EMBEDDED

Question : l and m are two parallel lines intersected by another pair of parallel lines p and $q(figure\ 1)$,show that $\triangle ABC \cong \triangle CDA$.

Figure :

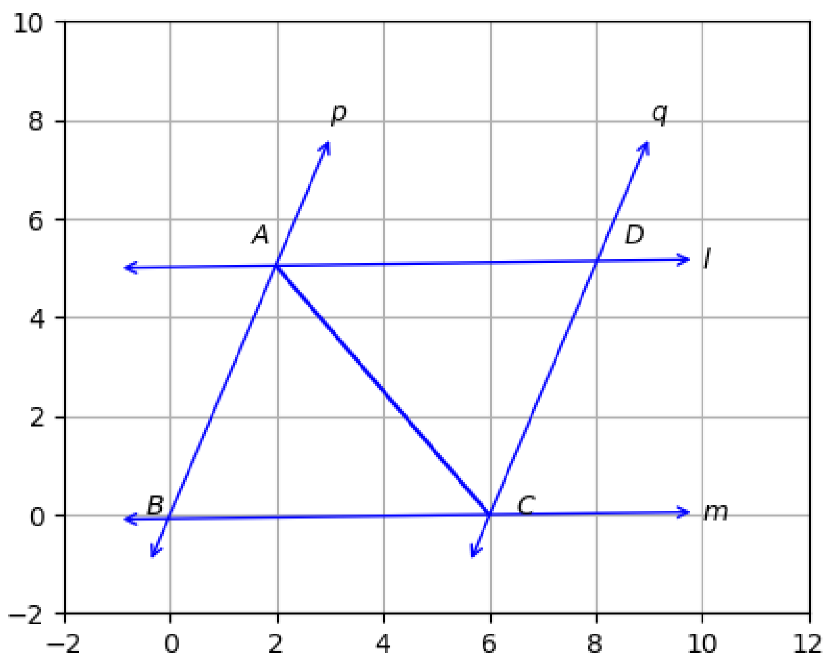

Figure 1: Required parallelogram

Solution :

Symbol	**Description**	**Value**
$\mathbf{B}$	Vertex at origin	$\mathbf{0}$
a	Side of the parallelogram,$BC = DA$	6
b	Side of the parallelogram,$AB = CD$	$\sqrt{29}$
θ	Angle of the parallelogram, $\angle ABC$	$\sin^{-1}\left(\frac{5}{\sqrt{29}}\right)$

Table 1: Table of input parameters

Symbol	**Description**	**Value**
$\mathbf{C}$	Vertex of parallelogram	ae_1
$\mathbf{A}$	Vertex of parallelogram	$b\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$
$\mathbf{D}$	Vertex of parallelogram	$\mathbf{C}+\mathbf{A}$
CA	Side of both $\triangle ACB$ and $\triangle CDA$	$\sqrt{a^2 - 2ab\cos\theta + b^2}$

Table 2: Table of output parameters

From $figure$ 1 between $\triangle ABC$ and $\triangle CDA$

$$\begin{aligned}
\cos\angle BAC &= \frac{AB.CA}{\|AB\|\,\|CA\|} && (1)\\
&= \frac{ab\cos\theta + b\sin\theta}{b\sqrt{a^2-2ab\cos\theta+b^2}} && (2)\\
&= \frac{17}{\sqrt{29}\sqrt{41}} && (3)\\
\cos\angle ACD &= \frac{CD.CA}{\|CD\|\,\|CA\|} && (4)\\
&= \frac{b^2-ab\cos\theta}{b\sqrt{a^2-2ab\cos\theta+b^2}} && (5)\\
&= \frac{17}{\sqrt{29}\sqrt{41}} && (6)\\
So, \angle BAC = \angle ACD. & && (7)\\
\cos\angle ACB &= \frac{CB.CA}{\|CB\|\,\|CA\|} && (8)\\
&= \frac{a^2-ab\cos\theta}{a\sqrt{a^2-2ab\cos\theta+b^2}} && (9)\\
&= \frac{24}{6\sqrt{41}} && (10)\\
\cos\angle CAD &= \frac{DA.CA}{\|DA\|\,\|CA\|} && (11)\\
&= \frac{a^2-ab\cos\theta}{a\sqrt{a^2-2ab\cos\theta+b^2}} && (12)\\
&= \frac{24}{6\sqrt{41}} && (13)\\
So, \angle ACB = \angle CAD. & && (14)
\end{aligned}$$

And CA is common side .

So,$\triangle ABC \cong \triangle CDA. \, (byA-A-S)\,(proved)$