

9.10.5.3

Question : In figure 1 or,figure 2 $\angle PQR = 100^\circ$, where P, Q and R are points on a circle with centre O . Find $\angle OPR$.

Solution :

Input Parameters	Description	Value
O	Center(at origin)	0
r	Radius	1
θ	$\angle PQR$	100°
θ_1	$\angle NOQ$	θ_1°
θ_2	$\angle NOP$	165°
θ_3	$\angle NOR$	5°

Table 1: Table of input parameters

Output Parameters	Description	Value
Q	Point	$\begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 \end{pmatrix}$
P	Point	$\begin{pmatrix} \cos \theta_2 \\ \sin \theta_2 \end{pmatrix}$
R	Point	$\begin{pmatrix} \cos \theta_3 \\ \sin \theta_3 \end{pmatrix}$

Table 2: Table of output parameters

For getting the value of the $\angle NOQ$

$$\cos \theta = \frac{(\mathbf{R} - \mathbf{Q})^\top (\mathbf{P} - \mathbf{Q})}{\|\mathbf{R} - \mathbf{Q}\| \|\mathbf{P} - \mathbf{Q}\|} \quad (1)$$

$$\text{or, } \cos \theta = \frac{\sin \frac{\theta_1 + \theta_2}{2} \cos \frac{\theta_2 + \theta_3}{2}}{\sin \frac{\theta_2 - \theta_1}{2}} \quad (2)$$

$$\text{so, } \theta_1 = 2 \tan^{-1} \left(\tan \left(\frac{\theta_2}{2} \right) \left(\frac{\cos \theta + \cos \frac{\theta_2 + \theta_3}{2}}{\cos \theta - \cos \frac{\theta_2 + \theta_3}{2}} \right) \right) \quad (3)$$

$$\implies \theta_1 = 136.696^\circ \quad (4)$$

$$\text{or, } \theta_1 = 2 \tan^{-1} \left(\tan \left(\frac{\theta_2}{2} \right) \left(\frac{\cos \theta - \cos \frac{\theta_2 + \theta_3}{2}}{\cos \theta + \cos \frac{\theta_2 + \theta_3}{2}} \right) \right) \quad (5)$$

$$\implies \theta_1 = 175^\circ \quad (6)$$

$$(7)$$

For getting the value of the $\angle OPR$

$$\angle OQP + \angle OQR = \theta \quad (8)$$

$$\angle QOR = 180 - 2\angle OQR, (OQ = OR) \quad (9)$$

$$\angle QOP = 180 - 2\angle OQP, (OQ = OP) \quad (10)$$

$$\text{or, } \angle POR = 360 - (\angle QOP + \angle QOR) \quad (11)$$

$$\implies \angle POR = 2(\angle OQP + \angle OQR) \quad (12)$$

$$= 2\theta \quad (13)$$

$$\angle POR + \angle ORP + \angle OPR = 180^\circ \quad (14)$$

$$\angle POR + 2\angle OPR = 180^\circ, (OR = OP) \quad (15)$$

$$\angle OPR = \frac{180^\circ - 2\theta}{2} \quad (16)$$

$$\angle OPR = -10^\circ \quad (17)$$

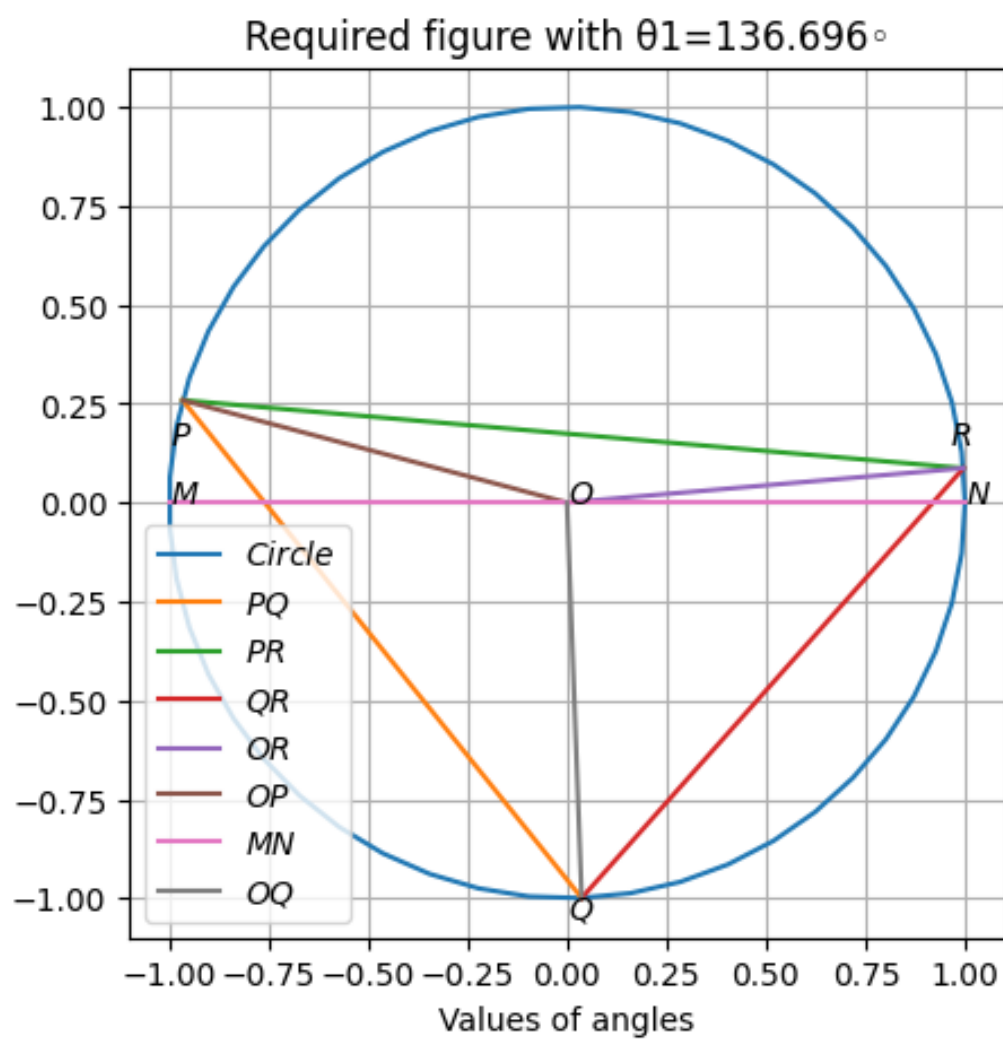


Figure 1:

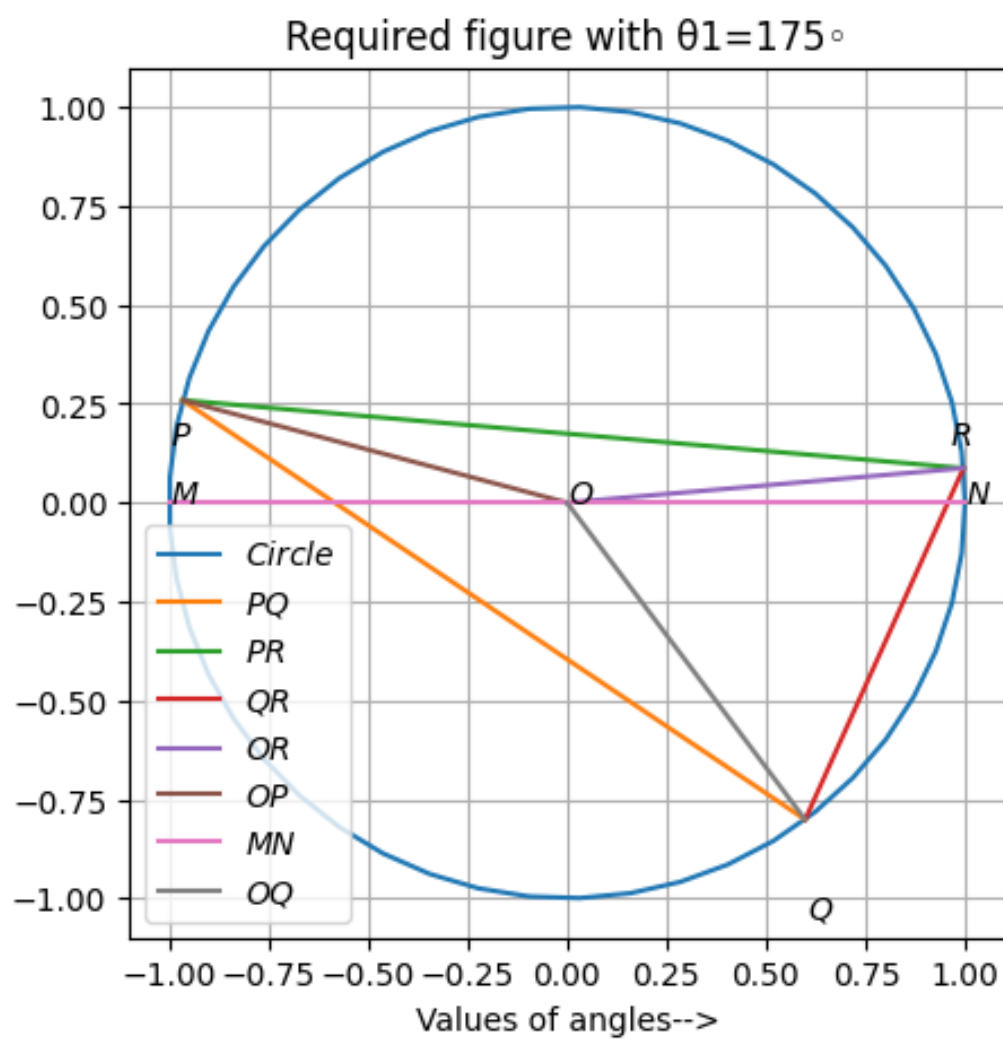


Figure 2: