VECTOR

Question: $\vec{P}$ and $\vec{Q}$ are any two points lying on the sides DC and AD respectively of a parallelogram $ABCD$. Show that, $ar(\triangle APB) = ar(\triangle BQC)$.

Figure:

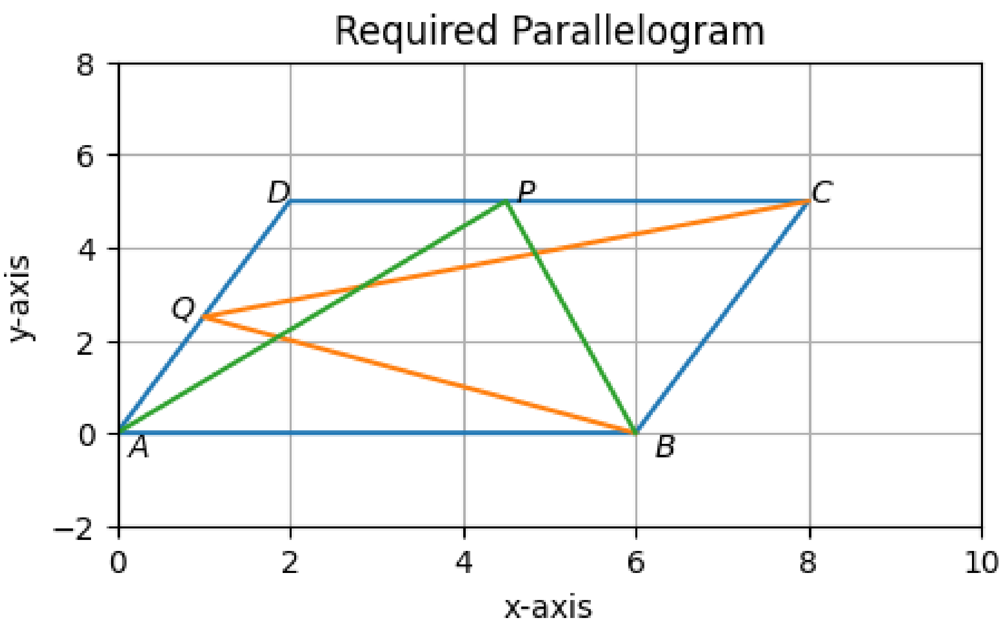

Figure 1

Solution:

Using formulas:

1. Area of a parallelogram with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$ is $= \frac{1}{2} \times \left| [(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)] \right|$ square units.

2. Area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is $= \frac{1}{2} \times \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$ square units.

3. If a triangle and a parallelogram are on the same base and between the same parallels, then area of the triangle is equal to half the area of the parallelogram.

From figure 1 , considering $\triangle BQC$ and parallelogram $ABCD$, having $\underline{\text{same base } BC}$ and, $\underline{AD \text{ is parallel to } BC}$.

$$\implies \triangle BQC = \frac{1}{2} \times ar ABCD. \tag{1}$$

Considering $\triangle APB$ and parallelogram $ABCD$, having $\underline{\text{same base } AB}$ and,$\underline{DC \text{ is parallel to } AB}$.

$$\implies \triangle APB = \frac{1}{2} \times ar ABCD. \tag{2}$$

By comparing equation (1) and equation (2) we obtain that,

$$ar(\triangle APB) = ar(\triangle BQC). \tag{3}$$

Hence proved.

Proof by the help of diagram :(figure 1)
For the $\triangle APB$, the vertices of the triangle are

$\vec{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \vec{P} = \begin{pmatrix} 4.5 \\ 5 \end{pmatrix}, \vec{B} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

$\implies \triangle APB = \frac{1}{2} \times \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$
$= \frac{1}{2} \times \left| 0(5-0) + 4.5(0-0) + 6(0-5) \right|$
$= \frac{1}{2} \times \left| 0 + 0 - 30 \right|$
$= \frac{1}{2} \times 30$
$= 15$ square units.

For the $\triangle BQC$, the vertices of the triangle are

$\vec{B} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \vec{Q} = \begin{pmatrix} 1 \\ 2.5 \end{pmatrix}, \vec{C} = \begin{pmatrix} 8 \\ 5 \end{pmatrix}$

$\implies \triangle BQC = \frac{1}{2} \times \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$
$= \frac{1}{2} \times \left| 6(2.5-5) + 1(5-0) + 8(0-2.5) \right|$
$= \frac{1}{2} \times \left| -15 + 5 - 20 \right|$

$= \frac{1}{2} \times 30$
$= 15$ square units.

For parallelogram $ABCD$,the vertices are

$\vec{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \vec{B} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \vec{C} = \begin{pmatrix} 8 \\ 5 \end{pmatrix}, \vec{D} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$\implies arABCD = \frac{1}{2} \times \left|[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)]\right|$
$= \frac{1}{2} \times \left|[(0.0 - 6.0) + (6.5 - 8.0) + (8.5 - 2.5) + (2.0 - 0.5)]\right| = \frac{1}{2} \times \left|30 + 30\right|$
$= \frac{1}{2} \times 60$
$= 30$ square units.

Therefore,$arABCD = \frac{1}{2} \times ar\triangle APB = \frac{1}{2} \times ar\triangle BQC$(proved).