VECTOR

Question: **P** and **Q** are any two points lying on the sides DC and AD respectively of a parallelogram $ABCD$.Show that, $ar\left(\triangle APB\right) = ar\left(\triangle BQC\right)$.

Figure:

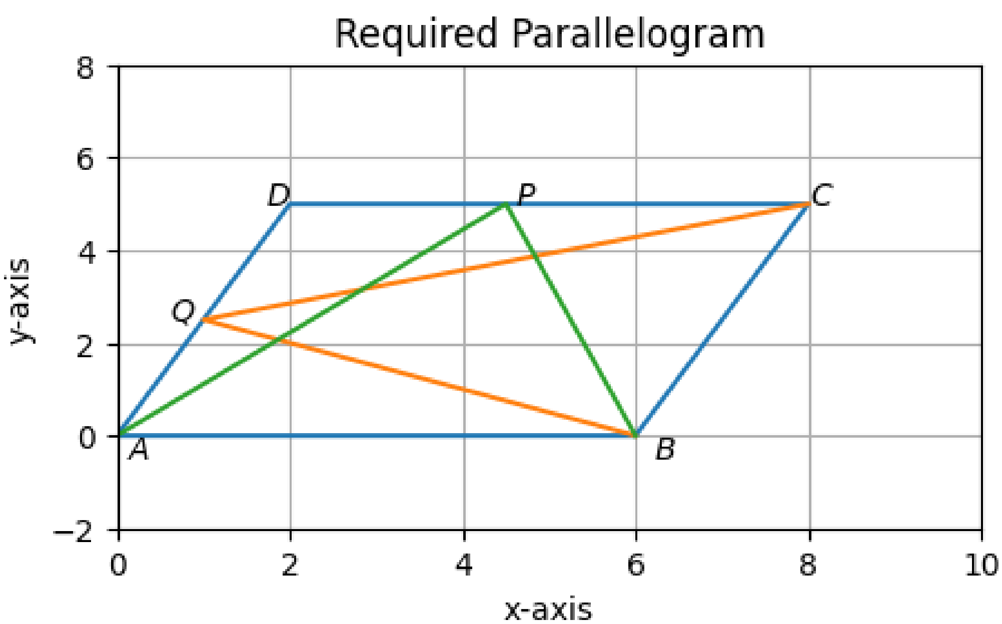

Figure 1

Solution: From figure 1 , considering $\triangle BQC$ and parallelogram $ABCD$, having <u>same base BC</u> and, <u>AD is parallel to BC</u>.

$$\implies \triangle BQC = \frac{1}{2} \times arABCD. \tag{1}$$

Considering $\triangle APB$ and parallelogram $ABCD$, having $\underline{\text{same base } AB}$ and,$\underline{DC \text{ is parallel to } AB}$.

$$\implies \triangle APB = \frac{1}{2} \times ar ABCD. \tag{2}$$

By comparing equation (1) and equation (2) we obtain that,

$$ar\left(\triangle APB\right) = ar\left(\triangle BQC\right). \tag{3}$$

Hence proved.

Proof by the help of diagram :($figure\ 1, table\ 1, table\ 2$)
Let the points **Q** and **P** divide AD and CD by $k_1 : 1$ and $k_2 : 1$ ratio respectively.

Input Parameters	**Description**	**Value**
A	Vertex(at origin)	$\mathbf{0}$
a	Side of the parallelogram,$AB = DC$	6
b	Side of the parallelogram,$AD = BC$	$\sqrt{29}$
θ	Angle of parallelogram,$\angle BAD$	$\sin^{-1}\left(\frac{5}{\sqrt{29}}\right)$
k_1	$AQ : QD$	k_1
k_2	$DP : PC$	k_2

Table 1: Table of input parameters

Output Parameters	**Description**	**Value**
B	Vertex of parallelogram	$a\begin{pmatrix}1\\0\end{pmatrix}$
D	Vertex of parallelogram	$b\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$
C	Vertex of parallelogram	$\mathbf{B}+\mathbf{D}$
Q	Vertex of $\triangle BQC$	$\frac{k_1\mathbf{D}+\mathbf{A}}{k_1+1}$
P	Vertex of $\triangle APB$	$\frac{k_2\mathbf{C}+\mathbf{D}}{k_2+1}$

Table 2: Table of output parameters

For the $\triangle BQC$, the vertices of the triangle are taken from table 1 and table 2.

$$\implies \triangle BQC = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 6 & \frac{2k_1}{k_1+1} & 8 \\ 0 & \frac{5k_1}{k_1+1} & 5 \end{vmatrix} \tag{4}$$

$$\xrightarrow{C_2'=C_2-C_1, C_3'=C_3-C_1} \frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 6 & \frac{-4k_1-6}{k_1+1} & 2 \\ 0 & \frac{5k_1}{k_1+1} & 5 \end{vmatrix} \tag{5}$$

$$= \frac{1}{2} \times 30 \tag{6}$$

$$= 15 \tag{7}$$

For the $\triangle APB$, the vertices of the triangle are taken from table 1 and table 2.

$$\implies \triangle APB = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 0 & \frac{8k_2+2}{k_2+1} & 6 \\ 0 & 5 & 0 \end{vmatrix} \tag{8}$$

$$= \frac{1}{2} \times 30 \tag{9}$$

$$= 15 \tag{10}$$

For parallelogram $ABCD$,the vertices are taken from table 1 nd table 2.

$$\implies arABCD = ab\sin\theta \tag{11}$$

$$= 6\sqrt{29}\sin\left(\sin^{-1}\frac{5}{\sqrt{29}}\right) \tag{12}$$

$$= 6\sqrt{29}\frac{5}{\sqrt{29}} \tag{13}$$

$$= 30 \tag{14}$$

Therefore,$arABCD = 2 \times ar\triangle APB = 2 \times ar\triangle BQC(proved)$.