

EMBEDDED

Question : l and m are two parallel lines intersected by another pair of parallel lines p and q (figure 1),show that $\triangle ABC \cong \triangle CDA$.

Figure :

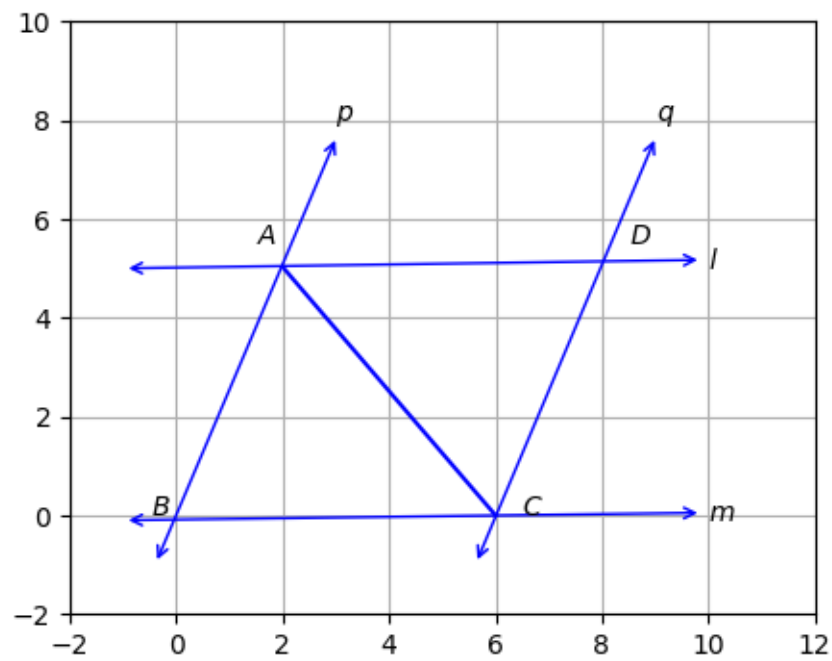


Figure 1: Required parallelogram

Solution :

Symbol	Description	Value
B	Vertex at origin	0
a	Side of the parallelogram, $BC = DA$	6
b	Side of the parallelogram, $AB = CD$	$\sqrt{29}$
θ	Angle of the parallelogram, $\angle ABC$	$\sin^{-1}\left(\frac{5}{\sqrt{29}}\right)$

Table 1: Table of input parameters

Symbol	Description	Value
C	Vertex of parallelogram	$a\mathbf{e}_1$
A	Vertex of parallelogram	$b \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$
D	Vertex of parallelogram	C + A

Table 2: Table of output parameters

From *figure 1* between $\triangle ABC$ and $\triangle CDA$

$$\cos \angle BAC = \frac{\langle B - A, A - C \rangle}{\|B - A\| \|A - C\|} \quad (1)$$

$$= \frac{ab \cos \theta + b \sin \theta}{b\sqrt{a^2 - 2ab \cos \theta + b^2}} \quad (2)$$

$$= \frac{17}{\sqrt{29}\sqrt{41}} \quad (3)$$

$$\cos \angle ACD = \frac{\langle D - C, A - C \rangle}{\|D - C\| \|A - C\|} \quad (4)$$

$$= \frac{b^2 - ab \cos \theta}{b\sqrt{a^2 - 2ab \cos \theta + b^2}} \quad (5)$$

$$= \frac{17}{\sqrt{29}\sqrt{41}} \quad (6)$$

$$\text{So, } \angle BAC = \angle ACD. \quad (7)$$

$$\cos \angle ACB = \frac{\langle B - C, A - C \rangle}{\|B - C\| \|A - C\|} \quad (8)$$

$$= \frac{a^2 - ab \cos \theta}{a\sqrt{a^2 - 2ab \cos \theta + b^2}} \quad (9)$$

$$= \frac{24}{6\sqrt{41}} \quad (10)$$

$$\cos \angle CAD = \frac{\langle A - D, A - C \rangle}{\|A - D\| \|A - C\|} \quad (11)$$

$$= \frac{a^2 - ab \cos \theta}{a\sqrt{a^2 - 2ab \cos \theta + b^2}} \quad (12)$$

$$= \frac{24}{6\sqrt{41}} \quad (13)$$

$$\text{So, } \angle ACB = \angle CAD. \quad (14)$$

And CA is common side .

So, $\triangle ABC \cong \triangle CDA$. (by $A - A - S$) (proved)