9.10.4.3

Question : If two equal chords of a circle intersect within the circle,prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Solution :

Input Parameters	Description	Value
$\mathbf{O}$	Center(at origin)	$\mathbf{0}$
d	Length of the chords	2
r	Radius	1
θ_1	-	$180°$
θ_2	-	$-47.9°$
θ_3	-	$0°$
θ_4	-	$-132°$

Table 1: Table of input parameters

Output Parameters	Description	Value
$\mathbf{P}$	Point	$\begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix}$
$\mathbf{Q}$	Point	$\begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix}$
$\mathbf{R}$	Point	$\begin{pmatrix} \cos\theta_3 \\ \sin\theta_3 \end{pmatrix}$
$\mathbf{S}$	Point	$\begin{pmatrix} \cos\theta_4 \\ \sin\theta_4 \end{pmatrix}$

Table 2: Table of output parameters

The equation of PQ and RS is obtained by

$$\mathbf{n_1}^\top(\mathbf{x}-\mathbf{P}) = 0 \tag{1}$$

$$\mathbf{n_2}^\top(\mathbf{x}-\mathbf{R}) = 0 \tag{2}$$

$$or, \mathbf{n_1} = \begin{pmatrix} \sin\theta_2 - \sin\theta_1 \\ \cos\theta_1 - \cos\theta_2 \end{pmatrix} \tag{3}$$

$$or, \mathbf{n_2} = \begin{pmatrix} \sin\theta_4 - \sin\theta_3 \\ \cos\theta_3 - \cos\theta_4 \end{pmatrix} \tag{4}$$

$$\tag{5}$$

The value of the point of the intersection is

$$\mathbf{T} = \begin{pmatrix} 0 \\ -0.45 \end{pmatrix} \tag{6}$$

$$\cos\angle OTP = \frac{(\mathbf{T}-\mathbf{P})^\top(\mathbf{O}-\mathbf{T})}{||\mathbf{T}-\mathbf{P}||||\mathbf{T}-\mathbf{O}||} \tag{7}$$

$$or, \angle OTP = 65.8^\circ \tag{8}$$

Similarly,

$$\cos\angle OTR = \frac{(\mathbf{T}-\mathbf{R})^\top(\mathbf{T}-\mathbf{O})}{||\mathbf{T}-\mathbf{R}||||\mathbf{T}-\mathbf{O}||} \tag{9}$$

$$or, \angle OTR = 65.8^\circ \tag{10}$$

$$So, \angle OTP = \angle OTR\,(proved) \tag{11}$$

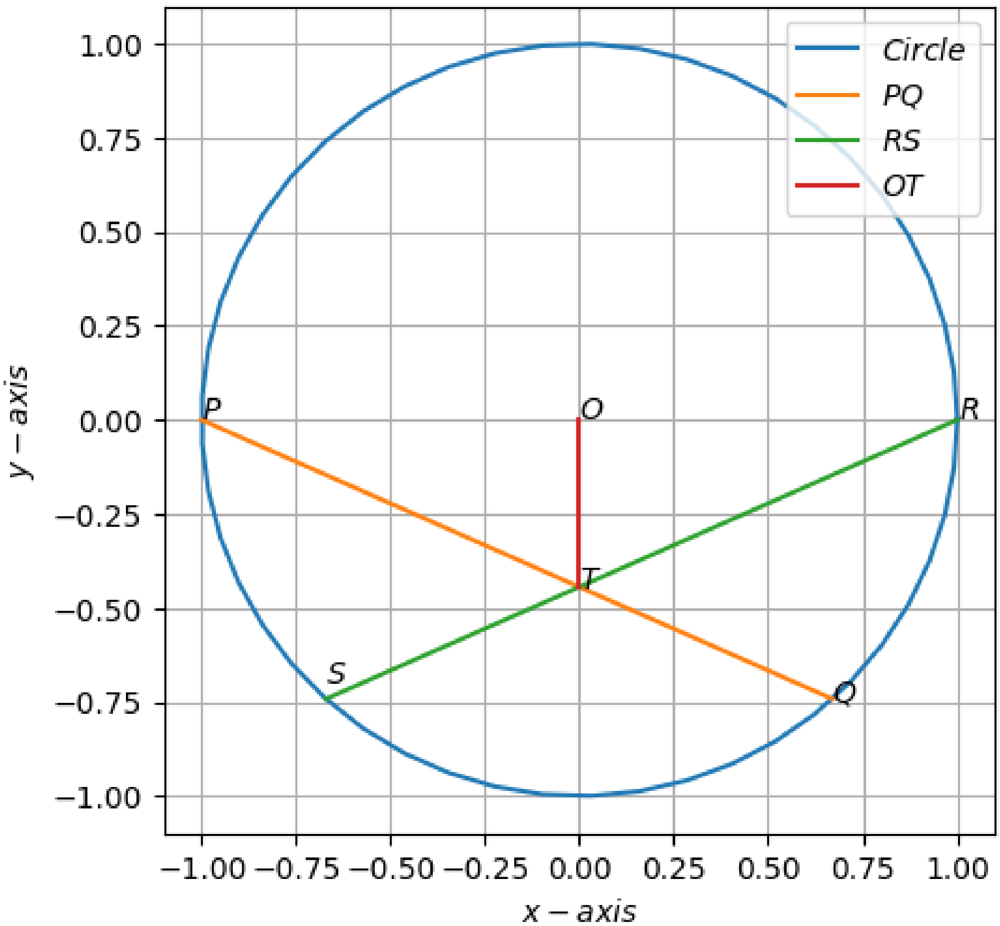
Circle
PQ
RS
OT
P
O
R
T
S
Q
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
x − axis
y − axis

Figure 1: