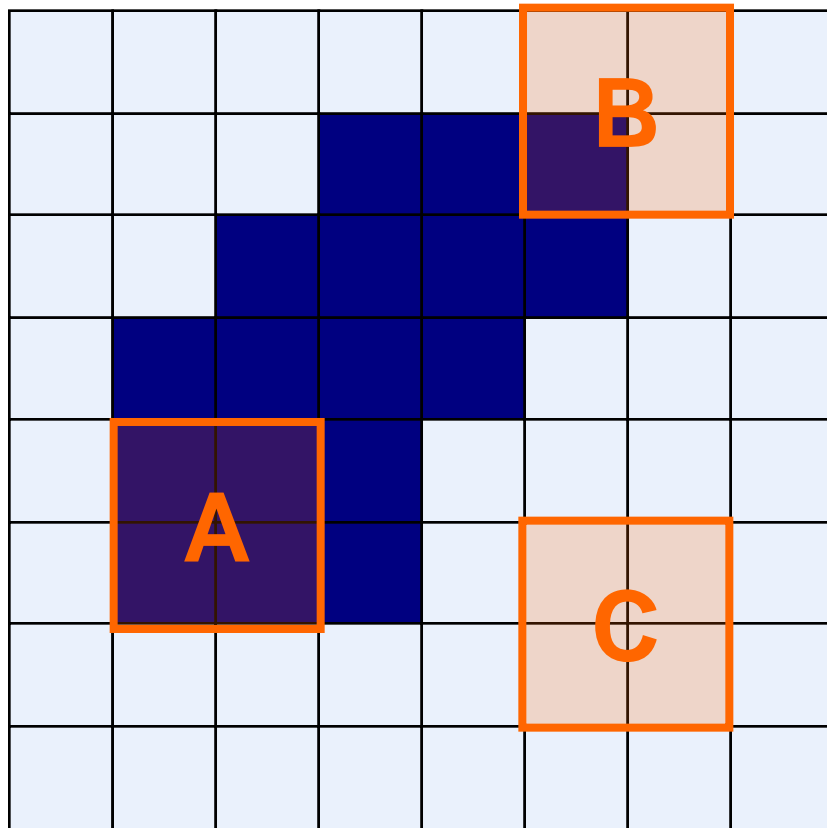
(a) Translation of A by z .

(b) Reflection of B . The sets A and B are from Fig. 9.1.

- Structuring elements are small sets/sub-images used to probe an image under study
- For each SE, define its origin
- shape and size must be adapted to geometric properties for the objects
 - For simplicity we will use rectangular structuring elements with their origin at the middle pixel



Structuring Elements, Hits & Fits



Structuring Element

Fit: All *on pixels* in the structuring element cover *on pixels* in the image

Hit: Any *on pixel* in the structuring element covers an *on pixel* in the image

Basic morphological processing operations are based on these simple ideas (informally)

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0	0	0	0	0
0	0	1	B	1	1	1	0	C	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	0	0	0	0
0	0	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	A	1	1	1	0
0	0	0	0	0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0	0

1	1	1
1	1	1
1	1	1

Structuring
Element 1

0	1	0
1	1	1
0	1	0

Structuring
Element 2

- Fundamentally morphological image processing is very much like spatial filtering
- The structuring element is moved across every pixel in the original image to give a pixel in a new processed image
- The value of this new pixel depends on the operation performed
- There are two basic morphological operations: **erosion** and **dilation**

Erosion of image A by structuring element B represented as sets in \mathbb{Z}^2 , is defined as:

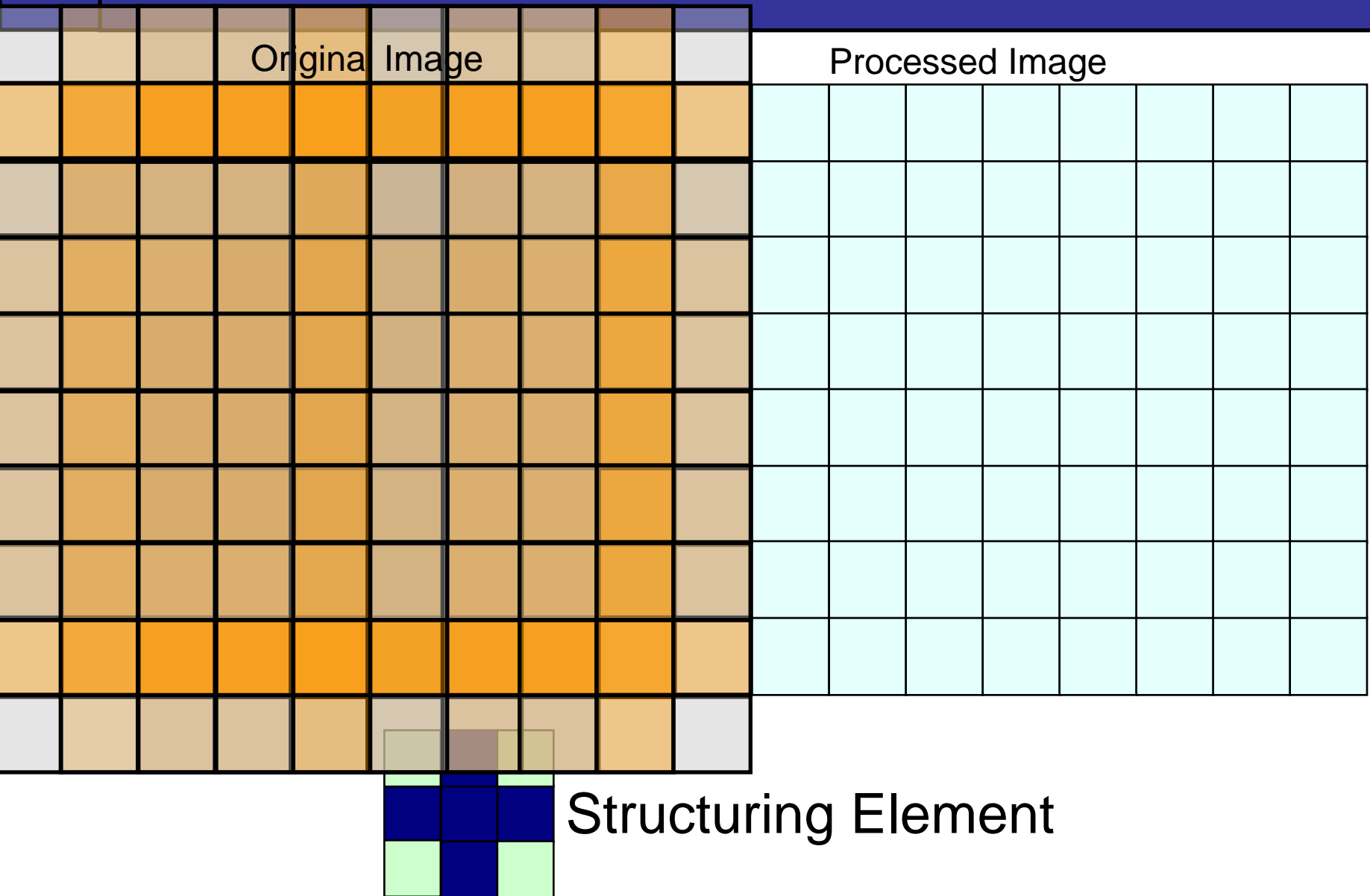
$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

The set of all points z such that B , translated by z , is contained by A .

Informally, the structuring element B is positioned with its origin at (x, y) and the new pixel value is determined using the rule:

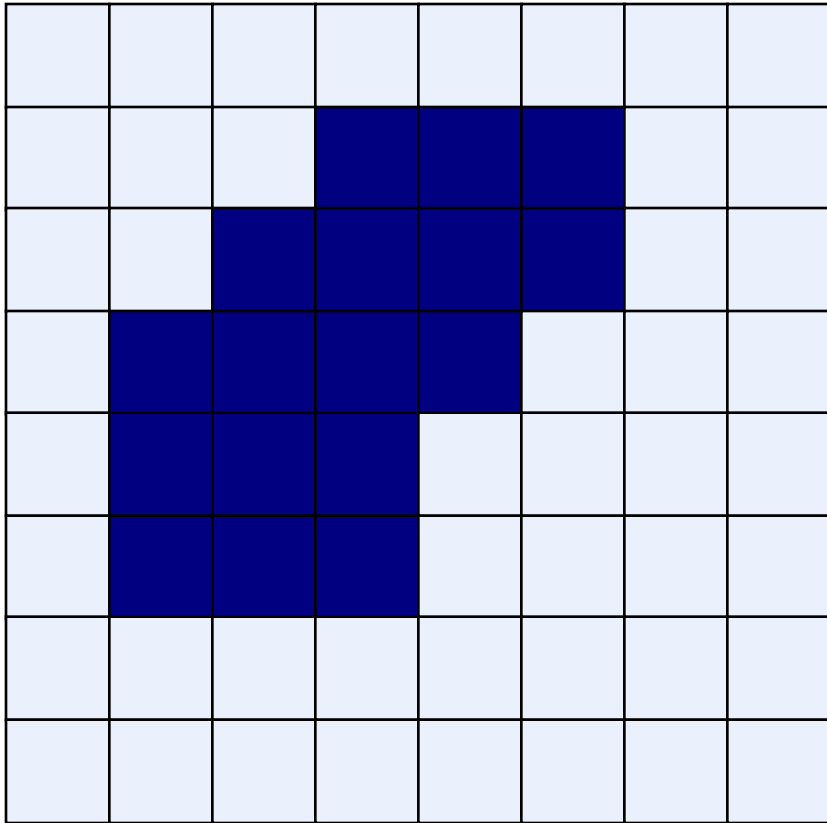
$$g(x, y) = \begin{cases} 1 & \text{if } B \text{ fits } A \\ 0 & \text{otherwise} \end{cases}$$

Erosion Example

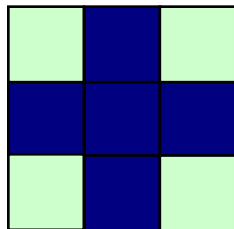
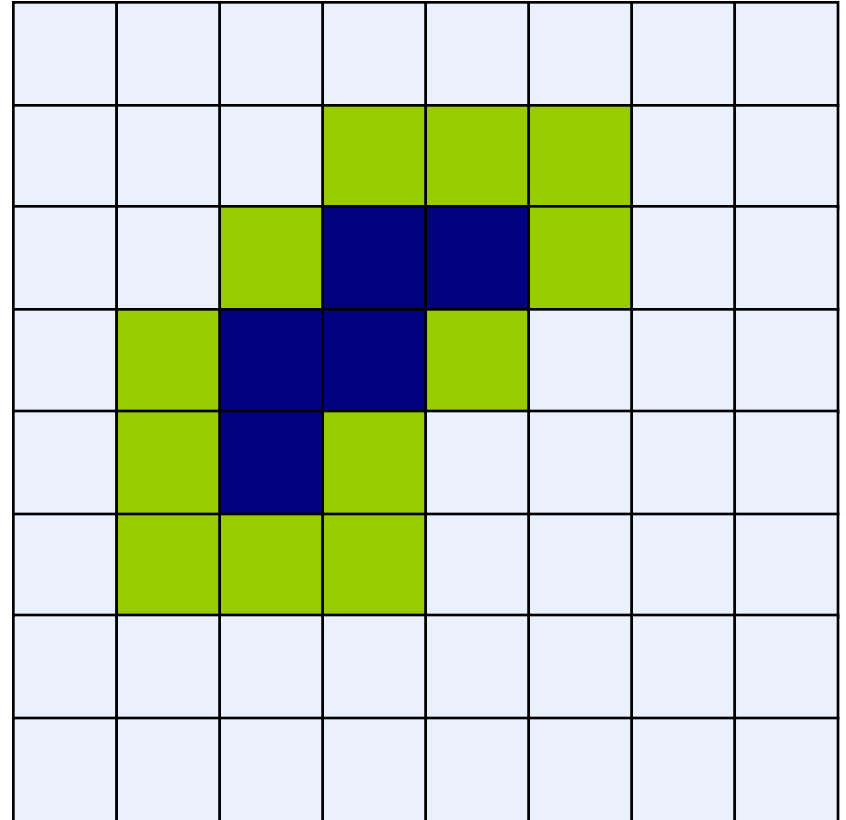


Erosion Example

Original Image



Processed Image With Eroded Pixels

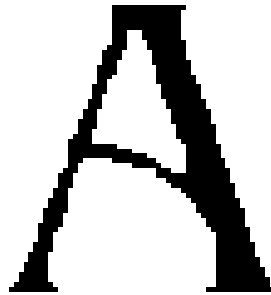


Structuring Element

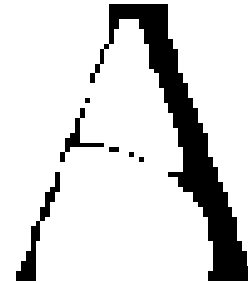
Erosion Example 1



Original image



Erosion by 3*3
square structuring
element

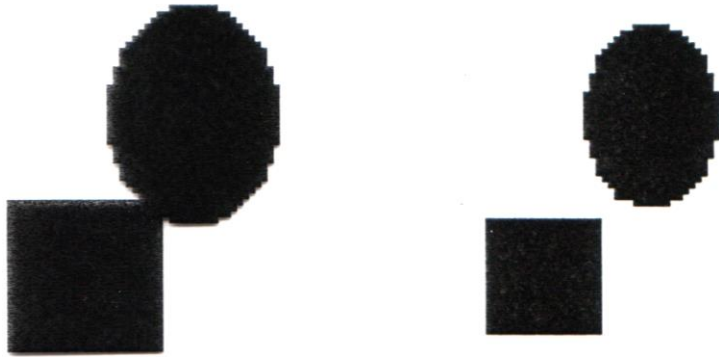


Erosion by 5*5
square structuring
element

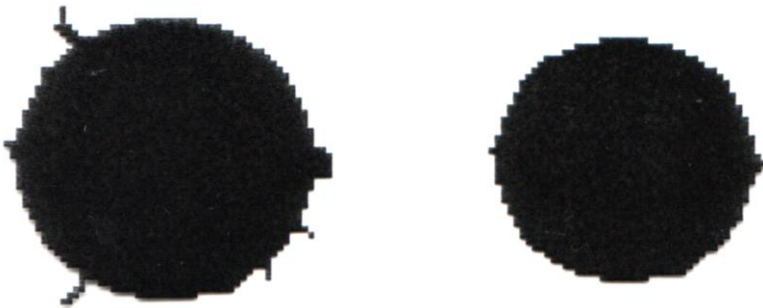
Watch out: In these examples a 1 refers to a black pixel!

What Is Erosion For?

