EMBEDDED

Question : l and m are two parallel lines intersected by another pair of parallel lines p and $q(figure\ 1)$,show that $\triangle ABC \cong \triangle CDA$.

Figure :

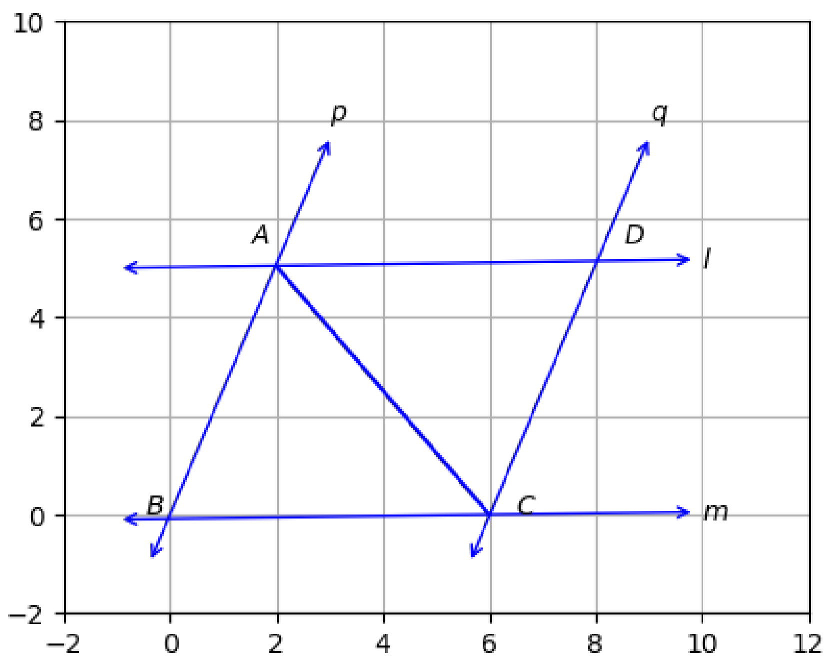

Figure 1: Required parallelogram

Solution :

Symbol	Description	Value
$\mathbf{B}$	Vertex at origin	$\mathbf{0}$
a	Side of the parallelogram,$BC = DA$	6
b	Side of the parallelogram,$AB = CD$	$\sqrt{29}$
θ	Angle of the parallelogram, $\angle ABC$	$\sin^{-1}\left(\frac{5}{\sqrt{29}}\right)$

Table 1: Table of input parameters

Symbol	Description	Value
$\mathbf{C}$	Vertex of parallelogram	$a\mathbf{e_1}$
$\mathbf{A}$	Vertex of parallelogram	$b\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$
$\mathbf{D}$	Vertex of parallelogram	$\mathbf{C}+\mathbf{A}$

Table 2: Table of output parameters

From $figure$ 1 between $\triangle ABC$ and $\triangle CDA$

$$\cos\angle BAC = \frac{AB.CA}{\|AB\|\,\|CA\|} \tag{1}$$

$$= \frac{ab\cos\theta + b\sin\theta}{b\sqrt{a^2 - 2ab\cos\theta + b^2}} \tag{2}$$

$$= \frac{17}{\sqrt{29}\sqrt{41}} \tag{3}$$

$$\cos\angle ACD = \frac{CD.CA}{\|CD\|\,\|CA\|} \tag{4}$$

$$= \frac{b^2 - ab\cos\theta}{b\sqrt{a^2 - 2ab\cos\theta + b^2}} \tag{5}$$

$$= \frac{17}{\sqrt{29}\sqrt{41}} \tag{6}$$

$$So, \angle BAC = \angle ACD. \tag{7}$$

$$\cos\angle ACB = \frac{CB.CA}{\|CB\|\,\|CA\|} \tag{8}$$

$$= \frac{a^2 - ab\cos\theta}{a\sqrt{a^2 - 2ab\cos\theta + b^2}} \tag{9}$$

$$= \frac{24}{6\sqrt{41}} \tag{10}$$

$$\cos\angle CAD = \frac{DA.CA}{\|DA\|\,\|CA\|} \tag{11}$$

$$= \frac{a^2 - ab\cos\theta}{a\sqrt{a^2 - 2ab\cos\theta + b^2}} \tag{12}$$

$$= \frac{24}{6\sqrt{41}} \tag{13}$$

$$So, \angle ACB = \angle CAD. \tag{14}$$

And CA is common side .

So,$\triangle ABC \cong \triangle CDA. \,(by A - A - S)\,(proved)$