

# A Comparision of Three Network Portfolio Selection Methods – Evidence from the Dow Jones

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## Abstract

We compare three network portfolio selection methods; hierarchical clustering trees, minimum spanning trees and neighbor-Nets, with random and industry group selection methods on twelve years of data from the 30 Dow Jones Industrial Average stocks from 2001 to 2013 for very small private investor sized portfolios. We find that the three network methods perform on par with randomly selected portfolios.

**Keywords:** Graph theory, hierarchical clustering trees, minimum spanning trees, neighbor-Net, portfolio selection, diversification

**JEL Codes:** G11

## 1 Introduction

Portfolio diversification is critical for risk management because it aims to reduce the variance of returns compared with a portfolio of a single stock (or similarly undiversified portfolio). The academic literature on diversification is vast, stretching back at least as far as Lowenfeld (1909). The modern science of diversification is usually traced to Markowitz (1952) which was expanded upon in great detail in Markowitz (1991).

The literature covers a wide range of approaches to portfolio diversification, such as; the number of stocks required to form a well diversified portfolio, which

has increased from eight stocks in the late 1960's (Evans and Archer, 1968) to over 100 stocks in the late 2000's (Domian et al., 2007), what types of risks should be considered, (Cont, 2001; Goyal and Santa-Clara, 2003; Bali et al., 2005), risk factors intrinsic to each stock (Fama and French, 1992; French and Fama, 1993), the age of the investor, (Benzoni et al., 2007), whether international diversification is beneficial, (Jorion, 1985; Bai and Green, 2010), among other risks.

In recent years a significant number of papers have appeared which apply graph theoretical methods to the study of a stock or other financial market, see, for example, Mantegna (1999), Onnela et al. (2003a), Onnela et al. (2003b), Bonanno et al. (2004), Micciché et al. (2006), Naylor et al. (2007), Kenett et al. (2010), and Djauhari (2012) among others.

On the pragmatic side, DiMiguel et al. (2009) lists 15 different methods for forming portfolios and report results from their study which evaluated 13 of these. Absent among these 15 methods were any which utilized the above-mentioned graph theory approaches. This leaves as an open question whether these graph theory approaches can usefully be applied to the problem of portfolio selection.

In one sense, the approach of Markowitz (1952) is optimal and cannot be bettered in the case that either the correlations and expected returns of the assets are not time-varying thus can be accurately estimated from historical data or, alternatively, they can be forecast accurately. Unfortunately, neither of these conditions hold in real markets. Indeed, Michaud (1989) studied the limitations of the mean-variance approach and claimed that the mean-variance optimizer was an "estimation error maximizer". These implementation problems have left the door open to other approaches and hence the large literature addressing this issue.

It is well-understood that the expected returns and variances of the individual stocks available for selection into a portfolio are insufficient for making an informed decision because selecting a portfolio also requires an understanding of the correlations between each of the stocks. However, the number of correlations between stocks rises in proportion to the square of the number of stocks meaning that for all but the smallest of stock markets the very large number of correlations are beyond the human ability to comprehend them. It is precisely in this area of understanding the correlation structure of the market that the graph theory methods have been usefully applied.

The goal of this paper is to compare three simple network methods – hierarchical clustering trees (HCT), minimum spanning trees (MST) and neighbor-Nets (NN) – with two simple portfolio selection methods for small private-investor sized portfolios. There are two motivations for looking at very small portfolios sizes.

The first is that, despite the recommendation of authorities like Domian et al. (2007), Barber and Odean (2008) reported that in a large sample of American private investors the average stock portfolio size of individual investors was only

4.3. Thus there is a practical need to find way of maximising the diversification benefits for these investors, and network methods remain largely unexplored. The second is that testing the methods on small portfolios gives us a chance to evaluate the potential benefits of the network methods because the larger the portfolio size, the more closely the portfolio resembles the whole market and the less likely any potential benefit is to be discernible.

We apply these methods to the problem of stock selection confining our investible universe to the 30 stocks of the Dow Jones Industrial Average Index. The choice of such a small group of stocks is motivated by the fact that stock networks are often interpreted by eye, thus having a small number of stocks means the networks are comparatively simple and easy to interpret. A follow-on study of a much larger set of stocks is in progress and we hope to be able to present the results of that study in the near future.

Our primary motivation is to investigate five portfolio selection strategies, three of which involve clustering or network algorithms. The five strategies are forming portfolios by picking stocks;

1. at random;
2. from different industry groups;
3. from different correlation clusters identified by MSTs
4. from different correlation clusters identified by HCTs
5. from different correlation clusters identified by neighbor-Net splits graphs.

HCT and MST have a long history of application to the study of financial markets. Recently Rea and Rea (2014) presented a method to visualise the correlation matrix using nieghbor-Net networks (Bryant and Moulton, 2004), yielding insights into the relationships between the stocks.

The outline of this paper is as follows; Section (2) discusses the data and methods used in this paper, Section (3) discusses identifying the correlation clusters, Section (4) applies the methods and results of the previous two sections to the problem of forming a diversified portfolio of stocks, and Section (5) contains the discussion, our conclusions and some suggestions for directions for future research.

## 2 Data and Methods

### 2.1 Data

We downloaded weekly closing prices along with dividend rate and dividend payment date for each of the 30 stocks for the period 2 January 2001 to 14 May 2013

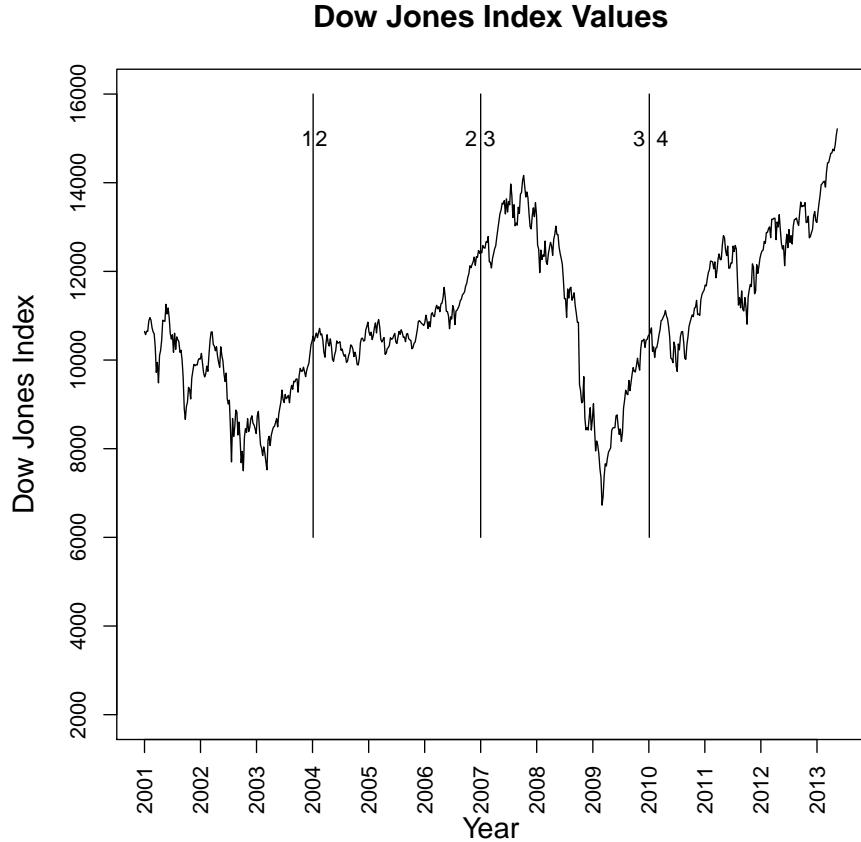


Figure 1: Dow Jones Industrial Average with the boundaries of the four study periods marked.

from DataStream. We calculated the returns by assuming that the dividends paid were reinvested into the stock which issued them, at the closing price on the day the dividend payment was made.

We defined four study periods, they were

1. 2 January 2001 to 6 January 2004.
2. 6 January 2004 to 2 January 2007.
3. 2 January 2007 to 5 January 2010.
4. 5 January 2010 to 14 May 2013.

The first three study periods are all three years in length and are used below for model building and in-sample testing. Periods two to four were used for out-of-

sample testing. The stocks in the sample, their ticker symbols and industry group is given in Appendix D.

Of the four study periods defined, period 3 contains the financial crisis of 2008 and the subsequent very steep fall and rebound in the market. Both in-sample and out-of-sample testing for period 3 represents a very severe test of a portfolio selection method.

## 2.2 Clustering Diagrams

The return series were used to generate correlation matrices using the function `cor` in base R (R Core Team, 2014). The correlation matrices were converted to distance matrices for use with the various clustering algorithms using the ultra-metric

$$d_{ij} = \sqrt{2(1 - \rho_{ij})}$$

where  $d_{ij}$  is the distance and  $\rho_{ij}$  is the correlation between stocks  $i$  and  $j$ . For further details on the ultra-metric see Mantegna (1999).

The hierarchical clustering trees were generated from the distance matrix using `hclust` from the R `stats` package using average linkage.

The minimum spanning trees were generated using functions in the `igraph` package (Csardi and Nepusz, 2006) within R.

To generate the neighbor-Net splits graphs we formatted the distance matrix and augmented it with the appropriate stock codes for reading into the SplitsTree software which implements the neighbor-Net algorithm. The neighbor-Net splits graphs were produced with `SplitsTree4` Version 4.13.1 available from <http://www.splitstree.org>.

Levene's test of equality of variances was performed with functions implemented in the R package `lawstat` (Gastwirth et al., 2013).

## 2.3 Simulated Portfolios

This section describes the three methods used to simulate portfolios. Below results of the simulations are presented based on 1,000 replications of each portfolio selection method.

**Random Selection:** The stocks were selected at random using a uniform distribution without replacement. In other words each stock was given equal chance of being selected but with no stock being selected twice within a single portfolio.

**By Industry Groups:** In the data extracted from DataStream there were 21 different industry groups among the 30 stocks. These were grouped into

four super-groups for the purposes of the portfolio simulations. The original industry groups of the stocks and their assignment to the four super-groups are listed in Appendix D.

If the portfolio size was four or less, the industries were chosen at random using a uniform distribution without replacement. From each of the selected industry groups one stock was selected. If the desired portfolio size was eight stocks, each group had at two stocks selected, again using a uniform distribution without replacement.

**By Correlation Clusters:** There were three clustering algorithms used to assign stocks to correlation clusters. The methods used to assign stocks to clusters is described in detail in Section (3) below and further results from the clustering algorithms are in Appendices A and B below. All the portfolio simulations used the same selection method.

The correlation clusters were determined by examining the graphical output from the clustering algorithm and stocks divided into between two and four correlation groups based on the needs of the simulations. The clusters determined in periods one, two and three were used to generate the in-sample portfolios for the same periods and the out-of-sample testing in periods two, three and four respectively. Because the goal of portfolio building is to reduce risk each cluster was paired with another cluster which was considered most distant from it where this was feasible. With the NN splits graphs this could always be done, with the MST groups sometimes this could be done, HCT groups could not be paired in this way.

## 3 Picking Correlation Clusters

### 3.1 Hierarchical Clustering Trees

Hierarchical clustering trees are very simple to read. One starts at the top of the tree and notes where the splits are. For example, Figures (2) and (3) show an HCT for the DJIA stocks in study period two with two and four clusters respectively. If one wants to divide the stocks into two clusters one simply finds the first split and the stocks fall naturally into the two clusters desired. If four clusters are desired, then it is simply a matter of moving down the tree until the splits give four clusters.

Figures (2) and (3) show us a problem with HCTs which are not present to the same extent with either MSTs or neighbor-Net splits graphs, that is, the HCT may produce highly unbalanced clusters. In Figure (2) the two cluster sizes are six and 24 respectively. The problem is even worse in Figure (3) in which the

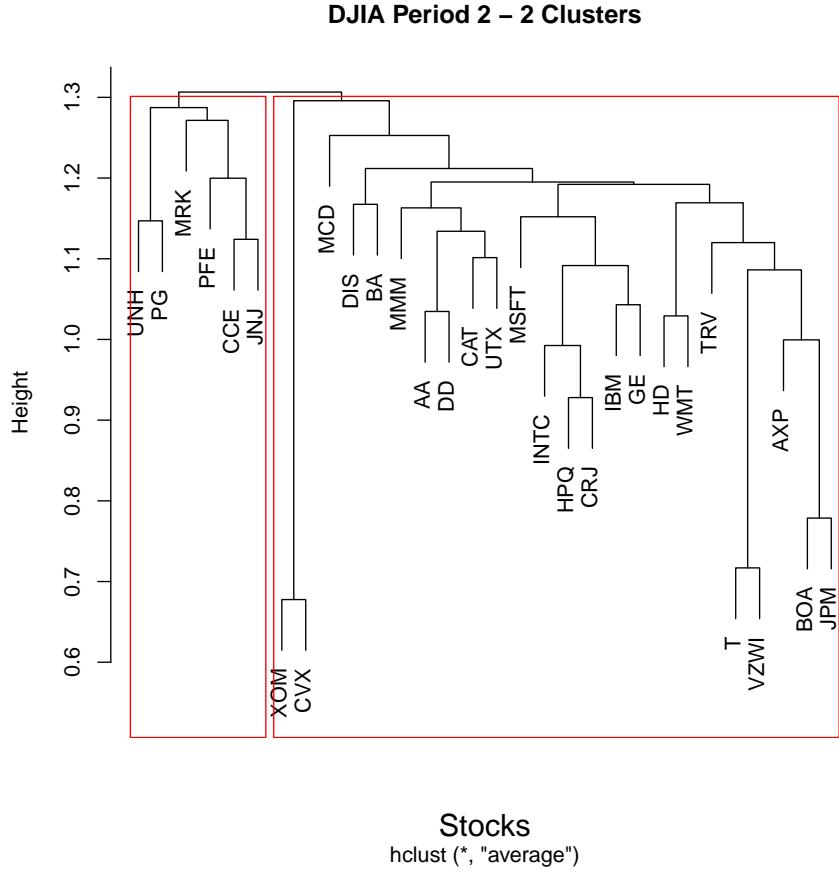


Figure 2: The HCT for Period 2 with two clusters. The unbalanced cluster sizes are clearly seen with clusters of six and 24 stocks respectively.

cluster sizes are two, two, four and 22. When running the portfolio simulations such unbalanced clusters may artificially depress the standard deviations. Given that the standard deviation is in the denominator of the widely used Sharpe ratio (Sharpe, 1964), this may inflate the apparent reward per unit risk. Thus we need to take care when interpreting the results from the simulations when using HCTs.

A second problem with HCT clusters is that in the simulations we wished to pick stocks from clusters which were most distant from each other. With a linear layout of the stocks and reading the clusters from left to right, in a four cluster case, pairing the left most cluster with the right most because it is most distant, leaves the other pair adjacent to each other. Because of this problem and the unbalanced cluster sizes, in the simulations we paired a large cluster with a small cluster. Further HCTs are in Appendices A and B.

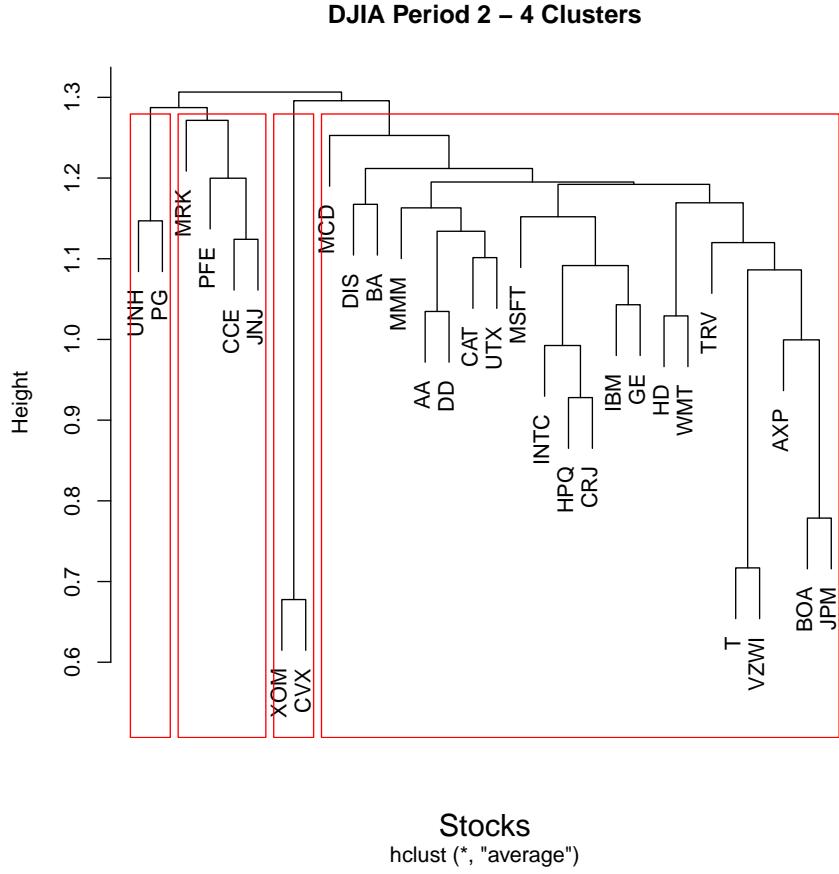


Figure 3: The HCT for Period 2 with four clusters. The unbalanced cluster sizes are clearly seen with clusters of two, four, two and 22 stocks respectively.

### 3.2 Minimum Spanning Trees

Figure (4) shows the MST for the DJIA for Period 2. When we divide the MST into correlation clusters, there are some natural breaks in the tree structure which allow us to easily assign some stocks to one particular cluster. Depending on how many clusters we need, the tree can be divided in more than one way. If we require two clusters it is clear that there are three distinct groupings, the branches with roots at JPM, DD and GE respectively. The two stock branch of TRV and AXP must go with BOA whichever cluster it is assigned to.

In Table (4) we have the distances from BOA to the five nearest stocks. It is clear from this that BOA must be assigned to the same cluster as JPM rather than the one with GE.

The task of dividing the MST into clusters can be difficult if the MST's struc-

Stock	Distance
JPM	0.779
AXP	0.961
GE	1.010
HD	1.045
DD	1.047

Table 1: Table of Distances to decide which cluster to put BOA into for period 2.

Stock	Distance	Stock	Distance
	to BOA		to AXP
JPM	0.779	BOA	0.961
AXP	0.961	JPM	1.038
GE	1.009	VZWI	1.043
DD	1.045	GE	1.047

Table 2: Table of Distances to decide which cluster to put BOA and AXP into for period four.

ture does not easily lend itself to being split into the desired number of clusters. In Figure (5) we have the same MST with four clusters identified. It would be more natural to only have three clusters with the branches rooted at GE and IBM combined into a single cluster.

The stock BOA is central within the MST so there is a question of which cluster should it be assigned to, and, also which cluster should the two stock branch of AXP and TRV be assigned to. Clearly AXP and TRV must go to whatever cluster BOA is assigned. In Table (2) we have the distances between BOA and AXP and each of their four nearest neighbors. The first column tells us that BOA should be assigned to the brown cluster. The second column also shows that the three nearest neighbors to AXP are in the brown cluster hence taking it with BOA was reasonable.

