Financial Time Series Final Project

PAIRS TRADING STOCK STRATEGY

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Abstract

Pair trading, a statistical arbitrage trading strategy, was firstly developed by Nunzio Tartaglia at Morgan Stanley in 1980s.

We propose the spread model and a strategy for the initial model. And we give the definition of cointegration and some important statistic tests for our project.

When we select the data, we choose two set of data: The first set is the BRK.A and BRK.B which are the sibling of the same company. The second set is a combination of three stocks chosen from the US stock market. The characteristics of these two set of data is that they both can formulate a stationary spread which is essential for us to proceed our strategy. For this case, we are taking PP- test and ADF test to show if the initial data are non-stationary and if the spread is stationary. It turns out that our set of data fulfill the requirement we are asking for. Then, we can go further.

Initial model is developed with two trading indicators, and it will perform talk long position and short position on the pair stocks according to the move of the current spread and the mean spread. By optimize the ratios, we can construct better portfolio for predicting market performance. Initial model is following by two revised models, which we set gain limitation and loss limitation on the trading strategies. The limitations help the model to reduce volatility risks. As the results indicating, the risks are reduced while the return shrink.

We also applied our model to the 2007-2008 finance crisis, and it surprisingly outperform the market return significantly since the market was nose diving at that moment. Our success is due to the correct prediction of the moving direction of spread. Therefore, our strategy may have a counter-risk mechanism.

Finally, we conclude that our model is basically following on the market-neutral principle while we cannot eliminate all risk since the source of risk is diversified. In addition, the revised models give us some constructive solution to constrain the risk and consolidate the profit. Indeed, it may not work well in an exact period. However, they are some useful solutions for the future improvement.

Contents

I Introduction

Pair trading, a statistical arbitrage trading strategy, was firstly developed by Nunzio Tartaglia at Morgan Stanley in 1980s. The Philosophy of Goldman Sachs Asset Management is specified by Littterman. The asset management assumed traders have a motivation to take maximum advantage from deviations from equilibrium, while the market seemly does not in equilibrium, and over time they move to a rational equilibrium. Since the pair trading is market neutral, the strategy does not be influenced by absolute stock price, which has a great advantage in this critical environment and unstable market performance due to COV-19 rapidly spreading in the world.

Relative pricing is proposed by the idea that if securities have comparable qualities, they should be priced at the same level. It should also be noted that the pairs trading is not risk-free at all. The critical requirements of well implement of pair trading are identifications pairs as well as efficient trading algorithm.

In this project, we build the spread model, and the O-U mean-reverting model to develop a trading strategy and utilize the model to BRK/A and BRK/B stock. To obtain a better performance on the trading, we constructed different strategies and build several trading rules. Different trading indicators are constructed by two models, the entry-exit line and the loss-limit line.

A critical requirement for the pair trading strategy is that stock selection should ensure that the high correlation, or cointegration exists between selected stocks. Generally, if the prices of two prices always move in the same direction, they may be used for pair trading. However, this explicit relationship between two stocks may not sufficiently justify the role of being a pair. A good pair should share as many same intrinsic characteristics as possible. However, in reality, it may be difficult to identify those stocks which have same features. Several technical indicators may help the stock selections, while it should be noted that the limitation of using these indicators.

II Initial Model, Strategy and Test

This section present road map for our project. It gives the initial model and strategies to apply pair trading. Also, some important and useful test for the results are discussed.

2.1. Spread Model

According to the threshold of pairs trading, assume stock A and B are two stocks having similar characteristics, the spread model can be expressed as following,

$$\frac{dA_t}{A_t} = \alpha dt + \beta \frac{dB_t}{B_t} + dX_t$$

where A_t is the price of stock A at time t, and B_t is the price of stock B at time t. X_t is the residual, which is the spread in our model. Since the market is assumed to drift towards the efficient equilibrium, we assert that the spread is suggested to be a mean reverting Ornstein - Uhlenback process (O-U process). The O-U process is the solution to the following stochastic differential equation,

$$dX_t = \theta(\mu - X_t)dt + \sigma dW_t$$

where we would like to estimate the parameter θ, μ and σ. W_t is a standard Brownian Motion, or known as a Wiener process. We can solve the equation for X_t, which is the residual, and the spread of the stock in our model.

2.2. Initial Strategy

In order to find the solution to the initial model, we utilize a new process as following.

$$u_t = \frac{s_t}{\sqrt{\mathrm{var}(s_t)}}$$

where u_t is the process that describe how far we are away from the standard deviation of the history record of s_t, which is the spread.

The trading strategy try to realize the old sayings, *buy at low and sell at high*. When the condition satisfied as $u_t > \psi$ and $u_t < -\psi$, we buy in the target shares which are undervalued, and short sell the stock that is overvalued, respectively.

Our sell strategy comes as when u_t is smaller than $|\phi|$, we sell our holding stocks. This is the simplest trading strategy, which we will try to try to optimize the two parameters of ψ and ϕ.

2.3. Cointegration and Tests

The analysis of models with unite roots have a major impact in time series field. The concept of cointegration was introduced by Granger[1, 2], in 1981 and 1986. The concept recognizes that a linear combination may exist in the several series, which all have unit roots. In other words, cointegration captures the notion that, in the long-run, nonstationary variables have a tendency to move together. Engle and Granger[3] proposal a theorem giving the presentation of cointegrated series, and the procedures of tests and estimation. By then, cointegration has been used to describe the relationship between consumption and income[4], stock prices and dividends[5], and money demands[6].

Cointegration identifies the degree to which variables are sensitive to the same average price in a period of time. Cointegration does not reflect the pairs would move in the same or opposite direction, while the correlation does, but it exams the distance between the pairs whether keep on a same level in that specified time period. For model traders, the movement of the pair variables are not related, but they may identify a long-run equilibrium is maintained.

Johansen test, introduced by Johansen[7] in 1991, is a procedure for testing cointegration of three or more time series.

The augmented Dickey-Fully test, introduced by Dicky and Fuller[8], and the Philliops-Perron (PP) test, introduced by Phillips[9], and Perron[10], are two widely accepted unit-root test tool. However, Davidson and Mackinnon[11] find that the PP test performs are not good as the augmented Dickey-Fuller test in finite samples.

III Data Preprocessing

In this section, we present the sources of our data and the selection of the data. Data analysis, stock section are presented as well.

3.1. Data Acquirement

In our project, we prefer to use daily data instead of intraday data because there would be too many factors are supposed to be considered in the model, for example, repetitive commission fee, if intraday data used. Since we are trying to find out two stock with the characteristic of cointegration, the higher volatility for intraday data could jeopardize our pair trading strategy. In this project, we use the daily data to test if our strategy was effective.

Also, we should select what kind of data is more sufficient. We use the dividend adjusted closing price. Firstly, the closing price is the most apparent price to show the trend for a stock comparing to open price or the price at other time point since the closing price can get rid of the effect from after-hours trade and flooding speculation. Secondly, we have to consider dividend in our model since the price will go straight down when dividend issued. In addition, two companies have a mere chance to issue dividend in the same date. Therefore, in case of this dropdown, we should include dividend.

We use the price data from the Center for Research in Securities Prices (CRSP) database and Macrotrends.net dataset with data of the dividend adjusted closing prices.

The data is collected through the time period from January 2007 to the end of Feb 2020.

3.2. Data Cleansing

Since we are using stocks from same market to formulate the strategy, the trading window between two adjunct data from two stocks must be identical and trading in same currency. Thus, we don’t have to transform our data into other ways. However, for the missing data and incorrect data, if existed, we still have to figure out a way to deal with it.

For missing data, linear interpolation is a decent way to compensate the flaw. Linear interpolation involves estimating a new value by connecting two adjacent known values with a straight line. Because only few data are deficient and our sample is relatively big, we can use linear interpolation to approximately fill out the blank.

There could be a chance that one of our stock is suspended from trading for a period from which a time mismatch will occur. Under this circumstance, we should match the period of two stocks instead of using multiple linear interpolation. Hence, time series should be cut into two pieces for the consistency of our strategy. Indeed, we can "re-stitch" them if we figure out a consistent cointegration both before and after suspension.

For the back-testing purpose, we shall divide our sample into two pieces as training set and testing set to see if our strategy is effectively making profit and has a great attribute of prediction.

3.3. Stock Selection

To process the pair trading, two stocks having similar characteristics and performances should be selected. For our initial step, we select two stocks for the same company. In this case, we select BRK/A and BRK/B for our model. These two class shares are both for same company, Berkshire Hathaway. While the Class A is trading for around $270,000 per share, the Class B is trading around at $180. The Class A shares have the privilege to convert into an equivalent amount of Class B shares any time. The conversion privilege does not exist. The Class B shares can only be sold and then buy the equivalent in Class A.

We obtained our data from CRSP database, the time was from 2007 to 2019. We would choose subsets of the datasets to see the result. The data are all in the format {PERMNO, date, TICKER, TSYMBOL, PERMCO, PRC, BID, ASK, OPENPRC, vwretd}.

For our project, the most valuable data is the PRC which represents the price of stock. Therefore, we extract price data of two stocks from our dataset for our phase one model.

The total data have 3,354 observations from 2007 to 2020. However, a relatively long period of time could have negative effect on our strategy. Hence, we first would take a latest one-year data in which the stock market has an obvious trend as well as stability. There are total 252 observations. We divide this period into two equal length part and use the first part as training set and the other part as testing set for our phase one strategy implementation.

	Min	**Mean**	**Max**	**Standard deviation**	**Observations**
BRKA	72400	184677	344970	75414.03	3354
BRKB	46	122.91	230.2	50.33151	3354

Table 1 Statistical Description of Two Stocks

Then we plot the graphs of our two stocks:

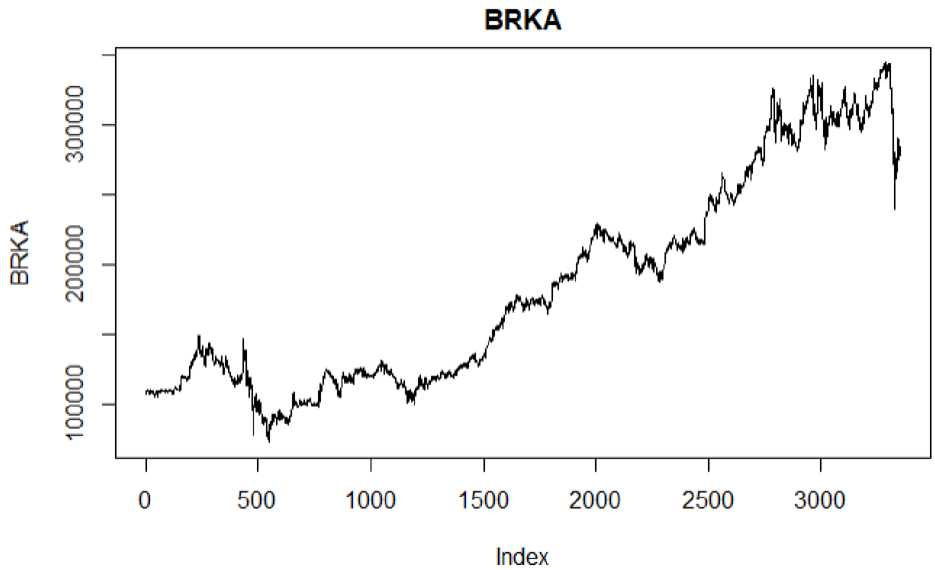

Figure 1 Stock Price Trend of BRK.A

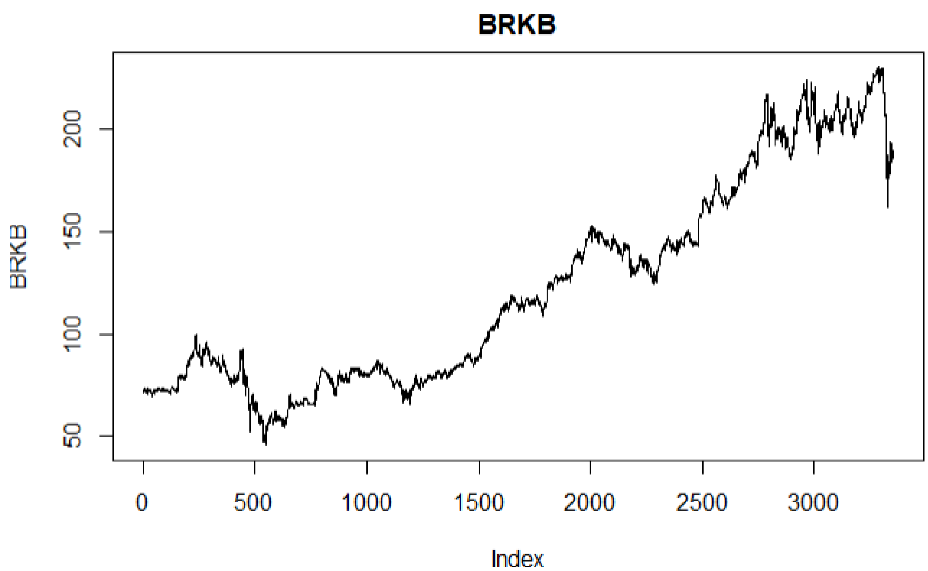

Figure 2 Stock Price Trend of BRK.B

Firstly, from these two graphs, we are easily able to find out an upward trend in this time period. In addition, no matter how big difference between two stock prices, the subtraction of these two stocks is seemingly stable since the fluctuation of two stocks are basically going in the same direction as well as degree of fluctuation which is what we want to see the characteristics of our sample stocks.

Then we test if two time series are stationary or not:

```
        Augmented Dickey-Fuller Test

data:  brkasample
Dickey-Fuller = -2.0096, Lag order = 6, p-value = 0.5717
alternative hypothesis: stationary


        Augmented Dickey-Fuller Test

data:  brkbsample
Dickey-Fuller = -2.0736, Lag order = 6, p-value = 0.5448
alternative hypothesis: stationary
```

It turns out that both of these two time series are not stationary which is perfectly fulfilled our goal that we need two non-stationary to formulate a stationary spread time series.

The next step is two check out if our new spread time series is stationary:

```
        Phillips-Perron Unit Root Test

data:  spr$spread
Dickey-Fuller = -32.071, Truncation lag parameter = 9, p-value = 0.01

p-value smaller than printed p-value
        Augmented Dickey-Fuller Test

data:  spr$spread
Dickey-Fuller = -6.3399, Lag order = 14, p-value = 0.01
alternative hypothesis: stationary
```

The p-value of both tests are below 0.05 which indicates that we can prove that our time series is stationary. Therefore, from this on, we are able to test if our pair has the property of cointegration:

```
######################
# Johansen-Procedure #
######################

Test type: maximal eigenvalue statistic (lambda max) , with linear trend

Eigenvalues (lambda):
[1] 0.0829114331 0.0001211247

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 1 |   0.41  6.50  8.18 11.65
r = 0  | 290.12 12.91 14.90 19.19

Eigenvectors, normalised to first column:
(These are the cointegration relations)

          BRKA1.l2 BRKB1.l2
BRKA1.l2     1.000  1.00000
BRKB1.l2 -1498.421 54.02774

Weights W:
(This is the loading matrix)

             BRKA1.l2      BRKB1.l2
BRKA1.d -1.086175e-01 -3.536707e-04
BRKB1.d  8.931932e-05 -2.407835e-07
```

From this result, since we reject the hypothesis that ‘r=0’, we may conclude that there is an evidence of cointegration property on our pair.

Then we can extract out the training set from one-year period and formulate the phase one strategy. The statistical description is presented as below:

Column1	Min	Mean	Max	Median	Standard deviation	Observations
BRKA	295000	318021	344970	314450	13973.94	252
BRKB	195.8	211.9	230.2	209.3	9.5117	252

Table 2 Statistical Description of Sample

We can see that the medium is around the mean, they seem to be at a proportion, and we would like to see that number. We calculated the average ratio of BRKA and BRKB for the training period by a simple formulation:

$$\bar{r} = \frac{1}{128}\sum_{t=0}^{127}\frac{y_t}{x_t}$$

In this training period, the value is 1500.328. We can say that, 1 stock of BRKA is approximately 1500.328 shares of BRKB in that period.

After the average ratio, we would like to calculate the spread for each time t.

$$s_t = y_t - \bar{r}x_t$$

If the two stocks are perfectly correlated, then the spread is always close to zero. In such cases we would not be able to make money since the two assets are all at the correct price. We plot the s_t to check how is the spread looks like.

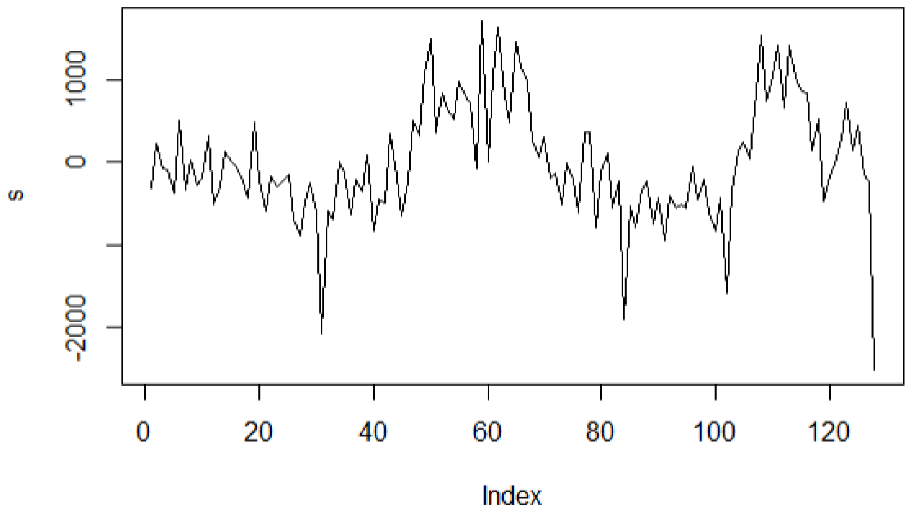

Table 3 Spread Trend

From the spread graph, it shows that the spread of two stocks is zigzagging around 0. However, between any two zero points, the spread goes up or down and turns out gets back to 0. The essence of our strategy is to catch up these situations and make profit from it which is what we are going to implement in phase one model.

In order to have a better understanding of the spread, we would like to see the ACF plot and PACF plot.

The ACF plot of spread is as following:

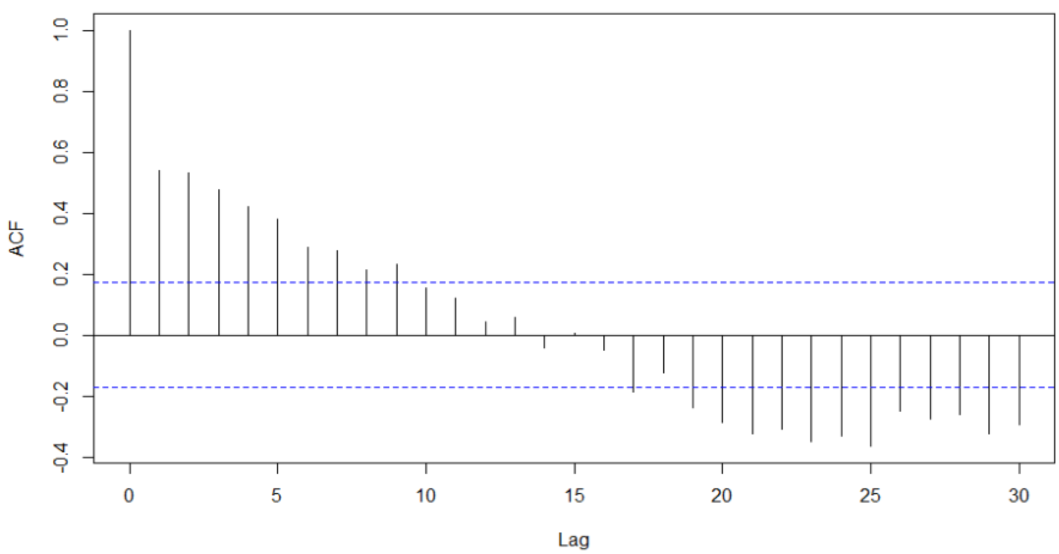

Figure 3 ACF Plot of Spread

The PACF plot of spread is as following:

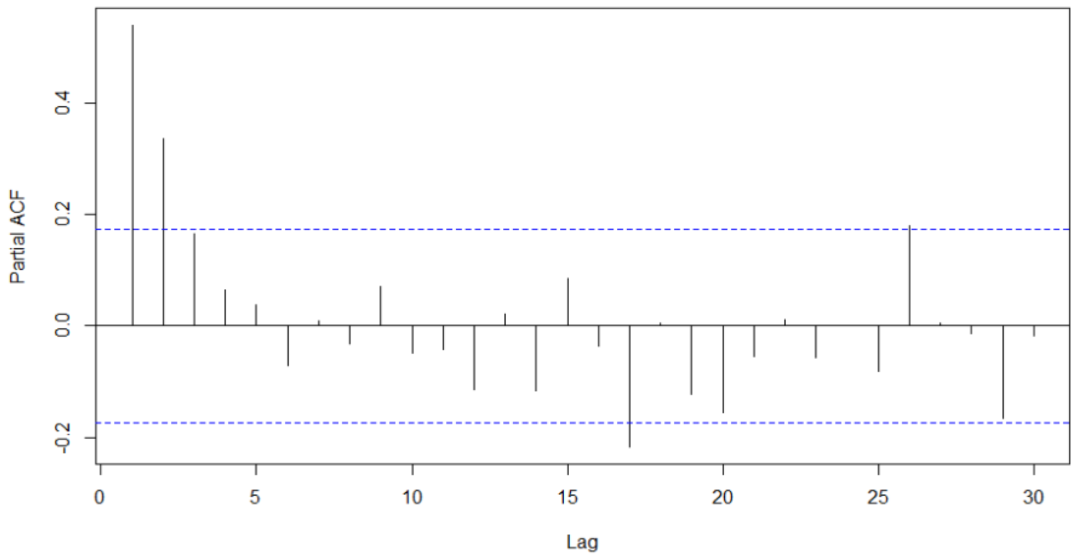

Figure 4 PACF Plot of Spread

From the spread, we clearly find out this time series is autocorrelated. Therefore, a further step to test cointegration is needed.

Targeting on the training set, we implement the same methods to test if our sample is stationary or cointegrated.

```
        Phillips-Perron Unit Root Test

data:  spr$spread
Dickey-Fuller = -32.071, Truncation lag parameter = 9, p-value = 0.01

p-value smaller than printed p-value
        Augmented Dickey-Fuller Test

data:  spr$spread
Dickey-Fuller = -6.3399, Lag order = 14, p-value = 0.01
alternative hypothesis: stationary


######################
# Johansen-Procedure #
######################

Test type: maximal eigenvalue statistic (lambda max) , with linear trend

Eigenvalues (lambda):
[1] 0.13945652 0.04093096

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 1 |  5.27  6.50  8.18 11.65
r = 0  | 18.92 12.91 14.90 19.19

Eigenvectors, normalised to first column:
(These are the cointegration relations)

              brka1train.l2 brkb1train.l2
brka1train.l2         1.000         1.000
brkb1train.l2     -1437.309      8175.155

Weights W:
(This is the loading matrix)

             brka1train.l2 brkb1train.l2
brka1train.d  -1.529137369 -7.784419e-03
brkb1train.d  -0.000818741 -6.266103e-06
```

We are able to confirm the stationary and cointegration of the training set based on the result of tests. Therefore, we can further proceed.

Also, we use the same dataset for our second and third strategy. There will be a slight difference according to the selection of windows.

In the fourth strategy, we will introduce 3 stocks to formulate a pair trading.

Here we use 3 stocks: “SPY”, “IVV”, “VOO”

```
        Augmented Dickey-Fuller Test

data:  SPY$SPY.Adjusted
Dickey-Fuller = -2.4025, Lag order = 6, p-value = 0.4063
alternative hypothesis: stationary


        Augmented Dickey-Fuller Test

data:  IVV$IVV.Adjusted
Dickey-Fuller = -2.4047, Lag order = 6, p-value = 0.4054
alternative hypothesis: stationary


        Augmented Dickey-Fuller Test

data:  VOO$VOO.Adjusted
Dickey-Fuller = -2.3929, Lag order = 6, p-value = 0.4103
alternative hypothesis: stationary
```

Here we firstly test if the original data is non-stationary. It turns out that these three stocks are actually non-stationary which is helpful for our next step:

```
######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend

Eigenvalues (lambda):
[1] 0.33778957 0.25099848 0.01433841

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 2 |   3.60  6.50  8.18 11.65
r <= 1 |  75.56 15.66 17.95 23.52
r = 0  | 178.19 28.71 31.52 37.22

Eigenvectors, normalised to first column:
(These are the cointegration relations)

                SPY.Adjusted.l2 IVV.Adjusted.l2 VOO.Adjusted.l2
SPY.Adjusted.l2         1.00000       1.0000000        1.000000
IVV.Adjusted.l2       -37.30683      -0.5620832        1.120679
VOO.Adjusted.l2        39.57477      -0.4684759       -2.341890

Weights W:
(This is the loading matrix)

               SPY.Adjusted.l2 IVV.Adjusted.l2 VOO.Adjusted.l2
SPY.Adjusted.d       0.1318767       -7.899260       0.4225361
IVV.Adjusted.d       0.1460998       -7.201796       0.4267413
VOO.Adjusted.d       0.1109656       -6.618501       0.3912208

[1] -16.03942

        Phillips-Perron Unit Root Test

data:  spread.three
Dickey-Fuller = -15.621, Truncation lag parameter = 5, p-value = 0.01

p-value smaller than printed p-value
        Augmented Dickey-Fuller Test

data:  spread.three
Dickey-Fuller = -5.3862, Lag order = 6, p-value = 0.01
alternative hypothesis: stationary
```

Here we use three tests which are identical with the above to show that this combination is plausible to formulate a pair trading strategy since we find out a stationary spread time series with a good property of cointegration.

Then for the better understanding, we will also plot the ACF and PACF of this spread:

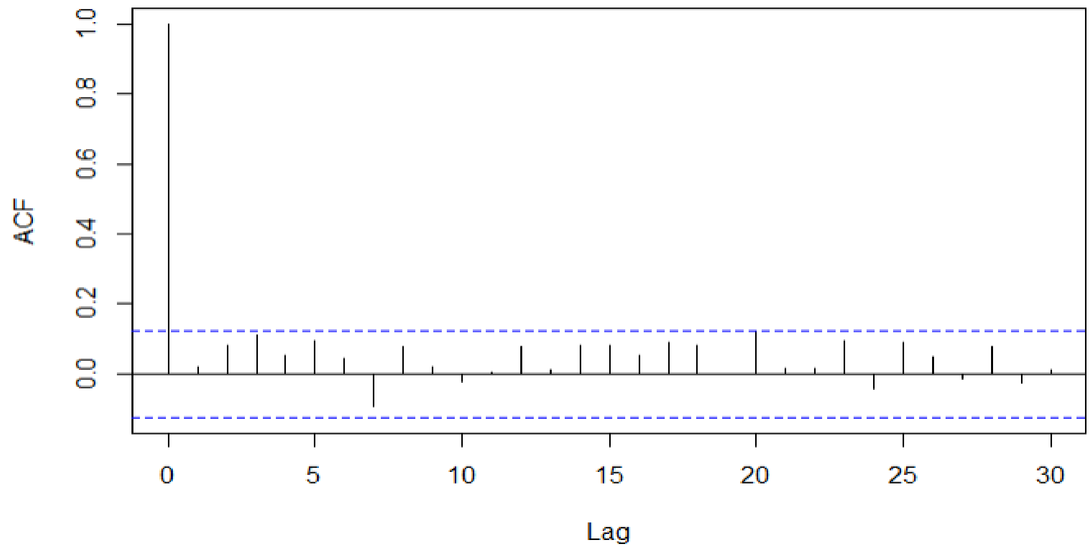

Figure 5 ACF of Spread of Three Stocks

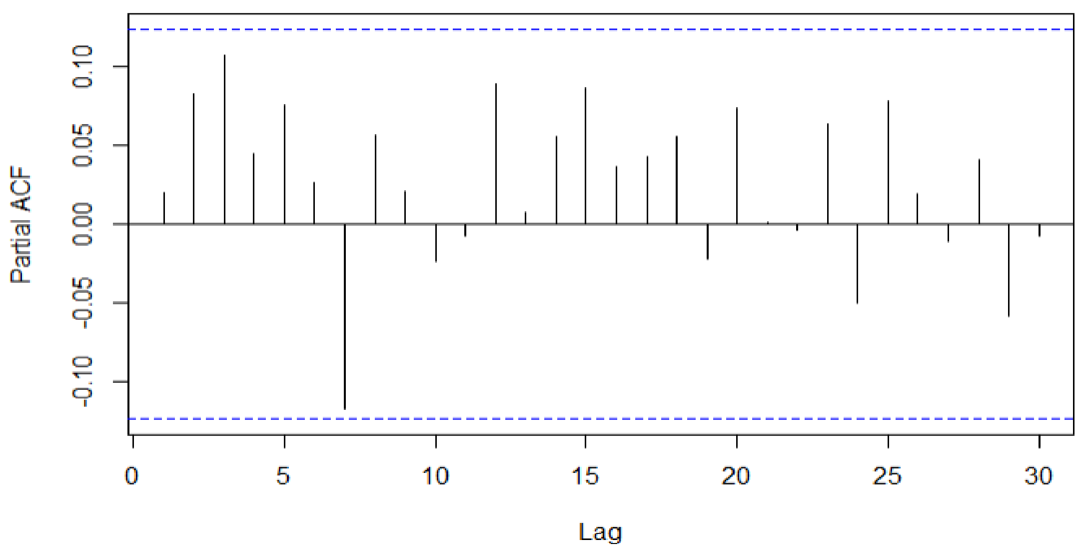

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Figure 6 PACF of Spread of Three Stocks

The autocorrelation is not obvious, whereas our stocks have fulfilled the requirements we want. Therefore, these data are plausible to proceed our strategy.

IV Initial Model

In this section, we present how we calculate the parameters in the section II. And we give the performance of the trading strategy in the market, from Sep 2019 to the end of Feb 2020. Then we would analysis the result of the model.

4.1. The Structure of Initial Model

We have the datasets of closing prices of the stocks, from Sep 2019 to the end of Feb 2020, s_t is defined as the spread for $t = 0,1,\cdots,T$. The first half, $t = 0, \ldots, \lfloor T/2 \rfloor$ is considered as historical data and is used to shape the parameters ratio ϕ.

The rest of samples is treated as the future and is used to calculate the profit and loss by applying our model and previous generated ratios with the pair stocks.

In this project, the trading window is set as 40, representing 40 trading days, around two months. The number is set to reflect a reasonable stock prices trend limitation. If the trading window setting was set too larger, the mean spread may not be able to reflect the current trend. If the trading windows was too small, the period may not be long enough to contain enough information to estimate a stable mean of the spreads.

In practice, the hedge ratio $\bar{r}$ and spread is derived from linear regression as following.

$$A = \bar{r}B + s_t$$

The mean spread is defined as the average value of residual in giving period, denoted as u.

Our trading strategy is presented as following.

When the spread s_t at time t first hit the line $u - \phi\sigma$, we set one share long position on stock A and short $\bar{r}$ share position on B. Then, when the spread hit the line $u + \phi\sigma$, we close out the previous portfolio.

When the spread s_t at time t first hit the line $u + \phi\sigma$, we set one share short position on stock A and long $\bar{r}$ share position on B. Then, when the spread hit the line $u - \phi\sigma$, we close out the previous portfolio.

When the spread s_t at time t is between the upper line $u + \phi\sigma$ and lower line $u - \phi\sigma$, we maintain the previous position.

In this simulation trading, no transaction fee or trade cost is counted. The reason to do this is that the cost of transactions or trading is usually small and can be neglected.

In this model, the BRKA is the A stock, and the BRKB is the B stock.

4.2. Performance Analysis

Firstly, we would like to see the optimized performance of the portfolio in the whole period. We first use the whole period as the training set to see the result. With the optimized parameters, we can see the graph is like this.

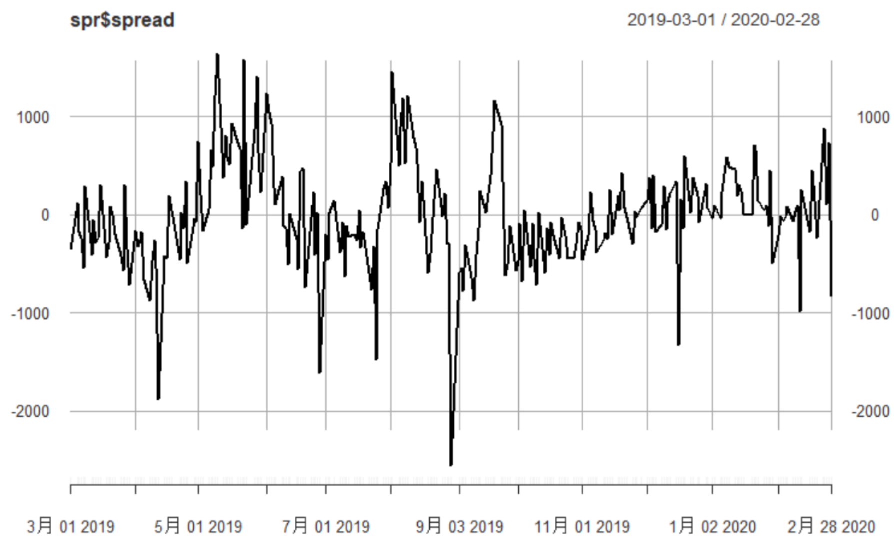

Figure 7 Spread Trend

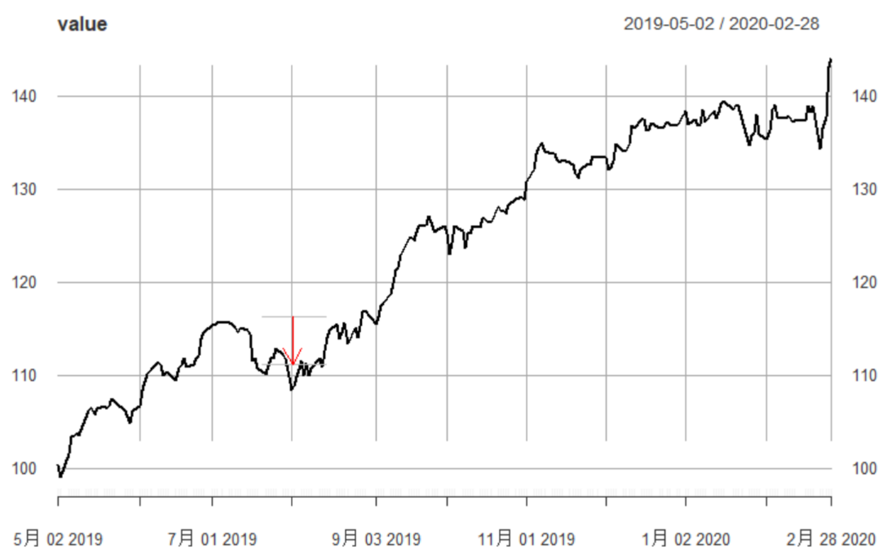

Figure 8 The Performance of the Whole Dataset

We can see from the graph that out return is 43.98% in this nine-month period while the maximum drawdown is 7.27%, which is very promising when formulate a trading strategy.

After having a glance at the total performance on the whole set, we divided the whole set into two parts equally. The first is the training period, we would try to get the best parameter of the

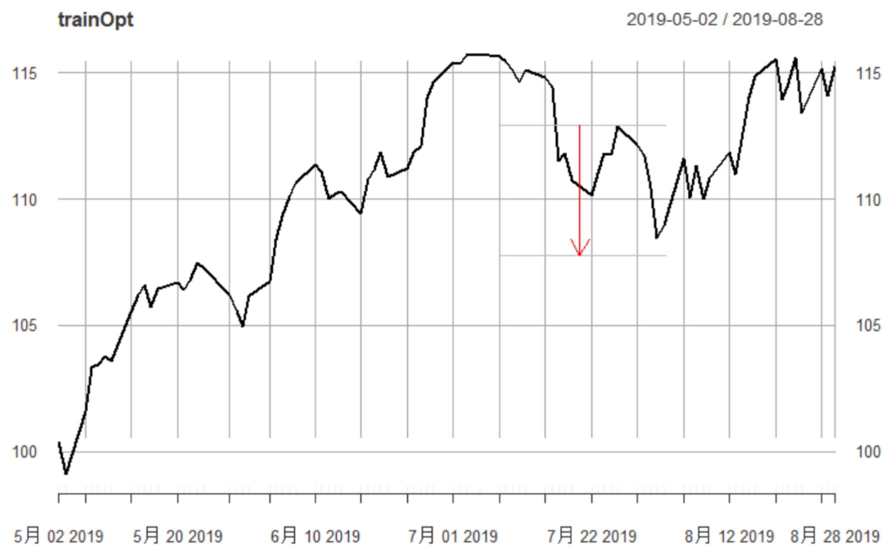

Figure 9 The Train Optimization of Initial Model

In the training set, our strategy is 15.28%, which is profitable. In addition, the drawdown is 7.27%, the same as the whole set.

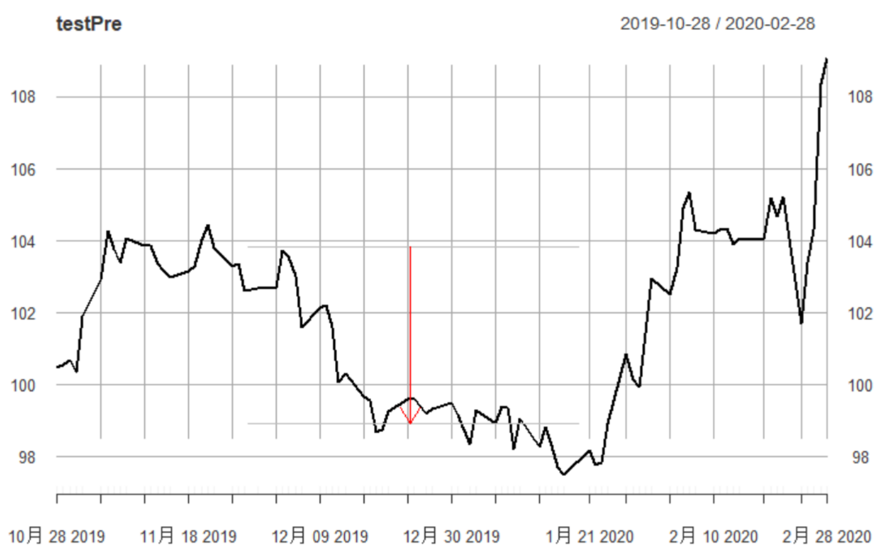

Figure 10 The Test Prediction by Using Test Optimized Ratios

In this case, the return in the whole time period is 9.07%. In addition, the maximum drawdown is 6.93% which is acceptable for our tolerance of risk.

Then we run the strategy in the test set which shows a great excess return. However, the drawdown is a bit out of control for this case. Therefore, a future optimization is required.

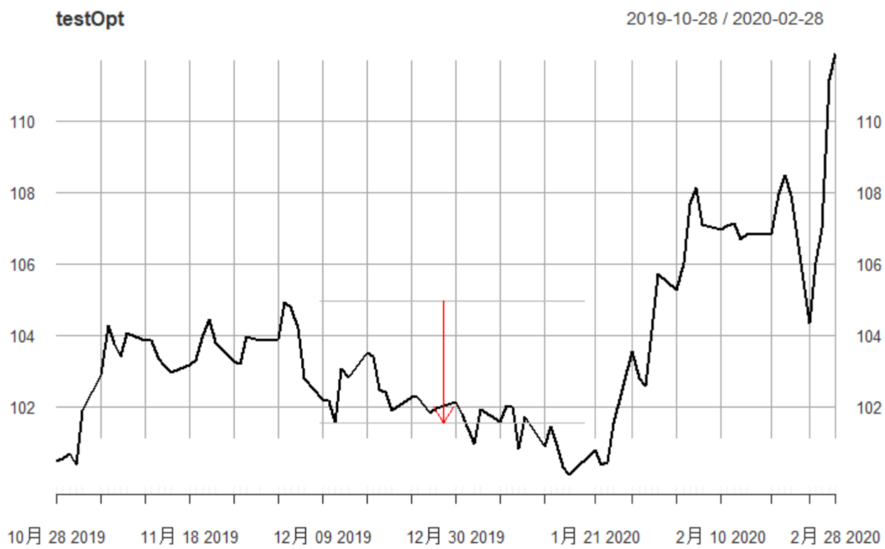

Figure 11 The Optimized Test of Initial Model

In addition, the maximum drawdown is which is acceptable for our tolerance of risk. We also get the optimized parameter of the testing period. We would like to compare it with the predicted testing period. We see that the return is 11.89%, while the max drawdown is 4.82%. We can see that the optimized parameter will give a higher return and a lower risk compared to the predicted parameters.

V Revised Model

5.1. With Gain Limitation

To reduce the trading risk, we set a gain limitation on trading strategy as ε. Which means we would close out the portfolio when the s_t hit the line $u + \phi\sigma - \varepsilon$ or the line $u - \phi\sigma + \varepsilon$. By setting this limitation, we could reduce the volatility risks.

From this new strategy (phase two), we introduce the risk management mechanism from which we with no loss of profit, we surprisingly reduce the drawdown of portfolio during this time on training set.

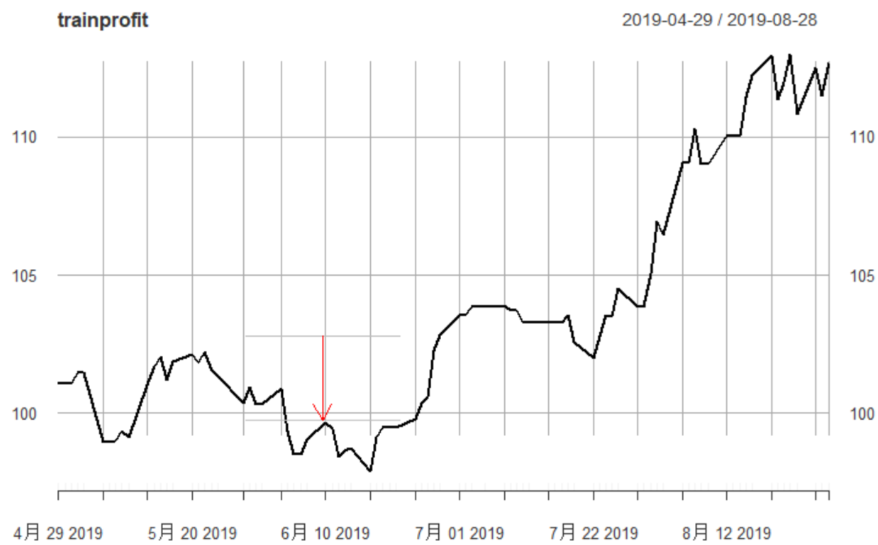

Figure 12 The Optimized Train of Revised Model I

We see that with profit bound, our return is 12.67%, and the max drawdown is 4.30%. Both of which are smaller than the initial model, which show us that there is a trade off between the profit and the risk.

