# Price response in correlated financial markets: empirical results

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Previous studies of the stock price response to individual trades focused on single stocks. We empirically investigate the price response of one stock to the trades of other stocks. How large is the impact of one stock on others and vice versa? — This impact of trades on the price change across stocks appears to be transient instead of permanent. Performing different averages, we distinguish active and passive responses. The two average responses show different characteristic dependences on the time lag. The passive response exhibits a shorter response period with sizeable volatilities, and the active response a longer period. We also study the response for a given stock with respect to different sectors and to the whole market. Furthermore, we compare the self–response with the various cross–responses. The correlation of the trade signs is a short–memory process for a pair of stocks, but it turns into a long–memory process when averaged over different pairs of stocks.

## I. INTRODUCTION

The trading at stock exchanges is organized by the order book whose main purpose is to provide the same information to all market participants. Although often ignored in the model building, it has a large impact on the price dynamics and thus on the stylized facts as well as on the more specific features [1–9]. The stock price is determined via a continuous double auction [10], in which some traders submit market orders for immediate trans-actions at the best available price, while other traders submit limit orders which specify an acceptable price for the trade. The limit orders are listed in the order book. Most of them do not immediately lead to trades. The buy limit orders are referred to as bids and the sell limit orders as asks. The best ask and best bid prices are the quotes. Market orders do not appear in the order book. When a market order is executed, it can either keep the quote unchanged, raise the best ask price in case of a buy market order or lower the best bid price for a sell market order. The prices change persistently as they are affected by the incoming market orders. To profit from the price difference between ask and bid, traders provide the limit orders which leads to an anti–persistence of prices. As a result of a detailed balance between persistent and anti–persistent, *i.e.*, between super– and subdiffusive behavior, the price on an intraday scale moves diffusively like a random walk [11].

In recent years, a high autocorrelation of the order flow was empirically found [11–14]. The splitting of orders over longer times introduces the long memory of the order flow [13] with remarkable persistence. Buy (sell) orders are often followed by more buy (sell) orders. Furthermore, the relation between trades and price changes has received considerable attention [2, 4, 11, 15–21] The Efficient Market Hypothesis (EMH) [22] states that all available information is processed and encoded in the current price, which would rule out any (statistical) arbitrage opportunities. While this is in conflict with the very different time scales on which, first, relevant new information arrives and, second, the prices change, the model of Zero Intelligence Trading (ZIT) [23] simply assumes randomly acting trader, but also arrives at a memory–less random walk.

Based on the EMH, there are two major approaches to explain the impact of trades on the stock price change. The first approach put forward by Lillo and Farmer (LF) [12], suggests that the price impact is permanent, but fluctuates with order size. The impact is caused by an asymmetry in liquidity which is induced by the trade. The response exhibits a power–law relation between order size and price change [12, 24–26]. In the second approach, Bouchaud, Gefen, Wyart and Potters (BGPW) [11] argue that the price impact is transient, but fixed with order size. The fact that the impact decays with time is a result of price mean reversion. Moreover, they identify the relation between order size and price response as logarithmic [27]. Gerig [28] suggests that the two approaches LF and BGPW are equivalent and can be related by exchanging variables. He also argues that the impact comes from the asymmetric liquidity rather than the price mean reversion.

There are numerous studies devoted to the price response, but they all focus on one single stock. Here, we go beyond this and investigate the role of correlations. We carry out a large–scale empirical study of real–time trade data and find a non–vanishing price response across different stocks. We shed light on the price impact from trades in different stocks by discussing the efficiency of the financial market and by analyzing how the stocks respond to the whole market and to different economic sectors. We thereby present a first complete view of the response in the market as a whole and identify several structural characteristics.

The paper is organized as follows. In Sect. II, we

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present our data set of stocks and provide some basic definitions. In Sect. III, we show the empirical results, which indicate the existence of trade sign correlation and price response in different stocks, we also estimate the response noise. In Sect. IV, we discuss the trade impact on the prices from the viewpoint of market efficiency. We introduce and work out two types of average response in Sect. V, an active and a passive one. In Sect. VI, we compare the self–response with the various cross–responses. We give our conclusions in Sect. VII.

## II.   DATA DESCRIPTION AND TIME CONVENTION

In Sect. II A, we present the data set that we use in our analysis. We discuss the proper choice of time convention in Sect. II B.

### A.   Data set

Our study is based on the data from NASDAQ stock market in the year 2008. NASDAQ is a purely electronic stock exchange, whose Trades and Quotes (TAQ) data set contains the time, price and volume. This information is not only given for the trades with all successive transactions, but also for the quotes with all successive best buy and sell limit orders.

To investigate the response across different stocks in Sect. III, we select six companies from three different economic sectors traded in the NASDAQ stock market in 2008. The stocks we analyzed are listed in Table I together with their acronyms and the corresponding economic sectors.

When studying the market response in Sect. IV, we select the first ten stocks with the largest average market capitalization in each economic sector of the S&P 500 index in 2008, except for the telecommunications services where only nine stocks were available in that year. We recall that the market capitalization is the trade price multiplied with the traded volume, and the average is performed over every trade during the year 2008. The selected 99 stocks are listed in App. A. The 99 stocks are also ranked according to strongest passive and active responses in Sect. V B.

TABLE I. Company information

| Company | Symbol | Sector |
|---|---|---|
| Apple Inc. | AAPL | Information technology |
| Microsoft Corp. | MSFT | Information technology |
| Goldman Sachs Group | GS | Financials |
| JPMorgan Chase | JPM | Financials |
| Exxon Mobil Corp. | XOM | Energy |
| Chevron Corp. | CVX | Energy |

For the average responses of an individual stock in Sect. V, the stocks AAPL, GS, XOM are selected as examples. The necessary average are performed over the remaining 495 stocks in the S&P 500 index or over the stocks in a given economic sector. Here, we neglect the self-response of the stocks.

We only consider the common trading days in which the trading of stocks $i$ and $j$ took place. This is so because the trades of one stock $j$ in one day would not impact the intraday price of another stock $j$ without any trade in that day, and *vice versa.*

### B.   Physical versus trading time

While studies on the response in single stocks typically employ trading time as time axis, this is not useful when studying the response across different stocks, because each stock has its own trading time. Hence, we have to use the real, physical time. We project the data set to a discrete time axis. The quote data and the trade data of each stock are in two separate files with a time–stamp accuracy of one second. However, more than one quote or trade may be recorded in the same second. Due to the one–second accuracy of the time–stamps, it is not possible to match each trade with the directly preceding quote. Hence, we cannot determine the trade sign by comparing the traded price and the preceding midpoint price. This latter definition of the trade sign was employed by Lee and Ready [29]. Instead, we here define the trade signs similarly to the tick rule of Holthausen, Leftwich and Mayers [30]. They define the trade as buyer–initiated (seller–initiated) if the trade is carried out at a price above (below) the prior price. Zero tick trades are not classified in general. The tick rule has an accuracy of 52.8% [30]. For our study, we further develop this method: as our data has a one–second accuracy in time, we consider the consecutive time intervals of length one second. Let $t$ label such an interval and let $N(t)$ be the number of trades in that interval. The individual trades carried out in this interval are numbered $n = 1, \ldots, N(t)$ and the corresponding prices are $S(t; n)$. We define the sign of the price change between consecutive trades as

$$\varepsilon(t;n) = \begin{cases} \mathrm{sgn}\big(S(t;n) \ - \ S(t;n-1)\big) , \\ \qquad\qquad \text{if } S(t;n) \neq S(t;n-1), \\ \varepsilon(t;n-1) \qquad , \text{ otherwise.} \end{cases} \tag{1}$$

If two consecutive trades of the same trading direction together did not exhaust all the available volume at the best price, the price of both trades would be the same. Thus, we set the trade sign equal to the previous trade sign in this case. If there is more than one trade in the interval denoted $t$, we average the corresponding trade signs,

$$\varepsilon(t) = \begin{cases} \mathrm{sgn}\left(\sum\limits_{n=1}^{N(t)} \varepsilon(t;n)\right) , & \text{if } N(t) > 0 , \\ 0 , & \text{if } N(t) = 0 , \end{cases} \tag{2}$$

which formally also includes the case $N(t) = 1$. Consequently $\varepsilon(t) = +1$ implies that the majority of trades in second $t$ was triggered by a market order to buy and, a value $\varepsilon(t) = -1$ indicates the majority of sell market orders. We have $\varepsilon(t) = 0$, whenever trading did not take place in the time interval $t$ or if there was a balance of buy and sell market orders. In order to avoid overnight effects and any artifacts at the opening and closing of the market, we consider only trades of the same day from 9:40 to 15:50 New York local time.

## III. RESPONSE FOR PAIRS OF STOCKS

To study the mutual dependences between stocks, we consider sets of two different stocks, to which we refer as pairs. We introduce a response function as well as a trade sign correlator for such stock pairs in Sects. III A and Sect. III B, respectively. After an empirical analysis of these quantities, we discuss a certain noise in Sect. III C. In the sequel, all quantities referring to a particular stock carry its indexed $i$, quantities referring to a pair carry two such indices. We consider eight pairs of the stocks listed in Table I, four within the same economic sector, four across different economic sectors.

### A. Response functions

To measure how a buy or sell order of stock with indexed $j$ at time $t$ influences the prices of the stock $i$ at a later time $t+\tau$, we introduce a new response function. We employ the logarithmic price differences or log–returns for stock $i$ and time lag $\tau$, defined via the midpoint prices $m_i(t)$,

$$r_i(t,\tau) = \log m_i(t+\tau) - \log m_i(t) = \log \frac{m_i(t+\tau)}{m_i(t)} \quad (3)$$

at a given time $t$, keeping in mind the one–second accuracy. To acquire statistical significance, the response function is the time average

$$R_{ij}(\tau) = \left\langle r_i(t,\tau)\varepsilon_j(t) \right\rangle_t \quad (4)$$

of the product of time–lagged returns and trade signs for stocks $i$ and $j$, respectively. Our empirical results of this response function for different stock pairs $(i, j)$ are shown in Fig. 1 versus the time lag. In all cases, an increase to a maximum is followed by a decrease, i.e. the trend in the response is eventually always reversed. This does not depend on whether or not the pairs are in the same economic sector or extend over two sectors.

The stocks face similar systematic risks, leading to stronger response in the same sector than across different sectors. However, strong responses for the stock pairs from different sectors also exist, e.g. for (GS, AAPL). Apart from reasons specific for the stock pair considered,

this might also be related to how investors assemble their portfolios. To disperse the investment risks, the portfolios often comprise stocks from different sectors since they are exposed to different economic risks and are less correlated than stocks within the same sector. When investors buy or sell the stocks in their portfolios gradually, it may produce responses and sign correlations in different stocks. We measure the strength of the sign correlation in Sect. III B.

Fig. 1 shows that the response increases again after decreasing back at large time lag $\tau$, i.e., $\tau$ close to 1000 s. We attribute this to the response noise that we introduce in Sect. III C.

### B. Trade sign correlator

The existence of sign correlations is the main reason that causes the response in a single stock [11]. For pairs of stocks, we introduce the trade sign correlator

$$\Theta_{ij}(\tau) = \left\langle \varepsilon_i(t+\tau)\varepsilon_j(t) \right\rangle_t \quad (5)$$

as a function of the time lag $\tau$. As we demonstrate in Fig. 1, there are a non–zero correlations across stocks. It turns out that the empirical results can be fitted well by the power law

$$\Theta_{ij}(\tau) = \frac{\vartheta_{ij}}{\left(1 + (\tau/\tau_{ij}^{(0)})^2\right)^{\gamma_{ij}/2}} . \quad (6)$$

To estimate the error, we use the normalized $\chi_{ij}^2$ (see App. B). The parameters for the best fit as well as the $\chi_{ij}^2$ values for the analyzed eight stock pairs are listed in Table II.

In contrast to the sign correlation in one single stock, the stock pair correlations exhibit short memory with exponents $\gamma_{ij} \geq 1$ rather than long memory, which usually is defined as corresponding to exponents smaller than unity [31]. This indicates that the price change of one

TABLE II. Fit parameters and normalized $\chi_{ij}^2$ for the trade sign correlators.

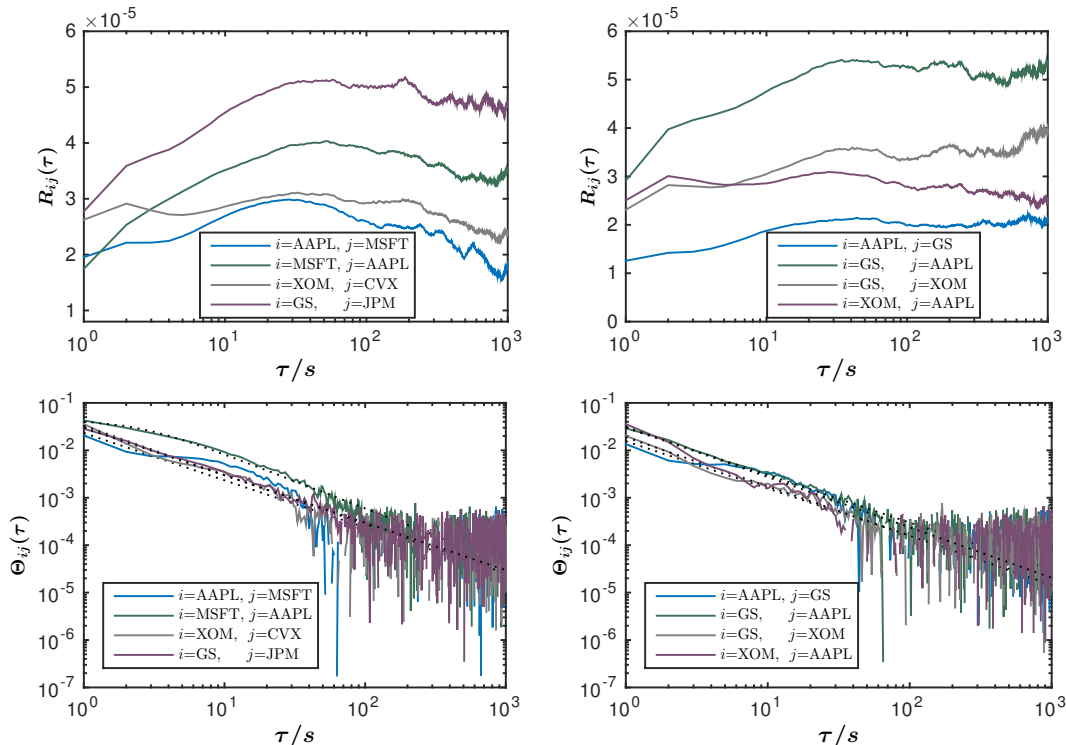| stock $i$ | stock $j$ | $\vartheta_{ij}$ | $\tau_{ij}^{(0)}$ [s] | $\gamma_{ij}$ | $\chi_{ij}^2$ ($\times 10^{-6}$) |
|---|---|---|---|---|---|
| AAPL | MSFT | 0.46 | 0.05 | 1.00 | 0.23 |
| MSFT | AAPL | 0.04 | 2.34 | 1.15 | 0.10 |
| XOM | CVX | 0.61 | 0.06 | 1.04 | 0.07 |
| GS | JPM | 0.45 | 0.07 | 1.00 | 0.04 |
| AAPL | GS | 0.46 | 0.03 | 1.00 | 0.11 |
| GS | AAPL | 0.49 | 0.06 | 1.00 | 0.05 |
| GS | XOM | 0.61 | 0.04 | 1.04 | 0.04 |
| XOM | AAPL | 1.18 | 0.03 | 1.06 | 0.13 |

FIG. 1. Response functions $R_{ij}(\tau)$ in 2008 versus time lag $\tau$ on a logarithmic scale (top left and right). Corresponding trade sign correlator $\Theta_{ij}(\tau)$ for different stock pairs on a doubly logarithmic scale, fits as black dotted lines (bottom, left and right). Stock pairs from the same economic sector (left), pairs of stocks from different sectors (right).

stock responding to the trades of another stock only persist for shorter times, and the response reverses at relatively small time lags $\tau$. We notice the large fluctuations of the trade sign correlator at larger lags $\tau$. They are partly due to the decrease of the response signal, but also to the limited statistics. The larger the time lag $\tau$, the larger is the overlap of the lag $\tau$ for different times $t$. When averaging the sign correlation over every second $t$ with large $\tau$, the result has poor statistics.

### C. Response noise

As pointed out above, the response functions and the sign correlators strongly fluctuate during the decay. Here, we address this point by introducing the response noise $\nu_{ij}(\tau)$ as an estimator: We determine the number $T_{ij}^{(c)}$ common to stocks $i$ and $j$ in which trading took place. We label these days with a running integer number and separate our data into two sets, for days with even and odd numbers, respectively. We work out the corresponding response functions $R_{ij}^{(k)}(\tau)$ with $k = 1, 2$ for the averages over odd or even days. Each of these two functions should be very close to the response func-

tion $R_{ij}(\tau)$ averaged over all days. Thus, we introduce a response noise as some kind of normalized Euclidian distance

$$\nu_{ij}(\tau) \;=\; \frac{1}{|R_{ij}(\tau)|}\sqrt{\frac{1}{2}\sum_{k=1}^{2}\left(R_{ij}^{(k)}(\tau) - R_{ij}(\tau)\right)^2} \quad (7)$$

for each value of the time lag $\tau$. In Fig. 2 we present the empirical results for the response noise during the year 2008. Obvious, most stock pairs do not suffer from large response noise for time lags smaller than about 120 seconds. During this period, the noise lies below a value of about 0.06. With increasing time lag, the noise becomes much strong, indicating unstable response. The largest noise reaches values of more than 0.25 for lags tending towards 1000 seconds. This is the reason why some stock pairs show upwards trends after reversing back. As the sign correlator weakens in the regime of large time lag, other factors dominate leading to the large response fluctuations. Limited statistics blurs the picture, since there are only 22200 seconds of effective trading time in each trading day. This clearly demonstrates that, when looking at the response of a stock pair, the lags considered must not be too large to obtain meaningful results. In the sequel, we overcome the problems related to the lim-
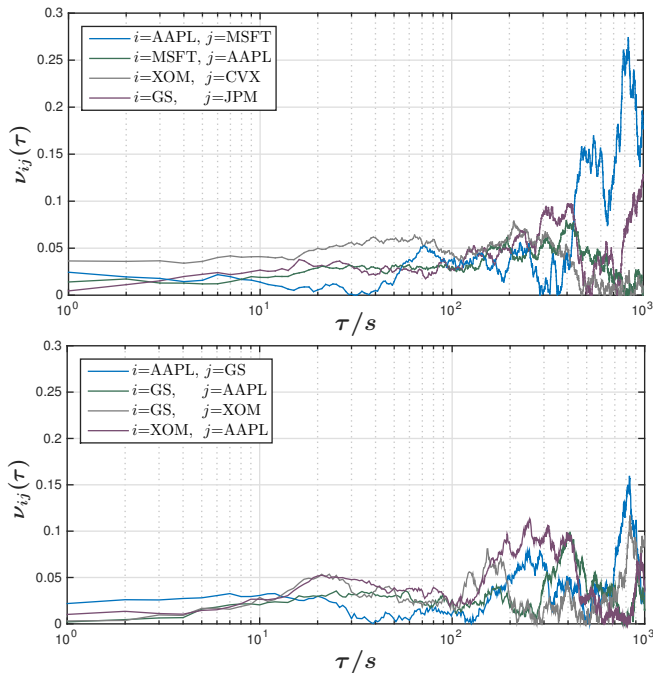
FIG. 2. Response noise $\nu_{ij}(\tau)$ for stock different pairs during the year 2008 versus the time lag $\tau$ measured on a logarithmic scale. Stock pairs from the same economic sector (top), pairs of stocks from different sectors (bottom).

ited statistics, by further averaging the response functions over different stock pairs.

## IV. MARKET RESPONSE

The response functions and the trade sign correlators we considered up to now give us a kind of microscopic information for stock pairs. It is equally important to investigate how the trading of individual stocks influences the market as a whole. In a first step, we tackle this question by introducing the market response as the matrix $\rho(\tau)$ whose entries are the normalized response functions at a given time lag,

$$\rho_{ij}(\tau) \;=\; \frac{R_{ij}(\tau)}{\max\left(|R_{ij}(\tau)|\right)} \;, \tag{8}$$

where the denominator is the maximum over all stock pairs $(i, j)$ for fixed $\tau$. This object is reminiscent of, but should not be mixed up with a correlation matrix. Importantly, the matrix of the market response is not symmetric, $R_{ij}(\tau) \neq R_{ji}(\tau)$, as to different quantities, the returns and the trade signs, enter the definition Eq. (4). Furthermore, the market response reveals information about the time evolution.

Our empirical analysis is depicted in Fig. 3 for a market with 99 stocks (see App. A). In Fig. 3 we show the $99 \times 99$ matrices of the market response for different time

lags $\tau = 1, 2, 60, 300, 1800, 7200$ s in the year 2008. The diagonal strip is simply the response of the stock to itself. In general, the price change of one stock is always affected by the trading of all others, and *vice versa*. The stocks are ordered according to the economic sectors, and the six matrices of the market response in Fig.3 feature striking patterns of strips which can be associated with these sectors. For example, the information technology (IT) sector produces a visibly strong strip over almost all other sectors. This effect is quite stable over time. It is worth mentioning that the price responses vary from sector to sector. For example, the energy (E) also has strong response but utilities (U) have weaker response.

As seen in Fig. 3, the market response is mainly positive up to time lags of about $\tau = 7200$ s, while negative
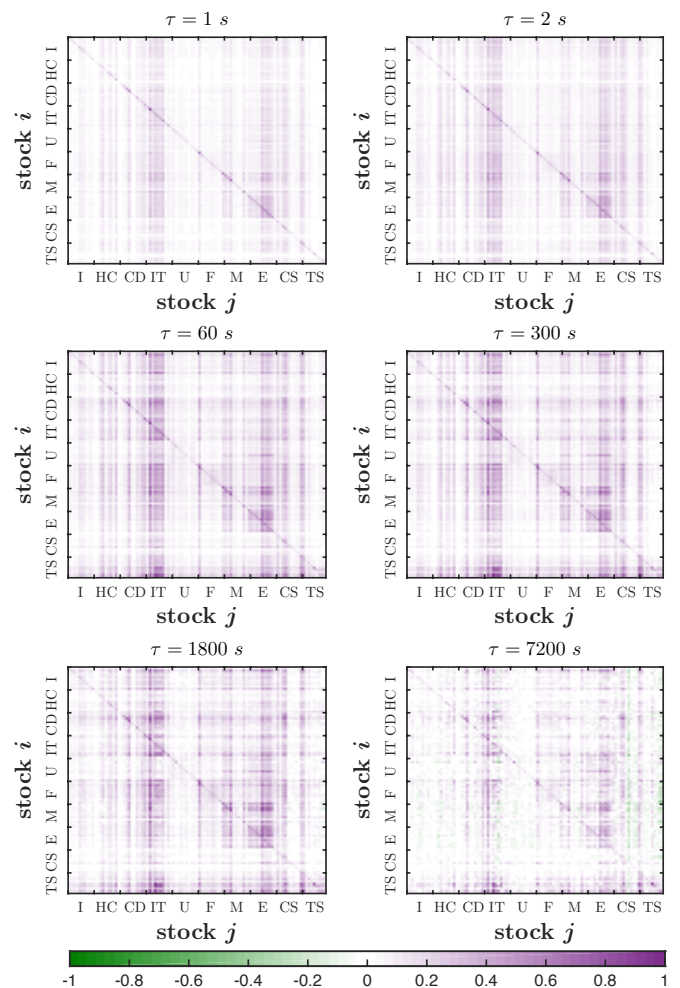


FIG. 3. Matrices of market response with entries $\rho_{ij}(\tau)$ for $i, j = 1, \ldots, 99$ at different time lags $\tau = 1, 2, 60, 300, 1800, 7200$ s in the year 2008. The stocks pairs $(i, j)$ belong to the sectors industrials (I), health care (HC), consumer discretionary (CD), information technology (IT), utilities (U), financials (F), materials (M), energy (E), consumer staples (CS), and telecommunications services (TS).

responses show up later. According to the Efficient Market Hypothesis (EMH) [22], the price encodes all available information, implying that arbitrage opportunities do not exist. As the response functions measure the price changes caused by trade signs, they should according to the EMH be zero, for one single stock as well as across different stocks. However, the empirical response functions for one single stock already demonstrated that the existence of non–zero response value [11]. Here, we go beyond this. The non–zero response functions across different stocks that we find show the lack of efficiency for the market as a whole. Our results allow us to extend the interpretation put forward in Ref. [11]. The impact of trades on the prices is transient with lag–dependent characteristics. The trends due to potentially information–driven trading by one or several of the market participants will be reversed by others who act as arbitrageurs until a state that is compatible with the EMH is reached again. This process involves the market as a whole, not only the stock that is traded because of potential information. The market needs more time to respond to all the potential information before becoming efficient again. We showed in Sect. III that the responses fluctuate at large time lags due and interpreted this as a noise effect. However, for the whole market, these fluctuations are washed out by a self–averaging process amounting to

$$\overline{R}(\tau) \ = \ \langle\langle R_{ij}(\tau)\rangle_j\rangle_i \ , \qquad (9)$$

where $i = j$ is excluded. The average response $\overline{R}(\tau)$ for the whole market is shown in Fig. 4 versus the time lag $\tau$. An increase of $\overline{R}(\tau)$ is followed by a decrease similar to the response for a stock pair. For the whole market, however, the decay takes longer and is observable for time scales up to three hours. We conclude that the market impact is transient. Efficiency is restored only after these decay processes.

## V.   AVERAGE RESPONSE FUNCTIONS

We define passive and active response functions in Sect. V A, and then analyze them for the whole market and for economic sectors in Sect. V B and V C.

### A.   Passive and active response functions

As seen in Fig. 1, the responses of one stock to other stocks varies quite a bit. For example, the three stock pairs $(GS, j)$ with $j = $ JPM, AAPL, XOM, show different responses dependent on $\tau$. There are similar differences for the three stock pairs $(i, AAPL)$ with $i = $ MSFT, GS, XOM. Moreover, the market response displayed in Fig. 3, quantifies the mutual impacts. Typically, a given stock is related to several or many others by trading. As already mentioned, that is partly due to the grouping of investments in portfolios, but there are
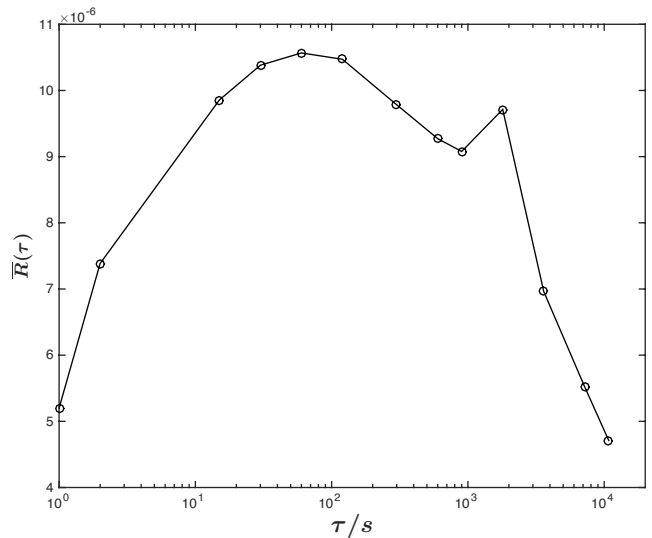


FIG. 4. Average response function $\overline{R}(\tau)$ for the whole market in 2008 versus time lag $\tau$ on a logarithmic scale.

may other reasons for the mutual impact. Suppose, for example, a trader who considers AAPL as presently underpriced and likely to raise in the near future. To buy many shares of AAPL he might use the profit from selling other stocks. If many others act correspondingly, an impact results: buying (selling) AAPL affects the stocks which are sold (bought). By discussing this scenario, we want to motivate that averaging the response functions over different stocks that are paired with the same stock can yield interesting new observations. Furthermore, such averages will also to some extent smoothen the drastic fluctuations of the sign correlations at large time lags, cf., Fig. 1, and reduce the response noise, cf., Fig. 2. As the definition Eq. (4) of the response functions is not symmetric with respect to the indices, we can perform two conceptually different averages,

$$R_i^{(p)}(\tau) = \langle R_{ij}(\tau)\rangle_j \quad \text{and} \quad R_j^{(a)}(\tau) = \langle R_{ij}(\tau)\rangle_i, \quad (10)$$

to which we refer as passive and active response functions. Importantly, the self–responses for $(i, i)$ or $(j, j)$ are excluded in these averages. The passive response function $R_i^{(p)}(\tau)$ measures, how the price of stock $i$ changes due to the trading of all other stocks, while the active response function $R_j^{(a)}(\tau)$ quantifies which effect the trading of stock $j$ has on the prices of all other stocks. Correspondingly, we also introduce

$$\Theta_i^{(p)}(\tau) = \langle \Theta_{ij}(\tau)\rangle_j \quad \text{and} \quad \Theta_j^{(a)}(\tau) = \langle \Theta_{ij}(\tau)\rangle_i, \quad (11)$$

as passive and active trade sign correlators. We notice that they are not symmetric either, because the time lag only enters the trade sign with index $i$.

## B. Average responses of an individual stock to the market

We carry out the empirical analysis for the stocks AAPL, GS, XOM by averaging their response functions over other 495 stocks in the S&P500 index. The results for passive and active response functions as well as the corresponding passive and active trade sign correlators are presented in Fig. 5. We checked that these empirical results are similar to those from averaging over the other 98 stocks seen in App. A. To facilitate the computation, we only calculate the average response values at several time lags, at the marked positions shown in Fig. 5, rather than at every second as in Sect. III.

The passive and active response functions clearly show different behaviors. The passive response reverses faster than the active one. It only persists dozens of seconds and then reverses to drop down quickly with sizeable volatility. In contrast, the active response reverses at time lags of some hundreds of seconds and the price changes slowly. An obvious reason for this difference is the easier detectability of price changes in one stock than of those dispersed over different stocks. Again, with our results, we can extend the previous interpretations based on the study of single stocks. The passive response function reflects the price dynamics on short time scales. When the price goes up, less market orders to buy will be emitted and more limit orders to sell. Thus, the price reverses [32] without a need to evoke new information as cause. Moreover, liquidity induced mean reversion attracts more buyers, which motivates liquidity providers to raise the price again, while the volatilities in this process of responding decline. Thus, we conclude for the market as a whole that the mean reversion accentuates the short–period price volatility, which is consistent with the single–stock analysis [33, 34]. The active response reflects the dispersion of the trade impacts over the prices of different stocks. It is conceivable that this process takes longer than compared with the time scales of the passive response. Furthermore, this dispersion is accompanied by a spreading out of the volatilities.

As visible in Fig. 5, the active response is about five times stronger than the passive one at the maximum values. The reason for different strength of passive and passive responses is the existence of strongly influential stocks. In Fig. 3, we observe that the vertical stripes are much more pronounced than the horizontal ones. More specifically, there are groups of stocks which have a strong influence across most of the market, in particular, this is evident for the stocks in the IT sector. Consequently, the active response of these stocks averaged over the market shows a strong signal. To the passive response, however, fewer influenced stocks really contribute, which leads to reduced average.

To identify the strongly influential and influenced stocks, we rank the 99 stocks in App. A according to the numerical values of passive and active response functions, normalized according to Eq. (8), at a given time lag $\tau$. The first fifteen stocks with strongest average response at $\tau = 1, 2, 60, 300$ s are shown in Fig.6. As seen, FTR has stronger passive responses for $\tau \geq 60$ s, implying that its price is more easily impacted by the trades of other stocks. As a result, it has stronger passive response than active response, opposite to the cases of AAPL, GS and XOM in Fig. 5. In contrast, AAPL, XOM, MSFT and other stocks have stronger active responses, which means the trades of these stocks are more likely to impact the prices of other stocks. Interestingly, the ranking of individual stocks in the active responses looks similar at different time lags. This matches the relatively stable response structure visible in Fig. 3.

To analyze the average trade sign correlators depicted in Fig. 5, we use the power law Eq. (6) to fit the empirical results. The fitting parameters and errors are shown in Table III. The remarkable result is that the volatile short memory of the individual correlators turns into a long memory with exponents smaller than one after averaging. The only exception is the passive trade sign correlator for the stock XOM. We thus infer that the price changes caused by trade sign correlations in different stocks can accumulate to persist over longer times.

## C. Average responses of an individual stock to economic sectors

Another observation which can be made in Fig. 3 is that the responses vary for different economic sectors. In other words, the stocks from different sectors may produce different average responses to a given stock. We calculate the average response of the stocks AAPL, GS and XOM to ten economic sectors in the S&P 500 index. The passive and active responses are displayed in Figs. 7 and 8, respectively. Clear differences are seen.

Regarding the passive responses, the prices of the three stocks considered are all affected by the trades within their own sectors, especially XOM, which is not surprising due to common economic effects. Moreover, the price of the stock AAPL is also easily influenced by both en-

TABLE III. Fit parameters and $\chi^2$ of the average trade sign correlators.

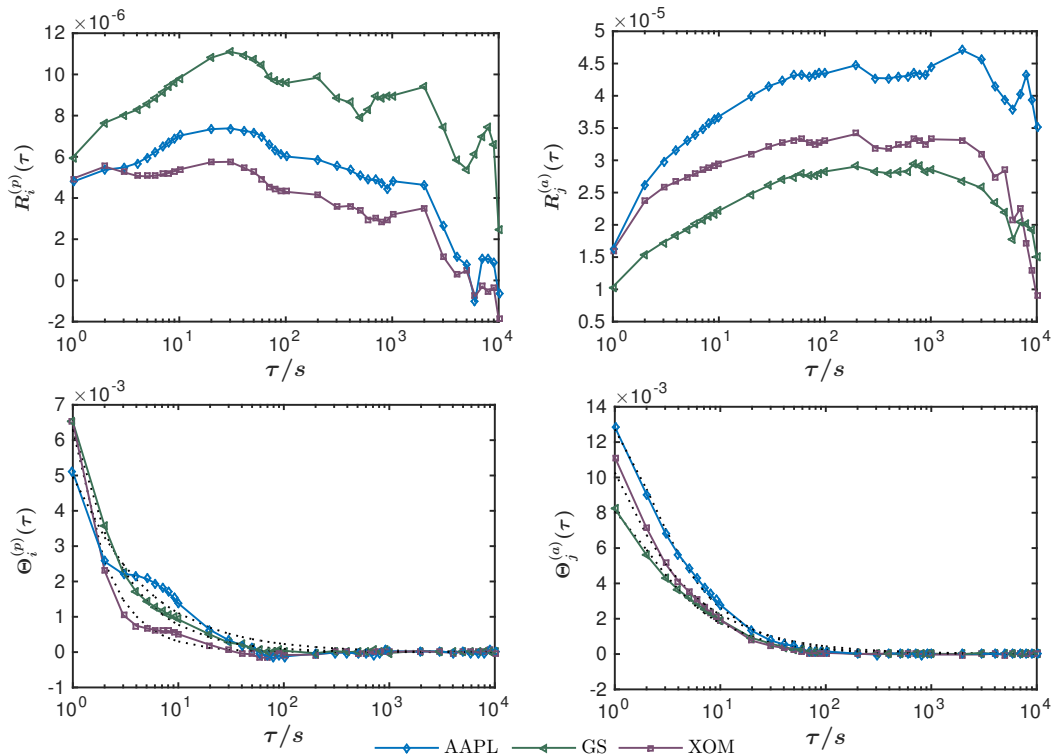| Sign correlators | Parameters and errors | Stocks $i, j$ | | |
|---|---|---|---|---|
| | | AAPL | GS | XOM |
| $\Theta_i^{(p)}(\tau)$ | $\vartheta_i$ | 0.01 | 0.03 | 0.27 |
| | $\tau_i^{(0)}$ [ s ] | 0.47 | 0.23 | 0.06 |
| | $\gamma_i$ | 0.68 | 0.92 | 1.32 |
| | $\chi_i^2 (\times 10^{-7})$ | 0.70 | 0.07 | 0.19 |
| $\Theta_j^{(a)}(\tau)$ | $\vartheta_j$ | 0.02 | 0.01 | 0.02 |
| | $\tau_j^{(0)}$ [ s ] | 1.44 | 1.31 | 0.55 |
| | $\gamma_j$ | 0.90 | 0.85 | 0.71 |
| | $\chi_j^2 (\times 10^{-7})$ | 0.28 | 0.18 | 1.06 |

FIG. 5. Passive and active response functions $R_i^{(p)}(\tau)$ and $R_j^{(a)}(\tau)$ for $i, j = \text{AAPL}, \text{GS}, \text{XOM}$ in the year 2008 versus time lag $\tau$ on a logarithmic scale (top left and right). Corresponding passive and active trade sign correlators $\Theta_i^{(p)}(\tau)$ and $\Theta_j^{(a)}(\tau)$, fits as black dotted lines (bottom, left and right).

ergy (E) and financials (F), similar observation hold for the price of GS in relation to information technology (IT) and energy (E). Regarding the active response, the trades of AAPL and GS have a significant impact on the prices of the stocks from financials (F) significantly, but a lesser one utilities (U), health care (HC), and consumer staples (CS). This might be due to the stability of these sectors which serve the needs of daily life. The trades of XOM are more likely to influence energy (E), but have a lesser effect on health care (HC) and consumer staples (CS). This is so because utilities (U) are economically more strongly coupled to energy (E) than to health care (HC) and consumer staples (CS).

## VI. COMPARISONS BETWEEN SELF-AND CROSS-RESPONSES

It is important to compare the self–response to the various cross-responses. In Fig. 9, we show the self–responses for AAPL, GS and XOM together with cross-responses as well as together with the active and passive, *i.e.* with averaged responses.

Typically, the self–response is stronger than the cross–response for two stocks due to the strong auto-correlation

of trade signs. The example of XOM, however, shows that the cross–response can be stronger than the self–response. From Fig. 6, we know the AAPL and CVX are influential stocks. Their trades are more likely to impact the price change of other stocks, such that the influenced stock, *i.e.* XOM, responds to them more strongly than to itself.

The average responses are always weaker than the self–response, implying that cases such as XOM are rare. Due to the noise reduction we can follow the average responses over a longer time interval. On shorter scales of the time lag $\tau$, both, self– and cross–responses should be considered when looking at individual stocks, but for investigating response stability, persistence or efficiency of the market as a whole, the average quantities give useful information on longer scales.

Fig. 10 shows the trade sign self– and cross–correlators for AAPL, GS and XOM corresponding to Fig. 9. For the correlators between two stocks, the correlation differences are observable for time lags of less than 10 seconds. With the lag increasing, the self–correlators are close to the cross–correlators for AAPL and GS. The exception of XOM, however, is visible again in the sign correlators. In the research of the self–response [11], a 'bare' impact function with a power–law decay for a single trade is
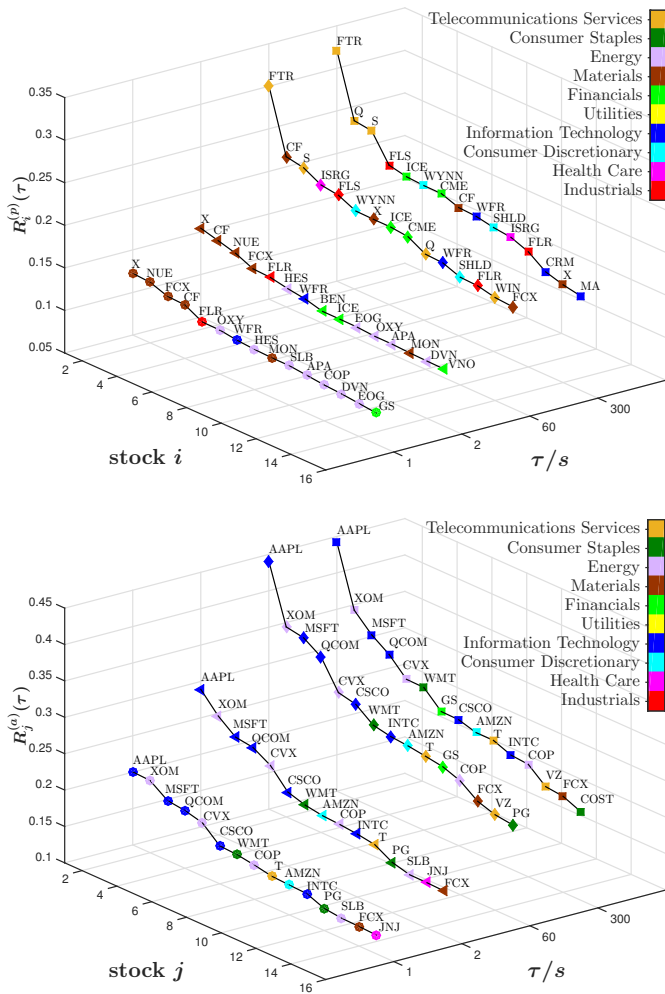
FIG. 6. The first fifteen stocks with strongest passive (top) and active (bottom) response functions $R_i^{(p)}(\tau)$ and $R_j^{(a)}(\tau)$ versus stock index $i$ or $j$ and time lags $\tau = 1$ s ($\circ$), 2 s ($\triangleleft$), 60 s ($\diamond$), and 300 s ($\square$). The ordinates of top and bottom graphs extend over different intervals.

proposed to offset the amplification effect of sign self–correlations, which is derived from the correlation accumulation with the time lag increasing. Thus, both the impact function and the sign self–correlation mutually describe the price self–response. For XOM, the sign self–correlator is larger than the cross–correlators before the appearance of correlation fluctuations, but its cross–response is stronger than the self–response. It implies that the impact functions of the self– and cross–responses are different. In other words, there are different impact mechanisms between the self–and cross–responses. The influential stocks, *i.e.* AAPL and CVX, amplify the impact difference in the case of XOM.

The average synthesizes the impacts of different stocks on individual stocks. The impacts cannot be observed directly. Instead, we can observe the decay of the average

sign correlators. The passive and active correlators are always smaller than the self–correlators for AAPL, GS and XOM before the appearance of correlation fluctuations, which is consistent with the case of the responses. It implies that the synthesized impact functions of the individual stocks are more stable than the ones in each stock pair.

## VII. CONCLUSIONS

We extended the study of the responses of stock prices to trading from individual stocks to a whole correlated market. We empirically investigated the price responses to the trading of different stocks as functions of the time lag. The response functions increase and then reverse back. Thus, the impact of the trades on the prices appears to be transient. Pictorially speaking, the market needs time to react to the distortion of efficiency caused by the potentially informed traders. In this period of distortion, arbitrageurs drive the price to a reversion and thereby help to restore market efficiency. The price response is clearly related to the trade sign correlations for different stocks. These correlators decays in a power–law fashion, revealing a short–memory process with exponents larger than one for a stock pair.

We also analyzed the market as a whole by setting up a matrix, the market response, that collects the normalized information of all response functions. Several characteristic features show up which are visible in patterns having a remarkable stability in time. The market response provides quantitative information about how the trading of one stock affects the prices of other stocks, and how its own price is influenced by the trades of other stocks stocks.

After this somehow microscopic view, we introduced average response functions, a passive and an active one, measuring the average price change of a given stock due to the trades of all others and the impact of trading a specific stock on the average price change of the other ones, respectively. Interestingly, the passive response reverses at a relatively short time lag of dozens of seconds or so and then declines rather quickly in a volatile way, while the active response reverses at a longer time lag of some hundreds of seconds with less volatility. This is so, because the price change in one stock easily alerts the market participants. The dispersion over different stocks makes it more difficult to detect an effect. We identified the response noise as a criterion for the stability. The averaged responses considerably reduce this noise, and make generic effects visible. We also introduced the corresponding active and passive trade sign correlators. It is quite remarkable that the above mentioned short memory turns into a long memory when averaged over different stock pairs.

Some stocks dominate the price responses of the market in a very stable fashion during the first 7200 s. By ranking the stocks, we found that some influential stocks
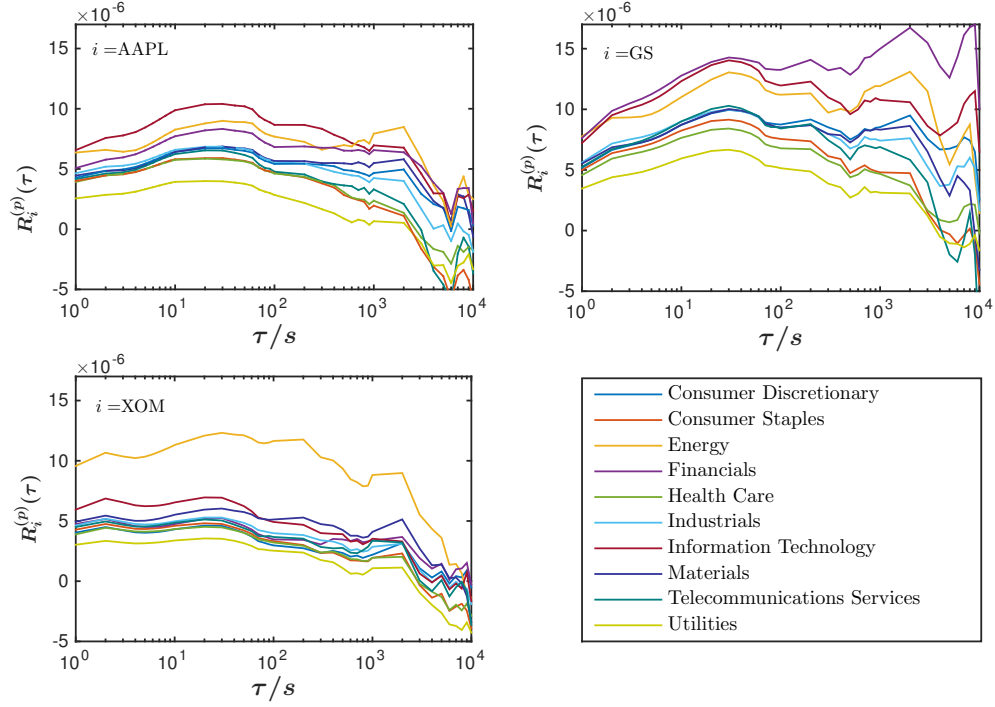
FIG. 7. Passive response functions $R_i^{(p)}(\tau)$ of the stocks $i = \mathrm{AAPL, GS, XOM}$ to ten different economic sectors in the year 2008 versus time lag $\tau$ on a logarithmic scale.
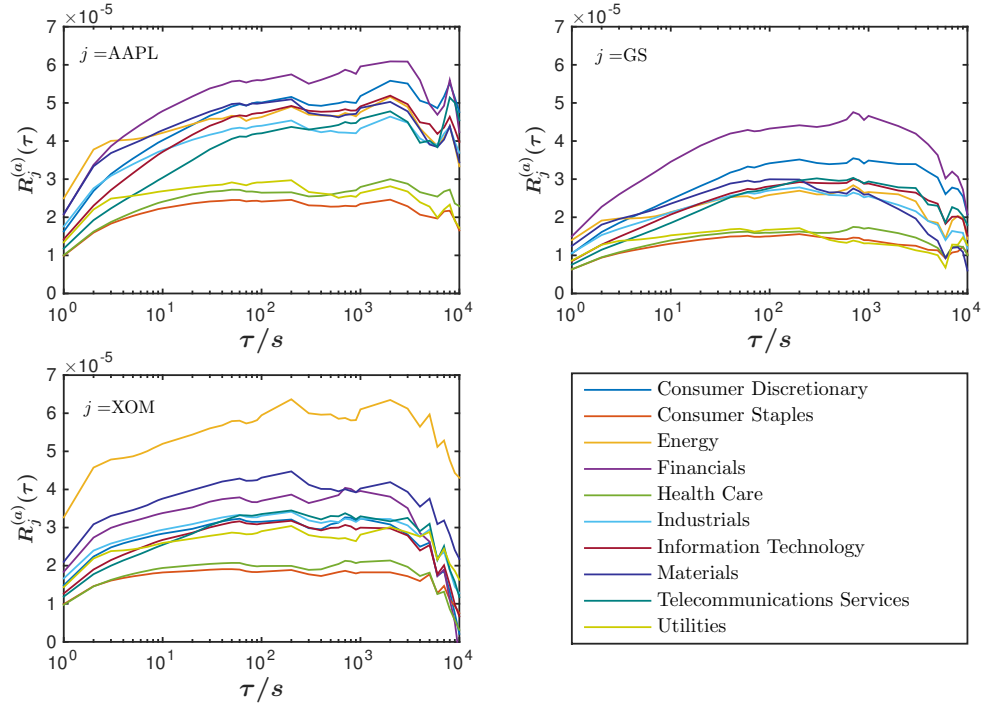


FIG. 8. Active response functions $R_j^{(a)}(\tau)$ of the stocks $j = \mathrm{AAPL, GS, XOM}$ to ten different economic sectors in the year 2008 versus time lag $\tau$ on a logarithmic scale.
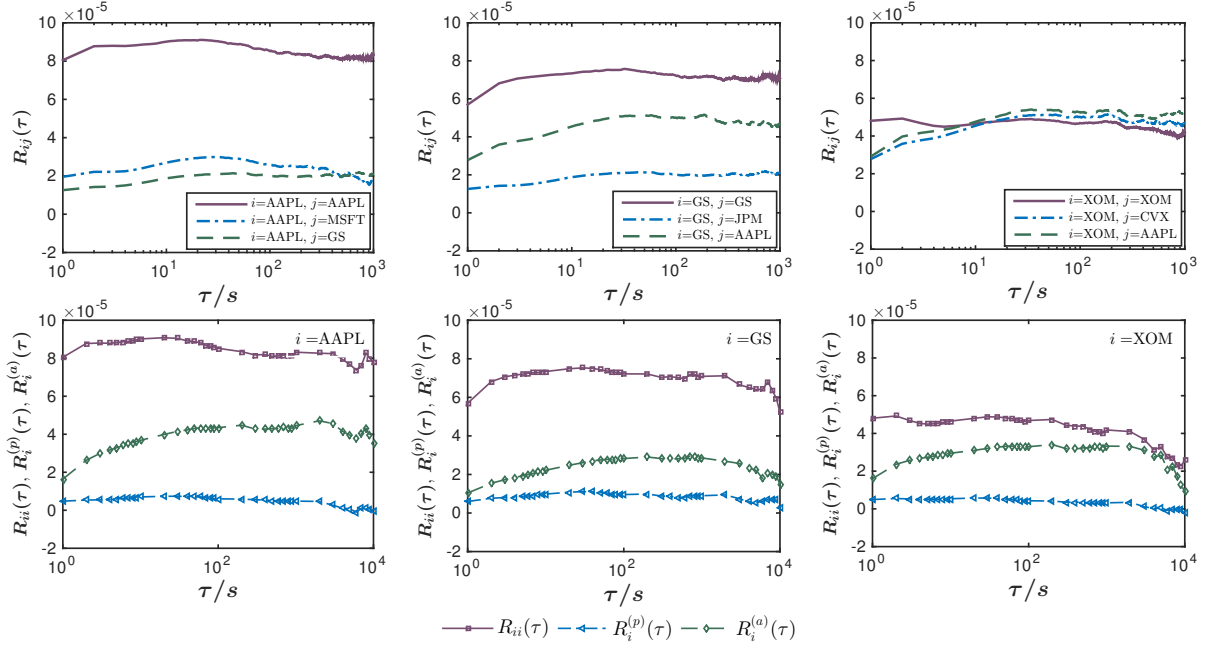
FIG. 9. Comparisons of the self-responses for AAPL, GS and XOM with cross-responses to different stocks (top) on a logarithmic scale. Comparisons of self-responses, passive responses, and active responses for the same stocks on a logarithmic scale (bottom).
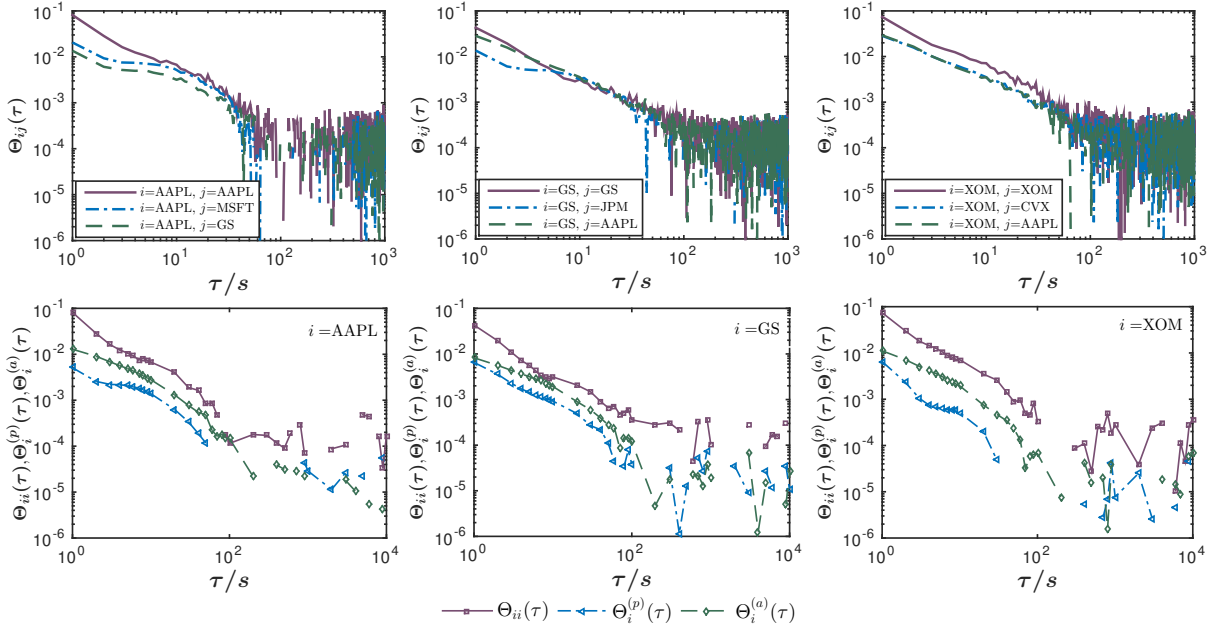


FIG. 10. Comparisons of the trade sign self-correlators for AAPL, GS and XOM with cross-correlators with different stocks on a doubly logarithmic scale (top). Comparisons of the self-correlators, passive correlators, and active correlators for the same stocks on a doubly logarithmic scale (bottom). The negative correlators are not shown in the graphs.

exhibit strong active response, but we also identified an example for a strong passive response. Here, it is also important that the responses in the market vary from sector to sector.

Last, the self–response is compared with the various cross–responses. On shorter scales of the time lag, both the self- and cross–responses should be considered for individual stocks. But on longer scales, the average responses of individual stocks give useful information for investigating the response stability, persistence or the efficiency of the market as a whole. On the other hand, the comparison of sign self– and cross–correlators implies the existence of different impact mechanisms between self– and cross–responses. By averaging the responses of individual stocks across the whole market, the impacts of individual stocks become stable.

## ACKNOWLEDGEMENTS

## Appendix A: Stocks used for analyzing the market response

We evaluated the market response for the 99 stocks from ten economic sectors: industrials (I), health care (HC), consumer discretionary (CD), information technology (IT), utilities (U), financials (F), materials (M), energy (E), consumer staples (CS), and telecommunications services (TS) as listed in Table IV. The acronym AMC in Table IV stands for averaged market capitalization.

## Appendix B: Error estimation

Suppose we measured or numerically simulated a set of $M$ data points $y(\tau_m)$ at positions $\tau_m$, $m = 1, \ldots, M$. We want to describe the data with a function $f(\tau)$ by fitting its $M_P$ parameters. To assess the quality of the fit, the normalized $\chi^2$ [35]

$$\chi^2 = \frac{1}{M - M_P} \sum_{m=1}^{M} \left( f(\tau_m) - y(\tau_m) \right)^2, \qquad (B1)$$

is used. Here, $M - M_P$ is referred to as the number of degrees of freedom. In our case, we have $M = 1000$, $M_P = 3$ for the fitting of trade sign correlators in stock pairs, and $M = 34$, $M_P = 3$ for the fitting of average trade sign correlators of individual stocks.

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TABLE IV. Information of 99 stocks from ten economic sectors

| Industrials (I) | | | Financials (F) | | |
|---|---|---|---|---|---|
| Symbol | Company | AMC | Symbol | Company | AMC |
| FLR | Fluor Corp. (New) | 14414.4 | CME | CME Group Inc. | 49222.9 |
| LMT | Lockheed Martin Corp. | 12857.8 | GS | Goldman Sachs Group | 21524.3 |
| FLS | Flowserve Corporation | 12670.2 | ICE | Intercontinental Exchange Inc. | 14615.3 |
| PCP | Precision Castparts | 12447.0 | AVB | AvalonBay Communities | 11081.6 |
| LLL | L-3 Communications Holdings | 12170.8 | BEN | Franklin Resources | 10966.2 |
| UNP | Union Pacific | 11920.9 | BXP | Boston Properties | 10893.0 |
| BNI | Burlington Northern Santa Fe C | 11837.5 | SPG | Simon Property Group Inc | 10862.4 |
| FDX | FedEx Corporation | 10574.7 | VNO | Vornado Realty Trust | 10802.3 |
| GWW | Grainger (W.W.) Inc. | 10416.8 | PSA | Public Storage | 10147.9 |
| GD | General Dynamics | 10035.6 | MTB | M&T Bank Corp. | 9920.2 |

| Health Care (HC) | | | Materials (M) | | |
|---|---|---|---|---|---|
| Symbol | Company | AMC | Symbol | Company | AMC |
| ISRG | Intuitive Surgical Inc. | 31355.9 | X | United States Steel Corp. | 15937.7 |
| BCR | Bard (C.R.) Inc. | 11362.7 | MON | Monsanto Co. | 14662.6 |
| BDX | Becton Dickinson | 10298.4 | CF | CF Industries Holdings Inc | 14075.5 |
| GENZ | Genzyme Corp. | 9728.8 | FCX | Freeport-McMoran Cp & Gld | 11735.7 |
| JNJ | Johnson & Johnson | 9682.6 | APD | Air Products & Chemicals | 10246.4 |
| LH | Laboratory Corp. of America Holding | 9035.7 | PX | Praxair Inc. | 10234.5 |
| ESRX | Express Scripts | 8864.6 | VMC | Vulcan Materials | 8700.4 |
| CELG | Celgene Corp. | 8783.1 | ROH | Rohm & Haas | 8527.1 |
| ZMH | Zimmer Holdings | 8681.7 | NUE | Nucor Corp. | 7997.4 |
| AMGN | Amgen | 8543.0 | PPG | PPG Industries | 7336.7 |

| Consumer Discretionary (CD) | | | Energy (E) | | |
|---|---|---|---|---|---|
| Symbol | Company | AMC | Symbol | Company | AMC |
| WPO | Washington Post | 61856.1 | RIG | Transocean Inc. (New) | 16409.5 |
| AZO | AutoZone Inc. | 14463.7 | APA | Apache Corp. | 13981.9 |
| SHLD | Sears Holdings Corporation | 11759.2 | EOG | EOG Resources | 13095.0 |
| WYNN | Wynn Resorts Ltd. | 11507.9 | DVN | Devon Energy Corp. | 12499.7 |
| AMZN | Amazon Corp. | 10939.2 | HES | Hess Corporation | 11990.4 |
| WHR | Whirlpool Corp. | 9501.9 | XOM | Exxon Mobil Corp. | 11460.3 |
| VFC | V.F. Corp. | 9051.2 | SLB | Schlumberger Ltd. | 11241.1 |
| APOL | Apollo Group | 8495.8 | CVX | Chevron Corp. | 11100.0 |
| NKE | NIKE Inc. | 8149.5 | COP | ConocoPhillips | 10215.3 |
| MCD | McDonald's Corp. | 8025.6 | OXY | Occidental Petroleum | 9758.4 |

| Information Technology (IT) | | | Consumer Staples (CS) | | |
|---|---|---|---|---|---|
| Symbol | Company | AMC | Symbol | Company | AMC |
| GOOG | Google Inc. | 62971.6 | BUD | Anheuser-Busch | 9780.6 |
| MA | Mastercard Inc. | 28287.8 | PG | Procter & Gamble | 9711.5 |
| AAPL | Apple Inc. | 22104.1 | CL | Colgate-Palmolive | 9549.2 |
| IBM | International Bus. Machines | 15424.9 | COST | Costco Co. | 9545.9 |
| MSFT | Microsoft Corp. | 10845.1 | WMT | Wal-Mart Stores | 9325.7 |
| CSCO | Cisco Systems | 8731.4 | PEP | PepsiCo Inc. | 9180.7 |
| INTC | Intel Corp. | 8385.8 | LO | Lorillard Inc. | 8919.0 |
| QCOM | QUALCOMM Inc. | 7739.4 | UST | UST Inc. | 8433.1 |
| CRM | Salesforce Com Inc. | 7691.9 | GIS | General Mills | 8243.3 |
| WFR | MEMC Electronic Materials | 7392.8 | KMB | Kimberly-Clark | 8069.5 |

| Utilities (U) | | | Telecommunications Services (TS) | | |
|---|---|---|---|---|---|
| Symbol | Company | AMC | Symbol | Company | AMC |
| ETR | Entergy Corp. | 12798.7 | T | AT&T Inc. | 6336.2 |
| EXC | Exelon Corp. | 9738.8 | VZ | Verizon Communications | 5732.5 |
| CEG | Constellation Energy Group | 9061.5 | EQ | Embarq Corporation | 5318.7 |
| FE | FirstEnergy Corp. | 8689.4 | AMT | American Tower Corp. | 5195.6 |
| FPL | FPL Group | 7742.8 | CTL | Century Telephone | 4333.8 |
| SRE | Sempra Energy | 6940.6 | S | Sprint Nextel Corp. | 2533.7 |
| STR | Questar Corp. | 6520.4 | Q | Qwest Communications Int | 2201.3 |
| TEG | Integrys Energy Group Inc. | 5978.4 | WIN | Windstream Corporation | 2089.1 |
| EIX | Edison Int'l | 5877.5 | FTR | Frontier Communications | 1580.9 |
| AYE | Allegheny Energy | 5864.9 | | | |

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