9.10.4.3

Question : If two equal chords of a circle intersect within the circle,prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Solution :

Input Parameters	**Description**	**Value**
$\mathbf{O}$	Center(at origin)	$\mathbf{0}$
d	Length of the chords	2
r	Radius	1
θ_1	$\angle ROP$	θ_1°
θ_2	$\angle ROQ$	-48°
θ_3	$\angle ROR$	0°
θ_4	$\angle ROS$	-132°

Table 1: Table of input parameters

Output Parameters	**Description**	**Value**
$\mathbf{P}$	Point	$\begin{pmatrix}\cos\theta_1\\ \sin\theta_1\end{pmatrix}$
$\mathbf{Q}$	Point	$\begin{pmatrix}\cos\theta_2\\ \sin\theta_2\end{pmatrix}$
$\mathbf{R}$	Point	$\begin{pmatrix}\cos\theta_3\\ \sin\theta_3\end{pmatrix}$
$\mathbf{S}$	Point	$\begin{pmatrix}\cos\theta_4\\ \sin\theta_4\end{pmatrix}$

Table 2: Table of output parameters

$$
\begin{aligned}
\cos\angle RQS &= \frac{(\mathbf{R}-\mathbf{Q})^\top(\mathbf{Q}-\mathbf{S})}{||\mathbf{R}-\mathbf{Q}||||\mathbf{Q}-\mathbf{S}||} && (1)\\
&= \cos\frac{\theta_3-\theta_4}{2} && (2)\\
\cos\angle PSQ &= \frac{(\mathbf{P}-\mathbf{S})^\top(\mathbf{S}-\mathbf{Q})}{||\mathbf{P}-\mathbf{S}||||\mathbf{S}-\mathbf{Q}||} && (3)\\
&= \cos\frac{\theta_1-\theta_2}{2} && (4)\\
\cos\angle RQS &= \cos\angle PSQ && (5)\\
or, \theta_1 &= \theta_2-\theta_3+\theta_4 && (6)\\
&= -180^\circ && (7)
\end{aligned}
$$

The equation of PQ and RS is obtained by

$$
\begin{aligned}
\mathbf{n_1}^\top(\mathbf{x}-\mathbf{P}) &= 0 && (8)\\
\mathbf{n_2}^\top(\mathbf{x}-\mathbf{R}) &= 0 && (9)\\
or, \mathbf{n_1} &= \begin{pmatrix}\sin\theta_2-\sin\theta_1\\ \cos\theta_1-\cos\theta_2\end{pmatrix} && (10)\\
or, \mathbf{n_2} &= \begin{pmatrix}\sin\theta_4-\sin\theta_3\\ \cos\theta_3-\cos\theta_4\end{pmatrix} && (11)\\
& && (12)
\end{aligned}
$$

The value of the point of the intersection is

$$
\begin{aligned}
\begin{pmatrix}\mathbf{n_1}^\top\\ \mathbf{n_2}^\top\end{pmatrix}\mathbf{x} &= \begin{pmatrix}\mathbf{n_1}^\top\mathbf{P}\\ \mathbf{n_2}^\top\mathbf{R}\end{pmatrix} && (13)\\
\mathbf{x} &= \begin{pmatrix}\mathbf{n_1}^\top\\ \mathbf{n_2}^\top\end{pmatrix}^{-1}\begin{pmatrix}\mathbf{n_1}^\top\mathbf{P}\\ \mathbf{n_2}^\top\mathbf{R}\end{pmatrix} && (14)\\
or, \mathbf{T} &= \begin{pmatrix}0\\ -0.45\end{pmatrix} && (15)\\
\cos\angle OTP &= \frac{(\mathbf{T}-\mathbf{P})^\top(\mathbf{O}-\mathbf{T})}{||\mathbf{T}-\mathbf{P}||||\mathbf{T}-\mathbf{O}||} && (16)\\
or, \angle OTP &= 66^\circ && (17)
\end{aligned}
$$

Similarly,

$$\cos \angle OTR = \frac{(\mathbf{T}-\mathbf{R})^{\top}(\mathbf{T}-\mathbf{O})}{||\mathbf{T}-\mathbf{R}||||\mathbf{T}-\mathbf{O}||} \tag{18}$$

$$or, \angle OTR = 66^{\circ} \tag{19}$$

$$So, \angle OTP = \angle OTR\,(proved) \tag{20}$$

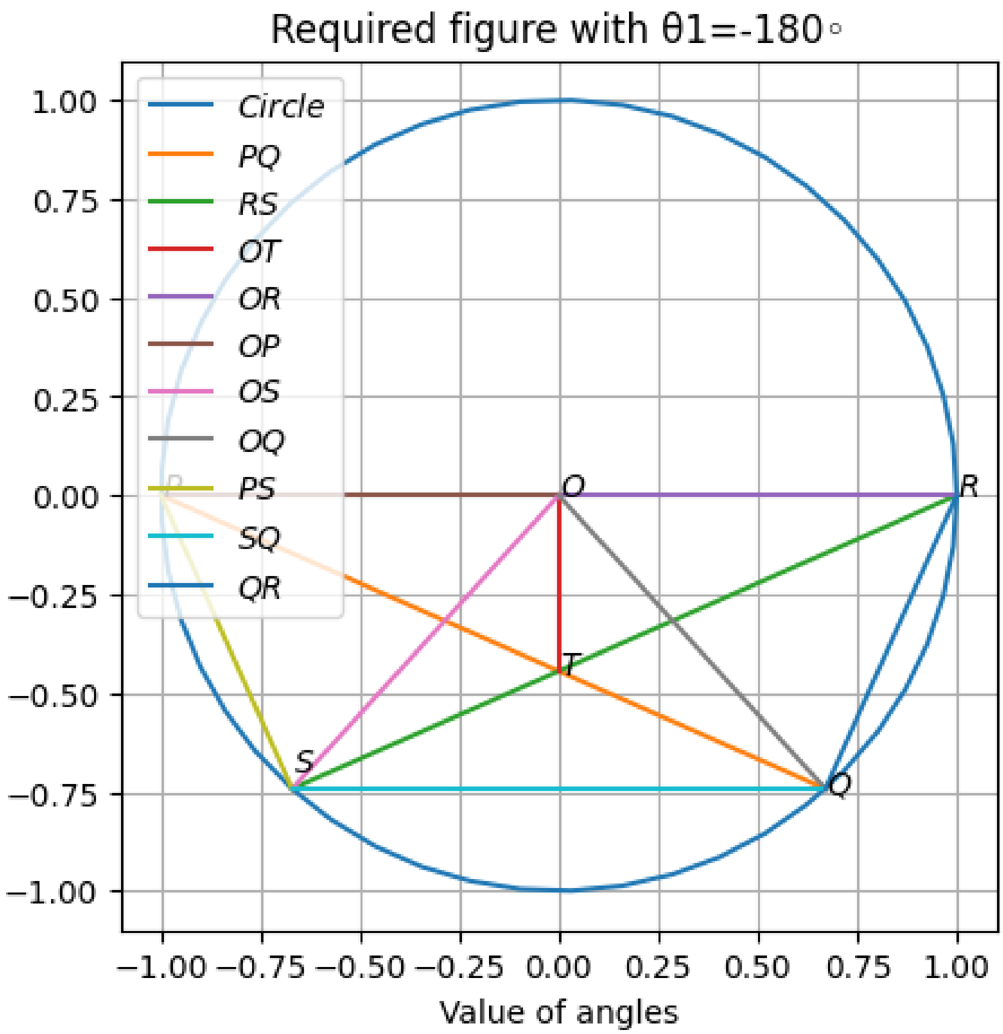

Figure 1: