9.10.4.3

Question : If two equal chords of a circle intersect within the circle,prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Solution :

Input Parameters	Description	Value
$\mathbf{O}$	Center(at origin)	$\mathbf{0}$
d	Length of the chords	2
r	Radius	1
θ_1	$\angle ROP$	θ_1°
θ_2	$\angle ROQ$	-48°
θ_3	$\angle ROR$	0°
θ_4	$\angle ROS$	-132°

Table 1: Table of input parameters

Output Parameters	Description	Value
$\mathbf{P}$	Point	$\begin{pmatrix}\cos\theta_1\\ \sin\theta_1\end{pmatrix}$
$\mathbf{Q}$	Point	$\begin{pmatrix}\cos\theta_2\\ \sin\theta_2\end{pmatrix}$
$\mathbf{R}$	Point	$\begin{pmatrix}\cos\theta_3\\ \sin\theta_3\end{pmatrix}$
$\mathbf{S}$	Point	$\begin{pmatrix}\cos\theta_4\\ \sin\theta_4\end{pmatrix}$

Table 2: Table of output parameters

$$\cos\angle RQS = \frac{(\mathbf{R}-\mathbf{Q})^\top(\mathbf{Q}-\mathbf{S})}{||\mathbf{R}-\mathbf{Q}||||\mathbf{Q}-\mathbf{S}||} \tag{1}$$

$$= \cos\frac{\theta_3-\theta_4}{2} \tag{2}$$

$$\cos\angle PSQ = \frac{(\mathbf{P}-\mathbf{S})^\top(\mathbf{S}-\mathbf{Q})}{||\mathbf{P}-\mathbf{S}||||\mathbf{S}-\mathbf{Q}||} \tag{3}$$

$$= \cos\frac{\theta_1-\theta_2}{2} \tag{4}$$

$$\cos\angle RQS = \cos\angle PSQ \tag{5}$$

$$or, \theta_1 = \theta_2-\theta_3+\theta_4 \tag{6}$$

$$= -180^\circ \tag{7}$$

The equation of PQ and RS is obtained by

$$\mathbf{n_1}^\top(\mathbf{x}-\mathbf{P}) = 0 \tag{8}$$

$$\mathbf{n_2}^\top(\mathbf{x}-\mathbf{R}) = 0 \tag{9}$$

$$or, \mathbf{n_1} = \begin{pmatrix}\sin\theta_2-\sin\theta_1\\ \cos\theta_1-\cos\theta_2\end{pmatrix} \tag{10}$$

$$or, \mathbf{n_2} = \begin{pmatrix}\sin\theta_4-\sin\theta_3\\ \cos\theta_3-\cos\theta_4\end{pmatrix} \tag{11}$$

$$\tag{12}$$

The value of the point of the intersection is

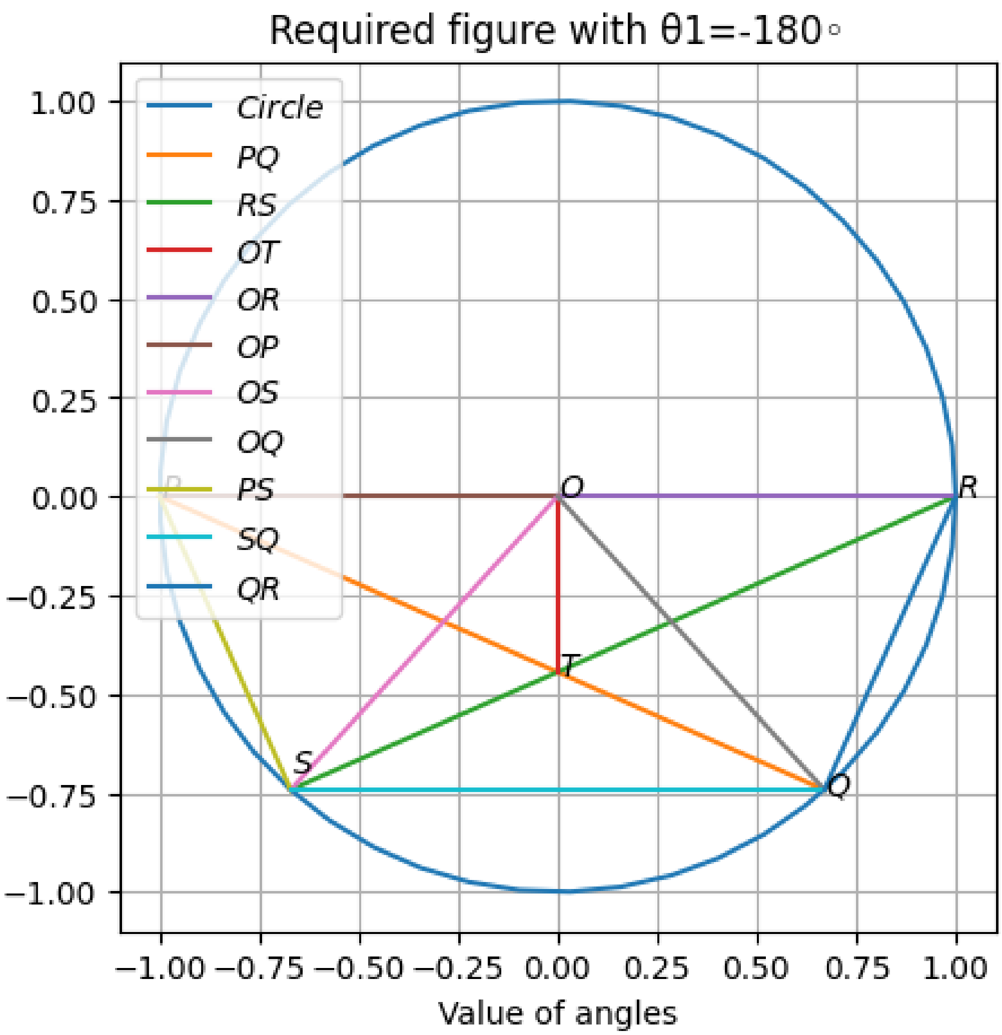
$$\begin{pmatrix}\mathbf{n_1}^\top\\ \mathbf{n_2}^\top\end{pmatrix}\mathbf{X} = \begin{pmatrix}\mathbf{n_1}^\top\mathbf{P}\\ \mathbf{n_2}^\top\mathbf{R}\end{pmatrix} \tag{13}$$

$$\mathbf{X} = \frac{1}{\begin{vmatrix}\cos\theta_3-\cos\theta_4 & \cos\theta_2-\cos\theta_1\\ \sin\theta_3-\sin\theta_4 & \sin\theta_2-\sin\theta_1\end{vmatrix}} \times \begin{pmatrix}\cos\theta_3-\cos\theta_4 & \cos\theta_2-\cos\theta_1\\ \sin\theta_3-\sin\theta_4 & \sin\theta_2-\sin\theta_1\end{pmatrix}\begin{pmatrix}\sin\theta_2-\theta_1\\ \sin\theta_4-\theta_3\end{pmatrix} \tag{14}$$

$$
\begin{aligned}
or, \mathbf{T} &= \begin{pmatrix} 0 \\ -0.45 \end{pmatrix} && (15) \\
\cos \angle OTP &= \frac{(\mathbf{T}-\mathbf{P})^\top (\mathbf{O}-\mathbf{T})}{||\mathbf{T}-\mathbf{P}||||\mathbf{T}-\mathbf{O}||} && (16) \\
or, \angle OTP &= 66^\circ && (17)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\cos \angle OTR &= \frac{(\mathbf{T}-\mathbf{R})^\top (\mathbf{T}-\mathbf{O})}{||\mathbf{T}-\mathbf{R}||||\mathbf{T}-\mathbf{O}||} && (18) \\
or, \angle OTR &= 66^\circ && (19) \\
So, \angle OTP &= \angle OTR \, (proved) && (20)
\end{aligned}
$$



Required figure with θ1=-180∘
Circle
PQ
RS
OT
OR
OP
OS
OQ
PS
SQ
QR
O
R
T
S
Q
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
Value of angles

Figure 1: