

A Multi Agent Model for the Limit Order Book Dynamics

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Abstract. In the present work we introduce a novel multi-agent model with the aim to reproduce the dynamics of a double auction market at microscopic time scale through a faithful simulation of the matching mechanics in the limit order book. The agents follow a noise decision making process where their actions are related to a stochastic variable, *the market sentiment*, which we define as a mixture of public and private information. The model, despite making just few basic assumptions over the trading strategies of the agents, is able to reproduce several empirical features of the high-frequency dynamics of the market microstructure not only related to the price movements but also to the deposition of the orders in the book.

PACS. Market Microstructure – Econophysics – Detrended Fluctuation Analysis – Multi-Agent Models

1 Introduction

In the past few years, following the increasing power of technological infrastructures, high-frequency trading, which broadly speaking includes every strategies which holding period is shorter than a day, has bloomed among the major financial institutions around the globe and, nowadays, it accounts for about 70% of all the volume traded in US equities. The same trend is evident also in Europe where the number of transactions for the most liquid contracts has recently experienced an exponential growth: in the Eurex Stoxx futures index, for example, the number of daily trades has gone from 12500 in January 2005 up to a maximum of 150000 in November 2008 (Bartolozzi et al., 2007a). As a consequence for the growing interest in the short time scales, the exchanges has started to provide live feeds of every single order submitted (buy or sell), therefore, making the market microstructure a primary field of interest for the financial practitioners.

The progressive shift towards the very high-frequencies has not been unnoticed in the Econophysics community which, in the meanwhile, has become an established field of study among physicists (Bouchaud and Potters, 1999; Mantegna and Stanley, 1999; Paul and Baschnagel, 1999; Voit, 2005). In the multi-agent framework, a common feature in the physicist's approach to the market microstructure of *order driven markets* is the lack of a proper *utility function*, which is often invoked in Economics/Econometric literature in order to study the rational behaviour of the agents (Bailey, 2005), and, consequently, a simple statistical approach to the problem is preferred (Bak et al., 1997;

Maslov, 2000; Matassini and Franci, 2001; Roberto et al., 2001; Smith et al., 2003; Daniels et al., 2003; Iori et al., 2003; Farmer et al., 2005; Mike and Farmer, 2008; Zaccaria et al., 2010). This line of thought has been referred to as *zero intelligence* (Farmer et al., 2005): a recent review on some of the models proposed so far can be found in (Slanina, 2008). For an Econometric approach to the market microstructure, instead, the reader can refer to (O'Hara, 1997).

Order driven markets are based on the principle of *continuous double auction* where the price of an asset is continuously adjusted in order to account for the inflow of demand and supply. In particular, the orders placed by the traders, that is the number of contracts they want to buy or sell along with the corresponding price, are organized and matched in the *limit order book* (LOB) which ultimately defines the microstructure of the market. At every instant in time, the LOB, a sketch of which is given for clarity in Fig. 1, is described by two sets of *limit orders* in opposite "directions", one for the long orders (buy side) and one for the short (sell side). Each order is characterized by a *limit price*, that is the price the trader is willing to buy/sell, and the number of contracts requested/offered, or volume. The minimum price difference greater than zero between two orders is called *tick size* and, in real markets, depends on the specific contract. When more than one limit order is send to the same price then the exchange rank them in a stack by arrival time: the first to be executed will be the oldest¹. The *mid-point*

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¹ Note that while this ranking describes a typical situation, the execution of limit orders can slightly change depending on the rules of the specific exchange

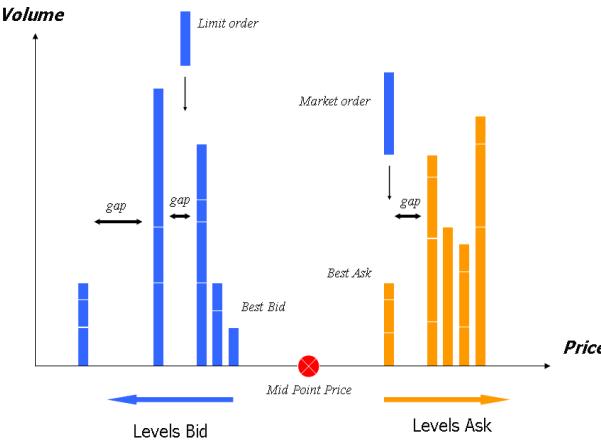


Fig. 1. Cartoon representation of the LOB. Note that gaps between price levels can be present, especially in non-liquid contracts.

price, P_m , which is usually referred as the “price” of an asset, is the mean value between the *best ask* and the *best bid*, being, respectively, the lowest price on the ask side and the higher price on the buy of the LOB. The execution of a limit order, that is the actual *trade*, is triggered if, and only if, an order in the opposite direction matches its price quote. The incoming order can either be another limit order, in which case the order executed is at best ask or bid, or a *market order*, that is a request to execute a certain volume, starting from the best price, *immediately* and until it has been completely filled. This latter type of order is considered very “aggressive” given that the price of the spread is paid upfront and it is usually associated with “impatience” traders. *Cancellations* also play an important role in the dynamics of the LOB. In fact, limit orders that have not been filled in a reasonable amount of time, according to the trader’s necessity, are usually removed from the LOB.

In the present work we introduce a novel multi-agent model² with the aim to reproduce the dynamics of the market microstructure. Our agents relate their actions to a stochastic variable, the “*market sentiment*”, which includes feedbacks from both public and private information. Besides, the model relies on a realistic LOB mechanics which takes into account every possible change on “event” base such as the arrival of limit orders, market orders or cancellations as well as a faithful order matching mechanism.

The paper is organized as following: in the next section we introduce our model while the numerical results

² For “agents” here we indicate all possible financial institutions such as investment banks, hedge funds etc... rather than the occasional trader.

of the simulations are presented in Sec. 3. Discussions and conclusion are left in the last section.

2 The Model

2.1 Brief outline

Our market model evolves in discrete time steps³ during which each agent may undertake a certain action or just wait for a more profitable opportunity. These actions can be broadly divided into *cancellation* and *active trading*, the latter including both limit and market orders. In particular, all decision steps are based on dynamical probabilities which are function of both the private and public information, the former being related to the “state” of the market as we will see in the following sections. Moreover, we assume that our agents can have just one open position at the time and, therefore, before issuing a new order they need to close their current position. By using this limitation we neglect explicit market making activity.

Another central part of our model is the *order generation step* where the specifications for each order such as *type* (limit or market), *price* (for limit orders) and *volume* are decided. In the next sections we explain in details all the fundamental building blocks.

2.2 Public and private information update

At the beginning of each time step, we assume that the agents have access to the current state of the LOB, that is they can “see” every order placed in the market: all the indicators derived by this knowledge, such as the mid-point price for example, are classified as *public information*. However, these are not used in their “raw” form but they get “smoothed”: this filtered information, that we refer as “perceived”, is the one which is actually used in the process of decision making. The reason behind this extra step is to take our simulations one step closer to reality where indicators are usually pre-processed in order to get rid of some noise⁴. The filter used in our simulations is an exponential moving average (EMA) (Fusai and Ronconi, 2008) which is defined for a generic time series $x(t)$ as

$$\hat{x}(t) = \frac{1}{L}x(t) + \left(1 - \frac{1}{L}\right)\hat{x}(t-1), \quad (1)$$

where L is proportional to the memory of the process. By using the previous equation Eq.(1), we define the *perceived volatility* as

$$\hat{\nu}(t) = \sqrt{\hat{r}^2(t)}, \quad (2)$$

³ It is important to stress that in real markets the arrival of new information at microscopic time scale is heterogeneous in time and often people consider the *trade time* as a more appropriate scale. While the output of the model is updated in discrete time steps for simplicity, the LOB is changed on event base.

⁴ As a consequence, short term memory is induced in the signal itself.

being r the one step return of the mid point price $r(t) = P_m(t) - P_m(t-1)$. Note that, for time being, we have not included in the public information any *news realize*, the importance of which has been questioned over time, see (Cutler et al., 1989) for example or (Joulin et al., 2008) for a more recent criticism.

Regarding the *private information*, instead, we assume that it can be represented by a simple Gaussian process, independent for each trader, with zero mean and standard deviation proportional to the perceived volatility of the market, $\hat{\nu}(t)$: this information can be thought to represent the convolution of the trading strategies used by the agent.

2.3 Cancellation of orders form the LOB

At each time step, agents having an outstanding order in the LOB will evaluate the possibility of removing it according to two different criteria. The first corresponds to a simple *time-out*, that is the order is automatically removed if it has not been executed in a T_{max} number of time steps relatively long if compared to the average time for a transaction and fixed to 100 time steps in our simulations. The second criterion, instead, is related to a strategic decision based on the current market condition. In particular, we define a *cancellation probability*, $\psi_{\hat{\nu}}(t) \in [0, 1]$, common to all the agents as

$$\psi_{\hat{\nu}}(t) = 1 - e^{-\gamma \hat{\nu}(t)}, \quad (3)$$

being $\gamma = 0.02$ a sensitivity parameter. The former expression, which relies just on the perceived volatility $\hat{\nu}(t)$ in terms of information, is justified by the empirical observation that the cancellation frequency increases when the market is highly volatile: this is clearly a case of self-reinforcing effect triggered by the herding behaviour of the market participants (Cont and Bouchaud, 2000; Bartolozzi 2004).

2.4 Active trading and market sentiment

While the cancellation step takes place, agents with no orders in the LOB evaluate the possibility to enter the market. Their decision is based on a stochastic variable which represents the “level of confidence” in their price forecast: the *market sentiment*, ϕ . This quantity, which is a core part of our model, relates the public and the private information through a multiplicative process and, specifically, for the i th agent at time t we define

$$\phi_i(t) = \phi_0 \cdot \kappa_i \cdot \psi_{\hat{\nu}}^*(t) \cdot \eta(t) \cdot \epsilon_i(t), \quad (4)$$

where each terms represents a different aspect which may impact on the decision making process as following:

- ϕ_0 is a strength parameter common to all the agents which, as we will see in the next section, fixes the trading frequency of the model and, therefore, the time scale.

- The parameter κ_i represents the agent’s degree of *risk aversion* and it used in order to diversify the appetite for risk. Its value is randomly selected, at the beginning of the simulation, from a uniform distribution bounded in $[0.25, 0.75]$.
- $\psi_{\hat{\nu}}^* = 1 - \psi_{\hat{\nu}}(t)$ is the *volatility risk* and mimics the fact that traders are more cautious to send orders during high volatility periods given that the risk associated to these is higher as well.
- $\eta(t)$, instead, is a proxy for the *liquidity risk*: a sparse LOB has a potential large negative impact on the execution of a trade and, therefore, it decreases the probability that an agent is willing to accept the risk. In the present work we assume $\eta(t) = \bar{N}(t)/N$ where N is the maximum number of orders in the LOB, equivalent also to the total number of agents in the simulation, and $\bar{N}(t)$ the number of orders in the LOB at time step t . From the definition we have that $\eta(t) \in [0, 1]$.
- $\epsilon_i(t)$ represents the *private information* which is drawn from a Gaussian distribution with zero mean and standard deviation equal to $\hat{\nu}(t)$, $\epsilon_i(t) = \hat{\nu}(t) \cdot G(0, 1)$.

The *market sentiment*, Eq.(4), can be thought as the convolution between the agent’s trading strategies, the private information, and the risk factors evaluated via the public information (volatility and liquidity in this case): the stronger is the signal the more likely will be for the trader to take a decision.

The next step involves the mapping of $\phi_i(t)$ into a *trading probability*, $p_i(t) \in [0, 1]$, via a the following transfer function

$$p_i(t) = \frac{2}{\pi} |\arctg [\phi_i(t)]|, \quad (5)$$

which represents the probability for the i th agent to submit an order at time step t . Conversely, an agent will not take any action with probability $1 - p_i(t)$.

The *direction* of a trade, assuming that short selling is allowed, $d_i(t) = +1$ for long orders (buy) and $d_i(t) = -1$ for short orders (sell), is also derived from the market sentiment according to

$$d_i(t) = \frac{\phi_i(t)}{|\phi_i(t)|}, \quad (6)$$

as if $\phi_i(t)$ was a *momentum* indicator for the market direction. Besides it is worth underlying that, according to the previous definition, the direction of a trade depends only on the sign of the private information, Eq.(4), and, therefore, there is 50% chances for a trade to be a buy or a sell⁵.

2.5 Order generation

Each time an agent submits an order to the market its specifications are defined in *order generation step* which addresses the following two points:

⁵ The market sentiment, $\phi_i(t)$, and consequently the trade direction, can be statistically skewed in one direction depending on different market factors and, therefore, leading to herding effects. The study of this phenomenon will be the topic of future investigations.

- *Limit or market order?* The decision to submit a *limit* or a *market order* is related to the impatience of the trader: if an order needs to be filled as soon as possible he/she may accept to pay the cost of the spread up-front and send a market one. The alternative would be a less aggressive limit order that will execute just at a pre-fixed price: the further away from the opposite best the more time the order will require to get filled.
- *Size of the order?* The number of lots that the trader is willing to buy or sell can be related to several factors such as to the specific execution strategy, to the available liquidity in the market, the volatility at that point in time etc... Moreover, in a real trading environment single large orders are usually splitted into smaller ones in order to minimize the cost related to their impact. While the latter is an important issue in practice, in order to keep things simple, we do not tackle this problem in the present work: orders are sent in just one single chunk.

In our model the first point is addressed by setting the submission price of an order, which can be interpreted as the degree of “aggressiveness”, probabilistically via a value drawn from log-normal distribution⁶, ξ , as following

$$P_l = P_b - (\xi - Q_P), \quad (7)$$

for a long order (buy) P_l while for a short (sell), P_s

$$P_s = P_a + (\xi - Q_P), \quad (8)$$

being P_b and P_a the best bid and the best ask, respectively, and Q_P the q -quantile, $q = 0.5$ in our case, of the distribution. Besides, the value of ξ is rounded up to an integer value and, therefore, fixing, without losing of generality, the tick size of our market to 1. It is also important to stress that following the former procedure we explicitly assume that limit orders can be submitted asymptotically far from the best price as shown in the sketched in Fig. 2.

After the submission price has been fixed, the *type* of the order is decided based on its relative position to the best prices: if the submission price results to be greater than the ask price and the order is long (or lower than the bid price and the trade is short) then we interpret this as a *market order* and all the volume will be completely filled starting from the best price. All the other orders are considered *limit orders* and, for a fixed price, they are organized in stacks from the oldest to the newest being the formers the first to be executed⁷. This modelling approach is motivated by two empirical observations: the distribution of the volumes in the LOB, when averaged over long periods, displays “fat tails” and that the frequency of market order represents just a fraction of that of limit orders submitted in the market (Bouchaud et al., 2002).

⁶ The parameters used in the simulations for the log-normal distribution are 7 for the mean and 10 for the standard deviation.

⁷ If the submission price coincides with the opposite best we give 50% probability for the trade to be a limit order or a market order.

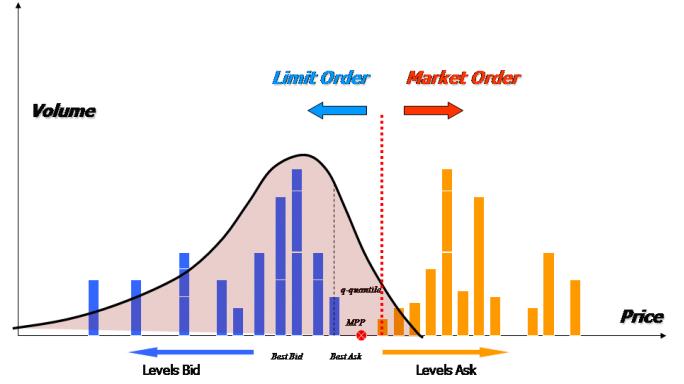


Fig. 2. The price at which an order is submitted, and therefore its “aggressiveness”, is drawn from a log-normal distribution which q -quantile is centered at the best bid (long order) or best ask (short order). The “direction” of the distribution is also chosen in accordance with the order direction. In the cartoon above we sketch the probability of price submission for a long (buy) order.

The second issue concerns the *order size*, that is the number of contracts, or volume, that an agent is willing to buy or sell. Coherently with the stochastic nature of this process, this is drawn from a log-normal distribution⁸, rounded to integer, with the following constrains: the order has to be greater or equal than one and smaller than one quarter of the total volume contained in the appropriate side of the LOB⁹.

Before moving to the next section, we wish to underline that the order generation step used in our model is just a first order approximation of what really happens in real execution strategies. In particular, we do not address the problem of splitting large volumes into smaller ones, typically used in order to minimize the transaction costs: this issue, despite being very important for market impact, goes beyond the scope of this work at present.

⁸ The parameters of the distribution used in the simulations are the same as for the price submission that is 7 for the mean and 10 for the standard deviation.

⁹ In the simulation this limit is hardly reached: usually, single orders tend to be relatively small compared to the total volume available in the LOB. Moreover, before submitting a market order we also perform a liquidity check: if the number of lots in the requested side of the LOB is less or equal N_{min} , with $N_{min} = \min(50, N/10)$, then the order is not issued. Liquidity checks on the volumes in the book are often done also in real trading in order to have an estimate of the risk associated with the trade itself.

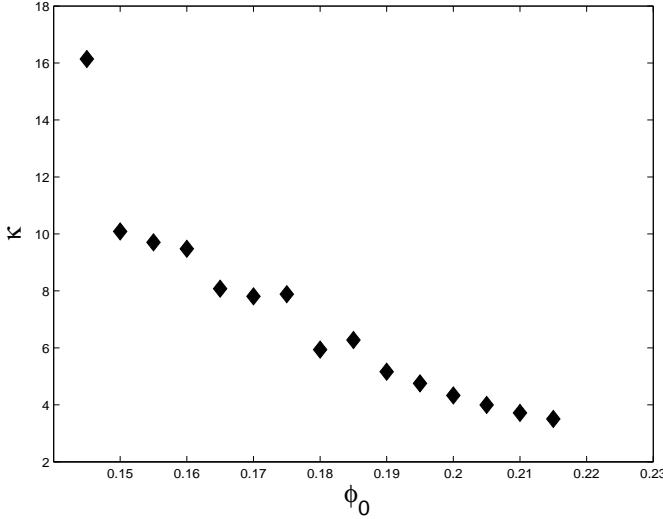


Fig. 3. Kurtosis of the one step returns estimated over an ensemble of three simulations each one consisting in 10^5 time steps. It is noticeable how this quantity converges to 3, the value for a Gaussian process, as ϕ_0 increases. By decreasing ϕ_0 , instead, the activity gets very low and the dynamics of the model starts to be ruled by few large market movements. The parameters for these simulation have been declared in the previous section except for the EMA history L , Eq.(1), which in this instance has been fixed to 5.

3 Numerical simulations at short time scales

In this section we report the results of the numerical simulations of our microscopic market model for $N = 10000$ agents¹⁰. The trading frequency, which can be interpreted as a time scale for the simulation, is fixed by the parameter ϕ_0 : as its value gets smaller the trading activity becomes relatively lower and gaps within the LOB, leading to large price changes, are more likely to appear. On the other hand, if the activity is very high, the book is almost always full and large fluctuations will occur more sporadically. The scaling of the fluctuations with the activity is emphasized in Fig. 3 where we report the kurtosis, κ , of the one step returns against ϕ_0 .

For $\phi_0 = 0.165$, the value that we use in the rest of this section, the activity resembles that of the market at very short time scales, from seconds to minutes depending on the specific contract, where non-Gaussian fluctuations play a fundamental role. In this regime, which is the one of interest in the current work, there is a probability of approximately 30% for the mid-point price to remain unchanged after one time step.

Another important parameter is the EMA history, L in Eq.(1). In fact, we have found that activity clustering, such as high volatile periods, are particular evident when

¹⁰ Numerical tests have shown that the outcomes are indifferent to the number of agents as long as their number is relatively large.

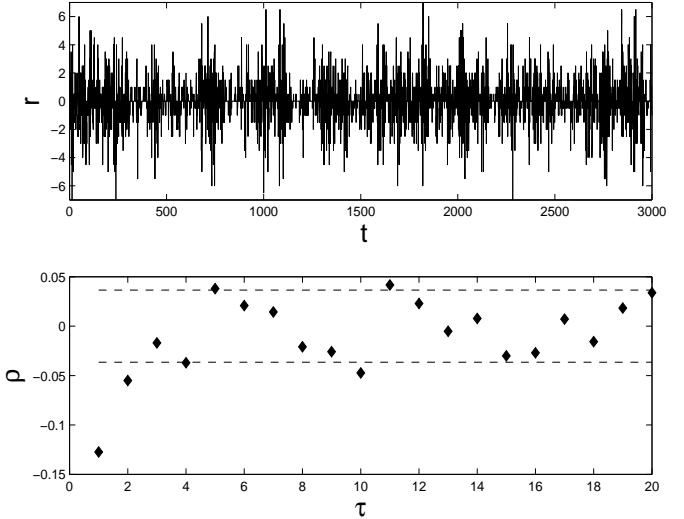


Fig. 4. (Top) A window of the one step returns time series generated by the model with $N = 10000$, $\phi_0 = 0.165$ and $L = 5$. The time series, standardized according to $r(t) \rightarrow (r(t) - \bar{r})/\sigma(r)$ being \bar{r} the average return and $\sigma(r)$ the standard deviation, displays an intermittent character. (Bottom) Autocorrelation function, $\rho(\tau)$, related to the time series on top: a significant anticorrelation is evident for the first time steps.

$3 \lesssim L \lesssim 10$. In the simulations we have fixed $L = 5$ for all the agents.

3.1 Price and volatility dynamics

A sample of the time series of one step returns, $r(t)$, generated by the model is reported in Fig. 4 along with its autocorrelation function, $\rho(\tau)$. From the plots we can notice an intermittent dynamics, characteristic of financial time series at short time scales, as well as a significant negative correlation up to few time steps. This latter effect, known empirically as “bid-ask bounce”, in the present model is purely a result of the order book mechanics. However, for real high-frequency data this effect can last up to few minutes depending on the specific market (Bouchaud and Potters, 1999): this is an important indication that, on top of the LOB mechanics, there must be other mechanisms, such as memory feedbacks or order splitting for example, responsible for the enhancement of the negative correlation in price changes.

The anticorrelated behaviour of the returns is also confirmed by an estimate of the *Hurst exponent*, H , done via the *detrended fluctuation analysis* algorithm (Peng et al., 1994; Bartolozzi et al., 2007b). According to this method, originally developed during the Nile’s river dam project (Hurst, 1951; Feder, 1988), a time series is *persistent* if $H > 0.5$, *anti-persistent* if $H < 0.5$ or *uncorrelated* if $H = 0.5$. The value found for the one step returns in our model

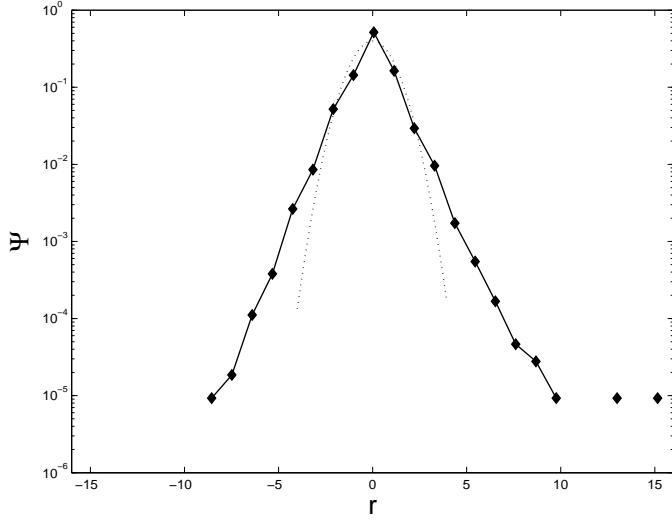


Fig. 5. Probability distribution function, Ψ , for the one step returns standardized as in Fig. 4 (solid line). This distribution is *leptokurtic*, that is the tails are “fatter” when compared to those of a Gaussian (dotted line).

is $H = 0.495(2)$ over an ensemble of 3 runs of 10^5 samples each, being the error on the last digit, reported in the brackets, estimated via the bootstrap method (Efron and Tibshirani, 1994). Noticeably, by removing the zero from the time series the value of the former exponent is statistically equivalent, in fact we find in this case $H_0 = 0.491(3)$

The probability distribution function or *pdf*, denoted as Ψ , of the one step returns is reported in Fig. 5 where the large fluctuations observed in Fig. 4 (top) give rise to a *leptokurtic* shape of the distribution, that is the tails are “fatter” than those of a Gaussian. This feature, persistent up to time scales of weeks, is well documented in different empirical works (Bouchaud and Potters, 1999; Mantegna and Stanley, 1999; Voit, 2005). However, while in most of the previous examples the asymptotic decay of the tails can be described by a power law, in our simulations this is more consistent with an exponential one. From the same plot it is also possible to notice a relatively large presence of zero returns, which is also a common feature of financial price time series at very high-frequencies.

In Fig. 6 (Top), instead, we report the *instantaneous volatility* defined as $\nu(t) = |r(t)|$ where it is noticeable the presence of *clustering*, that is periods of high activity tend to be *patched* together. This memory effect is highlighted by the slowly decaying autocorrelation function, Fig. 6 (Bottom), and by the Hurst exponent estimate which gives the value of $H = 0.61(2)$, indicating a significant persistency. Besides, we have verified that the correlation, despite being weaker, is still present after the zero returns removal, $H_0 = 0.56(1)$. A similar behaviour is also observed in real markets, see, for example, in (Liu et al., 1997; Gopikrishnan et al., 1999; Liu et al., 1999).

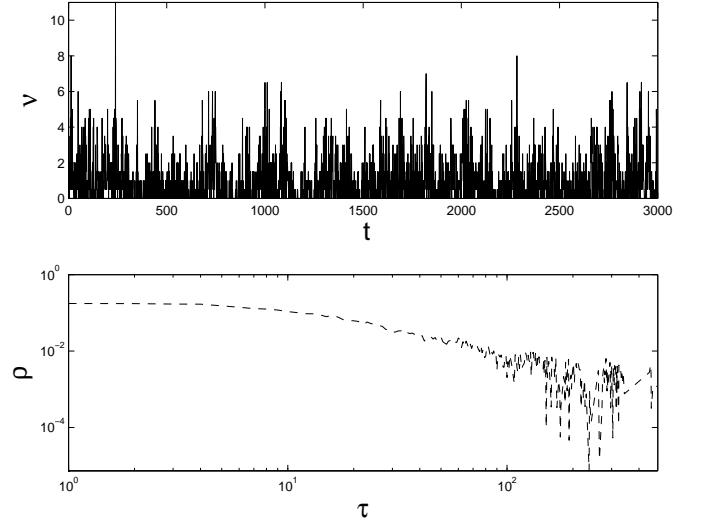


Fig. 6. (Top) Absolute value of returns, or instantaneous volatility, for $N = 10000$, $\phi_0 = 0.165$ and $L = 5$. Patches of high-volatility periods are visible. (Bottom) Autocorrelation function relative to the time series on top. The instantaneous volatility results to be correlated positively up to few hundreds steps.

As pointed out in the previous section, we would like to highlight the role played by the memory induced via the *perceived volatility*, Eq.(2), and, therefore, the parameter L , in the volatility clustering. In fact, the clustering phenomenon seems to be particularly relevant only when the former feedback is present: this is a strong indication that the use of short moving averages in high-frequency trading strategies, implicitly inducing memory effects, can be the source of the bursts of patched volatility observed at short time scales. Also note that, following the previous observation, in our model both the market sentiment, Eq.(4), and the cancellation process, Eq.(3), contribute to the volatility clustering. In fact, it can be shown via numerical experiments that by “switching off” the dependence from $\hat{\nu}(t)$ in one of these two terms, for example by substituting it with a constant, the clustering effect gets noticeably reduced.

3.2 Traded volume and impact

So far we have examined the behaviour of the price returns and volatility. However, both these quantities are related to other fundamental factors, one of the most important being the *traded volume*, V . Some authors, in fact, have recently argued that large transactions could be responsible for the non-Gaussian fluctuations observed in the fat tails of the returns distribution (Gabaix et al., 2003) even though more recent empirical work seems to suggest that *liquidity crises*, that is the temporary presence of gaps in the LOB, should be at the origin of those (Bouchaud et al., 2002; Daniels et al., 2003; Farmer et al., 2004; Lillo and Farmer,

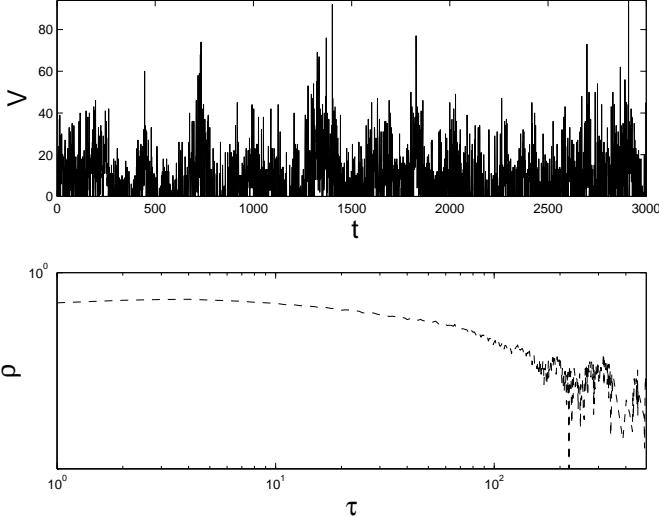


Fig. 7. (Top) Window of 3000 samples for the traded volume in the model along with the relative autocorrelation function (Bottom). The parameters used are $N = 10000$, $\phi_0 = 0.165$ and $L = 5$.

2005; Weber and Rosenow, 2005; Farmer, 2006; Weber and 2006). Moreover, the dynamics of the traded volume time series, like the volatility, has been observed to be characterized by clusters of intense activity followed by relatively quite periods (Lillo and Farmer, 2004).

For our model, the cumulative traded volume during each time step is reported in Fig. 7, where a behaviour very similar to that of volatility is evident. The intuitive relation between the two is further confirmed by their cross-correlation coefficient which we found greater than 30%. Moreover, the estimate Hurst exponent for the traded volume results $H = 0.70(2)$ ($H_0 = 0.66(1)$), matchings reasonably well the value found empirically in (Lillo and Farmer, 2004), namely $H = 0.73(7)$. It also worth pointing out that a possible introduction of order splitting in the execution part of the model, which is missing at the moment, should enhance this correlation.

We also estimate the average *instantaneous* change in price subsequent to a trade of a certain volume V , usually known as *impact function*. This quantity is very important in practical applications being a proxy for the liquidity of the market: its value indicates how, approximately, an order can penetrate “deep” into the LOB. The impact function for our model, Fig. 8, displays a linear behaviour as long as V is relatively small. However, due to the presence of “volume barriers”, as we will see in the next section, this proportionality is lost as the volume increases. While the same shape for the impact function is observed in real markets, see (Weber and Rosenow, 2005, 2006) for example, zero-intelligence models usually reproduce only the linear part, with the exception of (Farmer et al., 2004).

The values reported in Fig. 8 have also another important implication. In fact, the average price response to the

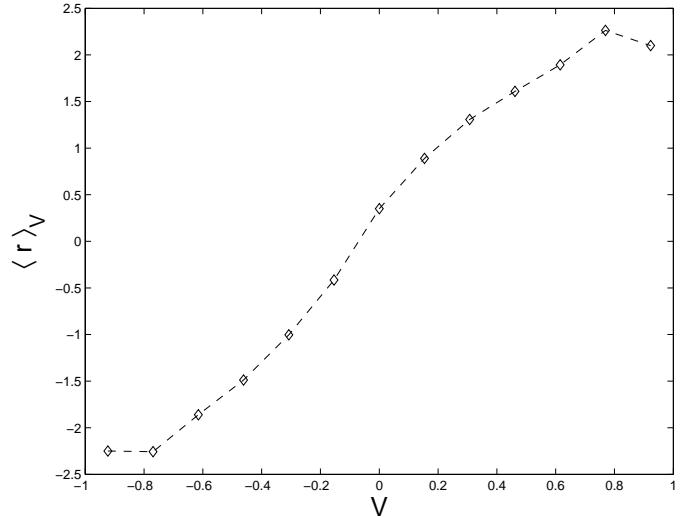


Fig. 8. Instantaneous impact function where the negative part corresponds to sell initialized orders. Note that while the impact is linear for $V \sim 0$, for large volumes this feature is lost. Also, the values of the traded volume in the plot have been rescaled by the maximum volume traded in order to fit the interval [-1 1]. In this case the rescaling has been made simply to facilitate the visual comparison of different impact functions in case of changing the parameters of the volume distribution, see Sec. 2.5. The simulation has been run with $N = 10000$, $\phi_0 = 0.165$ and $L = 5$.

large volumes is not enough to justify the extreme fluctuations observed in the pdf of returns that can be greater than 10 ticks, Fig. 5, which, therefore, have to be related or to rallies of orders in one direction or to a temporary lack of liquidity between the levels of the LOB. Of course nothing prevents both scenarios to be realized at the same time.

3.3 LOB average shape, spread and imbalance dynamics

In this final section we turn our attention to the way the orders are “on average” deployed in the LOB as well as the dynamics of some quantities related to the relative position of the orders such as the spread and the imbalance between buy and sell.

Firstly, we show the “mean” shape of the LOB, that is the average volume present at a fixed distance, $|P - P_m|$, from the mid-point price. From the plot, Fig. 9, it is clear that the liquidity increases steeply up to a maximum, located few ticks away from P_m , and then it starts to decay as we move away from it. The peak in volume can be thought as a sort of “volume barrier” which prevents large orders to get too deep into the LOB and, therefore, confirming the speculations made in the previous section. Very similar shapes have been found in empirical studies where it has been suggested that the asymptotic shape of

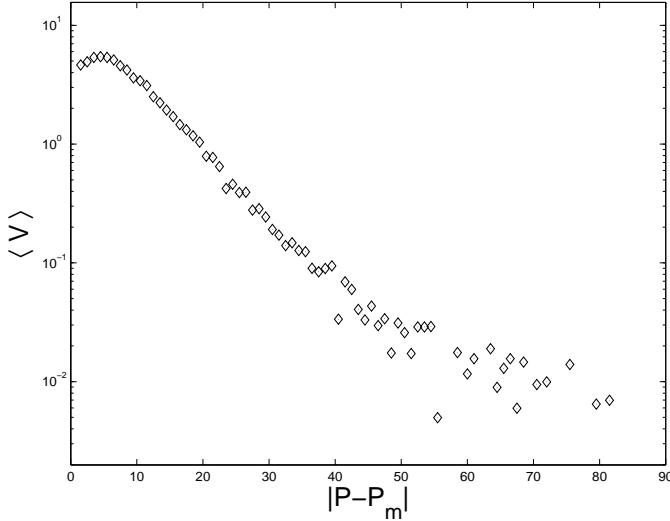


Fig. 9. Average shape of the limit order book reported against the distance from the mid-point price. Note that, given the symmetry of the model, we have aggregated the data on the ask and the bid side of the book so to have better statistics. The parameters used are the same as in Fig. 7.

the LOB, related to the “patient” traders, may follow a power law decay (Bouchaud et al., 2002). It is also important to stress that the smooth “mean” shape of the LOB is not significant for the “instantaneous” shape which, in fact, can be relatively sparse.

Another important quantity related to the placement of the orders in the LOB is the spread, S . This is the difference in price between the best ask and the best bid and it represents the cost that a trader has to pay up-front in order to execute a market order. Moreover, the spread is a further proxy for the liquidity: large spreads would indicate shallow markets while very liquid ones will tend to keep a very small spread at all times, possibly close to one tick. Dynamically, the spread displays persistency in time with $0.7 \lesssim H \lesssim 0.8$ (Plerou et al., 2005; Cajueiro and Tabak, 2007; Gu et al., 2007) depending on the market and the time scale of observation, while the asymptotic shape of its pdf can be characterized by a power law function (Plerou et al., 2005). The same two features appear also in our simulations, Fig. 10, even though the Hurst exponent result to be relatively smaller if compared to the empirical findings, namely $H = 0.56(2)$ ($H_0 = 0.56(2)$): this discrepancy may arise from the fact that at the moment we are considering only trivial execution algorithms.

Lastly, we focus our attention to the dynamic imbalance between buy and sell orders or simply the *volume imbalance*, $\Delta V(t)$, defined as

$$\Delta V(t) = \sum_{i=1}^{N_b(t)} V_i^b(t) - \sum_{i=1}^{N_a(t)} V_i^a(t), \quad (9)$$

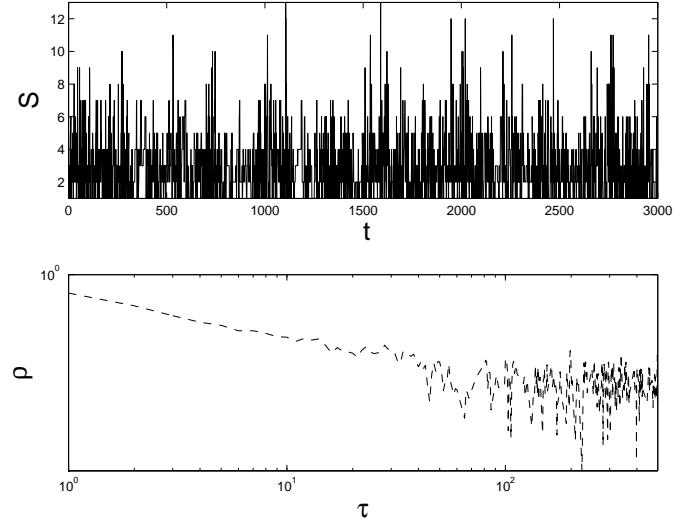


Fig. 10. (Top) A sample for the spread time series (in ticks): it is possible to notice how, occasionally, the spread can become relatively large. (Bottom) Autocorrelation function for the spread time series. The parameters used are the same as in Fig. 7.

where, for a time t , $V_i^{b,a}$ represents the volume of i th limit order and $N_{b,a}$ their total number, respectively, for the bid and the ask side of the LOB. The time series, Fig. 11, as expected for a market near-equilibrium, displays a mean reverting behaviour, that is the imbalance tends to fluctuate all time, more or less symmetrically, around a “pseudo-equilibrium” price, possibly close to criticality Bartolozzi et al. (2005, 2006). The same dynamics has been observed for ΔV in different futures contracts (Bartolozzi et al., 2007a).

4 Discussion and conclusions

In the present work we have developed a multi-agent framework characterized by a realistic order book keeping as a tool for the study of the activity of a double auction market at microscopic time scales. Inside this framework the model relies just on few basic assumptions related to the agent’s strategic behaviour. Among the most important ones, the order submission process makes use of a stochastic variable, the *market sentiment*, which is related to both the public and the private information. Besides, the agents lack of the concept of “utility maximization” often invoked in the economics literature and embodying, in mathematical terms, the concept of “rationality”.

Despite its simplicity, the model manages to reproduce several empirical features of the high-frequency dynamics of the stock market such as the negative correlation in market returns, clustering in the trading activity (such as volatility, traded volume and bid-ask spread) as well as a non-linear response of the price change to the volume

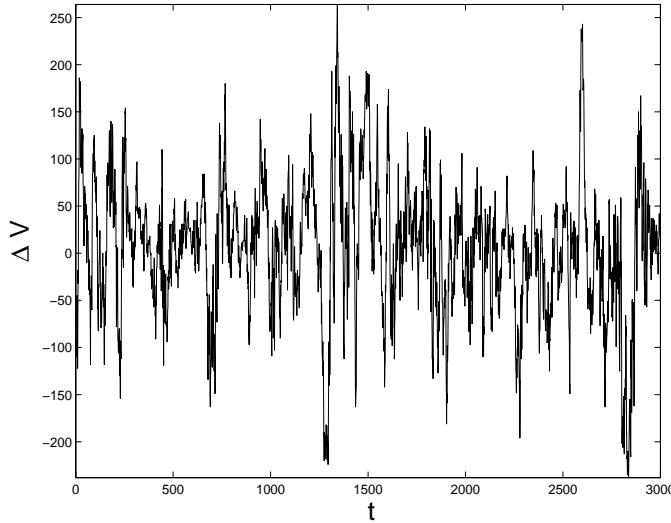


Fig. 11. (Top) Time series of imbalance between demand and supply displaying a clear mean reverting nature. The parameters used for the simulation are the same as in Fig. 7.

traded. Moreover, the similarities with the real markets extend also in the way the orders are deployed on the LOB as observed through the average shape of the book and the volume imbalance time series.

In conclusion, our model indicates that large part of the dynamics of the stock market at very short time scales can be explained without the requiring any particular rational approach from an agent prospective if not some memory feedback which, in our model, are represented by short term moving averages of the public information. Moreover, our results confirm that large price movements are more likely to be related to a temporary lack of liquidity in the LOB rather than to large volume transactions.

Our future work will involve adding more realistic features and feedbacks in the model. In particular, one interesting effect to take into consideration would be the *herding phenomenon* which, it has been argued, to be at the origin of dramatic liquidity crises which can ultimately lead to financial crashes (Sornette, 2004). Other factors that can claim to play a fundamental role at these time scales are *news realize* and *market making strategies*: both of them should to be taken into account.

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