

9.10.4.3

Question : If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Solution :

Input Parameters	Description	Value
O	Center(at origin)	0
d	Length of the chords	2
r	Radius	1
θ_1	$\angle ROP$	θ_1°
θ_2	$\angle ROQ$	-48°
θ_3	$\angle ROR$	0°
θ_4	$\angle ROS$	-132°

Table 1: Table of input parameters

Output Parameters	Description	Value
P	Point	$\begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 \end{pmatrix}$
Q	Point	$\begin{pmatrix} \cos \theta_2 \\ \sin \theta_2 \end{pmatrix}$
R	Point	$\begin{pmatrix} \cos \theta_3 \\ \sin \theta_3 \end{pmatrix}$
S	Point	$\begin{pmatrix} \cos \theta_4 \\ \sin \theta_4 \end{pmatrix}$

Table 2: Table of output parameters

$$\cos \angle RQS = \frac{(\mathbf{R} - \mathbf{Q})^\top (\mathbf{Q} - \mathbf{S})}{\|\mathbf{R} - \mathbf{Q}\| \|\mathbf{Q} - \mathbf{S}\|} \quad (1)$$

$$= \cos \frac{\theta_3 - \theta_4}{2} \quad (2)$$

$$\cos \angle PSQ = \frac{(\mathbf{P} - \mathbf{S})^\top (\mathbf{S} - \mathbf{Q})}{\|\mathbf{P} - \mathbf{S}\| \|\mathbf{S} - \mathbf{Q}\|} \quad (3)$$

$$= \cos \frac{\theta_1 - \theta_2}{2} \quad (4)$$

$$\cos \angle RQS = \cos \angle PSQ \quad (5)$$

$$or, \theta_1 = \theta_2 - \theta_3 + \theta_4 \quad (6)$$

$$= -180^\circ \quad (7)$$

The equation of PQ and RS is obtained by

$$\mathbf{n}_1^\top (\mathbf{x} - \mathbf{P}) = 0 \quad (8)$$

$$\mathbf{n}_2^\top (\mathbf{x} - \mathbf{R}) = 0 \quad (9)$$

$$or, \mathbf{n}_1 = \begin{pmatrix} \sin \theta_2 - \sin \theta_1 \\ \cos \theta_1 - \cos \theta_2 \end{pmatrix} \quad (10)$$

$$or, \mathbf{n}_2 = \begin{pmatrix} \sin \theta_4 - \sin \theta_3 \\ \cos \theta_3 - \cos \theta_4 \end{pmatrix} \quad (11)$$

$$(12)$$

The value of the point of the intersection is

$$\begin{pmatrix} \mathbf{n}_1^\top \\ \mathbf{n}_2^\top \end{pmatrix} \mathbf{X} = \begin{pmatrix} \mathbf{n}_1^\top \mathbf{P} \\ \mathbf{n}_2^\top \mathbf{R} \end{pmatrix} \quad (13)$$

$$\mathbf{X} = \frac{1}{\det \begin{pmatrix} \cos \theta_3 - \cos \theta_4 & \cos \theta_2 - \cos \theta_1 \\ \sin \theta_3 - \sin \theta_4 & \sin \theta_2 - \sin \theta_1 \end{pmatrix}} \begin{pmatrix} \cos \theta_3 - \cos \theta_4 & \cos \theta_2 - \cos \theta_1 \\ \sin \theta_3 - \sin \theta_4 & \sin \theta_2 - \sin \theta_1 \end{pmatrix} \begin{pmatrix} \sin \theta_2 - \theta_1 \\ \sin \theta_4 - \theta_3 \end{pmatrix} \quad (14)$$

$$or, \mathbf{T} = \begin{pmatrix} 0 \\ -0.45 \end{pmatrix} \quad (15)$$

$$\cos \angle OTP = \frac{(\mathbf{T} - \mathbf{P})^\top (\mathbf{O} - \mathbf{T})}{\|\mathbf{T} - \mathbf{P}\| \|\mathbf{T} - \mathbf{O}\|} \quad (16)$$

