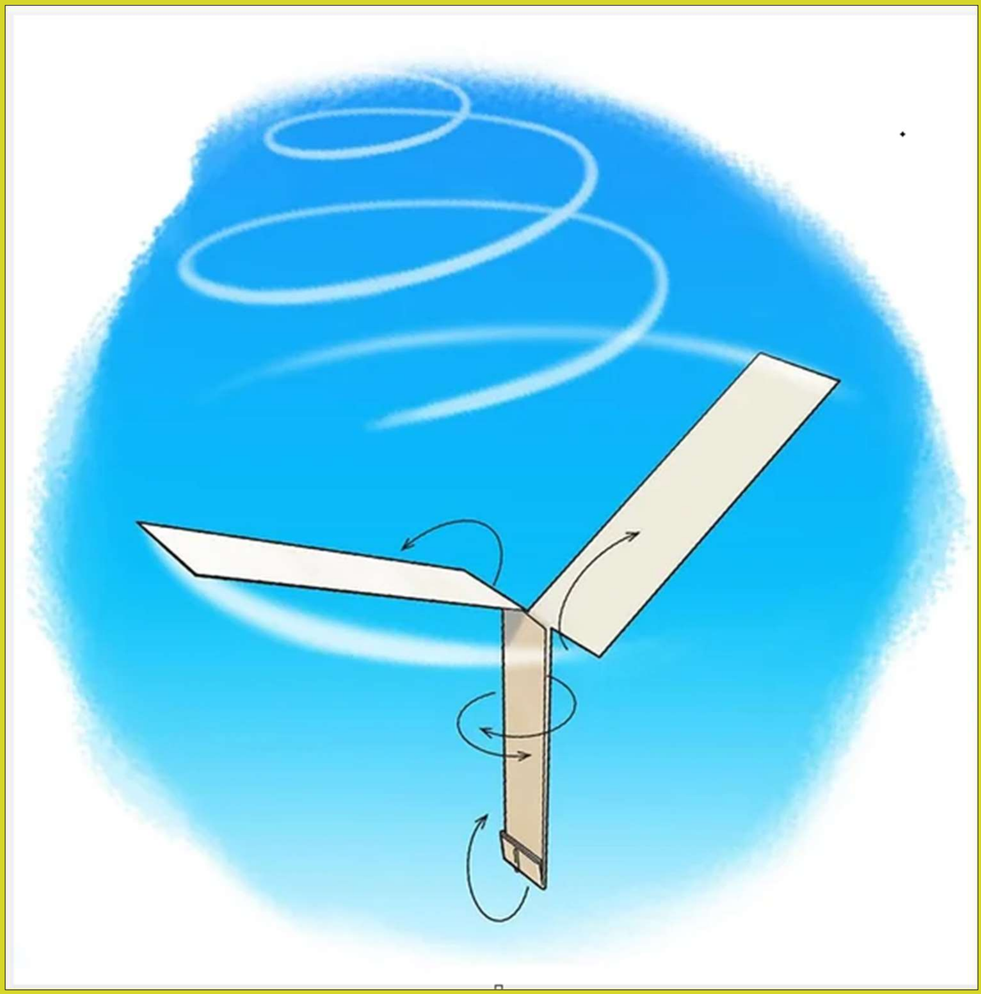

Experiment to identify optimal design parameters for Paper Helicopter

Group Members:

1. Ashit Ravi Poojari
2. Jyotsana Ashok Trivedi
3. Prithv Singh Pardhan

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Problem Statement

The purpose of this project is to utilize the orthogonal array technique to enhance the performance of the paper helicopter. In this project, we will be using Plackett- Burman Design and Hamada-Wu strategy to perform the experiment and to analyze and validate the data. This will help in identifying the significant variables and their level to maximize the flight time of our paper helicopters.

The input data for this experiment is the table with six factors and their level provided by Prof. Rui Tuo. We will be using this information to build our paper helicopters with replication and randomization. Using Plackett-Burman Design, we will perform the 12-run experiment and take the reading of flight time for each of those experiments. After that, we will use a half-normal plot and perform stepwise regression under the Hamnda-Wu strategy to remove the least important factor and identify the most significant factors and their level that will help us maximize the flight time of paper helicopters. For validating our results, we will use the identified significant factors and their level to perform a validation experiment.

Methodology

In this project, we are expected to design an experiment using the orthogonal array technique learned in the class. We have 6 factors to take into consideration for designing a paper helicopter and are required to perform this experiment in the minimum number of runs. The Plackett-Burman design is one such design that is used to identify the few important main effects.

Main and interaction effects: In statistics, the main effect is the effect of just one of the independent variables on the dependent variable. The number of main effects is the same as the independent variables. An interaction effect is an interaction between the independent variables that affect the dependent variable.

The fundamental principles guiding the variable selection are:

- Effect hierarchy:
  - Lower-order effects are more important compared to the higher-order effects.
  - Effects of the same order are equally important.
- Effect heredity:
  - An interaction is said to be significant if at least one of its parent factors is significant.
- Effect sparsity:
  - The number of relatively important factors is usually small.

Plackett-Burman design:

Plackett-Burman designs are non-regular orthogonal designs that are implemented to identify a few significant factors from a large set of available factors in a limited number of runs. For this experiment, we will be using OA (12,211) design that has 12 experimental runs and 11 factors each having 2 levels. The strength of the experiment (t) is taken as 2. 12 run design matrix is as shown below:

The purpose of this project is to utilize the orthogonal array technique to enhance the performance of the paper helicopter. In this project, we will be using Plackett- Burman Design and Hamada-Wu strategy to perform the experiment and to analyze and validate the data. This will help in identifying the significant variables and their level to maximize the flight time of our paper helicopters.

Original run	FACTOR										
RUN	A	B	C	D	E	F	G	8	9	10	11
1	+	+	-	+	+	+	-	-	-	+	-
2	+	-	+	+	+	-	-	-	+	-	+
3	-	+	+	+	-	-	-	+	-	+	+
4	+	+	+	-	-	-	+	-	+	+	-
5	+	+	-	-	-	+	-	+	+	-	+
6	+	-	-	-	+	-	+	+	-	+	+
7	-	-	-	+	-	+	+	-	+	+	+
8	-	-	+	-	+	+	-	+	+	+	-
9	-	+	-	+	+	-	+	+	+	-	-
10	+	-	+	+	-	+	+	+	-	-	-
11	-	+	+	-	+	+	+	-	-	-	+
12	-	-	-	-	-	-	-	-	-	-	-

We will be further optimizing the design matrix in consideration of the factors provided by our professor for this project in the following section.

.

Experiment

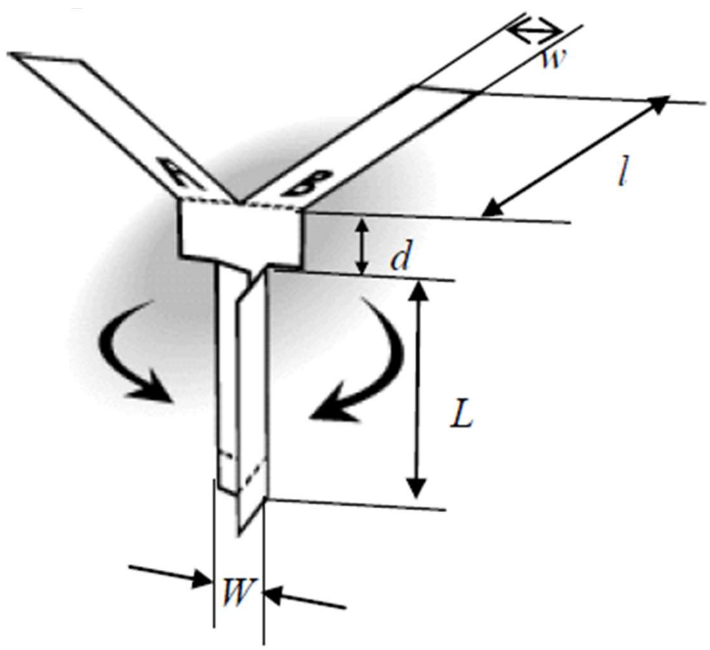
Input Data

We used the following input data shared by Prof. Rui Tuo to perform the experiment and identify the significant factors. We added design symbols to each of the factors and used that to refer to those factors while experimenting.

Sr No.	Factors	Symbol	Dimensions		Design Symbol
			- level	+ level	
1	Wing length	l	3 inches	4.5 inches	A
2	Wing width	w	1.8 inches	2.4 inches	B
3	Body length	L	3 inches	4.5 inches	C
4	Body width	W	1.25 inches	2 inches	D
5	Middle body length	d	1 inch	1.5 inches	E
6	Fold at tip	F	no	yes	F

Table 1.1

Following is the image provided to us along with input data to visualize the shape and dimensions of the paper helicopter.



Planning Matrix

In order to perform this experiment, we created a planning matrix using an orthogonal array based on a 12- run *Plackett-Burman design*, which can be denoted as OA(12, 2^{11}). Here, 12 refers to the number of runs, and 11 refers to the number of factors that can be used in the design.

This experiment required us to perform randomization while selecting six factors and a run-order sequence. To achieve that, we used R-code to randomly choose six factors out of 11, and columns no. "1", "10", "11", "5", "6", and "8" were chosen from table 1.1. Further, we used R-code for generating the random run sequence. The planning matrix used for the experiment is shown below:

STANDARD ORDER	RUN ORDER	Factors					
		A	B	C	D	E	F
1	9	1	1	-1	1	1	-1
2	4	1	-1	1	1	-1	-1
3	7	-1	1	1	-1	-1	1
4	1	1	1	-1	-1	-1	-1
5	2	1	-1	1	-1	1	1
6	5	1	1	1	1	-1	1
7	3	-1	1	1	-1	1	-1
8	8	-1	1	-1	1	1	1
9	6	-1	-1	-1	1	-1	1
10	11	1	-1	-1	-1	1	1
11	12	-1	-1	1	1	1	-1
12	10	-1	-1	-1	-1	-1	-1

Table 1.2

In the above table, +1 represents the high level and -1 represents the low level of the factors. The response variable (Y) for the experiment is flight time which needs to be maximized.

Experiment Methodology

Based on the factors given in table 1.2, we created 12 models of the paper helicopter on Solidworks which is a CAD software, and took the printouts of papers containing the dimensional layout and used scissors to create the required cut-out. After that, we folded the papers in the required shape to achieve 36 models (12 models for each person) of paper helicopters.

To drop the helicopter from a constant height, we created a reference by marking the wall at a distance of 100 inches from the ground. We dropped the helicopter from that constant height, three times each for replication. We used a stopwatch to record flight time during each 12-run with three replications. To create randomization while taking the flight reading, the stopwatch was used by each of the three team members in random order. Additionally, in order to counter variation due to environmental effects such as moisture content of air, airspeed, and temperature, we performed experiments at different times on days.

Data Collection

As per our experiment methodology, we dropped a paper helicopter from a constant height, three times each for replication, and measured flight time. Using the data from the experiment, we calculated the mean and variance for each run which is shown below in Table 1.3

STANDARD ORDER	RUN ORDER	FACTOR						Replications			Mean	Var s^2	$\ln s^2$
		A	B	C	D	E	F	1	2	3			
1	9	+	+	-	+	+	-	1.65	1.59	1.64	1.63	0.0010	-6.8750
2	4	+	-	+	+	-	-	1.65	1.64	1.78	1.69	0.0061	-5.0995
3	7	-	+	+	-	-	+	1.38	1.45	1.32	1.38	0.0042	-5.4648
4	1	+	+	-	-	-	-	2.1	2.18	2.18	2.15	0.0021	-6.1501
5	2	+	-	+	-	+	+	2.1	1.97	2.03	2.03	0.0042	-5.4648
6	5	+	+	+	+	-	+	1.98	1.97	1.91	1.95	0.0014	-6.5478
7	3	-	+	+	-	+	-	1.77	1.64	1.64	1.68	0.0056	-5.1791
8	8	-	+	-	+	+	+	1.64	1.59	1.52	1.58	0.0036	-5.6176
9	6	-	-	-	+	-	+	1.65	1.66	1.64	1.65	0.0001	-9.2103
10	11	+	-	-	-	+	+	1.95	1.98	1.91	1.95	0.0012	-6.6980
11	12	-	-	+	+	+	-	1.71	1.79	1.71	1.74	0.0021	-6.1501
12	10	-	-	-	-	-	-	1.66	1.72	1.65	1.68	0.0014	-6.5478

Table 1.3

Half-Normal Plot for Factor Analysis

After collecting data, we calculated the main effect of all six factors. The main effect of a factor is the difference between the average response of that factor at "high" factor level and the average response of that same factor at the "low" factor level. The table 1.4 and table 1.5 shows the main effect and absolute value for all six factors for location and dispersion.

Location	
Factor	**Absolute Value**
A	0.28
D	0.106
B	0.058
C	0.026
E	0.017
F	0.003

Table 1.4

Dispersion	
Factor	**Absolute Value**
C	1.199
D	0.666
B	0.634
E	0.506
F	0.5
A	0.22

Table 1.5

In order to obtain significant factors, we used half-normal plots which is a graphical tool used to help identify the factors that have a significant effect on the response. In the half-normal plot, the magnitude which is the absolute value of the main effect of factors is arranged in increasing magnitude along the X-axis. Figure 1.1 and 1.2 show that the factors are ranked according to their significance for location and dispersion respectively.

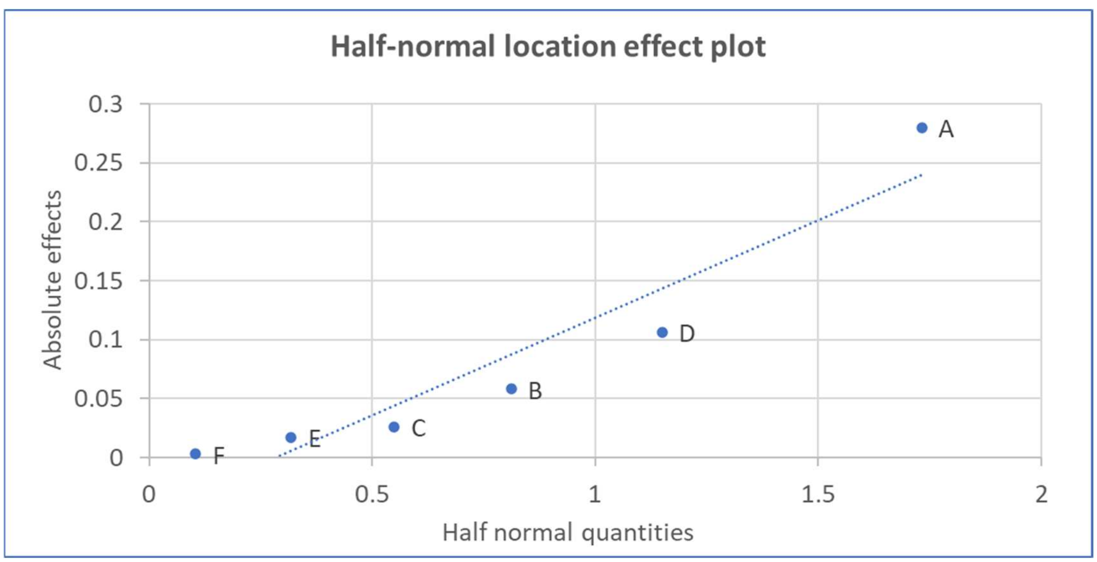


Figure 1.1

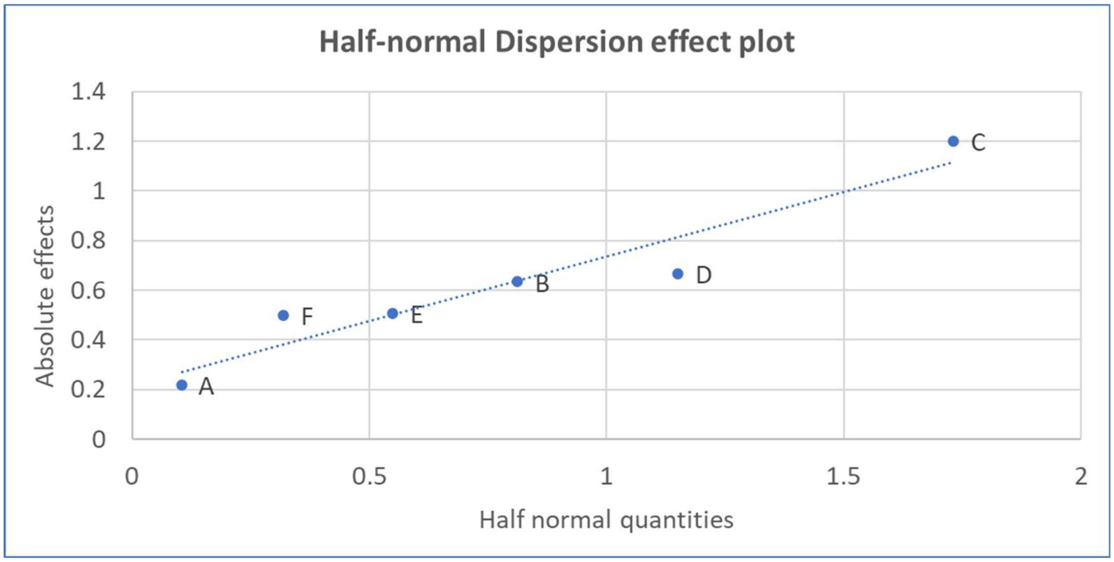


Figure 1.2

As per the Half-normal location effect plot, the most significant factor in this experiment is A i.e. wing length (l)

Stepwise Regression using Hamada-Wu Strategy

In the Plackett-Burman design, the analysis of design is mainly focused on the main effect under the assumption that the interactions are negligible. This is due to the complicated aliasing of the main effects with the interaction effect. Therefore, we used Hamada- Wu stepwise regression strategy. In the Hamada-Wu strategy, we can precisely identify the significant main effects and interaction effects by using the iterative analysis strategy which is based on the "principle of sparsity" and "principle of heredity".

Following is the procedure used to implement the Hamada-wu strategy in the effect analysis of our experiment:
Step-1: For each factor A, consider a model with main effect A and all two-factor interactions involving A i.e. AB, AC, AD, etc. After that, we apply stepwise regression to identify significant effects. We repeat this for the remaining five factors and keep the best model.

Step-2: In this step, we perform stepwise regression to identify the significant effects from a model which contains all the main effects and two-factor interactions from the best model identified in step 1.

Step-3: Using the effect heredity principle, we consider the significant effects identified in step 2 and two-factor interactions which have at least one factor that was identified as a significant main effect. After this, we perform stepwise regression again to identify the significant effects.

Step-4: We use the model obtained in step-2 and step-3 and perform an iteration until there are no changes in the significant factors identified.

Using the above procedure in R (refer to R-code attached in Appendix I), we identified A i.e. wing length (l) as the most significant factor for this experiment.

Results

From the experiment, we identified the significant factor A-wing length which affects our response variable i.e. flight time. Therefore, we will use this factor to maximize our flight time. The response variable can be represented using a regression equation as follow:

$$y = \alpha + \beta_A \, x_A$$

Here, α = intercept which is the global mean i.e. mean of all the observations, β_A is the effect of factor A and x_A represents the factor level.

For maximizing the response variable i.e. flight time, factor A is set to high i.e. + level. The value of β_A can be calculated as below:

$\beta_A = \frac{1}{2}$ * Main Effect of A = 0.14

Therefore, y = 1.76 + 0.12 = 1.90 seconds

Hence, as per our model, the maximum flight time is 1.90 seconds.

Validation

In the previous step, we obtained the expected value of flight time which is our response variable. In this section,n we validate our results using a confirmatory experiment keeping factor A level as high and other factors as high/low.

We have manufactured additionally two types of helicopters. One in which factor A is high and other factors as low and in the other factor A is high and other factors are high. The conditions of the experiment were kept the same i.e., there were dropped from the same height and the environmental conditions were also similar as before.

The results obtained from the experiment are as follows:

VALIDATION	FACTOR						Replications			Mean	Var s^2	$\ln s^2$
RUN ORDER	A	B	C	D	E	F	1	2	3			
1	+	+	+	+	+	+	2.13	2.04	2.09	2.09	0.0020	-6.1981
2	+	-	-	-	-	-	2.43	2.44	2.49	2.45	0.0010	-6.8750

From the validation experiment, we got a mean flight time of 2.09 sec with a variance of 0.002 sec corresponding to factor A, along with all other factors, set high for run 1. In run 2 we got a mean flight of 2.45 sec with a variance of 0.001 sec corresponding to factor A set high and all other factors set low.

If we compare means with our predicted value of flight time (fit of the experimental output of response from the model) obtained above, we can conclude that our analysis result is accurate and that irrespective of factors such as wing width, body length, body width, etc., the flight time is majorly affected by wing length.

The purpose of this project is to utilize the orthogonal array technique to enhance the performance of the paper helicopter. In this project, we will be using Plackett- Burman Design and Hamada-Wu strategy to perform the experiment and to analyze and validate the data. This will help in identifying the significant variables and their level to maximize the flight time of our paper helicopters.

Recommendation

We have done both the experiment and analysis. Deriving conclusions from them we can recommend that wing length must be set at a high level to maximize the flight time. Other factors are not so significant and therefore must be set at low levels to minimize the cost of construction and save materials.

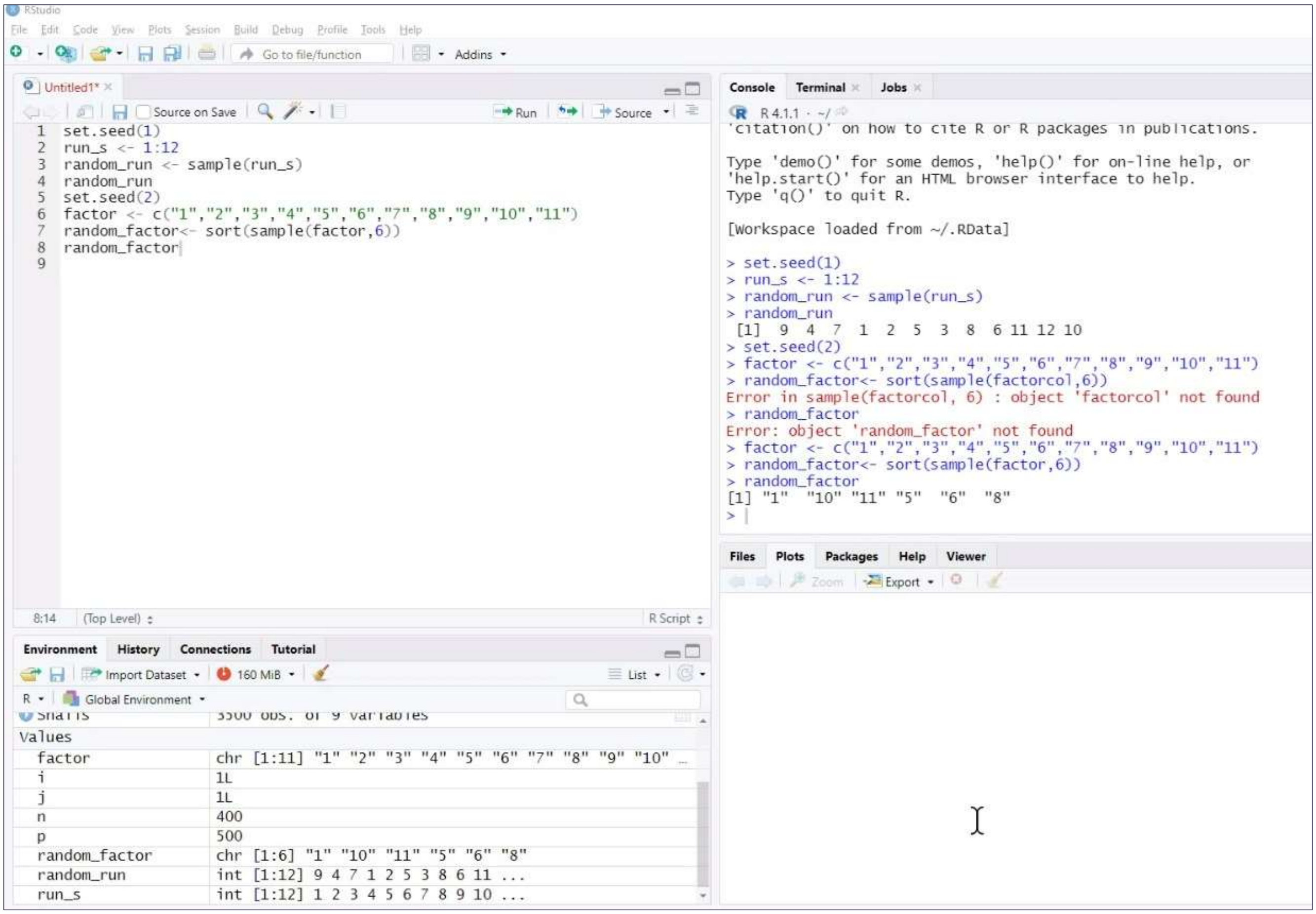
Finally, the optimal settings are as follows:

Symbol	Design Symbol	Factor	Optimal Levels	
			Uncoded	Coded
l	A	Wing Length	4.5 in	High
w	B	Wing Width	1.8 in	Low
L	C	Body Length	3 in	Low
W	D	Body Width	1.25 in	Low
d	E	Middle Body Length	1 in	Low
F	F	Fold At Tip	no	Low

The results obtained so far can be used for further analysis of other factors and de-aliased interactions which may affect the flight time in the experiment.

Appendix I

Part A: R code for six random factor selection and random order sequence



Part B: Stepwise Regression using Hamada-Wu Strategy

```
> trialF <- lm(Y ~ F + F:A + F:B + F:C + F:D + F:E, data = data)
> pF <- ols_step_both_p(trialF, details =T)
Stepwise Selection Method
---------------------------

Candidate Terms:

1. F
2. F:A
3. F:B
4. F:C
5. F:D
6. F:E

We are selecting variables based on p value...


Stepwise Selection: Step 1

+ F:B

                    Model Summary
--------------------------------------------------------------
R                    0.427       RMSE              0.207
R-Squared            0.182       Coef. Var        11.749
Adj. R-Squared       0.101       MSE               0.043
Pred R-Squared      -0.177       MAE               0.160
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                       ANOVA
-----------------------------------------------------------------
            Sum of
           Squares     DF    Mean Square     F       Sig.
-----------------------------------------------------------------
Regression    0.095      1         0.095    2.232    0.1661
Residual      0.427     10         0.043
Total         0.523     11
-----------------------------------------------------------------

                          Parameter Estimates
```

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.060		29.484	0.000	1.627	1.893
F:B	-0.089	0.060	-0.427	-1.494	0.166	-0.222	0.044

No more variables to be added/removed.

Final Model Output

Model Summary

R	0.427	RMSE	0.207
R-Squared	0.182	Coef. Var	11.749
Adj. R-Squared	0.101	MSE	0.043
Pred R-Squared	-0.177	MAE	0.160

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.095	1	0.095	2.232	0.1661
Residual	0.427	10	0.043		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.060		29.484	0.000	1.627	1.893
F:B	-0.089	0.060	-0.427	-1.494	0.166	-0.222	0.044

> pF

Stepwise Selection Summary

```
---------------------------------------------------------------------------------
                Added/                  Adj.
Step   Variable   Removed   R-Square   R-Square    C(p)     AIC      RMSE
---------------------------------------------------------------------------------
   1      F:B      addition      0.182       0.101    1.2640   0.0371    0.2068
---------------------------------------------------------------------------------
> ##Step 2(iteration1)
> trial2 <- lm(Y ~ A + A:D + A:F + B + C+ C:D +D+ D:C +E+ E:B +F+ F:B , data = data)
> p2 <- ols_step_both_p(trial2, details =T)
Stepwise Selection Method
---------------------------

Candidate Terms:

1. A
2. B
3. C
4. D
5. E
6. F
7. A:D
8. A:F
9. D:C
10. B:E
11. F:B

We are selecting variables based on p value...


Stepwise Selection: Step 1

+ A

                    Model Summary
--------------------------------------------------------------
R                   0.675       RMSE              0.169
R-Squared            0.455       Coef. Var          9.591
Adj. R-Squared        0.401       MSE                0.028
Pred R-Squared        0.215       MAE                0.126
--------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                         ANOVA
-------------------------------------------------------------------
```

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.238	1	0.238	8.355	0.0161
Residual	0.285	10	0.028		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.049		36.117	0.000	1.651	1.868
A	0.141	0.049	0.675	2.891	0.016	0.032	0.249

Stepwise Selection: Step 2

+ A:D

Model Summary

R	0.803	RMSE	0.144
R-Squared	0.645	Coef. Var	8.166
Adj. R-Squared	0.566	MSE	0.021
Pred R-Squared	0.368	MAE	0.113

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.337	2	0.169	8.16	0.0095
Residual	0.186	9	0.021		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper

(Intercept)	1.760	0.041		42.419	0.000	1.666	1.854
A	0.141	0.041	0.675	3.395	0.008	0.047	0.235
A:D	-0.091	0.041	-0.435	-2.190	0.056	-0.185	0.003

Model Summary

R	0.803	RMSE	0.144
R-Squared	0.645	Coef. Var	8.166
Adj. R-Squared	0.566	MSE	0.021
Pred R-Squared	0.368	MAE	0.113

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.337	2	0.169	8.16	0.0095
Residual	0.186	9	0.021		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.041		42.419	0.000	1.666	1.854
A	0.141	0.041	0.675	3.395	0.008	0.047	0.235
A:D	-0.091	0.041	-0.435	-2.190	0.056	-0.185	0.003

Stepwise Selection: Step 3

+ A:F

Model Summary

R	0.887	RMSE	0.118
R-Squared	0.786	Coef. Var	6.715
Adj. R-Squared	0.706	MSE	0.014
Pred R-Squared	0.519	MAE	0.079

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.411	3	0.137	9.816	0.0047
Residual	0.112	8	0.014		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.034		51.588	0.000	1.681	1.838
A	0.141	0.034	0.675	4.129	0.003	0.062	0.219
A:D	-0.091	0.034	-0.435	-2.663	0.029	-0.169	-0.012
A:F	0.079	0.034	0.377	2.305	0.050	0.000	0.157

Model Summary

R	0.887	RMSE	0.118
R-Squared	0.786	Coef. Var	6.715
Adj. R-Squared	0.706	MSE	0.014
Pred R-Squared	0.519	MAE	0.079

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.

Regression	0.411	3	0.137	9.816	0.0047
Residual	0.112	8	0.014		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.034		51.588	0.000	1.681	1.838
A	0.141	0.034	0.675	4.129	0.003	0.062	0.219
A:D	-0.091	0.034	-0.435	-2.663	0.029	-0.169	-0.012
A:F	0.079	0.034	0.377	2.305	0.050	0.000	0.157

Stepwise Selection: Step 4

+ E

Model Summary

R	0.924	RMSE	0.104
R-Squared	0.854	Coef. Var	5.935
Adj. R-Squared	0.771	MSE	0.011
Pred R-Squared	0.572	MAE	0.057

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.447	4	0.112	10.235	0.0047
Residual	0.076	7	0.011		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper

(Intercept)	1.760	0.030		58.369	0.000	1.688	1.831
A	0.141	0.030	0.675	4.671	0.002	0.070	0.212
E	-0.062	0.034	-0.295	-1.800	0.115	-0.142	0.019
A:D	-0.111	0.032	-0.533	-3.455	0.011	-0.188	-0.035
A:F	0.099	0.032	0.475	3.076	0.018	0.023	0.175

Model Summary

R	0.924	RMSE	0.104
R-Squared	0.854	Coef. Var	5.935
Adj. R-Squared	0.771	MSE	0.011
Pred R-Squared	0.572	MAE	0.057

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.447	4	0.112	10.235	0.0047
Residual	0.076	7	0.011		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.030		58.369	0.000	1.688	1.831
A	0.141	0.030	0.675	4.671	0.002	0.070	0.212
E	-0.062	0.034	-0.295	-1.800	0.115	-0.142	0.019
A:D	-0.111	0.032	-0.533	-3.455	0.011	-0.188	-0.035
A:F	0.099	0.032	0.475	3.076	0.018	0.023	0.175

Stepwise Selection: Step 5

+ B

Model Summary

R	0.953	RMSE	0.090
R-Squared	0.908	Coef. Var	5.089
Adj. R-Squared	0.831	MSE	0.008
Pred R-Squared	0.575	MAE	0.049

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.475	5	0.095	11.84	0.0046
Residual	0.048	6	0.008		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.026		68.070	0.000	1.696	1.823
A	0.141	0.026	0.675	5.448	0.002	0.078	0.204
E	-0.078	0.031	-0.373	-2.548	0.044	-0.153	-0.003
B	0.057	0.031	0.275	1.876	0.110	-0.017	0.132
A:D	-0.136	0.031	-0.651	-4.444	0.004	-0.211	-0.061
A:F	0.124	0.031	0.593	4.045	0.007	0.049	0.199

Model Summary

R	0.953	RMSE	0.090
R-Squared	0.908	Coef. Var	5.089
Adj. R-Squared	0.831	MSE	0.008
Pred R-Squared	0.575	MAE	0.049

RMSE: Root Mean Square Error
MSE: Mean Square Error

MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.475	5	0.095	11.84	0.0046
Residual	0.048	6	0.008		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.026		68.070	0.000	1.696	1.823
A	0.141	0.026	0.675	5.448	0.002	0.078	0.204
E	-0.078	0.031	-0.373	-2.548	0.044	-0.153	-0.003
B	0.057	0.031	0.275	1.876	0.110	-0.017	0.132
A:D	-0.136	0.031	-0.651	-4.444	0.004	-0.211	-0.061
A:F	0.124	0.031	0.593	4.045	0.007	0.049	0.199

Stepwise Selection: Step 6

+ F

Model Summary

R	0.983	RMSE	0.059
R-Squared	0.967	Coef. Var	3.325
Adj. R-Squared	0.928	MSE	0.003
Pred R-Squared	0.825	MAE	0.031

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.

Regression	0.506	6	0.084	24.616	0.0015
Residual	0.017	5	0.003		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.017		104.171	0.000	1.716	1.803
A	0.141	0.017	0.675	8.337	0.000	0.097	0.184
E	-0.089	0.020	-0.426	-4.380	0.007	-0.141	-0.037
B	0.068	0.020	0.328	3.368	0.020	0.016	0.121
F	-0.055	0.018	-0.265	-3.009	0.030	-0.103	-0.008
A:D	-0.162	0.022	-0.775	-7.437	0.001	-0.218	-0.106
A:F	0.131	0.020	0.628	6.510	0.001	0.079	0.183

Model Summary

R	0.983	RMSE	0.059
R-Squared	0.967	Coef. Var	3.325
Adj. R-Squared	0.928	MSE	0.003
Pred R-Squared	0.825	MAE	0.031

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.506	6	0.084	24.616	0.0015
Residual	0.017	5	0.003		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.017		104.171	0.000	1.716	1.803

A	0.141	0.017	0.675	8.337	0.000	0.097	0.184
E	-0.089	0.020	-0.426	-4.380	0.007	-0.141	-0.037
B	0.068	0.020	0.328	3.368	0.020	0.016	0.121
F	-0.055	0.018	-0.265	-3.009	0.030	-0.103	-0.008
A:D	-0.162	0.022	-0.775	-7.437	0.001	-0.218	-0.106
A:F	0.131	0.020	0.628	6.510	0.001	0.079	0.183

Stepwise Selection: Step 7

+ C

Model Summary

R	0.984	RMSE	0.065
R-Squared	0.967	Coef. Var	3.704
Adj. R-Squared	0.911	MSE	0.004
Pred R-Squared	0.756	MAE	0.031

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.506	7	0.072	17.009	0.0079
Residual	0.017	4	0.004		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.019		93.520	0.000	1.707	1.812
A	0.141	0.019	0.675	7.485	0.002	0.089	0.193
E	-0.089	0.023	-0.426	-3.926	0.017	-0.152	-0.026
B	0.068	0.023	0.327	3.018	0.039	0.005	0.131
F	-0.055	0.021	-0.263	-2.652	0.057	-0.112	0.003
C	-0.004	0.022	-0.018	-0.173	0.871	-0.064	0.056
A:D	-0.160	0.026	-0.768	-6.243	0.003	-0.232	-0.089

A:F	0.132	0.023	0.634	5.643	0.005	0.067	0.197

Model Summary

R	0.984	RMSE	0.065
R-Squared	0.967	Coef. Var	3.704
Adj. R-Squared	0.911	MSE	0.004
Pred R-Squared	0.756	MAE	0.031

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.506	7	0.072	17.009	0.0079
Residual	0.017	4	0.004		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.019		93.520	0.000	1.707	1.812
A	0.141	0.019	0.675	7.485	0.002	0.089	0.193
E	-0.089	0.023	-0.426	-3.926	0.017	-0.152	-0.026
B	0.068	0.023	0.327	3.018	0.039	0.005	0.131
F	-0.055	0.021	-0.263	-2.652	0.057	-0.112	0.003
C	-0.004	0.022	-0.018	-0.173	0.871	-0.064	0.056
A:D	-0.160	0.026	-0.768	-6.243	0.003	-0.232	-0.089
A:F	0.132	0.023	0.634	5.643	0.005	0.067	0.197

Stepwise Selection: Step 8

x B

Model Summary

R	0.945	RMSE	0.106
R-Squared	0.893	Coef. Var	5.998
Adj. R-Squared	0.766	MSE	0.011
Pred R-Squared	0.349	MAE	0.059