Assigning cluster groups to pairs was relatively straight-forward for the MSTs. In Figure (5) the green cluster is opposite the red cluster so are paired and the remaining two lie opposite each other and are paired as well. Further MSTs are in Appendices A and B.

### 3.3 Neighbor-Net Splits Graph

In Figure (6) shows the neighbor-Nets splits graph for period two with four clusters identified. When defining correlation cluster the neighbor-Nets splits graphs have

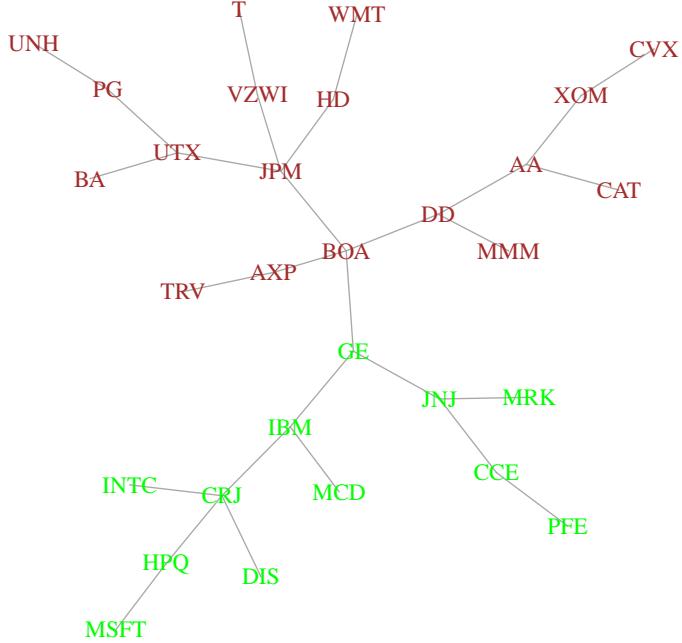


Figure 4: Minimum spanning tree for Period two DJIA with two clusters.

a distinct advantage over either an HCT or an MST because the taxa, here stocks, are given a position in a circular ordering. The network structure has some fairly clear breaks in its structure which allow us to assign the stocks to clusters. Perhaps the clearest is the red cluster; there is a clear break between GE and PG and again between HD and T. Sometimes it is ambiguous which cluster a stock should be assigned to, for example, DD could easily have been assigned to the khaki cluster rather than green cluster. Because of the circular ordering its two nearest neighbors are MMM and CAT and probably no harm would have been done had the assignment of this stock been done differently. It is also possible to examine the distance matrix as an aid in the decision of which cluster to assign an isolated stock to, such as DD. However, this may not be an easy task because of the way neighbor-Nets generates the network.

Table (3) presents the five nearest stocks to DD. The closest, AA, is indeed in the green cluster, but the second, fourth, and fifth closest, BOA, AXP, and JPM

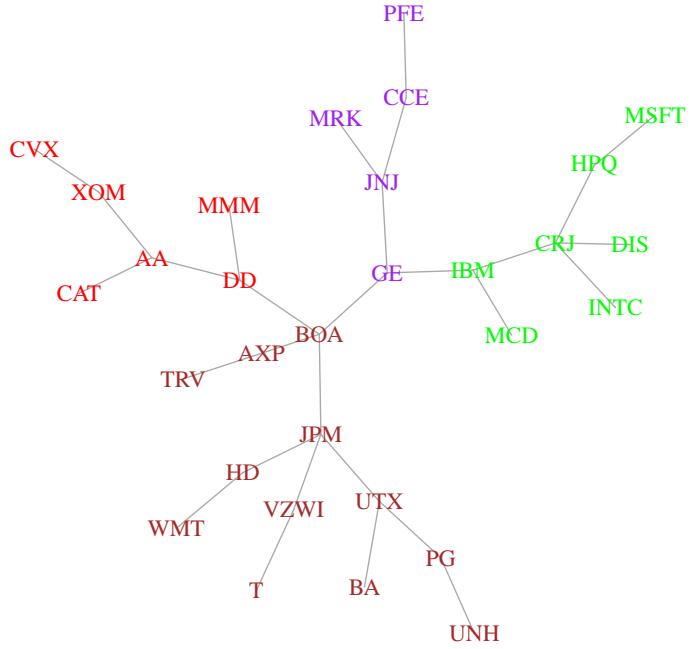


Figure 5: MST Period two DJIA with four clusters.

are in the purple on the opposite side of the network. The third closest is in the khaki. The examination of nearest neighbors is not as helpful as it was in the MST case, nevertheless the assignment to the green cluster seems reasonable.

While Figure (6) shows four clusters it is fairly easy to see how these could be combined into two clusters, the red and purple would be combined as would the green and khaki alternatively the red and khaki can be combined as would the green and purple.

The circular ordering of the stocks makes the pairing of the clusters straight forward. The khaki group would be paired with the purple and the green with the red. Further NN splits graphs are in Appendices A and B.

Stock	Distance
AA	1.035
BOA	1.047
GE	1.067
AXP	1.070
JPM	1.071

Table 3: Table of Distances to decide which cluster to put DD into for Period 2.

Cluster	All	HCT-MST	HCT-NN	MST-NN	HCT	MST	NN
1	PG		CCE	AXP	AA		
	UNH		JNJ	BOA	BA		
			MRK	HD	CAT		
			PFE	JPM	CVX		
				T	DD		
				VZWI	MMM		
				WMT	TRV		
					UTX		
					XOM		
2	CRJ		AA	AXP	CCE		
	DIS		BA	BOA	JNJ		
	GE		CAT	HD	MRK		
	HPQ		CXV	JPM	PFE		
	IBM		DD	T			
	INTC		MMM	VZWI			
	MCD		TRV	WMT			
	MSFT		UTX				
			XOM				

Table 4: The distribution of stocks during period two across two clusters for the three different methods of cluster identification. The column “All” is the stocks that are have been assigned to the same cluster by all three methods. The next three columns are the stocks which assigned to the same cluster by two of the methods, the remaining three columns are the stocks that are unique to each method.

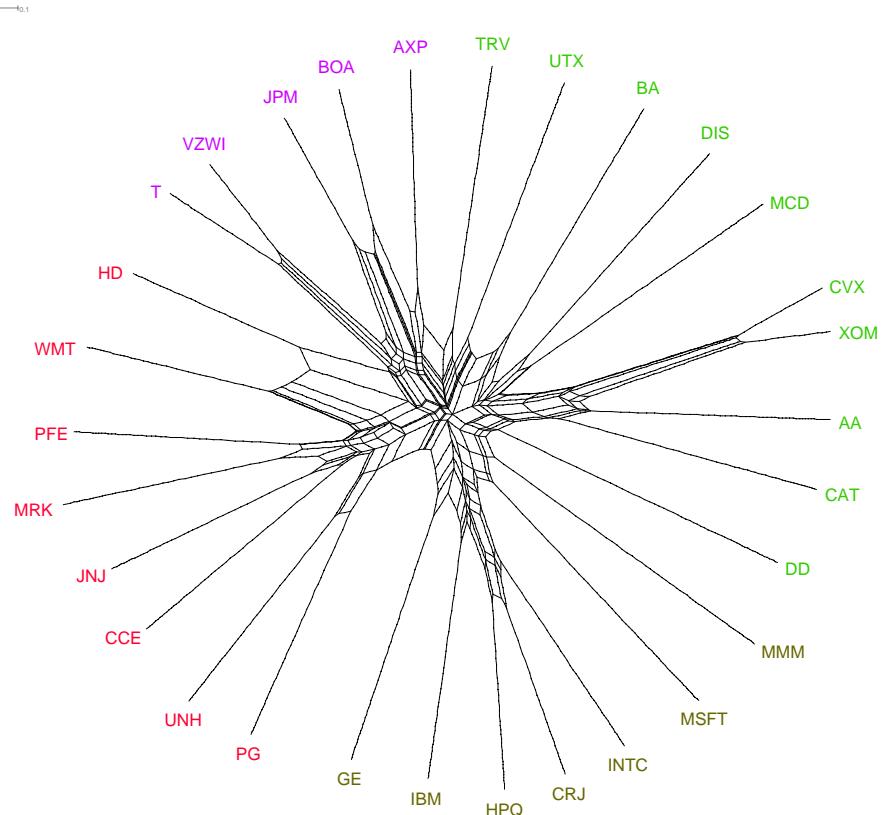


Figure 6: Neighbor-Net splits graph for Period 2 DJIA with four clusters.

### 3.4 Comparison of Clusters

Table (4) shows the assignment of the stocks to two clusters, see also Figures (2), (4) and (6). While the assignment of the labels 1 and 2 are arbitrary it is clear from Table (4) that each method has both stocks in common and stocks which are different from the other two methods. A similar analysis can be applied to the four clusters. The question then is – does this make any difference in portfolio selection for very small, private investor sized, portfolios?

## 4 Results

In this section we discuss the results in Table (5) which gives the results of the in-sample and out-of-sample portfolio simulations for period two clusters for periods two and three. Results for other periods are in Appendix C.

Period 2

Simulation results	Random	N-Net	HCT	MST	Industry Group	Levene p-value
<b>Mean return</b>						
(2-stock portfolios)	60.80	62.77	76.49	64.77	65.84	
(4-stock portfolios)	62.50	61.96	76.61	63.31	67.17	
(8-stock portfolios)	62.31	62.66	76.33	62.54	66.32	
<b>Standard Deviation</b>						
(2-stock portfolios)	30.57	28.10	32.97	32.89	32.99	$10^{-16}$
(4-stock portfolios)	21.90	19.41	22.02	21.21	21.35	0.006
(8-stock portfolios)	14.19	13.08	11.95	14.03	13.72	0.109
<b>Sharpe Ratios</b>						
(2-stock portfolios)	1.92	2.16	2.25*	1.90	1.93	
(4-stock portfolios)	2.75	3.08	3.38*	2.88	3.04	
(8-stock portfolios)	4.24	4.62	6.20*	4.30	4.67	

Period 3

Simulation results	Random	N-Net	HCT	MST	Industry Group	Levene p-value
<b>Mean return</b>						
(2-stock portfolios)	24.72	22.09	35.48	24.46	24.52	
(4-stock portfolios)	24.41	22.65	35.97	24.60	25.28	
(8-stock portfolios)	25.40	21.93	36.49	25.09	25.29	
<b>Standard Deviation</b>						
(2-stock portfolios)	23.52	24.52	18.93	23.54	22.55	0.012
(4-stock portfolios)	16.07	16.20	12.35	15.83	14.65	0.042
(8-stock portfolios)	9.88	10.86	7.39	10.50	9.40	0.029
<b>Sharpe Ratios</b>						
(2-stock portfolios)	0.86	0.72	1.64*	0.85	0.89	
(4-stock portfolios)	1.25	1.13	2.56*	1.28	1.43	
(8-stock portfolios)	2.13	1.61	4.34*	1.97	2.22	

Table 5: Mean portfolio returns and the standard deviations of 1000 replications of five different portfolio selection methods for in-sample (period two) and out-of-sample (period three). The Sharpe ratios used a period return of 2.2% and 4.4% for periods two and three respectively. The highest Sharpe ratios within each portfolio size are marked with an asterisk. The Levene test p-values exclude the HCT because all results which included the HCT were very highly significant.

Period	Cluster	N-Net	HCT	MST	Industry	
2	1	74.50 (8)	76.0 (2)	78.9 (9)	69.6 (8)	
In	2	63.00 (5)	38.5 (4)	59.0 (8)	102.6 (5)	
Sample	3	41.90 (10)	145.0 (2)	35.6 (7)	49.0 (8)	
	4	73.44 (7)	59.1 (22)	77.8 (6)	48.2 (9)	
3	1	26.75 (8)	-5.00 (2)	24.1 (9)	7.9 (8)	
Out of	2	15.60 (5)	43.74 (4)	13.6 (8)	20.6 (5)	
Sample	3	27.30 (10)	43.50 (2)	33.6 (7)	30.6 (10)	
	4	24.14 (7)	21.91 (22)	28.8 (6)	37.4 (7)	
3	1	27.50 (6)	56.5 (2)	17.2 (5)	7.9 (8)	
In	2	29.27 (11)	20.73 (22)	29.8 (5)	20.6 (5)	
Sample	3	-4.14 (7)	-40.00 (1)	34.8 (13)	30.6 (10)	
	4	46.0 (6)	41.00 (5)	6.6 (7)	37.4 (7)	
4	1	122.0 (6)	234.0 (2)	119.2 (5)	101.9 (8)	
Out of	2	64.45 (11)	89.90 (22)	38.4 (5)	77.4 (5)	
Sample	3	84.28 (7)	127.0 (1)	139.3 (13)	131.5 (10)	
	4	191.0 (6)	121.0 (5)	82.6 (7)	94.3 (7)	

Table 6: The per-period returns and cluster sizes for the three network methods and the industry groups.

In all cases the random selection method, as expected, gave mean returns which were statistically indistinguishable from the mean return of the thirty stocks. The random selection method is the only one of the five in which all stocks are equally likely to be chosen for a simulated portfolio. In the other four methods stocks in small clusters are more likely to be included in a simulated portfolio than stocks in large clusters. Thus the mean return for a simulated portfolio depends on the mean returns of the clusters.

Table (6) presents some of these values of the returns for periods two and three in-sample and periods three and four out-of-sample. The exceptionally high returns of the HCT seen in period two in Table (5) are due to the fact that one of the two-stock clusters had a mean return of 145%. This is an unexpected result because the returns were not used directly to generate the HCTs. Instead, the returns were used to generate the correlations and it is on the basis of the correlations that the HCTs were generated. Thus, we have no reason to expect that the portfolios selected by the HCTs would have higher than average returns. Yet that is what we see both for in-sample and out-of-sample testing for all periods.

In all but two cases the Levene test of equality of variances were statistically significant. The results in Tables (5), (10), and (11) exclude the HCT from the Levene test because we were concerned that the highly unbalanced cluster sizes may depress the standard deviations and make the results look statistically signif-

icant when, in fact, they were not.

In the in-sample testing neighbor-Net portfolios all had lower standard deviation compared to random selection, though these were only statistically significant in seven of the nine cases. Figures for the HCT, MST and Industry selection methods were seven, six and eight out of nine respectively. This shows that each of these methods was extracting useful financial information from the data but, with the possible exception of neighbor-Net, was no better than dividing stocks into groups by industry.

Unfortunately this did not carry over to the out-of-sample tests where the reduction in standard deviation occurred four times for neighbor-Net, eight times for HCT and once for MST out of nine sets of simulations.

For the industry group selection there is no difference between in-sample and out-of-sample tests because the industry groups were constant across all simulations. They showed a reduction variance with respect to random selection in eight out of 12 sets of simulations.

## 5 Discussion and Conclusions

The results of the simulated portfolios show that the portfolios selected using the correlation clusters identified by the HCTs outperformed all other portfolio selection methods based on the Sharpe ratio, with only one exception. This was, in part, because of high returns (the numerator of the Sharpe ratio) relative to the other portfolio selection methods. Other times a low standard deviation (the denominator) also contributed to the high Sharpe ratio. The low standard deviations are understandable given the way the simulations were run and the highly unbalanced cluster sizes, but the high returns were unexpected. This needs to be checked with other markets.

Despite these excellent results, the highly unbalanced clusters sizes of the HCT clusters in all periods calls into question whether HCTs should be used in practice. For example, if one was choosing a four stock portfolio based on period 2 clusters one would always include one of UNH and PG and one of XOM and CVX, leaving the remaining two stocks to be selected from 26. It is questionable whether an investor would accept such a severe restriction in their stock selection.

Numerous papers have demonstrated the value of graph theory methods in shedding light on the correlation structure of a stock market. This paper in no way negates the value of those papers. Indeed, the fact that the neighbor-Net portfolios all had lower standard deviation compared to random selection in the in-sample testing shows that neighbor-Net was extracting useful financial information from the data. However, it is clear that it is not straight forward to apply these insights in portfolio selection. Thus, a potential direction for future research is to explore

ways to combine the insights provided by the graph theory methods with other information about the stocks in order to improve stock selection.

The industry groups used here may not be particularly meaningful because the original 21 industry sectors represented by the 30 stocks were combined into four super-groups. In these circumstances one may argue that random selection without replacement would make better use of the 21 industries represented than the four super-groups we did use. Future research should use a larger number of stocks in order to give a more meaningful set of industry groups for selection in the portfolio simulations.

The type of reduction in portfolio standard deviation used in these simulations is important because if an investor chose to hold a portfolio that was under-diversified relative to the broad market a significant concern would be that their portfolio would under-perform the market. A low standard deviation means that in the case of under-performance their actual performance would likely be closer to that of the market than a method which had a high standard deviation.

In our results the differences in standard deviations seem so small that they are unlikely to be of economic significance. Consequently, our results show that the standard advice to hold a well-diversified portfolio, which in practice means a larger portfolio than we have tested here, or better, buy a low cost index fund, is good advice.

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## A Period One Graphs

### A.1 Period One Hierarchical Clustering Trees

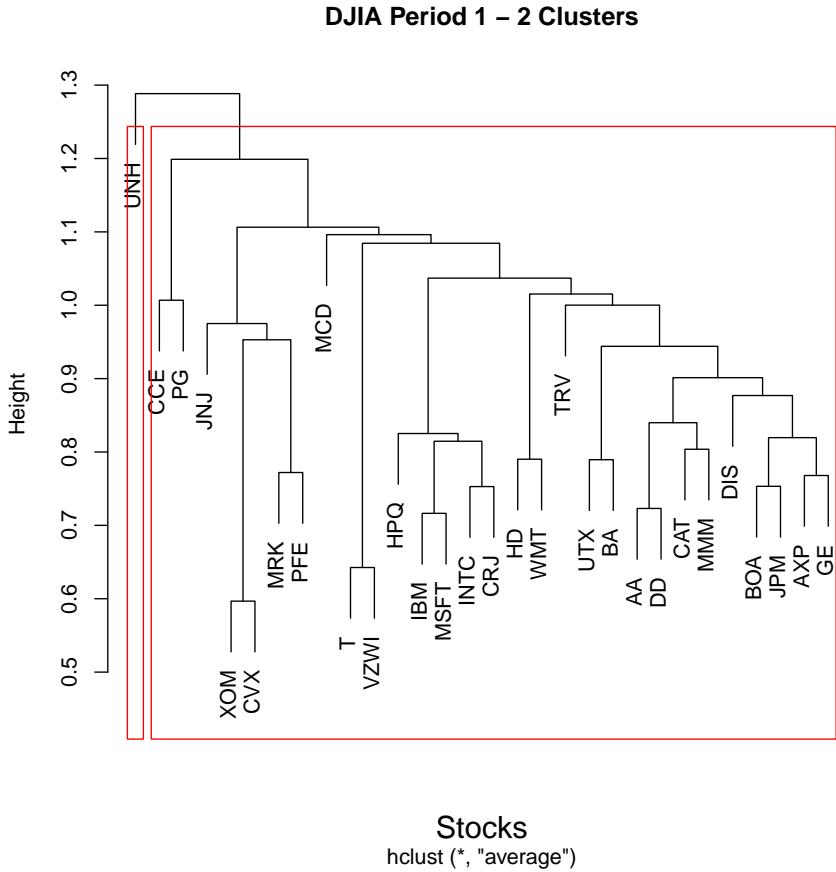


Figure 7: HCT Period 1 DJIA with two clusters. The unbalanced cluster sizes are clearly seen with clusters of one and 29 stocks respectively.

### A.2 Period One Minimum Spanning Trees

The MST in Figure (10) shows AXP as the pivot stock in the Dow 30. It has nine stocks to which it joins as nearest neighbors. Such a structure does not easily lend itself to dividing the 30 stocks into groups. If we arbitrarily set the minimum branch size for a group to be five stocks, there are three such branches in the MST. Of these branches for the ones rooted at JPM and XOM it is clear which stocks are members of these groups. For the branch rooted at HPQ there is a question

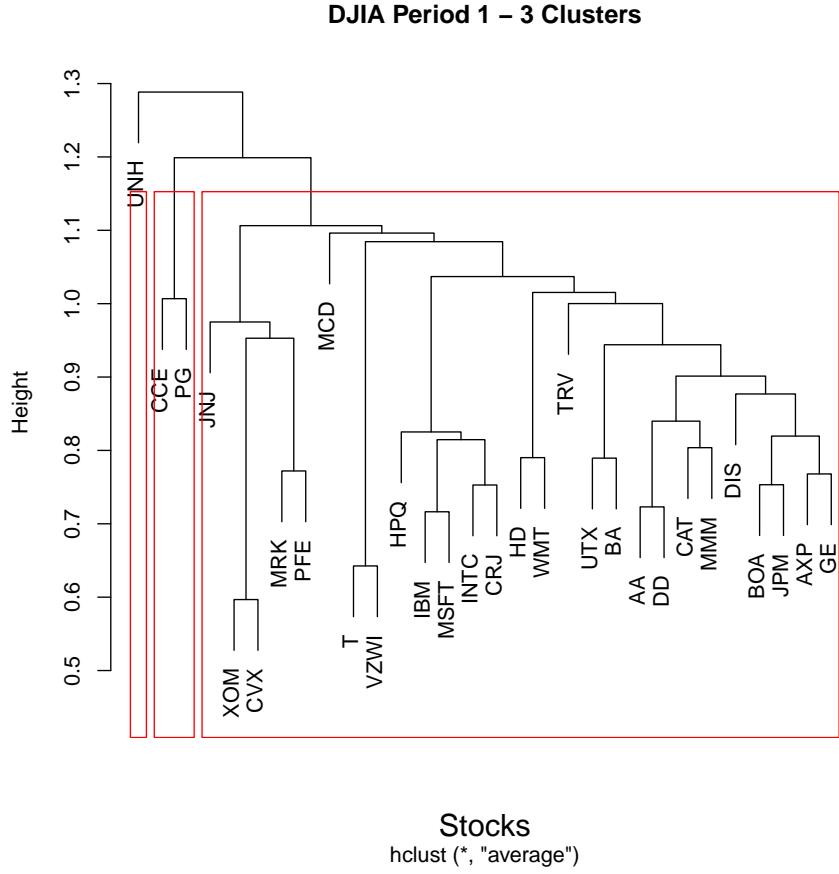


Figure 8: HCT Period 1 DJIA with three clusters. The unbalanced cluster sizes are clearly seen with clusters of one, two and 27 stocks respectively.

of whether HPQ should be assigned to the group containing AXP or INTC. Table (7) shows the distances to the three nearest stocks to HPQ. From this it is clear HPQ should be assigned to the group with INTC.

### A.3 Period One Neighbor-Net Splits Graphs

Figures (11) and (12) represent the neighbor-Nets splits graphs for period one. The network structure has as many as eight breaks allowing considerable flexibility in splitting the stocks into groups. However, it is clear that the network structure is elongated in the horizontal direction meaning that if the stocks are to be split into two groups the most natural way to do that would be to use a vertical cut. We have one way of doing that in Figure (11) where the stocks have been split into two groups of size 14 and 16.

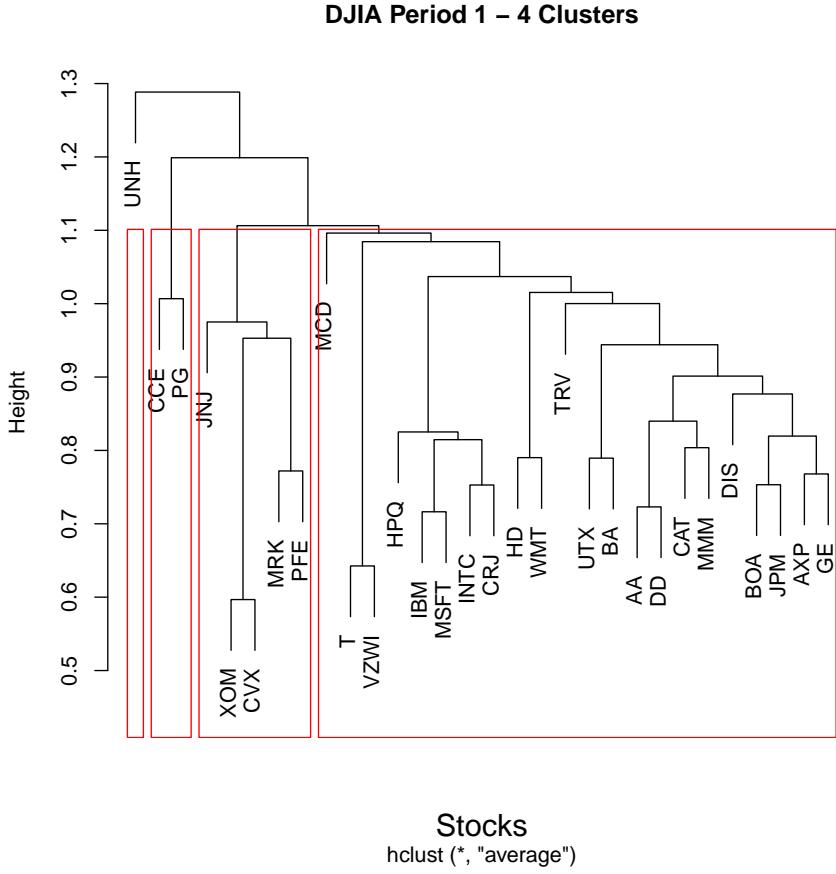


Figure 9: HCT Period 1 DJIA with four clusters. The unbalanced cluster sizes are clearly seen with clusters of one, two, five and 22 stocks respectively.

In Figure (12) we have split the stocks into four groups. The network structure does not lend itself to easily making four groups because there are isolated stocks such as TRV and MCD which are not close to any other stocks. Also there are some pairs such as VZMI and T on the lower side of the graphs and HD and WMT on the upper which appear as isolated groups. In this case the circular ordering helps because each of these isolated individual stocks or pairs of stocks must be grouped with those either to the left or the right. In such a case it can be helpful to look at a few of the stocks to see which group is should be assigned to. For example, we consider TRV.

In Table (8) it is clear that the three closest stocks are in the group on the opposite side of the network while the fourth and fifth closest are to the right. Thus TRV together with MCD, HD, and WMT should all be included in the green rather than the red group.

Stock	Distance
INTC	0.793
MSFT	0.804
CRJ	0.850

Table 7: Table of the distances of the three stocks closest to HPQ used to decide which cluster to assign HPQ to.

Stock	Distance
AXP	0.783
JPM	0.859
AA	0.866
GE	0.870
DD	0.874

Table 8: The five closest stocks to TRV in Period 1 and their distance.

Had the distances in Table (8) been taken into account in the two group case (Figure 11) it could have been argued that TRV, MCD, HD, and WMT should be in the red group rather than the black. Had this been done the clusters would be more unbalanced with sizes of 11 and 19 stocks rather than the 14 and 16 we chose.

**MST for DJIA for Period 1 – 4 Clusters**

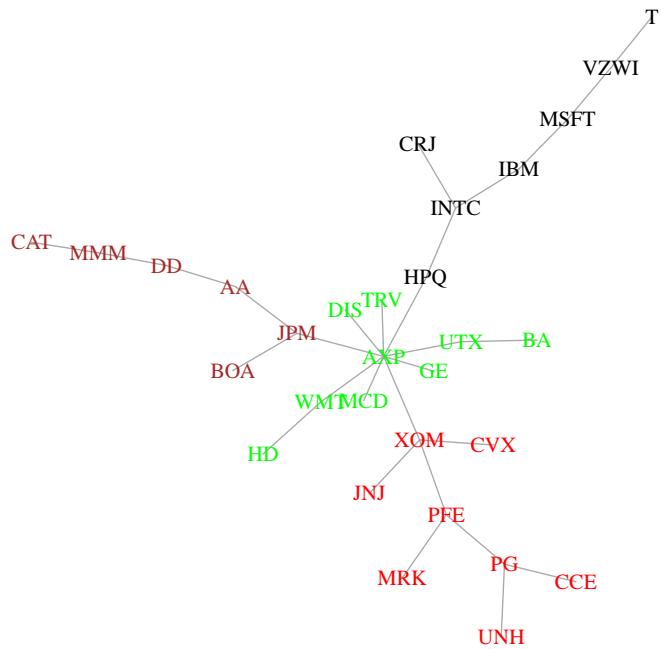


Figure 10: MST Period 1 DJIA with four clusters. The cluster sizes are well balanced with clusters of six, seven, eight and nine stocks respectively.

