12.11.4.3

Question: Find the angle between the lines whose direction ratios are a, b, c and b - c, c - a, a - b.

Solution:

$$\mathbf{m_1} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{1}$$

$$\mathbf{m_2} = \begin{pmatrix} b - c \\ c - a \\ a - b \end{pmatrix}$$

$$\cos \theta = \frac{(\mathbf{m_1}^\top) \mathbf{m_2}}{||\mathbf{m_1}|| ||\mathbf{m_2}||}$$
(3)

$$\cos \theta = \frac{\left(\mathbf{m_1}^\top\right) \mathbf{m_2}}{||\mathbf{m_1}||||\mathbf{m_2}||} \tag{3}$$

$$= \frac{(a \ b \ c) \begin{pmatrix} b-c \\ c-a \\ a-b \end{pmatrix}}{\sqrt{a^2 + b^2 + c^2} \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}}$$
(4)

$$=0 (5)$$

$$or, \theta = \frac{\pi}{2} \tag{6}$$