9.10.5.3

Question : In figure 1 or,figure 2,$\angle PQR = 100^\circ$,where P, Q and R are points on a circle with centre O.Find $\angle OPR$.

Solution :

Input Parameters	**Description**	**Value**
O	Center(at origin)	**0**
r	Radius	1
θ	$\angle PQR$	100°
θ_1	$\angle NOQ$	θ_1°
θ_2	$\angle NOP$	165°
θ_3	$\angle NOR$	5°

Table 1: Table of input parameters

Output Parameters	**Description**	**Value**
Q	Point	$\begin{pmatrix}\cos\theta_1\\ \sin\theta_1\end{pmatrix}$
P	Point	$\begin{pmatrix}\cos\theta_2\\ \sin\theta_2\end{pmatrix}$
R	Point	$\begin{pmatrix}\cos\theta_3\\ sin\theta_3\end{pmatrix}$

Table 2: Table of output parameters

For getting the value of the $\angle NOQ$

$$\cos\theta = \frac{(\mathbf{R}-\mathbf{Q})^\top(\mathbf{P}-\mathbf{Q})}{||\mathbf{R}-\mathbf{Q}||||\mathbf{P}-\mathbf{Q}||} \tag{1}$$

$$or, \cos\theta = \frac{\sin\frac{\theta_1+\theta_2}{2}\cos\frac{\theta_2+\theta_3}{2}}{\sin\frac{\theta_2-\theta_1}{2}} \tag{2}$$

$$so, \theta_1 = 2\tan^{-1}\left(\tan\left(\frac{\theta_2}{2}\right)\left(\frac{\cos\theta+\cos\frac{\theta_2+\theta_3}{2}}{\cos\theta-\cos\frac{\theta_2+\theta_3}{2}}\right)\right) \tag{3}$$

$$or, \theta_1 = 2\tan^{-1}\left(\tan\left(\frac{\theta_2}{2}\right)\left(\frac{\cos\theta-\cos\frac{\theta_2+\theta_3}{2}}{\cos\theta+\cos\frac{\theta_2+\theta_3}{2}}\right)\right) \tag{4}$$

$$\implies \theta_1 = 136.696^\circ \tag{5}$$

$$= 175^\circ \tag{6}$$

For getting the value of the $\angle OPR$

$$\cos\angle OPR = \frac{(\mathbf{O}-\mathbf{P})^\top(\mathbf{R}-\mathbf{P})}{||\mathbf{O}-\mathbf{P}||||\mathbf{R}-\mathbf{P}||} \tag{7}$$

$$= \sqrt{\frac{1-\cos(\theta_2-\theta_3)}{2}} \tag{8}$$

$$\angle OPR = 10^\circ \tag{9}$$

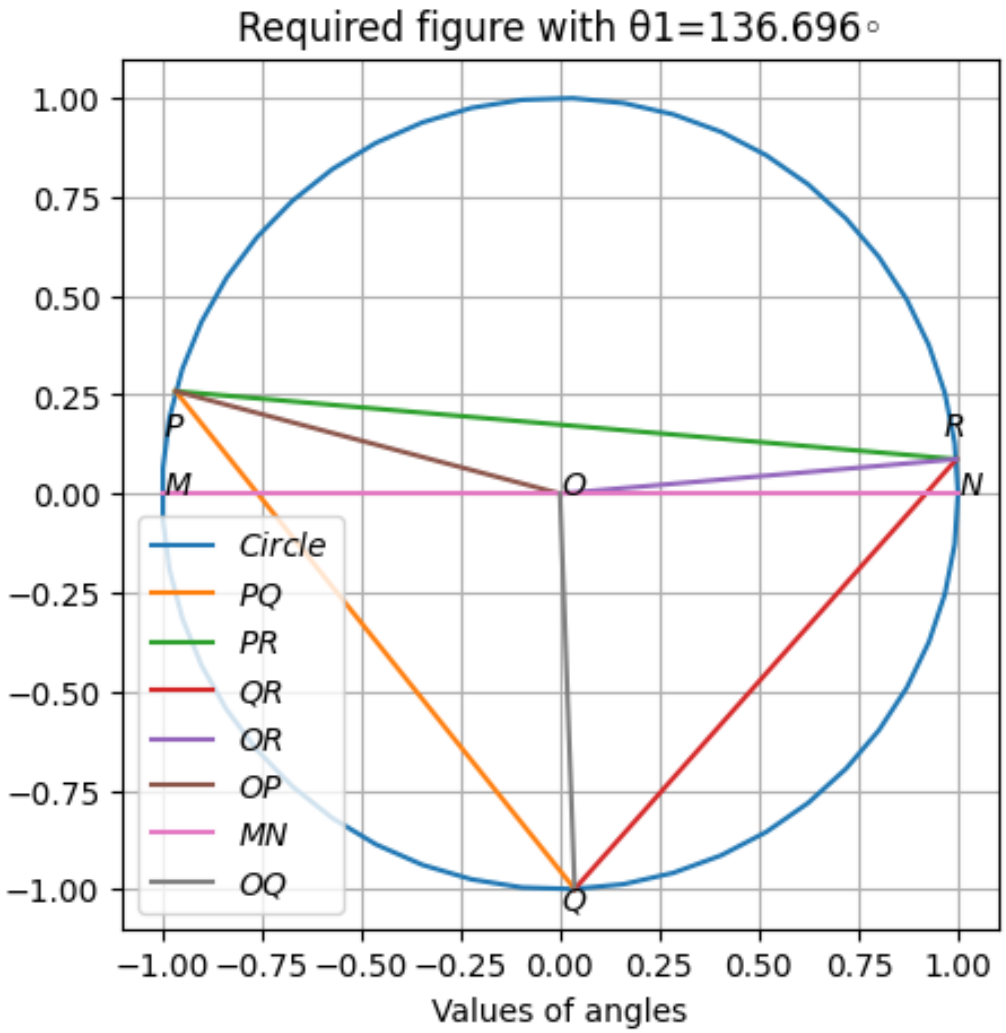
Required figure with θ1=136.696◦
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
Values of angles
P
R
M
O
N
Q
Circle
PQ
PR
QR
OR
OP
MN
OQ

Figure 1:

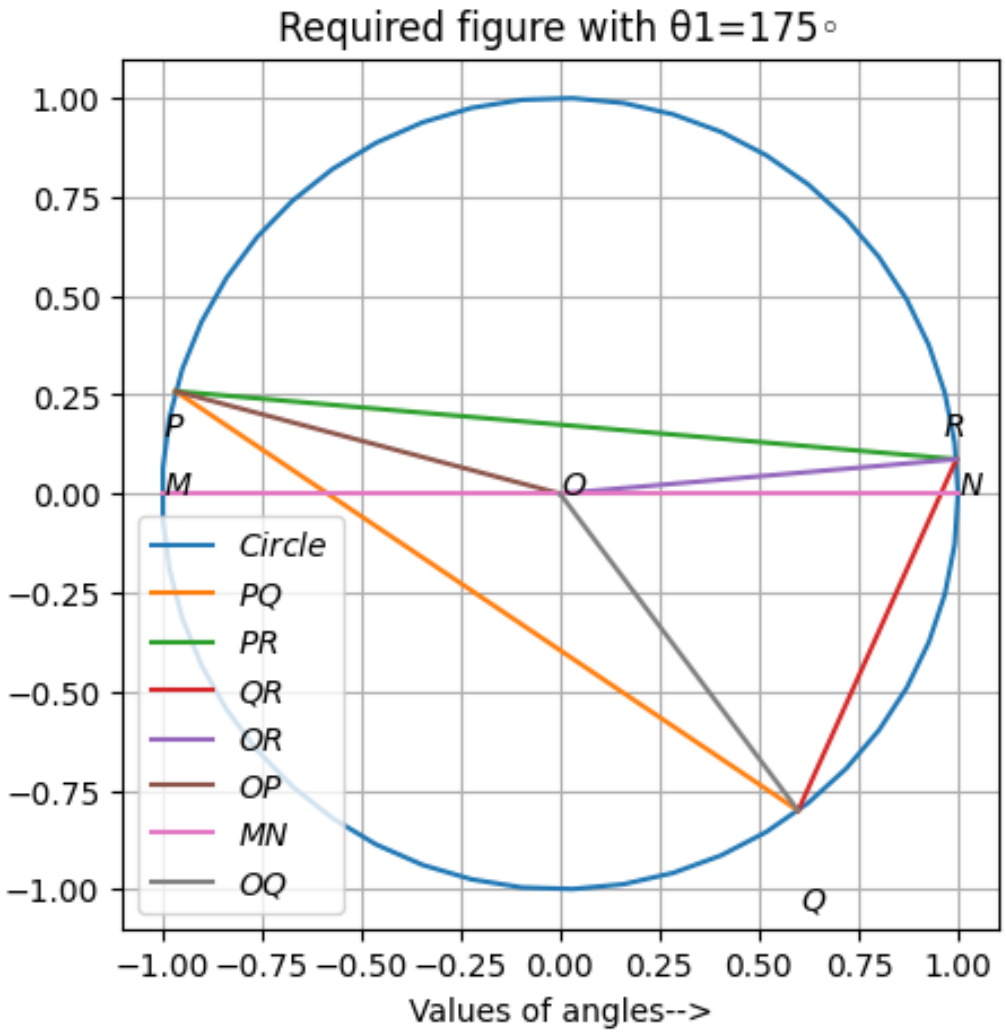
Required figure with θ1=175◦
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
P
R
M
O
N
Q
Circle
PQ
PR
QR
OR
OP
MN
OQ
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
Values of angles-->

Figure 2: