

ASSIGNMENT 2

Question : Two lines passing through the point(2,3) intersect each other at an angle of 60° . If slope of one line is 2, find equation of the other line.

Solution : The direction vector of one line is, $\mathbf{m}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

The direction vector of another line is, $\mathbf{m}_2 = \begin{pmatrix} 1 \\ m \end{pmatrix}$

So,

$$\cos 60^\circ = \frac{\mathbf{m}_1 \mathbf{m}_2}{\|\mathbf{m}_1\| \|\mathbf{m}_2\|} \quad (1)$$

$$\text{or, } \frac{1}{2} = \frac{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix}}{\sqrt{5} \sqrt{m^2 + 1}} \quad (2)$$

$$\text{or, } \frac{1}{2} = \frac{1 + 2m}{\sqrt{5m^2 + 5}} \quad (3)$$

$$\text{or, } m = \frac{-8 \pm 5\sqrt{3}}{11} \quad (4)$$

Therefore, the direction vector is, $\mathbf{m}_2 = \begin{pmatrix} 1 \\ \frac{-8+5\sqrt{3}}{11} \end{pmatrix}$ or, $\begin{pmatrix} 1 \\ \frac{-8-5\sqrt{3}}{11} \end{pmatrix}$

The normal vector is, $\mathbf{n} = \begin{pmatrix} \frac{-8+5\sqrt{3}}{11} \\ -1 \end{pmatrix}$ or, $\begin{pmatrix} \frac{-8-5\sqrt{3}}{11} \\ -1 \end{pmatrix}$

So, the equation of the line is

$$\mathbf{n}^T \mathbf{x} = c \quad (5)$$

$$\begin{pmatrix} \frac{-8 \pm 5\sqrt{3}}{11} & -1 \end{pmatrix} \mathbf{x} = c \quad (6)$$

Passes through the point $\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$$\begin{pmatrix} \frac{-8 \pm 5\sqrt{3}}{11} & -1 \end{pmatrix} \mathbf{P} = c \quad (7)$$

$$\text{or, } c = \frac{-49 \pm 16\sqrt{3}}{11} \quad (8)$$

So, the equation of the line is

$$y = \frac{-8 + 5\sqrt{3}}{11}x + \frac{49 - 16\sqrt{3}}{11} \quad (9)$$

$$\text{or, } y = \frac{-8 - 5\sqrt{3}}{11}x + \frac{49 + 16\sqrt{3}}{11} \quad (10)$$

Figure :

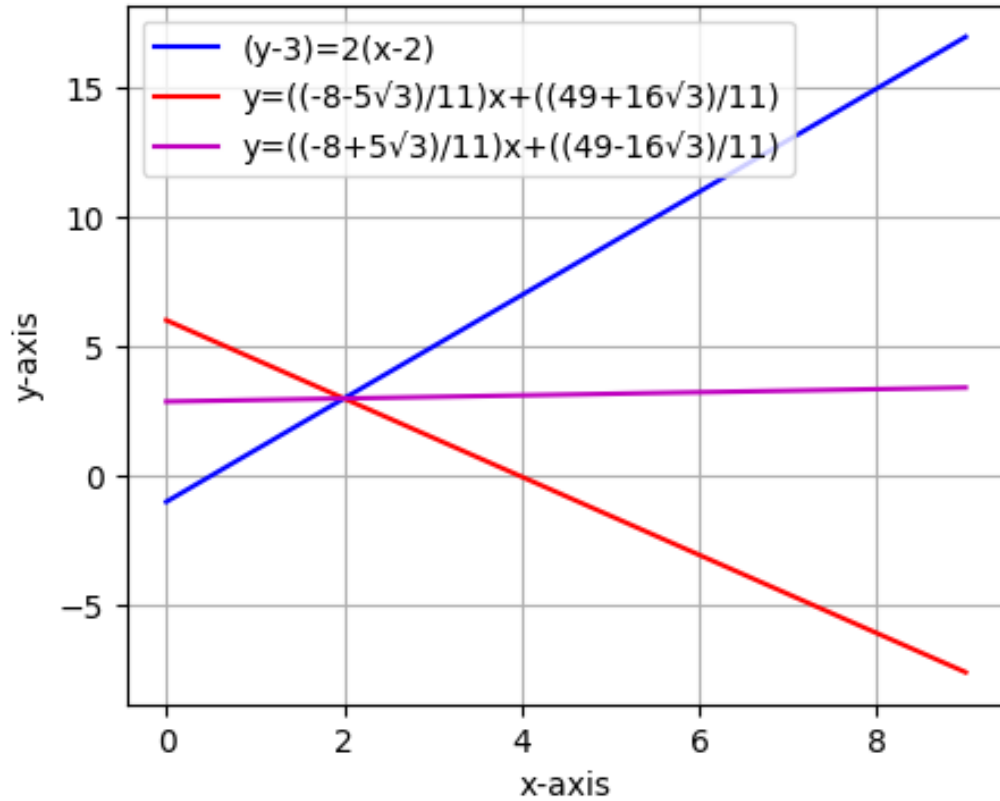


Figure 1: Required Figure