Consolidated Report

Prithvi Poddar 17191

<u>Note:</u> Report 2 and 3 are the reports on CNN and autoencoder respectively. Those were made on Google Colab as it provides GPU facility. Thus, the .ipynb files regarding the same might not run on the local jupyter notebook. Although I've added those python notebooks in the report directory, I'll leave a link to the Colab notebook here as well . You can visit the link to view the code in Colab.

To run the code, you'll have to make a copy of it in your own drive as my codes are in view only format.

Report 2 (CNN) link:
https://colab.research.google.com/drive/1GxHUK_lYsqA3YGl_Ts4SltuB5q7tVE4L?usp=sharing

Report 3 (Autoencoder) link:
https://colab.research.google.com/drive/1_qcXU7SAz_ZQjyYU9bw3jxThv6E6d30V?usp=sharing

Starting with Report-1 now.

Report-1 (Neural network)

Files: Prithvi_Poddar_17191_Report_1.ipynb and
Prithvi_Poddar_17191_Report_1.pdf

Part -1:

I have defined the network as a class in the python
file named **network.py** in the parent directory. The
code to load the data is in **data_loader.py** in the
parent directory.

In part 1, I start off with using the neural network
with normal training and testing data splits and do
the hyper parameter tuning for the network. At the
end, we find out the the best parameter choices are:

Weight_initializer = default
Cost = CrossEntropy
Activation = sigmoid
Epochs = 50
Batch_size = 10
learning_rate = 0.1
Regularization_parameter = 7.0

With this we got an accuracy of around 95% on the
training data and 88% on the testing data that was
provided.

Finally we used **10-fold cross validation** and got training accuracy of 97% and testing accuracy of 96% on the given data.
Then I ran the model on the entire MNIST dataset with cross validation, getting an accuracy of 95% in both training and testing sets.

Part-2 (PCA data):

Here we ran the same model on the PCA data provided and ended up getting a maximum accuracy of 88% on the test set and 87% on training set. This showed that the PCA data wasn't as good as the original data.

P.S. I'd suggest you to look at the python notebook for report 1 as the pdf is too long because the pdf contains the outputs in their entirety. The python notebooks have the outputs too but in smaller windows and hence is easy to follow.

Report-2 (CNN):

Files: Prithvi_Poddar_17191_report_2_CNN.ipynb and Prithvi_Poddar_17191_report_2_CNN.pdf

Colab link:
https://colab.research.google.com/drive/1GxHUK_l
YsqA3YGl_Ts4SltuB5q7tVE4L?usp=sharing

In this report, for the architecture of the CNN, I took help form
https://www.kaggle.com/cdeotte/how-to-choose-c
nn-architecture-mnist
as this site has already done some of the experiments to find the optimal parameters. I avoided doing these experiments manually as the CNN in itself takes lot of time to train and hence it would be a highly time taking process to train the model for each parameter.
The architecture of the model is:

```
Network_CNN(
  (conv1): Conv2d(1, 32, kernel_size=(5, 5), stride=(1, 1))
  (conv2): Conv2d(32, 64, kernel_size=(5, 5), stride=(1, 1))
  (fc1): Linear(in_features=1024, out_features=500, bias=True)
  (dropout1): Dropout(p=0.4, inplace=False)
  (fc2): Linear(in_features=500, out_features=10, bias=True)
)
```

At the end of the report, I have also derived the outputs of individual convolutional filters of the CNN to look at what kind of information does the CNN learn.

Report-3 (CNN):

Files:
Prithvi_Poddar_17191_Report_3_Autoencoders.ipynb and
Prithvi_Poddar_17191_Report_3_Autoencoders.pdf
Colab link:
https://colab.research.google.com/drive/1_qcXU7SAz_ZQjyYU9bw3jxThv6E6d30V?usp=sharing

In this, I built a sparse autoencoder inspired by the notes of Andrew NG on sparse autoencoder.
The model has the following parameters:

```
SparseAutoencoder(
  (en1): Linear(in_features=784, out_features=256, bias=True)
  (en2): Linear(in_features=256, out_features=128, bias=True)
  (en3): Linear(in_features=128, out_features=64, bias=True)
  (en4): Linear(in_features=64, out_features=32, bias=True)
  (en5): Linear(in_features=32, out_features=16, bias=True)
  (de1): Linear(in_features=16, out_features=32, bias=True)
  (de2): Linear(in_features=32, out_features=64, bias=True)
  (de3): Linear(in_features=64, out_features=128, bias=True)
  (de4): Linear(in_features=128, out_features=256, bias=True)
  (de5): Linear(in_features=256, out_features=784, bias=True)
)
```

At the end, I've regenerated 20 input images from the validation set, using the autoencoder.

Prithvi_Poddar_17191_Report_1

June 22, 2020

1 Assignment 4

2 Prithvi Poddar 17191

2.1 Report: Neural Network

In this report, I'll be running a neural network based completely on numpy and inspired from http://neuralnetworksanddeeplearning.com/index.html

The codes can be found in the same directory as this python notebook is.

P.S. Initially I'll be doing all the parameter tuning using train and test set splitting as this model converges quicker than Cross Validation. This will allow us to get better results in less number of epochs. Once we've decided our parameters, we'll finally use Cross validation to see if it gives better accuracy

2.2 Part 1

2.2.1 Importing the neural network module and the data loader

```
[1]: import matplotlib.pyplot as plt
     import numpy as np
     import network
     import data_loader
     %matplotlib inline
     plt.rcParams['figure.figsize']=(20,10)
```

2.2.2 Before starting, lets take a look at tunable parameters of this linear network model

weight_initializer = 'default' or 'large'

cost = 'CrossEntropy' or 'Quadratic'

activation = 'sigmoid' or 'tanh' or 'relu'

size = [n1, n2, ...] {where n1=no. of neurons in first layer, n2=no. of neurons in second layer, and so on}

2.2.3 Parameters for training

training_data, epochs, batch_size, eta(learning rate), lmbda(regularization factor), test_data

2.2.4 Loading the training and testing data

We have total 3000 data entries. We'll be dividing it in 2500 for training and 500 for testing

```
[2]: training_data, testing_data = data_loader.load_full_data()
```

2.2.5 Now we call the neural network from the network.py file and start tuning our parameters

2.2.6 First, no hidden layer

Here we'll determine which cost function, activation function and weight initializer to use:

1. Choosing the weight initializer

'default' sets the weights randomly from gaussian distribution with mean 0 and standard deviation 1/sqrt(number of training data)

'large' sets the weights randomly from gaussian distribution with mean 0 and standard deviation 1

```
[3]: net = network.Linear_Network([784,10], weight_initializer='default')
```

```
[4]: net.SGD(training_data, 30, 10, 0.5, lmbda = 5.0, test_data=testing_data)
```

```
Epoch 1
Training loss: 1.3272248517655656637
Training accuracy: 82.36%
Test accuracy: 76.4%

Epoch 2
Training loss: 0.71936398376920923024
Training accuracy: 90.88000000000001%
Test accuracy: 84.0%

Epoch 3
Training loss: 0.7287727998241616091
Training accuracy: 90.08%
Test accuracy: 81.6%

Epoch 4
```

2

```
Training loss: 0.8360652745009499089
Training accuracy: 89.12%
Test accuracy: 82.6%


Epoch 5
Training loss: 0.6425745589650592904
Training accuracy: 92.04%
Test accuracy: 86.6%


Epoch 6
Training loss: 0.6241366896945118083
Training accuracy: 92.0%
Test accuracy: 83.0%


Epoch 7
Training loss: 0.59985113185178247337
Training accuracy: 92.96%
Test accuracy: 84.2%


Epoch 8
Training loss: 0.68241411488689877727
Training accuracy: 91.52%
Test accuracy: 83.8%


Epoch 9
Training loss: 0.6178573554240523534
Training accuracy: 92.72%
Test accuracy: 84.0%


Epoch 10
Training loss: 0.76310250868291614135
Training accuracy: 89.28%
Test accuracy: 83.2%


Epoch 11
Training loss: 0.625601111409811858
Training accuracy: 91.60000000000001%
Test accuracy: 84.0%


Epoch 12
Training loss: 0.7855628262008626621
Training accuracy: 87.88%
Test accuracy: 82.8%


Epoch 13
Training loss: 0.5769184174395629903
Training accuracy: 93.56%
Test accuracy: 85.0%
```

```
Epoch 14
Training loss: 0.5563355127692802643
Training accuracy: 93.67999999999999%
Test accuracy: 85.39999999999999%

Epoch 15
Training loss: 0.5379903095588351084
Training accuracy: 93.2%
Test accuracy: 87.0%

Epoch 16
Training loss: 0.72949905138729781543
Training accuracy: 90.64%
Test accuracy: 83.2%

Epoch 17
Training loss: 0.6482803319992067948
Training accuracy: 91.64%
Test accuracy: 83.2%

Epoch 18
Training loss: 0.6061266507828651662
Training accuracy: 92.67999999999999%
Test accuracy: 85.8%

Epoch 19
Training loss: 0.59069717762839155654
Training accuracy: 92.4%
Test accuracy: 85.2%

Epoch 20
Training loss: 0.61283203722174617886
Training accuracy: 92.2%
Test accuracy: 84.39999999999999%

Epoch 21
Training loss: 0.57973076543837441326
Training accuracy: 93.56%
Test accuracy: 85.6%

Epoch 22
Training loss: 0.84151611942292329506
Training accuracy: 88.36%
Test accuracy: 81.8%

Epoch 23
Training loss: 0.70880471330908518813
```

Training accuracy: 89.96%
Test accuracy: 83.6%

Epoch 24
Training loss: 0.5461793207742172807
Training accuracy: 92.92%
Test accuracy: 85.39999999999999%

Epoch 25
Training loss: 0.5364074030537711383
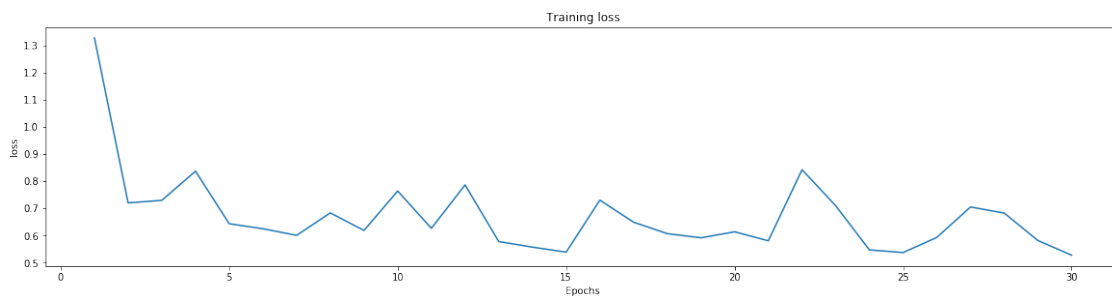Training accuracy: 92.84%
Test accuracy: 86.2%

Epoch 26
Training loss: 0.5921190950516245869
Training accuracy: 92.47999999999999%
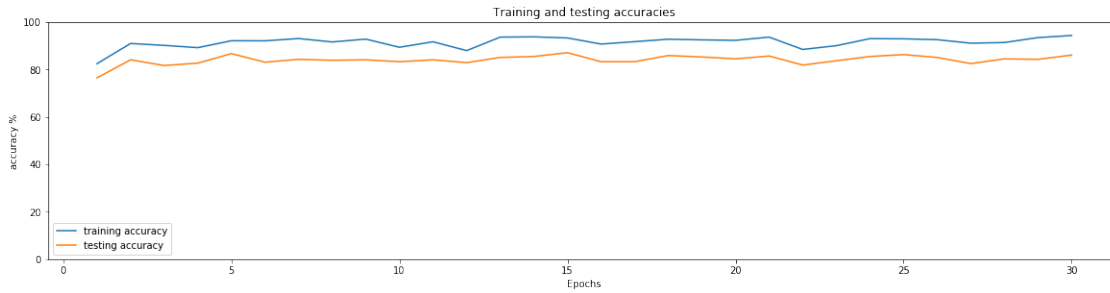Test accuracy: 85.0%

Epoch 27
Training loss: 0.7042730509569844694
Training accuracy: 91.0%
Test accuracy: 82.39999999999999%

Epoch 28
Training loss: 0.68189131938133071563
Training accuracy: 91.28%
Test accuracy: 84.39999999999999%

Epoch 29
Training loss: 0.5808135480988476286
Training accuracy: 93.36%
Test accuracy: 84.2%

Epoch 30
Training loss: 0.5267710394529381758
Training accuracy: 94.24%
Test accuracy: 86.0%

Training and testing accuracies

We get top training accuracy of 93% and test accuracy of 84%

Checking the large weight initializer

```
[5]: net = network.Linear_Network([784,10], weight_initializer='large')
```

```
[6]: net.SGD(training_data, 30, 10, 0.5, lmbda = 5.0, test_data=testing_data)
```

Epoch 1
Training loss: 1.4578713093857943548
Training accuracy: 83.56%
Test accuracy: 76.0%

Epoch 2
Training loss: 0.89294562377752571017
Training accuracy: 88.44%
Test accuracy: 81.2%

Epoch 3
Training loss: 0.6914466719289004403
Training accuracy: 90.72%
Test accuracy: 83.39999999999999%

Epoch 4
Training loss: 0.70416204213083758145
Training accuracy: 91.47999999999999%
Test accuracy: 82.0%

Epoch 5
Training loss: 0.6498968144804360314
Training accuracy: 91.32000000000001%
Test accuracy: 83.0%

Epoch 6
Training loss: 0.7011865655904408734

Training accuracy: 91.56%
Test accuracy: 83.39999999999999%

Epoch 7
Training loss: 0.70585561103012643816
Training accuracy: 90.32%
Test accuracy: 82.6%

Epoch 8
Training loss: 0.7205639502113299712
Training accuracy: 89.52%
Test accuracy: 81.2%

Epoch 9
Training loss: 0.601893005602259888
Training accuracy: 92.92%
Test accuracy: 84.6%

Epoch 10
Training loss: 0.64886662271131890844
Training accuracy: 91.4%
Test accuracy: 83.8%

Epoch 11
Training loss: 0.58694003587963932534
Training accuracy: 92.24%
Test accuracy: 84.8%

Epoch 12
Training loss: 0.61938680469315643656
Training accuracy: 92.12%
Test accuracy: 84.0%

Epoch 13
Training loss: 0.5812377080604218361
Training accuracy: 92.47999999999999%
Test accuracy: 84.2%

Epoch 14
Training loss: 0.5568395041875347067
Training accuracy: 93.16%
Test accuracy: 87.0%

Epoch 15
Training loss: 0.59555967628173598065
Training accuracy: 93.04%
Test accuracy: 84.8%

```
Epoch 16
Training loss: 0.6654119533684015922
Training accuracy: 91.52%
Test accuracy: 82.0%


Epoch 17
Training loss: 0.5900631852942883275
Training accuracy: 92.88%
Test accuracy: 85.2%


Epoch 18
Training loss: 0.555863960001395426
Training accuracy: 93.84%
Test accuracy: 84.8%


Epoch 19
Training loss: 0.57954733397477828925
Training accuracy: 92.84%
Test accuracy: 85.8%


Epoch 20
Training loss: 0.59664816990770241906
Training accuracy: 92.92%
Test accuracy: 83.8%


Epoch 21
Training loss: 0.61190289303504129405
Training accuracy: 92.12%
Test accuracy: 83.6%


Epoch 22
Training loss: 0.7103513545524271277
Training accuracy: 90.32%
Test accuracy: 84.2%


Epoch 23
Training loss: 0.61178790046544688855
Training accuracy: 92.36%
Test accuracy: 83.39999999999999%


Epoch 24
Training loss: 0.60308399103683367525
Training accuracy: 92.08%
Test accuracy: 84.8%


Epoch 25
Training loss: 0.5386263573189866411
Training accuracy: 93.36%
```

Test accuracy: 84.0%

Epoch 26
Training loss: 0.601493377321824918
Training accuracy: 92.44%
Test accuracy: 83.6%

Epoch 27
Training loss: 0.5593667069569982776
Training accuracy: 92.96%
Test accuracy: 83.8%

Epoch 28
Training loss: 0.58097065791399862047
Training accuracy: 93.04%
Test accuracy: 86.6%

Epoch 29
Training loss: 0.63557041976418269966
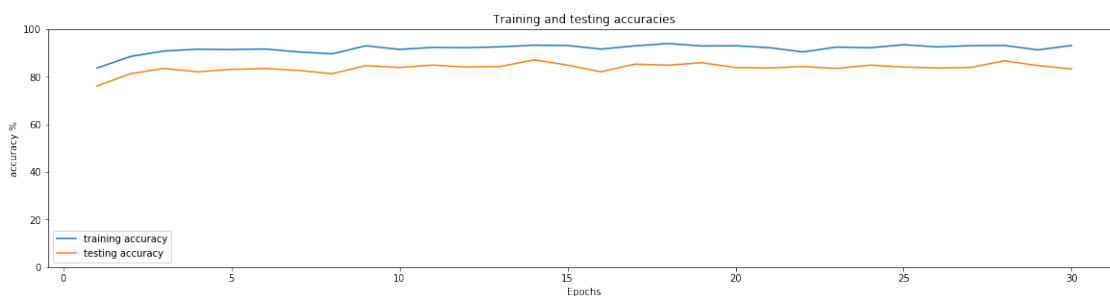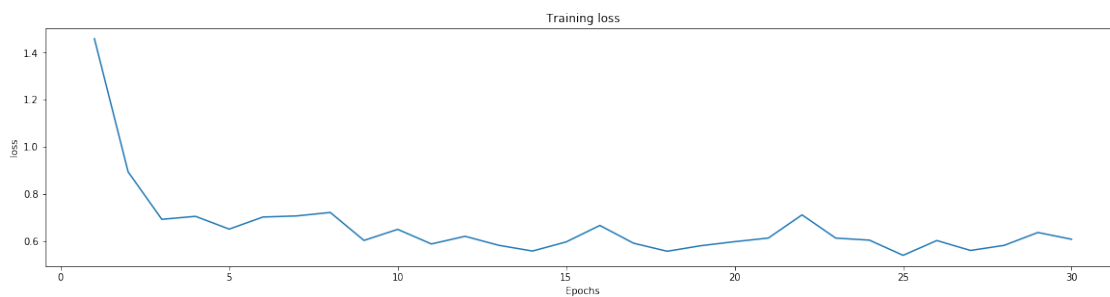Training accuracy: 91.2%
Test accuracy: 84.6%

Epoch 30
Training loss: 0.6067781788950730768
Training accuracy: 93.04%
Test accuracy: 83.2%

'large' weight initializer performs similar to the 'default' initializer with max test accuracy of 84% and training accuracy of 93%

Hence we stick to 'default' weight initializer

2. Choosing cost function

First we try out the 'Quadratic' cost

```
[7]: net = network.Linear_Network([784,10], weight_initializer='default',␣
     ↪cost='Quadratic')
```

```
[8]: net.SGD(training_data, 30, 10, 0.5, lmbda = 5.0, test_data=testing_data)
```

```
Epoch 1
Training loss: 0.14430774391452515734
Training accuracy: 87.12%
Test accuracy: 81.6%

Epoch 2
Training loss: 0.12434602091434443095
Training accuracy: 89.44%
Test accuracy: 83.2%

Epoch 3
Training loss: 0.115573315212080905765
Training accuracy: 89.96%
Test accuracy: 84.39999999999999%

Epoch 4
Training loss: 0.10971693140937655258
Training accuracy: 90.44%
Test accuracy: 84.39999999999999%

Epoch 5
Training loss: 0.10820816462001523951
Training accuracy: 90.4%
Test accuracy: 85.2%

Epoch 6
Training loss: 0.10918127544937454743
Training accuracy: 90.36%
Test accuracy: 84.2%

Epoch 7
Training loss: 0.10515369190497852874
```

```
Training accuracy: 91.24%
Test accuracy: 85.39999999999999%


Epoch 8
Training loss: 0.10510713413921377183
Training accuracy: 91.12%
Test accuracy: 84.8%


Epoch 9
Training loss: 0.10571742606670785589
Training accuracy: 90.8%
Test accuracy: 85.39999999999999%


Epoch 10
Training loss: 0.103638973411906549415
Training accuracy: 91.28%
Test accuracy: 86.0%


Epoch 11
Training loss: 0.100888317744808204255
Training accuracy: 92.08%
Test accuracy: 84.39999999999999%


Epoch 12
Training loss: 0.101918052915637961766
Training accuracy: 91.0%
Test accuracy: 84.8%


Epoch 13
Training loss: 0.10527601839393982388
Training accuracy: 90.84%
Test accuracy: 84.8%


Epoch 14
Training loss: 0.10327954535826308309
Training accuracy: 91.72%
Test accuracy: 85.39999999999999%


Epoch 15
Training loss: 0.10295923682437099695
Training accuracy: 91.60000000000001%
Test accuracy: 84.2%


Epoch 16
Training loss: 0.099238975557068284096
Training accuracy: 91.60000000000001%
Test accuracy: 85.6%
```

```
Epoch 17
Training loss: 0.100934193076398050036
Training accuracy: 91.64%
Test accuracy: 84.2%


Epoch 18
Training loss: 0.10092141939111313164
Training accuracy: 91.64%
Test accuracy: 84.2%


Epoch 19
Training loss: 0.101285341750599067025
Training accuracy: 91.72%
Test accuracy: 86.0%


Epoch 20
Training loss: 0.10120532614748231064
Training accuracy: 91.28%
Test accuracy: 85.0%


Epoch 21
Training loss: 0.100735801236247800164
Training accuracy: 91.56%
Test accuracy: 85.39999999999999%


Epoch 22
Training loss: 0.099846189579484215685
Training accuracy: 91.08000000000001%
Test accuracy: 85.6%


Epoch 23
Training loss: 0.10024747364784468513
Training accuracy: 91.4%
Test accuracy: 85.39999999999999%


Epoch 24
Training loss: 0.10022707787123517257
Training accuracy: 91.92%
Test accuracy: 85.0%


Epoch 25
Training loss: 0.098565199902015944624
Training accuracy: 91.88%
Test accuracy: 85.39999999999999%


Epoch 26
Training loss: 0.100775069707188423974
Training accuracy: 91.60000000000001%
```

```
Test accuracy: 84.6%

Epoch 27
Training loss: 0.09849728591976504718
Training accuracy: 91.8%
Test accuracy: 85.6%

Epoch 28
Training loss: 0.102366198436458460494
Training accuracy: 91.4%
Test accuracy: 85.2%

Epoch 29
Training loss: 0.10038632306984616855
Training accuracy: 91.28%
Test accuracy: 85.39999999999999%

Epoch 30
Training loss: 0.099725184937177996566
Training accuracy: 91.60000000000001%
Test accuracy: 85.0%
```
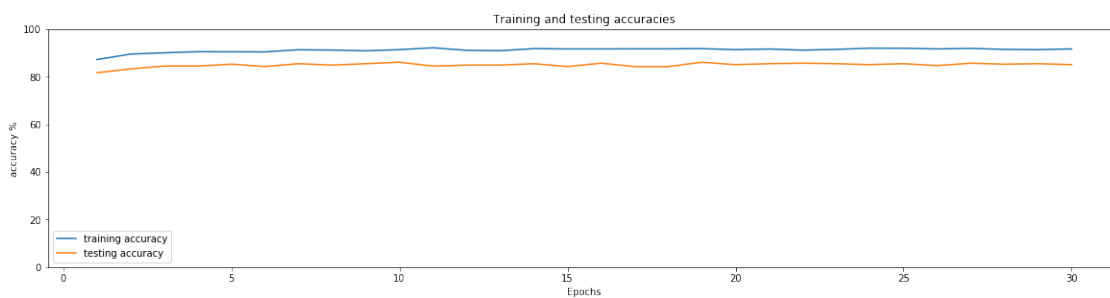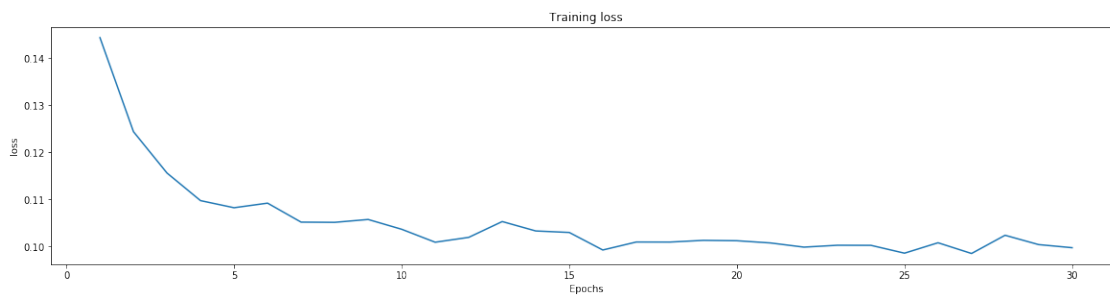

'Quadratic' cost gives training accuracy of 91% and test accuracy of 85%

Trying 'CrossEntropy'

13

```
[9]: net = network.Linear_Network([784,10], weight_initializer='default',␣
     ↪cost='CrossEntropy')
```

```
[10]: net.SGD(training_data, 30, 10, 0.5, lmbda = 5.0, test_data=testing_data)
```

Epoch 1
Training loss: 1.1400620321095680596
Training accuracy: 84.84%
Test accuracy: 78.8%

Epoch 2
Training loss: 1.1351648544076420024
Training accuracy: 81.72%
Test accuracy: 72.2%

Epoch 3
Training loss: 0.65034009401556294516
Training accuracy: 91.64%
Test accuracy: 85.0%

Epoch 4
Training loss: 0.7527512377906450604
Training accuracy: 90.32%
Test accuracy: 82.8%

Epoch 5
Training loss: 0.6661909609644937047
Training accuracy: 91.24%
Test accuracy: 84.2%

Epoch 6
Training loss: 0.63578586763901611236
Training accuracy: 92.60000000000001%
Test accuracy: 84.6%

Epoch 7
Training loss: 0.64226355571199650633
Training accuracy: 91.64%
Test accuracy: 81.39999999999999%

Epoch 8
Training loss: 0.5922074087873146851
Training accuracy: 92.56%
Test accuracy: 84.39999999999999%

Epoch 9
Training loss: 0.63043149207374716317

```
Training accuracy: 92.75999999999999%
Test accuracy: 85.2%


Epoch 10
Training loss: 0.8473234829533657159
Training accuracy: 89.4%
Test accuracy: 81.6%


Epoch 11
Training loss: 0.60644628140989874333
Training accuracy: 91.92%
Test accuracy: 83.0%


Epoch 12
Training loss: 1.0088905407902601304
Training accuracy: 82.67999999999999%
Test accuracy: 76.0%


Epoch 13
Training loss: 0.54999673055911138666
Training accuracy: 93.52000000000001%
Test accuracy: 85.2%


Epoch 14
Training loss: 0.5722264926432652073
Training accuracy: 93.04%
Test accuracy: 86.8%


Epoch 15
Training loss: 0.65501607543581852203
Training accuracy: 91.72%
Test accuracy: 82.39999999999999%


Epoch 16
Training loss: 0.5986362680342832776
Training accuracy: 91.8%
Test accuracy: 84.39999999999999%


Epoch 17
Training loss: 0.5761357506820018151
Training accuracy: 92.64%
Test accuracy: 84.8%


Epoch 18
Training loss: 0.56294678282739809406
Training accuracy: 92.64%
Test accuracy: 84.8%
```

```
Epoch 19
Training loss: 0.7116680829999793707
Training accuracy: 90.44%
Test accuracy: 82.39999999999999%

Epoch 20
Training loss: 0.61512079966337630707
Training accuracy: 92.36%
Test accuracy: 86.2%

Epoch 21
Training loss: 0.8908456305872533953
Training accuracy: 87.64%
Test accuracy: 81.0%

Epoch 22
Training loss: 0.5851243713355452483
Training accuracy: 92.04%
Test accuracy: 85.0%

Epoch 23
Training loss: 0.55932565572145434823
Training accuracy: 93.84%
Test accuracy: 86.4%

Epoch 24
Training loss: 0.73933033889102701497
Training accuracy: 90.96%
Test accuracy: 84.0%

Epoch 25
Training loss: 0.5862781926799022349
Training accuracy: 93.24%
Test accuracy: 84.0%

Epoch 26
Training loss: 0.63679211979251014507
Training accuracy: 92.04%
Test accuracy: 83.8%

Epoch 27
Training loss: 0.5848565171300773471
Training accuracy: 93.24%
Test accuracy: 85.39999999999999%

Epoch 28
Training loss: 0.5411677063095106016
Training accuracy: 93.72%
```

```
Test accuracy: 86.0%

Epoch 29
Training loss: 0.7150090699505372132
Training accuracy: 89.52%
Test accuracy: 84.39999999999999%

Epoch 30
Training loss: 0.7116959135353462823
Training accuracy: 90.03999999999999%
Test accuracy: 82.0%
```
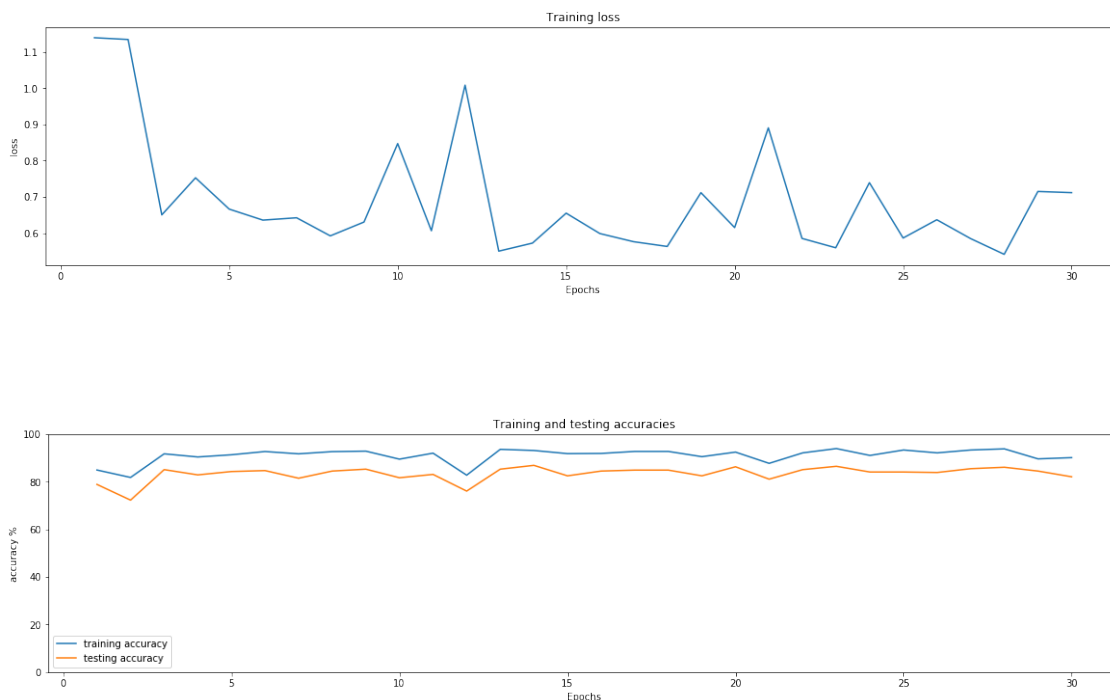

We see that 'CrossEntropy' gives a slightly better performance of 93% traing and 85% testing accuracy.

But we notice from the loss graph that the drop in loss for Quadratic is much smoother than for CrossEntropy.

For now we'll stick to CrossEntropy and hope that the loss graph for the CrossEntropy function will smoothen out as we do further parameter tunning. If not then we'll visti this again and do another comparison between these two after the parameters have been set.

3. Choosing activation function

First we try 'sigmoid'

```
[11]: net = network.Linear_Network([784,10], weight_initializer='default',␣
      ↪cost='Quadratic',
                                 activation = 'sigmoid')
```

```
[12]: net.SGD(training_data, 30, 10, 0.5, lmbda = 5.0, test_data=testing_data)
```

```
Epoch 1
Training loss: 0.14122971164088507967
Training accuracy: 87.56%
Test accuracy: 81.6%

Epoch 2
Training loss: 0.12183825604439538239
Training accuracy: 89.88000000000001%
Test accuracy: 82.6%

Epoch 3
Training loss: 0.11069726174276199918
Training accuracy: 90.64%
Test accuracy: 84.6%

Epoch 4
Training loss: 0.111057099726467961
Training accuracy: 90.72%
Test accuracy: 84.0%

Epoch 5
Training loss: 0.10838269080122015666
Training accuracy: 90.32%
Test accuracy: 83.2%

Epoch 6
Training loss: 0.11343002714292998762
Training accuracy: 89.84%
Test accuracy: 83.6%

Epoch 7
Training loss: 0.105037625832915775505
Training accuracy: 91.24%
Test accuracy: 84.8%

Epoch 8
Training loss: 0.103404341202121716296
Training accuracy: 91.96%
Test accuracy: 84.8%

Epoch 9
```

Training loss: 0.10419560153325619538
Training accuracy: 91.16%
Test accuracy: 83.8%

Epoch 10
Training loss: 0.10144401874467356361
Training accuracy: 91.8%
Test accuracy: 84.6%

Epoch 11
Training loss: 0.102604120742263210374
Training accuracy: 91.16%
Test accuracy: 85.2%

Epoch 12
Training loss: 0.102964981346572091526
Training accuracy: 90.84%
Test accuracy: 85.8%

Epoch 13
Training loss: 0.1066126987253369264
Training accuracy: 90.96%
Test accuracy: 85.0%

Epoch 14
Training loss: 0.10742839994708550137
Training accuracy: 91.2%
Test accuracy: 83.39999999999999%

Epoch 15
Training loss: 0.10107866748084083409495
Training accuracy: 91.36%
Test accuracy: 85.0%

Epoch 16
Training loss: 0.10167868850154676023
Training accuracy: 91.72%
Test accuracy: 85.0%

Epoch 17
Training loss: 0.10119786940045463477
Training accuracy: 91.84%
Test accuracy: 85.0%

Epoch 18
Training loss: 0.10217698647409774651
Training accuracy: 91.16%
Test accuracy: 85.39999999999999%

```
Epoch 19
Training loss: 0.10139874662149048952
Training accuracy: 91.0%
Test accuracy: 84.2%


Epoch 20
Training loss: 0.0987169394456307345
Training accuracy: 91.8%
Test accuracy: 84.8%


Epoch 21
Training loss: 0.103417567356149265744
Training accuracy: 91.36%
Test accuracy: 85.0%


Epoch 22
Training loss: 0.1077361743222739469
Training accuracy: 91.4%
Test accuracy: 83.8%


Epoch 23
Training loss: 0.098269939442113426556
Training accuracy: 91.96%
Test accuracy: 85.39999999999999%


Epoch 24
Training loss: 0.10101376421813314317
Training accuracy: 91.4%
Test accuracy: 84.0%


Epoch 25
Training loss: 0.10143763757343051315
Training accuracy: 91.92%
Test accuracy: 85.0%


Epoch 26
Training loss: 0.09920781098870465662
Training accuracy: 92.0%
Test accuracy: 85.0%


Epoch 27
Training loss: 0.101642044614305744825
Training accuracy: 91.47999999999999%
Test accuracy: 85.0%


Epoch 28
Training loss: 0.102940995343994103054
```

Training accuracy: 91.60000000000001%
Test accuracy: 85.2%

Epoch 29
Training loss: 0.10024126340416166763
Training accuracy: 91.88%
Test accuracy: 85.6%

Epoch 30
Training loss: 0.1006897247175448399
Training accuracy: 91.92%
Test accuracy: 86.0%
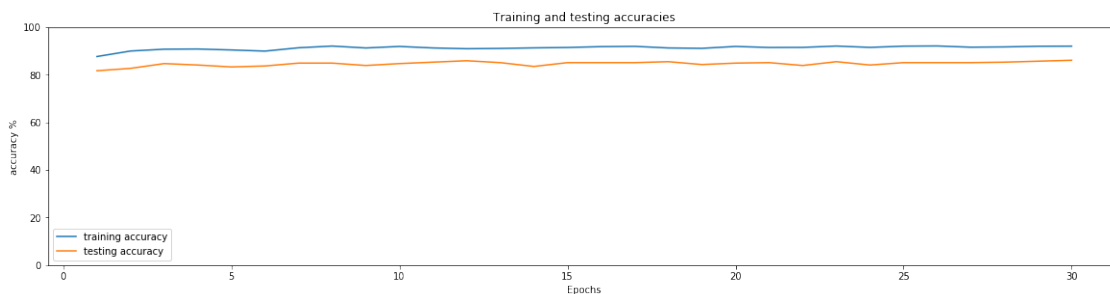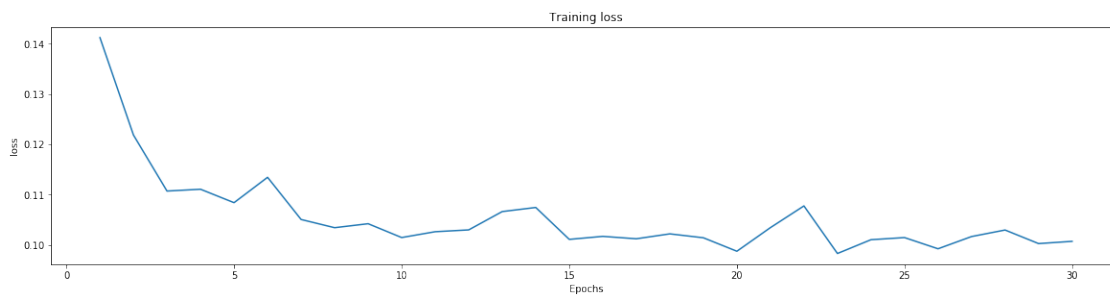
Next we try 'tanh'

```
[13]: net = network.Linear_Network([784,10], weight_initializer='default',␣
      ↪cost='Quadratic',
                              activation = 'tanh')
```

```
[14]: net.SGD(training_data, 30, 10, 0.5, lmbda = 5.0, test_data=testing_data)
```

Epoch 1
Training loss: 3.2303686435523601016
Training accuracy: 23.76%

Test accuracy: 25.8%

Epoch 2
Training loss: 2.8391497719870323316
Training accuracy: 51.519999999999996%
Test accuracy: 47.199999999999996%

Epoch 3
Training loss: 3.0352400608258571068
Training accuracy: 52.72%
Test accuracy: 54.6%

Epoch 4
Training loss: 2.935100450170637811
Training accuracy: 18.2%
Test accuracy: 18.6%

Epoch 5
Training loss: 3.1471953116887459011
Training accuracy: 37.6%
Test accuracy: 35.0%

Epoch 6
Training loss: 3.0969188531546436255
Training accuracy: 16.16%
Test accuracy: 16.2%

Epoch 7
Training loss: 2.8658553208136144576
Training accuracy: 45.4%
Test accuracy: 44.4%

Epoch 8
Training loss: 2.9958681236315074536
Training accuracy: 56.56%
Test accuracy: 53.2%

Epoch 9
Training loss: 2.8750594990381577432
Training accuracy: 40.64%
Test accuracy: 39.4%

Epoch 10
Training loss: 2.821630697405130503
Training accuracy: 54.120000000000005%
Test accuracy: 49.2%

Epoch 11

```
Training loss: 2.6034395529350308773
Training accuracy: 61.040000000000006%
Test accuracy: 56.00000000000001%


Epoch 12
Training loss: 2.6777453698931522534
Training accuracy: 58.32000000000001%
Test accuracy: 54.6%


Epoch 13
Training loss: 2.6481718717010123277
Training accuracy: 57.24%
Test accuracy: 55.2%


Epoch 14
Training loss: 2.7441753971279190145
Training accuracy: 22.6%
Test accuracy: 22.2%


Epoch 15
Training loss: 2.7118954620663923759
Training accuracy: 39.36%
Test accuracy: 34.0%


Epoch 16
Training loss: 2.8885067015254884827
Training accuracy: 24.279999999999998%
Test accuracy: 23.599999999999998%


Epoch 17
Training loss: 3.2009306612989373016
Training accuracy: 40.48%
Test accuracy: 41.0%


Epoch 18
Training loss: 2.8446437104100511053
Training accuracy: 50.239999999999995%
Test accuracy: 46.0%


Epoch 19
Training loss: 3.065784714651874284
Training accuracy: 53.44%
Test accuracy: 49.8%


Epoch 20
Training loss: 2.762009200160105247
Training accuracy: 51.2%
Test accuracy: 46.2%
```

```
Epoch 21
Training loss: 2.9784520175320587086
Training accuracy: 51.04%
Test accuracy: 48.0%


Epoch 22
Training loss: 2.6870582312597372815
Training accuracy: 52.32%
Test accuracy: 43.4%


Epoch 23
Training loss: 3.1708376953627454556
Training accuracy: 44.84%
Test accuracy: 39.4%


Epoch 24
Training loss: 2.836906156888301584
Training accuracy: 41.32%
Test accuracy: 37.4%


Epoch 25
Training loss: 3.015421873947301406
Training accuracy: 41.199999999999996%
Test accuracy: 38.4%


Epoch 26
Training loss: 3.1471990475947490733
Training accuracy: 31.319999999999997%
Test accuracy: 28.799999999999997%


Epoch 27
Training loss: 3.3073441086690658689
Training accuracy: 51.6%
Test accuracy: 45.2%


Epoch 28
Training loss: 2.521124851750993633
Training accuracy: 68.24%
Test accuracy: 66.2%


Epoch 29
Training loss: 2.7465482805800106004
Training accuracy: 47.0%
Test accuracy: 45.0%


Epoch 30
Training loss: 2.8205753850486988596
```

Training accuracy: 57.04%
Test accuracy: 53.800000000000004%
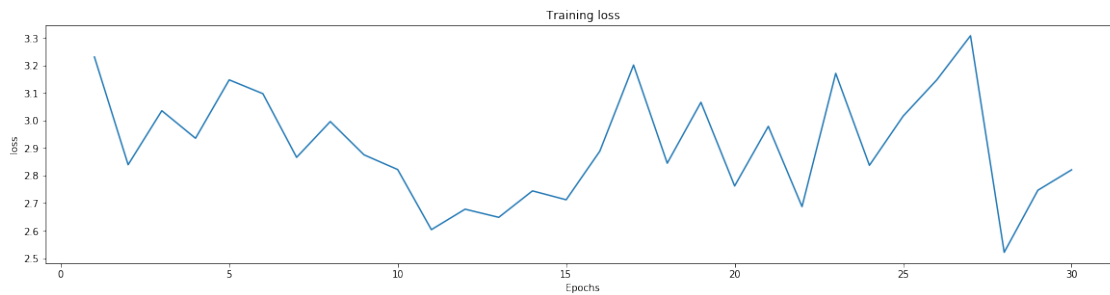
Next is 'relu'

```
[15]: net = network.Linear_Network([784,10], weight_initializer='default',␣
      ↪cost='Quadratic',
                                activation = 'relu')
```

```
[16]: net.SGD(training_data, 30, 10, 0.5, lmbda = 5.0, test_data=testing_data)
```

Epoch 1
Training loss: 0.4998753750037329
Training accuracy: 10.16%
Test accuracy: 11.600000000000001%

Epoch 2
Training loss: 0.5
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%

Epoch 3
Training loss: 0.5
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%

```
Epoch 4
Training loss: 0.49998475007594734
Training accuracy: 10.280000000000001%
Test accuracy: 11.600000000000001%

Epoch 5
Training loss: 0.4999076350386032
Training accuracy: 10.16%
Test accuracy: 11.600000000000001%

Epoch 6
Training loss: 0.4995488523378649
Training accuracy: 10.24%
Test accuracy: 11.600000000000001%

Epoch 7
Training loss: 0.5000017563420155
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%

Epoch 8
Training loss: 0.49999794468983255
Training accuracy: 10.12%
Test accuracy: 11.799999999999999%

Epoch 9
Training loss: 0.500003075029748
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%

Epoch 10
Training loss: 0.4995272756174767
Training accuracy: 10.36%
Test accuracy: 11.600000000000001%

Epoch 11
Training loss: 0.4994626165659598
Training accuracy: 10.4%
Test accuracy: 11.600000000000001%

Epoch 12
Training loss: 0.5000008708440793
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%

Epoch 13
Training loss: 0.500000195778715
```

```
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%


Epoch 14
Training loss: 0.5000006908692093
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%


Epoch 15
Training loss: 0.5000002145128356
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%


Epoch 16
Training loss: 0.5000022365878521
Training accuracy: 10.08%
Test accuracy: 11.600000000000001%


Epoch 17
Training loss: 0.4996294796616622
Training accuracy: 10.36%
Test accuracy: 11.799999999999999%


Epoch 18
Training loss: 0.49972734299694477
Training accuracy: 10.2%
Test accuracy: 11.600000000000001%


Epoch 19
Training loss: 0.49960776562871306
Training accuracy: 10.16%
Test accuracy: 11.600000000000001%


Epoch 20
Training loss: 0.4996009639194086
Training accuracy: 10.2%
Test accuracy: 11.600000000000001%


Epoch 21
Training loss: 0.4987192194041874
Training accuracy: 10.639999999999999%
Test accuracy: 11.799999999999999%


Epoch 22
Training loss: 0.5000006661991736
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%
```

```
Epoch 23
Training loss: 0.5000005869100168
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%

Epoch 24
Training loss: 0.49740447488982736
Training accuracy: 10.92%
Test accuracy: 12.4%

Epoch 25
Training loss: 0.5000008572344935
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%

Epoch 26
Training loss: 0.5000002644267925
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%

Epoch 27
Training loss: 0.4999995713251814
Training accuracy: 10.16%
Test accuracy: 11.600000000000001%

Epoch 28
Training loss: 0.5000008954464134
Training accuracy: 10.040000000000001%
Test accuracy: 11.600000000000001%

Epoch 29
Training loss: 0.500000089223235
Training accuracy: 10.12%
Test accuracy: 11.600000000000001%

Epoch 30
Training loss: 0.49980072677276055
Training accuracy: 10.16%
Test accuracy: 11.600000000000001%
```
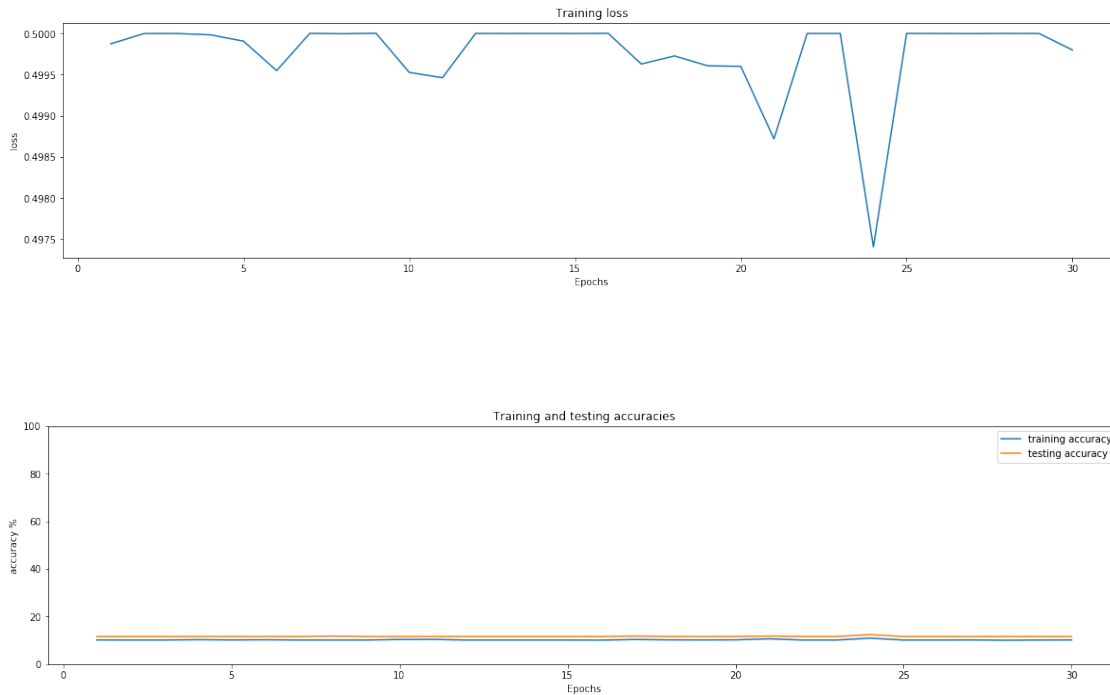
Training loss

Training and testing accuracies

Hence we can clearly see that 'sigmoid' is the best option we have

Since our initial parameters are set, we can move on to the number of layers and the stochastic gradient parameter tuning. I haqve already set the above parameters as default for our network so that we don't have to call them agian and again

2.2.7 Now we decide the number of neurons and the number of hidden layers

1. 1 hidden layer. Testing for 15 to 50 neurons:

```
[22]: n = [15,20,25,30,35,40,45,50]
      train_accuracies=[]
      test_accuracies=[]
      for neurons in n:
          net = network.Linear_Network([784, neurons, 10])
          tr_a, te_a = net.SGD(training_data, 30, 10, 0.5, lmbda = 5.0,
       →test_data=testing_data,
                    show_eval=False, show_graph=False, return_accuracies=True)
          train_accuracies.append(tr_a)
          test_accuracies.append(te_a)
          print(str(neurons)+" neurons:")
          print("Training acc.: "+str(tr_a)+"% Testing acc.: "+str(te_a)+"%")
          print()
```

```
plt.subplot(2, 1, 1)
plt.plot(n, train_accuracies, label='training accuracy')
plt.plot(n, test_accuracies, label='test accuracy')
plt.legend()
plt.title("Training and testing accuracies")
plt.xlabel("neurons")
plt.ylabel("accuracy %")
plt.ylim(0, 100)
plt.show()
```

```
15 neurons:
Training acc.: 94.096% Testing acc.: 85.83999999999999%

20 neurons:
Training acc.: 94.576% Testing acc.: 86.32%

25 neurons:
Training acc.: 95.47200000000001% Testing acc.: 87.16%

30 neurons:
Training acc.: 95.26400000000001% Testing acc.: 86.32%

35 neurons:
Training acc.: 95.58399999999999% Testing acc.: 87.16%

40 neurons:
Training acc.: 95.44800000000001% Testing acc.: 86.68%

45 neurons:
Training acc.: 95.55199999999999% Testing acc.: 86.56%

50 neurons:
Training acc.: 95.27199999999999% Testing acc.: 86.47999999999999%
```
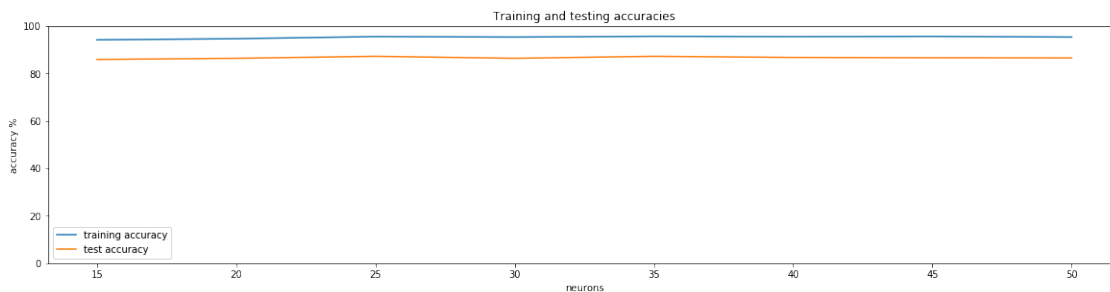

The result is pretty uniform. So lets stick to 35 neurons in the first layer.

Now we test the second hidden layer

```
[23]: n = [15,20,25,30]
      train_accuracies=[]
      test_accuracies=[]
      for neurons in n:
          net = network.Linear_Network([784, 35, neurons, 10])
          tr_a, te_a = net.SGD(training_data, 30, 10, 0.5, lmbda = 5.0,␣
       ↪test_data=testing_data,
                    show_eval=False, show_graph=False, return_accuracies=True)
          train_accuracies.append(tr_a)
          test_accuracies.append(te_a)
          print(str(neurons)+" neurons:")
          print("Training acc.: "+str(tr_a)+"% Testing acc.: "+str(te_a)+"%")
          print()

      plt.subplot(2, 1, 1)
      plt.plot(n, train_accuracies, label='training accuracy')
      plt.plot(n, test_accuracies, label='test accuracy')
      plt.legend()
      plt.title("Training and testing accuracies")
      plt.xlabel("neurons")
      plt.ylabel("accuracy %")
      plt.ylim(0, 100)
      plt.show()
```

```
15 neurons:
Training acc.: 95.84% Testing acc.: 87.44000000000001%

20 neurons:
Training acc.: 95.6% Testing acc.: 87.36000000000001%

25 neurons:
Training acc.: 95.208% Testing acc.: 87.52000000000001%

30 neurons:
Training acc.: 95.96000000000001% Testing acc.: 87.24000000000001%
```
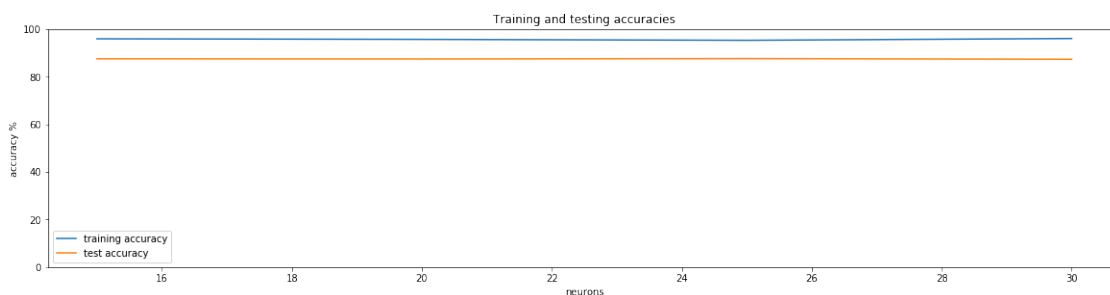
Here we stick to 25 neurons in 2nd layer

Testing for 3rd hidden layer

```
[26]: n = [15,20]
train_accuracies=[]
test_accuracies=[]
for neurons in n:
    net = network.Linear_Network([784, 35, 25, neurons, 10])
    tr_a, te_a = net.SGD(training_data, 30, 10, 0.5, lmbda = 5.0,␣
 ↪test_data=testing_data,
            show_eval=False, show_graph=False, return_accuracies=True)
    train_accuracies.append(tr_a)
    test_accuracies.append(te_a)
    print(str(neurons)+" neurons:")
    print("Training acc.: "+str(tr_a)+"% Testing acc.: "+str(te_a)+"%")
    print()

plt.subplot(2, 1, 1)
plt.plot(n, train_accuracies, label='training accuracy')
plt.plot(n, test_accuracies, label='test accuracy')
plt.legend()
plt.title("Training and testing accuracies")
plt.xlabel("neurons")
plt.ylabel("accuracy %")
plt.ylim(0, 100)
plt.show()
```

```
15 neurons:
Training acc.: 92.744% Testing acc.: 85.28%

20 neurons:
Training acc.: 92.92% Testing acc.: 85.8%
```
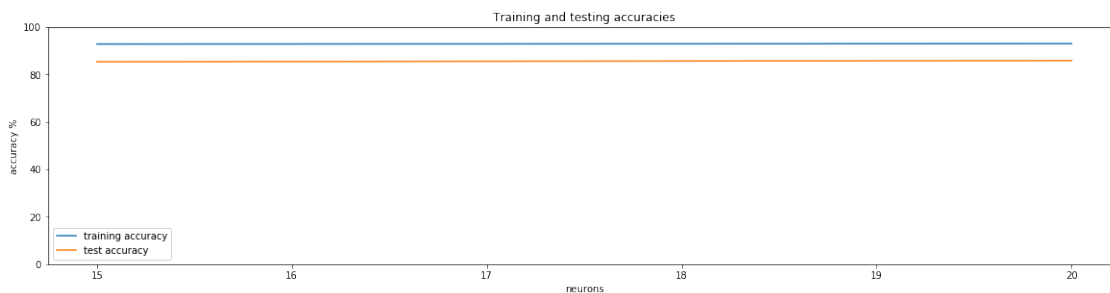
With a decrease in accuracies, we finally stick to 2 hidden layers.

First hidden layer has 35 neurons and seconf hidden layer has 25 neurons

Checking the learning rate now. We increase the number of epochs and reduce the learning rate to see where the accuracies saturate Initially we try 50 epochs with eta=0.1

```
[27]: net = network.Linear_Network([784, 35, 25, 10])
      net.SGD(training_data, 50, 10, 0.1, lmbda = 5.0, test_data=testing_data)
```

```
Epoch 1
Training loss: 3.2032931311622133827
Training accuracy: 13.600000000000001%
Test accuracy: 13.8%

Epoch 2
Training loss: 2.979357053247336601
Training accuracy: 41.88%
Test accuracy: 43.8%

Epoch 3
Training loss: 2.3166261117557005891
Training accuracy: 64.96%
Test accuracy: 59.199999999999996%

Epoch 4
Training loss: 1.9229003357879172885
Training accuracy: 71.16%
Test accuracy: 65.4%

Epoch 5
Training loss: 1.6672206102524891513
Training accuracy: 72.64%
Test accuracy: 67.80000000000001%

Epoch 6
Training loss: 1.4578467746512805906
Training accuracy: 80.64%
Test accuracy: 78.4%

Epoch 7
Training loss: 1.3022954671634116282
Training accuracy: 79.03999999999999%
Test accuracy: 75.8%

Epoch 8
Training loss: 1.1923338158331425587
Training accuracy: 84.2%
```

```
Test accuracy: 81.39999999999999%


Epoch 9
Training loss: 1.104272894144313197
Training accuracy: 83.96000000000001%
Test accuracy: 79.2%


Epoch 10
Training loss: 1.0260431540807957934
Training accuracy: 88.2%
Test accuracy: 84.8%


Epoch 11
Training loss: 0.9518740348069485484
Training accuracy: 89.4%
Test accuracy: 85.6%


Epoch 12
Training loss: 0.9026574478291545418
Training accuracy: 89.36%
Test accuracy: 84.8%


Epoch 13
Training loss: 0.8368121554262523727
Training accuracy: 90.36%
Test accuracy: 85.8%


Epoch 14
Training loss: 0.8015666519443689536
Training accuracy: 90.56%
Test accuracy: 86.2%


Epoch 15
Training loss: 0.74793894548781152023
Training accuracy: 91.64%
Test accuracy: 86.8%


Epoch 16
Training loss: 0.7202715751531060989
Training accuracy: 92.32000000000001%
Test accuracy: 87.0%


Epoch 17
Training loss: 0.6819053678877821798
Training accuracy: 92.60000000000001%
Test accuracy: 87.8%


Epoch 18
```

```
Training loss: 0.6675191944689199315
Training accuracy: 93.04%
Test accuracy: 86.6%


Epoch 19
Training loss: 0.6438959099241531791
Training accuracy: 93.04%
Test accuracy: 85.2%


Epoch 20
Training loss: 0.6301744153544253214
Training accuracy: 93.36%
Test accuracy: 85.39999999999999%


Epoch 21
Training loss: 0.6082534277698551203
Training accuracy: 93.4%
Test accuracy: 87.6%


Epoch 22
Training loss: 0.5855477356087875859
Training accuracy: 93.64%
Test accuracy: 87.6%


Epoch 23
Training loss: 0.58102895297055570987
Training accuracy: 94.08%
Test accuracy: 86.4%


Epoch 24
Training loss: 0.56245226340447469743
Training accuracy: 94.0%
Test accuracy: 87.6%


Epoch 25
Training loss: 0.5477023301539234164
Training accuracy: 94.6%
Test accuracy: 88.0%


Epoch 26
Training loss: 0.54806605583476812615
Training accuracy: 94.28%
Test accuracy: 87.6%


Epoch 27
Training loss: 0.53581361380082646775
Training accuracy: 94.6%
Test accuracy: 86.8%
```

```
Epoch 28
Training loss: 0.52250660036221330485
Training accuracy: 94.84%
Test accuracy: 87.6%


Epoch 29
Training loss: 0.5322095325667175552
Training accuracy: 95.04%
Test accuracy: 87.4%


Epoch 30
Training loss: 0.5401955676587451312
Training accuracy: 94.12%
Test accuracy: 87.8%


Epoch 31
Training loss: 0.50370827997148981316
Training accuracy: 95.04%
Test accuracy: 87.8%


Epoch 32
Training loss: 0.5024204703821459328
Training accuracy: 95.44%
Test accuracy: 87.6%


Epoch 33
Training loss: 0.48950475770784996677
Training accuracy: 95.24000000000001%
Test accuracy: 87.6%


Epoch 34
Training loss: 0.49309646067488661992
Training accuracy: 95.6%
Test accuracy: 88.0%


Epoch 35
Training loss: 0.4854083650874701685
Training accuracy: 95.44%
Test accuracy: 88.4%


Epoch 36
Training loss: 0.48967493190505669443
Training accuracy: 95.39999999999999%
Test accuracy: 87.6%


Epoch 37
Training loss: 0.4715832638366463035
```

```
Training accuracy: 95.96000000000001%
Test accuracy: 88.2%


Epoch 38
Training loss: 0.47664284314299774835
Training accuracy: 95.84%
Test accuracy: 87.0%


Epoch 39
Training loss: 0.470897145197710012
Training accuracy: 95.8%
Test accuracy: 87.8%


Epoch 40
Training loss: 0.48417765487660274814
Training accuracy: 95.08%
Test accuracy: 87.8%


Epoch 41
Training loss: 0.46822515180435101073
Training accuracy: 96.08%
Test accuracy: 88.0%


Epoch 42
Training loss: 0.4570609948228599176
Training accuracy: 96.32%
Test accuracy: 87.0%


Epoch 43
Training loss: 0.44977651379688632152
Training accuracy: 96.39999999999999%
Test accuracy: 88.4%


Epoch 44
Training loss: 0.45223173702288123118
Training accuracy: 95.76%
Test accuracy: 87.8%


Epoch 45
Training loss: 0.45139669997598413754
Training accuracy: 95.96000000000001%
Test accuracy: 88.8%


Epoch 46
Training loss: 0.45710616916202450057
Training accuracy: 96.36%
Test accuracy: 87.8%
```

```
Epoch 47
Training loss: 0.44614847732084361088
Training accuracy: 96.0%
Test accuracy: 87.8%

Epoch 48
Training loss: 0.44101348263886575344
Training accuracy: 96.39999999999999%
Test accuracy: 87.4%

Epoch 49
Training loss: 0.44976422677027937648
Training accuracy: 96.2%
Test accuracy: 88.6%

Epoch 50
Training loss: 0.44304126623144652935
Training accuracy: 96.32%
Test accuracy: 88.6%
```
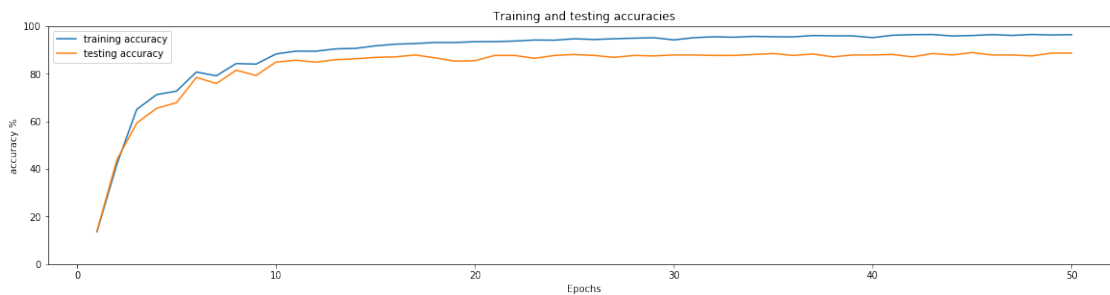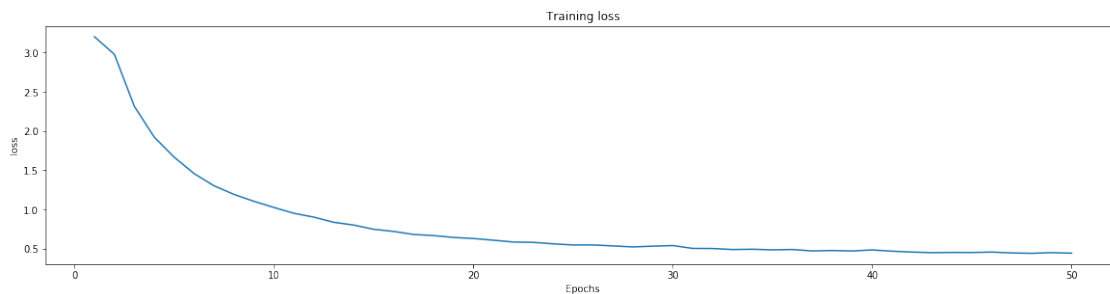

Trying eta=0.01 for 100 epochs

```
[28]: net = network.Linear_Network([784, 35, 25, 10])
      net.SGD(training_data, 100, 10, 0.01, lmbda = 5.0, test_data=testing_data)
```

```
Epoch 1
Training loss: 3.2460821917901008164
Training accuracy: 11.959999999999999%
Test accuracy: 12.4%

Epoch 2
Training loss: 3.239028594460498077
Training accuracy: 11.959999999999999%
Test accuracy: 12.4%

Epoch 3
Training loss: 3.235895415070075321
Training accuracy: 11.959999999999999%
Test accuracy: 12.4%

Epoch 4
Training loss: 3.232279988576710235
Training accuracy: 11.959999999999999%
Test accuracy: 12.4%

Epoch 5
Training loss: 3.2280075513184290456
Training accuracy: 11.959999999999999%
Test accuracy: 12.4%

Epoch 6
Training loss: 3.2248574488085505977
Training accuracy: 11.200000000000001%
Test accuracy: 8.799999999999999%

Epoch 7
Training loss: 3.219469202006140357
Training accuracy: 19.28%
Test accuracy: 16.400000000000002%

Epoch 8
Training loss: 3.2135988254186800164
Training accuracy: 20.44%
Test accuracy: 18.0%

Epoch 9
Training loss: 3.20811834918552255
Training accuracy: 19.88%
Test accuracy: 21.4%

Epoch 10
Training loss: 3.2010051766086239586
Training accuracy: 12.0%
```

Test accuracy: 12.6%

Epoch 11
Training loss: 3.1922122905115274695
Training accuracy: 16.08%
Test accuracy: 14.799999999999999%

Epoch 12
Training loss: 3.1828051604763835975
Training accuracy: 22.0%
Test accuracy: 22.8%

Epoch 13
Training loss: 3.171549337209243616
Training accuracy: 20.48%
Test accuracy: 22.2%

Epoch 14
Training loss: 3.1579692570564938745
Training accuracy: 18.279999999999998%
Test accuracy: 19.6%

Epoch 15
Training loss: 3.142504897605991459
Training accuracy: 40.12%
Test accuracy: 37.6%

Epoch 16
Training loss: 3.122797757948355835
Training accuracy: 37.72%
Test accuracy: 35.199999999999996%

Epoch 17
Training loss: 3.1004062036062451215
Training accuracy: 38.76%
Test accuracy: 36.199999999999996%

Epoch 18
Training loss: 3.074397703183800599
Training accuracy: 41.92%
Test accuracy: 41.199999999999996%

Epoch 19
Training loss: 3.044349118716373748
Training accuracy: 40.2%
Test accuracy: 37.2%

Epoch 20

```
Training loss: 3.0091303008163201964
Training accuracy: 46.6%
Test accuracy: 45.4%


Epoch 21
Training loss: 2.9684991816499108842
Training accuracy: 46.44%
Test accuracy: 46.0%


Epoch 22
Training loss: 2.92469611200130771
Training accuracy: 48.32%
Test accuracy: 48.0%


Epoch 23
Training loss: 2.8759409221530301218
Training accuracy: 51.160000000000004%
Test accuracy: 48.199999999999996%


Epoch 24
Training loss: 2.8240886380030860686
Training accuracy: 49.68%
Test accuracy: 47.4%


Epoch 25
Training loss: 2.7697902628774254458
Training accuracy: 49.84%
Test accuracy: 48.8%


Epoch 26
Training loss: 2.7162277572661148433
Training accuracy: 49.36%
Test accuracy: 49.6%


Epoch 27
Training loss: 2.6597446797016051545
Training accuracy: 53.04%
Test accuracy: 51.4%


Epoch 28
Training loss: 2.6061288412169825428
Training accuracy: 55.84%
Test accuracy: 53.800000000000004%


Epoch 29
Training loss: 2.5530383081132731824
Training accuracy: 55.32%
Test accuracy: 52.800000000000004%
```

```
Epoch 30
Training loss: 2.5018998306884877557
Training accuracy: 53.72%
Test accuracy: 52.800000000000004%

Epoch 31
Training loss: 2.4508792215258656015
Training accuracy: 55.67999999999999%
Test accuracy: 53.2%

Epoch 32
Training loss: 2.4012865598298434712
Training accuracy: 55.559999999999995%
Test accuracy: 54.400000000000006%

Epoch 33
Training loss: 2.3541694968697522428
Training accuracy: 56.44%
Test accuracy: 54.800000000000004%

Epoch 34
Training loss: 2.3078569588949350783
Training accuracy: 56.120000000000005%
Test accuracy: 54.6%

Epoch 35
Training loss: 2.2637557417282180897
Training accuracy: 56.599999999999994%
Test accuracy: 54.800000000000004%

Epoch 36
Training loss: 2.221228771235827473
Training accuracy: 58.199999999999996%
Test accuracy: 57.599999999999994%

Epoch 37
Training loss: 2.1798856352725289572
Training accuracy: 58.32000000000001%
Test accuracy: 56.99999999999999%

Epoch 38
Training loss: 2.1404746908608168305
Training accuracy: 59.96%
Test accuracy: 57.8%

Epoch 39
Training loss: 2.1037621162467799647
```

```
Training accuracy: 59.24%
Test accuracy: 57.99999999999999%


Epoch 40
Training loss: 2.0685769382234162049
Training accuracy: 62.03999999999999%
Test accuracy: 59.599999999999994%


Epoch 41
Training loss: 2.0359159212131779585
Training accuracy: 61.12%
Test accuracy: 58.8%


Epoch 42
Training loss: 2.0048847311586729329
Training accuracy: 61.28%
Test accuracy: 58.8%


Epoch 43
Training loss: 1.974848235023085078
Training accuracy: 63.160000000000004%
Test accuracy: 60.4%


Epoch 44
Training loss: 1.9473420165813056105
Training accuracy: 63.63999999999999%
Test accuracy: 61.6%


Epoch 45
Training loss: 1.9208079851927383835
Training accuracy: 63.63999999999999%
Test accuracy: 61.199999999999996%


Epoch 46
Training loss: 1.8957143576823315464
Training accuracy: 64.28%
Test accuracy: 60.8%


Epoch 47
Training loss: 1.8717288913158866705
Training accuracy: 64.52%
Test accuracy: 61.199999999999996%


Epoch 48
Training loss: 1.848219909449681893
Training accuracy: 65.92%
Test accuracy: 62.4%
```

```
Epoch 49
Training loss: 1.8260729254400167509
Training accuracy: 66.67999999999999%
Test accuracy: 62.6%


Epoch 50
Training loss: 1.8047938109639787926
Training accuracy: 67.2%
Test accuracy: 63.2%


Epoch 51
Training loss: 1.7834129408990705926
Training accuracy: 67.72%
Test accuracy: 63.2%


Epoch 52
Training loss: 1.7629601038715065655
Training accuracy: 67.88%
Test accuracy: 63.4%


Epoch 53
Training loss: 1.7434503943780405013
Training accuracy: 69.08%
Test accuracy: 65.4%


Epoch 54
Training loss: 1.7237760193977251957
Training accuracy: 69.56%
Test accuracy: 65.8%


Epoch 55
Training loss: 1.7055133364057228889
Training accuracy: 70.52000000000001%
Test accuracy: 67.4%


Epoch 56
Training loss: 1.6856116623531849346
Training accuracy: 71.12%
Test accuracy: 67.60000000000001%


Epoch 57
Training loss: 1.6670932176954086682
Training accuracy: 71.36%
Test accuracy: 67.60000000000001%


Epoch 58
Training loss: 1.6490286058734865947
Training accuracy: 71.28%
```

Test accuracy: 68.4%

Epoch 59
Training loss: 1.6305001383306866711
Training accuracy: 72.36%
Test accuracy: 68.2%

Epoch 60
Training loss: 1.6130062561568574205
Training accuracy: 73.24000000000001%
Test accuracy: 68.4%

Epoch 61
Training loss: 1.5946559071688159602
Training accuracy: 73.68%
Test accuracy: 69.19999999999999%

Epoch 62
Training loss: 1.5770382621481562106
Training accuracy: 73.8%
Test accuracy: 68.8%

Epoch 63
Training loss: 1.5588948405875222796
Training accuracy: 75.36%
Test accuracy: 70.19999999999999%

Epoch 64
Training loss: 1.5417844324714328183
Training accuracy: 75.16000000000001%
Test accuracy: 70.39999999999999%

Epoch 65
Training loss: 1.5241947466488462496
Training accuracy: 75.8%
Test accuracy: 71.6%

Epoch 66
Training loss: 1.5066104174884922006
Training accuracy: 77.08%
Test accuracy: 72.6%

Epoch 67
Training loss: 1.4899628374798290287
Training accuracy: 77.64%
Test accuracy: 73.6%

Epoch 68

Training loss: 1.4724902043963055593
Training accuracy: 77.28%
Test accuracy: 72.8%

Epoch 69
Training loss: 1.4548622997156738217
Training accuracy: 78.0%
Test accuracy: 74.4%

Epoch 70
Training loss: 1.4377368092651069656
Training accuracy: 78.72%
Test accuracy: 75.2%

Epoch 71
Training loss: 1.4208452418719656946
Training accuracy: 79.16%
Test accuracy: 75.4%

Epoch 72
Training loss: 1.4038049094047345091
Training accuracy: 79.36%
Test accuracy: 75.8%

Epoch 73
Training loss: 1.3872365292509588534
Training accuracy: 80.16%
Test accuracy: 76.6%

Epoch 74
Training loss: 1.3704860215398357733
Training accuracy: 80.72%
Test accuracy: 77.2%

Epoch 75
Training loss: 1.3542485419195613612
Training accuracy: 80.96%
Test accuracy: 77.60000000000001%

Epoch 76
Training loss: 1.3389831627064600793
Training accuracy: 81.8%
Test accuracy: 78.0%

Epoch 77
Training loss: 1.3219832630852059864
Training accuracy: 82.12%
Test accuracy: 79.0%

```
Epoch 78
Training loss: 1.3063851778332541608
Training accuracy: 82.84%
Test accuracy: 79.60000000000001%

Epoch 79
Training loss: 1.291205768735197076
Training accuracy: 83.2%
Test accuracy: 79.60000000000001%

Epoch 80
Training loss: 1.2756564658444705095
Training accuracy: 83.76%
Test accuracy: 80.80000000000001%

Epoch 81
Training loss: 1.260856611725711458
Training accuracy: 83.52000000000001%
Test accuracy: 80.80000000000001%

Epoch 82
Training loss: 1.2454534042063571398
Training accuracy: 84.2%
Test accuracy: 81.6%

Epoch 83
Training loss: 1.2308656061274645392
Training accuracy: 84.39999999999999%
Test accuracy: 81.39999999999999%

Epoch 84
Training loss: 1.2165548989951122977
Training accuracy: 84.76%
Test accuracy: 82.19999999999999%

Epoch 85
Training loss: 1.2028314733363346907
Training accuracy: 84.96000000000001%
Test accuracy: 82.39999999999999%

Epoch 86
Training loss: 1.1886340126235066866
Training accuracy: 85.64%
Test accuracy: 82.19999999999999%

Epoch 87
Training loss: 1.1748772795262187512
```

```
Training accuracy: 86.04%
Test accuracy: 83.6%


Epoch 88
Training loss: 1.1616291866280881721
Training accuracy: 85.96000000000001%
Test accuracy: 82.8%


Epoch 89
Training loss: 1.1485587851601321016
Training accuracy: 86.08%
Test accuracy: 83.0%


Epoch 90
Training loss: 1.1357762153106187144
Training accuracy: 86.56%
Test accuracy: 84.0%


Epoch 91
Training loss: 1.1236120774360980917
Training accuracy: 86.32%
Test accuracy: 83.8%


Epoch 92
Training loss: 1.1112726251411067034
Training accuracy: 86.88%
Test accuracy: 83.8%


Epoch 93
Training loss: 1.0990041061046098446
Training accuracy: 86.92%
Test accuracy: 84.2%


Epoch 94
Training loss: 1.0874187265099264566
Training accuracy: 87.08%
Test accuracy: 84.0%


Epoch 95
Training loss: 1.0760050708785375866
Training accuracy: 87.44%
Test accuracy: 84.0%


Epoch 96
Training loss: 1.0646318248633173022
Training accuracy: 87.76%
Test accuracy: 84.39999999999999%
```

```
Epoch 97
Training loss: 1.0539327871630847478
Training accuracy: 88.2%
Test accuracy: 84.6%

Epoch 98
Training loss: 1.0437634386080763659
Training accuracy: 88.4%
Test accuracy: 84.39999999999999%

Epoch 99
Training loss: 1.0328500565623754981
Training accuracy: 88.03999999999999%
Test accuracy: 84.6%

Epoch 100
Training loss: 1.0225673635624652969
Training accuracy: 88.48%
Test accuracy: 85.2%
```
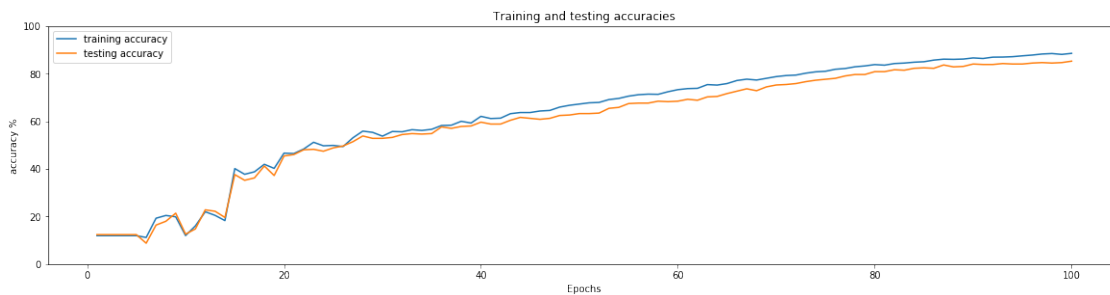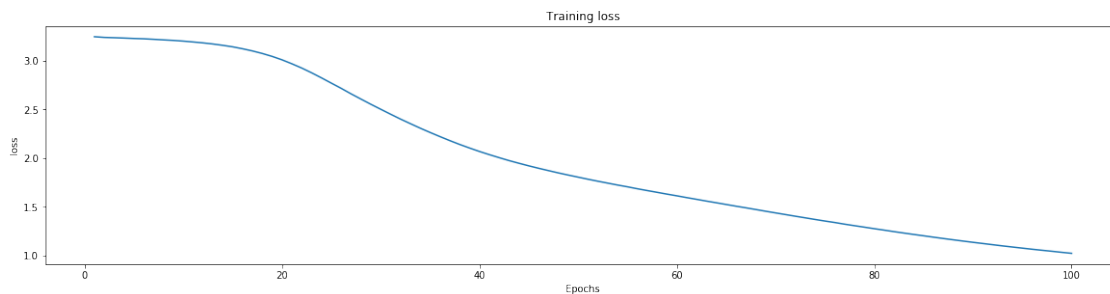

Since we didnt reach saturation, let's try with 200 epochs

```
[29]: net = network.Linear_Network([784, 35, 25, 10])
      net.SGD(training_data, 200, 10, 0.01, lmbda = 5.0, test_data=testing_data)
```

```
Epoch 1
Training loss: 3.2448648598402080197
Training accuracy: 11.959999999999999%
Test accuracy: 12.4%

Epoch 2
Training loss: 3.2373135604231627444
Training accuracy: 11.959999999999999%
Test accuracy: 12.4%

Epoch 3
Training loss: 3.2316508006551143098
Training accuracy: 18.4%
Test accuracy: 15.2%

Epoch 4
Training loss: 3.2263156506888010654
Training accuracy: 18.240000000000002%
Test accuracy: 15.4%

Epoch 5
Training loss: 3.2183598531551415537
Training accuracy: 16.400000000000002%
Test accuracy: 17.0%

Epoch 6
Training loss: 3.210233225292959681
Training accuracy: 13.239999999999998%
Test accuracy: 13.8%

Epoch 7
Training loss: 3.2010835726235456465
Training accuracy: 19.040000000000003%
Test accuracy: 20.4%

Epoch 8
Training loss: 3.1905204702988827826
Training accuracy: 29.92%
Test accuracy: 29.799999999999997%

Epoch 9
Training loss: 3.17673840059719404
Training accuracy: 28.360000000000003%
Test accuracy: 28.000000000000004%

Epoch 10
Training loss: 3.1617877300963245946
Training accuracy: 29.080000000000002%
```

Test accuracy: 28.199999999999996%

Epoch 11
Training loss: 3.1439759453097912707
Training accuracy: 28.88%
Test accuracy: 28.000000000000004%

Epoch 12
Training loss: 3.1229978301627211515
Training accuracy: 32.28%
Test accuracy: 30.4%

Epoch 13
Training loss: 3.0981339858470488106
Training accuracy: 31.080000000000002%
Test accuracy: 31.2%

Epoch 14
Training loss: 3.0698266047872981837
Training accuracy: 27.48%
Test accuracy: 28.199999999999996%

Epoch 15
Training loss: 3.036665429925219199
Training accuracy: 29.720000000000002%
Test accuracy: 31.6%

Epoch 16
Training loss: 2.9997690267258597739
Training accuracy: 36.120000000000005%
Test accuracy: 34.2%

Epoch 17
Training loss: 2.9596269673826975566
Training accuracy: 35.36%
Test accuracy: 34.599999999999994%

Epoch 18
Training loss: 2.9154516722581708269
Training accuracy: 39.2%
Test accuracy: 36.6%

Epoch 19
Training loss: 2.8684099699877610638
Training accuracy: 44.04%
Test accuracy: 40.8%

Epoch 20

Training loss: 2.8189767979938176908
Training accuracy: 42.32%
Test accuracy: 37.8%

Epoch 21
Training loss: 2.7675733086435321336
Training accuracy: 44.72%
Test accuracy: 40.400000000000006%

Epoch 22
Training loss: 2.7131023592527464367
Training accuracy: 44.64%
Test accuracy: 39.6%

Epoch 23
Training loss: 2.6567429832600327336
Training accuracy: 48.72%
Test accuracy: 43.8%

Epoch 24
Training loss: 2.5985438096183832908
Training accuracy: 48.76%
Test accuracy: 42.0%

Epoch 25
Training loss: 2.5375138751746777331
Training accuracy: 54.04%
Test accuracy: 47.4%

Epoch 26
Training loss: 2.4755215038864055888
Training accuracy: 52.959999999999994%
Test accuracy: 47.4%

Epoch 27
Training loss: 2.412296344343621339
Training accuracy: 56.599999999999994%
Test accuracy: 51.4%

Epoch 28
Training loss: 2.349044874459079787
Training accuracy: 58.68%
Test accuracy: 53.400000000000006%

Epoch 29
Training loss: 2.287425276675670176
Training accuracy: 60.440000000000005%
Test accuracy: 57.199999999999996%

```
Epoch 30
Training loss: 2.227593976773718862
Training accuracy: 61.519999999999996%
Test accuracy: 56.8%

Epoch 31
Training loss: 2.171830491115984002
Training accuracy: 63.160000000000004%
Test accuracy: 59.8%

Epoch 32
Training loss: 2.118947942669142957
Training accuracy: 63.480000000000004%
Test accuracy: 59.0%

Epoch 33
Training loss: 2.070304883854367748
Training accuracy: 66.0%
Test accuracy: 62.6%

Epoch 34
Training loss: 2.0234420703287843942
Training accuracy: 64.48%
Test accuracy: 61.0%

Epoch 35
Training loss: 1.9815780164013248907
Training accuracy: 64.52%
Test accuracy: 61.6%

Epoch 36
Training loss: 1.9421671172035465943
Training accuracy: 65.68%
Test accuracy: 62.0%

Epoch 37
Training loss: 1.9048386327800478276
Training accuracy: 67.04%
Test accuracy: 63.4%

Epoch 38
Training loss: 1.870198086286800898
Training accuracy: 67.84%
Test accuracy: 63.0%

Epoch 39
Training loss: 1.8371729189072062828
```

```
Training accuracy: 68.47999999999999%
Test accuracy: 64.2%


Epoch 40
Training loss: 1.8059812594806316281
Training accuracy: 69.56%
Test accuracy: 65.60000000000001%


Epoch 41
Training loss: 1.7766110573477318171
Training accuracy: 69.56%
Test accuracy: 65.0%


Epoch 42
Training loss: 1.7493029259867606846
Training accuracy: 71.24000000000001%
Test accuracy: 66.0%


Epoch 43
Training loss: 1.7221326063389069891
Training accuracy: 72.24000000000001%
Test accuracy: 67.4%


Epoch 44
Training loss: 1.6963154362727693296
Training accuracy: 71.64%
Test accuracy: 66.2%


Epoch 45
Training loss: 1.6719976240586162215
Training accuracy: 72.0%
Test accuracy: 65.60000000000001%


Epoch 46
Training loss: 1.6478905005764351338
Training accuracy: 73.52%
Test accuracy: 67.0%


Epoch 47
Training loss: 1.625622332341793169
Training accuracy: 72.92%
Test accuracy: 67.0%


Epoch 48
Training loss: 1.6026926432925977457
Training accuracy: 74.72%
Test accuracy: 68.2%
```

```
Epoch 49
Training loss: 1.5810781387832224743
Training accuracy: 74.92%
Test accuracy: 68.2%

Epoch 50
Training loss: 1.5602948782306045695
Training accuracy: 75.76%
Test accuracy: 68.8%

Epoch 51
Training loss: 1.5401601152220052131
Training accuracy: 75.84%
Test accuracy: 69.8%

Epoch 52
Training loss: 1.5201670845523957551
Training accuracy: 76.2%
Test accuracy: 68.8%

Epoch 53
Training loss: 1.5009311419924430381
Training accuracy: 77.24%
Test accuracy: 70.19999999999999%

Epoch 54
Training loss: 1.4819905806030879341
Training accuracy: 77.60000000000001%
Test accuracy: 69.8%

Epoch 55
Training loss: 1.4638336856048307897
Training accuracy: 77.56%
Test accuracy: 71.8%

Epoch 56
Training loss: 1.4451548647403959276
Training accuracy: 78.68%
Test accuracy: 72.0%

Epoch 57
Training loss: 1.4273283430360277579
Training accuracy: 78.56%
Test accuracy: 71.8%

Epoch 58
Training loss: 1.4097489279625173905
Training accuracy: 78.64%
```

Test accuracy: 72.0%

Epoch 59
Training loss: 1.3927237857007958264
Training accuracy: 78.68%
Test accuracy: 71.39999999999999%

Epoch 60
Training loss: 1.3755970593340858997
Training accuracy: 79.28%
Test accuracy: 72.0%

Epoch 61
Training loss: 1.3593934381885453783
Training accuracy: 79.60000000000001%
Test accuracy: 73.2%

Epoch 62
Training loss: 1.3433029808291595912
Training accuracy: 79.72%
Test accuracy: 74.4%

Epoch 63
Training loss: 1.3270371784703640441
Training accuracy: 81.0%
Test accuracy: 76.0%

Epoch 64
Training loss: 1.3113332840049581442
Training accuracy: 81.36%
Test accuracy: 75.8%

Epoch 65
Training loss: 1.2970018796200158722
Training accuracy: 81.39999999999999%
Test accuracy: 76.8%

Epoch 66
Training loss: 1.2812429799740605171
Training accuracy: 82.12%
Test accuracy: 77.2%

Epoch 67
Training loss: 1.2666396076368702449
Training accuracy: 82.67999999999999%
Test accuracy: 78.2%

Epoch 68

Training loss: 1.2533590894455205921
Training accuracy: 83.36%
Test accuracy: 79.2%

Epoch 69
Training loss: 1.2378935003378603682
Training accuracy: 82.56%
Test accuracy: 79.4%

Epoch 70
Training loss: 1.2248447174763241473
Training accuracy: 83.72%
Test accuracy: 79.4%

Epoch 71
Training loss: 1.2117561228666764345
Training accuracy: 84.28%
Test accuracy: 78.60000000000001%

Epoch 72
Training loss: 1.1975948763352138132
Training accuracy: 83.6%
Test accuracy: 80.80000000000001%

Epoch 73
Training loss: 1.1846408183400811487
Training accuracy: 84.56%
Test accuracy: 80.2%

Epoch 74
Training loss: 1.1720948207842566809
Training accuracy: 84.28%
Test accuracy: 80.80000000000001%

Epoch 75
Training loss: 1.1602747021176393314
Training accuracy: 84.44%
Test accuracy: 80.60000000000001%

Epoch 76
Training loss: 1.1476667480533851068
Training accuracy: 85.16%
Test accuracy: 81.39999999999999%

Epoch 77
Training loss: 1.1365720097217338381
Training accuracy: 85.24000000000001%
Test accuracy: 81.8%

```
Epoch 78
Training loss: 1.1247255847502848179
Training accuracy: 85.84%
Test accuracy: 81.6%

Epoch 79
Training loss: 1.1149158336767258999
Training accuracy: 85.68%
Test accuracy: 81.39999999999999%

Epoch 80
Training loss: 1.1039917171596402458
Training accuracy: 85.08%
Test accuracy: 81.0%

Epoch 81
Training loss: 1.0920787058580155763
Training accuracy: 85.84%
Test accuracy: 81.8%

Epoch 82
Training loss: 1.081715399744579795
Training accuracy: 86.56%
Test accuracy: 82.19999999999999%

Epoch 83
Training loss: 1.0724316371901005924
Training accuracy: 86.64%
Test accuracy: 82.0%

Epoch 84
Training loss: 1.0618783918153546165
Training accuracy: 86.4%
Test accuracy: 82.19999999999999%

Epoch 85
Training loss: 1.0523028605508784877
Training accuracy: 86.56%
Test accuracy: 82.19999999999999%

Epoch 86
Training loss: 1.0428309317496282559
Training accuracy: 86.72%
Test accuracy: 82.39999999999999%

Epoch 87
Training loss: 1.0335930147888180488
```

```
Training accuracy: 87.28%
Test accuracy: 82.8%


Epoch 88
Training loss: 1.0251894726401689612
Training accuracy: 87.72%
Test accuracy: 83.6%


Epoch 89
Training loss: 1.0157816026483921518
Training accuracy: 87.64%
Test accuracy: 83.0%


Epoch 90
Training loss: 1.0076949990941851434
Training accuracy: 87.96000000000001%
Test accuracy: 83.39999999999999%


Epoch 91
Training loss: 0.9990189168665997991
Training accuracy: 87.92%
Test accuracy: 83.39999999999999%


Epoch 92
Training loss: 0.9906598544992383033
Training accuracy: 88.03999999999999%
Test accuracy: 83.0%


Epoch 93
Training loss: 0.9829733850150172254
Training accuracy: 88.03999999999999%
Test accuracy: 83.2%


Epoch 94
Training loss: 0.9753109816219269748
Training accuracy: 88.68%
Test accuracy: 84.0%


Epoch 95
Training loss: 0.9673540793529894392
Training accuracy: 88.8%
Test accuracy: 84.0%


Epoch 96
Training loss: 0.9597514970576607972
Training accuracy: 89.03999999999999%
Test accuracy: 83.8%
```

```
Epoch 97
Training loss: 0.95234425147898404333
Training accuracy: 89.03999999999999%
Test accuracy: 84.2%


Epoch 98
Training loss: 0.9451673288892534151
Training accuracy: 88.96%
Test accuracy: 84.39999999999999%


Epoch 99
Training loss: 0.93805962067274864513
Training accuracy: 88.88000000000001%
Test accuracy: 84.2%


Epoch 100
Training loss: 0.9315577093472887503
Training accuracy: 89.28%
Test accuracy: 83.8%


Epoch 101
Training loss: 0.9241622647584131939
Training accuracy: 89.12%
Test accuracy: 84.0%


Epoch 102
Training loss: 0.9180485451998891938
Training accuracy: 89.36%
Test accuracy: 83.8%


Epoch 103
Training loss: 0.9109929078513542769
Training accuracy: 89.44%
Test accuracy: 83.8%


Epoch 104
Training loss: 0.9048270072692862695
Training accuracy: 89.75999999999999%
Test accuracy: 84.0%


Epoch 105
Training loss: 0.899356816572868636
Training accuracy: 89.56%
Test accuracy: 84.6%


Epoch 106
Training loss: 0.89225139936257858846
Training accuracy: 89.60000000000001%
```

Test accuracy: 84.39999999999999%

Epoch 107
Training loss: 0.886676226204396169
Training accuracy: 89.84%
Test accuracy: 84.0%

Epoch 108
Training loss: 0.88030074843402830335
Training accuracy: 89.8%
Test accuracy: 84.39999999999999%

Epoch 109
Training loss: 0.8746853764703772553
Training accuracy: 89.84%
Test accuracy: 84.6%

Epoch 110
Training loss: 0.86912751639407075734
Training accuracy: 89.92%
Test accuracy: 84.6%

Epoch 111
Training loss: 0.8632107469139967564
Training accuracy: 89.96%
Test accuracy: 84.6%

Epoch 112
Training loss: 0.8576764708850190823
Training accuracy: 90.08%
Test accuracy: 85.0%

Epoch 113
Training loss: 0.85230878540240242243
Training accuracy: 90.32%
Test accuracy: 85.2%

Epoch 114
Training loss: 0.84686883539164960715
Training accuracy: 90.52%
Test accuracy: 85.2%

Epoch 115
Training loss: 0.8416985475681299664
Training accuracy: 90.60000000000001%
Test accuracy: 85.0%

Epoch 116

Training loss: 0.83747999645107525774
Training accuracy: 90.64%
Test accuracy: 85.0%

Epoch 117
Training loss: 0.8314217745867625897
Training accuracy: 90.4%
Test accuracy: 85.39999999999999%

Epoch 118
Training loss: 0.82679126937779197115
Training accuracy: 90.72%
Test accuracy: 85.39999999999999%

Epoch 119
Training loss: 0.82153390364633254014
Training accuracy: 90.8%
Test accuracy: 85.39999999999999%

Epoch 120
Training loss: 0.81688116694402794596
Training accuracy: 90.88000000000001%
Test accuracy: 85.39999999999999%

Epoch 121
Training loss: 0.81219536999729301344
Training accuracy: 90.60000000000001%
Test accuracy: 85.39999999999999%

Epoch 122
Training loss: 0.8071922856760298857
Training accuracy: 91.0%
Test accuracy: 85.39999999999999%

Epoch 123
Training loss: 0.80294261598008158837
Training accuracy: 91.2%
Test accuracy: 85.8%

Epoch 124
Training loss: 0.7987991118468399861
Training accuracy: 91.12%
Test accuracy: 86.0%

Epoch 125
Training loss: 0.79391852202519442157
Training accuracy: 91.24%
Test accuracy: 86.2%

```
Epoch 126
Training loss: 0.7896756589047557638
Training accuracy: 91.32000000000001%
Test accuracy: 86.0%

Epoch 127
Training loss: 0.78607630643014459924
Training accuracy: 91.28%
Test accuracy: 85.8%

Epoch 128
Training loss: 0.7811274433262855312
Training accuracy: 91.4%
Test accuracy: 86.4%

Epoch 129
Training loss: 0.7767869721471583545
Training accuracy: 91.60000000000001%
Test accuracy: 85.8%

Epoch 130
Training loss: 0.77240243738995738434
Training accuracy: 91.47999999999999%
Test accuracy: 86.4%

Epoch 131
Training loss: 0.7691039853575153948
Training accuracy: 92.04%
Test accuracy: 86.2%

Epoch 132
Training loss: 0.7649061175713949493
Training accuracy: 91.96%
Test accuracy: 86.0%

Epoch 133
Training loss: 0.7608901847267768064
Training accuracy: 91.64%
Test accuracy: 86.6%

Epoch 134
Training loss: 0.75866994663786567374
Training accuracy: 92.04%
Test accuracy: 86.6%

Epoch 135
Training loss: 0.752960017202732477
```

```
Training accuracy: 91.88%
Test accuracy: 86.4%


Epoch 136
Training loss: 0.75055359193349202644
Training accuracy: 91.88%
Test accuracy: 86.8%


Epoch 137
Training loss: 0.7455113116188975216
Training accuracy: 92.2%
Test accuracy: 86.4%


Epoch 138
Training loss: 0.7436768530742874325
Training accuracy: 92.08%
Test accuracy: 86.8%


Epoch 139
Training loss: 0.7383576093752210722
Training accuracy: 91.96%
Test accuracy: 86.0%


Epoch 140
Training loss: 0.7348899017214693404
Training accuracy: 92.12%
Test accuracy: 86.8%


Epoch 141
Training loss: 0.73145469834716003417
Training accuracy: 92.2%
Test accuracy: 86.8%


Epoch 142
Training loss: 0.728139260403335867
Training accuracy: 92.36%
Test accuracy: 86.4%


Epoch 143
Training loss: 0.7246311330291147691
Training accuracy: 92.24%
Test accuracy: 86.4%


Epoch 144
Training loss: 0.72155424234031550725
Training accuracy: 92.47999999999999%
Test accuracy: 87.0%
```

```
Epoch 145
Training loss: 0.7183018109222813925
Training accuracy: 92.24%
Test accuracy: 86.8%

Epoch 146
Training loss: 0.7156373309897406
Training accuracy: 92.44%
Test accuracy: 87.2%

Epoch 147
Training loss: 0.71161071769414887205
Training accuracy: 92.24%
Test accuracy: 86.6%

Epoch 148
Training loss: 0.70867872488624437607
Training accuracy: 92.47999999999999%
Test accuracy: 86.8%

Epoch 149
Training loss: 0.70617170926885193717
Training accuracy: 92.64%
Test accuracy: 87.0%

Epoch 150
Training loss: 0.70274955892325414786
Training accuracy: 92.47999999999999%
Test accuracy: 86.8%

Epoch 151
Training loss: 0.6997328141277668819
Training accuracy: 92.52%
Test accuracy: 86.6%

Epoch 152
Training loss: 0.69629242198541990857
Training accuracy: 92.52%
Test accuracy: 87.0%

Epoch 153
Training loss: 0.6941443313843758697
Training accuracy: 92.60000000000001%
Test accuracy: 87.4%

Epoch 154
Training loss: 0.6908471337415322669
Training accuracy: 92.64%
```

Test accuracy: 86.8%

Epoch 155
Training loss: 0.68853430768362481034
Training accuracy: 92.64%
Test accuracy: 87.0%

Epoch 156
Training loss: 0.68514003060513119995
Training accuracy: 92.88%
Test accuracy: 87.0%

Epoch 157
Training loss: 0.6825781068075870057
Training accuracy: 92.84%
Test accuracy: 87.2%

Epoch 158
Training loss: 0.6802750108143772526
Training accuracy: 92.72%
Test accuracy: 86.8%

Epoch 159
Training loss: 0.6770158656481711667
Training accuracy: 92.96%
Test accuracy: 87.0%

Epoch 160
Training loss: 0.6746036332645691025
Training accuracy: 92.72%
Test accuracy: 87.4%

Epoch 161
Training loss: 0.6720427757983118415
Training accuracy: 92.88%
Test accuracy: 87.6%

Epoch 162
Training loss: 0.66940962661114800097
Training accuracy: 92.92%
Test accuracy: 87.4%

Epoch 163
Training loss: 0.6669632219010615594
Training accuracy: 93.08%
Test accuracy: 87.4%

Epoch 164

```
Training loss: 0.6648791954771500335
Training accuracy: 92.96%
Test accuracy: 87.4%


Epoch 165
Training loss: 0.66190244500986940865
Training accuracy: 93.08%
Test accuracy: 87.4%


Epoch 166
Training loss: 0.65944103395232555405
Training accuracy: 93.04%
Test accuracy: 87.4%


Epoch 167
Training loss: 0.65691821273306173854
Training accuracy: 93.2%
Test accuracy: 87.8%


Epoch 168
Training loss: 0.65498798182789156897
Training accuracy: 93.16%
Test accuracy: 87.6%


Epoch 169
Training loss: 0.65238680611439229254
Training accuracy: 93.28%
Test accuracy: 87.4%


Epoch 170
Training loss: 0.6501144064130033243
Training accuracy: 93.32000000000001%
Test accuracy: 87.8%


Epoch 171
Training loss: 0.6483810013036173176
Training accuracy: 93.32000000000001%
Test accuracy: 87.4%


Epoch 172
Training loss: 0.64608502843123692656
Training accuracy: 93.24%
Test accuracy: 87.4%


Epoch 173
Training loss: 0.64436892396917374757
Training accuracy: 93.28%
Test accuracy: 87.6%
```

Epoch 174
Training loss: 0.64165690900774034197
Training accuracy: 93.24%
Test accuracy: 87.6%

Epoch 175
Training loss: 0.63952935322323386337
Training accuracy: 93.36%
Test accuracy: 87.6%

Epoch 176
Training loss: 0.63837159085818753605
Training accuracy: 93.47999999999999%
Test accuracy: 87.6%

Epoch 177
Training loss: 0.63519229436790445974
Training accuracy: 93.28%
Test accuracy: 87.4%

Epoch 178
Training loss: 0.63332676032971372713
Training accuracy: 93.52000000000001%
Test accuracy: 87.4%

Epoch 179
Training loss: 0.63094469646691017146
Training accuracy: 93.44%
Test accuracy: 87.6%

Epoch 180
Training loss: 0.62926729031650513611
Training accuracy: 93.56%
Test accuracy: 87.6%

Epoch 181
Training loss: 0.62713882196761277555
Training accuracy: 93.4%
Test accuracy: 87.4%

Epoch 182
Training loss: 0.6253841666763890473
Training accuracy: 93.44%
Test accuracy: 87.4%

Epoch 183
Training loss: 0.624090923552555987

Training accuracy: 93.64%
Test accuracy: 87.4%

Epoch 184
Training loss: 0.6214213639034449173
Training accuracy: 93.52000000000001%
Test accuracy: 87.8%

Epoch 185
Training loss: 0.61996924809412455446
Training accuracy: 93.56%
Test accuracy: 87.4%

Epoch 186
Training loss: 0.618022682014588171
Training accuracy: 93.8%
Test accuracy: 88.0%

Epoch 187
Training loss: 0.616361126888142262
Training accuracy: 93.52000000000001%
Test accuracy: 87.6%

Epoch 188
Training loss: 0.6143469426363560454
Training accuracy: 93.76%
Test accuracy: 87.8%

Epoch 189
Training loss: 0.6131238347822947631
Training accuracy: 93.72%
Test accuracy: 87.6%

Epoch 190
Training loss: 0.61127910880563941993
Training accuracy: 93.84%
Test accuracy: 88.0%

Epoch 191
Training loss: 0.60890157521974988634
Training accuracy: 93.92%
Test accuracy: 87.4%

Epoch 192
Training loss: 0.60758724443976687315
Training accuracy: 93.8%
Test accuracy: 87.6%

```
Epoch 193
Training loss: 0.6056600340362162441
Training accuracy: 93.84%
Test accuracy: 88.0%

Epoch 194
Training loss: 0.60412219758631545516
Training accuracy: 93.96%
Test accuracy: 87.8%

Epoch 195
Training loss: 0.602337904903135703
Training accuracy: 93.88%
Test accuracy: 88.0%

Epoch 196
Training loss: 0.60057651468884508877
Training accuracy: 93.84%
Test accuracy: 87.6%

Epoch 197
Training loss: 0.5990361561873028138
Training accuracy: 93.92%
Test accuracy: 87.6%

Epoch 198
Training loss: 0.597834502980628766
Training accuracy: 93.92%
Test accuracy: 87.6%

Epoch 199
Training loss: 0.5966503912371351327
Training accuracy: 94.16%
Test accuracy: 87.6%

Epoch 200
Training loss: 0.5945048589134503131
Training accuracy: 94.04%
Test accuracy: 87.4%
```
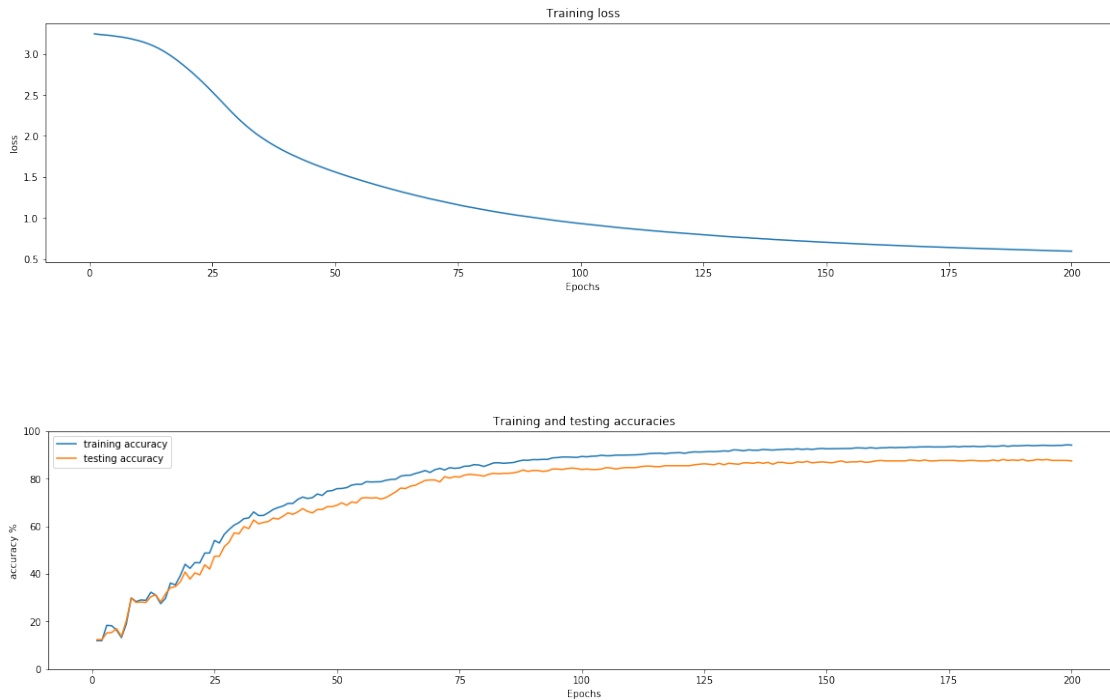
Since we tend to get similar results for both eta=0.1 and eta=0.01, we continue with eta=0.1

Finally we check if regularization is helpful or not   First with no regularization

```
[30]: net = network.Linear_Network([784, 35, 25, 10])
      net.SGD(training_data, 50, 10, 0.1, lmbda = 0.0, test_data=testing_data)
```

Epoch 1
Training loss: 3.1566247878512270909
Training accuracy: 19.16%
Test accuracy: 19.2%

Epoch 2
Training loss: 2.638599112819415536
Training accuracy: 53.2%
Test accuracy: 48.8%

Epoch 3
Training loss: 1.9805994656752686879
Training accuracy: 63.56%
Test accuracy: 59.4%

Epoch 4
Training loss: 1.608633046737969135
Training accuracy: 76.84%

71

Test accuracy: 74.2%

Epoch 5
Training loss: 1.3345860039540976656
Training accuracy: 81.04%
Test accuracy: 77.8%

Epoch 6
Training loss: 1.1284489039738173178
Training accuracy: 83.52000000000001%
Test accuracy: 79.60000000000001%

Epoch 7
Training loss: 1.0026843595724710446
Training accuracy: 86.96000000000001%
Test accuracy: 83.0%

Epoch 8
Training loss: 0.86874549684692934
Training accuracy: 89.16%
Test accuracy: 84.6%

Epoch 9
Training loss: 0.7653506640010482795
Training accuracy: 90.48%
Test accuracy: 85.6%

Epoch 10
Training loss: 0.6954871826036289916
Training accuracy: 91.0%
Test accuracy: 86.4%

Epoch 11
Training loss: 0.635146010930845314
Training accuracy: 91.8%
Test accuracy: 87.0%

Epoch 12
Training loss: 0.57202693843374002706
Training accuracy: 93.36%
Test accuracy: 86.8%

Epoch 13
Training loss: 0.52869511146036359293
Training accuracy: 94.0%
Test accuracy: 86.8%

Epoch 14

```
Training loss: 0.49522335450114631322
Training accuracy: 94.52000000000001%
Test accuracy: 87.2%


Epoch 15
Training loss: 0.4555449436507736489
Training accuracy: 94.64%
Test accuracy: 88.0%


Epoch 16
Training loss: 0.42898493685530982074
Training accuracy: 95.24000000000001%
Test accuracy: 87.6%


Epoch 17
Training loss: 0.399284533070007757
Training accuracy: 95.56%
Test accuracy: 87.4%


Epoch 18
Training loss: 0.38202419382888428833
Training accuracy: 95.76%
Test accuracy: 87.0%


Epoch 19
Training loss: 0.35723311841749348362
Training accuracy: 96.24000000000001%
Test accuracy: 87.0%


Epoch 20
Training loss: 0.34810640722746674254
Training accuracy: 96.16%
Test accuracy: 87.6%


Epoch 21
Training loss: 0.3231433343829978936
Training accuracy: 96.28%
Test accuracy: 87.8%


Epoch 22
Training loss: 0.2905622877744582346
Training accuracy: 97.0%
Test accuracy: 88.2%


Epoch 23
Training loss: 0.27556723798554143794
Training accuracy: 97.28%
Test accuracy: 88.0%
```

```
Epoch 24
Training loss: 0.26657158046482363966
Training accuracy: 97.39999999999999%
Test accuracy: 87.6%

Epoch 25
Training loss: 0.24837797672056164642
Training accuracy: 97.84%
Test accuracy: 87.4%

Epoch 26
Training loss: 0.23185170300059365869
Training accuracy: 97.96000000000001%
Test accuracy: 88.0%

Epoch 27
Training loss: 0.22045941944642101013
Training accuracy: 98.04%
Test accuracy: 87.0%

Epoch 28
Training loss: 0.20808602338657359802
Training accuracy: 98.16%
Test accuracy: 87.8%

Epoch 29
Training loss: 0.19481388368811271211
Training accuracy: 98.32%
Test accuracy: 87.6%

Epoch 30
Training loss: 0.18746235075330992874
Training accuracy: 98.52%
Test accuracy: 87.8%

Epoch 31
Training loss: 0.17624785879693008033
Training accuracy: 98.68%
Test accuracy: 88.2%

Epoch 32
Training loss: 0.16684809755391320649
Training accuracy: 98.83999999999999%
Test accuracy: 87.8%

Epoch 33
Training loss: 0.15794250167174995801
```

```
Training accuracy: 98.83999999999999%
Test accuracy: 88.2%


Epoch 34
Training loss: 0.14858422332373452581
Training accuracy: 99.0%
Test accuracy: 87.2%


Epoch 35
Training loss: 0.14277124008601227345
Training accuracy: 99.11999999999999%
Test accuracy: 88.0%


Epoch 36
Training loss: 0.13485401159964168761
Training accuracy: 99.11999999999999%
Test accuracy: 88.0%


Epoch 37
Training loss: 0.12945302037582401789
Training accuracy: 99.2%
Test accuracy: 87.4%


Epoch 38
Training loss: 0.12693830053463929211
Training accuracy: 99.36%
Test accuracy: 87.4%


Epoch 39
Training loss: 0.12491073151483849345
Training accuracy: 99.32%
Test accuracy: 88.0%


Epoch 40
Training loss: 0.11309156008954263679
Training accuracy: 99.4%
Test accuracy: 87.8%


Epoch 41
Training loss: 0.10671730965437816927
Training accuracy: 99.48%
Test accuracy: 88.6%


Epoch 42
Training loss: 0.10223956193534999181
Training accuracy: 99.52%
Test accuracy: 88.4%
```

```
Epoch 43
Training loss: 0.09873792074343048724
Training accuracy: 99.52%
Test accuracy: 88.2%

Epoch 44
Training loss: 0.096304470109465864944
Training accuracy: 99.6%
Test accuracy: 88.0%

Epoch 45
Training loss: 0.09017947684613037218
Training accuracy: 99.64%
Test accuracy: 88.0%

Epoch 46
Training loss: 0.087573065413591265116
Training accuracy: 99.68%
Test accuracy: 88.6%

Epoch 47
Training loss: 0.084378827461023778266
Training accuracy: 99.68%
Test accuracy: 88.4%

Epoch 48
Training loss: 0.07969449837067826522
Training accuracy: 99.68%
Test accuracy: 88.4%

Epoch 49
Training loss: 0.078168595932950542455
Training accuracy: 99.68%
Test accuracy: 88.0%

Epoch 50
Training loss: 0.07624919357889873241
Training accuracy: 99.72%
Test accuracy: 88.4%
```

Training loss

Training and testing accuracies
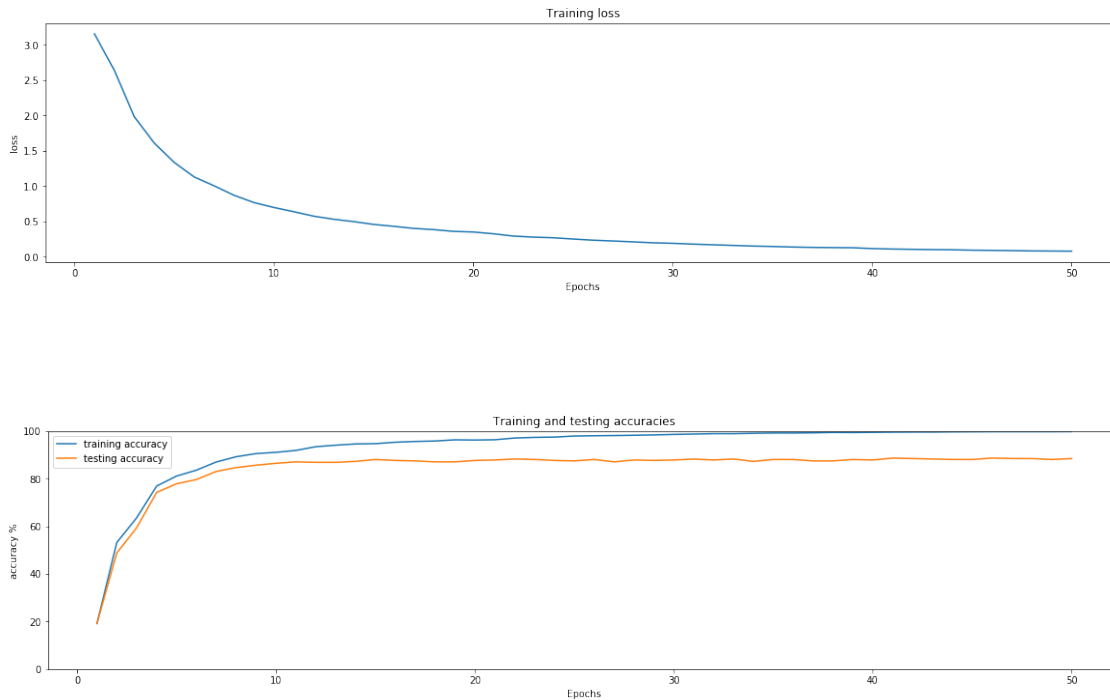
Clearly there has been over fitting as the training accuracy is 99%. It doesn't show up on the test accuracy as the testing set is too small. Hence we try different regularization factors to see which fits best

```python
[34]: regs = [1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0]
      train_accuracies=[]
      test_accuracies=[]
      for reg in regs:
          net = network.Linear_Network([784, 35, 25, 10])
          tr_a, te_a = net.SGD(training_data, 50, 10, 0.1, lmbda = reg,␣
       ↪test_data=testing_data,
                    show_eval=False, show_graph=False, return_accuracies=True)
          train_accuracies.append(tr_a)
          test_accuracies.append(te_a)
          print("regularization factor:"+str(reg))
          print("Training acc.: "+str(tr_a)+"% Testing acc.: "+str(te_a)+"%")
          print()

      plt.subplot(2, 1, 1)
      plt.plot(regs, train_accuracies, label='training accuracy')
      plt.plot(regs, test_accuracies, label='test accuracy')
      plt.legend()
      plt.title("Training and testing accuracies")
      plt.xlabel("neurons")
      plt.ylabel("accuracy %")
```

```
plt.ylim(0, 100)
plt.show()
```

```
regularization factor:1.0
Training acc.: 99.512% Testing acc.: 88.67999999999999%

regularization factor:2.0
Training acc.: 99.02399999999999% Testing acc.: 88.67999999999999%

regularization factor:3.0
Training acc.: 98.016% Testing acc.: 87.67999999999999%

regularization factor:4.0
Training acc.: 97.088% Testing acc.: 87.24000000000001%

regularization factor:5.0
Training acc.: 96.48% Testing acc.: 87.80000000000001%

regularization factor:6.0
Training acc.: 95.64800000000001% Testing acc.: 87.36%

regularization factor:7.0
Training acc.: 94.912% Testing acc.: 87.72%

regularization factor:8.0
Training acc.: 93.928% Testing acc.: 87.0%

regularization factor:9.0
Training acc.: 93.4% Testing acc.: 87.2%
```
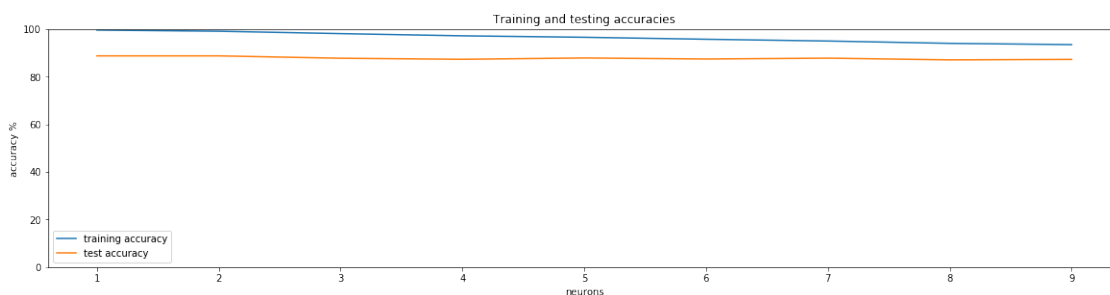

We see the training accuracy constantly dropping while the testing accuracy remained constant. This is indicative of over fitting. But we can't be too sure as the training and testing data is too small. We'd need atleast a bigger testing data to be entirely sure.

So for now, we stick to regularization of 7.0

We'll revist the regularization tuning when we look at Cross Validation Now that we finally have all our parameters, let's take a list them down. Then we'll visualize the results on the testing data

1. size = [784, 35, 25, 10]
2. weight_initializer = 'default'
3. cost = 'CrossEntropy'
4. activation = 'sigmoid'
5. epochs = 50
6. batch_size = 10
7. eta = 0.1
8. lmbda = 7.0

Final run of the mdoel

```
[3]: net = network.Linear_Network([784,35,25,10])
     net.SGD(training_data, 50, 10, 0.1, lmbda = 7.0, test_data = testing_data)
```

Epoch 1
Training loss: 3.2326841205725281342
Training accuracy: 10.280000000000001%
Test accuracy: 11.799999999999999%

Epoch 2
Training loss: 2.9537459618658661222
Training accuracy: 39.04%
Test accuracy: 36.199999999999996%

Epoch 3
Training loss: 2.4236604054472806506
Training accuracy: 50.96000000000001%
Test accuracy: 49.0%

Epoch 4
Training loss: 2.0560941779011762964
Training accuracy: 61.760000000000005%
Test accuracy: 58.4%

Epoch 5
Training loss: 1.7851758473356474966
Training accuracy: 73.64%
Test accuracy: 68.2%

Epoch 6
Training loss: 1.5828159690652832289
Training accuracy: 75.48%
Test accuracy: 71.2%

Epoch 7

```
Training loss: 1.4069334791200266226
Training accuracy: 79.75999999999999%
Test accuracy: 75.2%


Epoch 8
Training loss: 1.2701079597692875129
Training accuracy: 84.28%
Test accuracy: 81.2%


Epoch 9
Training loss: 1.1523511466166390834
Training accuracy: 86.88%
Test accuracy: 83.6%


Epoch 10
Training loss: 1.0689898801611050113
Training accuracy: 88.36%
Test accuracy: 84.2%


Epoch 11
Training loss: 0.9958489060768214889
Training accuracy: 89.24%
Test accuracy: 85.6%


Epoch 12
Training loss: 0.94237863925684892477
Training accuracy: 89.24%
Test accuracy: 85.2%


Epoch 13
Training loss: 0.89018769680228264287
Training accuracy: 90.8%
Test accuracy: 85.8%


Epoch 14
Training loss: 0.8528112852957638264
Training accuracy: 91.08000000000001%
Test accuracy: 86.0%


Epoch 15
Training loss: 0.8458220951912395739
Training accuracy: 91.56%
Test accuracy: 85.2%


Epoch 16
Training loss: 0.79749523820170468784
Training accuracy: 90.92%
Test accuracy: 86.2%
```

```
Epoch 17
Training loss: 0.78889735394966301713
Training accuracy: 90.64%
Test accuracy: 86.6%

Epoch 18
Training loss: 0.7622515288662194772
Training accuracy: 91.92%
Test accuracy: 87.4%

Epoch 19
Training loss: 0.7348530839081167861
Training accuracy: 91.92%
Test accuracy: 87.2%

Epoch 20
Training loss: 0.7299510551153513726
Training accuracy: 92.16%
Test accuracy: 87.6%

Epoch 21
Training loss: 0.71238620434335595336
Training accuracy: 92.96%
Test accuracy: 87.8%

Epoch 22
Training loss: 0.6891577904927793396
Training accuracy: 93.28%
Test accuracy: 87.4%

Epoch 23
Training loss: 0.69107512165084638786
Training accuracy: 93.12%
Test accuracy: 87.4%

Epoch 24
Training loss: 0.68156985734990621346
Training accuracy: 93.36%
Test accuracy: 87.0%

Epoch 25
Training loss: 0.6626684695361602232
Training accuracy: 93.16%
Test accuracy: 88.0%

Epoch 26
Training loss: 0.65078858219642967206
```

Training accuracy: 93.76%
Test accuracy: 88.2%

Epoch 27
Training loss: 0.6458221216025693004
Training accuracy: 94.08%
Test accuracy: 87.8%

Epoch 28
Training loss: 0.63586314620361638754
Training accuracy: 94.24%
Test accuracy: 87.6%

Epoch 29
Training loss: 0.63400881903736921677
Training accuracy: 93.84%
Test accuracy: 87.8%

Epoch 30
Training loss: 0.6251042179150910963
Training accuracy: 94.52000000000001%
Test accuracy: 88.2%

Epoch 31
Training loss: 0.61695678820882455173
Training accuracy: 94.64%
Test accuracy: 88.0%

Epoch 32
Training loss: 0.6177354067088967172
Training accuracy: 94.48%
Test accuracy: 87.8%

Epoch 33
Training loss: 0.61754338812817334466
Training accuracy: 94.44%
Test accuracy: 87.6%

Epoch 34
Training loss: 0.6165443884876816758
Training accuracy: 94.12%
Test accuracy: 87.6%

Epoch 35
Training loss: 0.6033791667883454335
Training accuracy: 94.44%
Test accuracy: 87.6%

```
Epoch 36
Training loss: 0.6021970325391033135
Training accuracy: 94.56%
Test accuracy: 87.8%


Epoch 37
Training loss: 0.5987250311007248646
Training accuracy: 94.64%
Test accuracy: 88.2%


Epoch 38
Training loss: 0.6112678590840087999
Training accuracy: 93.84%
Test accuracy: 88.4%


Epoch 39
Training loss: 0.60098091201283159623
Training accuracy: 94.16%
Test accuracy: 88.0%


Epoch 40
Training loss: 0.5943690540855281772
Training accuracy: 94.39999999999999%
Test accuracy: 88.2%


Epoch 41
Training loss: 0.59540718384031455964
Training accuracy: 95.12%
Test accuracy: 87.6%


Epoch 42
Training loss: 0.5805668478565324346
Training accuracy: 94.67999999999999%
Test accuracy: 87.4%


Epoch 43
Training loss: 0.59134911657571848293
Training accuracy: 94.32000000000001%
Test accuracy: 87.6%


Epoch 44
Training loss: 0.58343780479105319026
Training accuracy: 94.76%
Test accuracy: 88.2%


Epoch 45
Training loss: 0.5813726071401272093
Training accuracy: 94.8%
```

Test accuracy: 88.6%

Epoch 46
Training loss: 0.5746104797288835689
Training accuracy: 95.12%
Test accuracy: 87.8%

Epoch 47
Training loss: 0.57736468906669823493
Training accuracy: 95.19999999999999%
Test accuracy: 87.4%

Epoch 48
Training loss: 0.57321975893967532943
Training accuracy: 95.19999999999999%
Test accuracy: 87.0%

Epoch 49
Training loss: 0.57746910449444494555
Training accuracy: 94.96%
Test accuracy: 87.6%

Epoch 50
Training loss: 0.56415261698794610885
Training accuracy: 95.16%
Test accuracy: 88.0%
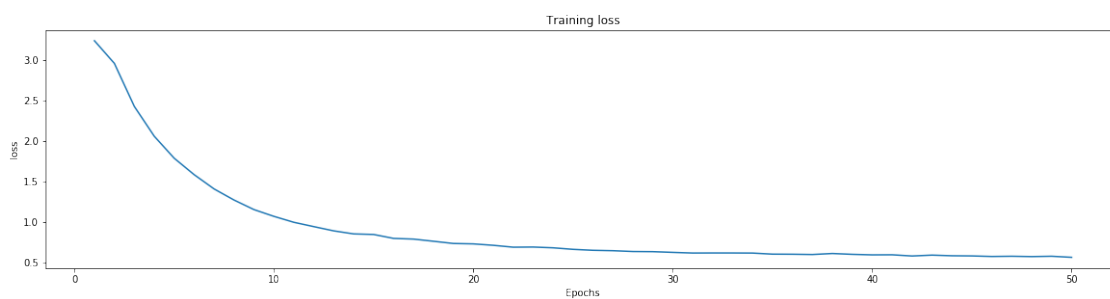
Now we are ready to visualize some results: Let's run it on 20 test datas

```python
from PIL import Image
test_set = testing_data[:20]
prediction=[]
label=[]
images=[]
for (x,y) in test_set:
    prediction.append(np.argmax(net.feedforward(x)))
    label.append(y)
    image = x[0]
    image = x.reshape(28,28)*255
    images.append(image)

fig = plt.figure(figsize=(25, 5))

for idx in np.arange(20):
    ax = fig.add_subplot(2, 10, idx+1, xticks=[], yticks=[])
    plt.imshow(images[idx])
    ax.set_title("{} ({})".format(str(prediction[idx]), str(label[idx].item())),
                 color=("green" if prediction[idx]==label[idx] else "red"))
```
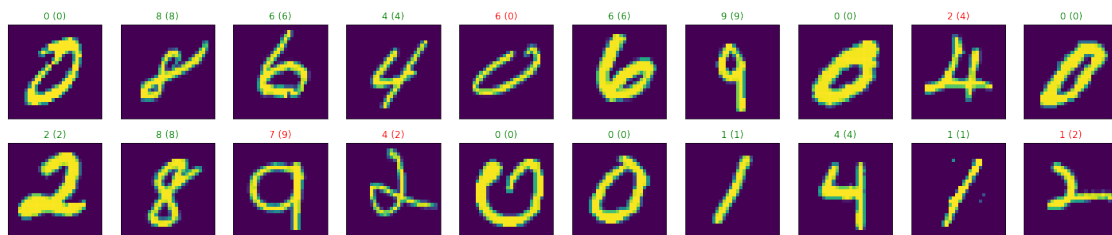

2.2.8 Now, finally lets use Cross validation to see if we get any improvement

We'll be using k-fold cross validation with k=10

```python
training_data_CV = data_loader.load_full_data_cross_validate()
```

First, without any regularization

```python
net2 = network.Linear_Network_CrossValidation([784,35,25,10])
```

```python
net2.SGD(training_data_CV, 200, 10, 0.5, lmbda=0.0)
```

```
Epoch 1
Training loss: 3.233537854495628635
Training accuracy: 14.416666666666668%
Test accuracy: 11.666666666666668%


Epoch 2
Training loss: 3.2136444536432516718
Training accuracy: 17.833333333333336%
Test accuracy: 14.333333333333332%


Epoch 3
Training loss: 3.1936151538179284432
Training accuracy: 22.583333333333336%
Test accuracy: 17.166666666666664%


Epoch 4
Training loss: 3.1697761742039304717
Training accuracy: 27.583333333333332%
Test accuracy: 26.666666666666664%


Epoch 5
Training loss: 3.1344249098431653114
Training accuracy: 35.54166666666667%
Test accuracy: 30.166666666666664%


Epoch 6
Training loss: 3.0894104136105461755
Training accuracy: 40.45833333333333%
Test accuracy: 38.666666666666664%


Epoch 7
Training loss: 3.0208982340898541958
Training accuracy: 44.24999999999999%
Test accuracy: 45.0%


Epoch 8
Training loss: 2.924544118978684489
Training accuracy: 45.95833333333333%
Test accuracy: 43.66666666666667%


Epoch 9
Training loss: 2.811105203930599877
Training accuracy: 46.125%
Test accuracy: 45.16666666666667%


Epoch 10
Training loss: 2.6667257286226191304
Training accuracy: 49.75%
```

Test accuracy: 48.00000000000001%

Epoch 11
Training loss: 2.5389568290337344732
Training accuracy: 52.833333333333336%
Test accuracy: 55.5%

Epoch 12
Training loss: 2.3855806663028148716
Training accuracy: 55.375%
Test accuracy: 51.5%

Epoch 13
Training loss: 2.271666654296425065
Training accuracy: 58.70833333333333%
Test accuracy: 61.0%

Epoch 14
Training loss: 2.163025813302426088
Training accuracy: 62.583333333333336%
Test accuracy: 61.66666666666667%

Epoch 15
Training loss: 2.0537074913546677784
Training accuracy: 65.20833333333334%
Test accuracy: 61.833333333333336%

Epoch 16
Training loss: 1.9638222763344113231
Training accuracy: 67.66666666666666%
Test accuracy: 67.0%

Epoch 17
Training loss: 1.8835723106574945352
Training accuracy: 68.75%
Test accuracy: 70.5%

Epoch 18
Training loss: 1.8045931964493152448
Training accuracy: 71.79166666666667%
Test accuracy: 72.0%

Epoch 19
Training loss: 1.7324466340130144214
Training accuracy: 73.12499999999999%
Test accuracy: 73.33333333333333%

