

VECTOR

Question: \vec{P} and \vec{Q} are any two points lying on the sides DC and AD respectively of a parallelogram $ABCD$. Show that, $ar(\triangle APB) = ar(\triangle BQC)$.

Figure:

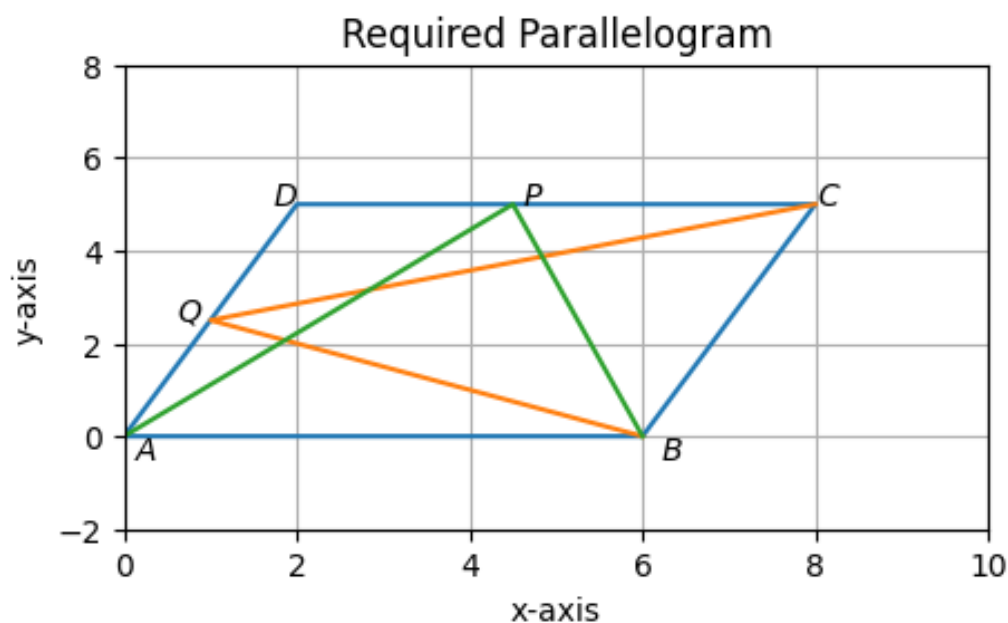


Figure 1

Solution:

Using formulas:

1. Area of a parallelogram with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$ is $= \frac{1}{2} \times |[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)]|$ square units.

2. Area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is $= \frac{1}{2} \times |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$ square units.
3. If a triangle and a parallelogram are on the same base and between the same parallels, then area of the triangle is equal to half the area of the parallelogram.

From figure 1 , considering $\triangle BQC$ and parallelogram $ABCD$, having same base BC and, AD is parallel to BC .

$$\implies \triangle BQC = \frac{1}{2} \times arABCD. \quad (1)$$

Considering $\triangle APB$ and parallelogram $ABCD$, having same base AB and, DC is parallel to AB .

$$\implies \triangle APB = \frac{1}{2} \times arABCD. \quad (2)$$

By comparing equation (1) and equation (2) we obtain that,

$$ar(\triangle APB) = ar(\triangle BQC). \quad (3)$$

Hence proved.

Proof by the help of diagram :(figure 1,table 1)

Points	Description	Value
\vec{A}	vertex	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
\vec{B}	vertex	$\begin{pmatrix} 6 \\ 0 \end{pmatrix}$
\vec{C}	vertex	$\begin{pmatrix} 8 \\ 5 \end{pmatrix}$
\vec{D}	vertex	$\begin{pmatrix} 2 \\ 5 \end{pmatrix}$
\vec{P}	vertex($\triangle APB$)	$\begin{pmatrix} 4.5 \\ 5 \end{pmatrix}$
\vec{Q}	vertex($\triangle BQC$)	$\begin{pmatrix} 1 \\ 2.5 \end{pmatrix}$

Table 1: Parameters of the diagram.

For the $\triangle APB$, the vertices of the triangle are

$$\begin{aligned}\vec{A} &= \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \vec{P} = \begin{pmatrix} 4.5 \\ 5 \end{pmatrix}, \vec{B} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \\ \Rightarrow \triangle APB &= \frac{1}{2} \times |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2} \times |0(5 - 0) + 4.5(0 - 0) + 6(0 - 5)| \\ &= \frac{1}{2} \times |0 + 0 - 30| \\ &= \frac{1}{2} \times 30 \\ &= 15 \text{ square units.}\end{aligned}$$

For the $\triangle BQC$, the vertices of the triangle are

$$\begin{aligned}\vec{B} &= \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \vec{Q} = \begin{pmatrix} 1 \\ 2.5 \end{pmatrix}, \vec{C} = \begin{pmatrix} 8 \\ 5 \end{pmatrix} \\ \Rightarrow \triangle BQC &= \frac{1}{2} \times |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| \\ &= \frac{1}{2} \times |6(2.5 - 5) + 1(5 - 0) + 8(0 - 2.5)| \\ &= \frac{1}{2} \times |-15 + 5 - 20| \\ &= \frac{1}{2} \times 30 \\ &= 15 \text{ square units.}\end{aligned}$$

For parallelogram $ABCD$, the vertices are

$$\begin{aligned}\vec{A} &= \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \vec{B} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \vec{C} = \begin{pmatrix} 8 \\ 5 \end{pmatrix}, \vec{D} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} \\ \Rightarrow arABCD &= \frac{1}{2} \times |[x_1y_2 - x_2y_1] + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + \\ &\quad (x_4y_1 - x_1y_4)]| \\ &= \frac{1}{2} \times |[0(0 - 6.0) + (6.5 - 8.0) + (8.5 - 2.5) + (2.0 - 0.5)]| \\ &= \frac{1}{2} \times |30 + 30| \\ &= \frac{1}{2} \times 60 \\ &= 30 \text{ square units.}\end{aligned}$$

Therefore, $arABCD = \frac{1}{2} \times ar\triangle APB = \frac{1}{2} \times ar\triangle BQC$ (proved).