MATLAB ASSIGNMENT 1

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1. MATLAB code to solve a nonlinear equation $f(x) = 0$ using the following methods and test it for $f(x) = sin(x) + x^2 - 1$ take the interval $[0, 1]$.

   (a) **Bisection Method**
   Code:

```
%Bisection method
close all;
close all;

upper_bound="Let's choose an upper bound--(a1)--"
a1=input(upper_bound)
lower_bound="Let's choose an lower bound--(b1)--"
b1=input(lower_bound)
fun1=@(x) sin(x)+x^2-1;
c=(a1+b1)/2;
i=0;
data=[i a1 b1 c fun1(c)];
while (abs(fun1(c)) > 10^(-4))
if fun1(a1)*fun1(c)<0
    b1=c;
else
    a1=c;
end
i=i+1;
c=(a1+b1)/2;
data=[i a1 b1 c fun1(c)]

end
```

   Result: data=[7.0000 0.6406 0.6328 0.6367 -0.0000]

   (b) **Newton-Raphson Method**
   Code:

```
%Newton-Raphson Method
close all;
close all;
```

```
nearest_point="Let's choose a nearest point--(x0)--"
x0=input(nearest_point)
fun1 =@(x) sin(x)+x^2-1.0;
fun1(x0)
syms f(x)
f(x) = sin(x)+x^2;
Df = diff(f,x);
D=double(Df(x0))
i=0;
x=x0-(fun1(x0)/D)
abs(fun1(x))
while (abs(fun1(x))>10^(-8))
i=i+1;
x=x-(fun1(x)/D);
data=[i x fun1(x)]
end
```

Result: data=[10.0000 0.6367 0.0000]

(c) **Secant Methods**

Code:

```
%Secant Method
close all;
close all;
a1="Let's choose a nearest point--(x0)--"
x0=input(a1)
a2="Let's choose a nearest point--(x1)--"
x1=input(a2)
fun1 =@(x) sin(x)+x^2-1.0;
i=0;
x=x1-(((x0-x1)/(fun1(x0)-fun1(x1)))*fun1(x1));
while (abs(fun1(x)) > 10^(-4))
if fun1(x0)*fun1(x1)<0
    x0=x;
else
    x1=x;
end
i=i+1;
x=x1-((x0-x1)/(fun1(x0)-fun1(x1)))*fun1(x1)
data=[i x0 x1 x fun1(x)]

end
```

Result: data=[4.0000 0.6366 1.0000 0.6367 -0.0000]

(d) **Regula-Falsi Method**

Code:

```
%Regula-falsi method
close all;
close all;
upper_bound="Let's choose an upper bound--(a1)--"
a1=input(upper_bound)
lower_bound="Let's choose an lower bound--(b1)--"
b1=input(lower_bound)
fun1=@(x) sin(x)+x^2-1;
x=a1-(fun1(a1)*((b1-a1)/(fun1(b1)-fun1(a1))))
i=0;
data=[i a1 b1 x fun1(x)];
while (abs(fun1(x)) > 10^(-4))
if fun1(a1)*fun1(x)<0
    b1=x;
else
    a1=x;
end
i=i+1;
x=a1-(fun1(a1)*((b1-a1)/(fun1(b1)-fun1(a1))))
data=[i a1 b1 x fun1(x)]

end
```

Result: data=[4.0000 1.0000 0.6366 0.6367 -0.0000]

(e) **Fixed-point Iteration Method**

Code:

```
%Fixed-point Iteration Method
close all;
close all;
upper_bound="Let's choose any number--(x0)--"
x0=input(upper_bound)
fun1=@(x) sin(x)+x^2-1.0;
g=@(x) sqrt(1-sin(x));
i=0;
fun1(x0)
x1=g(x0);
data=[i x1 fun1(x1)]
while (abs(fun1(x0))>10^(-4))
x0=g(x0)
x1=g(x0)
i=i+1;
data=[i   x1    fun1(x1)]
end
```

Result: data=[20.0000 0.6367 -0.0000]

2. Apply Newton-Raphson method to approximate the root of equation $f(x) = x^3 - x - 3 = 0$ with initial guess $x0 = 0$.Show that the sequence diverges. Further,perform the Newton the Newton-Raphson method with initial guess sufficiently close to the root $r \approx 1.6717$ and discuss the convergence in this case.