Epoch 20

```
Training loss: 1.6648586208553211743
Training accuracy: 75.54166666666666%
Test accuracy: 76.5%


Epoch 21
Training loss: 1.5946132950316805122
Training accuracy: 76.87500000000001%
Test accuracy: 75.99999999999999%


Epoch 22
Training loss: 1.535241905843814365
Training accuracy: 77.54166666666667%
Test accuracy: 76.16666666666667%


Epoch 23
Training loss: 1.470905251606416574
Training accuracy: 79.70833333333334%
Test accuracy: 75.50000000000001%


Epoch 24
Training loss: 1.4430616623719659966
Training accuracy: 79.58333333333334%
Test accuracy: 82.5%


Epoch 25
Training loss: 1.3864291292202437616
Training accuracy: 81.33333333333334%
Test accuracy: 82.49999999999999%


Epoch 26
Training loss: 1.3301467053309693075
Training accuracy: 82.91666666666667%
Test accuracy: 81.0%


Epoch 27
Training loss: 1.2926144981573606048
Training accuracy: 83.20833333333334%
Test accuracy: 83.5%


Epoch 28
Training loss: 1.2445197852773281304
Training accuracy: 84.37499999999999%
Test accuracy: 82.83333333333334%


Epoch 29
Training loss: 1.1908512600625459742
Training accuracy: 85.33333333333333%
Test accuracy: 82.5%
```

```
Epoch 30
Training loss: 1.1489348588688546372
Training accuracy: 86.95833333333333%
Test accuracy: 81.83333333333334%

Epoch 31
Training loss: 1.1161859154366940527
Training accuracy: 86.33333333333333%
Test accuracy: 83.83333333333333%

Epoch 32
Training loss: 1.0884191005382822382
Training accuracy: 86.95833333333333%
Test accuracy: 83.33333333333333%

Epoch 33
Training loss: 1.0557087938613798154
Training accuracy: 87.0%
Test accuracy: 86.66666666666666%

Epoch 34
Training loss: 1.026124732991103635
Training accuracy: 87.25%
Test accuracy: 87.5%

Epoch 35
Training loss: 0.99993741448852918466
Training accuracy: 87.58333333333334%
Test accuracy: 87.33333333333334%

Epoch 36
Training loss: 0.9602884423170300631
Training accuracy: 88.33333333333333%
Test accuracy: 85.5%

Epoch 37
Training loss: 0.94301490964553000794
Training accuracy: 88.33333333333334%
Test accuracy: 88.83333333333334%

Epoch 38
Training loss: 0.9045795184717783384
Training accuracy: 88.99999999999999%
Test accuracy: 86.83333333333333%

Epoch 39
Training loss: 0.8872087088640832861
```

```
Training accuracy: 88.33333333333333%
Test accuracy: 88.5%


Epoch 40
Training loss: 0.86917929699570480116
Training accuracy: 89.16666666666667%
Test accuracy: 88.33333333333334%


Epoch 41
Training loss: 0.82985020283456606676
Training accuracy: 89.95833333333333%
Test accuracy: 85.0%


Epoch 42
Training loss: 0.81448734432867398516
Training accuracy: 89.875%
Test accuracy: 87.66666666666666%


Epoch 43
Training loss: 0.803156388806158129
Training accuracy: 90.41666666666666%
Test accuracy: 87.5%


Epoch 44
Training loss: 0.80250260321638110917
Training accuracy: 90.08333333333333%
Test accuracy: 90.33333333333334%


Epoch 45
Training loss: 0.7772046803427827947
Training accuracy: 90.33333333333333%
Test accuracy: 89.83333333333333%


Epoch 46
Training loss: 0.7637192572759200975
Training accuracy: 90.20833333333333%
Test accuracy: 89.83333333333334%


Epoch 47
Training loss: 0.7446162045200841144
Training accuracy: 90.875%
Test accuracy: 89.33333333333334%


Epoch 48
Training loss: 0.7441090282189275952
Training accuracy: 90.25000000000001%
Test accuracy: 92.0%
```

```
Epoch 49
Training loss: 0.71002135769709309786
Training accuracy: 90.95833333333333%
Test accuracy: 89.33333333333333%


Epoch 50
Training loss: 0.70909111842985812345
Training accuracy: 90.875%
Test accuracy: 90.83333333333334%


Epoch 51
Training loss: 0.68323298413620381093
Training accuracy: 91.20833333333334%
Test accuracy: 90.33333333333334%


Epoch 52
Training loss: 0.66851598356334323447
Training accuracy: 91.45833333333334%
Test accuracy: 90.83333333333334%


Epoch 53
Training loss: 0.64968153618653084574
Training accuracy: 91.70833333333333%
Test accuracy: 89.0%


Epoch 54
Training loss: 0.6398616372196474671
Training accuracy: 91.62500000000001%
Test accuracy: 90.16666666666667%


Epoch 55
Training loss: 0.6412475480804507115
Training accuracy: 91.75%
Test accuracy: 90.33333333333333%


Epoch 56
Training loss: 0.62619662360283175393
Training accuracy: 92.08333333333333%
Test accuracy: 90.16666666666666%


Epoch 57
Training loss: 0.614212794631739877
Training accuracy: 92.25000000000001%
Test accuracy: 91.0%


Epoch 58
Training loss: 0.6136251767230409955
Training accuracy: 92.0%
```

Test accuracy: 92.5%

Epoch 59
Training loss: 0.59304443407742042275
Training accuracy: 92.20833333333334%
Test accuracy: 91.49999999999999%

Epoch 60
Training loss: 0.57930135996502677673
Training accuracy: 92.41666666666667%
Test accuracy: 90.83333333333333%

Epoch 61
Training loss: 0.57294108257769131814
Training accuracy: 92.45833333333334%
Test accuracy: 92.0%

Epoch 62
Training loss: 0.5642136022420478326
Training accuracy: 92.58333333333334%
Test accuracy: 91.0%

Epoch 63
Training loss: 0.56422032895730070466
Training accuracy: 92.58333333333333%
Test accuracy: 92.16666666666666%

Epoch 64
Training loss: 0.5393062154227270558
Training accuracy: 93.0%
Test accuracy: 90.83333333333334%

Epoch 65
Training loss: 0.5346263021869130283
Training accuracy: 93.125%
Test accuracy: 91.5%

Epoch 66
Training loss: 0.53614029488283986713
Training accuracy: 93.20833333333334%
Test accuracy: 91.83333333333333%

Epoch 67
Training loss: 0.5238537524958866116
Training accuracy: 93.37499999999999%
Test accuracy: 92.16666666666666%

Epoch 68

```
Training loss: 0.5141374183858615948
Training accuracy: 93.54166666666667%
Test accuracy: 90.66666666666666%


Epoch 69
Training loss: 0.5034502230425772184
Training accuracy: 93.54166666666666%
Test accuracy: 91.5%


Epoch 70
Training loss: 0.49332982763752986497
Training accuracy: 93.95833333333334%
Test accuracy: 91.66666666666667%


Epoch 71
Training loss: 0.4981555185247075186
Training accuracy: 93.62500000000001%
Test accuracy: 92.16666666666666%


Epoch 72
Training loss: 0.4859485794092234187
Training accuracy: 93.79166666666667%
Test accuracy: 93.16666666666666%


Epoch 73
Training loss: 0.49292245857212593703
Training accuracy: 93.58333333333334%
Test accuracy: 95.33333333333333%


Epoch 74
Training loss: 0.48058538978474888274
Training accuracy: 94.0%
Test accuracy: 94.0%


Epoch 75
Training loss: 0.46670350525377085426
Training accuracy: 94.45833333333334%
Test accuracy: 92.83333333333333%


Epoch 76
Training loss: 0.46580880439297319
Training accuracy: 94.04166666666667%
Test accuracy: 94.5%


Epoch 77
Training loss: 0.45025278126030846286
Training accuracy: 94.41666666666666%
Test accuracy: 93.83333333333333%
```

```
Epoch 78
Training loss: 0.44568515329730115244
Training accuracy: 94.625%
Test accuracy: 93.16666666666%

Epoch 79
Training loss: 0.45728493769409120825
Training accuracy: 94.04166666666%
Test accuracy: 95.0%

Epoch 80
Training loss: 0.43893024364561242532
Training accuracy: 94.41666666666%
Test accuracy: 94.16666666666%

Epoch 81
Training loss: 0.4350648032407386571
Training accuracy: 94.95833333333%
Test accuracy: 94.83333333333334%

Epoch 82
Training loss: 0.42379938359434590018
Training accuracy: 94.95833333333%
Test accuracy: 92.83333333333333%

Epoch 83
Training loss: 0.42013993720283877297
Training accuracy: 94.66666666666%
Test accuracy: 94.5%

Epoch 84
Training loss: 0.4110694180304460797
Training accuracy: 94.95833333333%
Test accuracy: 94.50000000000001%

Epoch 85
Training loss: 0.396175139717582081
Training accuracy: 95.29166666666%
Test accuracy: 93.33333333333333%

Epoch 86
Training loss: 0.40289144932859024913
Training accuracy: 95.0%
Test accuracy: 94.0%

Epoch 87
Training loss: 0.3942100646964119145
```

```
Training accuracy: 95.41666666666666%
Test accuracy: 93.66666666666667%


Epoch 88
Training loss: 0.40213892306792189125
Training accuracy: 95.00000000000001%
Test accuracy: 95.00000000000001%


Epoch 89
Training loss: 0.3930557160175613247
Training accuracy: 95.54166666666667%
Test accuracy: 94.16666666666666%


Epoch 90
Training loss: 0.39824597951079950358
Training accuracy: 95.29166666666667%
Test accuracy: 95.49999999999999%


Epoch 91
Training loss: 0.3860232166290985269
Training accuracy: 95.33333333333333%
Test accuracy: 95.5%


Epoch 92
Training loss: 0.37552515954703147158
Training accuracy: 95.66666666666667%
Test accuracy: 94.16666666666666%


Epoch 93
Training loss: 0.3674534473840604852
Training accuracy: 95.625%
Test accuracy: 95.16666666666666%


Epoch 94
Training loss: 0.3760295920381473143
Training accuracy: 95.29166666666667%
Test accuracy: 96.0%


Epoch 95
Training loss: 0.36306118467090412694
Training accuracy: 95.79166666666667%
Test accuracy: 94.66666666666667%


Epoch 96
Training loss: 0.3668929916001822132
Training accuracy: 95.83333333333334%
Test accuracy: 95.5%
```

```
Epoch 97
Training loss: 0.35835375228697677834
Training accuracy: 95.70833333333334%
Test accuracy: 96.33333333333333%

Epoch 98
Training loss: 0.34991033829118221109
Training accuracy: 95.95833333333334%
Test accuracy: 95.5%

Epoch 99
Training loss: 0.34653206556367312252
Training accuracy: 96.16666666666666%
Test accuracy: 95.49999999999999%

Epoch 100
Training loss: 0.34749667747319227767
Training accuracy: 96.12499999999999%
Test accuracy: 96.16666666666666%

Epoch 101
Training loss: 0.33248805795580390975
Training accuracy: 96.50000000000001%
Test accuracy: 94.5%

Epoch 102
Training loss: 0.33296794195569076542
Training accuracy: 96.41666666666666%
Test accuracy: 95.50000000000001%

Epoch 103
Training loss: 0.3230726531059142974
Training accuracy: 96.5%
Test accuracy: 95.16666666666666%

Epoch 104
Training loss: 0.3186702769406441511
Training accuracy: 96.70833333333334%
Test accuracy: 95.0%

Epoch 105
Training loss: 0.31552180475221269874
Training accuracy: 96.37500000000001%
Test accuracy: 96.33333333333333%

Epoch 106
Training loss: 0.30715800825228927538
Training accuracy: 96.70833333333334%
```

Test accuracy: 95.16666666666%

Epoch 107
Training loss: 0.31372719525712916458
Training accuracy: 96.54166666666667%
Test accuracy: 96.33333333333334%

Epoch 108
Training loss: 0.31147468389433702747
Training accuracy: 96.58333333333334%
Test accuracy: 97.00000000000001%

Epoch 109
Training loss: 0.30401946100437874022
Training accuracy: 96.75000000000001%
Test accuracy: 95.33333333333333%

Epoch 110
Training loss: 0.29473213582329932522
Training accuracy: 97.04166666666666%
Test accuracy: 95.0%

Epoch 111
Training loss: 0.30458225851461759233
Training accuracy: 96.70833333333334%
Test accuracy: 96.66666666666667%

Epoch 112
Training loss: 0.3024291566177774124
Training accuracy: 96.83333333333334%
Test accuracy: 96.16666666666667%

Epoch 113
Training loss: 0.2959282481177537407
Training accuracy: 96.83333333333333%
Test accuracy: 96.33333333333333%

Epoch 114
Training loss: 0.29864434406749244862
Training accuracy: 96.45833333333333%
Test accuracy: 97.83333333333334%

Epoch 115
Training loss: 0.27537320018474425207
Training accuracy: 97.54166666666666%
Test accuracy: 94.99999999999999%

Epoch 116

Training loss: 0.28973314538900529308
Training accuracy: 97.04166666666%
Test accuracy: 96.33333333333334%

Epoch 117
Training loss: 0.27828360515138396428
Training accuracy: 97.29166666666667%
Test accuracy: 96.33333333333333%

Epoch 118
Training loss: 0.2748774380718776696
Training accuracy: 97.12499999999999%
Test accuracy: 96.66666666666666%

Epoch 119
Training loss: 0.27885319655724924705
Training accuracy: 97.37500000000001%
Test accuracy: 98.0%

Epoch 120
Training loss: 0.2607242075577945485
Training accuracy: 97.625%
Test accuracy: 96.33333333333333%

Epoch 121
Training loss: 0.27190881512309718004
Training accuracy: 97.41666666666666%
Test accuracy: 97.83333333333334%

Epoch 122
Training loss: 0.27499029498764918783
Training accuracy: 97.41666666666666%
Test accuracy: 97.5%

Epoch 123
Training loss: 0.26492011557657773056
Training accuracy: 97.54166666666667%
Test accuracy: 96.33333333333334%

Epoch 124
Training loss: 0.26281708558338961248
Training accuracy: 97.79166666666666%
Test accuracy: 97.0%

Epoch 125
Training loss: 0.2594877697893804848
Training accuracy: 97.5%
Test accuracy: 97.33333333333333%

```
Epoch 126
Training loss: 0.24801046766115878418
Training accuracy: 97.58333333333333%
Test accuracy: 97.99999999999999%

Epoch 127
Training loss: 0.2544819256691145019
Training accuracy: 97.66666666666666%
Test accuracy: 97.33333333333334%

Epoch 128
Training loss: 0.2510110560142390494
Training accuracy: 97.83333333333334%
Test accuracy: 97.50000000000001%

Epoch 129
Training loss: 0.24983823585143704555
Training accuracy: 97.74999999999999%
Test accuracy: 97.83333333333334%

Epoch 130
Training loss: 0.23796188689254582613
Training accuracy: 97.75%
Test accuracy: 97.16666666666666%

Epoch 131
Training loss: 0.24288715871391874937
Training accuracy: 97.79166666666666%
Test accuracy: 98.5%

Epoch 132
Training loss: 0.23844946243967035447
Training accuracy: 98.20833333333334%
Test accuracy: 97.16666666666666%

Epoch 133
Training loss: 0.23248424737769048774
Training accuracy: 98.16666666666667%
Test accuracy: 97.00000000000001%

Epoch 134
Training loss: 0.23234957393184391429
Training accuracy: 98.16666666666666%
Test accuracy: 97.33333333333333%

Epoch 135
Training loss: 0.23069132606995833797
```

```
Training accuracy: 98.16666666666666%
Test accuracy: 97.66666666666666%


Epoch 136
Training loss: 0.22076983262285283384
Training accuracy: 98.25%
Test accuracy: 98.0%


Epoch 137
Training loss: 0.22856290434131055765
Training accuracy: 98.25%
Test accuracy: 97.99999999999999%


Epoch 138
Training loss: 0.23152964668479385666
Training accuracy: 98.375%
Test accuracy: 98.33333333333334%


Epoch 139
Training loss: 0.21639159129902917635
Training accuracy: 98.5%
Test accuracy: 96.99999999999999%


Epoch 140
Training loss: 0.21353158560195480684
Training accuracy: 98.375%
Test accuracy: 97.83333333333334%


Epoch 141
Training loss: 0.2068775154896613747
Training accuracy: 98.66666666666666%
Test accuracy: 97.0%


Epoch 142
Training loss: 0.21651357599337643696
Training accuracy: 98.25%
Test accuracy: 98.66666666666666%


Epoch 143
Training loss: 0.21344324591054655433
Training accuracy: 98.29166666666666%
Test accuracy: 98.33333333333333%


Epoch 144
Training loss: 0.2050188983531804092
Training accuracy: 98.375%
Test accuracy: 98.33333333333334%
```

```
Epoch 145
Training loss: 0.19500051027553397246
Training accuracy: 98.83333333333334%
Test accuracy: 97.66666666666667%

Epoch 146
Training loss: 0.21241827381397513069
Training accuracy: 98.375%
Test accuracy: 98.66666666666667%

Epoch 147
Training loss: 0.19445402009707637442
Training accuracy: 98.58333333333334%
Test accuracy: 97.99999999999999%

Epoch 148
Training loss: 0.20635011804810885895
Training accuracy: 98.58333333333333%
Test accuracy: 98.99999999999999%

Epoch 149
Training loss: 0.19648311167542076017
Training accuracy: 98.45833333333333%
Test accuracy: 98.66666666666666%

Epoch 150
Training loss: 0.20080591498319494352
Training accuracy: 98.41666666666667%
Test accuracy: 99.33333333333334%

Epoch 151
Training loss: 0.18069974580529949233
Training accuracy: 98.95833333333333%
Test accuracy: 97.33333333333334%

Epoch 152
Training loss: 0.19249646562606721745
Training accuracy: 98.54166666666666%
Test accuracy: 98.83333333333334%

Epoch 153
Training loss: 0.19488623774093451176
Training accuracy: 98.58333333333333%
Test accuracy: 99.0%

Epoch 154
Training loss: 0.18104430865038142136
Training accuracy: 98.62499999999999%
```

Test accuracy: 98.33333333333334%

Epoch 155
Training loss: 0.1800293033728428389
Training accuracy: 98.66666666666666%
Test accuracy: 98.5%

Epoch 156
Training loss: 0.17708346129552741228
Training accuracy: 98.95833333333334%
Test accuracy: 97.99999999999999%

Epoch 157
Training loss: 0.16836035965957944655
Training accuracy: 98.875%
Test accuracy: 98.16666666666666%

Epoch 158
Training loss: 0.17230847323947600167
Training accuracy: 98.91666666666666%
Test accuracy: 98.33333333333333%

Epoch 159
Training loss: 0.17912693189021032777
Training accuracy: 98.75%
Test accuracy: 98.49999999999999%

Epoch 160
Training loss: 0.17388458647384169331
Training accuracy: 98.83333333333333%
Test accuracy: 99.16666666666667%

Epoch 161
Training loss: 0.1730708292116527965
Training accuracy: 98.79166666666667%
Test accuracy: 98.83333333333333%

Epoch 162
Training loss: 0.17582647651474587496
Training accuracy: 98.79166666666666%
Test accuracy: 98.83333333333334%

Epoch 163
Training loss: 0.16697540169145183065
Training accuracy: 99.08333333333334%
Test accuracy: 98.33333333333333%

Epoch 164

```
Training loss: 0.17309629249206558117
Training accuracy: 98.79166666666667%
Test accuracy: 99.0%


Epoch 165
Training loss: 0.1612094477521055103
Training accuracy: 98.95833333333333%
Test accuracy: 98.66666666666666%


Epoch 166
Training loss: 0.16564558997883734772
Training accuracy: 98.87499999999999%
Test accuracy: 99.0%


Epoch 167
Training loss: 0.16820310344190631736
Training accuracy: 99.0%
Test accuracy: 99.16666666666667%


Epoch 168
Training loss: 0.1532430830000172007
Training accuracy: 99.0%
Test accuracy: 98.66666666666667%


Epoch 169
Training loss: 0.15882823152176912601
Training accuracy: 98.95833333333333%
Test accuracy: 98.66666666666666%


Epoch 170
Training loss: 0.1548341771018890666
Training accuracy: 99.0%
Test accuracy: 98.33333333333334%


Epoch 171
Training loss: 0.15523995129799627125
Training accuracy: 99.0%
Test accuracy: 98.83333333333334%


Epoch 172
Training loss: 0.14957948841058119952
Training accuracy: 98.95833333333334%
Test accuracy: 98.83333333333333%


Epoch 173
Training loss: 0.14611684124963931899
Training accuracy: 99.125%
Test accuracy: 98.33333333333334%
```

```
Epoch 174
Training loss: 0.15140465326693490838
Training accuracy: 99.12499999999999%
Test accuracy: 98.66666666666666%

Epoch 175
Training loss: 0.14590294970227477
Training accuracy: 99.125%
Test accuracy: 98.83333333333334%

Epoch 176
Training loss: 0.15298028938280705454
Training accuracy: 98.83333333333334%
Test accuracy: 99.66666666666666%

Epoch 177
Training loss: 0.14639683234569530505
Training accuracy: 99.04166666666666%
Test accuracy: 99.5%

Epoch 178
Training loss: 0.14749199265373726139
Training accuracy: 99.16666666666666%
Test accuracy: 99.0%

Epoch 179
Training loss: 0.14646913557516882468
Training accuracy: 99.16666666666667%
Test accuracy: 99.33333333333333%

Epoch 180
Training loss: 0.14253620112813249294
Training accuracy: 99.25000000000001%
Test accuracy: 99.16666666666666%

Epoch 181
Training loss: 0.14190517383155897439
Training accuracy: 99.16666666666667%
Test accuracy: 99.5%

Epoch 182
Training loss: 0.14018644935665947645
Training accuracy: 99.20833333333333%
Test accuracy: 99.5%

Epoch 183
Training loss: 0.14080945784592921594
```

```
Training accuracy: 99.29166666666667%
Test accuracy: 98.83333333333333%


Epoch 184
Training loss: 0.13775736094223302348
Training accuracy: 99.25000000000001%
Test accuracy: 99.16666666666666%


Epoch 185
Training loss: 0.13406607285524639679
Training accuracy: 99.375%
Test accuracy: 98.83333333333333%


Epoch 186
Training loss: 0.13258924166296546687
Training accuracy: 99.29166666666666%
Test accuracy: 99.16666666666666%


Epoch 187
Training loss: 0.13080697710423674828
Training accuracy: 99.25000000000001%
Test accuracy: 99.0%


Epoch 188
Training loss: 0.13426622844138323114
Training accuracy: 99.33333333333333%
Test accuracy: 98.99999999999999%


Epoch 189
Training loss: 0.12549673862456182001
Training accuracy: 99.45833333333334%
Test accuracy: 98.83333333333334%


Epoch 190
Training loss: 0.13033045967816450953
Training accuracy: 99.20833333333334%
Test accuracy: 99.5%


Epoch 191
Training loss: 0.12399850096138438214
Training accuracy: 99.45833333333333%
Test accuracy: 98.66666666666667%


Epoch 192
Training loss: 0.12030170758503202798
Training accuracy: 99.41666666666667%
Test accuracy: 98.83333333333333%
```

```
Epoch 193
Training loss: 0.12077861821056685112
Training accuracy: 99.375%
Test accuracy: 99.16666666666%

Epoch 194
Training loss: 0.123122047850643746134
Training accuracy: 99.33333333333334%
Test accuracy: 99.33333333333334%

Epoch 195
Training loss: 0.1244189069402086804
Training accuracy: 99.50000000000001%
Test accuracy: 99.00000000000001%

Epoch 196
Training loss: 0.11702673569830103874
Training accuracy: 99.33333333333334%
Test accuracy: 99.5%

Epoch 197
Training loss: 0.12590261000093152491
Training accuracy: 99.16666666666666%
Test accuracy: 100.0%

Epoch 198
Training loss: 0.12202674122549884145
Training accuracy: 99.33333333333333%
Test accuracy: 99.33333333333333%

Epoch 199
Training loss: 0.10853645496922166513
Training accuracy: 99.54166666666666%
Test accuracy: 98.66666666666666%

Epoch 200
Training loss: 0.11863103602889296698
Training accuracy: 99.37500000000001%
Test accuracy: 99.33333333333333%
```

Training loss

Training and testing accuracies
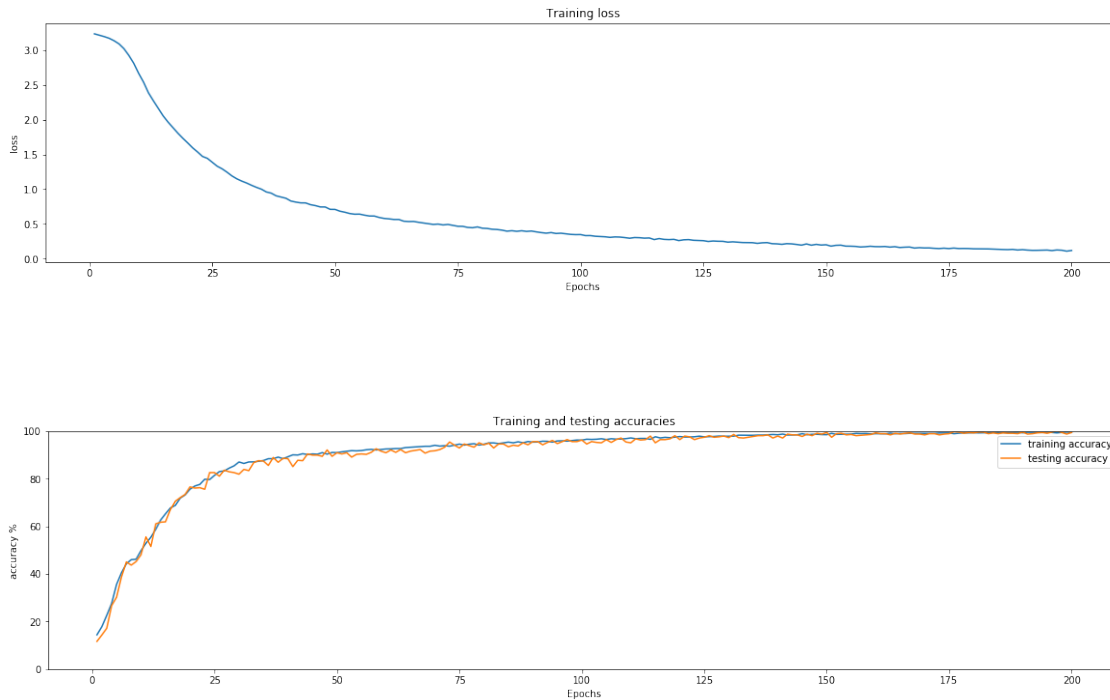
Now trying with a regularizatin factor:

```
[26]: net2 = network.Linear_Network_CrossValidation([784,35,25,10])
```

```
[27]: net2.SGD(training_data_CV, 200, 10, 0.5, lmbda=5.0)
```

```
Epoch 1
Training loss: 3.2479787792207839177
Training accuracy: 13.708333333333334%
Test accuracy: 13.0%

Epoch 2
Training loss: 3.2154712958092353582
Training accuracy: 16.083333333333336%
Test accuracy: 15.833333333333334%

Epoch 3
Training loss: 3.2042990612529214736
Training accuracy: 18.750000000000004%
Test accuracy: 17.166666666666664%

Epoch 4
Training loss: 3.182185956432032023
Training accuracy: 23.666666666666664%
Test accuracy: 19.000000000000004%
```

```
Epoch 5
Training loss: 3.1563591483696204476
Training accuracy: 26.5%
Test accuracy: 23.499999999999996%

Epoch 6
Training loss: 3.1190120285228041847
Training accuracy: 30.958333333333332%
Test accuracy: 30.0%

Epoch 7
Training loss: 3.069249316275351229
Training accuracy: 34.041666666666664%
Test accuracy: 35.16666666666667%

Epoch 8
Training loss: 2.9886785482640016581
Training accuracy: 36.291666666666664%
Test accuracy: 35.0%

Epoch 9
Training loss: 2.8987725837657495876
Training accuracy: 37.208333333333336%
Test accuracy: 34.166666666666664%

Epoch 10
Training loss: 2.7956088561542536055
Training accuracy: 37.41666666666667%
Test accuracy: 31.499999999999993%

Epoch 11
Training loss: 2.7089090824446049124
Training accuracy: 40.541666666666664%
Test accuracy: 39.33333333333333%

Epoch 12
Training loss: 2.6062313580128571404
Training accuracy: 44.708333333333336%
Test accuracy: 46.83333333333333%

Epoch 13
Training loss: 2.5099035431732771692
Training accuracy: 47.79166666666667%
Test accuracy: 49.333333333333336%

Epoch 14
Training loss: 2.4219925465016178784
```

```
Training accuracy: 48.541666666666664%
Test accuracy: 42.333333333333336%


Epoch 15
Training loss: 2.3499499555415663823
Training accuracy: 52.08333333333333%
Test accuracy: 48.333333333333336%


Epoch 16
Training loss: 2.276330190651422131
Training accuracy: 56.0%
Test accuracy: 54.833333333333336%


Epoch 17
Training loss: 2.2000531576148051117
Training accuracy: 57.25%
Test accuracy: 54.666666666666664%


Epoch 18
Training loss: 2.1370256995314808892
Training accuracy: 61.04166666666667%
Test accuracy: 59.333333333333336%


Epoch 19
Training loss: 2.0848511237533405892
Training accuracy: 62.79166666666667%
Test accuracy: 65.5%


Epoch 20
Training loss: 2.0137205909015272482
Training accuracy: 67.04166666666667%
Test accuracy: 66.16666666666666%


Epoch 21
Training loss: 1.9533093302620241818
Training accuracy: 67.62499999999999%
Test accuracy: 67.83333333333334%


Epoch 22
Training loss: 1.8898338987303976112
Training accuracy: 69.58333333333333%
Test accuracy: 71.16666666666666%


Epoch 23
Training loss: 1.8307328454711889418
Training accuracy: 70.87500000000001%
Test accuracy: 67.66666666666666%
```

```
Epoch 24
Training loss: 1.7765021115508926877
Training accuracy: 72.33333333333%
Test accuracy: 72.49999999999999%

Epoch 25
Training loss: 1.7024188140817697199
Training accuracy: 74.20833333333333%
Test accuracy: 73.16666666666666%

Epoch 26
Training loss: 1.6527680905120517129
Training accuracy: 75.58333333333334%
Test accuracy: 71.66666666666666%

Epoch 27
Training loss: 1.5972874361736717142
Training accuracy: 76.29166666666666%
Test accuracy: 72.83333333333334%

Epoch 28
Training loss: 1.5564497143070425788
Training accuracy: 76.95833333333334%
Test accuracy: 77.0%

Epoch 29
Training loss: 1.5136278775985920208
Training accuracy: 78.0%
Test accuracy: 78.33333333333334%

Epoch 30
Training loss: 1.4587659710007716225
Training accuracy: 80.125%
Test accuracy: 78.33333333333334%

Epoch 31
Training loss: 1.4288245516540630599
Training accuracy: 79.62499999999999%
Test accuracy: 80.16666666666666%

Epoch 32
Training loss: 1.3919085680526149597
Training accuracy: 80.5%
Test accuracy: 79.33333333333333%

Epoch 33
Training loss: 1.3573732137636988203
Training accuracy: 81.37500000000001%
```

```
Test accuracy: 80.66666666666%


Epoch 34
Training loss: 1.3169464158206277386
Training accuracy: 82.33333333333334%
Test accuracy: 80.16666666666667%


Epoch 35
Training loss: 1.3033419912520667014
Training accuracy: 82.875%
Test accuracy: 83.5%


Epoch 36
Training loss: 1.2511141410983288981
Training accuracy: 83.20833333333333%
Test accuracy: 81.33333333333334%


Epoch 37
Training loss: 1.2312799379665704251
Training accuracy: 83.25%
Test accuracy: 83.0%


Epoch 38
Training loss: 1.1893164088582373971
Training accuracy: 84.20833333333334%
Test accuracy: 81.83333333333334%


Epoch 39
Training loss: 1.1833968778192231737
Training accuracy: 83.875%
Test accuracy: 85.0%


Epoch 40
Training loss: 1.1497179650784195005
Training accuracy: 84.375%
Test accuracy: 84.33333333333334%


Epoch 41
Training loss: 1.1216700277295455729
Training accuracy: 85.66666666666666%
Test accuracy: 83.16666666666666%


Epoch 42
Training loss: 1.0936943182485305148
Training accuracy: 85.16666666666666%
Test accuracy: 84.83333333333334%


Epoch 43
```

```
Training loss: 1.068147430165589693
Training accuracy: 86.75%
Test accuracy: 84.33333333333333%


Epoch 44
Training loss: 1.0534425528084259783
Training accuracy: 86.66666666666666%
Test accuracy: 85.66666666666667%


Epoch 45
Training loss: 1.036295909911595317
Training accuracy: 86.95833333333334%
Test accuracy: 84.66666666666666%


Epoch 46
Training loss: 1.0191970198548043363
Training accuracy: 87.29166666666666%
Test accuracy: 86.16666666666666%


Epoch 47
Training loss: 1.0244400105473987117
Training accuracy: 86.91666666666666%
Test accuracy: 89.50000000000001%


Epoch 48
Training loss: 0.98375971192371858655
Training accuracy: 87.70833333333334%
Test accuracy: 87.49999999999999%


Epoch 49
Training loss: 0.97154038585373679593
Training accuracy: 87.54166666666666%
Test accuracy: 89.33333333333334%


Epoch 50
Training loss: 0.9463017908414572534
Training accuracy: 88.625%
Test accuracy: 87.66666666666667%


Epoch 51
Training loss: 0.92899965423135429706
Training accuracy: 88.20833333333334%
Test accuracy: 89.0%


Epoch 52
Training loss: 0.91672584990988335954
Training accuracy: 89.29166666666666%
Test accuracy: 88.16666666666666%
```

112

```
Epoch 53
Training loss: 0.9005839156835550208
Training accuracy: 89.12499999999%
Test accuracy: 88.33333333333333%

Epoch 54
Training loss: 0.88746445295095612287
Training accuracy: 89.16666666666667%
Test accuracy: 89.66666666666666%

Epoch 55
Training loss: 0.87706860489942942773
Training accuracy: 89.625%
Test accuracy: 89.66666666666667%

Epoch 56
Training loss: 0.86393505047190934879
Training accuracy: 90.16666666666666%
Test accuracy: 89.66666666666666%

Epoch 57
Training loss: 0.8433923231931204901
Training accuracy: 90.08333333333333%
Test accuracy: 88.5%

Epoch 58
Training loss: 0.82969915666658642685
Training accuracy: 90.25%
Test accuracy: 89.83333333333333%

Epoch 59
Training loss: 0.81998067367123828866
Training accuracy: 90.25%
Test accuracy: 88.83333333333333%

Epoch 60
Training loss: 0.80508808089076278405
Training accuracy: 90.70833333333334%
Test accuracy: 88.33333333333334%

Epoch 61
Training loss: 0.805538576371802753
Training accuracy: 90.5%
Test accuracy: 89.66666666666666%

Epoch 62
Training loss: 0.7851636643906947198
```

```
Training accuracy: 90.625%
Test accuracy: 89.5%


Epoch 63
Training loss: 0.78104992318246836854
Training accuracy: 90.66666666666666%
Test accuracy: 90.33333333333333%


Epoch 64
Training loss: 0.78488597126647265166
Training accuracy: 90.70833333333333%
Test accuracy: 91.66666666666666%


Epoch 65
Training loss: 0.76586877184553158544
Training accuracy: 91.04166666666666%
Test accuracy: 91.0%


Epoch 66
Training loss: 0.7627415076328802056
Training accuracy: 90.70833333333334%
Test accuracy: 91.5%


Epoch 67
Training loss: 0.7394111681129755427
Training accuracy: 91.12500000000001%
Test accuracy: 89.0%


Epoch 68
Training loss: 0.7414956570038536551
Training accuracy: 90.99999999999999%
Test accuracy: 92.00000000000001%


Epoch 69
Training loss: 0.73016313568748646167
Training accuracy: 91.33333333333334%
Test accuracy: 91.66666666666666%


Epoch 70
Training loss: 0.710853685886049901
Training accuracy: 91.50000000000001%
Test accuracy: 89.66666666666666%


Epoch 71
Training loss: 0.7012255025330277849
Training accuracy: 92.0%
Test accuracy: 89.5%
```

```
Epoch 72
Training loss: 0.69835240969558038207
Training accuracy: 91.875%
Test accuracy: 90.83333333333333%

Epoch 73
Training loss: 0.7096119642473644569
Training accuracy: 91.45833333333333%
Test accuracy: 92.0%

Epoch 74
Training loss: 0.68662704130133062606
Training accuracy: 91.79166666666666%
Test accuracy: 90.66666666666666%

Epoch 75
Training loss: 0.6794407135588605874
Training accuracy: 92.125%
Test accuracy: 90.16666666666666%

Epoch 76
Training loss: 0.6791475667787470243
Training accuracy: 92.0%
Test accuracy: 90.50000000000001%

Epoch 77
Training loss: 0.6777194222773522526
Training accuracy: 91.79166666666666%
Test accuracy: 92.5%

Epoch 78
Training loss: 0.66271087886373034666
Training accuracy: 92.20833333333333%
Test accuracy: 91.66666666666666%

Epoch 79
Training loss: 0.67312596809601807346
Training accuracy: 92.12500000000001%
Test accuracy: 92.16666666666666%

Epoch 80
Training loss: 0.66089598602693623737
Training accuracy: 92.12499999999999%
Test accuracy: 92.66666666666667%

Epoch 81
Training loss: 0.63810989381418171766
Training accuracy: 92.41666666666666%
```

Test accuracy: 90.16666666666667%

Epoch 82
Training loss: 0.64148968075877913587
Training accuracy: 92.33333333333334%
Test accuracy: 90.83333333333333%

Epoch 83
Training loss: 0.6265699034142066583
Training accuracy: 93.125%
Test accuracy: 89.0%

Epoch 84
Training loss: 0.6392864012220427994
Training accuracy: 92.66666666666666%
Test accuracy: 92.83333333333334%

Epoch 85
Training loss: 0.6266006267387549588
Training accuracy: 92.58333333333333%
Test accuracy: 92.0%

Epoch 86
Training loss: 0.62211655318230423104
Training accuracy: 92.95833333333333%
Test accuracy: 91.16666666666666%

Epoch 87
Training loss: 0.60962395522159163467
Training accuracy: 92.70833333333334%
Test accuracy: 92.0%

Epoch 88
Training loss: 0.61555908748795964784
Training accuracy: 92.66666666666666%
Test accuracy: 91.83333333333333%

Epoch 89
Training loss: 0.6206448106112517699
Training accuracy: 92.70833333333334%
Test accuracy: 94.0%

Epoch 90
Training loss: 0.62019830209994132966
Training accuracy: 92.75%
Test accuracy: 93.83333333333333%

Epoch 91

Training loss: 0.6065650652854656403
Training accuracy: 93.12499999999999%
Test accuracy: 93.0%

Epoch 92
Training loss: 0.58744448276939657263
Training accuracy: 93.625%
Test accuracy: 90.66666666666666%

Epoch 93
Training loss: 0.600956259946204607
Training accuracy: 93.5%
Test accuracy: 93.16666666666666%

Epoch 94
Training loss: 0.58942465346894128
Training accuracy: 93.20833333333334%
Test accuracy: 91.83333333333334%

Epoch 95
Training loss: 0.5875020532472406944
Training accuracy: 93.5%
Test accuracy: 93.00000000000001%

Epoch 96
Training loss: 0.58659921324732605024
Training accuracy: 93.37500000000001%
Test accuracy: 92.66666666666666%

Epoch 97
Training loss: 0.59270500196454628056
Training accuracy: 93.33333333333334%
Test accuracy: 94.33333333333333%

Epoch 98
Training loss: 0.57404966506087506137
Training accuracy: 93.5%
Test accuracy: 93.16666666666666%

Epoch 99
Training loss: 0.5603962451653039363
Training accuracy: 93.95833333333333%
Test accuracy: 92.0%

Epoch 100
Training loss: 0.58139950112372748667
Training accuracy: 93.66666666666666%
Test accuracy: 94.33333333333334%

```
Epoch 101
Training loss: 0.5504932939981991767
Training accuracy: 94.33333333333334%
Test accuracy: 90.66666666666667%


Epoch 102
Training loss: 0.56695328661168041204
Training accuracy: 93.875%
Test accuracy: 93.33333333333333%


Epoch 103
Training loss: 0.57478326237261645915
Training accuracy: 93.75%
Test accuracy: 94.5%


Epoch 104
Training loss: 0.5490889575171057555
Training accuracy: 94.41666666666666%
Test accuracy: 93.33333333333333%


Epoch 105
Training loss: 0.5546534616126897231
Training accuracy: 93.91666666666666%
Test accuracy: 93.16666666666667%


Epoch 106
Training loss: 0.5520286629063114423
Training accuracy: 94.04166666666666%
Test accuracy: 94.83333333333334%


Epoch 107
Training loss: 0.5460721922486810489
Training accuracy: 94.50000000000001%
Test accuracy: 93.0%


Epoch 108
Training loss: 0.55054706705771070394
Training accuracy: 94.25%
Test accuracy: 94.33333333333334%


Epoch 109
Training loss: 0.5269611817405436965
Training accuracy: 95.04166666666666%
Test accuracy: 92.16666666666666%


Epoch 110
Training loss: 0.5317618483310746433
```

```
Training accuracy: 94.83333333333334%
Test accuracy: 93.33333333333334%


Epoch 111
Training loss: 0.5335269180206591611
Training accuracy: 94.58333333333333%
Test accuracy: 93.83333333333333%


Epoch 112
Training loss: 0.53408761113067549655
Training accuracy: 94.625%
Test accuracy: 93.16666666666667%


Epoch 113
Training loss: 0.5348281387170761161
Training accuracy: 94.33333333333334%
Test accuracy: 93.83333333333333%


Epoch 114
Training loss: 0.5250364345596446813
Training accuracy: 94.41666666666667%
Test accuracy: 94.16666666666666%


Epoch 115
Training loss: 0.5234101195985151651
Training accuracy: 94.70833333333333%
Test accuracy: 94.66666666666667%


Epoch 116
Training loss: 0.5217382568419288064
Training accuracy: 94.87499999999999%
Test accuracy: 94.33333333333333%


Epoch 117
Training loss: 0.51962051617765403315
Training accuracy: 94.70833333333333%
Test accuracy: 93.66666666666667%


Epoch 118
Training loss: 0.52130206091476524255
Training accuracy: 94.95833333333334%
Test accuracy: 93.5%


Epoch 119
Training loss: 0.51533498021104942084
Training accuracy: 94.58333333333334%
Test accuracy: 95.5%
```

```
Epoch 120
Training loss: 0.51609079989613041383
Training accuracy: 94.625%
Test accuracy: 94.83333333333333%

Epoch 121
Training loss: 0.50384150804798183155
Training accuracy: 94.87500000000001%
Test accuracy: 94.16666666666666%

Epoch 122
Training loss: 0.50643285932262460805
Training accuracy: 95.20833333333334%
Test accuracy: 93.66666666666666%

Epoch 123
Training loss: 0.51582140268006260186
Training accuracy: 94.66666666666666%
Test accuracy: 95.5%

Epoch 124
Training loss: 0.5091117830962293215
Training accuracy: 95.20833333333334%
Test accuracy: 94.16666666666667%

Epoch 125
Training loss: 0.50716910626601717537
Training accuracy: 94.83333333333334%
Test accuracy: 94.66666666666667%

Epoch 126
Training loss: 0.5071110830179545531
Training accuracy: 95.04166666666667%
Test accuracy: 94.16666666666666%

Epoch 127
Training loss: 0.5039549932007919334
Training accuracy: 94.91666666666667%
Test accuracy: 94.66666666666666%

Epoch 128
Training loss: 0.50361859516030878464
Training accuracy: 94.91666666666667%
Test accuracy: 95.83333333333333%

Epoch 129
Training loss: 0.49259359032396152932
Training accuracy: 95.29166666666666%
```

Test accuracy: 94.66666666666666%

Epoch 130
Training loss: 0.4907284238405285068
Training accuracy: 95.45833333333334%
Test accuracy: 94.83333333333333%

Epoch 131
Training loss: 0.483104574042329242468
Training accuracy: 95.66666666666666%
Test accuracy: 93.0%

Epoch 132
Training loss: 0.48791446557895463304
Training accuracy: 95.41666666666666%
Test accuracy: 95.0%

Epoch 133
Training loss: 0.49101799830916978995
Training accuracy: 95.5%
Test accuracy: 94.16666666666666%

Epoch 134
Training loss: 0.49048526167185543665
Training accuracy: 95.45833333333334%
Test accuracy: 94.33333333333333%

Epoch 135
Training loss: 0.47997909318411970857
Training accuracy: 95.54166666666667%
Test accuracy: 95.16666666666666%

Epoch 136
Training loss: 0.47579368600119046465
Training accuracy: 95.45833333333333%
Test accuracy: 95.66666666666667%

Epoch 137
Training loss: 0.47108000085166319392
Training accuracy: 95.95833333333334%
Test accuracy: 94.16666666666666%

Epoch 138
Training loss: 0.47646555171408798445
Training accuracy: 95.66666666666667%
Test accuracy: 94.49999999999999%

Epoch 139

```
Training loss: 0.4822893022515563341
Training accuracy: 95.45833333333333%
Test accuracy: 95.5%


Epoch 140
Training loss: 0.4762280371219583381
Training accuracy: 95.66666666666667%
Test accuracy: 94.66666666666667%


Epoch 141
Training loss: 0.47495190126325154963
Training accuracy: 95.45833333333334%
Test accuracy: 96.0%


Epoch 142
Training loss: 0.4758138936910502766
Training accuracy: 95.70833333333333%
Test accuracy: 95.83333333333333%


Epoch 143
Training loss: 0.47046317324474728276
Training accuracy: 95.5%
Test accuracy: 94.83333333333333%


Epoch 144
Training loss: 0.46512828504929561052
Training accuracy: 95.66666666666667%
Test accuracy: 94.99999999999999%


Epoch 145
Training loss: 0.4580644160030732293
Training accuracy: 96.125%
Test accuracy: 94.16666666666667%


Epoch 146
Training loss: 0.47095079387958331557
Training accuracy: 95.66666666666667%
Test accuracy: 95.83333333333334%


Epoch 147
Training loss: 0.4687127681252226289
Training accuracy: 95.875%
Test accuracy: 96.0%


Epoch 148
Training loss: 0.45951635549687651399
Training accuracy: 96.16666666666667%
Test accuracy: 95.16666666666666%
```

```
Epoch 149
Training loss: 0.46718194888642835738
Training accuracy: 95.875%
Test accuracy: 95.5%

Epoch 150
Training loss: 0.464452477329352695
Training accuracy: 95.75%
Test accuracy: 95.83333333333334%

Epoch 151
Training loss: 0.4610188101660739184
Training accuracy: 96.0%
Test accuracy: 95.66666666666667%

Epoch 152
Training loss: 0.45564314486874410524
Training accuracy: 96.0%
Test accuracy: 95.16666666666666%

Epoch 153
Training loss: 0.4601041060101812616
Training accuracy: 95.95833333333333%
Test accuracy: 95.83333333333334%

Epoch 154
Training loss: 0.46589302702050002682
Training accuracy: 95.95833333333334%
Test accuracy: 96.16666666666666%

Epoch 155
Training loss: 0.45420338458819624095
Training accuracy: 95.83333333333333%
Test accuracy: 95.66666666666666%

Epoch 156
Training loss: 0.44855392484374034855
Training accuracy: 95.83333333333334%
Test accuracy: 95.33333333333334%

Epoch 157
Training loss: 0.46783527992994196118
Training accuracy: 95.83333333333334%
Test accuracy: 96.66666666666666%

Epoch 158
Training loss: 0.44617249531030554245
```

Training accuracy: 96.08333333333334%
Test accuracy: 95.50000000000001%

Epoch 159
Training loss: 0.44695191723628128672
Training accuracy: 96.16666666666667%
Test accuracy: 95.33333333333334%

Epoch 160
Training loss: 0.45036137108818487636
Training accuracy: 96.125%
Test accuracy: 96.0%

Epoch 161
Training loss: 0.4580546348782063869
Training accuracy: 96.04166666666667%
Test accuracy: 96.83333333333334%

Epoch 162
Training loss: 0.45555172418512725527
Training accuracy: 96.16666666666666%
Test accuracy: 96.66666666666667%

Epoch 163
Training loss: 0.44272595571754029116
Training accuracy: 96.41666666666666%
Test accuracy: 94.83333333333333%

Epoch 164
Training loss: 0.4442842545900404404
Training accuracy: 96.20833333333334%
Test accuracy: 95.66666666666666%

Epoch 165
Training loss: 0.43802554099214790318
Training accuracy: 96.37500000000001%
Test accuracy: 95.16666666666667%

Epoch 166
Training loss: 0.44936497505826862903
Training accuracy: 96.5%
Test accuracy: 95.5%

Epoch 167
Training loss: 0.44641058052415321457
Training accuracy: 96.25%
Test accuracy: 95.83333333333333%

```
Epoch 168
Training loss: 0.43520848574175497914
Training accuracy: 96.62499999999999%
Test accuracy: 94.66666666666666%

Epoch 169
Training loss: 0.43161511494384770276
Training accuracy: 96.5%
Test accuracy: 95.33333333333334%

Epoch 170
Training loss: 0.43887963752914026493
Training accuracy: 96.66666666666667%
Test accuracy: 95.5%

Epoch 171
Training loss: 0.43442849268067308053
Training accuracy: 96.5%
Test accuracy: 95.66666666666666%

Epoch 172
Training loss: 0.43207938416741564582
Training accuracy: 96.45833333333333%
Test accuracy: 95.5%

Epoch 173
Training loss: 0.43798534118285132273
Training accuracy: 96.375%
Test accuracy: 95.49999999999999%

Epoch 174
Training loss: 0.43959339467954436394
Training accuracy: 96.375%
Test accuracy: 96.49999999999999%

Epoch 175
Training loss: 0.43344800167307012494
Training accuracy: 96.66666666666667%
Test accuracy: 96.0%

Epoch 176
Training loss: 0.41954139758632181642
Training accuracy: 96.41666666666667%
Test accuracy: 95.66666666666666%

Epoch 177
Training loss: 0.42110946009268479306
Training accuracy: 97.08333333333333%
```

Test accuracy: 93.83333333333334%

Epoch 178
Training loss: 0.42806099112787142705
Training accuracy: 96.66666666666666%
Test accuracy: 95.16666666666666%

Epoch 179
Training loss: 0.41451573947503286447
Training accuracy: 97.08333333333333%
Test accuracy: 94.0%

Epoch 180
Training loss: 0.42837178988459138066
Training accuracy: 96.66666666666666%
Test accuracy: 96.0%

Epoch 181
Training loss: 0.41777400980930845076
Training accuracy: 96.625%
Test accuracy: 95.5%

Epoch 182
Training loss: 0.4300578992498257296
Training accuracy: 96.375%
Test accuracy: 96.83333333333334%

Epoch 183
Training loss: 0.42260262215924090161
Training accuracy: 96.58333333333333%
Test accuracy: 95.83333333333334%

Epoch 184
Training loss: 0.41978967039299038634
Training accuracy: 97.08333333333334%
Test accuracy: 95.66666666666667%

Epoch 185
Training loss: 0.4124393061934362509
Training accuracy: 97.20833333333334%
Test accuracy: 94.5%

Epoch 186
Training loss: 0.417288354227210665
Training accuracy: 97.0%
Test accuracy: 95.0%

Epoch 187

Training loss: 0.4206368764987213967
Training accuracy: 96.91666666666666%
Test accuracy: 94.5%

Epoch 188
Training loss: 0.42209755188837812825
Training accuracy: 96.83333333333333%
Test accuracy: 95.66666666666667%

Epoch 189
Training loss: 0.4145557828829357251
Training accuracy: 96.50000000000001%
Test accuracy: 96.5%

Epoch 190
Training loss: 0.420670286377289013
Training accuracy: 96.37500000000001%
Test accuracy: 96.5%

Epoch 191
Training loss: 0.43247128160499316018
Training accuracy: 96.62500000000001%
Test accuracy: 98.16666666666666%

Epoch 192
Training loss: 0.41842589671255519542
Training accuracy: 96.83333333333334%
Test accuracy: 96.33333333333334%

Epoch 193
Training loss: 0.42039387861725794484
Training accuracy: 96.87499999999999%
Test accuracy: 95.66666666666666%

Epoch 194
Training loss: 0.41658119987596791442
Training accuracy: 96.79166666666666%
Test accuracy: 97.16666666666667%

Epoch 195
Training loss: 0.4084725959622829171
Training accuracy: 97.16666666666666%
Test accuracy: 96.00000000000001%

Epoch 196
Training loss: 0.41613037961262712774
Training accuracy: 97.0%
Test accuracy: 96.83333333333334%

```
Epoch 197
Training loss: 0.41428437566116258127
Training accuracy: 96.875%
Test accuracy: 96.33333333333334%

Epoch 198
Training loss: 0.4027377666574514209
Training accuracy: 96.91666666666667%
Test accuracy: 96.5%

Epoch 199
Training loss: 0.41719520111587007888
Training accuracy: 97.0%
Test accuracy: 97.5%

Epoch 200
Training loss: 0.40886217833198734307
Training accuracy: 97.16666666666666%
Test accuracy: 96.16666666666666%
```
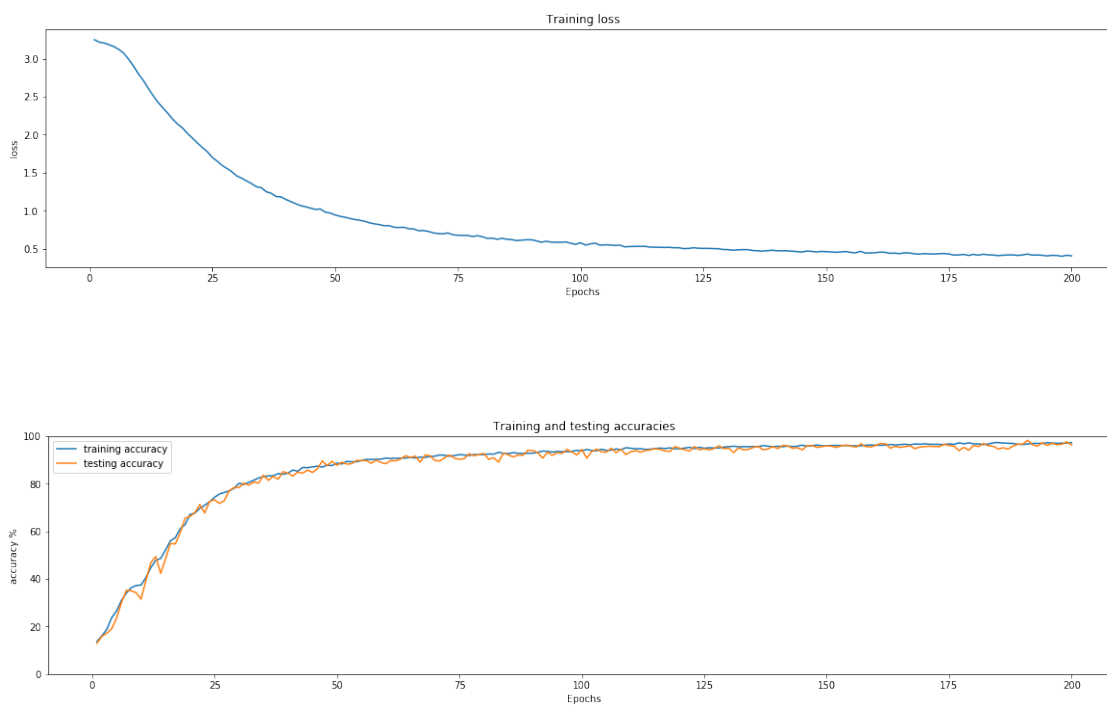

We see that there has been a little drop in both training and testing accuracy. But we can't be sure it's overfitting as the train and test accuracies are the same.

2.2.9 So here I take the liberty to test my model using Cross Validation, on the entire MNIST data set.

```
[3]: import mnist_loader
```

```
[5]: training_data_MNIST, validation_data_MNIST, __ = mnist_loader.
     ↪load_data_wrapper()
```

```
[6]: net_mnist = network.Linear_Network_CrossValidation([784,35,25,10])
```

```
[7]: """No Regularization"""
     net_mnist.SGD(training_data_MNIST, 200, 10, 0.5, lmbda=0.0)
```

```
Epoch 1
Training loss: 3.2549137677927903634
Training accuracy: 12.0%
Test accuracy: 12.370000000000001%

Epoch 2
Training loss: 3.2303653631804196348
Training accuracy: 13.15%
Test accuracy: 13.62%

Epoch 3
Training loss: 3.21403511856918898
Training accuracy: 15.615%
Test accuracy: 15.780000000000001%

Epoch 4
Training loss: 3.1937961060341875327
Training accuracy: 26.5625%
Test accuracy: 27.370000000000005%

Epoch 5
Training loss: 3.1650126603191624968
Training accuracy: 29.375%
Test accuracy: 28.360000000000003%

Epoch 6
Training loss: 3.12670138108243522
Training accuracy: 39.94%
Test accuracy: 39.910000000000004%

Epoch 7
Training loss: 3.0709740305697261484
Training accuracy: 46.9375%
Test accuracy: 46.42999999999999%
```

```
Epoch 8
Training loss: 2.9931596486721955392
Training accuracy: 50.13999999999999%
Test accuracy: 49.290000000000006%

Epoch 9
Training loss: 2.891747902007158164
Training accuracy: 52.81750000000001%
Test accuracy: 52.08%

Epoch 10
Training loss: 2.7685683583480938375
Training accuracy: 54.3925%
Test accuracy: 54.25%

Epoch 11
Training loss: 2.630258383457266856
Training accuracy: 56.735%
Test accuracy: 56.48%

Epoch 12
Training loss: 2.4923512324388638832
Training accuracy: 58.602500000000006%
Test accuracy: 59.129999999999995%

Epoch 13
Training loss: 2.3600355157016473136
Training accuracy: 60.625%
Test accuracy: 60.529999999999994%

Epoch 14
Training loss: 2.240377761388976314
Training accuracy: 62.4775%
Test accuracy: 61.80999999999999%

Epoch 15
Training loss: 2.1316149126602059413
Training accuracy: 64.3875%
Test accuracy: 64.59%

Epoch 16
Training loss: 2.0391558080466419735
Training accuracy: 65.94500000000001%
Test accuracy: 65.78999999999999%

Epoch 17
Training loss: 1.9522488193406969867
```

Training accuracy: 67.2625%
Test accuracy: 67.6%

Epoch 18
Training loss: 1.8733498855591442873
Training accuracy: 68.945%
Test accuracy: 68.66999999999999%

Epoch 19
Training loss: 1.805443993400869609
Training accuracy: 70.2725%
Test accuracy: 70.68%

Epoch 20
Training loss: 1.7426149503634943496
Training accuracy: 71.7375%
Test accuracy: 71.74%

Epoch 21
Training loss: 1.6820376008064805446
Training accuracy: 73.07%
Test accuracy: 73.28%

Epoch 22
Training loss: 1.6244922816915998697
Training accuracy: 74.365%
Test accuracy: 74.14999999999999%

Epoch 23
Training loss: 1.571081227523225908
Training accuracy: 75.81750000000001%
Test accuracy: 74.94000000000001%

Epoch 24
Training loss: 1.5243752162732040671
Training accuracy: 76.64500000000001%
Test accuracy: 76.2%

Epoch 25
Training loss: 1.480156117464390088
Training accuracy: 77.7975%
Test accuracy: 77.53%

Epoch 26
Training loss: 1.4385026077754121182
Training accuracy: 78.39750000000001%
Test accuracy: 78.53%

```
Epoch 27
Training loss: 1.3957791996356729744
Training accuracy: 79.42750000000001%
Test accuracy: 78.95%

Epoch 28
Training loss: 1.3602404962390186127
Training accuracy: 80.1%
Test accuracy: 79.62%

Epoch 29
Training loss: 1.3283987004161619605
Training accuracy: 80.495%
Test accuracy: 81.29%

Epoch 30
Training loss: 1.2930813677184594752
Training accuracy: 81.18749999999999%
Test accuracy: 80.9%

Epoch 31
Training loss: 1.261496540479844636
Training accuracy: 81.775%
Test accuracy: 81.5%

Epoch 32
Training loss: 1.2327775380045271845
Training accuracy: 82.3825%
Test accuracy: 82.0%

Epoch 33
Training loss: 1.2060662840206091873
Training accuracy: 82.79999999999998%
Test accuracy: 82.53999999999999%

Epoch 34
Training loss: 1.1844482896911789679
Training accuracy: 83.2775%
Test accuracy: 83.1%

Epoch 35
Training loss: 1.161595883817496025
Training accuracy: 83.47749999999999%
Test accuracy: 83.81%

Epoch 36
Training loss: 1.1392114834219438607
Training accuracy: 83.82249999999999%
```

```
Test accuracy: 84.04999999999998%


Epoch 37
Training loss: 1.1166716869641731766
Training accuracy: 84.32%
Test accuracy: 84.28%


Epoch 38
Training loss: 1.0979606377663168706
Training accuracy: 84.70750000000001%
Test accuracy: 84.82000000000001%


Epoch 39
Training loss: 1.0785392624043462338
Training accuracy: 85.02499999999999%
Test accuracy: 84.91999999999999%


Epoch 40
Training loss: 1.0612535013347289558
Training accuracy: 85.255%
Test accuracy: 85.05000000000001%


Epoch 41
Training loss: 1.0426605148625425263
Training accuracy: 85.54999999999998%
Test accuracy: 85.51%


Epoch 42
Training loss: 1.0284946224904879825
Training accuracy: 85.77000000000001%
Test accuracy: 85.67%


Epoch 43
Training loss: 1.0157374918889092406
Training accuracy: 85.975%
Test accuracy: 86.19999999999999%


Epoch 44
Training loss: 0.9951581636778840124
Training accuracy: 86.2475%
Test accuracy: 86.16%


Epoch 45
Training loss: 0.9801303680028433522
Training accuracy: 86.5325%
Test accuracy: 86.17999999999999%


Epoch 46
```

```
Training loss: 0.966785838606484092
Training accuracy: 86.60749999999999%
Test accuracy: 86.56%


Epoch 47
Training loss: 0.95068921172750765465
Training accuracy: 86.8925%
Test accuracy: 86.65%


Epoch 48
Training loss: 0.93652689937533321223
Training accuracy: 87.05%
Test accuracy: 86.9%


Epoch 49
Training loss: 0.9254670877761319656
Training accuracy: 87.3125%
Test accuracy: 86.80000000000001%


Epoch 50
Training loss: 0.9140372425036052727
Training accuracy: 87.3675%
Test accuracy: 87.57%


Epoch 51
Training loss: 0.904290560709954509
Training accuracy: 87.55749999999999%
Test accuracy: 87.83%


Epoch 52
Training loss: 0.8867661538703144436
Training accuracy: 87.86000000000001%
Test accuracy: 87.26%


Epoch 53
Training loss: 0.875880878884457069
Training accuracy: 87.9525%
Test accuracy: 87.72%


Epoch 54
Training loss: 0.8615725011764432524
Training accuracy: 88.08749999999999%
Test accuracy: 88.01%


Epoch 55
Training loss: 0.85341504151270984396
Training accuracy: 88.25%
Test accuracy: 88.10999999999999%
```

```
Epoch 56
Training loss: 0.8417671681075545337
Training accuracy: 88.36%
Test accuracy: 88.35%

Epoch 57
Training loss: 0.8289953714605576363
Training accuracy: 88.51750000000001%
Test accuracy: 88.32%

Epoch 58
Training loss: 0.81829261046099114
Training accuracy: 88.695%
Test accuracy: 88.36%

Epoch 59
Training loss: 0.80942514720686124503
Training accuracy: 88.75250000000001%
Test accuracy: 88.91000000000001%

Epoch 60
Training loss: 0.7956351261920827223
Training accuracy: 88.9975%
Test accuracy: 88.45%

Epoch 61
Training loss: 0.78813499901126612214
Training accuracy: 89.06249999999999%
Test accuracy: 89.09%

Epoch 62
Training loss: 0.7788519710412564244
Training accuracy: 89.19500000000001%
Test accuracy: 89.09%

Epoch 63
Training loss: 0.7689662174097436863
Training accuracy: 89.34%
Test accuracy: 89.21000000000001%

Epoch 64
Training loss: 0.7575235873248800461
Training accuracy: 89.4525%
Test accuracy: 89.13%

Epoch 65
Training loss: 0.75145383160936609853
```

Training accuracy: 89.49%
Test accuracy: 89.53%

Epoch 66
Training loss: 0.7419267448887730292
Training accuracy: 89.63%
Test accuracy: 89.52000000000001%

Epoch 67
Training loss: 0.7322442811956738567
Training accuracy: 89.76750000000001%
Test accuracy: 89.57000000000002%

Epoch 68
Training loss: 0.72794642162758390637
Training accuracy: 89.745%
Test accuracy: 90.18%

Epoch 69
Training loss: 0.7177915802939758386
Training accuracy: 89.95000000000002%
Test accuracy: 89.97999999999999%

Epoch 70
Training loss: 0.70870041609145359397
Training accuracy: 90.0925%
Test accuracy: 89.88000000000001%

Epoch 71
Training loss: 0.6999011838930103235
Training accuracy: 90.09750000000001%
Test accuracy: 90.38000000000001%

Epoch 72
Training loss: 0.69583124688280737613
Training accuracy: 90.2325%
Test accuracy: 90.41%

Epoch 73
Training loss: 0.68438552760432424946
Training accuracy: 90.425%
Test accuracy: 89.83%

Epoch 74
Training loss: 0.67960758771305266126
Training accuracy: 90.35249999999999%
Test accuracy: 90.53%

```
Epoch 75
Training loss: 0.66858719771266861184
Training accuracy: 90.59%
Test accuracy: 90.03%

Epoch 76
Training loss: 0.66460862669553705126
Training accuracy: 90.63250000000001%
Test accuracy: 90.3%

Epoch 77
Training loss: 0.65845408640708696067
Training accuracy: 90.72999999999999%
Test accuracy: 90.45%

Epoch 78
Training loss: 0.654727499811937681
Training accuracy: 90.675%
Test accuracy: 90.84%

Epoch 79
Training loss: 0.64757340471736984797
Training accuracy: 90.75000000000001%
Test accuracy: 90.92000000000002%

Epoch 80
Training loss: 0.63996706109492308386
Training accuracy: 90.9375%
Test accuracy: 90.56%

Epoch 81
Training loss: 0.6327073615411694417
Training accuracy: 91.0225%
Test accuracy: 90.63000000000001%

Epoch 82
Training loss: 0.62732354312631060137
Training accuracy: 91.00500000000001%
Test accuracy: 90.86000000000001%

Epoch 83
Training loss: 0.6256448658602083171
Training accuracy: 91.00750000000001%
Test accuracy: 91.3%

Epoch 84
Training loss: 0.62019132893406158497
Training accuracy: 91.14%
```

Test accuracy: 91.3%

Epoch 85
Training loss: 0.61244786160191979116
Training accuracy: 91.22749999999999%
Test accuracy: 91.36%

Epoch 86
Training loss: 0.60711826459307285323
Training accuracy: 91.3025%
Test accuracy: 91.20000000000002%

Epoch 87
Training loss: 0.600184055851650193
Training accuracy: 91.43%
Test accuracy: 91.08%

Epoch 88
Training loss: 0.5990108866896737519
Training accuracy: 91.27000000000001%
Test accuracy: 91.87000000000002%

Epoch 89
Training loss: 0.5900999045300778182
Training accuracy: 91.5375%
Test accuracy: 91.22%

Epoch 90
Training loss: 0.5852773988774215987
Training accuracy: 91.61250000000001%
Test accuracy: 91.27000000000001%

Epoch 91
Training loss: 0.58418047490896880857
Training accuracy: 91.58%
Test accuracy: 91.69000000000001%

Epoch 92
Training loss: 0.579592058875964512
Training accuracy: 91.66%
Test accuracy: 91.81000000000002%

Epoch 93
Training loss: 0.57360444315982017275
Training accuracy: 91.765%
Test accuracy: 91.62000000000002%

Epoch 94

```
Training loss: 0.56961162705048020506
Training accuracy: 91.78%
Test accuracy: 91.87%


Epoch 95
Training loss: 0.56548790206478602877
Training accuracy: 91.80749999999999%
Test accuracy: 92.07%


Epoch 96
Training loss: 0.5634193632574755771
Training accuracy: 91.86%
Test accuracy: 92.20000000000002%


Epoch 97
Training loss: 0.55950622641808686315
Training accuracy: 91.9875%
Test accuracy: 91.98%


Epoch 98
Training loss: 0.55115533455225003954
Training accuracy: 92.145%
Test accuracy: 91.75%


Epoch 99
Training loss: 0.5471877368783312996
Training accuracy: 92.155%
Test accuracy: 91.88000000000001%


Epoch 100
Training loss: 0.5448680766244118351
Training accuracy: 92.13250000000001%
Test accuracy: 92.16%


Epoch 101
Training loss: 0.54243870365330319565
Training accuracy: 92.06750000000001%
Test accuracy: 92.45000000000002%


Epoch 102
Training loss: 0.5370593625097435376
Training accuracy: 92.2625%
Test accuracy: 92.03999999999999%


Epoch 103
Training loss: 0.53105385026293428397
Training accuracy: 92.3125%
Test accuracy: 92.09%
```

```
Epoch 104
Training loss: 0.5297263652094429525
Training accuracy: 92.31500000000001%
Test accuracy: 92.32%

Epoch 105
Training loss: 0.5263967963182853283
Training accuracy: 92.43499999999999%
Test accuracy: 91.94000000000001%

Epoch 106
Training loss: 0.5274688864474547382
Training accuracy: 92.365%
Test accuracy: 92.72999999999999%

Epoch 107
Training loss: 0.5235953992051184295
Training accuracy: 92.405%
Test accuracy: 92.7%

Epoch 108
Training loss: 0.5175529612526807184
Training accuracy: 92.52000000000001%
Test accuracy: 92.65%

Epoch 109
Training loss: 0.5158665966011020421
Training accuracy: 92.50750000000002%
Test accuracy: 92.85%

Epoch 110
Training loss: 0.51481355013236593
Training accuracy: 92.57750000000001%
Test accuracy: 92.83%

Epoch 111
Training loss: 0.50925440215825335324
Training accuracy: 92.67750000000001%
Test accuracy: 92.72%

Epoch 112
Training loss: 0.50499963713168657606
Training accuracy: 92.695%
Test accuracy: 92.92%

Epoch 113
Training loss: 0.50213016499462745574
```

```
Training accuracy: 92.765%
Test accuracy: 92.73000000000002%


Epoch 114
Training loss: 0.49810210946167742847
Training accuracy: 92.7925%
Test accuracy: 92.82000000000001%


Epoch 115
Training loss: 0.49790211655841601942
Training accuracy: 92.85249999999999%
Test accuracy: 92.99000000000001%


Epoch 116
Training loss: 0.48859874138618185173
Training accuracy: 92.99500000000002%
Test accuracy: 92.44999999999999%


Epoch 117
Training loss: 0.48978641375085480393
Training accuracy: 92.95%
Test accuracy: 92.84%


Epoch 118
Training loss: 0.48704454981115405631
Training accuracy: 92.89%
Test accuracy: 93.16%


Epoch 119
Training loss: 0.484836892365944668
Training accuracy: 93.0575%
Test accuracy: 92.69000000000001%


Epoch 120
Training loss: 0.48621767385332914264
Training accuracy: 92.9225%
Test accuracy: 93.35%


Epoch 121
Training loss: 0.47850526845609744902
Training accuracy: 93.065%
Test accuracy: 93.0%


Epoch 122
Training loss: 0.47659508450624067256
Training accuracy: 93.07999999999998%
Test accuracy: 93.06%
```

```
Epoch 123
Training loss: 0.47475122446445212868
Training accuracy: 93.16%
Test accuracy: 93.02000000000001%

Epoch 124
Training loss: 0.46808274880120322436
Training accuracy: 93.24000000000001%
Test accuracy: 92.87000000000002%

Epoch 125
Training loss: 0.464224337508312318
Training accuracy: 93.32%
Test accuracy: 92.70000000000002%

Epoch 126
Training loss: 0.46988140038887290812
Training accuracy: 93.13500000000002%
Test accuracy: 93.56%

Epoch 127
Training loss: 0.46361561131306186644
Training accuracy: 93.2775%
Test accuracy: 93.21%

Epoch 128
Training loss: 0.45892261712413412377
Training accuracy: 93.33250000000001%
Test accuracy: 93.01%

Epoch 129
Training loss: 0.45826159034971505017
Training accuracy: 93.37750000000001%
Test accuracy: 93.01000000000002%

Epoch 130
Training loss: 0.45846006918214342476
Training accuracy: 93.41%
Test accuracy: 93.14999999999999%

Epoch 131
Training loss: 0.45815364481465175616
Training accuracy: 93.3425%
Test accuracy: 93.53999999999999%

Epoch 132
Training loss: 0.45448285490169881536
Training accuracy: 93.3575%
```

Test accuracy: 93.54%

Epoch 133
Training loss: 0.45174609817699773196
Training accuracy: 93.4075%
Test accuracy: 93.52000000000001%

Epoch 134
Training loss: 0.45057287792781927746
Training accuracy: 93.44249999999998%
Test accuracy: 93.62000000000002%

Epoch 135
Training loss: 0.4439435776773952469
Training accuracy: 93.5325%
Test accuracy: 93.25999999999999%

Epoch 136
Training loss: 0.44411678811575270052
Training accuracy: 93.525%
Test accuracy: 93.49%

Epoch 137
Training loss: 0.44324457879588654674
Training accuracy: 93.54249999999999%
Test accuracy: 93.49000000000001%

Epoch 138
Training loss: 0.44324319123878696525
Training accuracy: 93.5275%
Test accuracy: 93.71000000000001%

Epoch 139
Training loss: 0.43987613901745248817
Training accuracy: 93.5825%
Test accuracy: 93.78999999999999%

Epoch 140
Training loss: 0.436664713581823304
Training accuracy: 93.625%
Test accuracy: 93.77%

Epoch 141
Training loss: 0.4303227235650945576
Training accuracy: 93.68249999999999%
Test accuracy: 93.64%

Epoch 142

Training loss: 0.432802817843634215
Training accuracy: 93.71499999999999%
Test accuracy: 93.71000000000001%

Epoch 143
Training loss: 0.4317384451137185207
Training accuracy: 93.665%
Test accuracy: 93.91%

Epoch 144
Training loss: 0.43032012582326622176
Training accuracy: 93.815%
Test accuracy: 93.42%

Epoch 145
Training loss: 0.42626187542228994367
Training accuracy: 93.735%
Test accuracy: 93.88999999999999%

Epoch 146
Training loss: 0.42455500294497585572
Training accuracy: 93.80250000000001%
Test accuracy: 93.72%

Epoch 147
Training loss: 0.42492543858901883044
Training accuracy: 93.8%
Test accuracy: 93.98%

Epoch 148
Training loss: 0.41534818677721852632
Training accuracy: 93.99749999999999%
Test accuracy: 93.32000000000001%

Epoch 149
Training loss: 0.42087038405811670627
Training accuracy: 93.88749999999999%
Test accuracy: 94.00000000000001%

Epoch 150
Training loss: 0.41203653624302735985
Training accuracy: 94.04499999999999%
Test accuracy: 93.38%

Epoch 151
Training loss: 0.41643164182219858093
Training accuracy: 93.91499999999999%
Test accuracy: 94.11%

```
Epoch 152
Training loss: 0.41449397009616625697
Training accuracy: 93.9975%
Test accuracy: 93.89999999999999%

Epoch 153
Training loss: 0.41173198523883212108
Training accuracy: 94.0275%
Test accuracy: 94.0%

Epoch 154
Training loss: 0.41026183189993263158
Training accuracy: 94.11999999999999%
Test accuracy: 93.89%

Epoch 155
Training loss: 0.4054492057734267452
Training accuracy: 94.10499999999999%
Test accuracy: 93.79999999999998%

Epoch 156
Training loss: 0.40589666704218758153
Training accuracy: 94.1725%
Test accuracy: 93.77%

Epoch 157
Training loss: 0.404836185027750019
Training accuracy: 94.11500000000001%
Test accuracy: 94.22%

Epoch 158
Training loss: 0.402943400746825336
Training accuracy: 94.1525%
Test accuracy: 94.10999999999999%

Epoch 159
Training loss: 0.40145288672166645096
Training accuracy: 94.23999999999998%
Test accuracy: 94.0%

Epoch 160
Training loss: 0.40315426738565076397
Training accuracy: 94.16%
Test accuracy: 94.38%

Epoch 161
Training loss: 0.39737144454527497956
```

```
Training accuracy: 94.33500000000001%
Test accuracy: 94.03%


Epoch 162
Training loss: 0.39719513921885937996
Training accuracy: 94.2425%
Test accuracy: 94.41%


Epoch 163
Training loss: 0.3935560846615894485
Training accuracy: 94.3075%
Test accuracy: 94.1%


Epoch 164
Training loss: 0.39344206609875944522
Training accuracy: 94.3525%
Test accuracy: 94.04%


Epoch 165
Training loss: 0.39207711194504963484
Training accuracy: 94.31250000000001%
Test accuracy: 94.35%


Epoch 166
Training loss: 0.39158693397253236702
Training accuracy: 94.38250000000001%
Test accuracy: 94.05000000000001%


Epoch 167
Training loss: 0.3925627827420703906
Training accuracy: 94.38%
Test accuracy: 94.45%


Epoch 168
Training loss: 0.39159753003450063319
Training accuracy: 94.3%
Test accuracy: 94.77%


Epoch 169
Training loss: 0.3877796217170760142
Training accuracy: 94.39%
Test accuracy: 94.53%


Epoch 170
Training loss: 0.38908044454082560282
Training accuracy: 94.3375%
Test accuracy: 94.85%
```

Epoch 171
Training loss: 0.38192892321099931502
Training accuracy: 94.54249999999999%
Test accuracy: 94.28999999999999%

Epoch 172
Training loss: 0.38199241506695650637
Training accuracy: 94.53500000000001%
Test accuracy: 94.35%

Epoch 173
Training loss: 0.37985634652989393956
Training accuracy: 94.53500000000001%
Test accuracy: 94.39999999999999%

Epoch 174
Training loss: 0.38163356179768788897
Training accuracy: 94.49749999999999%
Test accuracy: 94.58000000000001%

Epoch 175
Training loss: 0.37882859869658468631
Training accuracy: 94.56249999999999%
Test accuracy: 94.48999999999998%

Epoch 176
Training loss: 0.36935909014981529018
Training accuracy: 94.7025%
Test accuracy: 94.02000000000001%

Epoch 177
Training loss: 0.37198621856431785004
Training accuracy: 94.61999999999999%
Test accuracy: 94.52%

Epoch 178
Training loss: 0.3778492781379183282
Training accuracy: 94.575%
Test accuracy: 94.77%

Epoch 179
Training loss: 0.3782577725950479073
Training accuracy: 94.565%
Test accuracy: 94.96000000000001%

Epoch 180
Training loss: 0.37036794274279403324
Training accuracy: 94.635%

```
Test accuracy: 94.68999999999998%


Epoch 181
Training loss: 0.36923994361741522622
Training accuracy: 94.68250000000002%
Test accuracy: 94.62%


Epoch 182
Training loss: 0.36914934109819437492
Training accuracy: 94.65%
Test accuracy: 94.74%


Epoch 183
Training loss: 0.36384758731292012377
Training accuracy: 94.79249999999999%
Test accuracy: 94.28999999999999%


Epoch 184
Training loss: 0.36348437821860411654
Training accuracy: 94.8%
Test accuracy: 94.45%


Epoch 185
Training loss: 0.36895496375140135573
Training accuracy: 94.6775%
Test accuracy: 95.08%


Epoch 186
Training loss: 0.36330459705155400798
Training accuracy: 94.79249999999999%
Test accuracy: 94.53999999999999%


Epoch 187
Training loss: 0.3606536572984903386
Training accuracy: 94.75249999999998%
Test accuracy: 94.78999999999999%


Epoch 188
Training loss: 0.361612120255389908
Training accuracy: 94.805%
Test accuracy: 94.80999999999999%


Epoch 189
Training loss: 0.35718861163897241737
Training accuracy: 94.88749999999999%
Test accuracy: 94.53%


Epoch 190
```

Training loss: 0.35862918062728819823
Training accuracy: 94.8225%
Test accuracy: 94.85%

Epoch 191
Training loss: 0.35949735031136498877
Training accuracy: 94.8125%
Test accuracy: 94.96%

Epoch 192
Training loss: 0.3563152715485924694
Training accuracy: 94.885%
Test accuracy: 94.64999999999999%

Epoch 193
Training loss: 0.35137338075906334616
Training accuracy: 95.005%
Test accuracy: 94.34%

Epoch 194
Training loss: 0.35013641888528973376
Training accuracy: 94.91749999999999%
Test accuracy: 94.67%

Epoch 195
Training loss: 0.34952663293369294828
Training accuracy: 94.9775%
Test accuracy: 94.61999999999999%

Epoch 196
Training loss: 0.35045629155387968004
Training accuracy: 94.95%
Test accuracy: 94.82000000000001%

Epoch 197
Training loss: 0.34899207640514613688
Training accuracy: 94.94749999999999%
Test accuracy: 94.85%

Epoch 198
Training loss: 0.34912806308276130186
Training accuracy: 94.975%
Test accuracy: 94.93999999999998%

Epoch 199
Training loss: 0.34675385339362467304
Training accuracy: 95.03%
Test accuracy: 94.66%

```
Epoch 200
Training loss: 0.347023505355116699
Training accuracy: 94.9725%
Test accuracy: 94.94%
```

Training loss

Training and testing accuracies
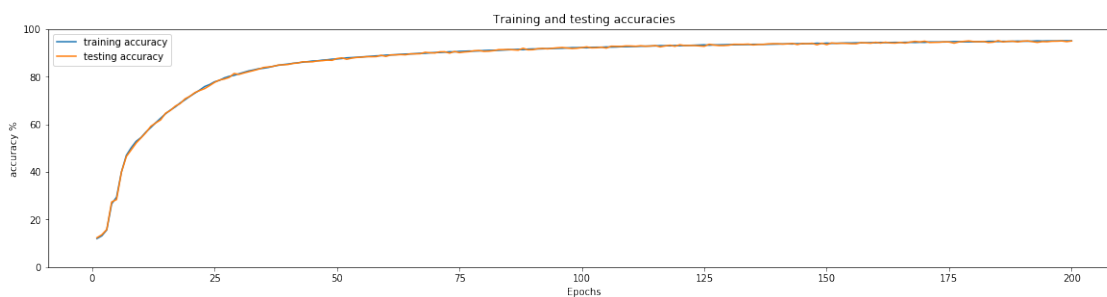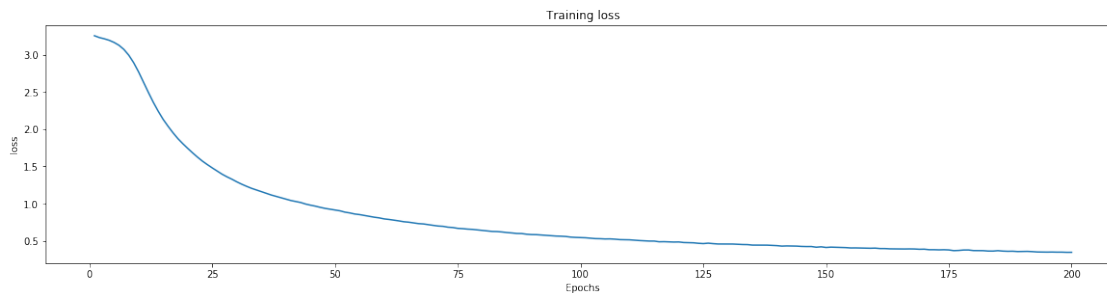
We can see that the model performs considerably well on the larger dataset of 50000 entries with 95% accuracy. It seems we could get an higher accuracy if we continued for more epochs but considering the enormous time it takes to train the model on the entire data set, we will leave it to this.

Let us now look at the results on the validation data:

```
[10]: print ("Validation accuracy: "+str(net_mnist.
      ↪evaluate_validation_accuracy(validation_data_MNIST))+"%")
```

```
Validation accuracy: 94.87%
```

Hence we can see that the Cross Validation has clearly improved the performance.

2.3   Part 2

Now we test how well does the PCA data work compared to the normal image data.
We'll use the same cost and activation here and play with the parameters of the Stochastic Gradient Descent to see which ones fit the best

Starting with no hidden layers:

```
[2]: training_data = data_loader.load_PCA()
```

```
[3]: net = network.Linear_Network_CrossValidation([25,10])
```

```
[4]: net.SGD(training_data, 200, 10, 0.1)
```

```
Epoch 1
Training loss: 9.22099801657356956
Training accuracy: 46.5%
Test accuracy: 45.00000000000001%

Epoch 2
Training loss: 5.6972104611961508326
Training accuracy: 64.29166666666666%
Test accuracy: 64.0%

Epoch 3
Training loss: 4.416983909376654337
Training accuracy: 69.66666666666666%
Test accuracy: 73.33333333333334%

Epoch 4
Training loss: 3.615743826552267308
Training accuracy: 73.87499999999999%
Test accuracy: 70.66666666666666%

Epoch 5
Training loss: 3.099086169043806279
Training accuracy: 76.91666666666666%
Test accuracy: 74.0%

Epoch 6
Training loss: 2.7268228464475887996
Training accuracy: 78.66666666666666%
Test accuracy: 76.83333333333334%

Epoch 7
Training loss: 2.4652329690654852014
Training accuracy: 79.20833333333334%
Test accuracy: 81.33333333333333%

Epoch 8
Training loss: 2.2423227203728069627
Training accuracy: 81.20833333333334%
Test accuracy: 81.16666666666667%
```

```
Epoch 9
Training loss: 2.0902640366876575906
Training accuracy: 81.875%
Test accuracy: 80.5%

Epoch 10
Training loss: 1.9570741335411768905
Training accuracy: 83.25000000000001%
Test accuracy: 81.33333333333334%

Epoch 11
Training loss: 1.8514596443420896478
Training accuracy: 83.16666666666666%
Test accuracy: 81.5%

Epoch 12
Training loss: 1.7771641398699509586
Training accuracy: 83.29166666666667%
Test accuracy: 82.50000000000001%

Epoch 13
Training loss: 1.6865713420050845176
Training accuracy: 83.87499999999999%
Test accuracy: 81.33333333333334%

Epoch 14
Training loss: 1.6438645911290386337
Training accuracy: 84.0%
Test accuracy: 80.66666666666667%

Epoch 15
Training loss: 1.5931499490888378794
Training accuracy: 84.04166666666666%
Test accuracy: 81.16666666666666%

Epoch 16
Training loss: 1.5275985931733456518
Training accuracy: 84.20833333333334%
Test accuracy: 81.33333333333334%

Epoch 17
Training loss: 1.5108335397047323175
Training accuracy: 83.45833333333334%
Test accuracy: 83.5%

Epoch 18
Training loss: 1.4578650939323272563
Training accuracy: 84.58333333333334%
```

```
Test accuracy: 81.83333333333334%


Epoch 19
Training loss: 1.4295217681960789051
Training accuracy: 84.33333333333334%
Test accuracy: 81.99999999999999%


Epoch 20
Training loss: 1.4023068684772099056
Training accuracy: 84.37499999999999%
Test accuracy: 82.16666666666667%


Epoch 21
Training loss: 1.393550409873582511
Training accuracy: 84.125%
Test accuracy: 83.83333333333334%


Epoch 22
Training loss: 1.3593415094239845594
Training accuracy: 84.58333333333334%
Test accuracy: 82.83333333333333%


Epoch 23
Training loss: 1.3367656897410729739
Training accuracy: 84.37500000000001%
Test accuracy: 83.16666666666666%


Epoch 24
Training loss: 1.3303301493884393016
Training accuracy: 84.08333333333333%
Test accuracy: 84.16666666666667%


Epoch 25
Training loss: 1.2997867553723576002
Training accuracy: 84.45833333333334%
Test accuracy: 83.5%


Epoch 26
Training loss: 1.2744088054515127466
Training accuracy: 85.08333333333333%
Test accuracy: 81.99999999999999%


Epoch 27
Training loss: 1.2643420714976450969
Training accuracy: 84.62499999999999%
Test accuracy: 82.66666666666666%


Epoch 28
```

```
Training loss: 1.2578427745448093097
Training accuracy: 84.66666666666666%
Test accuracy: 84.5%


Epoch 29
Training loss: 1.248363277501828601
Training accuracy: 85.0%
Test accuracy: 83.5%


Epoch 30
Training loss: 1.2087526776956383451
Training accuracy: 85.29166666666666%
Test accuracy: 82.66666666666667%


Epoch 31
Training loss: 1.2247311884333820144
Training accuracy: 84.91666666666666%
Test accuracy: 84.0%


Epoch 32
Training loss: 1.2078280365076548693
Training accuracy: 85.12500000000001%
Test accuracy: 83.16666666666666%


Epoch 33
Training loss: 1.2043542942152072343
Training accuracy: 84.95833333333333%
Test accuracy: 84.50000000000001%


Epoch 34
Training loss: 1.1901402066932745033
Training accuracy: 85.04166666666667%
Test accuracy: 83.66666666666666%


Epoch 35
Training loss: 1.1893266877239534126
Training accuracy: 85.125%
Test accuracy: 84.33333333333334%


Epoch 36
Training loss: 1.1753629308617575488
Training accuracy: 85.00000000000001%
Test accuracy: 83.50000000000001%


Epoch 37
Training loss: 1.15785837871610086
Training accuracy: 84.79166666666666%
Test accuracy: 82.5%
```

```
Epoch 38
Training loss: 1.1566425832193783563
Training accuracy: 84.75%
Test accuracy: 84.5%

Epoch 39
Training loss: 1.1495045274817770896
Training accuracy: 85.25%
Test accuracy: 83.83333333333334%

Epoch 40
Training loss: 1.1466338426937817961
Training accuracy: 84.91666666666666%
Test accuracy: 85.0%

Epoch 41
Training loss: 1.131884816837323815
Training accuracy: 85.16666666666666%
Test accuracy: 83.5%

Epoch 42
Training loss: 1.1207454266956132541
Training accuracy: 85.20833333333334%
Test accuracy: 84.66666666666667%

Epoch 43
Training loss: 1.1151955116606021295
Training accuracy: 85.04166666666666%
Test accuracy: 85.0%

Epoch 44
Training loss: 1.1286553414805337821
Training accuracy: 85.0%
Test accuracy: 85.5%

Epoch 45
Training loss: 1.1231122970894174319
Training accuracy: 84.70833333333334%
Test accuracy: 86.16666666666667%

Epoch 46
Training loss: 1.1006883708309570318
Training accuracy: 85.25%
Test accuracy: 84.33333333333334%

Epoch 47
Training loss: 1.1063091023874119528
```

```
Training accuracy: 85.0%
Test accuracy: 85.83333333333334%


Epoch 48
Training loss: 1.1020724523413657304
Training accuracy: 85.37499999999999%
Test accuracy: 83.66666666666666%


Epoch 49
Training loss: 1.1104249716803785487
Training accuracy: 84.75000000000001%
Test accuracy: 86.5%


Epoch 50
Training loss: 1.0818069533600869779
Training accuracy: 84.66666666666667%
Test accuracy: 84.83333333333334%


Epoch 51
Training loss: 1.0802805427946699336
Training accuracy: 85.45833333333333%
Test accuracy: 83.50000000000001%


Epoch 52
Training loss: 1.0781572948084795707
Training accuracy: 85.29166666666667%
Test accuracy: 84.0%


Epoch 53
Training loss: 1.0803223229246038665
Training accuracy: 85.0%
Test accuracy: 85.99999999999999%


Epoch 54
Training loss: 1.0777593083232875224
Training accuracy: 85.25%
Test accuracy: 84.16666666666666%


Epoch 55
Training loss: 1.0590637339114498695
Training accuracy: 85.62499999999999%
Test accuracy: 83.33333333333333%


Epoch 56
Training loss: 1.0485721456538875736
Training accuracy: 85.54166666666667%
Test accuracy: 82.0%
```

```
Epoch 57
Training loss: 1.0657101902259506693
Training accuracy: 85.41666666666667%
Test accuracy: 84.16666666666666%

Epoch 58
Training loss: 1.0646789183573849475
Training accuracy: 85.0%
Test accuracy: 85.66666666666666%

Epoch 59
Training loss: 1.0700191287991760078
Training accuracy: 84.79166666666667%
Test accuracy: 87.5%

Epoch 60
Training loss: 1.0425749785778933251
Training accuracy: 85.79166666666666%
Test accuracy: 83.33333333333334%

Epoch 61
Training loss: 1.0411661059297303879
Training accuracy: 85.50000000000001%
Test accuracy: 85.33333333333333%

Epoch 62
Training loss: 1.0275679917544947195
Training accuracy: 86.04166666666666%
Test accuracy: 82.33333333333334%

Epoch 63
Training loss: 1.0626175516479822297
Training accuracy: 85.20833333333333%
Test accuracy: 86.33333333333333%

Epoch 64
Training loss: 1.0268900450835347518
Training accuracy: 86.04166666666667%
Test accuracy: 84.16666666666666%

Epoch 65
Training loss: 1.0541205004489146379
Training accuracy: 84.95833333333334%
Test accuracy: 86.83333333333334%

Epoch 66
Training loss: 1.0261506166687908975
Training accuracy: 85.91666666666667%
```

Test accuracy: 84.33333333333333%

Epoch 67
Training loss: 1.0349572965866353538
Training accuracy: 85.5%
Test accuracy: 85.33333333333334%

Epoch 68
Training loss: 1.023071498461245923
Training accuracy: 85.66666666666666%
Test accuracy: 84.16666666666667%

Epoch 69
Training loss: 1.0166117901239781439
Training accuracy: 85.87499999999999%
Test accuracy: 83.33333333333333%

Epoch 70
Training loss: 1.021026728780364924
Training accuracy: 85.79166666666667%
Test accuracy: 84.16666666666666%

Epoch 71
Training loss: 1.0259281825614258763
Training accuracy: 85.49999999999999%
Test accuracy: 85.00000000000001%

Epoch 72
Training loss: 1.0116180602894958048
Training accuracy: 85.875%
Test accuracy: 83.66666666666666%

Epoch 73
Training loss: 1.0080787574609261215
Training accuracy: 85.875%
Test accuracy: 83.83333333333334%

Epoch 74
Training loss: 1.0194721232291883708
Training accuracy: 85.5%
Test accuracy: 85.50000000000001%

Epoch 75
Training loss: 1.0048028661834067646
Training accuracy: 85.58333333333334%
Test accuracy: 83.33333333333334%

Epoch 76

Training loss: 1.0187854349289774013
Training accuracy: 85.125%
Test accuracy: 86.5%

Epoch 77
Training loss: 1.0045413021616625495
Training accuracy: 85.91666666666666%
Test accuracy: 84.83333333333333%

Epoch 78
Training loss: 0.9959548748476411365
Training accuracy: 85.83333333333334%
Test accuracy: 84.33333333333334%

Epoch 79
Training loss: 1.0070186709150904116
Training accuracy: 85.45833333333334%
Test accuracy: 85.33333333333334%

Epoch 80
Training loss: 0.987267135534026435
Training accuracy: 86.125%
Test accuracy: 82.66666666666666%

Epoch 81
Training loss: 1.0089890770924260621
Training accuracy: 85.50000000000001%
Test accuracy: 85.66666666666666%

Epoch 82
Training loss: 0.9950429323735696089
Training accuracy: 85.95833333333333%
Test accuracy: 85.0%

Epoch 83
Training loss: 0.9976947280861072239
Training accuracy: 85.91666666666666%
Test accuracy: 85.50000000000001%

Epoch 84
Training loss: 0.9936223070099521555
Training accuracy: 86.20833333333334%
Test accuracy: 84.16666666666667%

Epoch 85
Training loss: 0.99902601892256380847
Training accuracy: 85.45833333333333%
Test accuracy: 86.50000000000001%

```
Epoch 86
Training loss: 0.98890967583754411246
Training accuracy: 86.16666666666667%
Test accuracy: 85.49999999999999%

Epoch 87
Training loss: 0.99209672280889362107
Training accuracy: 85.75%
Test accuracy: 84.5%

Epoch 88
Training loss: 0.9953416496848202492
Training accuracy: 85.875%
Test accuracy: 85.83333333333334%

Epoch 89
Training loss: 0.98016822976230116728
Training accuracy: 85.95833333333333%
Test accuracy: 83.83333333333334%

Epoch 90
Training loss: 0.9941278286603629355
Training accuracy: 85.45833333333334%
Test accuracy: 85.5%

Epoch 91
Training loss: 0.99759560550295738073
Training accuracy: 85.875%
Test accuracy: 85.5%

Epoch 92
Training loss: 0.9942328126805559842
Training accuracy: 85.75%
Test accuracy: 86.0%

Epoch 93
Training loss: 0.9649296148785307456
Training accuracy: 86.33333333333334%
Test accuracy: 83.66666666666666%

Epoch 94
Training loss: 0.9777514363994275353524
Training accuracy: 86.25%
Test accuracy: 85.83333333333333%

Epoch 95
Training loss: 0.980758190177110446
```

```
Training accuracy: 85.70833333333%
Test accuracy: 85.66666666666%


Epoch 96
Training loss: 0.97627445855370527644
Training accuracy: 86.12500000000001%
Test accuracy: 84.83333333333334%


Epoch 97
Training loss: 0.9694192008438162378
Training accuracy: 86.24999999999999%
Test accuracy: 84.66666666666667%


Epoch 98
Training loss: 0.9643773562921801708
Training accuracy: 86.41666666666666%
Test accuracy: 83.33333333333333%


Epoch 99
Training loss: 0.97539084866014406883
Training accuracy: 86.08333333333334%
Test accuracy: 85.16666666666666%


Epoch 100
Training loss: 0.9671893405417292775
Training accuracy: 85.58333333333334%
Test accuracy: 85.5%


Epoch 101
Training loss: 0.96451305736428271325
Training accuracy: 85.91666666666666%
Test accuracy: 86.00000000000001%


Epoch 102
Training loss: 0.96011113968634111083
Training accuracy: 85.91666666666666%
Test accuracy: 85.5%


Epoch 103
Training loss: 0.982912601711349907
Training accuracy: 85.5%
Test accuracy: 87.83333333333333%


Epoch 104
Training loss: 0.97641935471151834443
Training accuracy: 85.66666666666666%
Test accuracy: 86.16666666666666%
```

```
Epoch 105
Training loss: 0.9638803214258042194
Training accuracy: 86.20833333333333%
Test accuracy: 84.66666666666667%


Epoch 106
Training loss: 0.95477579559118251834
Training accuracy: 86.58333333333334%
Test accuracy: 82.50000000000001%


Epoch 107
Training loss: 0.9508399014948614521
Training accuracy: 86.70833333333334%
Test accuracy: 83.66666666666667%


Epoch 108
Training loss: 0.97508097963701102843
Training accuracy: 85.83333333333334%
Test accuracy: 85.66666666666666%


Epoch 109
Training loss: 0.97243272116623957773
Training accuracy: 85.70833333333333%
Test accuracy: 85.66666666666667%


Epoch 110
Training loss: 0.9715200685240141046
Training accuracy: 85.875%
Test accuracy: 85.16666666666667%


Epoch 111
Training loss: 0.9606236820461237213
Training accuracy: 85.70833333333333%
Test accuracy: 85.33333333333334%


Epoch 112
Training loss: 0.9530854339547007519
Training accuracy: 86.45833333333334%
Test accuracy: 84.49999999999999%


Epoch 113
Training loss: 0.9573774467594564696
Training accuracy: 85.875%
Test accuracy: 85.66666666666666%


Epoch 114
Training loss: 0.9591036677244270033
Training accuracy: 85.83333333333334%
```

