VECTOR

Question : Construct a triangle APB in which $a, \angle B = \theta$ and $b + c = k$ is given.

Solution :

From cosine rule,

$$b^2 = a^2 + c^2 - 2ac\cos\angle B \tag{1}$$
$$or, b^2 = a^2 + c^2 - 2ac\cos\theta \tag{2}$$
$$or, b^2 - c^2 = a^2 - 2ac\cos\theta \tag{3}$$
$$or, (b+c)(b-c) = a^2 - 2ac\cos\theta \tag{4}$$
$$or, k(b-c) = a^2 - 2ac\cos\theta \tag{5}$$
$$or, kb + (-k + 2a\cos\theta)c = a^2 \tag{6}$$
$$b + c = k \tag{7}$$
$$\tag{8}$$

From (6),(7)

$$\left(\begin{array}{cc|c} k & -k+2a\cos\theta & a^2 \\ 1 & 1 & k \end{array}\right) \tag{9}$$
$$\xrightarrow{R_1'=R_1/k} \left(\begin{array}{cc|c} 1 & \frac{-k+2a\cos\theta}{k} & \frac{a^2}{k} \\ 1 & 1 & k \end{array}\right) \tag{10}$$
$$\xrightarrow{R_2'=R_2-R_1} \left(\begin{array}{cc|c} 1 & \frac{-k+2a\cos\theta}{k} & \frac{a^2}{k} \\ 0 & \frac{2k-2a\cos\theta}{k} & \frac{k^2-a^2}{k} \end{array}\right) \tag{11}$$
$$\xrightarrow[R_2''=R_2'\left(\frac{k}{2k-2a\cos\theta}\right)]{R_1''=R_1'-R_2'\left(\frac{-k+2a\cos\theta}{2k-2a\cos\theta}\right)} \left(\begin{array}{cc|c} 1 & 0 & \frac{a^2+k^2-2ak\cos\theta}{2k-2a\cos\theta} \\ 0 & 1 & \frac{k^2-a^2}{2(k-a\cos\theta)} \end{array}\right) \tag{12}$$
$$\begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} \frac{a^2+k^2-2ak\cos\theta}{2k-2a\cos\theta} \\ \frac{k^2-a^2}{2(k-a\cos\theta)} \end{pmatrix} \tag{13}$$

Therefore,

$$\mathbf{A} = c \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{14}$$

$$\mathbf{B} = \mathbf{0} \tag{15}$$

$$\mathbf{C} = ae_1 \tag{16}$$