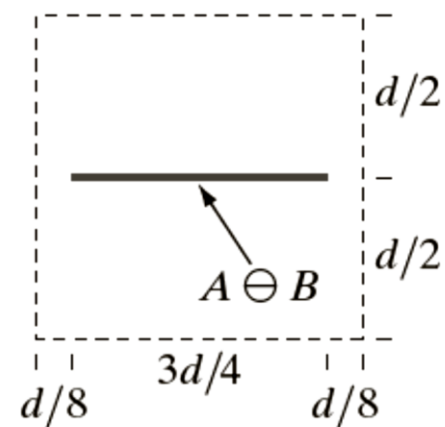
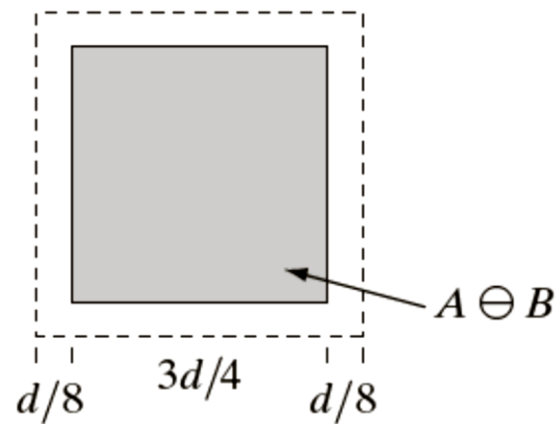
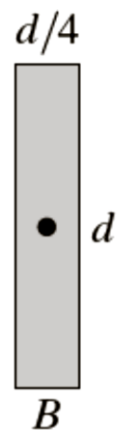
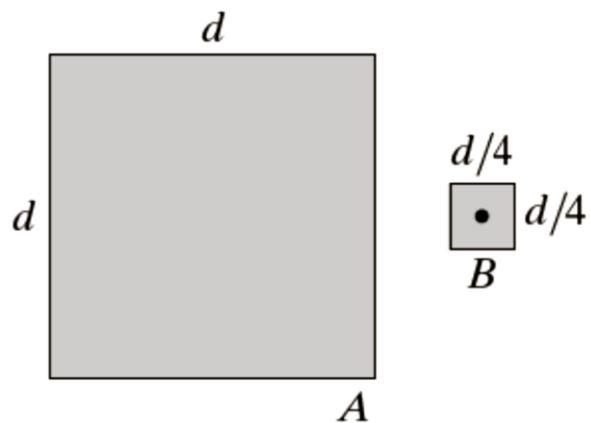
Erosion can split apart joined objects



Erosion can strip away extrusions

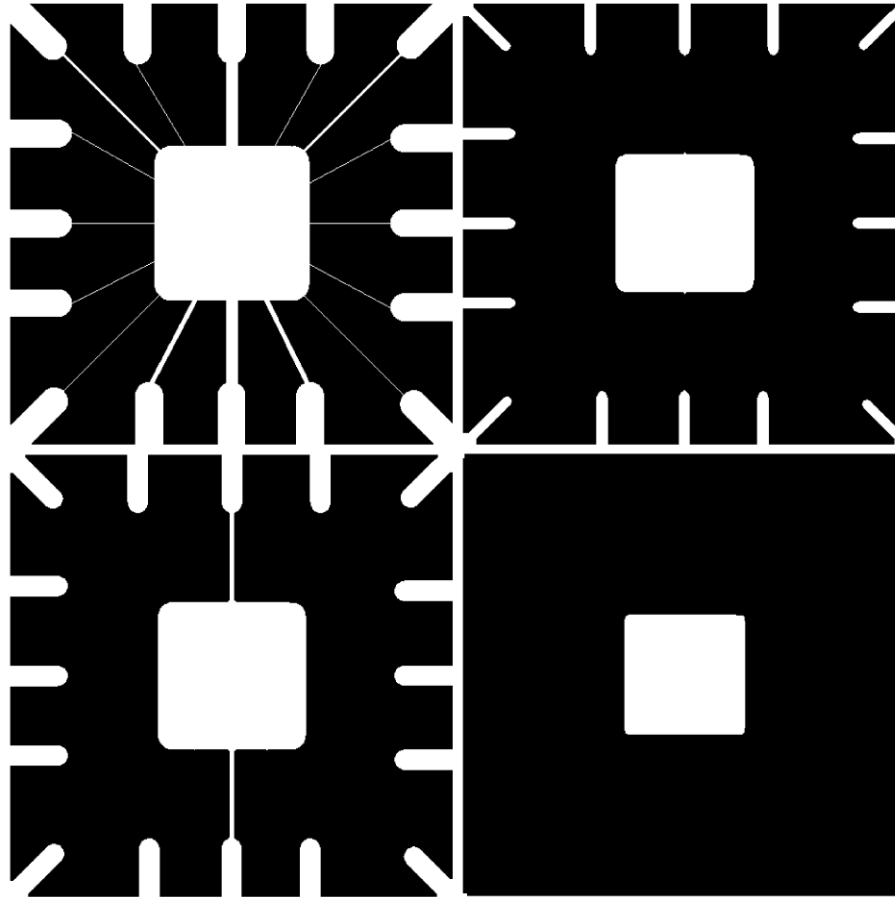


Watch out: Erosion shrinks objects



Erosion Example 3

Original image



After erosion
with a disc of
radius 15

After erosion
with a disc of
radius 11

After erosion
with a disc of
radius 45

Dilation of image A by structuring element B is given by:

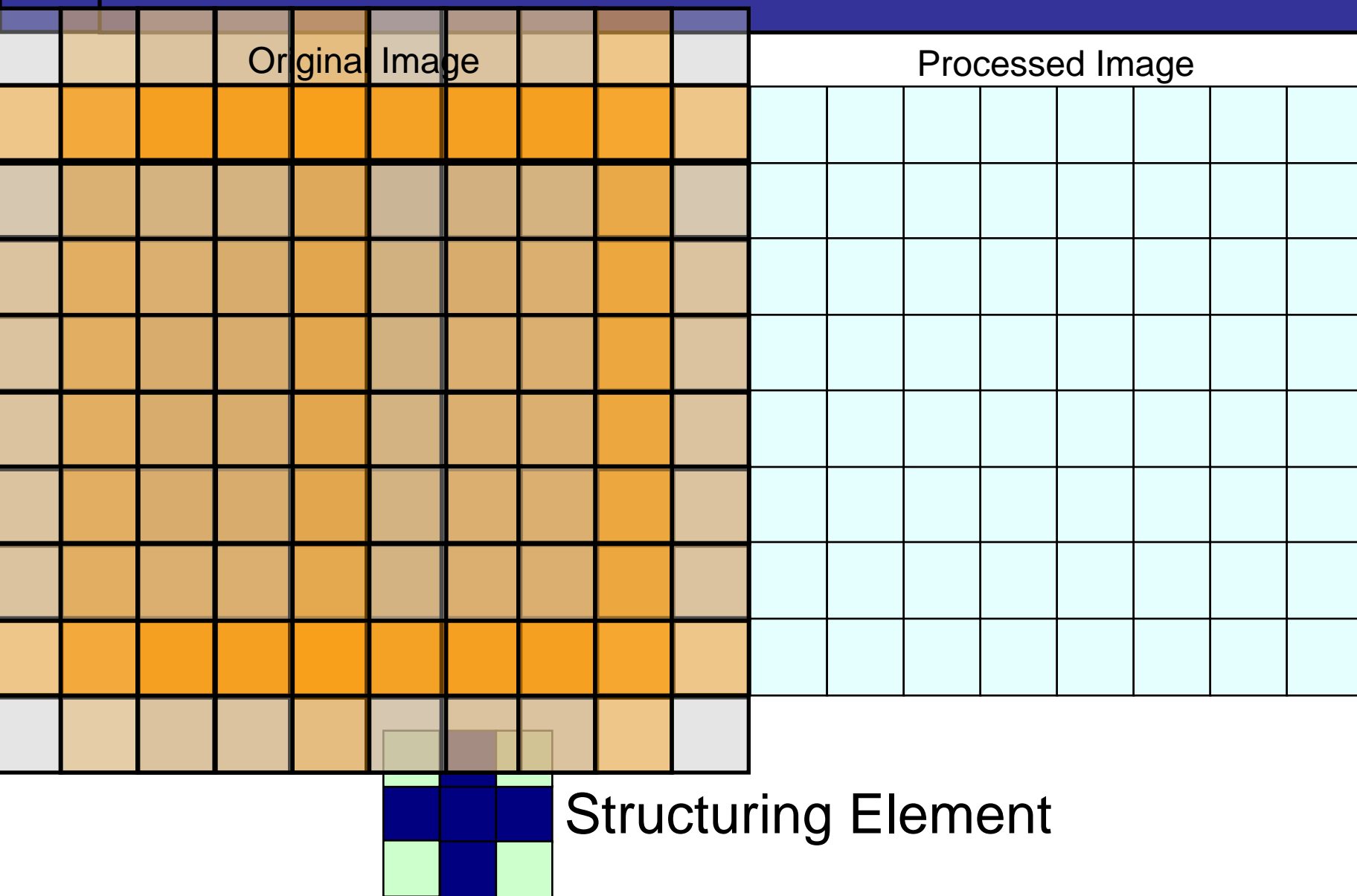
$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$$

The set of all displacements z , such that \hat{B} and A overlap by at least one element.

Informally: The structuring element B (if symmetric with origin at center, else reflected) is positioned with its origin at (x, y) and the new pixel value is determined using the rule:

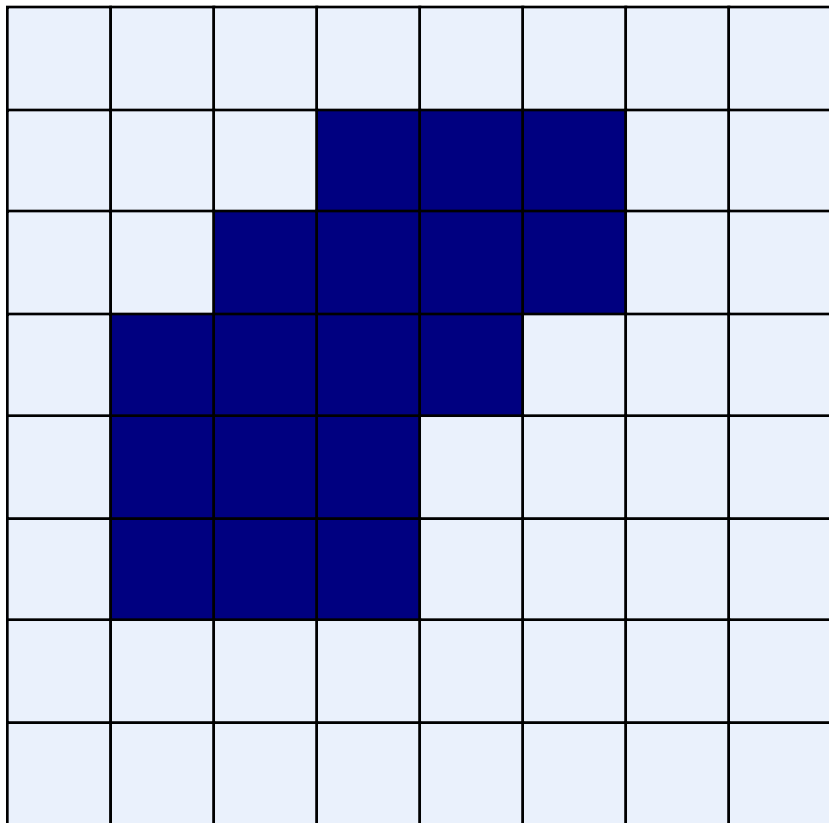
$$g(x, y) = \begin{cases} 1 & \text{if } B \text{ hits } A \\ 0 & \text{otherwise} \end{cases}$$

Dilation Example

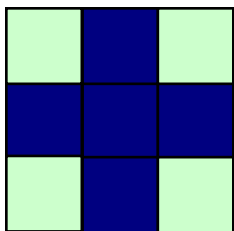
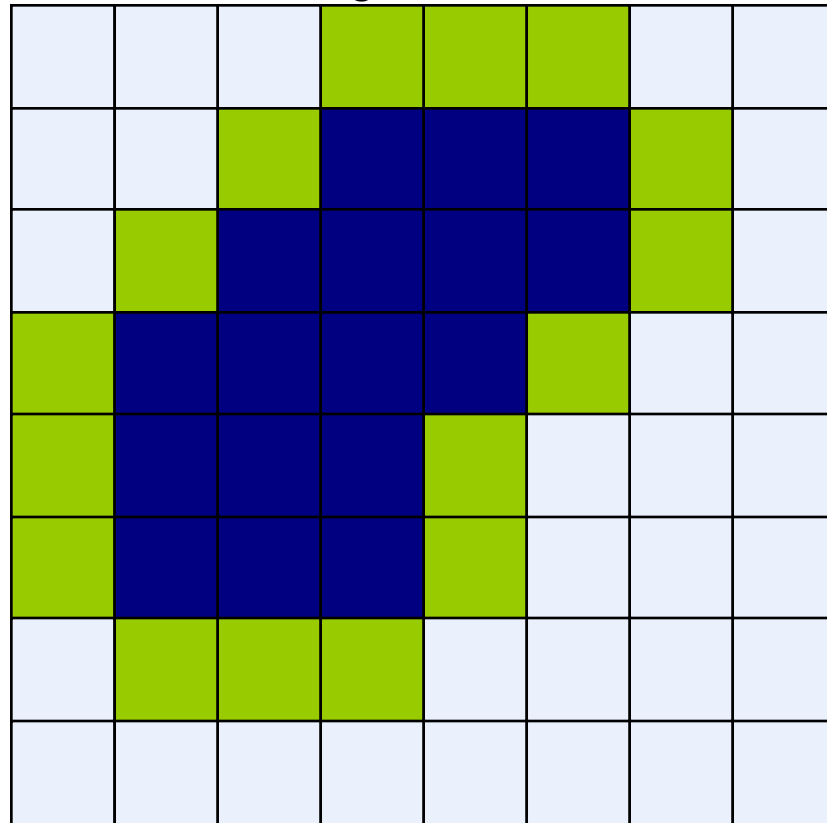


Dilation Example

Original Image



Processed Image With Dilated Pixels



Structuring Element

Dilation Example 1



Original image



Dilation by 3*3
square structuring
element



Dilation by 5*5
square structuring
element

Watch out: In these examples a 1 refers to a black pixel!

Dilation Example 2

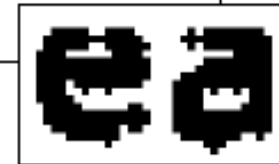
Original image

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



After dilation

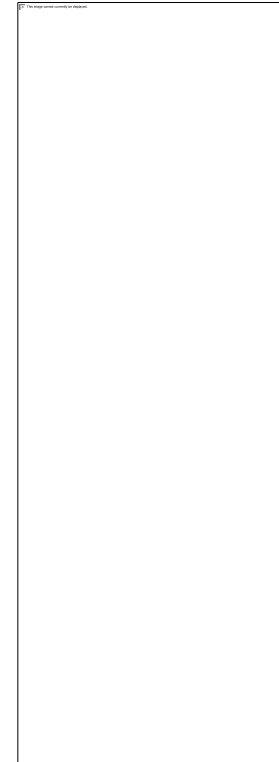
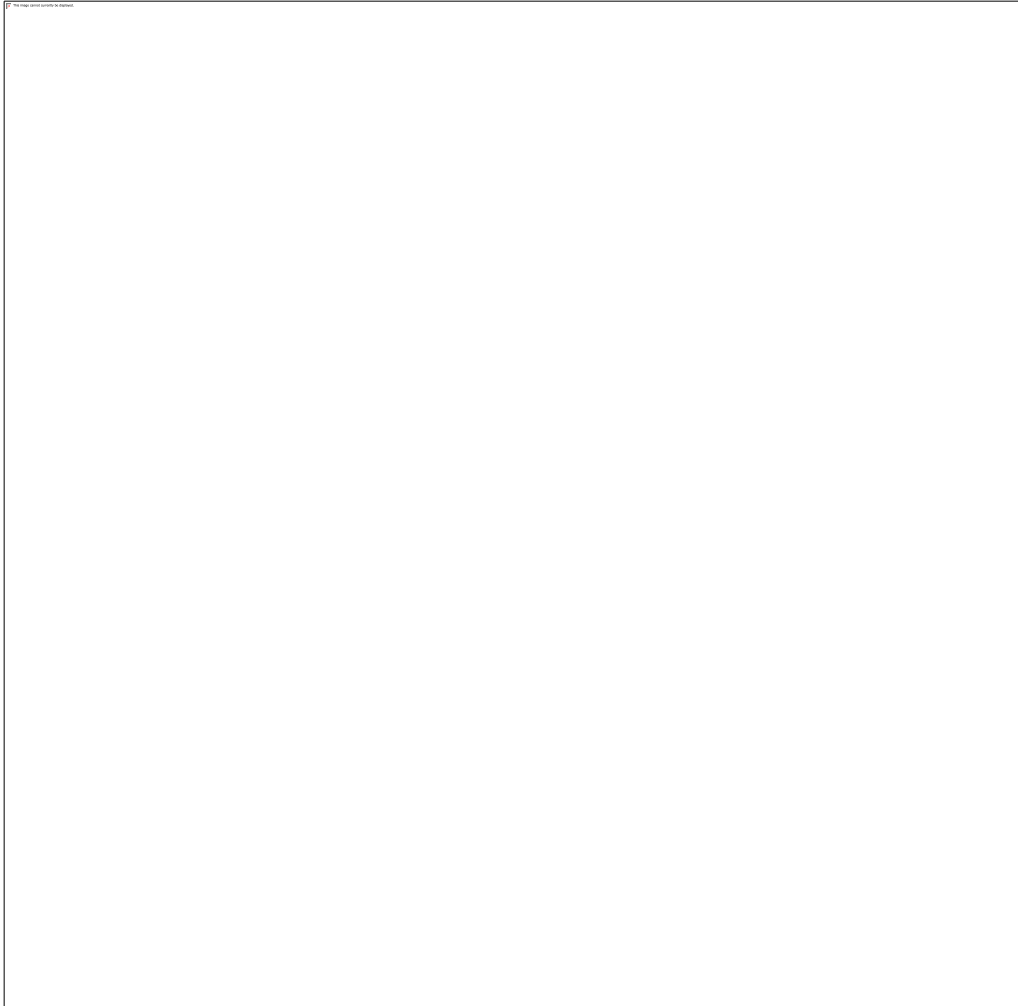
Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



0	1	0
1	1	1
0	1	0

Structuring element

Dilation Example 3



- What is the result of this dilation?

0	0	0	0		0	1	0		
0	1	1	0	\oplus	0	1	1	=	
0	0	0	0		0	0	0		

- What is the result of this dilation?

0	0	0	0		0	1	0		0	1	1	0
0	1	1	0	\oplus	0	1	1	=	0	1	1	1
0	0	0	0		0	0	0		0	0	0	0

What Is Dilation For?

Dilation can repair breaks



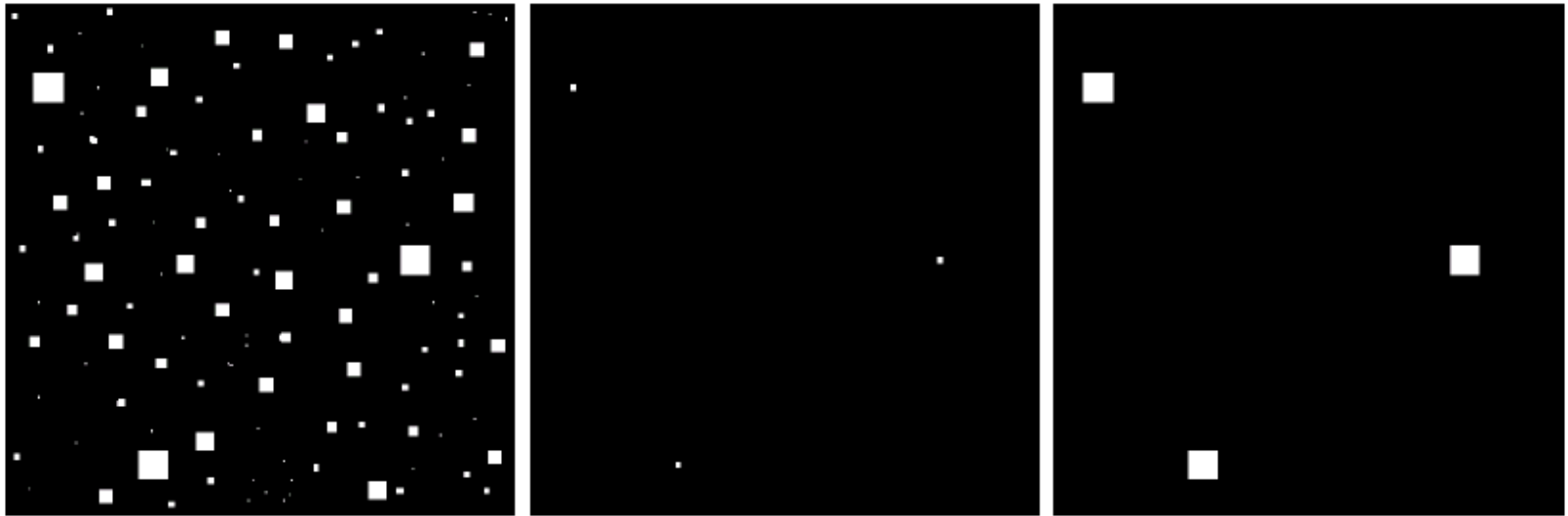
Dilation can repair intrusions



Watch out: Dilation enlarges objects

What Is Dilation For?

Combined with erosion: eliminate irrelevant details



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

structuring element $B = 13 \times 13$ pixels of gray level 1

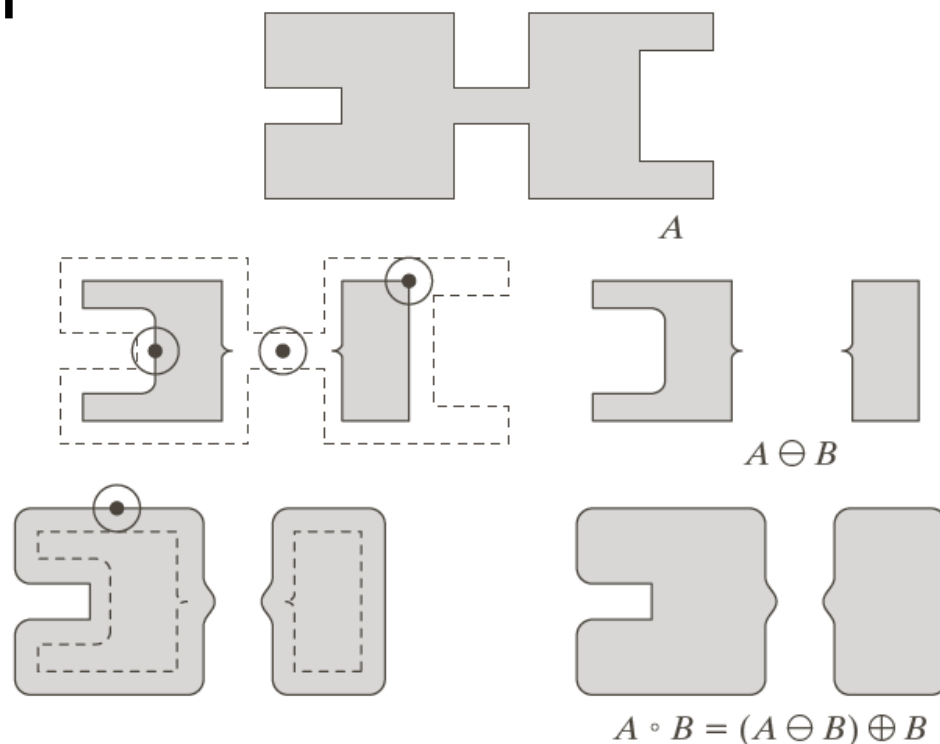
More interesting morphological operations can be performed by performing combinations of erosions and dilations

The most widely used of these *compound operations* are:

- Opening
- Closing

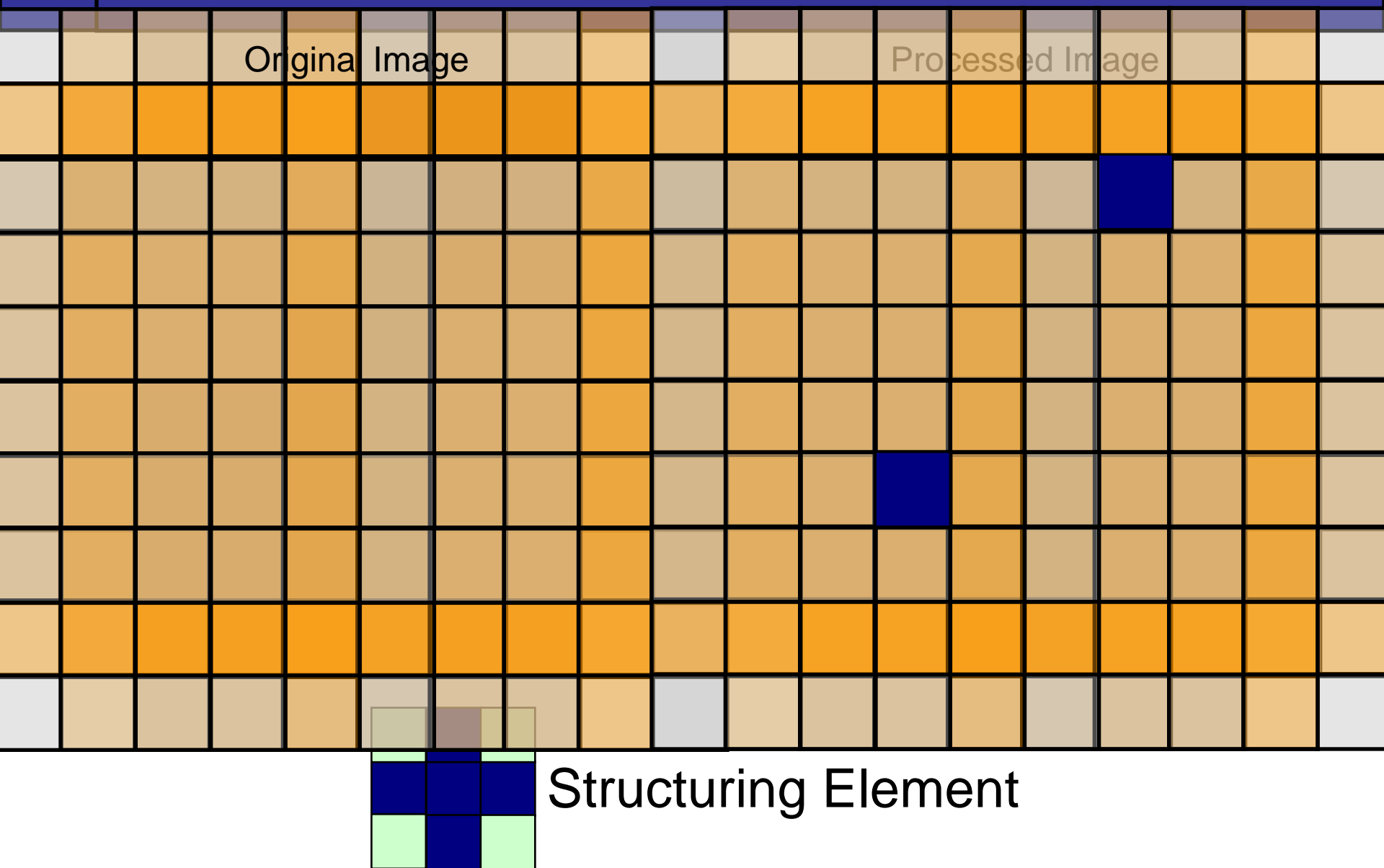
The opening of image f by structuring element s , denoted $f \circ s$ is simply an erosion followed by a dilation

$$f \circ s = (f \ominus s) \oplus s$$



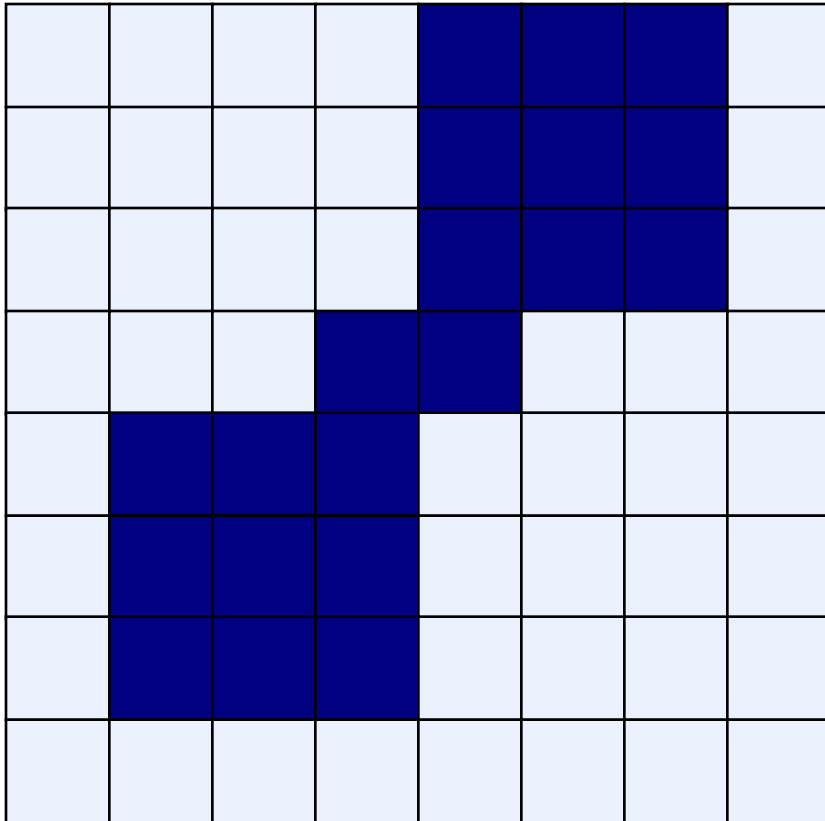
Note a disc shaped structuring element is used

Opening Example

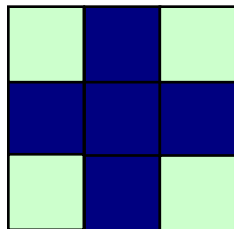
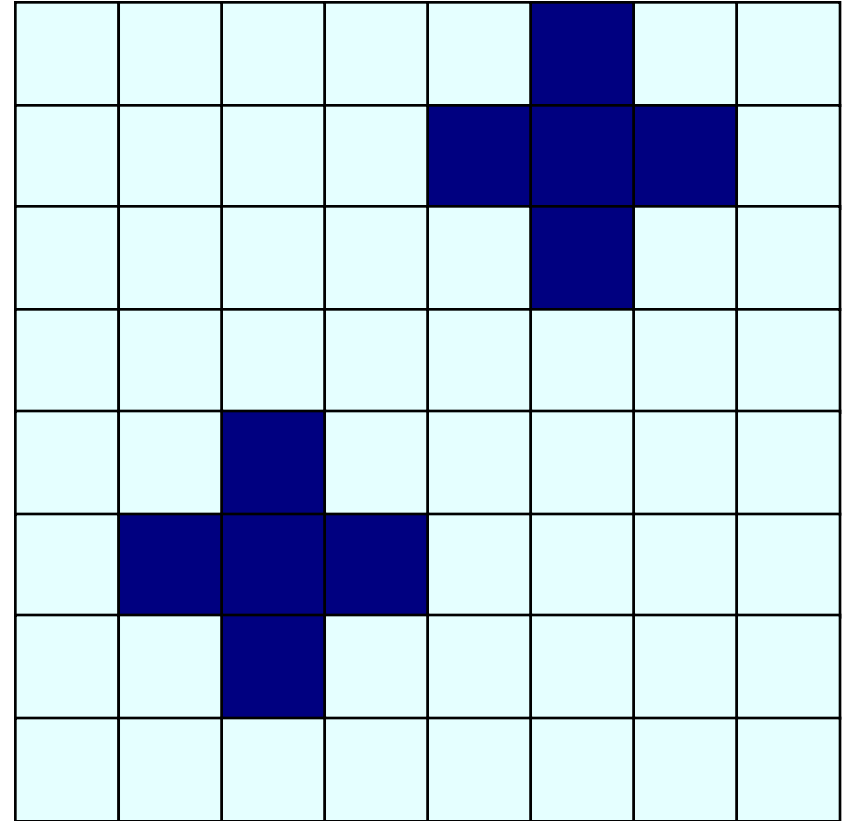


Opening Example

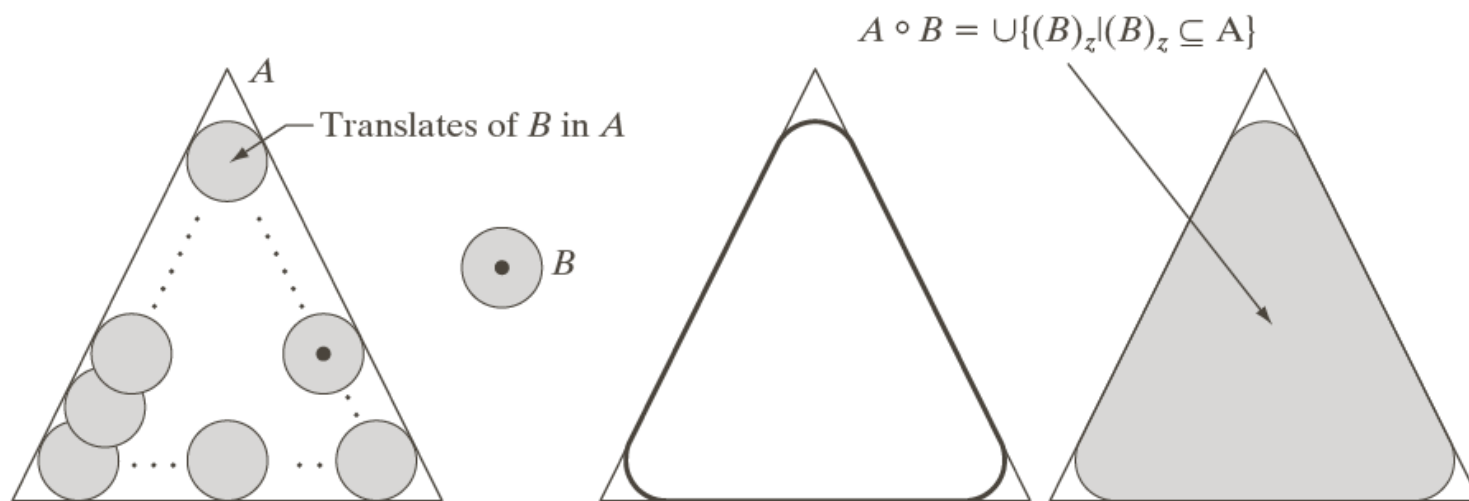
Original Image



Processed Image



Structuring Element

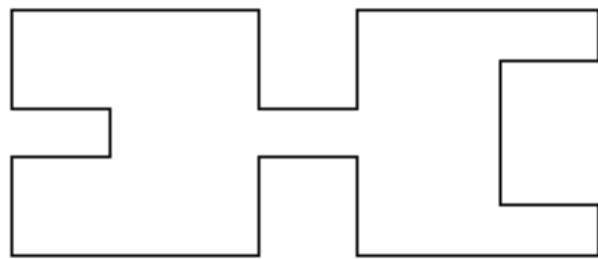


a b c d

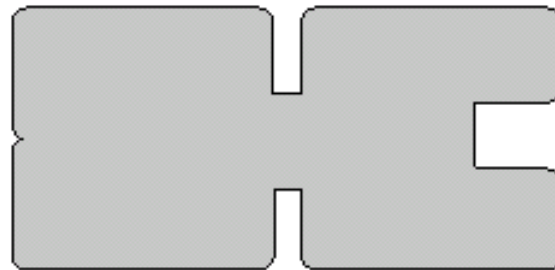
FIGURE 9.8 (a) Structuring element B “rolling” along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

The closing of image f by structuring element s , denoted $f \cdot s$ is simply a dilation followed by an erosion

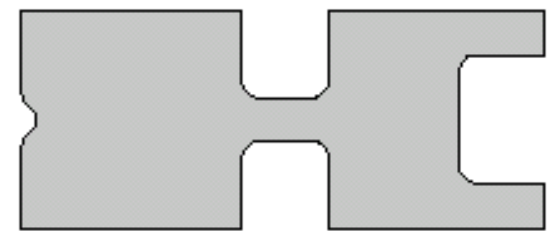
$$f \cdot s = (f \oplus s) \ominus s$$

 A

Original shape

 $A \oplus B$

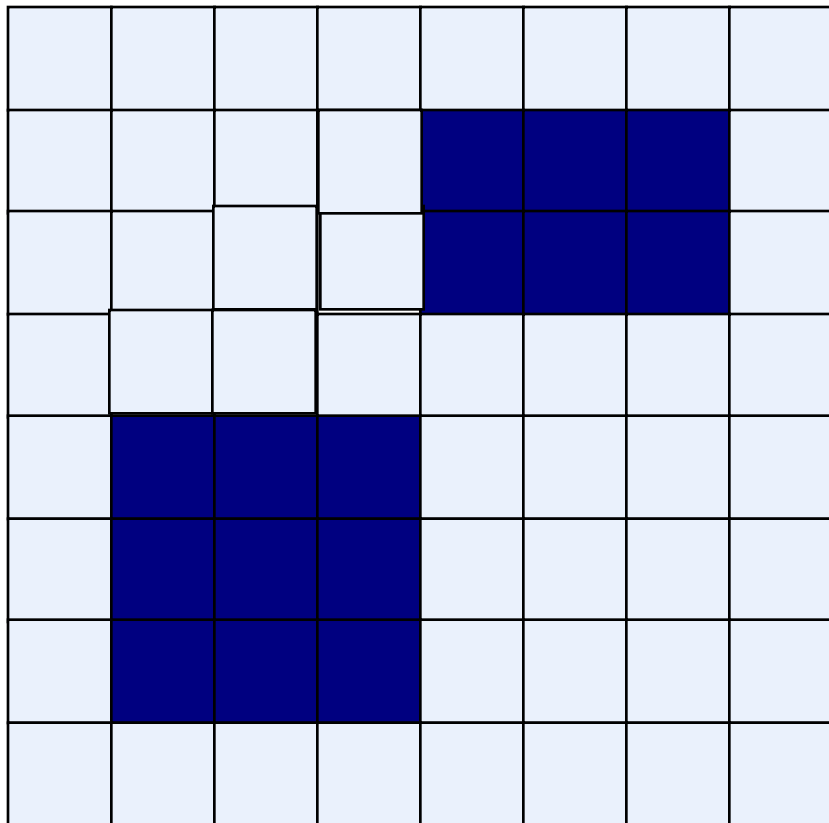
After dilation

 $A \cdot B = (A \oplus B) \ominus B$ After erosion
(closing)

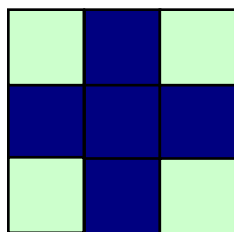
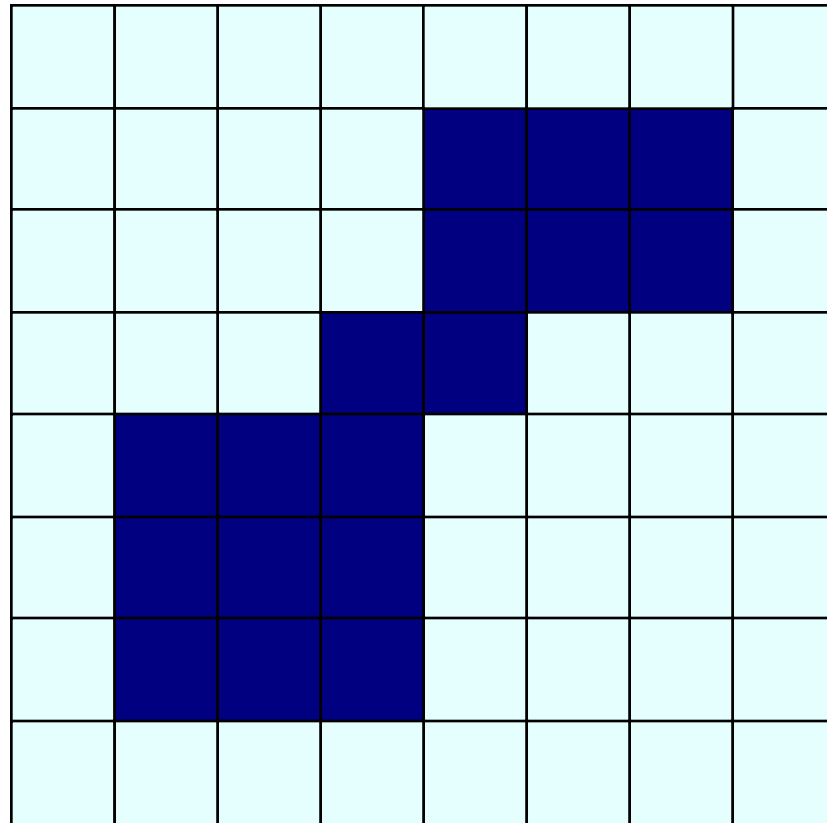
Note a disc shaped structuring element is used

Closing Example

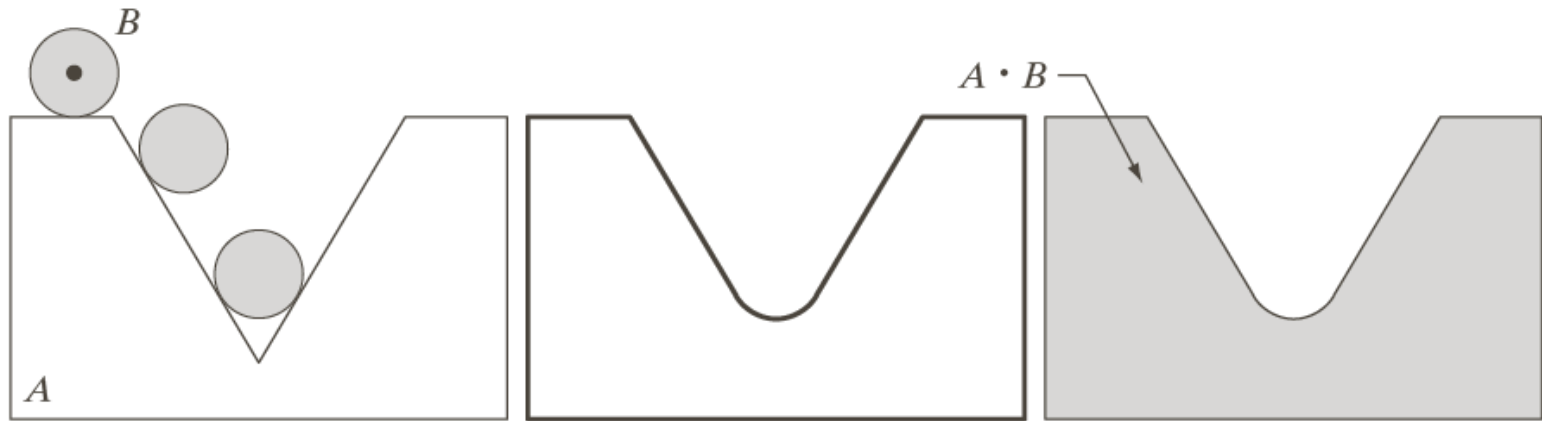
Original Image



Processed Image



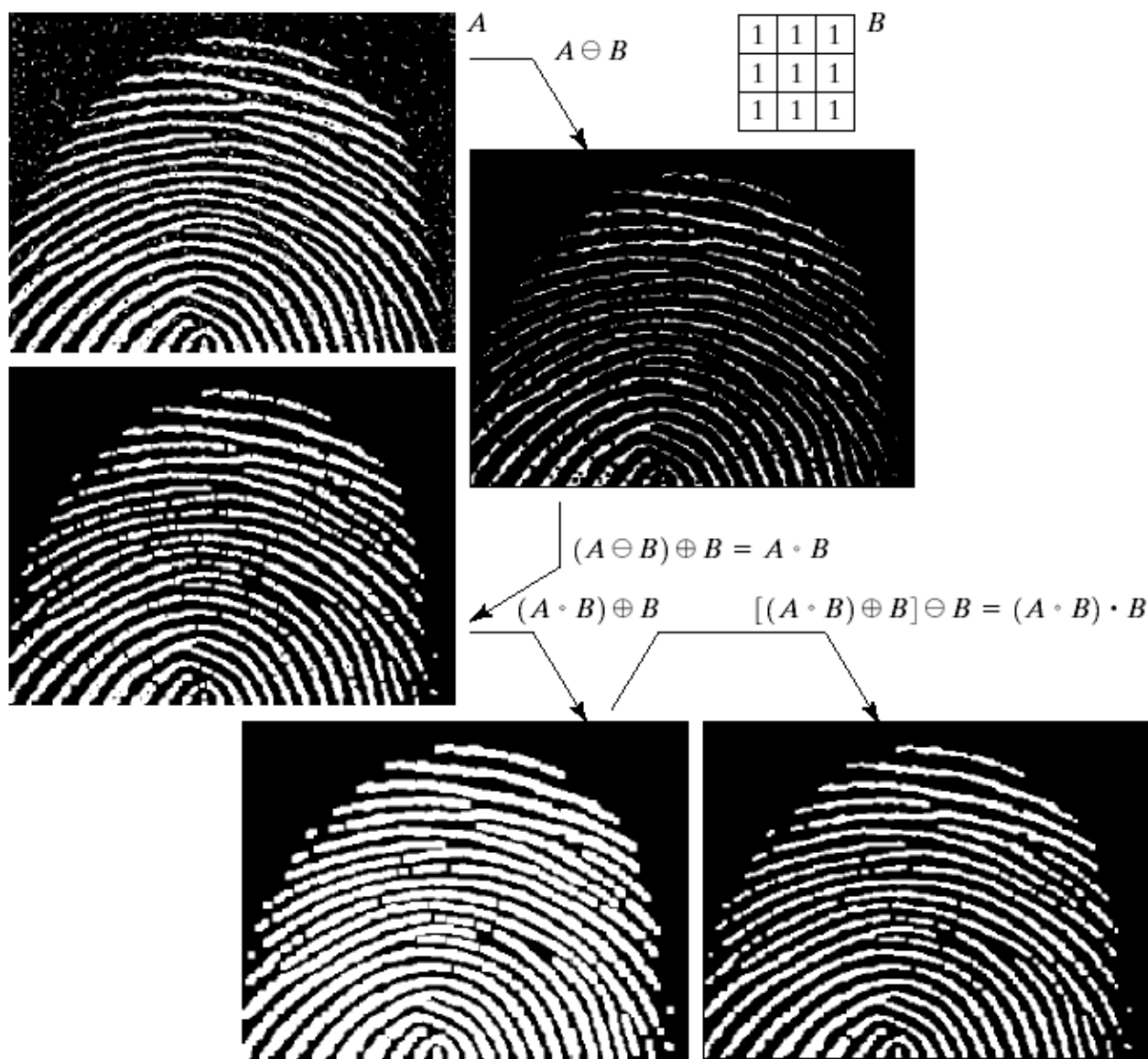
Structuring Element



a b c

FIGURE 9.9 (a) Structuring element B “rolling” on the outer boundary of set A . (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

Morphological Processing Example



Using the simple technique we have looked at so far we can begin to consider some more interesting morphological algorithms

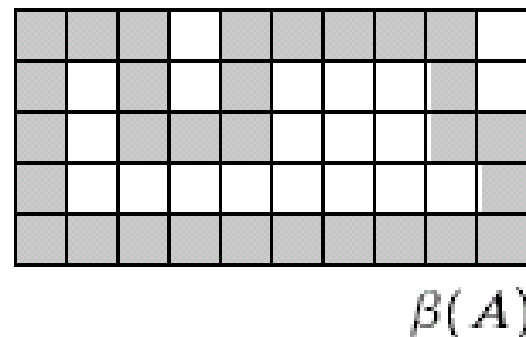
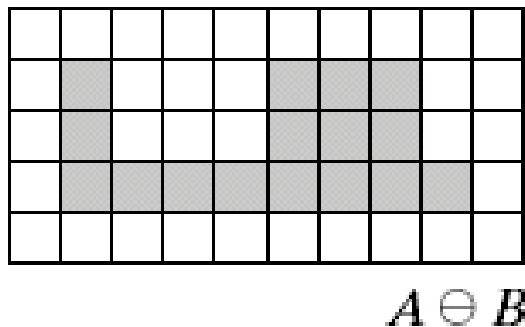
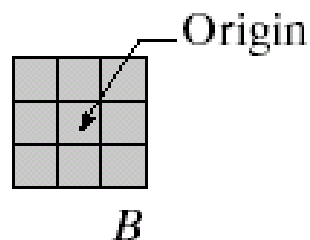
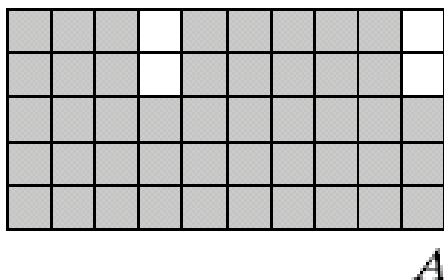
Such as:

- Boundary extraction
- Region filling
- Extraction of connected components
- Hit or Miss Transformation
- Convex Hull
- Thinning/thickening

Extracting the boundary (or outline) of an object is often extremely useful

The boundary can be given simply as

$$\beta(A) = A - (A \ominus B)$$



Boundary Extraction Example

A simple image and the result of performing boundary extraction using a square 3×3 structuring element



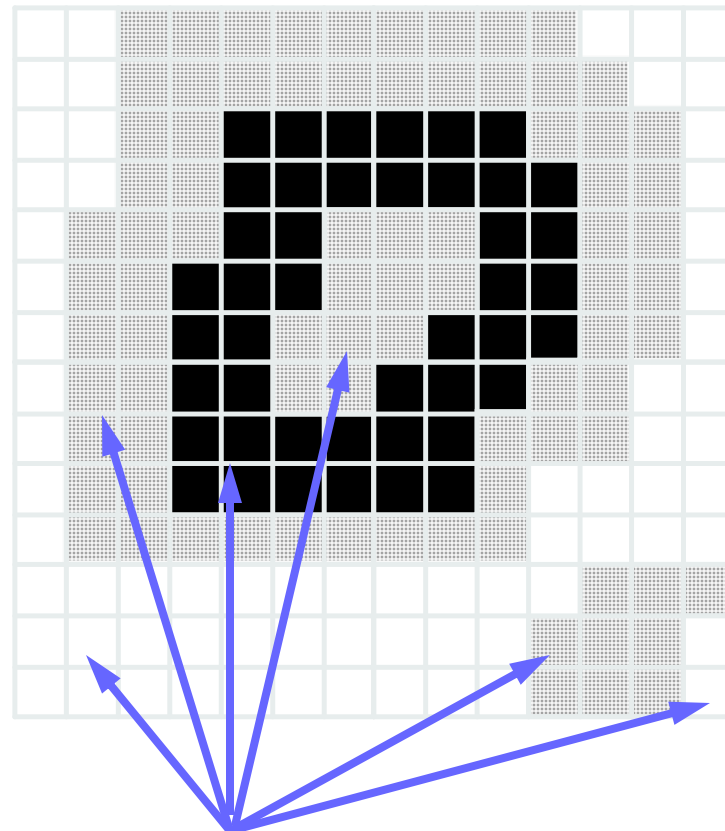
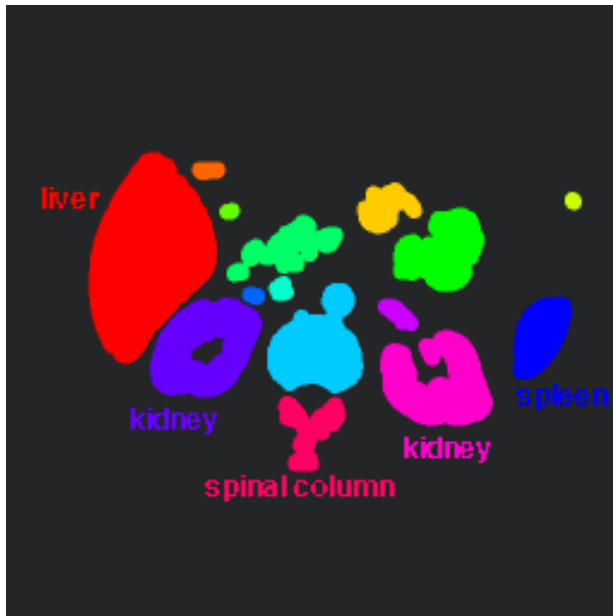
Original Image



Extracted Boundary

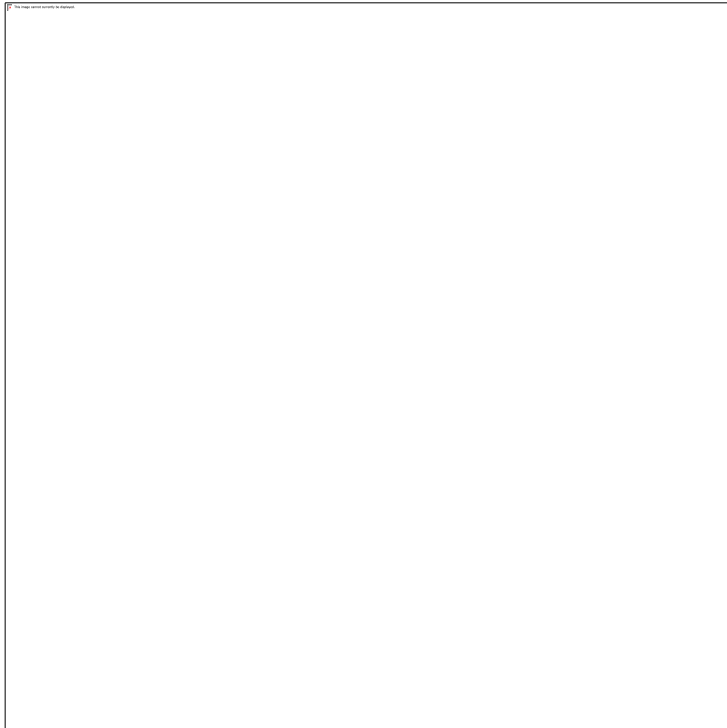
Connected Components

- Binary image with multiple 'objects'
- Separate 'objects' must be labeled individually



6 Connected Components

- Two points in an image are 'connected' if a path can be found for which the value of the image function is the same all along the path.



P_1 connected to P_2

P_3 connected to P_4

P_1 not connected to P_3 or P_4

P_2 not connected to P_3 or P_4

P_3 not connected to P_1 or P_2

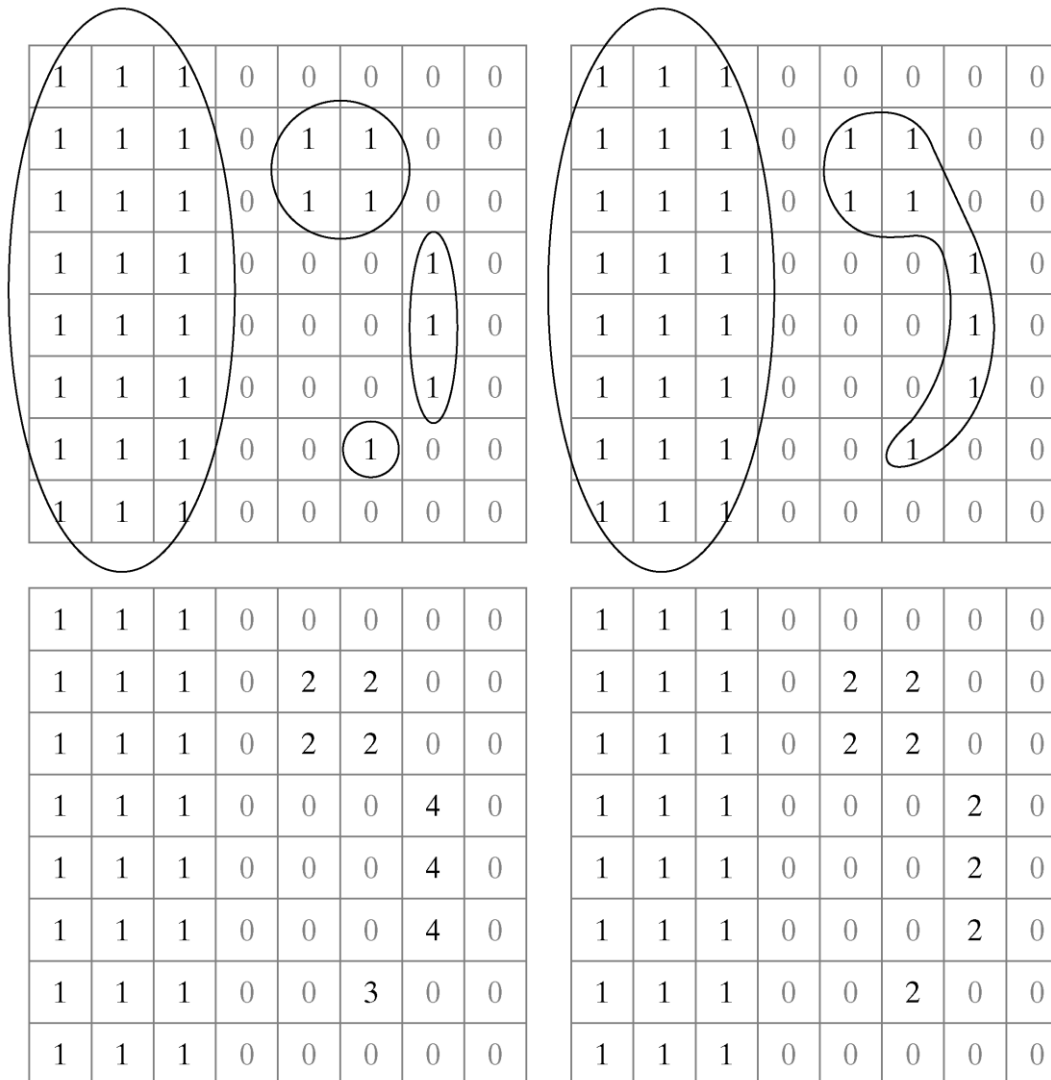
P_4 not connected to P_1 or P_2

Finding Connected Components

- Pick any pixel in the image and assign it a label
- Assign same label to any neighbor pixel with the same value of the image function
- Continue labeling neighbors until no neighbors can be assigned this label
- Choose another label and another pixel not already labeled and continue
- If no more unlabeled image points, stop.

Who's my neighbor?

Connected Components Labelling



a	b
c	d

FIGURE 9.19

Connected components
(a) Four
4-connected
components.

(b) Two
8-connected
components.

(c) Label matrix
obtained using
4-connectivity

(d) Label matrix
obtained using
8-connectivity.

Recursive algorithm for CCL

- Pseudo code:
 1. Scan the image to find an unlabeled unity valued pixel and assign it a new label L.
 2. Recursively assign a label L to all its unity valued neighbors.
 3. Stop if there are no more unlabeled unity valued pixels.
 4. Go to step 1.

Extraction of a connected component using morphology

$$X_k = (X_{k-1} \oplus B) \cap A$$

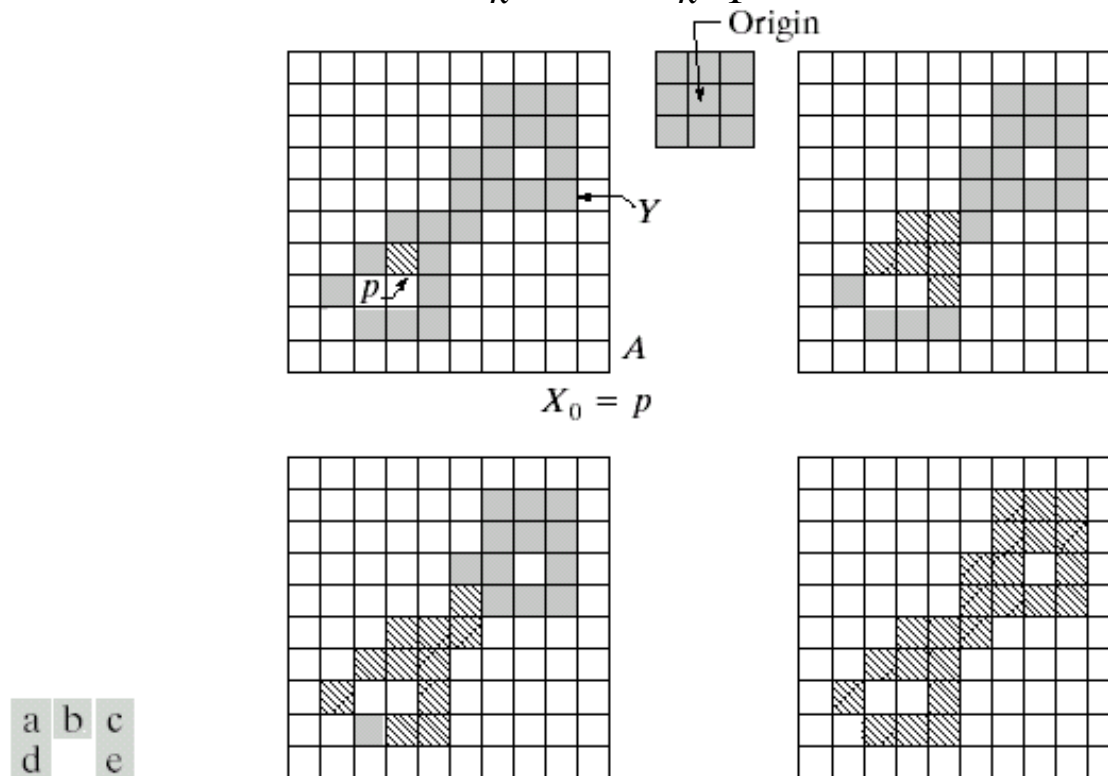
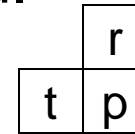


FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

Sequential algorithm for CCL

First Pass

- Initialize $CC=0$ and scan pixels across rows. So when we visit pixel p , pixel r and t have been visited and labeled.



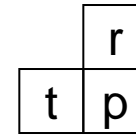
- If $p=0$: no action;
- If $p=1$: examine r and t .
 - both r and $t = 0$; $CC++$ and assign CC to p .
 - only one of r and t is 1; assign its label to p .
 - both r and t are 1:
 - * same label \Rightarrow assign it to p ;
 - * different label \Rightarrow assign lowest no. to p and Mark labels in neighbours as equivalent (i.e they are the same) and store in an equivalence table

0	0	0	1	1	1	0	0	0	0	1	1	1	1	0	1	1	1	1
0	0	0	1	1	1	1	0	0	0	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	0	0	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	0	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1

Sequential algorithm for CCL

First Pass

- Initialize $CC=0$ and scan pixels across rows. So when we visit pixel p , pixel r and t have been visited and labeled



- If $p=0$: no action;
- If $p=1$: examine r and t .
 - both r and $t = 0$; $CC++$ and assign CC to p .
 - only one of r and t is 1; assign its label to p .
 - both r and t are 1:
 - * same label \Rightarrow assign it to p ;
 - * different label \Rightarrow assign lowest no. to p and Mark labels in neighbours as equivalent (i.e they are the same) and store in an equivalence table

0	0	0	1	1	1	0	0	0	0	2	2	2	2	0	3	3	3	3
0	0	0	1	1	1	1	0	0	0	2	2	2	2	0	0	3	3	3
0	0	0	1	1	1	1	1	0	0	2	2	2	2	0	0	3	3	3
0	0	0	1	1	1	1	1	1	0	2	2	2	2	0	0	3	3	3
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	3	3	3
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	3	3	3
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1

Sequential algorithm for CCL

First Pass

- Initialize CC=0 and scan pixels across rows. So when we visit pixel p, pixel r and t have been visited and labeled

 $1 \equiv 2$
 $1 \equiv 3$

- If p=0: no action;

- If p=1: examine r and t.

- both r and t = 0; CC++ and assign CC to p.

- only one of r and t is 1; assign its label to p.

- both r and t are 1:

- * same label => assign it to p;

- * different label => assign lowest no. to p and Mark labels in neighbours as equivalent (i.e they are the same) and store in an equivalence table

0	0	0	1	1	1	0	0	0	0	2	2	2	2	0	3	3	3	3
0	0	0	1	1	1	1	0	0	0	2	2	2	2	0	0	3	3	3
0	0	0	1	1	1	1	1	0	0	2	2	2	2	0	0	3	3	3
0	0	0	1	1	1	1	1	1	0	2	2	2	2	0	0	3	3	3
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	3	3	3
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	3	3
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1

The Re-Labeling Process

Second Pass: simply replace the label with equivalent label stored in the table

1 \equiv 2
1 \equiv 3

0	0	0	1	1	1	0	0	0	0	1	1	1	1	0	1	1	1	1
0	0	0	1	1	1	1	0	0	0	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	0	0	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	0	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1

- Resolving Equivalences: is necessary between the two passes. Different methods:
 - **Union-Find** disjoint sets and operations
 - Obtain transitive closure using **Floyd-Warshall** algorithm

Why Equivalence Resolution?

	1					2	2				3	3	3	3	3	3	3
4	1	1			5	2	2	2			3						
	1			6	5			2	2		3		7	7	7	7	7
					5	5	5	2			3		7				7
8		9				5	5				3		7		10		7
8		9									3		7	7	10		7
8	8	9									3						7
	8										3	3	3	3	3	3	7

(a)

$$1 \equiv 4$$

$$8 \equiv 9$$

$$5 \equiv 6$$

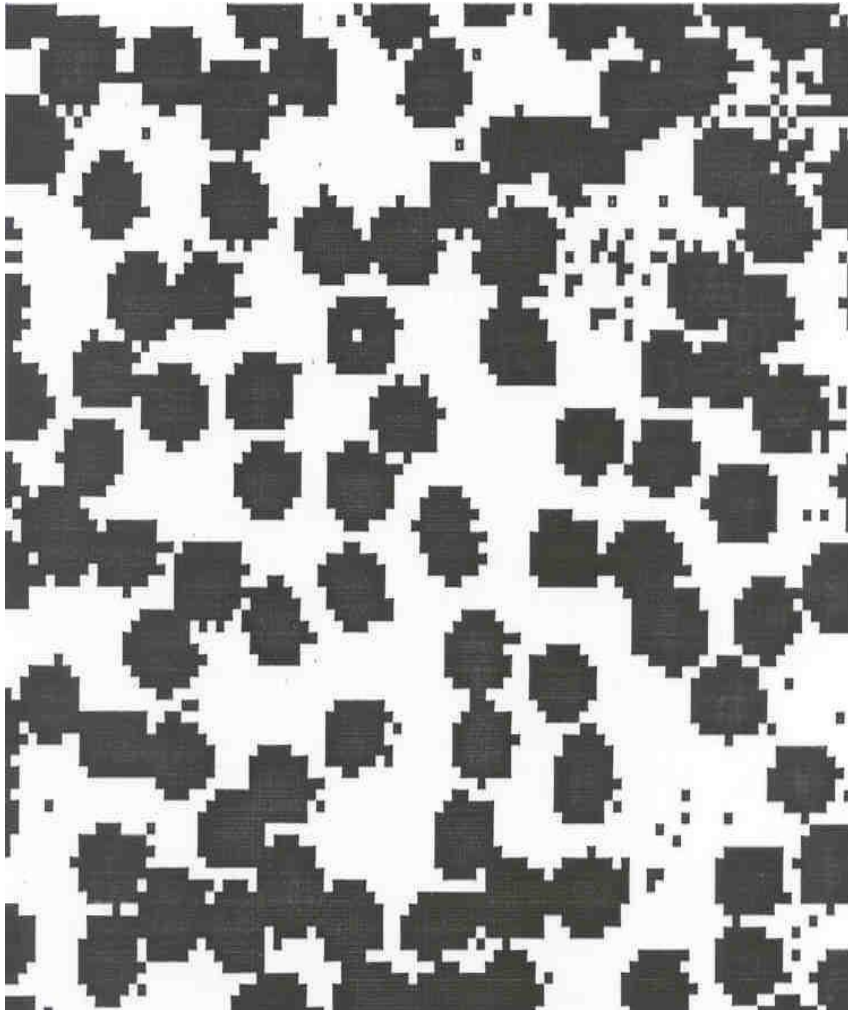
$$2 \equiv 5$$

$$3 \equiv 7$$

$$7 \equiv 10$$

	1					2	2				3	3	3	3	3	3	3
1	1	1				2	2	2	2		3						
	1			2	2			2	2		3		3	3	3	3	3
					2	2	2	2			3		3				3
8		8				2	2				3		3		3		3
8		8									3		3	3	3		3
8	8	8									3						3
	8										3	3	3	3	3	3	3

(d)

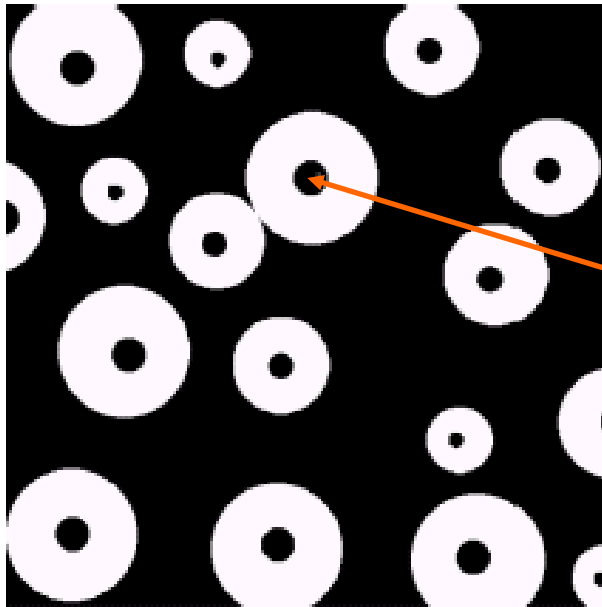


- Thresholded/binary Red blood cell image
- Many blood cells are separate objects
- Many touch – bad!
- Salt and pepper noise from thresholding
- What kind of analysis can be done ?

- 63 separate objects detected after CCL
- Compute features like area, BB, centroid etc.
- Single cells have area about 50
- Noise spots
- Gobs of cells
- What should be done to get better results?

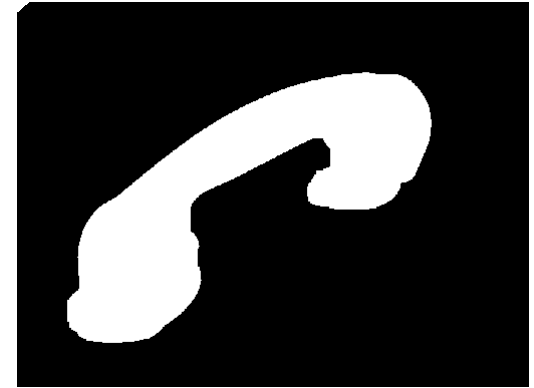
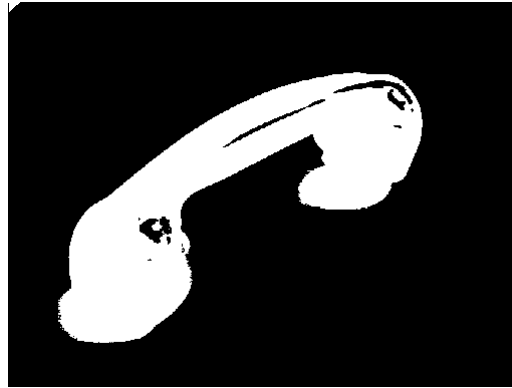
Object	Area	Centroid	Bounding Box	
=====				
1	383	(8.8 , 20)	[1 22 1 39]	
2	83	(5.8 , 50)	[1 11 42 55]	
3	11	(1.5 , 57)	[1 2 55 60]	
4	1	(1 , 62)	[1 1 62 62]	
5	1048	(19 , 75)	[1 40 35 100]	gobs
32	45	(43 , 32)	[40 46 28 35]	cell
33	11	(44 , 1e+02)	[41 47 98 100]	
34	52	(45 , 87)	[42 48 83 91]	cell
35	54	(48 , 53)	[44 52 49 57]	cell
60	44	(88 , 78)	[85 90 74 82]	
61	1	(85 , 94)	[85 85 94 94]	
62	8	(90 , 2.5)	[89 90 1 4]	
63	1	(90 , 6)	[90 90 6 6]	

Given a pixel inside a boundary, *region filling* attempts to fill that boundary with object pixels (1s)

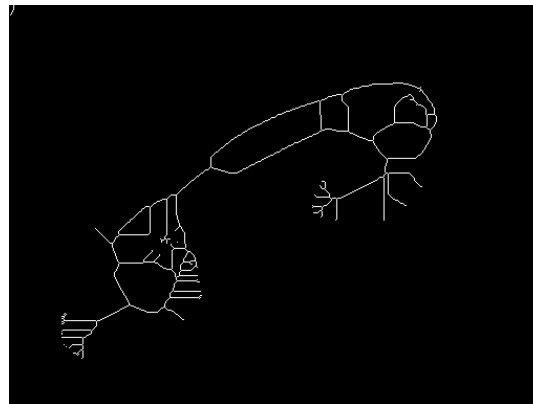


Given a point inside here, can we fill the whole circle?

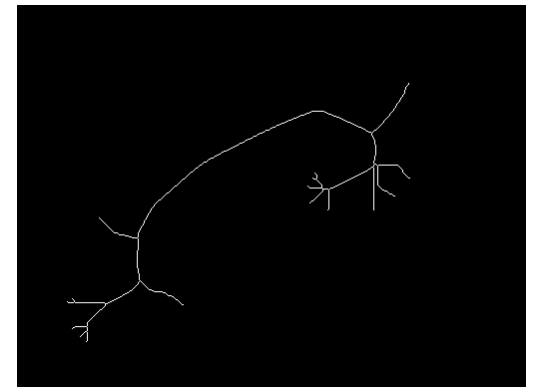
Why Region filling?



Helpful in the
later stages of
processing



skeleton before
closing



skeleton after
closing

Area fill and closing

Areafill Operation: extract connected components of dark regions and check their area . If the area is less than the threshold value they will be treated as holes. Then the dark holes can be removed by changing their intensity into light/white color.

Another criteria can be compactness (defined as $\text{perimeter}^2 / \text{area}$)

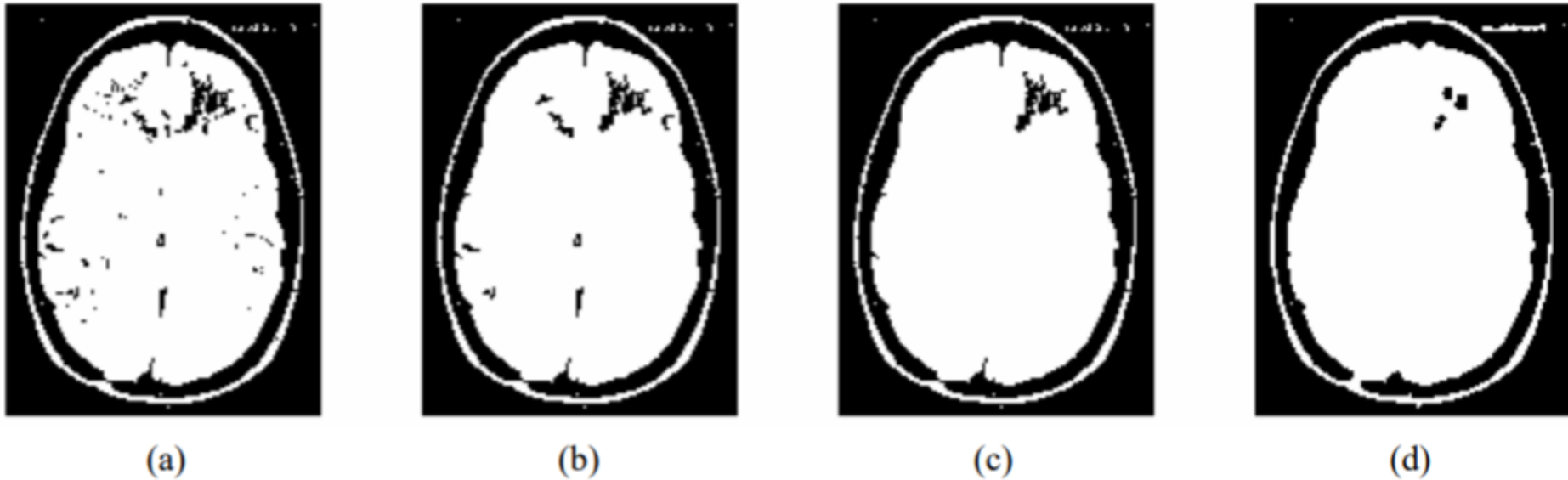


Fig. 1: Results of holes filling method done by AreaFill and Morphological operations (a) Binary form of MRI head scan (b) holes with having area less than 100 are filled (c) holes with having area less than 500 are filled (d) holes filled by morphological close operation

Region Filling (Floodfill Operation)

The key equation for region filling is

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, \dots$$

Where X_0 is simply the starting point inside the boundary, B is a simple structuring element and A^c is the complement of A

This equation is applied repeatedly until X_k is equal to X_{k-1}

Finally the result is unioned with the original boundary

Region Filling Step By Step

a	b	c
d	e	f
g	h	i

FIGURE 9.15

Region filling.

(a) Set A .

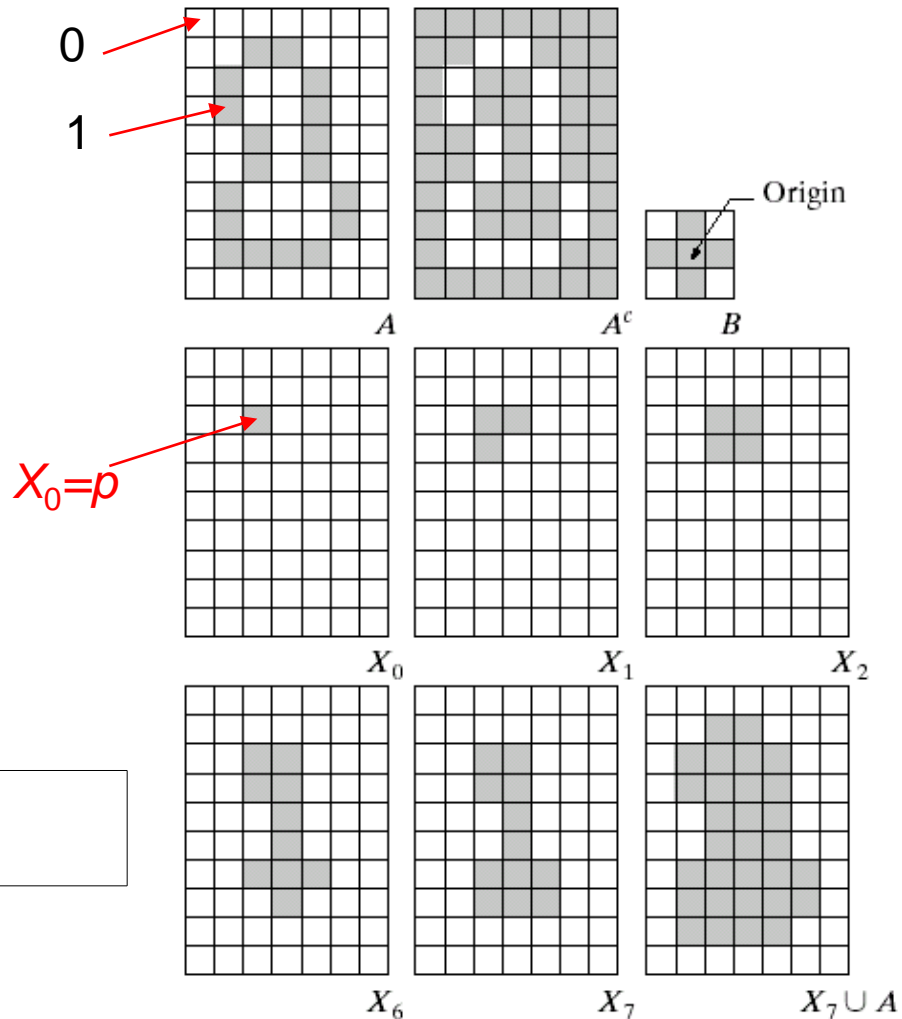
(b) Complement of A .

(c) Structuring element B .

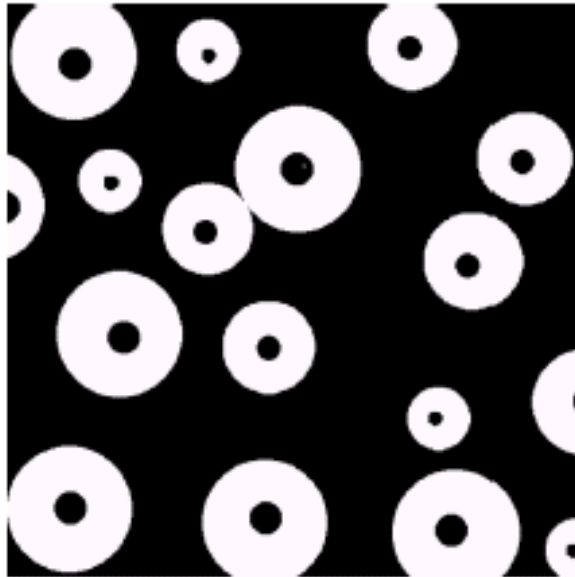
(d) Initial point inside the boundary.

(e)–(h) Various steps of Eq. (9.5-2).

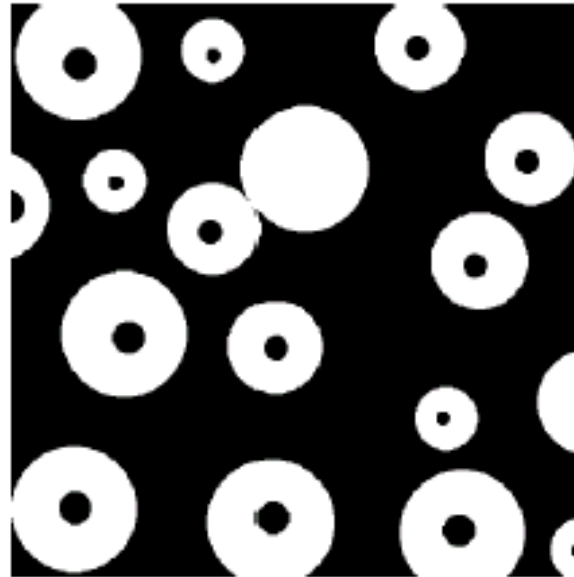
(i) Final result [union of (a) and (h)].



Region Filling Example



Original Image



One Region
Filled



All Regions
Filled

Automatic Initialization?

- How to automatically find the interior pixel in the hole region?
- Steps:
 1. Perform morphological boundary extraction $\beta(A) = A - (A \ominus B)$ to get both outer and inner boundary(s) of the object region
 2. Scan the image and first object pixel encountered must be part of outer boundary. Give unique label to all the connected pixels of the boundary (either tracing or using CCL algo)

Steps (continued) :

3. Continue scanning the image and any other object pixel encountered (within the outer boundary) will be on the inner boundary

4. Examine the neighbours of that pixel (belonging to the inner boundary) and select any background neighbour pixel (there must be at least one)

The Hit-or-Miss Transformation

- Useful as a Very basic tool for shape detection
- Can also be used to **look for particular patterns** of foreground and background pixels
- Uses a **Group of** Structuring Elements, containing 0s, 1s and sometimes **don't cares(!)**

The Hit-or-Miss Transformation

$$A \odot B = (A \ominus X) \cap [A^c \ominus (W - X)] \quad B = (X, W - X)$$

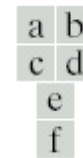
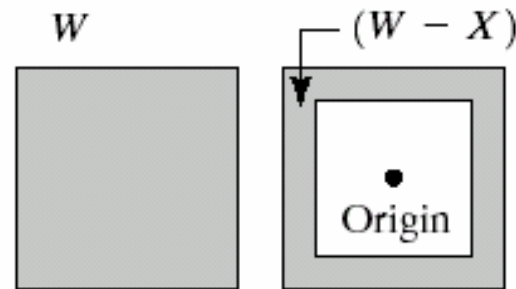
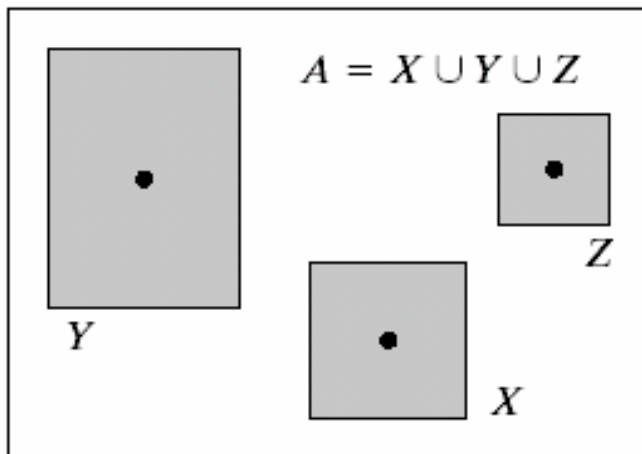
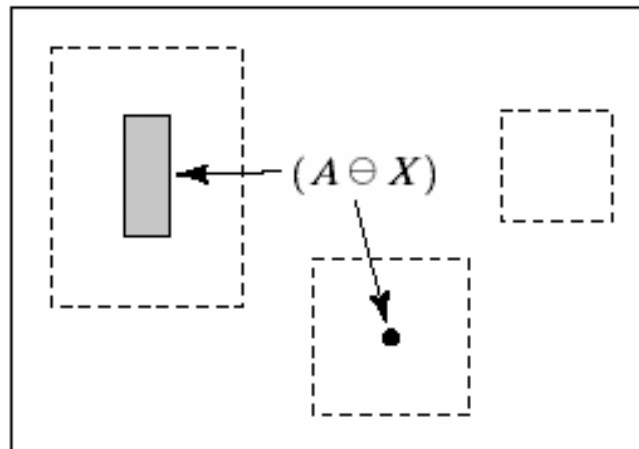
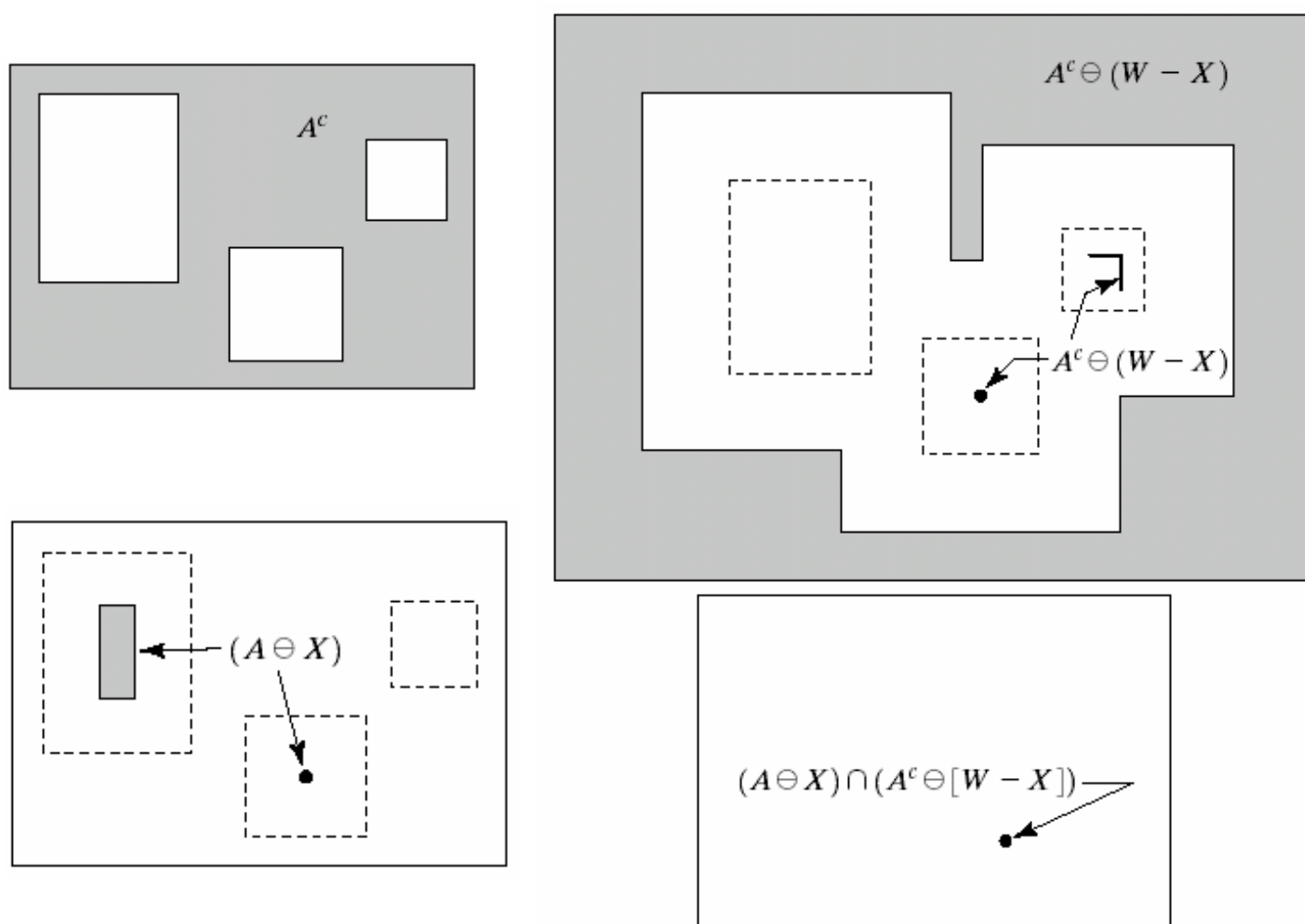


FIGURE 9.12
 (a) Set A . (b) A window, W , and the local background of X with respect to W , $(W - X)$.
 (c) Complement of A . (d) Erosion of A by X .
 (e) Erosion of A^c by $(W - X)$.
 (f) Intersection of (d) and (e), showing the location of the origin of X , as desired.



The Hit-or-Miss Transformation

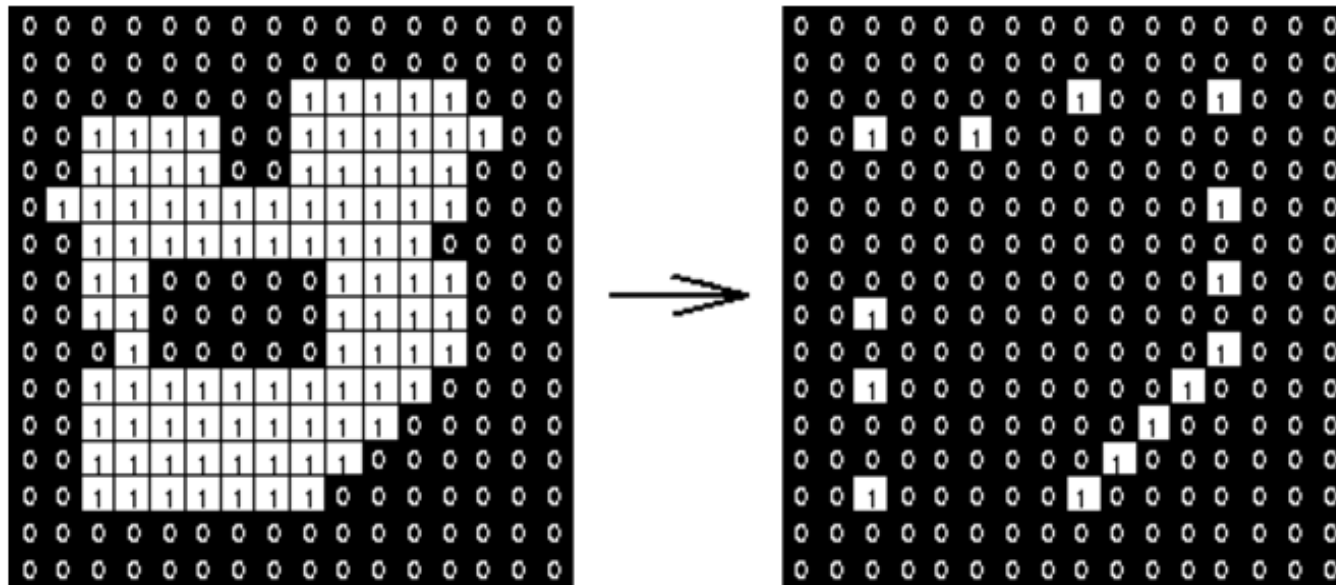
$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)] \quad B = (X, W - X)$$



a	b
c	d
e	
f	

FIGURE 9.12
 (a) Set A . (b) A window, W , and the local background of X with respect to W , $(W - X)$.
 (c) Complement of A . (d) Erosion of A by X .
 (e) Erosion of A^c by $(W - X)$.
 (f) Intersection of (d) and (e), showing the location of the origin of X , as desired.

- To find all the right angle convex corners of a region in a given image as shown below?



- Structuring Elements representing four corners
- Contains 0s, 1s and *don't care's*

x	1	x
0	1	1
0	0	x

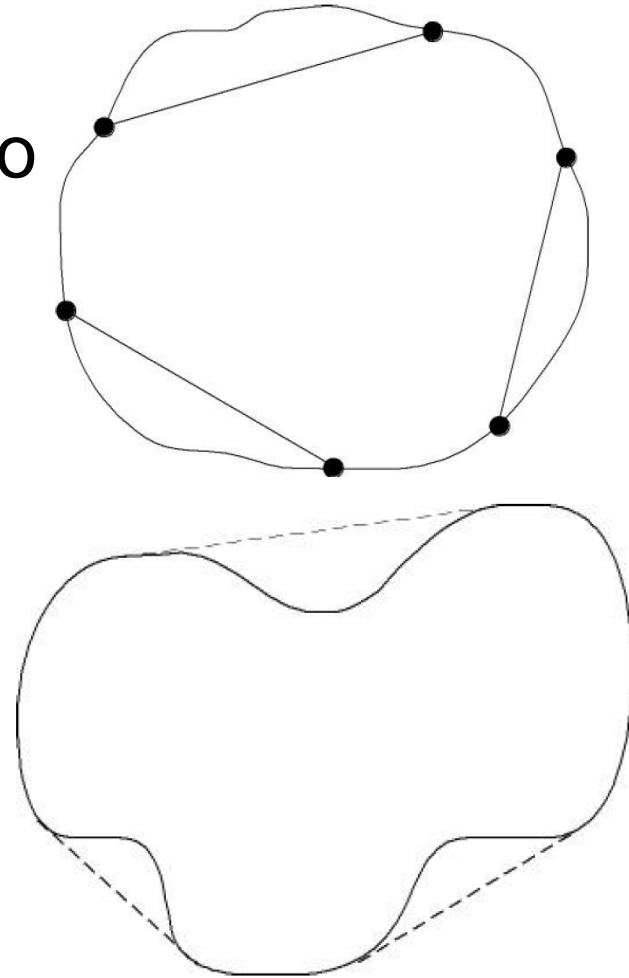
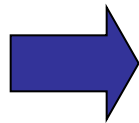
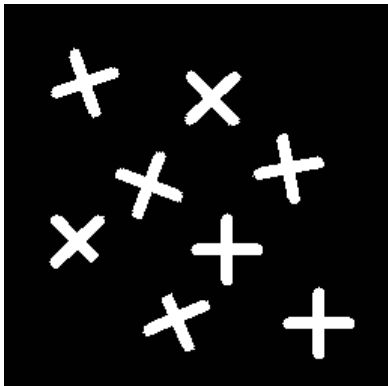
x	1	x
1	1	0
x	0	0

x	0	0
1	1	0
x	1	x

0	0	x
0	1	1
x	1	x

- Apply each Structuring Element
- Use OR operation to combine the four results

- A set A is said to be **convex** if the straight line segment joining any two points in A lies entirely within A .
- The **convex hull** H of an arbitrary set S is the smallest convex set containing S .
- Application in shape analysis, collision avoidance etc.



Let B^i , $i = 1, 2, 3, 4$, represent the four structuring elements.

The procedure consists of implementing the equation:

$$X_k^i = (X_{k-1} \circledast B^i) \cup A$$
$$i = 1, 2, 3, 4 \quad \text{and} \quad k = 1, 2, 3, \dots$$

with $X_0^i = A$.

When the procedure converges, or $X_k^i = X_{k-1}^i$, let $D^i = X_k^i$, the convex hull of A is

$$C(A) = \bigcup_{i=1}^4 D^i$$

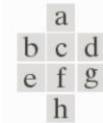
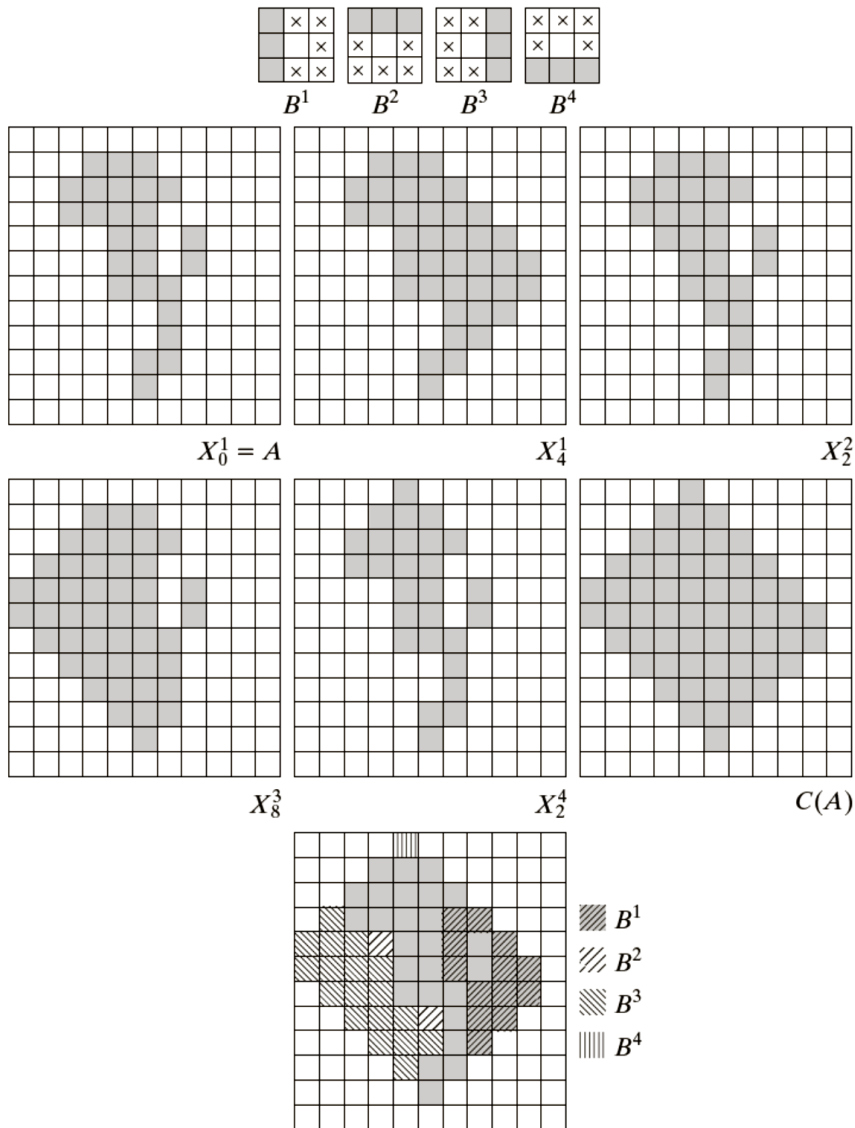
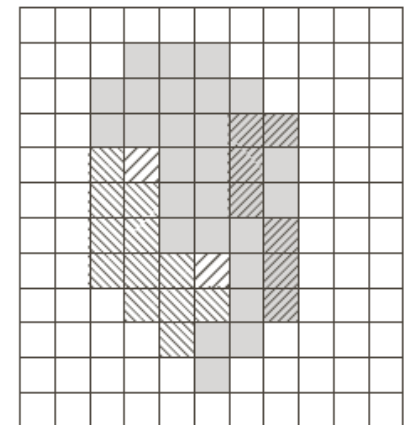


FIGURE 9.19
 (a) Structuring elements. (b) Set A . (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

FIGURE 9.20
 Result of limiting growth of the convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.



1. Used to **remove** selected **foreground pixels** from binary images
2. After edge detection, lines are often **thicker than one pixel**.
3. Thinning can be used to thin those line to **one pixel width**.
4. Several applications, but is particularly useful for **skeletonization**

- The thinning of a set A by a structuring element B , defined: $A \otimes B = A - (A \circledast B)$

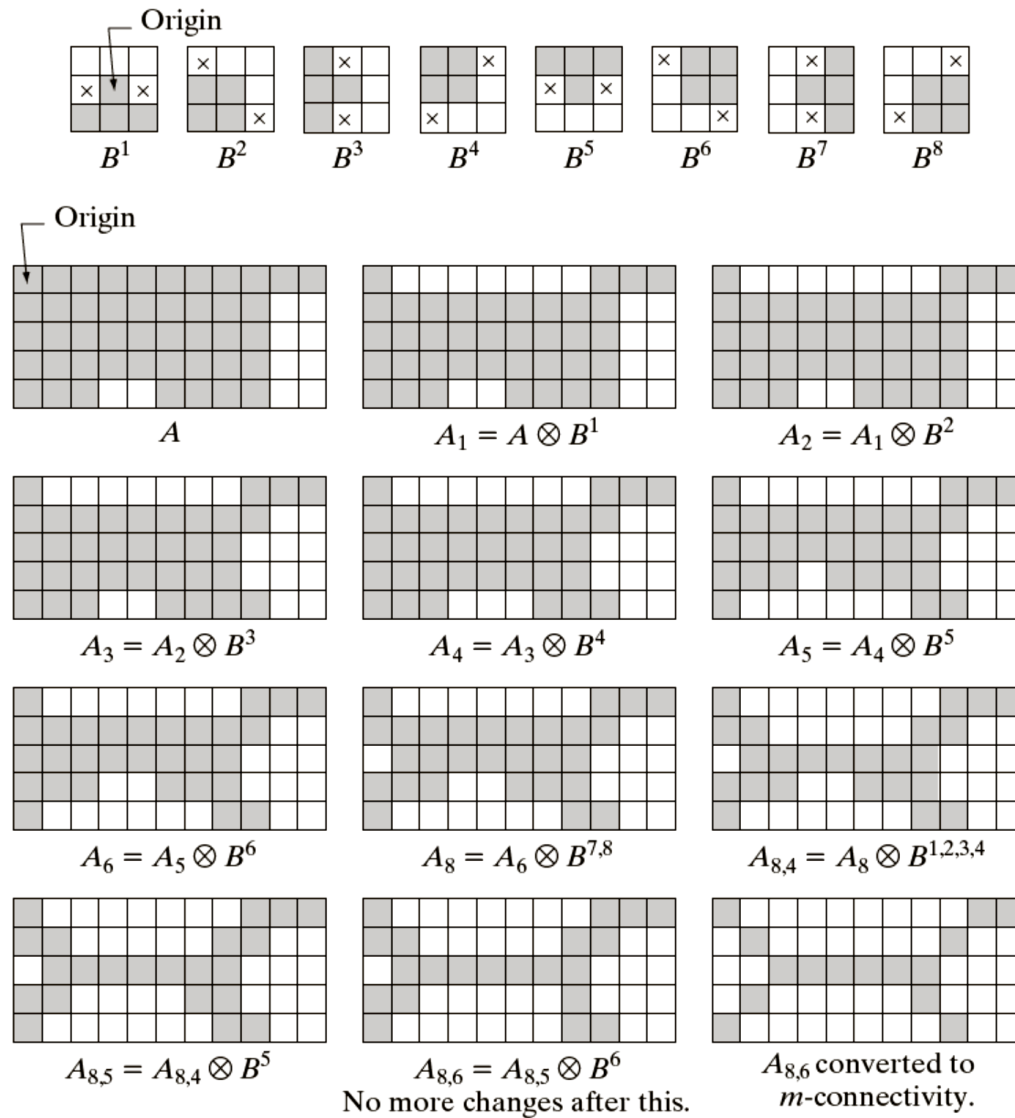
$$= A \cap (A \circledast B)^c$$
- A more useful expression for thinning A symmetrically is based on a sequence of structuring elements:

$$\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$$

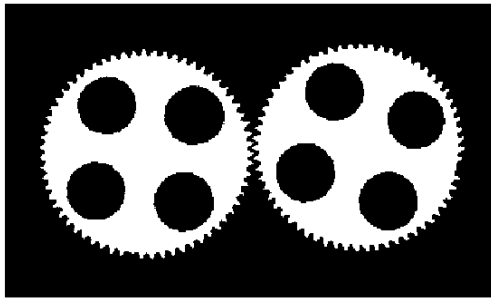
where B^i is a rotated version of B^{i-1}

The thinning of A by a sequence of structuring element $\{B\}$

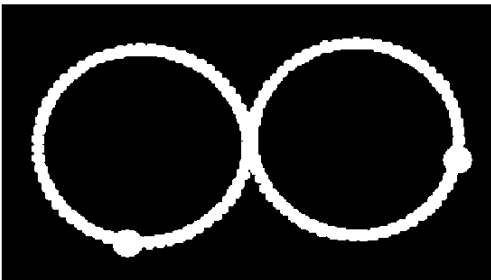
$$A \otimes \{B\} = (((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$$



- Gear Tooth Inspection



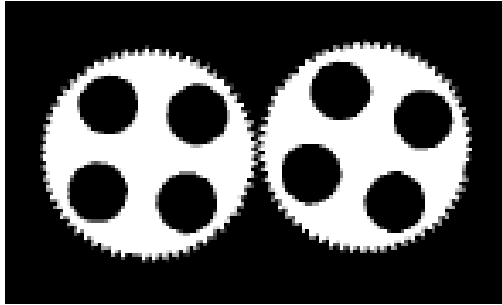
original
binary
image



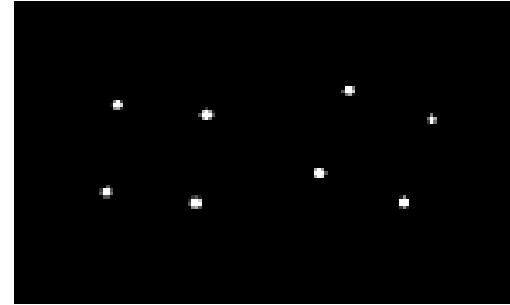
detected
defects

*How did
they do
that?*

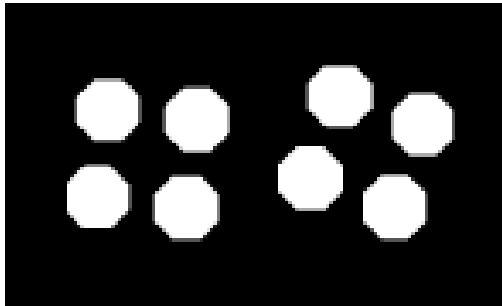
More Applications



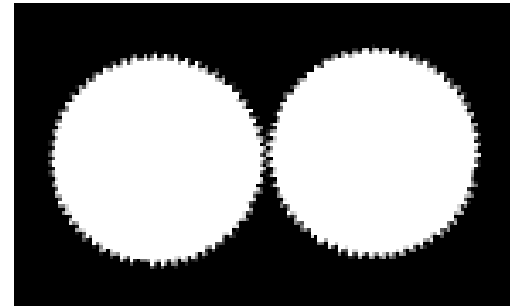
a) Original image B



b) $B1 = B \ominus hole_ring$

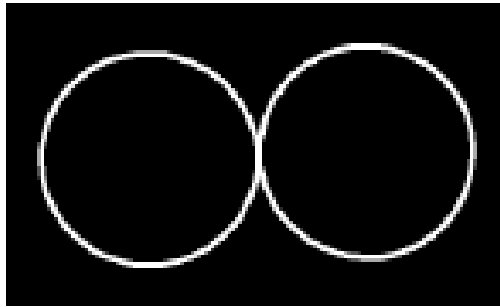


c) $B2 = B1 \oplus hole_mask$

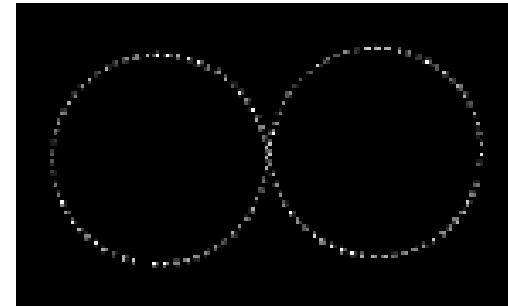


d) $B3 = B \text{ OR } B2$

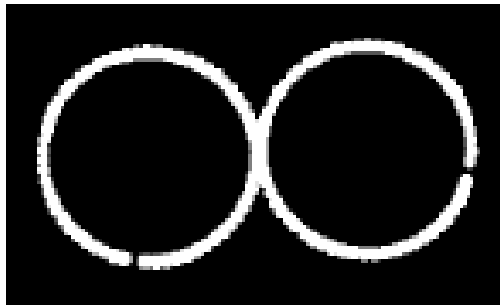
More Applications



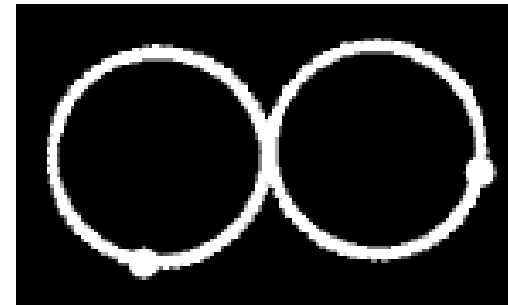
e) $B7$



f) $B8 = B \text{ AND } B7$

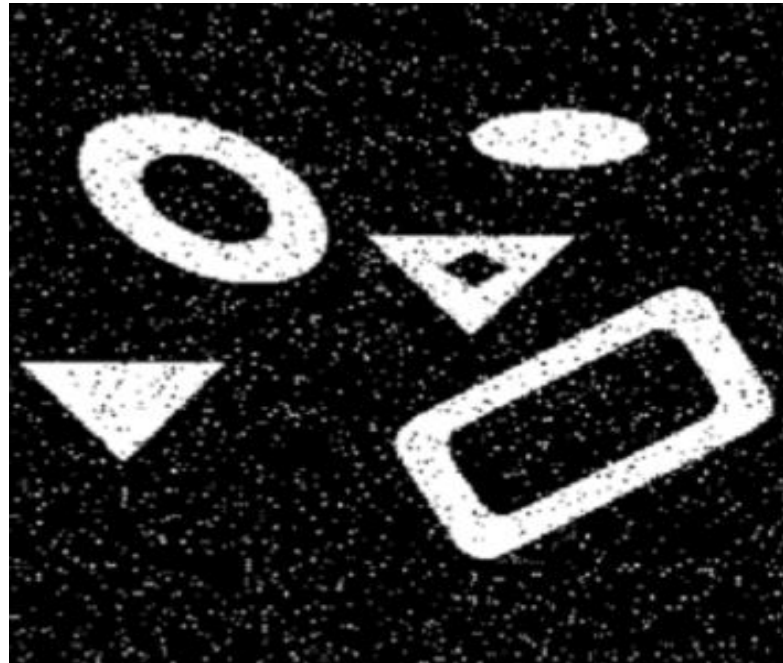


g) $B9 = B8 \oplus \text{tip_spacing}$



h) $RESULT = ((B7 - B9) \oplus \text{defect}_{cue}) \text{ OR } B9$

- Describe a morphological algorithm to count the number of objects with holes and the number of objects without holes in below image



The purpose of morphological processing is primarily to remove imperfections added during segmentation

The basic operations are *erosion* and *dilation*

Using the basic operations we can perform *opening* and *closing*

More advanced morphological operation can then be implemented using combinations of all of these