

# VECTOR

**Question :** Construct a triangle  $APB$  in which  $BC = 7\text{cm}$ ,  $\angle B = 75^\circ$  and  $AB + AC = 13\text{cm}$ .

**Figure :**

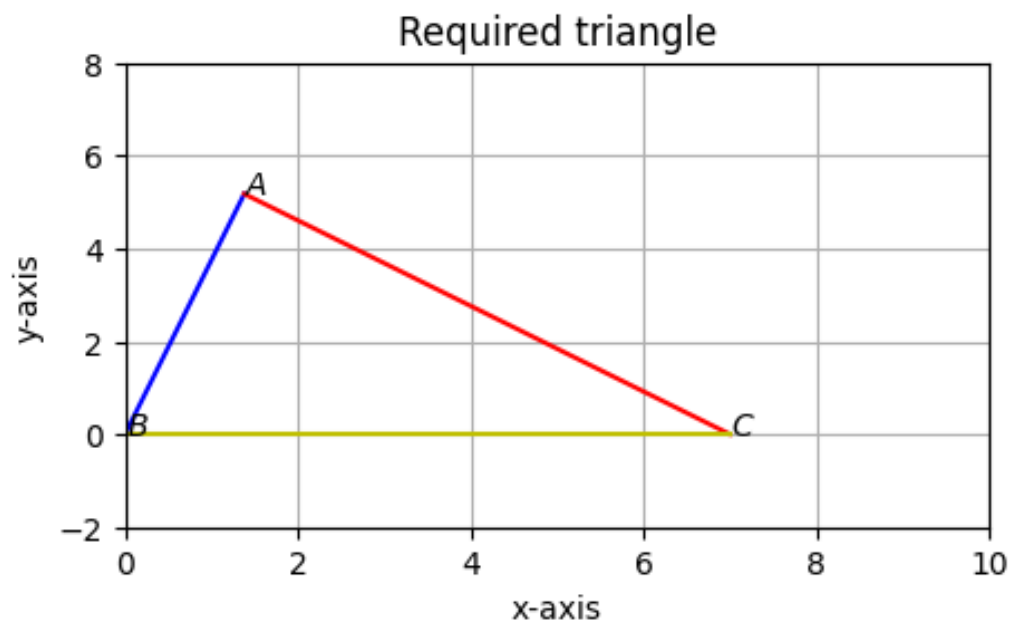


Figure 1:

**Solution :**

Input parameters	Description	Value
<b>B</b>	Vertex(at origin)	<b>0</b>
$a$	Side of $\triangle ABC, BC$	7
$b$	Side of $\triangle ABC, AB$	$b$
$c$	Side of $\triangle ABC, AC$	$c$
$\theta$	Angle of $\triangle ABC, \angle B$	$75^\circ$

Table 1: Table of input parameters

Output parameters	Description	Value
<b>C</b>	Vertex	$ae_1$
<b>A</b>	Vertex	$c \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$
$b + c$	$AB + AC$	13

Table 2: Table of output parameters

From cosine rule,

$$b^2 = a^2 + c^2 - 2ac \cos \angle B \quad (1)$$

$$or, b^2 = a^2 + c^2 - 2ac \left( \frac{\sqrt{3} - 1}{2\sqrt{2}} \right) \quad (2)$$

$$or, b^2 - c^2 = a^2 - ac \left( \frac{\sqrt{3} - 1}{\sqrt{2}} \right) \quad (3)$$

$$or, (b + c)(b - c) = a^2 - ac \left( \frac{\sqrt{3} - 1}{\sqrt{2}} \right) \quad (4)$$

$$or, 13(b - c) = 49 - 7c \left( \frac{\sqrt{3} - 1}{\sqrt{2}} \right) \quad (5)$$

$$or, 13b + \left( -13 + 7 \left( \frac{\sqrt{3} - 1}{\sqrt{2}} \right) \right) c = 49 \quad (6)$$

$$b + c = 13 \quad (7)$$

From (6),(7)

$$\left[ \begin{array}{cc|c} 13 & -13 + 7 \left( \frac{\sqrt{3}-1}{\sqrt{2}} \right) & 49 \\ 1 & 1 & 13 \end{array} \right] \quad (8)$$

$$\xrightarrow{R'_1=R_1/13} \left[ \begin{array}{cc|c} 1 & \frac{-13+7\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)}{13} & \frac{49}{13} \\ 1 & 1 & 13 \end{array} \right] \quad (9)$$

$$\xrightarrow{R'_2=R_2-R_1} \left[ \begin{array}{cc|c} 1 & \frac{-26+7\sqrt{6}-7\sqrt{2}}{26} & \frac{49}{13} \\ 0 & \frac{52-7\sqrt{6}+7\sqrt{2}}{26} & \frac{120}{13} \end{array} \right] \quad (10)$$

$$\xrightarrow[\begin{array}{c} R_1''=R'_1-R'_2\left(\frac{-26+7\sqrt{6}-7\sqrt{2}}{52-7\sqrt{6}+7\sqrt{2}}\right) \\ R_2''=R'_2\left(\frac{26}{52-7\sqrt{6}+7\sqrt{2}}\right) \end{array}]{\begin{array}{cc|c} 1 & 0 & \frac{436-91\sqrt{6}+91\sqrt{2}}{52-7\sqrt{6}+7\sqrt{2}} \\ 0 & 1 & \frac{240}{52-7\sqrt{6}+7\sqrt{2}} \end{array}} \quad (11)$$

$$\begin{bmatrix} b \\ c \end{bmatrix} = \begin{bmatrix} \frac{436-91\sqrt{6}+91\sqrt{2}}{52-7\sqrt{6}+7\sqrt{2}} \\ \frac{240}{52-7\sqrt{6}+7\sqrt{2}} \end{bmatrix} \quad (12)$$

Therefore,

$$\mathbf{A} = c \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \quad (13)$$

$$= \frac{240}{52-7\sqrt{6}+7\sqrt{2}} \begin{bmatrix} \cos 75^\circ \\ \sin 75^\circ \end{bmatrix} \quad (14)$$

$$= \begin{bmatrix} 1.388 \\ 5.18 \end{bmatrix} \quad (15)$$

$$(16)$$