$$or, \angle OTP = 66^\circ \quad (17)$$

Similarly,

$$\cos \angle OTR = \frac{(\mathbf{T} - \mathbf{R})^\top (\mathbf{T} - \mathbf{O})}{\|\mathbf{T} - \mathbf{R}\| \|\mathbf{T} - \mathbf{O}\|} \quad (18)$$

$$\text{or, } \angle OTR = 66^\circ \quad (19)$$

$$\text{So, } \angle OTP = \angle OTR (\text{proved}) \quad (20)$$

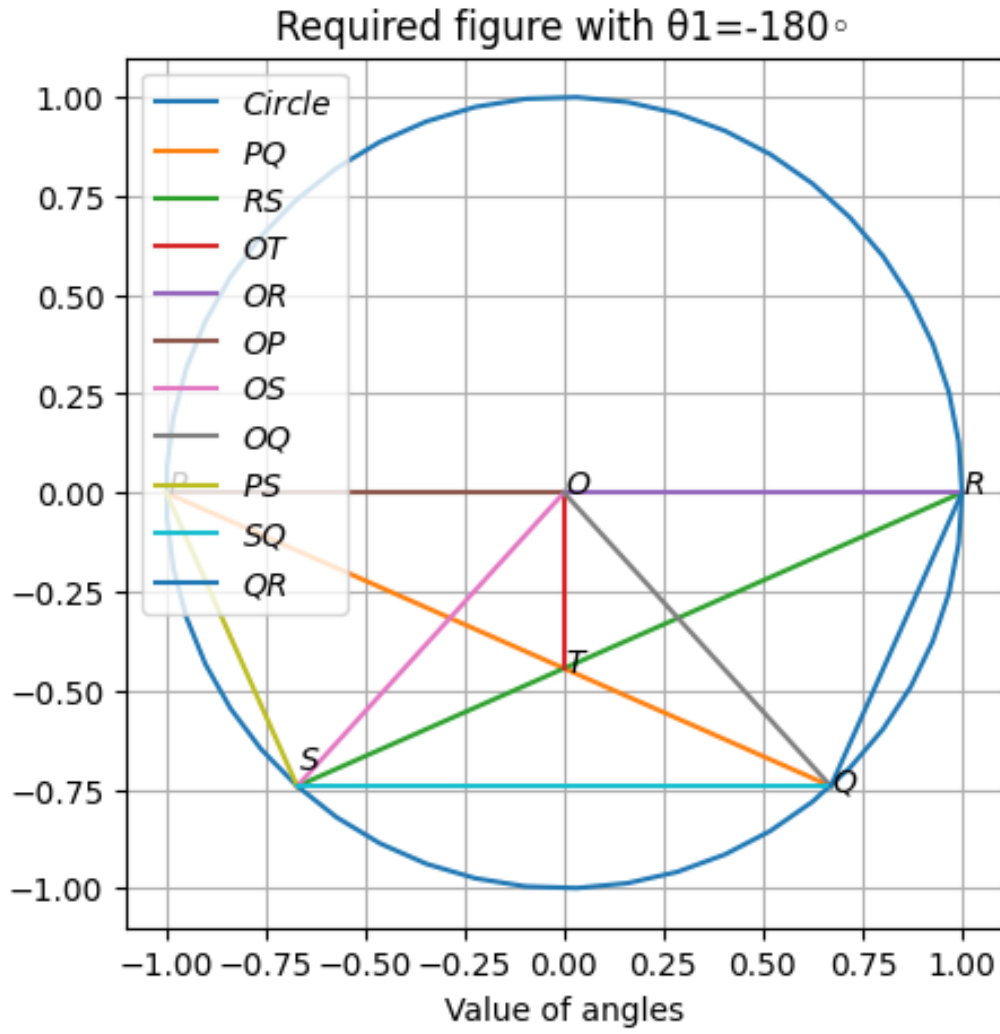


Figure 1: