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How many independent bets are there in an emerging market?

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Abstract

The benefits of portfolio diversification is a central tenet implicit to modern financial theory and practice. Linked to diversification is the notion of breadth. Breadth is correctly thought of as the number of independent bets available to an investor. Conventionally applications using breadth frequently assume only the number of separate bets. There may be a large discrepancy between these two interpretations. We utilize a simple singular-value decomposition (SVD) and the Keiser-Gutman stopping criterion to select the integer-valued effective dimensionality of the correlation matrix of returns. In an emerging market such as South African we document an estimated breadth that is considerably lower than anticipated. This lack of diversification may be because of market concentration, exposure to the global commodity cycle and local currency volatility. We discuss some practical extensions to a more statistically correct interpretation of market breadth, and its theoretical implications for both global and domestic investors.

Keywords: Effective dimensions; Covariance Estimation; Emerging Markets

1 Introduction

One of the most widely accepted tenets of financial theory is the principle that diversification is an essential component of any well-constructed portfolio. Diversification serves to mitigate specific sources or risk within any single asset class, and systemic sources of risk across asset classes. Hence holding long positions in two resource companies suchs as BHP Billiton and Rio Tinto, may go a good way towards lessening the impact of company-specific risk within the international resources sector.

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Similarly, being exposed to property within a balanced (mutual) portfolio lessens the threat that other asset classes under-perform if property rallies. The idea is that spreading ones bets results in value being unlocked slowly over time and that diversification is a way to deal with an uncertain and volatile investment universe. These are fairly convincing argument's to most.

On a simple mathematical level, through diversification, one enhances ones risk-adjusted return by nature of a principal impact on any 'risk' denominator, be it the standard deviation of a Sharpe ratio or the active risk of an information ratio [3, **?**]. A portfolio that is comfortably diversified is expected to have a higher Sharpe ratio and information ratio. Diversification is frequently lauded as the only free lunch that econometrics offers to fund managers, and "one ought to indulge heartily at the price" [6].

It is from this optimistic base that we enter the fray with the allegation that 'diversification' opportunities may be both limited and overstated. Diversification in its common pretext acts more to disguise value-add than to enhance it. Interestingly, the usefulness of diversification in the way it was originally intended is particularly limited in South Africa and possibly other emerging markets for some less-than obvious reasons, as we discuss later. As we illustrate, because diversification is a frequently misunderstood phenomenon it can clearly be a very mixed blessing.

To the skilled fund-manager diversification may actually be an impediment. Spreading ones bets too thinly across independent gambits condemns such talent towards the manifestation of mediocrity since there is little room to move efficiently in all dimensions. To the less prudent fund manager, however, diversification will often offer a safe-haven where poor bets amongst will be simultaneously countered by good ones in others. Furthermore, we show that in the context of conventional asset classes within Southern Africa, there is a lot less room to maneuver than most professional investors suspect due to an overriding communality of extraneous factors that impact similarly on a wide variety of asset classes.

This has both implications for those global investors naively treating emerging markets as an independent asset class, those seeking international diversification from within an emerging market, and attempts to understand the theoretical applicability of asset pricing models in emerging markets.

In section 1.1 we discuss 'breadth'; how it is understood, used, and typically misused. This is followed in section 1.2 by a discussion of well-known multivariate statistical technique that facilitates a more correct understanding of the available breadth within any universe of assets: using the singular value decomposition in conjunction with the Keiser-Gutman stopping criterion, to select the integer-valued effective dimensionality of a correlation matrix of returns, with eigenvalues greater-than or equal to 1. We are then able, in section 1.3 to illustrate the available breadth to investors in South African markets, by using some well known multivariate statistical techniques in relation to three examples: a portfolio of equity (See Figure's 1 and 2), an equity and bond portfolio (See Figure's 3 and 4) and last, a portfolio including, in addition to equity and bonds, cash, property and international bonds and international equity (See Figure 5

and 6).

Lastly, in light of the insights provided from these previous chapters, in section 2 we reconsider the role that asset allocation has within the context of a resident balanced portfolio and also focus our discussion within the context of the useful fundamental law of active management [1, 3].

1.1 Breadth - Independence rather then separateness?

Conventional theory suggests that an increase in diversification opportunities (N) is accompanied by an increase in one′s information ratio (IR) [3]. Hence, in the terminology of active management, an increase in N serves to enhance ones ability to exploit information. Note at the onset that N is defined and treated as the number of separate bets (sensu Clarke et al. 2002).

For example, assume we have a 60% chance of getting equity bets correct. A bet on one underlying will yield an IR of 0.2, a bet on five underlying securities, an IR of 0.45 and a bet on 20 underlying securities, an IR of 0.90. This situation is easily verified. The understanding stemming from this detail is universal. For example, Lee Thomas [6] notes the following implications for considering diversification in the selfsame light:

1. Since diversification has an obvious statistical basis, a larger number of bets will produce a higher information ratio.

2. A lot of the differences between fund managers performances are often ascribed to 'skill' whereas the differences may simply be an artifact of better diversified portfolios.

3. The search for higher quality investments should be superseded by the search for diversified investments.

4. Diversification is paramount to investment success across asset classes, styles and countries. Diversify, diversify, diversify!

The above-mentioned argument's are very appealing, and very well utilized, but we disagree with each and every contention since all omit two essential truisms, which when understood, shed a very different light on the benefits of diversification and the nature of breadth.

Lemma 1.1 (Independence is not separateness) *The square root of N in mathematical statistics implies 'independence' amongst statistical units (here bets) [5] rather than simply the notion of 'separate bets' as is most often implied.*

If I hold a portfolio of 10 single stocks, do I really have 10 'independent' bets and is my breadth really 3.16? If I increase this to 1000 single stocks (assuming I have as many available), is my breadth 31.6? This is the key theoretical question dealt with here.

Lemma 1.2 (Skill does not scale over breadth) *Skill is not generally or simply scalable over breadth. One requires considerable skill in preserving ones information coefficient (or IC) [1] across an increasingly diverse universe of investable underlying securities.*

There is no *a priori* reason to expect, for argument's sake, a South African fund manager or analyst to be as adept in understanding the earnings potential of a diversified industrial company, as in understanding the risks and upside of Chinese private equity; yet there is a continuity of forecasting skill invoked across both. This is a key practical implication considered here.

Taken to its logical extreme, there is no reason to expect the same information coefficients (IC) between any two underlying securities. IC is an average measure that is typically applied to the sum total of all bets in a portfolio. We need to disaggregate the measure to understand its scalability.

Understanding the reality and the benefits of diversification resides in understanding both truisms given in Lemma (1.1) and Lemma (1.2) concurrently. We focus on the following three pertinent questions:

Question #1 Just how many South African single stocks do I need to add to a portfolio before I start to replicate pre-existing elements of diversification (i.e. saturate all elements of independence)?

Question #2 Do I capture much more 'breadth' if I include other local and international asset classes?

Question #3 What are the implications of these findings to fund managers?

1.2 Methodology - The informational content in a SVD

The foundation of this research arises from the confusion between the notions of 'separate bets' and 'independent bets'; the two are not the same. The question really is - how alike are they, and are there better ways for understanding independence than through the manner in which [3, 1] imply? We believe there are several ways to better represent independence than through Grinolds original construct.

For our purposes, as well as for ease of replication, we make use of the principle of 'effective dimensionality': given a return matrix X, we us the singular-value decomposition to factorize X as $X = U\Sigma V^T$ for eigenvectors $\Sigma = \text{diag}(\Sigma)$ and eigenvalues U. The unitary matrix U spans the subspace where the variations in the data are the largest. Each eigenvalue has an associated eigenvector. We then utilize the Keiser-Gutman [4] stopping criterion to select those eigenvectors with eigenvalues greater than or equal to one, the number of such eigenvalues corresponds to an estimation of the effective dimensions of the subspace - the N in the fundamental equation of active management [3, **?**].

There are alternative means of defining the effective number of stocks in a portfolio using entropy measures [9]. These all ultimately revolve around the degree of localization of the portfolio controls under optimization, and hence

pivot on the appropriate definition independence as differentiated for separateness. Many definitions of entropy assume, *a priori* independence amongst the states (here bets) of the system, this as we argue may in fact end up being problematic in these new reformulations of the notion of effective number of bets if not dealt with sensibly.

1.3 Empirical example of the enhanced application

We utilize return data from the Johannesburg Stock Exchange (JSE) and the Bond Exchange of South Africa (BESA) for the purposes of demonstrating both our breadth computations as well as the evidenced effect of limited breadth within the South African marketplace. We use daily data for a period of 4.3 years (March 2003 - present) for 41 of the most liquid equity stocks on the JSE. The most liquid equity index is termed the Top-40 index. The period of 4.3 years is arbitrarily chosen as a cut off point where most of the equity counters currently trading are subsumed in the analysis.

We commence by computing the effective dimensionality of this sample of 41 single-stocks from the estimated correlation matrix. A projection of the single-stocks (variables) onto the 2-D eigenvector space is represented in Figure 1. The projection shows that some gold stocks (ANG, GFI and HAR) cluster together at the extremes of the first two eigenvectors. Similarly, most banking stocks (e.g. SBK, FSR, ASA and RMB) cluster at the opposite quadrant of the same two eigenvectors.

A scree-plot is used to map the decay of the eigenvalues by the dimensionality of the data set in Figure 2.

Using our Keiser-Gutman criterion, we compute the effective dimensionality of the dataset as no more than 8 dimensions. This translated into an effective breadth of about 3. The conventional use of the fundamental law of active management would estimate breadth here at $\sqrt{41} = 6$, twice that evidenced here.

Next, we repeat the selfsame exercise as above, but now consider jointly the selfsame period of eight total return series of the seven dominant government-issued bonds along with our 41 single-stocks. A projection of the underlying securities onto the 2D eigenvector space is noted in Figure 3.

In Figure 4 a scree-plot is once again used to map the decay of the eigenvalues by the dimensionality of the data set.

The effective dimensionality is estimated as no more than 9 dimensions, translating into an effective breadth of 3. Conventional analysis here would infer a breadth of 7. Note how the analysis suggests both that South African bonds do not present much of a diversification enhancement to an equity portfolio and that replication of the self-same communalities exist. The reasons for these anomalies are easily explained by the dominant role that the local exchange rate plays on both equity and bond valuation and the impact of interest rates and commodity pressures on both.

Lastly, for the purposes of illustration, and to ascertain the place that the South African marketplace assumes relative to international markets, we include

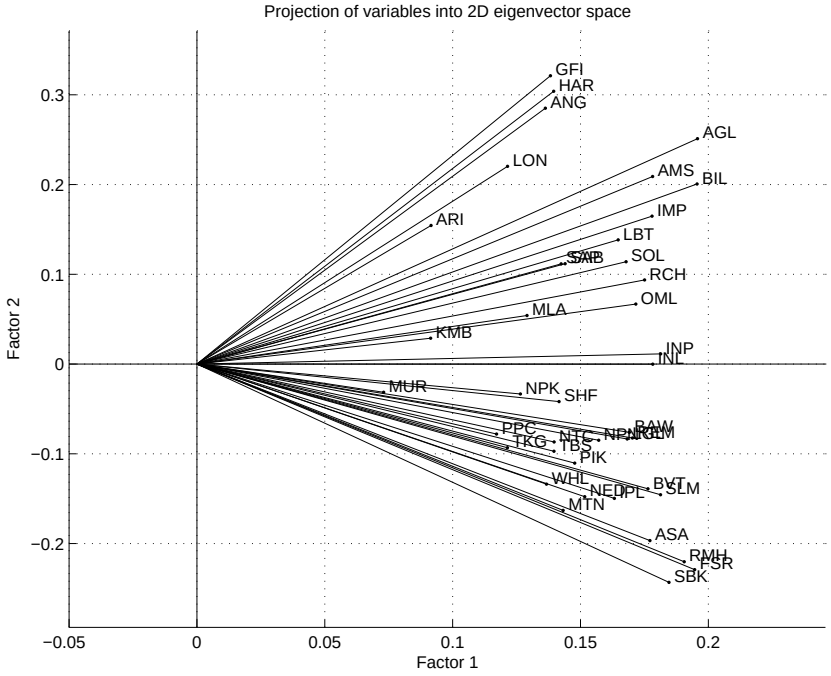

Figure 1: Example #1: The projection of the single-stocks (variables) onto the 2-D eigenvector space is provided as a bi-plot for. Note the prevalence of the bulk of the counters in two of the four quadrants. Note also that banking and gold-mining stocks cluster at the two end extremes.

13 new international assets to our analysis, all hard-currency denominated, including both bond indices and equity indices notably: the FT World Equity Index (USD), the MSCI World Equity Index (USD), four US Government bonds of varying duration (USD), Japanese inflation-linked bonds (JPY), a US corporate bond index (USD), an Emerging Market Index (USD), the FTSE 100 (GBP), the Russell 2000 (USD) , the Citigroup world government bond index (USD) and the UK government bond index (GBP). A projection of the underlyings onto the 2D eigenvector space is noted in Figure 5.

It is interesting to note from 5 that most of the new (international) assets span separate dimensions to the ones the South African securities occupy, apart for two obvious exceptions. First, US bonds are related to South African bonds - the correlations between these are strong and positive (as indicated by the acute angle between the bi-plot radians). Second, South African single-stock underlyings are inextricably tied to international equity markets - as evidenced by the presence of both the FTSE World and MSCI World Equity Indices in the same quadrant and in the same direction, despite the hard-currency differences here.

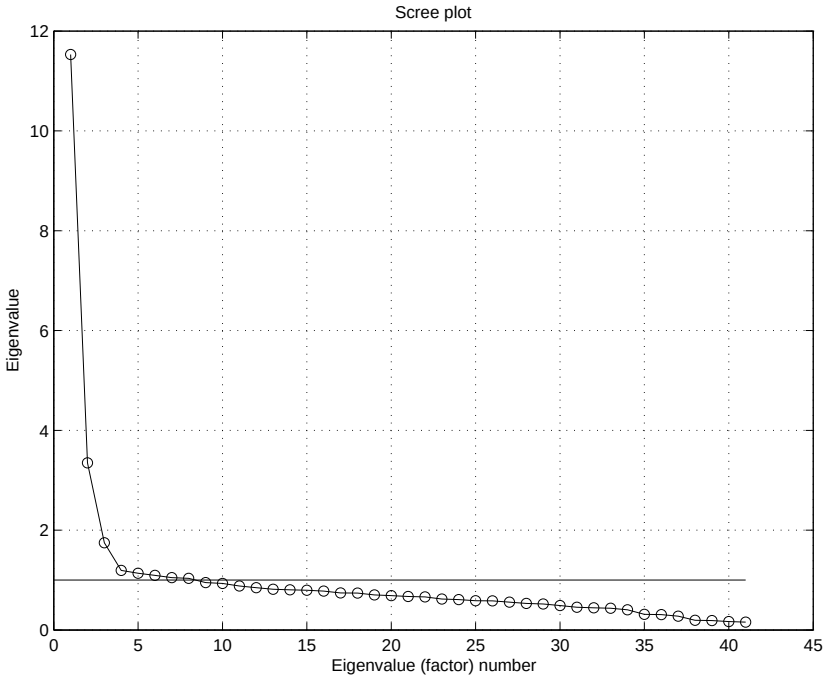

Figure 2: Example #1: The scree-plot mapping of the decay of the eigenvalues by the dimensionality of the single equity stock universe. The effective dimensionality of the data-set is found to be no more than 8 dimensions when using the Keiser-Gutman criterion. This is an effective breadth of approximately 3.

In Figure 6 a scree-plot is once again used to map the decay of the eigenvalues by the dimensionality of the data set. Including 13 of what most investors would consider to be a fairly diverse range of international asset classes to our original dataset of 41 South-African equity underlyings together with eight South African government issued bonds increases the effective dimensionality by only 4(from 9 to 13). Conventional methods would impute a breadth of closer to 8 whereas the actual breadth is closer to 4.

2 Conclusion

Breadth in our approach takes on a meaning that is quite different from that usually used, but is closer to the spirit of the idea, as it measures the benefit of real diversification. The SVD approach to the problem handles independence of the basis of spanning eigenvectors correctly whereas the notion typically assumed, that gambles are independent by their very nature, is simply incorrect.

One of the key results arising from this investigation into diversification possibilities in an emerging market such as South Africa is the significant lim-

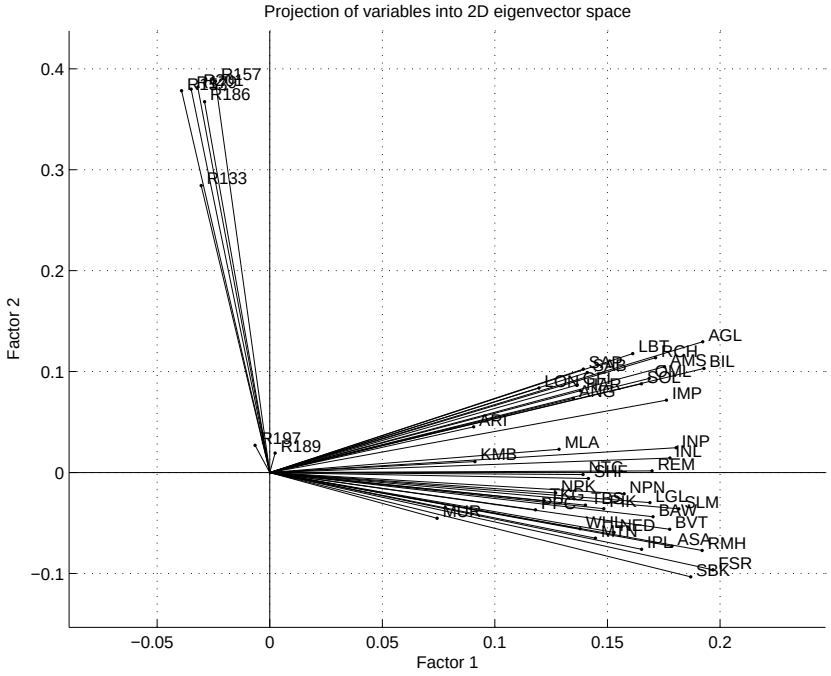

Figure 3: Example #2: A projection of the underlyings onto the 2-D eigenvector space. We include 8 dominant government-issued bonds along with the 41 single-stocks.

itation of breath in the opportunity set. This breadth limitation may well be due to a concentration of capital in a handful of local stocks within the South African market. Some 33% of the market capitalization is contained in the top-5 underlying securities. Interestingly, most of these stocks are dual-listed overseas, hence further limiting the breadth expansion when international assets are included in a resident South African portfolio. This breadth limitation has implications for the variety and nature of the risk associated with investment strategies and questions conventional wisdom in the construction of global funds.

A common misconception prevalent in the literature regarding the benefits of diversification is that skill is scaleable over breadth [6]. Hence, diversification is a free lunch offered to a 'diversified' portfolio in the sense that a larger number of bets effected with the same skill will produce higher IRs. However, the error is made in assuming that one's IC remains constant as breadth increases. Clearly, it cannot. For every added dimension of independence, one seems to require a novel skill-set. In the context of asset management, the implications of this error in the context of a limited breadth debate are threefold.

First, real diversification into breadth to achieve an optimal risk-adjusted return (via the IR) requires skill (as gauged by IC). If IC is compromised by

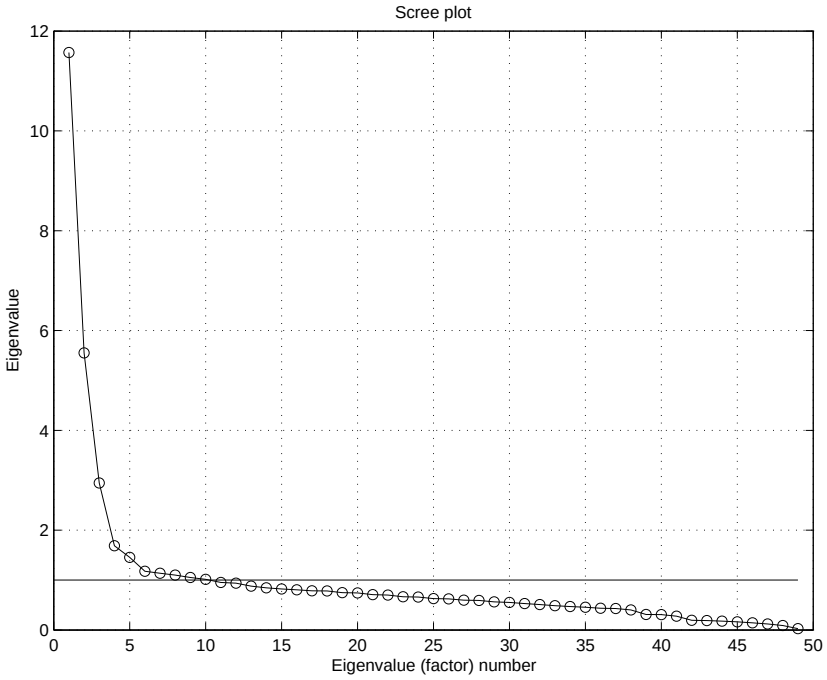

Figure 4: Example #2 : The scree-plot showing the decay of the eigenvalues in the equity-bond universe. The effective dimensionality is estimated as no more than 9 dimensions, translating into an effective breadth of 3.

breadth increasing, as we expect it to be, it can be argued that 'diversification' is actually a recipe for mediocrity amongst professional fund managers in the most general case. A prefatory analysis of several South African fund managers show different levels and persistence of IC for different sectoral bets. It would be of specific interest to quantify where the value resides within such institutions, and how to best extricate this value-add in the context of a balanced (mutual fund) mandate.

Second, less breadth will exist within any one asset class than the fundamental law implies. For a unit of capital, shifting allocation within an asset class will increase the breadth less (if at all) than shifting allocation across asset classes (the idea of tactical asset allocation). In the context of the South African marketplace, limited diversification exists within a highly concentrated equity market such that a shift from one security to another represents more of a bet about the relative spreads between their expected returns than it does anything about diversification. In fact, a pair-trading strategy (in its own right) creates a dimension of independence that is uncorrelated with either of the two original underlying securities, but may be correlated with other positions.

Lastly, it should be noted that tactical asset allocation can facilitate a rapid

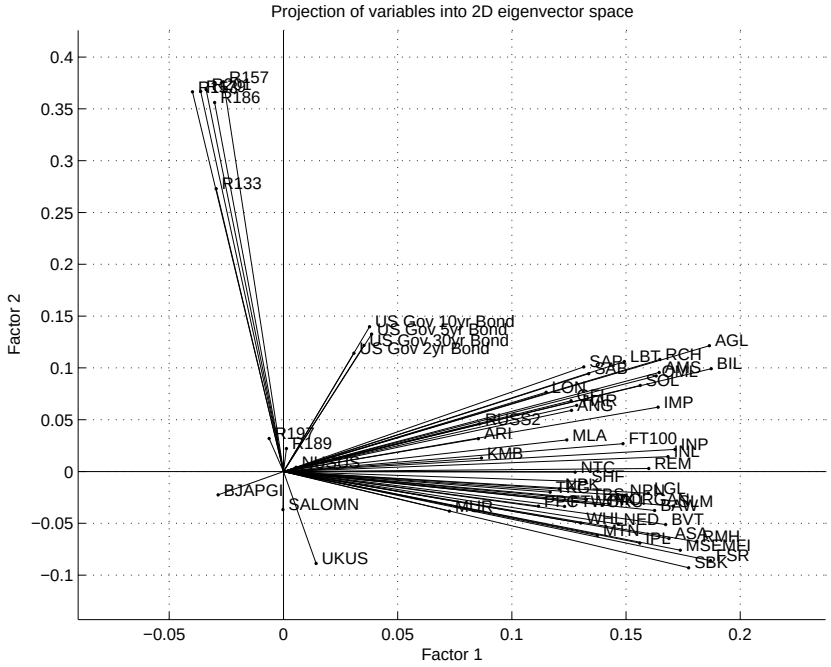

Figure 5: Example #3: A projection of the underlyings onto the 2-D eigenvector space. We include 13 international bonds and equity indices along with the 41 single-stocks and 8 government South African issued bonds.

breadth expansion by translating 'possible' breadth into 'realized' breadth. The pros and cons of asset allocation need to be considered in the selfsame context of the skill that managers have in timing the movements in various asset classes versus the diversification benefits of so doing. In this sense, our proposed modification to the fundamental law of active management provides a generalizable framework in which both static, dynamic and tactical asset allocation can be thoroughly and correctly investigated. In this context, IC(t), the information coefficient as a function of term, is the basis on which any analysis needs to be focussed.

The prospects of utilizing the fundamental law in this manner are particularly piquant and we hope that this research will stimulate some further work in this area. We note that the coefficients that are ultimately derived form the fundamental law of active management will not be comparable with previous studies, where breadth has not been estimated correctly.

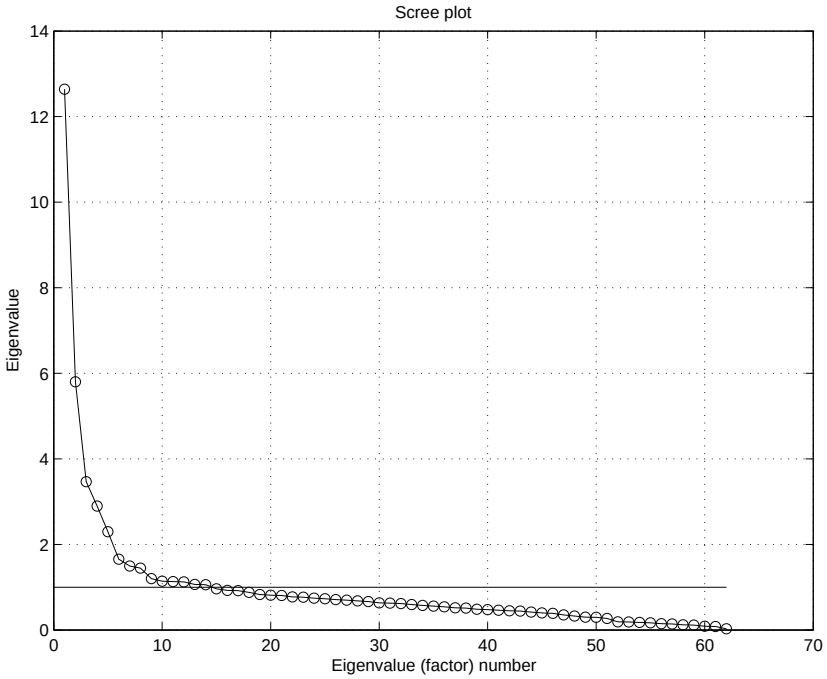

Figure 6: Example #3 : The scree-plot showing the decay of the eigenvalues in the mixed universe. The effective dimensionality is estimated as no more than 13 dimensions, translating into an effective breadth of close to 4.

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