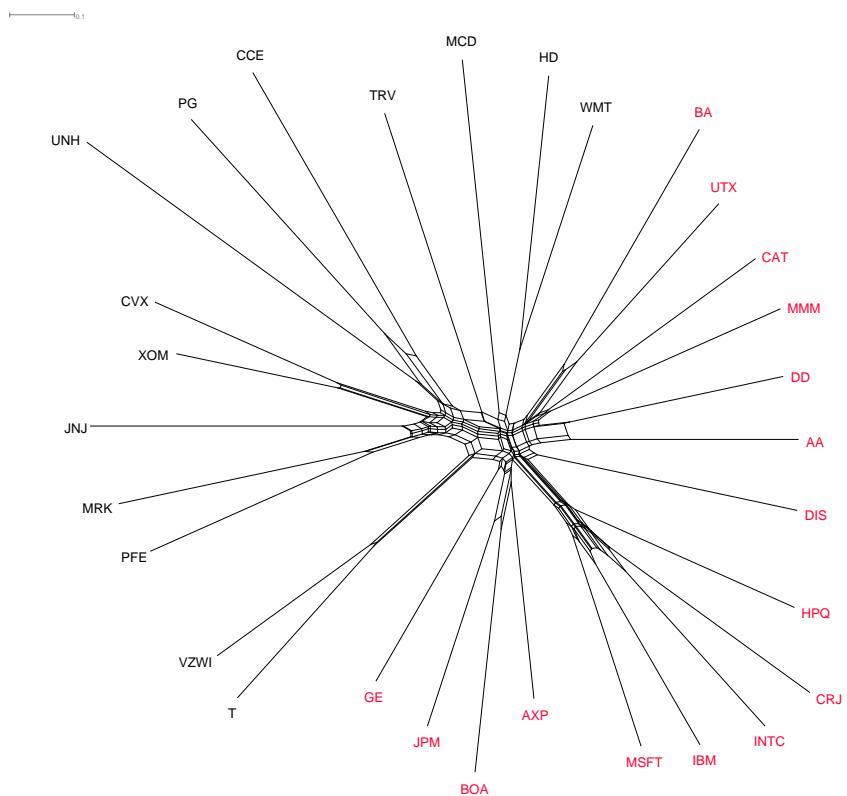


Figure 11: Neighbor-Net splits graph for Period 1 DJIA with two clusters. The cluster sizes are well balanced with clusters of 14 and 16 stocks respectively.

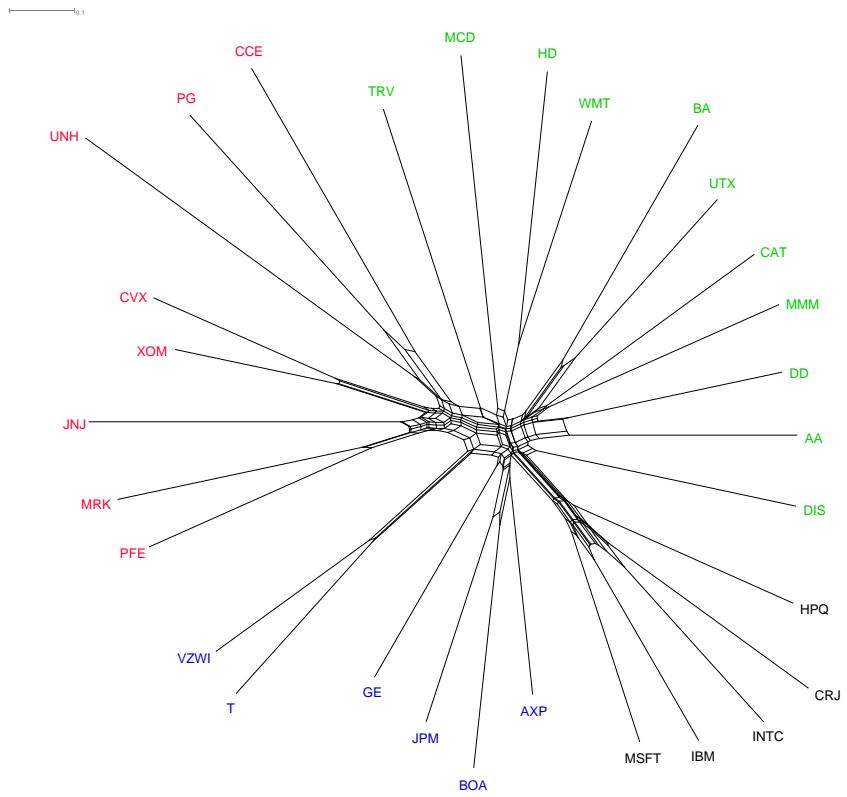


Figure 12: Neighbor-Net splits graph for Period 1 DJIA with four clusters. The cluster sizes are somewhat unbalanced with clusters of five, six, nine and 11 stocks respectively.

## B Period Three Graphs

### B.1 Period Three Hierarchical Clustering Trees

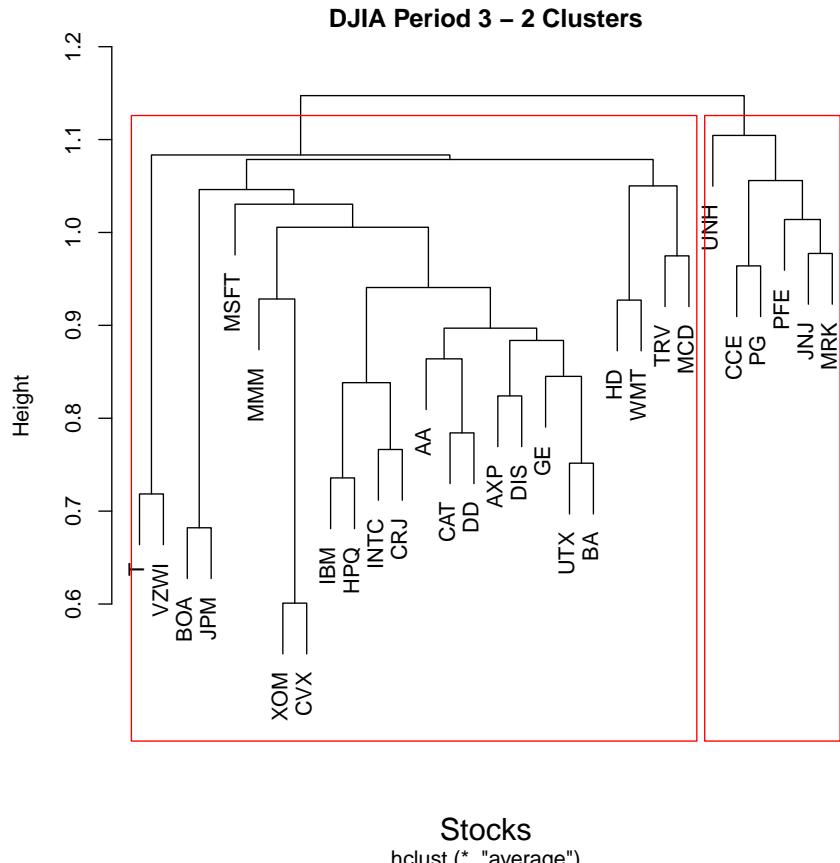


Figure 13: HCT Period 5 DJIA with two clusters. The unbalanced cluster sizes are clearly seen with clusters of six and 24 stocks respectively.

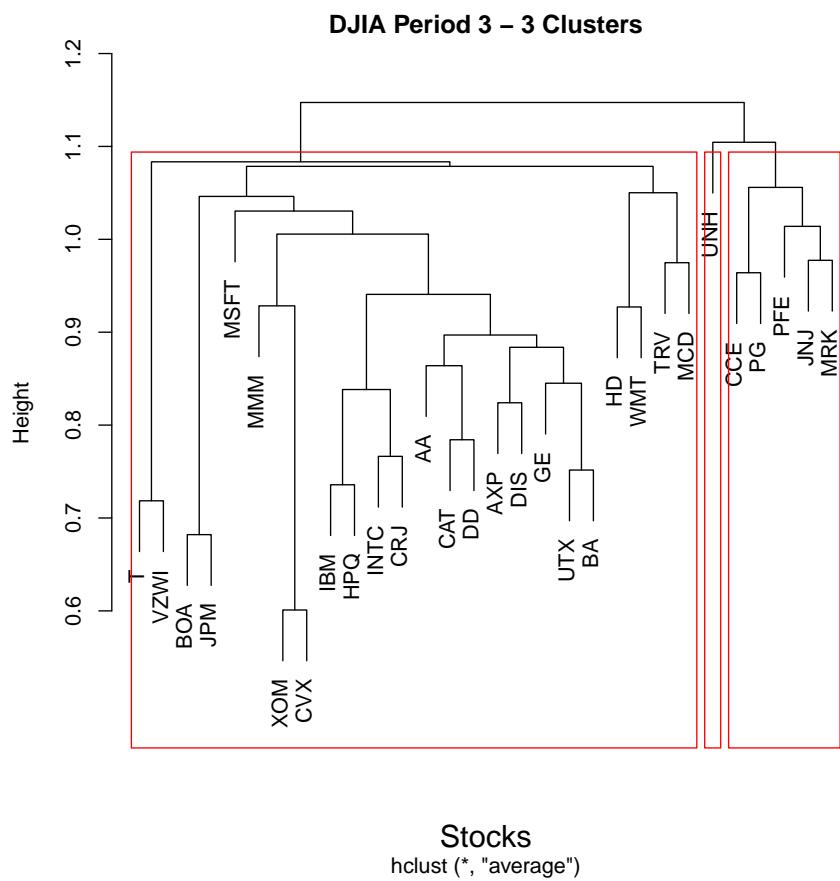


Figure 14: HCT Period 5 DJIA with three clusters. The unbalanced cluster sizes are clearly seen with clusters of one, five and 24 stocks respectively.

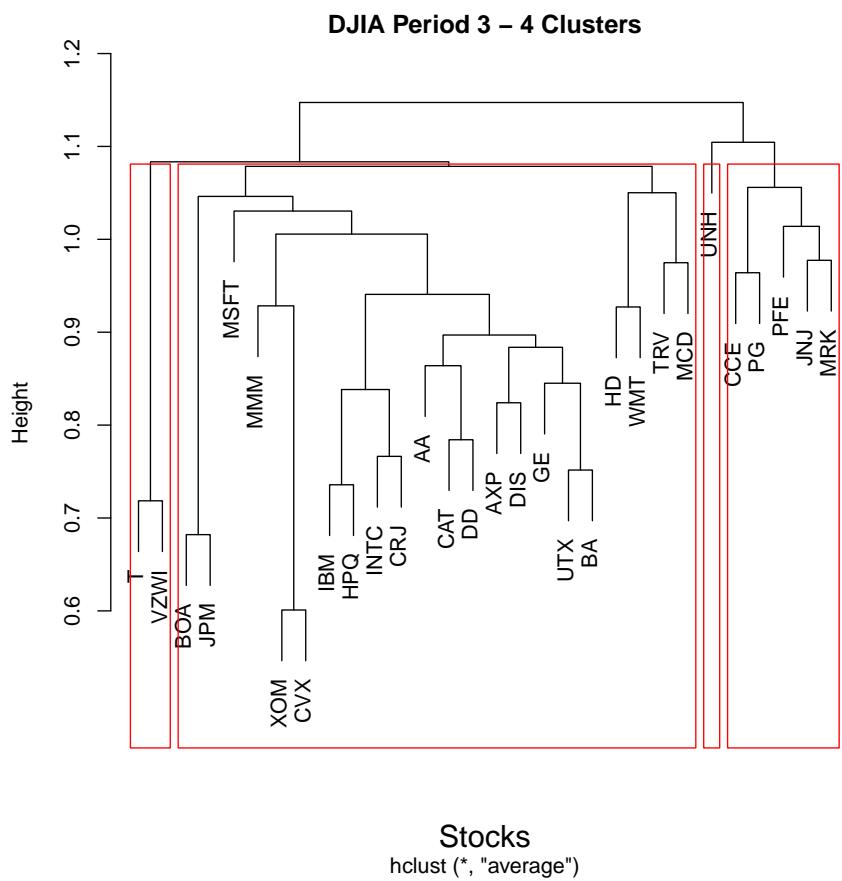


Figure 15: HCT Period 5 DJIA with four clusters. The unbalanced cluster sizes are clearly seen with clusters of one, two, five and 22 stocks respectively.

## B.2 Period Three Minimum Spanning Trees

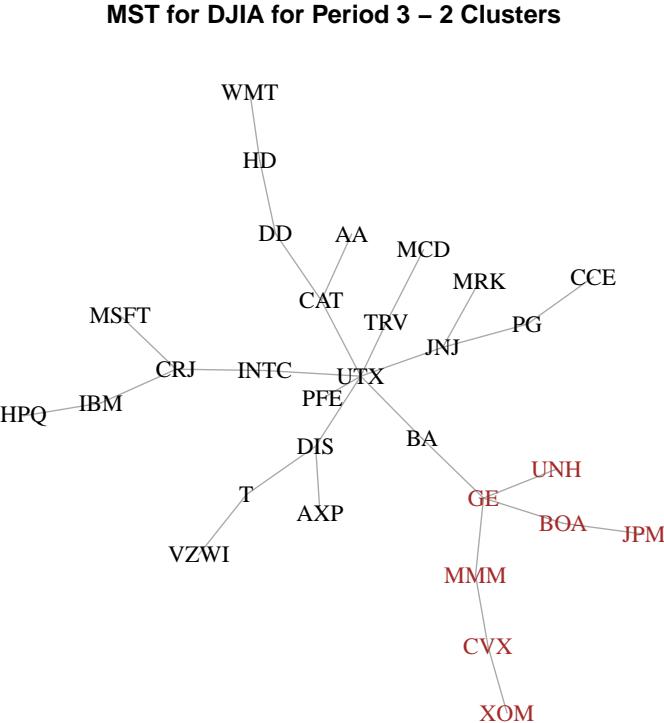


Figure 16: MST Period 3 DJIA with two clusters. The unbalanced cluster sizes are clearly seen with clusters of seven and 23 stocks respectively.

In Figure (16) the pivotal nature of UTX and the short branches extending from it give little flexibility in choosing two clusters. The largest branch is connected to UTX via BA, so the question is whether to assign BA to the larger or the smaller cluster. Table (9) gives the nearest four stocks to BA. The first two and fourth closest stocks are in the large cluster. So while it would be nice to assign BA to the smaller cluster in order to try to better balance the cluster sizes, this clearly cannot be justified based on the data in the table.

**MST for DJIA for Period 3 – 4 Clusters**

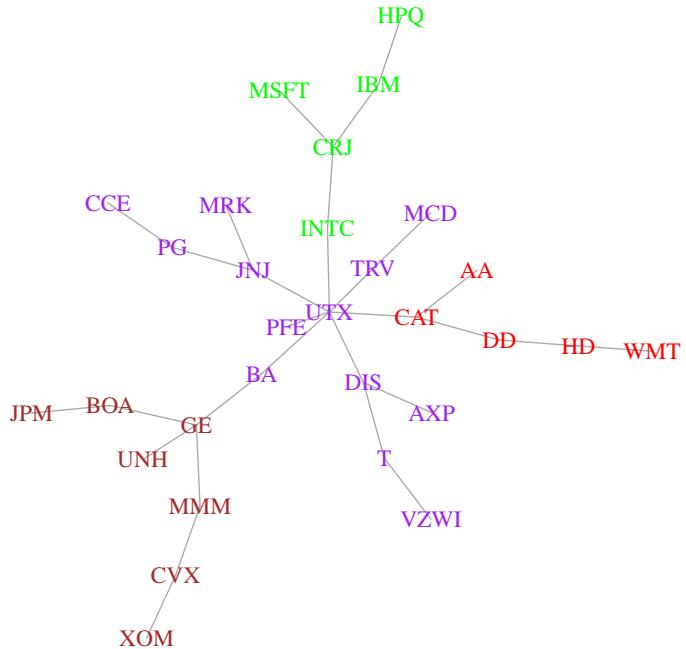


Figure 17: MST Period 3 DJIA with four clusters. The cluster sizes are better balanced with clusters of five, five, seven and 13 stocks respectively.

Stock	Distance
UTX	0.752
DIS	0.836
GE	0.842
AXP	0.901

Table 9: Table of Distances to decide which cluster to put BA into for period 3.

### B.3 Neighbor-Net Splits Graphs for Period Three

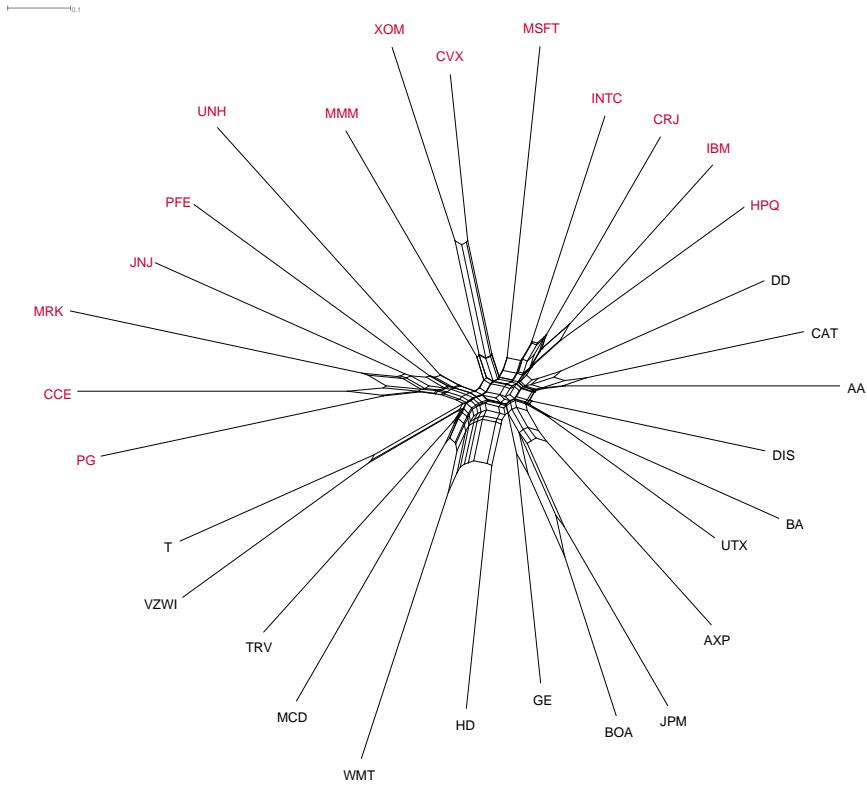


Figure 18: Neighbor-Net splits graph for Period 3 DJIA with two clusters. The cluster sizes are evenly balanced with clusters of 14 and 16 stocks respectively.

As indicated in the main body of this paper, the circular ordering of the stocks allows some flexibility in choosing the clusters. In Figures (18) and (19) shows as many as six breaks in the network structure giving the analyst some freedom in how they are combined to give the required number of clusters, in our case two and four. Figure (18) shows one possible way which yields a pair of clusters very similar in size. If one wished to combine two pairs of the four clusters in Figure (19) to form two clusters the most obvious choice is to group the stocks from T to DIS (reading counter-clockwise) into one cluster and AA to PG into a second cluster. This would yield clusters of sizes 13 and 17 stocks respectively.

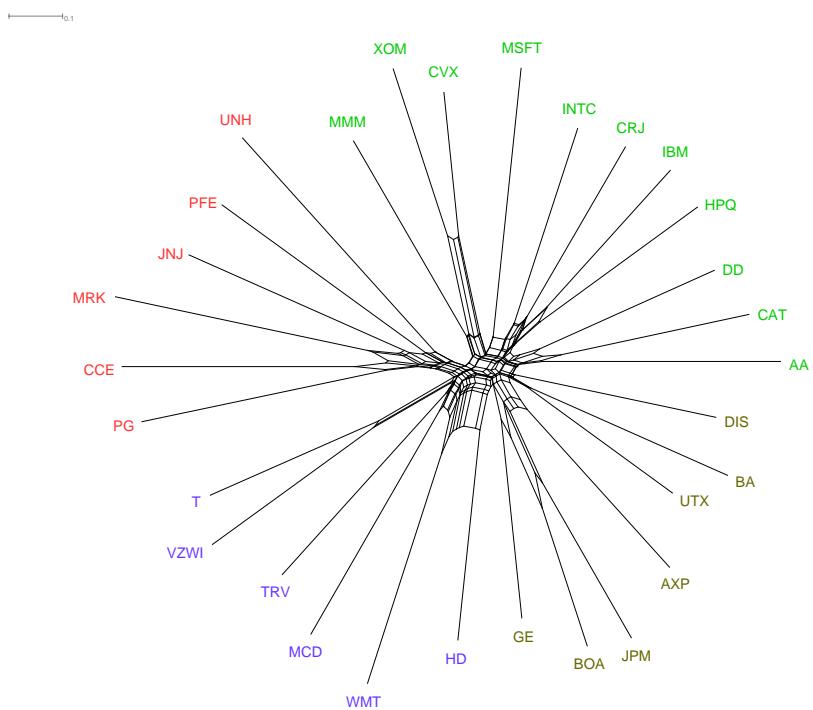


Figure 19: Neighbor-Net splits graph for Period 3 DJIA with four clusters. The cluster sizes are somewhat unbalanced with clusters of six, six, seven and 11 stocks respectively.

## C Extra Results

Table (10) presents the results of the simulations for groups determined by Period 1 data and tested against in-sample (Period 1) and out-of-sample (Period 2) data.

Table (11) presents the results of the simulations for groups determined by Period 3 data and tested against in-sample (Period 3) and out-of-sample (Period 4) data.

The other results are in the main body of the paper. However, some of the results here are discussed in the main body of the paper.

Period 1

Simulation results	Random	N-Net	HCT	MST	Industry Group	Levene p-value
Mean return						
(2-stock portfolios)	26.39	23.98	29.59	31.22	24.20	
(4-stock portfolios)	26.95	22.14	30.00	29.59	24.31	
(8-stock portfolios)	26.19	23.16	30.20	28.88	24.60	
Standard Deviation						
(2-stock portfolios)	32.44	30.12	28.91	32.57	31.22	0.005
(4-stock portfolios)	22.47	20.93	19.25	19.83	21.89	0.020
(8-stock portfolios)	14.40	14.30	13.11	13.01	13.91	0.002
Sharpe Ratios						
(2-stock portfolios)	0.72	0.70	0.91*	0.87	0.68	
(4-stock portfolios)	1.07	0.91	1.40*	1.34	0.97	
(8-stock portfolios)	1.61	1.41	2.07*	1.99	1.55	

Period 2

Simulation results	Random	N-Net	HCT	MST	Industry Group	Levene p-value
Mean return						
(2-stock portfolios)	60.80	61.25	67.93	64.38	66.35	
(4-stock portfolios)	61.06	59.23	67.59	63.05	68.79	
(8-stock portfolios)	61.65	59.27	67.13	62.60	67.90	
Standard Deviation						
(2-stock portfolios)	30.57	32.20	27.42	33.23	33.32	$9.9 \times 10^{-10}$
(4-stock portfolios)	21.16	21.87	20.46	23.17	21.06	0.008
(8-stock portfolios)	13.77	13.54	12.96	14.63	13.49	0.017
Sharpe Ratios						
(2-stock portfolios)	1.92	1.83	2.40*	1.87	1.92	
(4-stock portfolios)	2.78	2.60	3.20*	2.63	3.16	
(8-stock portfolios)	4.32	4.21	5.01*	4.13	4.87	

Table 10: Mean portfolio returns and the standard deviations of 1000 replications of five different portfolio selection methods for in-sample (period 1) and out-of-sample (period 2). The Sharpe ratios used a period return of 3.0% and 2.2% for periods 1 and 2 respectively. The highest Sharpe ratios within each portfolio size are marked with an asterisk. The Levene test p-values exclude the HCT because all results which included the HCT were very highly significant.

Period 3

Simulation results	Random	Neighbor-Net	HCT	MST	Industry Group	Levene p-value
Mean return						
(2-stock portfolios)	24.72	27.31	36.32	24.43	24.30	
(4-stock portfolios)	24.87	25.69	35.54	23.77	25.63	
(8-stock portfolios)	25.28	26.66	35.30	23.77	25.10	
Standard Deviation						
(2-stock portfolios)	23.52	22.30	21.86	21.99	22.58	0.109
(4-stock portfolios)	16.13	15.42	14.47	15.84	14.68	0.019
(8-stock portfolios)	9.98	8.98	8.44	10.26	9.40	$3.16 \times 10^{-5}$
Sharpe Ratios						
(2-stock portfolios)	0.86	1.03	1.46*	0.91	0.88	
(4-stock portfolios)	1.27	1.36	2.15*	1.22	1.44	
(8-stock portfolios)	2.09	2.48	3.68*	1.89	2.20	

Period 4

Simulation results	Random	Neighbor-Net	HCT	MST	Industry Group	Levene p-value
Mean return						
(2-stock portfolios)	104.51	115.60	121.98	93.84	106.37	
(4-stock portfolios)	104.35	115.29	121.89	96.31	107.04	
(8-stock portfolios)	105.86	116.03	120.67	96.24	107.63	
Standard Deviation						
(2-stock portfolios)	49.38	44.33	50.43	52.17	50.40	$5.27 \times 10^{-5}$
(4-stock portfolios)	34.15	30.32	33.57	35.35	34.68	0.0002
(8-stock portfolios)	22.34	19.74	19.62	22.18	22.60	$6.55 \times 10^{-5}$
Sharpe Ratios						
(2-stock portfolios)	2.08	2.57	2.39*	1.61	2.07	
(4-stock portfolios)	3.01	3.74*	3.58	2.68	3.04	
(8-stock portfolios)	4.66	5.79	6.06*	4.26	4.69	

Table 11: Mean portfolio returns and the standard deviations of 1000 replications of five different portfolio selection methods for in-sample (period 3) and out-of-sample (period 4). The Sharpe ratios used a period return of 4.4% and 1.7% for periods 3 and 4 respectively. The highest Sharpe ratios within each portfolio size are marked with an asterisk.

## D Stocks, Ticker Symbols and Industry Groups

Table (12) represents the 30 stocks in our sample in alphabetical order, with their exchanged-assigned ticket symbols and the industry group.

Table (13) has the stocks divided into the four super-groups which we used in the simulations.

Company Name	Symbol	Industry Group
3M	MMM	Diversified Industrials
Alcoa	AA	Aluminium
American Express	AXP	Insurance and Finance
AT&T	T	Telecom
Bank of America	BOA	Insurance and Finance
Boeing	BA	Aerospace
Caterpillar	CAT	Commercial Vehicles & Trucks
Chevron	CVX	Integrated Oil & Gas
Cisco Systems	CRJ	Telecom
Coca Cola	CCE	Soft Drinks
E I Du Pont de Nemours	DD	Commodity Chemicals
Exxon Mobil	XOM	Integrated Oil & Gas
General Electric	GE	Diversified Industrial
Hewlett-Packard	HPQ	Computer Hardware
Home Depot	HD	Home Improvement Retailers
Intel	INTC	Computer Hardware
International Bus.Mchs.	IBM	Computer Services
Johnson & Johnson	JNJ	Healthcare
JP Morgan Chase & Co.	JPM	Insurance and Finance
McDonalds	MCD	Resaurants & Bars
Merck & Co.	MRK	Pharmaceuticals
Microsoft	MSFT	Technology
Pfizer	PFE	Pharmaceuticals
Procter & Gamble	PG	Nondurable Household Products
Travelers Cos.	TRV	Insurance and Finance
United Technologies	UTX	Aerospace
United Health GP.	UNH	Healthcare
Verizon Communications	VZWI	Telecom
Wal Mart Stores	WMT	Retailers
Walt Disney	DIS	Broadcasting and Entertainment

Table 12: The 30 stocks in the Dow Jones sample with their ticker symbols and industry group.

Company Name	Symbol	Industry Group
American Express	AXP	Insurance and Finance
Bank of America	BOA	Insurance and Finance
Johnson & Johnson	JNJ	Healthcare
JP Morgan Chase & Co.	JPM	Insurance and Finance
Merck & Co.	MRK	Pharmaceuticals
Pfizer	PFE	Pharmaceuticals
Travelers Cos.	TRV	Insurance and Finance
United Health GP.	UNH	Healthcare
Alcoa	AA	Aluminium
Boeing	BA	Aerospace
Chevron	CVX	Integrated Oil & Gas
Exxon Mobil	XOM	Integrated Oil & Gas
United Technologies	UTX	Aerospace
3M	MMM	Diversified Industrials
Caterpillar	CAT	Commercial Vehicles & Trucks
Coca Cola	CCE	Soft Drinks
E I Du Pont de Nemours	DD	Commodity Chemicals
General Electric	GE	Diversified Industrial
Home Depot	HD	Home Improvement Retailers
McDonalds	MCD	Resaurants & Bars
Wal Mart Stores	WMT	Retailers
Walt Disney	DIS	Broadcasting and Entertainment
AT&T	T	Telecom
Cisco Systems	CRJ	Telecom
Hewlett-Packard	HPQ	Computer Hardware
Intel	INTC	Computer Hardware
International Bus.Mchs.	IBM	Computer Services
Microsoft	MSFT	Technology
Procter & Gamble	PG	Nondurable Household Products
Verizon Communications	VZWI	Telecom

Table 13: The 30 stocks in the Dow Jones sample with their ticker symbols and industry group with horizontal lines dividing the stocks into their four super-groups for the portfolio simulations using industry selection.