   (a) By Choosing nearest point $x0 = 0$
   Code:

```
%Newton-Raphson Method
close all;
close all;

nearest_point="Let's choose a nearest point--(x0)--"
x0=input(nearest_point)
fun1 =@(x) x^3-x-3;
fun1(x0)
syms f(x)
f(x) = sin(x)+x^2;
Df = diff(f,x);
D=double(Df(x0))
i=0;
x=x0-(fun1(x0)/D)
abs(fun1(x))
while (abs(fun1(x))>10^(-8))
i=i+1;
x=x-(fun1(x)/D);
data=[i x fun1(x)]
end
```

   Result: data=[7 -Inf NaN]

   (b) By choosing the point $x0 = 2$ as the root is given $r \approx 1.6717$ Code

```
%Newton-Raphson Method
close all;
close all;

nearest_point="Let's choose a nearest point--(x0)--"
x0=input(nearest_point)
fun1 =@(x) x^3-x-3;
fun1(x0)
syms f(x)
f(x) = sin(x)+x^2;
Df = diff(f,x);
D=double(Df(x0))
i=0;
x=x0-(fun1(x0)/D)
```

```
abs(fun1(x))
while (abs(fun1(x))>10^(-8))
i=i+1;
x=x-(fun1(x)/D);
data=[i x fun1(x)]
end
```

Result: data=[1.0e+05*1.9751 0.0000 0.0000]
The sequence $f(x) = x^3 - x - 3$ itself is a diverging sequence.

3. Consider the equation $x^2 - 6x + 5 = 0$.

   (a) Taking $x0 = 0$ and $x1 = 4.5$, generate first 7 terms of the iterative sequence of the secant method.

   (b) Take the initial interval as $[a0, b0] = [0, 4.5]$, generate the first 7 terms of the iterative sequence of the regula-falsi method. Observe to which roots of the given equation does the above two sequences converge?

   (a) Using Sccant Method
   Code:

```
%Secant Method
close all;
close all;
a1="Let's choose a nearest point--(x0)--"
x0=input(a1)
a2="Let's choose a nearest point--(x1)--"
x1=input(a2)
fun1 =@(x) x^2-6*x+5;
i=0;
x=x1-(((x0-x1)/(fun1(x0)-fun1(x1)))*fun1(x1));
while (abs(fun1(x)) > 10^(-4))
if fun1(x0)*fun1(x1)<0
    x0=x;
else
    x1=x;
end
i=i+1;
x=x1-((x0-x1)/(fun1(x0)-fun1(x1)))*fun1(x1)
data=[i x0 x1 x fun1(x)]

end
```

   Result:data=[7.0000 5.0000 5.4545 5.0000 -0.0000]
   Root converges to 5.0000.

   (b) Using Regula-Falsi Method
   Code:

```
%Regula-falsi method
close all;
close all;
upper_bound="Let's choose an upper bound--(a1)--"
a1=input(upper_bound)
lower_bound="Let's choose an lower bound--(b1)--"
b1=input(lower_bound)
fun1 =@(x) x^2-6*x+5;
x=a1-(fun1(a1)*((b1-a1)/(fun1(b1)-fun1(a1))))
i=0;
data=[i a1 b1 x fun1(x)];
while (abs(fun1(x)) > 10^(-4))
if fun1(a1)*fun1(x)<0
    b1=x;
else
    a1=x;
end
i=i+1;
x=a1-(fun1(a1)*((b1-a1)/(fun1(b1)-fun1(a1))))
data=[i a1 b1 x fun1(x)]

end
```

Result: data=[7.0000 1.0004 0 1.0001 -0.0003]
Root converges to 1.0001 .

4. Write a MATLAB code to solve a nonlinear equation f(x) = 0 using the Fixed-point iteration Method Consider the equation $f(x) = sin(x)+x^2-1$. Take the initial interval as [0,1]. There are three possible choices for the iteration functions namely

   (a) $g_1(x) = \sin^{-1}(1 - x^2)$,
   (b) $g_2(x) = -\sqrt{1 - sin(x)}$,
   (c) $g_3(x) = \sqrt{1 - sin(x)}$.

   Discuss the convergence or divergence of all the iterative sequences. Can you Justify theoretically?

   (a) Using $g_1(x) = \sin^{-1}(1 - x^2)$

```
% Fixed-point Iteration Method
close all;
close all;
upper_bound="Let's choose any number--(x0)--"
x0=input(upper_bound)
fun1=@(x) sin(x)+x^2-1.0;
g=@(x) asin(1-x^2)
```

```
i=0;
fun1(x0)
x1=g(x0);
data=[i x1 fun1(x1)]
while (abs(fun1(x0))>10^(-4))
x0=g(x0)
x1=x0
i=i+1;
data=[i   x1    fun1(x1)]
end
```

Result: data=[1.0e+04 * 4.0239 + 0.0000i 0.0001 + 0.0003i 0.0000 + 0.0013i x0 = Operation terminated by user during untitled2]
Not convergent.

Theoretical proof:

g_1 is not contraction map for $x \in [0, 1]$.

$$g'(x) = \frac{-2x}{\sqrt{1-(1-x^2)^2}} \tag{1}$$

$$= \frac{-2}{\sqrt{2-x^2}} \tag{2}$$

$$\lambda = \max_{0 \le x \le 1} |g'(x)| \not< 1 \tag{3}$$

(b) Using $g_2(x) = -\sqrt{1 - sin(x)}$

```
% Fixed-point Iteration Method
close all;
close all;
upper_bound="Let's choose any number--(x0)--"
x0=input(upper_bound)
fun1=@(x) sin(x)+x^2-1.0;
g=@(x) -sqrt(1-sin(x))
i=0;
fun1(x0)
x1=g(x0);
data=[i x1 fun1(x1)]
while (abs(fun1(x0))>10^(-4))
x0=g(x0)
x1=x0
i=i+1;
data=[i   x1    fun1(x1)]
end
```

Result: data=[6.0000 -1.4096 -0.0000]
Not Convergent.

Theoretical proof:
g_2 is not self map of $[0,1]$ to itself.

$$g_2(x) = -\sqrt{1 - sin(x)} \tag{4}$$
$$g_2'(x) = \frac{\sqrt{1+\sin x}}{2} \tag{5}$$
$$\tag{6}$$

(c) Using $g_3(x) = \sqrt{1 - sin(x)}$

```
% Fixed-point Iteration Method
close all;
close all;
upper_bound="Let's choose any number--(x0)--"
x0=input(upper_bound)
fun1=@(x) sin(x)+x^2-1.0;
g=@(x) sqrt(1-sin(x))
i=0;
fun1(x0)
x1=g(x0);
data=[i x1 fun1(x1)]
while (abs(fun1(x0))>10^(-4))
x0=g(x0)
x1=x0
i=i+1;
data=[i   x1    fun1(x1)]
end
```

Result: data=[20.0000 0.6368 0.0001]
Convergent.
Theoretical proof:
g_3 is converging to the root.

$$g_3(x) = \sqrt{1 - \sin x} \tag{7}$$
$$g_3'(x) = -\frac{\sqrt{1+\sin x}}{2} \tag{8}$$
$$|g_3'(x)| \leq \frac{1}{\sqrt{2}} < 1 \tag{9}$$