# EMBEDDED

**Question :** $l$ and $m$ are two parallel lines intersected by another pair of parallel lines $p$ and $q(figure\ 1)$,show that $\triangle ABC \cong \triangle CDA$.

**Figure :**

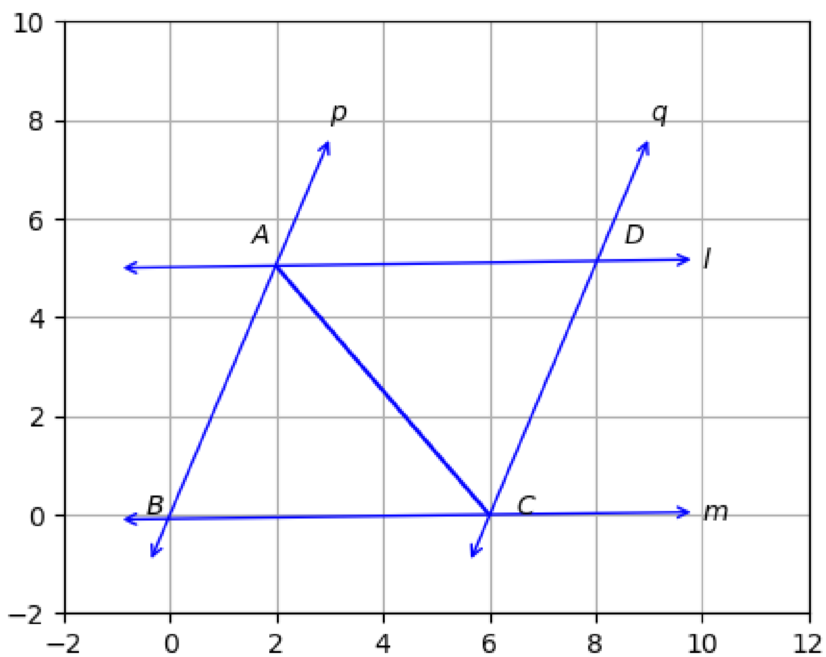


Figure 1: Required parallelogram

**Solution :**

| Symbol | Description | Value |
|---|---|---|
| $\mathbf{B}$ | Vertex at origin | $\mathbf{0}$ |
| $a$ | Side of the parallelogram,$BC = DA$ | 6 |
| $b$ | Side of the parallelogram,$AB = CD$ | $\sqrt{29}$ |
| $\theta$ | Angle of the parallelogram, $\angle ABC$ | $\sin^{-1}\left(\frac{5}{\sqrt{29}}\right)$ |

Table 1: Table of input parameters

| Symbol | Description | Value |
|---|---|---|
| $\mathbf{C}$ | Vertex of parallelogram | $a\mathbf{e_1}$ |
| $\mathbf{A}$ | Vertex of parallelogram | $b\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$ |
| $\mathbf{D}$ | Vertex of parallelogram | $\mathbf{C}+\mathbf{A}$ |

Table 2: Table of output parameters

From $figure$ 1 between $\triangle ABC$ and $\triangle CDA$

$$\begin{aligned}
\cos \angle BAC &= \frac{\langle \mathbf{B}-\mathbf{A}, \mathbf{A}-\mathbf{C} \rangle}{\|\mathbf{B}-\mathbf{A}\| \, \|\mathbf{A}-\mathbf{C}\|} && (1)\\
&= \frac{(\mathbf{B}-\mathbf{A})^T (\mathbf{A}-\mathbf{C})}{\|\mathbf{B}-\mathbf{A}\| \, \|\mathbf{A}-\mathbf{C}\|} && (2)\\
&= \frac{ab\cos\theta + b\sin\theta}{b\sqrt{a^2-2ab\cos\theta+b^2}} && (3)\\
&= \frac{17}{\sqrt{29}\sqrt{41}} && (4)\\
\cos \angle ACD &= \frac{\langle \mathbf{D}-\mathbf{C}, \mathbf{A}-\mathbf{C} \rangle}{\|\mathbf{D}-\mathbf{C}\| \, \|\mathbf{A}-\mathbf{C}\|} && (5)\\
&= \frac{(\mathbf{D}-\mathbf{C})^T (\mathbf{A}-\mathbf{C})}{\|\mathbf{D}-\mathbf{C}\| \, \|\mathbf{A}-\mathbf{C}\|} && (6)\\
&= \frac{b^2 - ab\cos\theta}{b\sqrt{a^2-2ab\cos\theta+b^2}} && (7)\\
&= \frac{17}{\sqrt{29}\sqrt{41}} && (8)\\
So, \angle BAC = \angle ACD. & && (9)\\
\cos \angle ACB &= \frac{\langle \mathbf{B}-\mathbf{C}, \mathbf{A}-\mathbf{C} \rangle}{\|\mathbf{B}-\mathbf{C}\| \, \|\mathbf{A}-\mathbf{C}\|} && (10)\\
&= \frac{(\mathbf{B}-\mathbf{C})^T (\mathbf{A}-\mathbf{C})}{\|\mathbf{B}-\mathbf{C}\| \, \|\mathbf{A}-\mathbf{C}\|} && (11)\\
&= \frac{a^2 - ab\cos\theta}{a\sqrt{a^2-2ab\cos\theta+b^2}} && (12)\\
&= \frac{24}{6\sqrt{41}} && (13)\\
\cos \angle CAD &= \frac{\langle \mathbf{A}-\mathbf{D}, \mathbf{A}-\mathbf{C} \rangle}{\|\mathbf{A}-\mathbf{D}\| \, \|\mathbf{A}-\mathbf{C}\|} && (14)\\
&= \frac{(\mathbf{A}-\mathbf{D})^T (\mathbf{A}-\mathbf{C})}{\|\mathbf{A}-\mathbf{D}\| \, \|\mathbf{A}-\mathbf{C}\|} && (15)\\
&= \frac{a^2 - ab\cos\theta}{a\sqrt{a^2-2ab\cos\theta+b^2}} && (16)\\
&= \frac{24}{6\sqrt{41}} && (17)\\
So, \angle ACB = \angle CAD. & && (18)
\end{aligned}$$

And $CA$ is common side .

So,$\triangle ABC \cong \triangle CDA. \, (by A - A - S) \, (proved)$