VECTOR

Question: **P** and **Q** are any two points lying on the sides DC and AD respectively of a parallelogram $ABCD$.Show that, $ar\left(\triangle APB\right) = ar\left(\triangle BQC\right)$.

Figure:

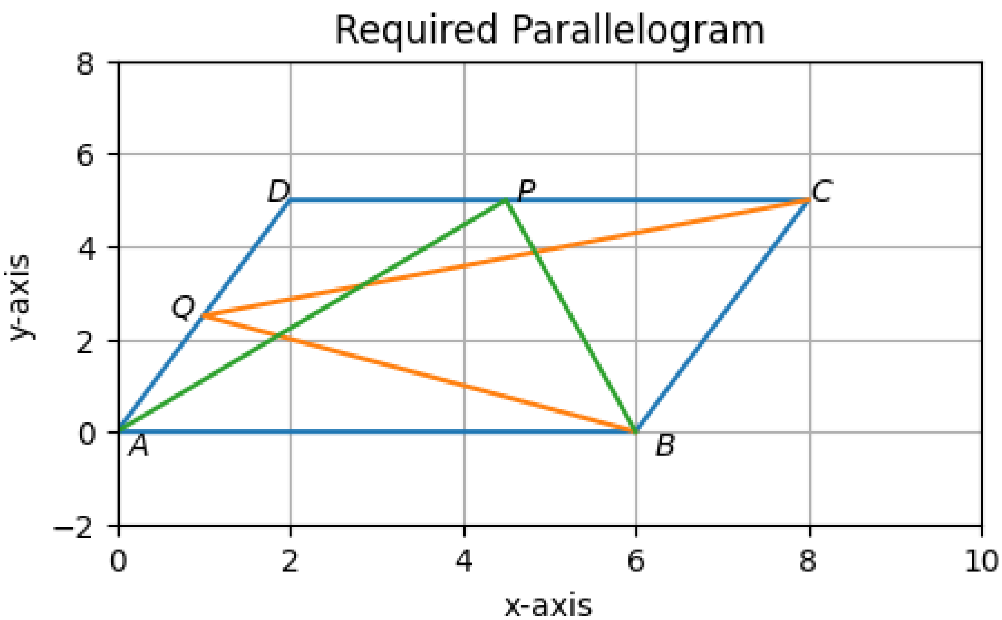

Figure 1

Solution:

Input Parameters	Description	Value
$\mathbf{A}$	Vertex(at origin)	$\mathbf{0}$
a	Side of the parallelogram,$AB = DC$	6
b	Side of the parallelogram,$AD = BC$	$\sqrt{29}$
θ	Angle of parallelogram,$\angle BAD$	$\sin^{-1}\left(\frac{5}{\sqrt{29}}\right)$
k_1	$AQ : QD$	k_1
k_2	$DP : PC$	k_2

Table 1: Table of input parameters

Output Parameters	Description	Value
$\mathbf{B}$	Vertex of parallelogram	$a\mathbf{e_1}$
$\mathbf{D}$	Vertex of parallelogram	$b\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$
$\mathbf{C}$	Vertex of parallelogram	$\mathbf{B}+\mathbf{D}$
$\mathbf{Q}$	Vertex of $\triangle BQC$	$\frac{k_1\mathbf{D}+\mathbf{A}}{k_1+1}$
$\mathbf{P}$	Vertex of $\triangle APB$	$\frac{k_2\mathbf{C}+\mathbf{D}}{k_2+1}$

Table 2: Table of output parameters

For the $\triangle BQC$, the vertices of the triangle are taken from table 1 and table 2.

$$
\begin{aligned}
\implies ar\,(\triangle BQC) &= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{Q} & \mathbf{C} \end{vmatrix} && (1)\\
&= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 6 & \frac{2k_1}{k_1+1} & 8 \\ 0 & \frac{5k_1}{k_1+1} & 5 \end{vmatrix} && (2)\\
\xrightarrow{C_2'=C_2-C_1,C_3'=C_3-C_1} & \frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 6 & \frac{-4k_1-6}{k_1+1} & 2 \\ 0 & \frac{5k_1}{k_1+1} & 5 \end{vmatrix} && (3)\\
&= \frac{1}{2}\left(1\begin{vmatrix} \frac{-4k_1-6}{k_1+1} & 2 \\ \frac{5k_1}{k_1+1} & 5 \end{vmatrix} + 0 + 0\right) && (4)\\
&= \frac{1}{2} \times 30 && (5)\\
&= 15 && (6)
\end{aligned}
$$

For the $\triangle APB$, the vertices of the triangle are taken from table 1 and table 2.

$$
\begin{aligned}
\implies ar\,(\triangle APB) &= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{P} & \mathbf{B} \end{vmatrix} && (7)\\
&= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 0 & \frac{8k_2+2}{k_2+1} & 6 \\ 0 & 5 & 0 \end{vmatrix} && (8)\\
&= \frac{1}{2} \times 30 && (9)\\
&= 15 && (10)
\end{aligned}
$$

(6) = (10)

So, ar($\triangle BQC$) = ar($\triangle APB$).(*proved*)