ASSIGNMENT 2

Question : Two lines passing through the point(2,3) intersect each other at an angle of 60°.If slope of one line is 2,find equation of the other line.

Solution :

Symbol	Description	Value
m_1	Slope of one line	2
m_2	Slope of another line	m
P	Intersecting point	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
θ	Angle between two lines	60°

Table 1: Table of input parameters

Symbol	Description	Value
$\mathbf{m'_1}$	Direction vector of one line	$\begin{pmatrix} 1 \\ m_1 \end{pmatrix}$
$\mathbf{n_1}$	Normal vector of one line	$\begin{pmatrix} m_1 \\ -1 \end{pmatrix}$
$\mathbf{m'_2}$	Direction vector of another line	$\begin{pmatrix} 1 \\ m_2 \end{pmatrix}$
$\mathbf{n_2}$	Normal vector of another line	$\begin{pmatrix} m_2 \\ -1 \end{pmatrix}$

Table 2: Table of output parameters

So,

$$\cos\theta = \frac{(\mathbf{m'^T_1})\mathbf{m'_2}}{\|\mathbf{m'_1}\|\|\mathbf{m'_2}\|} \tag{1}$$

$$or, \frac{1}{2} = \frac{(1 \quad 2)\begin{pmatrix}1\\m\end{pmatrix}}{\sqrt{5}\sqrt{m^2+1}} \tag{2}$$

$$or, \frac{1}{2} = \frac{1+2m}{\sqrt{5m^2+5}} \tag{3}$$

$$or, 11m^2 + 16m - 1 = 0 \tag{4}$$

$$or, m = \frac{-8 \pm 5\sqrt{3}}{11} \tag{5}$$

Therefore, the direction vector is, $\mathbf{m'_2} = \begin{pmatrix}1\\\frac{-8+5\sqrt{3}}{11}\end{pmatrix}$ or, $\begin{pmatrix}1\\\frac{-8-5\sqrt{3}}{11}\end{pmatrix}$

The normal vector is, $\mathbf{n_2} = \begin{pmatrix}\frac{-8+5\sqrt{3}}{11}\\-1\end{pmatrix}$ or, $\begin{pmatrix}\frac{-8-5\sqrt{3}}{11}\\-1\end{pmatrix}$

So, the equation of the line is

$$\mathbf{n^T}\mathbf{x} = c \tag{6}$$

$$\begin{pmatrix}\frac{-8\pm5\sqrt{3}}{11} & -1\end{pmatrix}\mathbf{x} = c \tag{7}$$

Passes through the point $\mathbf{P} = \begin{pmatrix}2\\3\end{pmatrix}$

$$\begin{pmatrix}\frac{-8\pm5\sqrt{3}}{11} & -1\end{pmatrix}\mathbf{P} = c \tag{8}$$

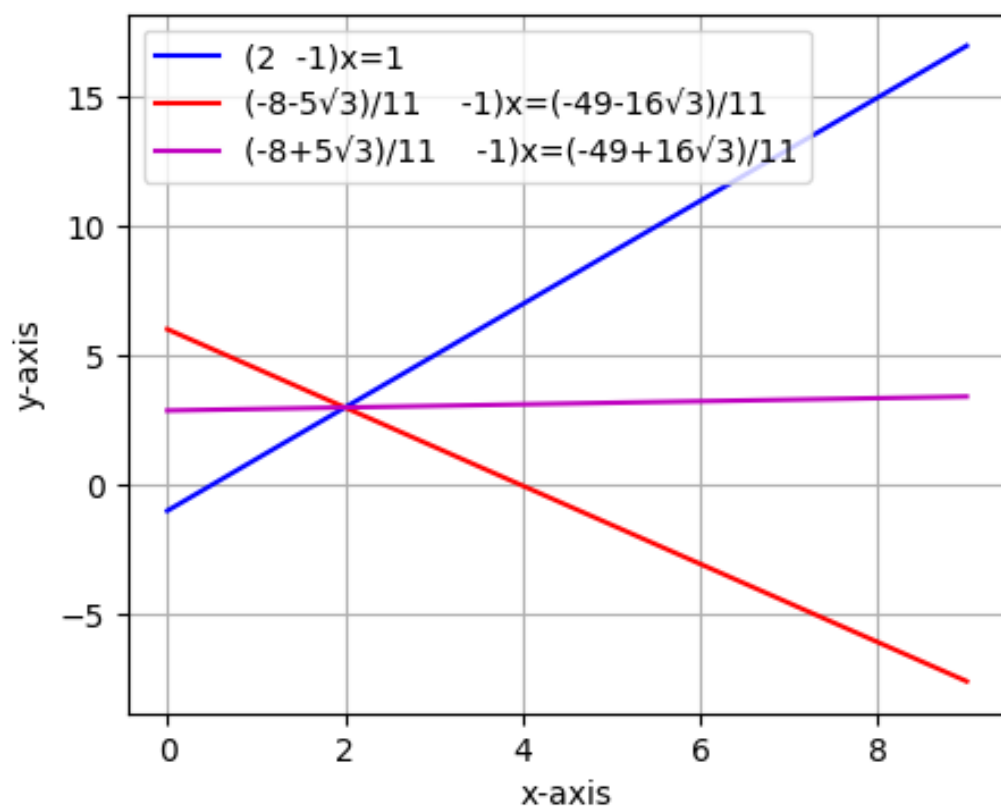$$or, c = \frac{-49 \pm 16\sqrt{3}}{11} \tag{9}$$

Figure :

2

Figure 1: