# EMBEDDED

**Question :** $l$ and $m$ are two parallel lines intersected by another pair of parallel lines $p$ and $q(figure\ 1)$,show that $\triangle ABC \cong \triangle CDA$.

**Figure :**

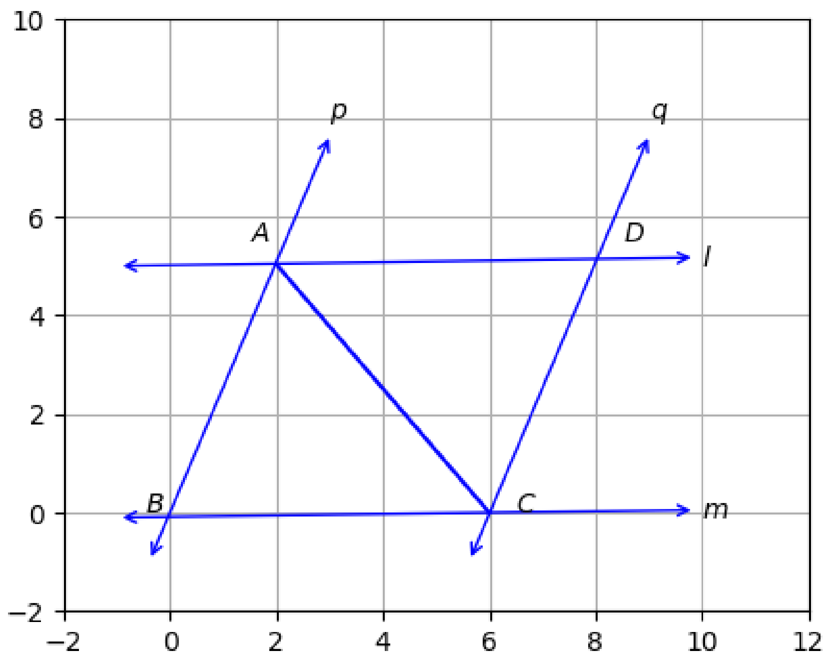


Figure 1: Required parallelogram

**Solution :**

| **Symbol** | **Description** | **Value** |
|---|---|---|
| $\mathbf{B}$ | Vertex at origin | $\mathbf{0}$ |
| $a$ | Side of the parallelogram,$BC = DA$ | 6 |
| $b$ | Side of the parallelogram,$AB = CD$ | $\sqrt{29}$ |
| $\theta$ | Angle of the parallelogram, $\angle ABC$ | $\sin^{-1}\left(\frac{5}{\sqrt{29}}\right)$ |

Table 1: Table of input parameters

| **Symbol** | **Description** | **Value** |
|---|---|---|
| $\mathbf{C}$ | Vertex of parallelogram | $a\mathbf{e_1}$ |
| $\mathbf{A}$ | Vertex of parallelogram | $b\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$ |
| $\mathbf{D}$ | Vertex of parallelogram | $\mathbf{C}+\mathbf{A}$ |

Table 2: Table of output parameters

From $figure$ 1 between $\triangle ABC$ and $\triangle CDA$

$$\begin{align}
\cos\angle BAC &= \frac{\langle B-A, A-C\rangle}{\|B-A\|\,\|A-C\|} \tag{1}\\
&= \frac{ab\cos\theta + b\sin\theta}{b\sqrt{a^2-2ab\cos\theta+b^2}} \tag{2}\\
&= \frac{17}{\sqrt{29}\sqrt{41}} \tag{3}\\
\cos\angle ACD &= \frac{\langle D-C, A-C\rangle}{\|D-C\|\,\|A-C\|} \tag{4}\\
&= \frac{b^2-ab\cos\theta}{b\sqrt{a^2-2ab\cos\theta+b^2}} \tag{5}\\
&= \frac{17}{\sqrt{29}\sqrt{41}} \tag{6}\\
So, \angle BAC = \angle ACD. \tag{7}\\
\cos\angle ACB &= \frac{\langle B-C, A-C\rangle}{\|B-C\|\,\|A-C\|} \tag{8}\\
&= \frac{a^2-ab\cos\theta}{a\sqrt{a^2-2ab\cos\theta+b^2}} \tag{9}\\
&= \frac{24}{6\sqrt{41}} \tag{10}\\
\cos\angle CAD &= \frac{\langle A-D, A-C\rangle}{\|A-D\|\,\|A-C\|} \tag{11}\\
&= \frac{a^2-ab\cos\theta}{a\sqrt{a^2-2ab\cos\theta+b^2}} \tag{12}\\
&= \frac{24}{6\sqrt{41}} \tag{13}\\
So, \angle ACB = \angle CAD. \tag{14}
\end{align}$$

And $CA$ is common side .

So,$\triangle ABC \cong \triangle CDA. (by A - A - S)\,(proved)$