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.467	6	0.078	6.989	0.0249
Residual	0.056	5	0.011		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.030		57.752	0.000	1.681	1.838
A	0.141	0.030	0.675	4.622	0.006	0.063	0.219
E	-0.068	0.035	-0.325	-1.943	0.110	-0.157	0.022
F	-0.043	0.033	-0.208	-1.319	0.244	-0.128	0.041
C	-0.005	0.035	-0.026	-0.153	0.884	-0.095	0.084
A:D	-0.126	0.037	-0.604	-3.379	0.020	-0.222	-0.030
A:F	0.103	0.035	0.493	2.980	0.031	0.014	0.192

Stepwise Selection: Step 8

+ D

Model Summary

R	0.950	RMSE	0.113
R-Squared	0.903	Coef. Var	6.404
Adj. R-Squared	0.733	MSE	0.013
Pred R-Squared	-0.186	MAE	0.056

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.472	7	0.067	5.311	0.0629
Residual	0.051	4	0.013		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.033		54.095	0.000	1.669	1.850
A	0.141	0.033	0.675	4.329	0.012	0.051	0.231
E	-0.065	0.038	-0.310	-1.721	0.160	-0.169	0.040
F	-0.043	0.035	-0.208	-1.236	0.284	-0.141	0.054
C	-0.002	0.038	-0.011	-0.059	0.956	-0.107	0.102
D	-0.022	0.035	-0.105	-0.622	0.568	-0.119	0.076
A:D	-0.126	0.040	-0.604	-3.165	0.034	-0.237	-0.015
A:F	0.094	0.040	0.448	2.350	0.079	-0.017	0.204

Model Summary

R	0.950	RMSE	0.113
R-Squared	0.903	Coef. Var	6.404
Adj. R-Squared	0.733	MSE	0.013
Pred R-Squared	-0.186	MAE	0.056

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.472	7	0.067	5.311	0.0629
Residual	0.051	4	0.013		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.033		54.095	0.000	1.669	1.850
A	0.141	0.033	0.675	4.329	0.012	0.051	0.231
E	-0.065	0.038	-0.310	-1.721	0.160	-0.169	0.040
F	-0.043	0.035	-0.208	-1.236	0.284	-0.141	0.054
C	-0.002	0.038	-0.011	-0.059	0.956	-0.107	0.102
D	-0.022	0.035	-0.105	-0.622	0.568	-0.119	0.076
A:D	-0.126	0.040	-0.604	-3.165	0.034	-0.237	-0.015
A:F	0.094	0.040	0.448	2.350	0.079	-0.017	0.204

Stepwise Selection: Step 10

x E

Model Summary

R	0.912	RMSE	0.133
R-Squared	0.831	Coef. Var	7.556
Adj. R-Squared	0.628	MSE	0.018
Pred R-Squared	-0.347	MAE	0.071

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.434	6	0.072	4.096	0.0716
Residual	0.088	5	0.018		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.038		45.848	0.000	1.661	1.858
A	0.141	0.038	0.675	3.669	0.014	0.042	0.239
F	-0.035	0.041	-0.169	-0.860	0.429	-0.141	0.070
C	-0.002	0.044	-0.011	-0.050	0.962	-0.116	0.112
D	-0.030	0.041	-0.143	-0.729	0.499	-0.136	0.076
A:D	-0.102	0.044	-0.488	-2.317	0.068	-0.215	0.011
A:F	0.069	0.044	0.332	1.578	0.175	-0.044	0.182

No more variables to be added/removed.

Final Model Output

Model Summary

R	0.912	RMSE	0.133
R-Squared	0.831	Coef. Var	7.556
Adj. R-Squared	0.628	MSE	0.018
Pred R-Squared	-0.347	MAE	0.071

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.434	6	0.072	4.096	0.0716
Residual	0.088	5	0.018		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper

```
-----------------------------------------------------------------------------------
(Intercept)    1.760       0.038              45.848    0.000     1.661    1.858
        A     0.141       0.038       0.675     3.669    0.014     0.042    0.239
        F    -0.035       0.041      -0.169    -0.860    0.429    -0.141    0.070
        C    -0.002       0.044      -0.011    -0.050    0.962    -0.116    0.112
        D    -0.030       0.041      -0.143    -0.729    0.499    -0.136    0.076
      A:D    -0.102       0.044      -0.488    -2.317    0.068    -0.215    0.011
      A:F     0.069       0.044       0.332     1.578    0.175    -0.044    0.182
-----------------------------------------------------------------------------------
There were 20 warnings (use warnings() to see them)
> p2

                      Stepwise Selection Summary
------------------------------------------------------------------------------
                  Added/                  Adj.
Step    Variable   Removed     R-Square   R-Square    C(p)      AIC        RMSE
------------------------------------------------------------------------------
   1       A       addition      0.455      0.401     NaN     -4.8331    0.1688
   2      A:D      addition      0.645      0.566     NaN     -7.9575    0.1437
   3      A:F      addition      0.786      0.706     NaN    -12.0672    0.1182
   4       E       addition      0.854      0.771     NaN    -14.6338    0.1044
   5       B       addition      0.908      0.831     NaN    -18.1735    0.0896
   6       F       addition      0.967      0.928     NaN    -28.5733    0.0585
   7       C       addition      0.967      0.911     NaN    -26.6623    0.0652
   8       B       removal       0.893      0.766     NaN    -14.4163    0.1056
   9       D       addition      0.903      0.733     NaN    -13.5238    0.1127
  10       E       removal       0.831      0.628     NaN     -8.8766    0.1330
------------------------------------------------------------------------------
> ##Step 3(iteration1)
> trial3 <- lm(Y ~ A:C+A:D+A:F+C:D+C:F+D:F, data = data)
> p3 <- ols_step_both_p(trial3, details =T)
Stepwise Selection Method
---------------------------

Candidate Terms:

1. A:C
2. A:D
3. A:F
4. C:D
5. C:F
6. D:F

We are selecting variables based on p value...
```

Stepwise Selection: Step 1

+ C:D

Model Summary

R	0.478	RMSE	0.201
R-Squared	0.228	Coef. Var	11.416
Adj. R-Squared	0.151	MSE	0.040
Pred R-Squared	-0.111	MAE	0.145

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.119	1	0.119	2.957	0.1162
Residual	0.404	10	0.040		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.058		30.345	0.000	1.631	1.889
C:D	0.100	0.058	0.478	1.720	0.116	-0.029	0.229

No more variables to be added/removed.

Final Model Output

Model Summary

R	0.478	RMSE	0.201
R-Squared	0.228	Coef. Var	11.416
Adj. R-Squared	0.151	MSE	0.040

```
Pred R-Squared        -0.111       MAE              0.145
---------------------------------------------------------------
 RMSE: Root Mean Square Error
 MSE: Mean Square Error
 MAE: Mean Absolute Error

                           ANOVA
------------------------------------------------------------------
              Sum of
             Squares        DF    Mean Square      F        Sig.
------------------------------------------------------------------
Regression      0.119        1         0.119    2.957    0.1162
Residual        0.404       10         0.040
Total           0.523       11
------------------------------------------------------------------

                             Parameter Estimates
--------------------------------------------------------------------------------------
      model    Beta    Std. Error    Std. Beta     t       Sig      lower    upper
--------------------------------------------------------------------------------------
(Intercept)    1.760        0.058                30.345    0.000    1.631    1.889
       C:D    0.100        0.058        0.478     1.720    0.116   -0.029    0.229
--------------------------------------------------------------------------------------
> p3

                          Stepwise Selection Summary
-----------------------------------------------------------------------------------
                 Added/                 Adj.
Step    Variable    Removed     R-Square    R-Square     C(p)       AIC        RMSE
-----------------------------------------------------------------------------------
   1      C:D       addition       0.228       0.151    2.4010    -0.6540    0.2009
-----------------------------------------------------------------------------------
> ##Step 2(iteration2)
> trial22 <- lm(Y ~A+B+C+D+E+F+C:D , data = data)
> p22 <- ols_step_both_p(trial22, details =T)
Stepwise Selection Method
---------------------------

Candidate Terms:

1. A
2. B
3. C
4. D
5. E
6. F
```

7. C:D

We are selecting variables based on p value...

Stepwise Selection: Step 1

+ A

Model Summary

R	0.675	RMSE	0.169
R-Squared	0.455	Coef. Var	9.591
Adj. R-Squared	0.401	MSE	0.028
Pred R-Squared	0.215	MAE	0.126

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.238	1	0.238	8.355	0.0161
Residual	0.285	10	0.028		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.049		36.117	0.000	1.651	1.868
A	0.141	0.049	0.675	2.891	0.016	0.032	0.249

No more variables to be added/removed.

Final Model Output

Model Summary

R	0.675	RMSE	0.169
R-Squared	0.455	Coef. Var	9.591
Adj. R-Squared	0.401	MSE	0.028
Pred R-Squared	0.215	MAE	0.126

RMSE: Root Mean Square Error
MSE: Mean Square Error
MAE: Mean Absolute Error

ANOVA

	Sum of Squares	DF	Mean Square	F	Sig.
Regression	0.238	1	0.238	8.355	0.0161
Residual	0.285	10	0.028		
Total	0.523	11			

Parameter Estimates

model	Beta	Std. Error	Std. Beta	t	Sig	lower	upper
(Intercept)	1.760	0.049		36.117	0.000	1.651	1.868
A	0.141	0.049	0.675	2.891	0.016	0.032	0.249

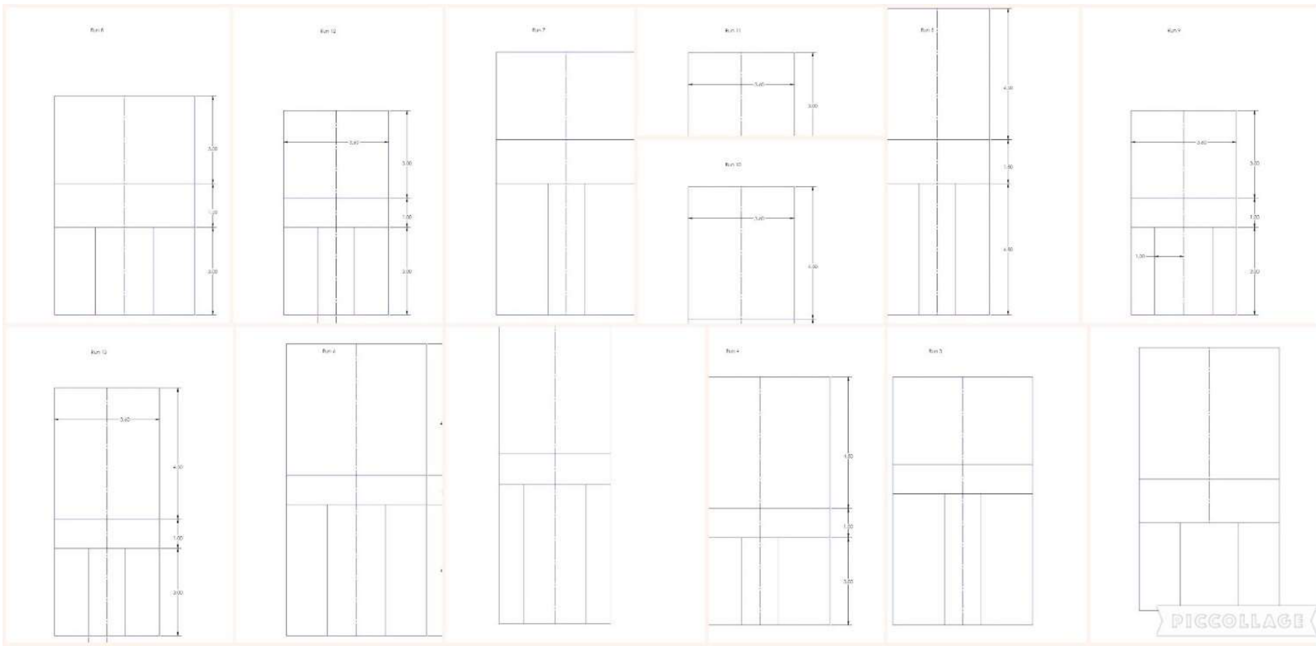
> p22#at this stage we are only getting only one main effect as important

Stepwise Selection Summary

Step	Variable	Added/ Removed	R-Square	Adj. R-Square	C(p)	AIC	RMSE
1	A	addition	0.455	0.401	-2.1040	-4.8331	0.1688

Appendix II

Part A: Layout of 12 models prepared in Solidworks



Part B

Following are the photos of paper helicopter made by our team for paper helicopter experiment:

References

1. Texas A&M logo: https://chotnsf.org/wp-content/uploads/2015/02/Atm-logo.png
2. Title page picture of the paper helicopter:
https://www.scientificamerican.com/article/make-a-whirlybird-from-paper/
3. [Wiley Series in Probability and Statistics] C. F. Jeff Wu, Michael S. Hamada - Experiments_ Planning, Analysis, and Optimization (2009, Wiley)