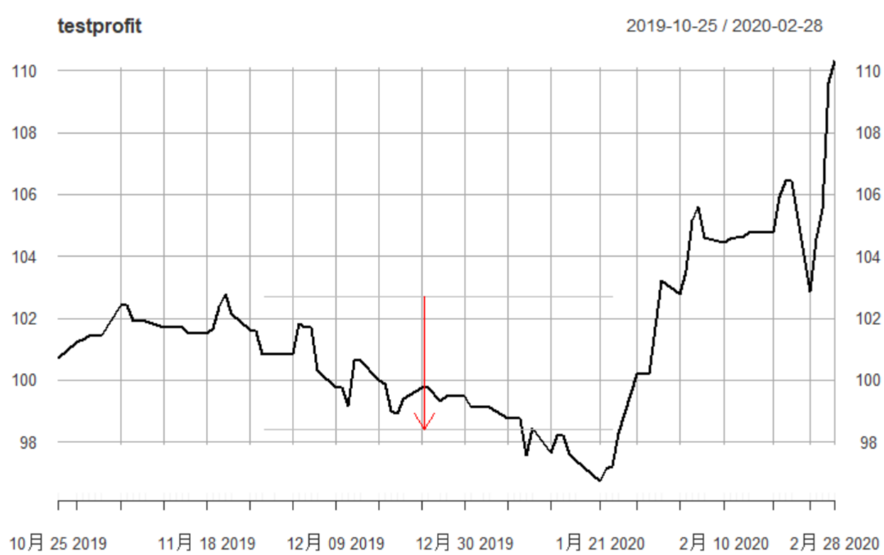

Figure 13 The Test Prediction by Using Test Optimized Ratios

Here, we can observe that the profit is 10.31%, which is kind of promising. In addition, the maximum drawdown is 6.03%, which is acceptable. Compare to the performance of the first model, we would see that the return is higher while the risk is lower, which may prove that the revised strategy would be a better strategy than our initial strategy.

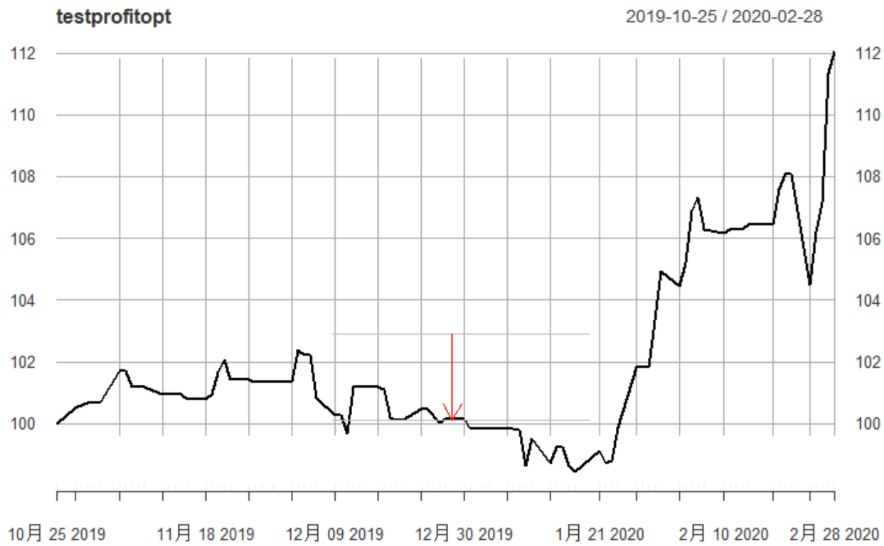

Figure 14 The Optimized Test of Revised Model I

When we compare this to the optimization situation, the profit is 12.02%, which is not much different. However, the maximum drawdown is 3.93%, which is much lower than our predicted strategy.

5.2. With Loss Limitation

Considering an unstable market with highly volatility, the previous strategy might be invalid. In a market with unusual volatility, the maintenance margin could exceed and thus trigger an automatically close out on our speculation position.

In the revised model, we set a loss-limit line for our trading model with a parameter ψ. The short position will be closed if the loss-limit line was reached and add a short position when the trend reversed. Specifically, the short position will be closed out when the spread s_t reached $u + (\psi + \phi)\sigma$. And the long position will be liquidated when the spread s_t reached $u - (\psi + \phi)\sigma$.

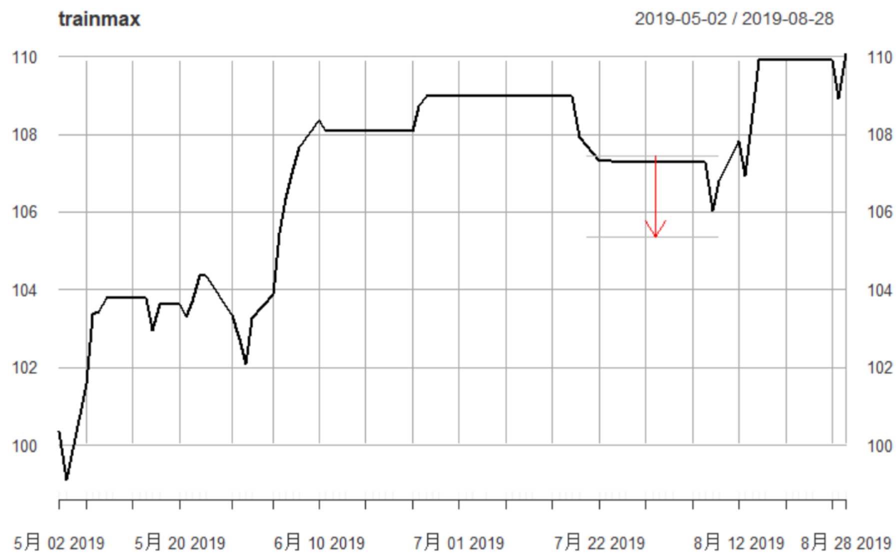

Figure 15 The Optimized Train of Revised Model II

When we add a loss limitation mechanism into our strategy, the drawdown is well controlled, which is 2.93%. And the profit goes to 10.05%, which is not bad.

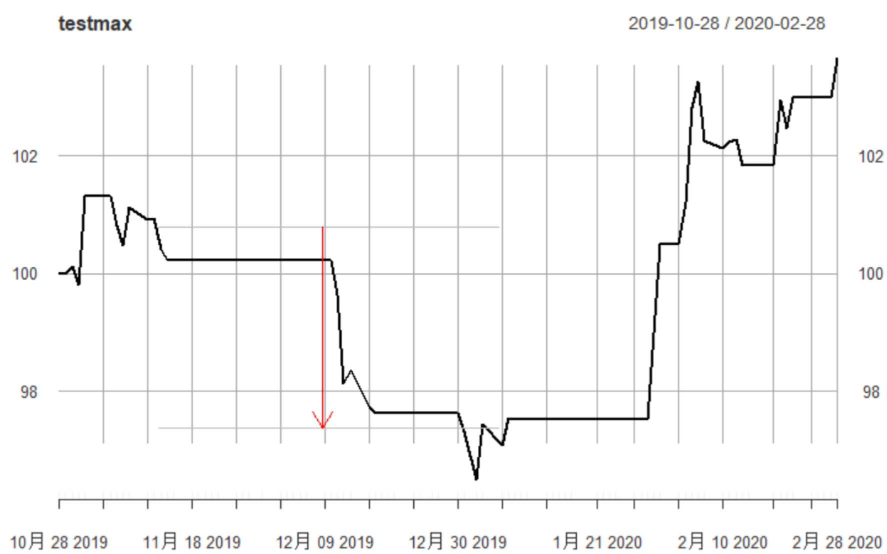

Figure 16 The Test Prediction by Using Test Optimized Ratios

It turns out the test set does not have an exciting outcome, which is only 3.64% while the max drawdown is only 4.81%. The result may be due to many different reasons, one of which could be restricted to the amount of sample data. The model may not be a bad model, but we can say the model do not work well on this period.

5.3. Initial Model with Three Time Series

In this section, we would discuss the result that we generate the initial model with additional company stock. In the previous chapter, we analyzed the stationary of SPY, IVV, and VOO. Then we looked into the cointegration between them. All of the companies has passed the ADF and PP test, as well as the Johansen test.

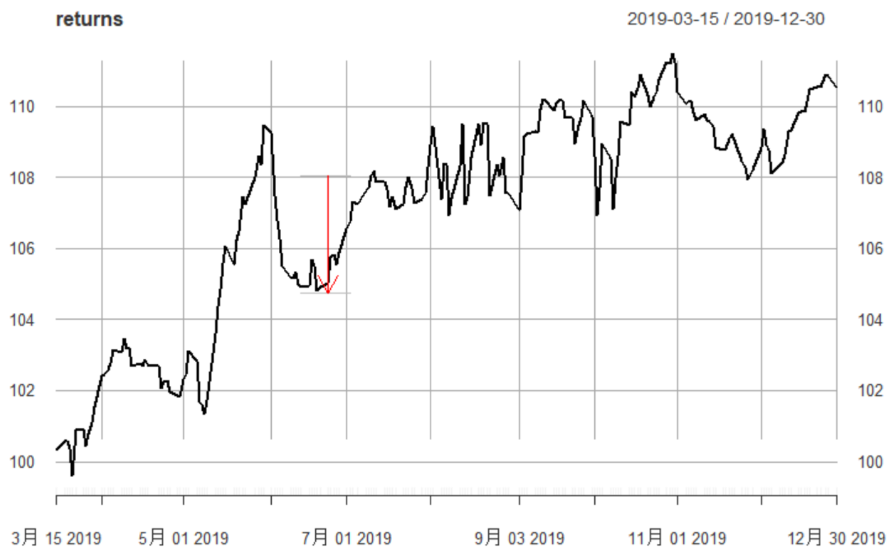

Figure 17 The Results of Revised Model III

The return during the nine-month period is 10.49%, a result that need to further improve. However, we can see from the result that the max drawdown is only 4.64%, which is not very high. The three time series model may have the effect of diversify the risk among the three stocks. Due to time issues, we do not further improve the model. But we see the potential in the improving of the three stock pairs model.

VI Back Testing in 2008

In this chapter, we applied the initial model into the data of 2007 to 2008, the period of finance crisis. The specific time to analysis is from the beginning of Oct 2007 to the end of Sep 2008.

We equally split the dataset into two sections, first is used as training dataset to obtain optimized parameters, and the second used as test dataset to evaluate the performance of our model.

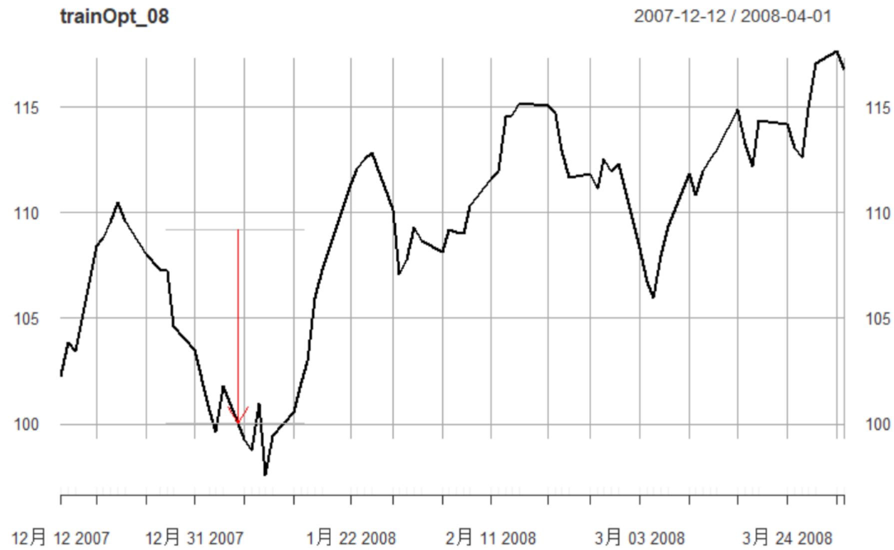

Figure 18 The Train Optimization by Initial Model

The training set return is 16.76% while the max drawdown is 6.34%.

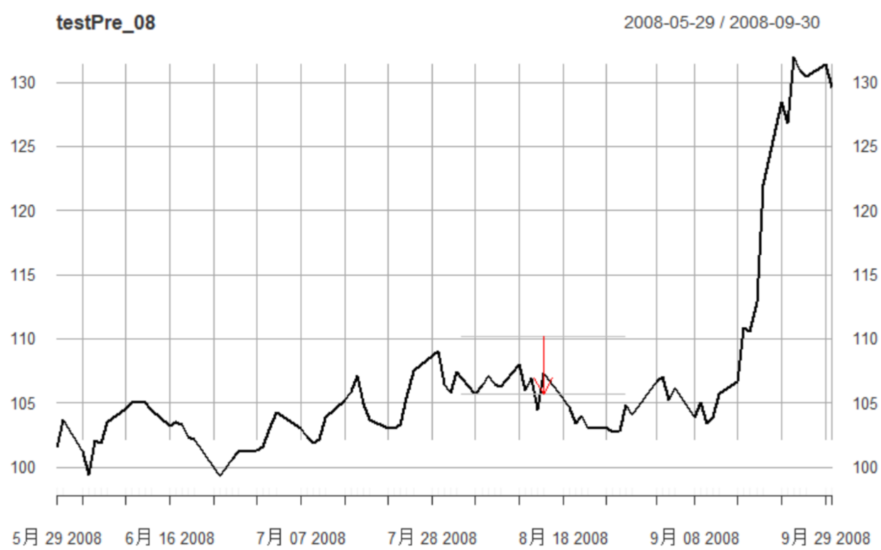

Figure 19 The Test Prediction by Using Test Optimized Ratios in Initial Model

On the testing period, we see that the return is 29.59% while the max drawdown is 6.34%.

We test the model into financial crisis in 2008. Surprisingly, the return is 'out of control' which means we make a great fortunate in financial crisis. We believe the success is derived from the successful prediction of the spread moving direction.

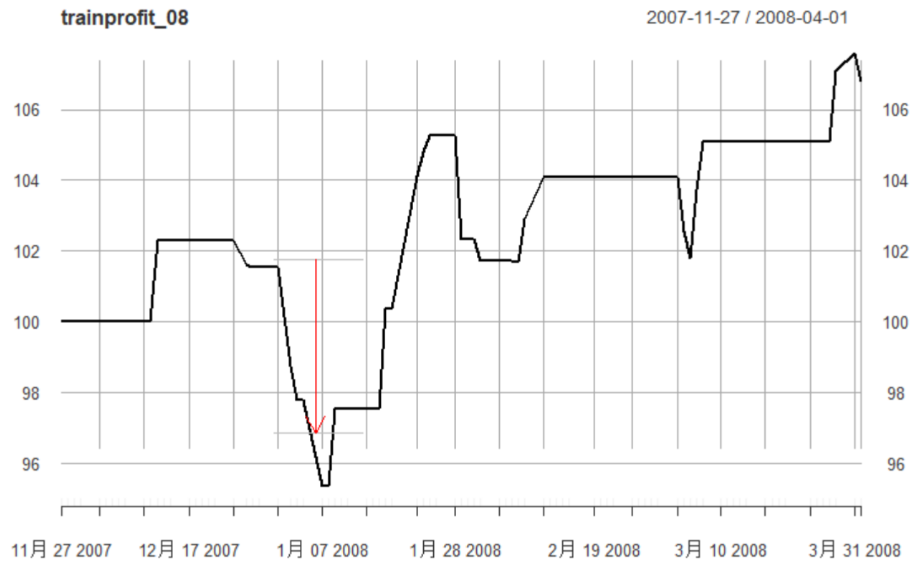

Figure 20 The Train Optimization by Revised Model I

Here, the profit is 6.78%, the maximum drawdown is 4.30%.

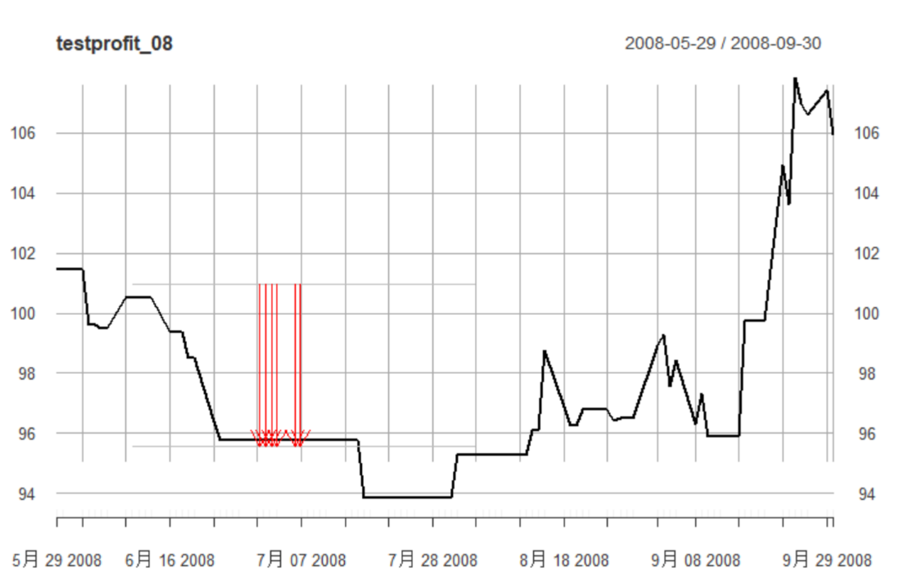

Figure 21 The Test Prediction by Using Optimized Ratios in Revised Model I

So we acquire the same statistics which the profit is 5.89%, and maximum drawdown is 7.59%.

We see that the revised model with the risk management strategy doesn't perform as we thought it would be. It proves that the risk management strategy may work well in the normal period, while in the financial crisis it doesn't work. We can argue that this may be the market is not rational and equilibrium in the crisis.

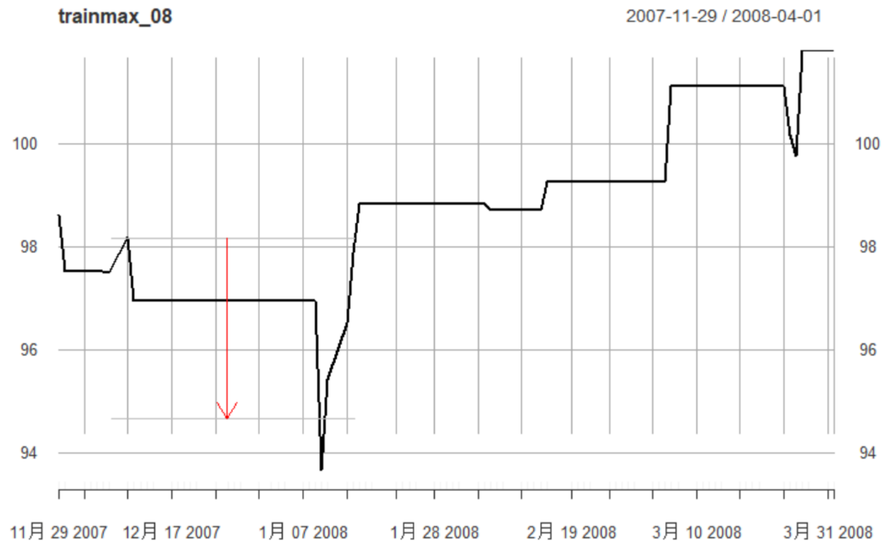

Figure 22 The Train Optimization by Revised Model II

Here in the end, we have profit equal to 1.80% and maximum drawdown equal to 4.95%.

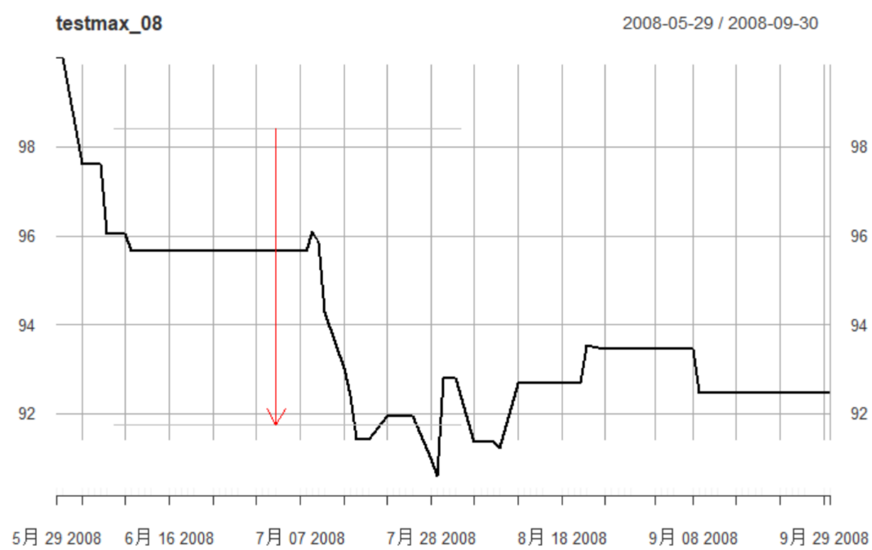

Figure 23 The Test Prediction by Using Optimized Ratios in Revised Model II

The profit is kind of ugly, we have the negative return equal to -7.56%; However, the maximum drawdown is lethal with the number of 9.40%. The strategy is not good in the crisis period, which tells us that we need to have a further looking at the risk factor in the crisis period.

VII Summary

Theoretically, the pairs trading is market-neutral; however, it cannot eliminate the risk at all. In this project, we developed three models for pair trading strategy.

The initial model gives the basic structure for all models. It performs the pairs trading by setting an entry and an exit line. When the spread hit the lower line, we hold a long position for A stocks, while take $\bar{r}$ short share on stock B.

While the initial model is good, we are concerned about the risk under the trading. So in order to reduce the risk of the trading, we first need to select a appropriate risk measure. In order to simplify the process, we select the max drawdown as the risk measure of our trading.

With the selected risk measure, we tried to revise the model a bit by introducing another strategy. We call it the closing strategy. In order to reduce future risk, we would execute the closing strategy when the condition is triggered.

The first revised model is with the closing strategy when we hit a proper gain line. We will close our position and wait for another entering signal is set. This strategy would let reduce our profit a little bit, which it will also reduce our risk exposure. We can see the trade off between the profit and the risk.

The second revised model is with the closing strategy when we hit a proper losing line. When there is the condition, we would close our strategy when we have a maximum lose. The strategy ensures us that when there is the case that the mean reverting is not working very well for a very short of time, we would suffer from significant losses caused by the short period un-stationary spread. We can see that with this strategy, the risk is lower than the initial model, while sometimes the return is also not that good.

After revising the model a bit, we would like to check the performance of the risk management when there is a financial crisis. So we choose the 2008 financial crisis as the period that we do the training and testing. We can see that our initial method and our model with gaining limit performed very well in the crisis, while the model with losing limit didn't perform very well. Maybe the model is not very fit for during the crisis.

We also developed a three stock pairs model which is similar to the initial model of the two pairs model. We hope to develop a further risk management on that, but we do not have the enough time. Hope we can do that later.

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Appendix

In this project, every teammate plays an important role to support the completeness of the project while everyone has different specific roles. To be specifically, everyone completes the following tasks alone or together.

Proposal Phase

All of the teammate discussed and shared their view on different topics for the project.

Shuke and Pritesh collected the initial dataset, and made the data analysis. Chang collected several informative papers on the topic of pairs trading, while both Chang and Yixiao studied these materials for the proposal. The initial proposal document was composed by Yixiao, revised by Chang and Yixiao. And the revised proposal was composed by Chang, and it was revised by all teammates.

Pritesh and Shuke studied the theories of pairs trading after proposal phases.

Project Implementation

Shuke and Pritesh collected the new dataset required by the project. Shuke performed data understanding and analysis. Pritesh worked with Shuke to complete the data preprocessing section.

Chang and Yixiao collected more studying materials for the project. All teammates studied all the materials and considering the models for the project.

The prototype of initial model is proposed by Pritesh and the modeling is discussed by all teammates. Chang and Shuke complete the code for the initial model.

Yixiao proposed a revised model, and discussed with Shuke and Chang. The implementation of the revised model is completed by Chang and Yixiao.

The third model is developed by Yixiao and Chang and discussed with Shuke, the code is completed by Chang and Yixiao.

Final Report

Shuke and Yixiao composed the abstract. Chang Revised it.

Yixiao composed the section I, II of the final project. Chang and Yixiao Revised it.

Shuke composed the section III for the final project. Yixiao revised it.

Yixiao and Shuke composed the section IV for the final project. Chang revised it.

Yixiao, Chang composed the section V for the final project. Yixiao revised it.

Chang and Yixiao Composed the section VII of the final project. Yixiao revised it.

Yixiao composed the other sections of the final project. Chang and Yixiao revised it.

Presentation file is composed by the Yixiao.