Test accuracy: 85.33333333333333%

Epoch 115
Training loss: 0.96221566654308287756
Training accuracy: 85.83333333333334%
Test accuracy: 85.0%

Epoch 116
Training loss: 0.9620331650043036054
Training accuracy: 85.91666666666667%
Test accuracy: 86.0%

Epoch 117
Training loss: 0.9480083571309839313
Training accuracy: 86.0%
Test accuracy: 85.16666666666666%

Epoch 118
Training loss: 0.95891523401212553366
Training accuracy: 86.25%
Test accuracy: 83.33333333333333%

Epoch 119
Training loss: 0.9549137199912463951
Training accuracy: 86.16666666666667%
Test accuracy: 85.33333333333334%

Epoch 120
Training loss: 0.95152916914622130885
Training accuracy: 86.29166666666667%
Test accuracy: 85.16666666666666%

Epoch 121
Training loss: 0.9327393379310809832
Training accuracy: 86.625%
Test accuracy: 83.83333333333333%

Epoch 122
Training loss: 0.95170439636192179987
Training accuracy: 86.08333333333334%
Test accuracy: 86.16666666666666%

Epoch 123
Training loss: 0.9479719040540984737
Training accuracy: 86.04166666666666%
Test accuracy: 85.50000000000001%

Epoch 124

Training loss: 0.9485797780305012789
Training accuracy: 85.70833333333334%
Test accuracy: 84.33333333333333%

Epoch 125
Training loss: 0.94987166012020494835
Training accuracy: 86.45833333333334%
Test accuracy: 84.83333333333334%

Epoch 126
Training loss: 0.9465243238367845628
Training accuracy: 86.20833333333333%
Test accuracy: 85.33333333333334%

Epoch 127
Training loss: 0.9532671469752907539
Training accuracy: 86.16666666666666%
Test accuracy: 85.99999999999999%

Epoch 128
Training loss: 0.94637989040867940855
Training accuracy: 85.95833333333334%
Test accuracy: 86.66666666666666%

Epoch 129
Training loss: 0.9536342726793946843
Training accuracy: 86.20833333333333%
Test accuracy: 85.66666666666666%

Epoch 130
Training loss: 0.94773663383420461747
Training accuracy: 86.125%
Test accuracy: 86.5%

Epoch 131
Training loss: 0.9381800498596907712
Training accuracy: 86.375%
Test accuracy: 85.49999999999999%

Epoch 132
Training loss: 0.94838897112323895165
Training accuracy: 86.37499999999999%
Test accuracy: 86.33333333333334%

Epoch 133
Training loss: 0.9475479501192993932
Training accuracy: 86.125%
Test accuracy: 86.5%

```
Epoch 134
Training loss: 0.9403978357626282752
Training accuracy: 85.95833333333333%
Test accuracy: 86.99999999999999%

Epoch 135
Training loss: 0.95148697735388235435
Training accuracy: 85.75%
Test accuracy: 86.66666666666667%

Epoch 136
Training loss: 0.92213030598836386275
Training accuracy: 86.54166666666666%
Test accuracy: 84.83333333333333%

Epoch 137
Training loss: 0.9483925306492080651
Training accuracy: 86.08333333333334%
Test accuracy: 85.83333333333333%

Epoch 138
Training loss: 0.9318572029531691334
Training accuracy: 86.16666666666666%
Test accuracy: 84.33333333333333%

Epoch 139
Training loss: 0.95558113786210473385
Training accuracy: 85.875%
Test accuracy: 86.83333333333334%

Epoch 140
Training loss: 0.9426267645017579676
Training accuracy: 85.91666666666666%
Test accuracy: 87.83333333333334%

Epoch 141
Training loss: 0.949690493423599071
Training accuracy: 86.04166666666667%
Test accuracy: 86.5%

Epoch 142
Training loss: 0.9393803347845028423
Training accuracy: 86.33333333333333%
Test accuracy: 85.16666666666667%

Epoch 143
Training loss: 0.9323541897388749656
```

```
Training accuracy: 86.74999999999999%
Test accuracy: 84.5%


Epoch 144
Training loss: 0.9239997508137448849
Training accuracy: 86.54166666666666%
Test accuracy: 84.00000000000001%


Epoch 145
Training loss: 0.93210789341292852786
Training accuracy: 86.08333333333333%
Test accuracy: 85.00000000000001%


Epoch 146
Training loss: 0.93173953082639658727
Training accuracy: 86.58333333333333%
Test accuracy: 84.16666666666666%


Epoch 147
Training loss: 0.92290222671662442187
Training accuracy: 86.75000000000001%
Test accuracy: 85.16666666666667%


Epoch 148
Training loss: 0.9564176076051543945
Training accuracy: 85.83333333333334%
Test accuracy: 87.66666666666667%


Epoch 149
Training loss: 0.9348165774269013373
Training accuracy: 86.41666666666667%
Test accuracy: 85.49999999999999%


Epoch 150
Training loss: 0.93596943290919236175
Training accuracy: 86.33333333333334%
Test accuracy: 85.50000000000001%


Epoch 151
Training loss: 0.9339130674301683927
Training accuracy: 86.50000000000001%
Test accuracy: 85.66666666666667%


Epoch 152
Training loss: 0.93320506152939738454
Training accuracy: 86.375%
Test accuracy: 85.16666666666666%
```

```
Epoch 153
Training loss: 0.919140210195542701
Training accuracy: 86.70833333333333%
Test accuracy: 84.83333333333334%

Epoch 154
Training loss: 0.92703292410881861713
Training accuracy: 86.24999999999999%
Test accuracy: 85.66666666666666%

Epoch 155
Training loss: 0.94086879093074386434
Training accuracy: 86.37500000000001%
Test accuracy: 85.33333333333334%

Epoch 156
Training loss: 0.95085953024332716234
Training accuracy: 85.66666666666667%
Test accuracy: 87.16666666666666%

Epoch 157
Training loss: 0.9468469458793463968
Training accuracy: 86.04166666666667%
Test accuracy: 87.0%

Epoch 158
Training loss: 0.94237196246204010477
Training accuracy: 86.125%
Test accuracy: 86.83333333333333%

Epoch 159
Training loss: 0.9297184877123993287
Training accuracy: 86.5%
Test accuracy: 85.16666666666666%

Epoch 160
Training loss: 0.9403964661932144125
Training accuracy: 86.33333333333333%
Test accuracy: 85.99999999999999%

Epoch 161
Training loss: 0.91467708801819335243
Training accuracy: 86.50000000000001%
Test accuracy: 85.0%

Epoch 162
Training loss: 0.9312914627904353087
Training accuracy: 86.0%
```

Test accuracy: 86.00000000000001%

Epoch 163
Training loss: 0.9329994720909179408
Training accuracy: 86.25%
Test accuracy: 84.66666666666667%

Epoch 164
Training loss: 0.9070743203536967148
Training accuracy: 86.58333333333334%
Test accuracy: 84.5%

Epoch 165
Training loss: 0.92750206907822564087
Training accuracy: 86.04166666666666%
Test accuracy: 87.0%

Epoch 166
Training loss: 0.9314286658320047433
Training accuracy: 86.125%
Test accuracy: 86.5%

Epoch 167
Training loss: 0.9072719704957024541
Training accuracy: 86.83333333333334%
Test accuracy: 84.16666666666666%

Epoch 168
Training loss: 0.92819261146233053865
Training accuracy: 86.12499999999999%
Test accuracy: 87.0%

Epoch 169
Training loss: 0.91111359618094969504
Training accuracy: 87.08333333333333%
Test accuracy: 83.0%

Epoch 170
Training loss: 0.9205717767795343373
Training accuracy: 86.375%
Test accuracy: 85.33333333333333%

Epoch 171
Training loss: 0.91284131819120961345
Training accuracy: 86.54166666666666%
Test accuracy: 84.33333333333334%

Epoch 172

Training loss: 0.9034718972975628766
Training accuracy: 86.375%
Test accuracy: 85.0%

Epoch 173
Training loss: 0.92236617145949156507
Training accuracy: 86.20833333333%
Test accuracy: 85.83333333333333%

Epoch 174
Training loss: 0.9163946539407847977
Training accuracy: 86.95833333333334%
Test accuracy: 84.00000000000001%

Epoch 175
Training loss: 0.93817536546166749084
Training accuracy: 86.0%
Test accuracy: 87.66666666666666%

Epoch 176
Training loss: 0.92294329129082237705
Training accuracy: 86.45833333333333%
Test accuracy: 85.16666666666667%

Epoch 177
Training loss: 0.92206947596340459216
Training accuracy: 86.83333333333334%
Test accuracy: 84.33333333333334%

Epoch 178
Training loss: 0.9445396987415937035
Training accuracy: 85.625%
Test accuracy: 87.83333333333333%

Epoch 179
Training loss: 0.9225894754227977433
Training accuracy: 86.66666666666666%
Test accuracy: 84.16666666666666%

Epoch 180
Training loss: 0.9300007719836669897
Training accuracy: 86.08333333333334%
Test accuracy: 86.83333333333334%

Epoch 181
Training loss: 0.91691251941907935517
Training accuracy: 86.5%
Test accuracy: 85.16666666666666%

```
Epoch 182
Training loss: 0.9099594058365245754
Training accuracy: 86.79166666666%
Test accuracy: 83.66666666666667%

Epoch 183
Training loss: 0.9402954934432730443
Training accuracy: 85.95833333333334%
Test accuracy: 87.66666666666666%

Epoch 184
Training loss: 0.905979100009847618844
Training accuracy: 86.58333333333333%
Test accuracy: 84.83333333333333%

Epoch 185
Training loss: 0.92376679380484247084
Training accuracy: 86.16666666666666%
Test accuracy: 86.33333333333333%

Epoch 186
Training loss: 0.91151696468287896747
Training accuracy: 86.62499999999999%
Test accuracy: 84.49999999999999%

Epoch 187
Training loss: 0.9239680807201916695
Training accuracy: 86.04166666666666%
Test accuracy: 86.66666666666666%

Epoch 188
Training loss: 0.9155973874335843458
Training accuracy: 86.33333333333334%
Test accuracy: 86.16666666666667%

Epoch 189
Training loss: 0.9276902048414148624
Training accuracy: 85.87499999999999%
Test accuracy: 87.0%

Epoch 190
Training loss: 0.9288258499646736957
Training accuracy: 86.16666666666666%
Test accuracy: 86.0%

Epoch 191
Training loss: 0.9207505904042493449
```

```
Training accuracy: 86.58333333333334%
Test accuracy: 85.33333333333334%


Epoch 192
Training loss: 0.92456407869531978565
Training accuracy: 86.16666666666667%
Test accuracy: 87.00000000000001%


Epoch 193
Training loss: 0.9079855519680257943
Training accuracy: 86.66666666666667%
Test accuracy: 85.5%


Epoch 194
Training loss: 0.91731855525541156624
Training accuracy: 86.33333333333334%
Test accuracy: 86.33333333333334%


Epoch 195
Training loss: 0.9089417328397642065
Training accuracy: 86.625%
Test accuracy: 84.33333333333333%


Epoch 196
Training loss: 0.9057281808393979813
Training accuracy: 86.95833333333333%
Test accuracy: 84.5%


Epoch 197
Training loss: 0.92285542674269693835
Training accuracy: 86.70833333333333%
Test accuracy: 86.33333333333333%


Epoch 198
Training loss: 0.9128849809650571227
Training accuracy: 86.87499999999999%
Test accuracy: 85.0%


Epoch 199
Training loss: 0.9224974515088569001
Training accuracy: 86.79166666666666%
Test accuracy: 85.83333333333333%


Epoch 200
Training loss: 0.9262812921787709626
Training accuracy: 86.0%
Test accuracy: 86.0%
```
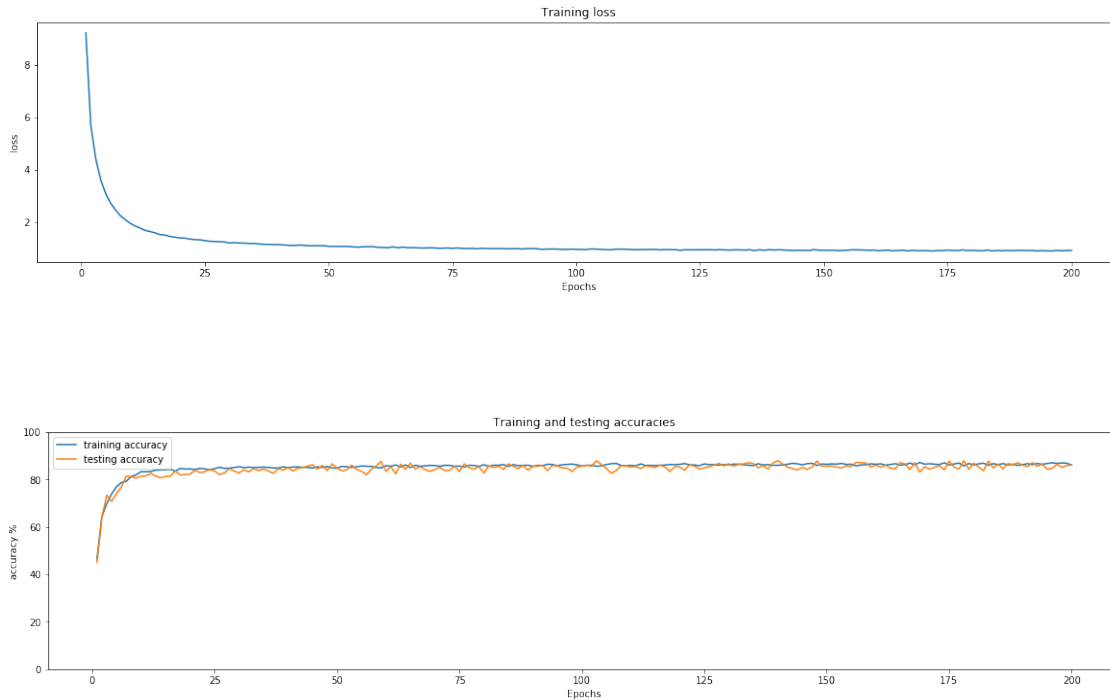
Now adding a hidden layer with 15 neurons:

```
[5]: net = network.Linear_Network_CrossValidation([25,15,10])
```

```
[6]: net.SGD(training_data, 200, 10, 0.1)
```

```
Epoch 1
Training loss: 4.792071241986174617
Training accuracy: 14.0%
Test accuracy: 14.166666666666666%

Epoch 2
Training loss: 3.3346337310360963358
Training accuracy: 33.04166666666667%
Test accuracy: 33.5%

Epoch 3
Training loss: 3.024566060762295849
Training accuracy: 44.791666666666664%
Test accuracy: 45.83333333333333%

Epoch 4
Training loss: 2.861589116469465267
Training accuracy: 51.75%
Test accuracy: 49.66666666666667%
```

```
Epoch 5
Training loss: 2.755135437759301802
Training accuracy: 55.25%
Test accuracy: 57.833333333333336%

Epoch 6
Training loss: 2.6496530787622441599
Training accuracy: 59.375%
Test accuracy: 57.833333333333336%

Epoch 7
Training loss: 2.5734595082181430433
Training accuracy: 61.75%
Test accuracy: 64.99999999999999%

Epoch 8
Training loss: 2.4751802065387753368
Training accuracy: 65.70833333333333%
Test accuracy: 61.66666666666667%

Epoch 9
Training loss: 2.4047869776722956285
Training accuracy: 66.875%
Test accuracy: 66.16666666666666%

Epoch 10
Training loss: 2.337064684430916995
Training accuracy: 69.54166666666666%
Test accuracy: 67.0%

Epoch 11
Training loss: 2.2634988026305364562
Training accuracy: 71.37500000000001%
Test accuracy: 68.16666666666666%

Epoch 12
Training loss: 2.2044487360756696262
Training accuracy: 72.54166666666667%
Test accuracy: 70.33333333333333%

Epoch 13
Training loss: 2.1526844679981901808
Training accuracy: 73.29166666666666%
Test accuracy: 73.99999999999999%

Epoch 14
Training loss: 2.0966026309364424907
```

```
Training accuracy: 74.25%
Test accuracy: 74.66666666666667%


Epoch 15
Training loss: 2.0494970646965249214
Training accuracy: 75.16666666666666%
Test accuracy: 76.83333333333334%


Epoch 16
Training loss: 1.9914875619643344253
Training accuracy: 76.79166666666666%
Test accuracy: 73.99999999999999%


Epoch 17
Training loss: 1.9422051684254647482
Training accuracy: 76.83333333333334%
Test accuracy: 77.16666666666667%


Epoch 18
Training loss: 1.8955869939104049791
Training accuracy: 77.79166666666667%
Test accuracy: 76.83333333333333%


Epoch 19
Training loss: 1.8665640969974049316
Training accuracy: 78.08333333333333%
Test accuracy: 77.33333333333334%


Epoch 20
Training loss: 1.8381849444997990589
Training accuracy: 78.41666666666667%
Test accuracy: 81.5%


Epoch 21
Training loss: 1.8003039329544767915
Training accuracy: 79.25%
Test accuracy: 81.16666666666667%


Epoch 22
Training loss: 1.759474149049852692
Training accuracy: 79.79166666666666%
Test accuracy: 81.66666666666667%


Epoch 23
Training loss: 1.7287966115834145813
Training accuracy: 80.45833333333333%
Test accuracy: 81.0%
```

```
Epoch 24
Training loss: 1.6986172921309353453
Training accuracy: 81.33333333333334%
Test accuracy: 80.5%

Epoch 25
Training loss: 1.664123405648994153
Training accuracy: 81.95833333333334%
Test accuracy: 80.0%

Epoch 26
Training loss: 1.6397804306801896705
Training accuracy: 81.91666666666666%
Test accuracy: 80.83333333333333%

Epoch 27
Training loss: 1.6170875797548805396
Training accuracy: 82.125%
Test accuracy: 81.5%

Epoch 28
Training loss: 1.5896991525325337858
Training accuracy: 82.37500000000001%
Test accuracy: 82.16666666666666%

Epoch 29
Training loss: 1.5642606610991159054
Training accuracy: 83.25%
Test accuracy: 80.0%

Epoch 30
Training loss: 1.5455033895480043297
Training accuracy: 82.58333333333334%
Test accuracy: 83.5%

Epoch 31
Training loss: 1.5125636832502982036
Training accuracy: 83.20833333333333%
Test accuracy: 81.83333333333334%

Epoch 32
Training loss: 1.4874199918974025626
Training accuracy: 83.79166666666667%
Test accuracy: 80.33333333333334%

Epoch 33
Training loss: 1.4943063733399869675
Training accuracy: 82.125%
```

```
Test accuracy: 86.0%

Epoch 34
Training loss: 1.4753324177960314486
Training accuracy: 82.83333333333334%
Test accuracy: 84.83333333333334%

Epoch 35
Training loss: 1.4457634506661197761
Training accuracy: 83.62500000000001%
Test accuracy: 83.33333333333334%

Epoch 36
Training loss: 1.4265547282899436502
Training accuracy: 83.66666666666667%
Test accuracy: 84.50000000000001%

Epoch 37
Training loss: 1.4211053928843640298
Training accuracy: 83.95833333333334%
Test accuracy: 83.5%

Epoch 38
Training loss: 1.3952727323563335106
Training accuracy: 84.29166666666667%
Test accuracy: 84.33333333333333%

Epoch 39
Training loss: 1.3818687002725903815
Training accuracy: 84.625%
Test accuracy: 83.0%

Epoch 40
Training loss: 1.3673264963782815201
Training accuracy: 84.45833333333334%
Test accuracy: 84.33333333333334%

Epoch 41
Training loss: 1.3510860143272757518
Training accuracy: 84.41666666666667%
Test accuracy: 85.5%

Epoch 42
Training loss: 1.3412508966567629617
Training accuracy: 84.58333333333334%
Test accuracy: 84.66666666666667%

Epoch 43
```

```
Training loss: 1.3354592836216542951
Training accuracy: 85.25%
Test accuracy: 83.0%


Epoch 44
Training loss: 1.3117213611373053009
Training accuracy: 85.33333333333333%
Test accuracy: 84.0%


Epoch 45
Training loss: 1.2978819332558615382
Training accuracy: 85.08333333333333%
Test accuracy: 84.83333333333333%


Epoch 46
Training loss: 1.2807342935883394653
Training accuracy: 85.0%
Test accuracy: 85.33333333333334%


Epoch 47
Training loss: 1.2832940053815598599
Training accuracy: 85.20833333333333%
Test accuracy: 85.5%


Epoch 48
Training loss: 1.2733372223314146567
Training accuracy: 84.91666666666667%
Test accuracy: 86.16666666666666%


Epoch 49
Training loss: 1.2611978321538024588
Training accuracy: 85.16666666666666%
Test accuracy: 85.0%


Epoch 50
Training loss: 1.2498402836722762714
Training accuracy: 85.16666666666666%
Test accuracy: 85.5%


Epoch 51
Training loss: 1.2443737628992837575
Training accuracy: 85.24999999999999%
Test accuracy: 85.83333333333333%


Epoch 52
Training loss: 1.2285847576162392171
Training accuracy: 85.29166666666667%
Test accuracy: 86.0%
```

```
Epoch 53
Training loss: 1.2180203783985894354
Training accuracy: 85.16666666666667%
Test accuracy: 87.33333333333333%

Epoch 54
Training loss: 1.2022887413734310714
Training accuracy: 85.91666666666666%
Test accuracy: 84.16666666666666%

Epoch 55
Training loss: 1.2013132219519657333
Training accuracy: 85.83333333333333%
Test accuracy: 85.16666666666666%

Epoch 56
Training loss: 1.1865733613617536035
Training accuracy: 85.87500000000001%
Test accuracy: 85.00000000000001%

Epoch 57
Training loss: 1.1835710885031180698
Training accuracy: 85.58333333333334%
Test accuracy: 86.83333333333333%

Epoch 58
Training loss: 1.1806409032128290632
Training accuracy: 85.58333333333334%
Test accuracy: 87.50000000000001%

Epoch 59
Training loss: 1.1526536949425053005
Training accuracy: 86.49999999999999%
Test accuracy: 84.0%

Epoch 60
Training loss: 1.1425263001911997493
Training accuracy: 86.75%
Test accuracy: 83.16666666666667%

Epoch 61
Training loss: 1.1426443532326102506
Training accuracy: 85.91666666666666%
Test accuracy: 86.33333333333333%

Epoch 62
Training loss: 1.1402445093914412693
```

```
Training accuracy: 86.29166666666666%
Test accuracy: 85.83333333333334%


Epoch 63
Training loss: 1.1205126387494043933
Training accuracy: 86.75000000000001%
Test accuracy: 83.33333333333334%


Epoch 64
Training loss: 1.130712958249890276
Training accuracy: 85.95833333333333%
Test accuracy: 86.83333333333333%


Epoch 65
Training loss: 1.1133509697931886723
Training accuracy: 86.20833333333333%
Test accuracy: 85.66666666666667%


Epoch 66
Training loss: 1.110849579724731241
Training accuracy: 86.125%
Test accuracy: 87.00000000000001%


Epoch 67
Training loss: 1.1086111833887634714
Training accuracy: 86.00000000000001%
Test accuracy: 87.0%


Epoch 68
Training loss: 1.0782887914912924358
Training accuracy: 86.66666666666667%
Test accuracy: 84.50000000000001%


Epoch 69
Training loss: 1.0784938539304441548
Training accuracy: 86.625%
Test accuracy: 85.16666666666667%


Epoch 70
Training loss: 1.0753656220372738554
Training accuracy: 86.41666666666666%
Test accuracy: 85.83333333333333%


Epoch 71
Training loss: 1.0869640692904350286
Training accuracy: 86.33333333333334%
Test accuracy: 87.0%
```

```
Epoch 72
Training loss: 1.0583872441487163007
Training accuracy: 87.08333333333333%
Test accuracy: 83.83333333333334%

Epoch 73
Training loss: 1.0583546080816946487
Training accuracy: 86.29166666666667%
Test accuracy: 86.5%

Epoch 74
Training loss: 1.073400021566969035
Training accuracy: 86.04166666666666%
Test accuracy: 88.16666666666667%

Epoch 75
Training loss: 1.0425174235419557928
Training accuracy: 87.08333333333333%
Test accuracy: 84.83333333333334%

Epoch 76
Training loss: 1.0497942674656644374
Training accuracy: 86.33333333333333%
Test accuracy: 87.66666666666667%

Epoch 77
Training loss: 1.0442352536381456231
Training accuracy: 86.74999999999999%
Test accuracy: 87.33333333333333%

Epoch 78
Training loss: 1.0485822954776624642
Training accuracy: 86.5%
Test accuracy: 87.33333333333333%

Epoch 79
Training loss: 1.0394737214361774839
Training accuracy: 86.875%
Test accuracy: 87.5%

Epoch 80
Training loss: 1.0266713662968041834
Training accuracy: 86.70833333333334%
Test accuracy: 87.0%

Epoch 81
Training loss: 1.0088097129781432113
Training accuracy: 87.25000000000001%
```

```
Test accuracy: 85.5%

Epoch 82
Training loss: 1.0030700666534626408
Training accuracy: 87.25000000000001%
Test accuracy: 85.5%

Epoch 83
Training loss: 1.0251100498694718426
Training accuracy: 86.58333333333334%
Test accuracy: 88.66666666666667%

Epoch 84
Training loss: 1.0187122554295327413
Training accuracy: 86.45833333333333%
Test accuracy: 88.83333333333333%

Epoch 85
Training loss: 0.9916018921327679249
Training accuracy: 87.125%
Test accuracy: 86.00000000000001%

Epoch 86
Training loss: 0.986182765179135335
Training accuracy: 87.04166666666667%
Test accuracy: 86.16666666666666%

Epoch 87
Training loss: 1.0004127638505596296
Training accuracy: 86.58333333333333%
Test accuracy: 88.33333333333333%

Epoch 88
Training loss: 0.9903955685781764636
Training accuracy: 87.20833333333333%
Test accuracy: 86.16666666666667%

Epoch 89
Training loss: 0.98822047562277363285
Training accuracy: 87.37499999999999%
Test accuracy: 86.16666666666667%

Epoch 90
Training loss: 0.9785798628633842007
Training accuracy: 87.41666666666667%
Test accuracy: 85.83333333333333%

Epoch 91
```

```
Training loss: 0.96252298184761960555
Training accuracy: 87.58333333333334%
Test accuracy: 84.83333333333334%


Epoch 92
Training loss: 0.97859248811936812914
Training accuracy: 86.70833333333334%
Test accuracy: 88.0%


Epoch 93
Training loss: 0.9846400518195798965
Training accuracy: 86.83333333333334%
Test accuracy: 87.99999999999999%


Epoch 94
Training loss: 0.9522857693034275841
Training accuracy: 87.74999999999999%
Test accuracy: 84.83333333333334%


Epoch 95
Training loss: 0.9708674870452022016
Training accuracy: 87.08333333333334%
Test accuracy: 87.16666666666666%


Epoch 96
Training loss: 0.9538011084673532119
Training accuracy: 87.29166666666666%
Test accuracy: 85.66666666666666%


Epoch 97
Training loss: 0.9620743374461834331
Training accuracy: 86.54166666666666%
Test accuracy: 89.66666666666666%


Epoch 98
Training loss: 0.9489315993769272317
Training accuracy: 87.29166666666666%
Test accuracy: 87.16666666666666%


Epoch 99
Training loss: 0.95448517119208038895
Training accuracy: 87.0%
Test accuracy: 87.33333333333334%


Epoch 100
Training loss: 0.9412384954137370943
Training accuracy: 87.45833333333334%
Test accuracy: 86.33333333333334%
```

```
Epoch 101
Training loss: 0.9359010258143066 7376
Training accuracy: 87.58333333333334%
Test accuracy: 86.5%

Epoch 102
Training loss: 0.9255851255313546605
Training accuracy: 87.875%
Test accuracy: 85.16666666666667%

Epoch 103
Training loss: 0.93184329146338837677
Training accuracy: 87.33333333333334%
Test accuracy: 87.0%

Epoch 104
Training loss: 0.9354958174191041429
Training accuracy: 87.16666666666666%
Test accuracy: 87.66666666666666%

Epoch 105
Training loss: 0.9254144451295545146
Training accuracy: 87.37499999999999%
Test accuracy: 87.33333333333334%

Epoch 106
Training loss: 0.93826832295926048354
Training accuracy: 86.87500000000001%
Test accuracy: 89.49999999999999%

Epoch 107
Training loss: 0.92281230631242051356
Training accuracy: 87.54166666666666%
Test accuracy: 87.00000000000001%

Epoch 108
Training loss: 0.92344866489474711907
Training accuracy: 87.5%
Test accuracy: 87.16666666666667%

Epoch 109
Training loss: 0.9160526559781779713
Training accuracy: 87.41666666666667%
Test accuracy: 88.16666666666667%

Epoch 110
Training loss: 0.9087690082938462748
```

```
Training accuracy: 87.91666666666666%
Test accuracy: 86.33333333333333%


Epoch 111
Training loss: 0.92184085554853523544
Training accuracy: 87.16666666666666%
Test accuracy: 89.16666666666666%


Epoch 112
Training loss: 0.898667110554932874
Training accuracy: 88.25%
Test accuracy: 84.83333333333334%


Epoch 113
Training loss: 0.8961106107936043808
Training accuracy: 87.875%
Test accuracy: 86.83333333333333%


Epoch 114
Training loss: 0.9124922442581743497
Training accuracy: 87.25%
Test accuracy: 89.49999999999999%


Epoch 115
Training loss: 0.90399932135205304806
Training accuracy: 87.66666666666667%
Test accuracy: 88.33333333333333%


Epoch 116
Training loss: 0.89135088281432103255
Training accuracy: 88.20833333333333%
Test accuracy: 86.33333333333334%


Epoch 117
Training loss: 0.8900109533701861539
Training accuracy: 87.95833333333333%
Test accuracy: 87.66666666666666%


Epoch 118
Training loss: 0.8933366739458444653
Training accuracy: 87.625%
Test accuracy: 88.0%


Epoch 119
Training loss: 0.8870837975972320764
Training accuracy: 87.87499999999999%
Test accuracy: 87.66666666666667%
```

```
Epoch 120
Training loss: 0.8894478913205205831
Training accuracy: 87.5%
Test accuracy: 88.16666666666%

Epoch 121
Training loss: 0.8858634435667877942
Training accuracy: 87.95833333333%
Test accuracy: 86.99999999999999%

Epoch 122
Training loss: 0.88436532299397705635
Training accuracy: 87.75%
Test accuracy: 88.66666666666%

Epoch 123
Training loss: 0.87278278124390786723
Training accuracy: 88.08333333333334%
Test accuracy: 86.83333333333334%

Epoch 124
Training loss: 0.86954010660260297146
Training accuracy: 88.00000000000001%
Test accuracy: 86.83333333333334%

Epoch 125
Training loss: 0.8570596032470416318
Training accuracy: 88.33333333333334%
Test accuracy: 86.16666666666667%

Epoch 126
Training loss: 0.84869443862327666504
Training accuracy: 88.50000000000001%
Test accuracy: 85.33333333333333%

Epoch 127
Training loss: 0.8597267919683197739
Training accuracy: 88.5%
Test accuracy: 85.83333333333334%

Epoch 128
Training loss: 0.86634196055481387433
Training accuracy: 88.04166666666667%
Test accuracy: 86.66666666666667%

Epoch 129
Training loss: 0.8614274878274532961
Training accuracy: 87.66666666666666%
```

```
Test accuracy: 88.16666666666%


Epoch 130
Training loss: 0.8675033232795675226
Training accuracy: 88.0%
Test accuracy: 88.00000000000001%


Epoch 131
Training loss: 0.8597440581395520389
Training accuracy: 87.95833333333333%
Test accuracy: 87.83333333333334%


Epoch 132
Training loss: 0.8438763199028173909
Training accuracy: 88.58333333333333%
Test accuracy: 85.83333333333334%


Epoch 133
Training loss: 0.85488098176505648997
Training accuracy: 88.29166666666666%
Test accuracy: 87.66666666666667%


Epoch 134
Training loss: 0.8485277062289370063
Training accuracy: 88.20833333333333%
Test accuracy: 87.33333333333334%


Epoch 135
Training loss: 0.85430917042471805987
Training accuracy: 88.20833333333334%
Test accuracy: 87.66666666666667%


Epoch 136
Training loss: 0.86120212578860751336
Training accuracy: 87.58333333333333%
Test accuracy: 90.33333333333334%


Epoch 137
Training loss: 0.8421445363747444247
Training accuracy: 88.29166666666666%
Test accuracy: 87.83333333333333%


Epoch 138
Training loss: 0.84943022814370667287
Training accuracy: 87.95833333333333%
Test accuracy: 89.16666666666666%


Epoch 139
```

```
Training loss: 0.8349318309690698501
Training accuracy: 88.29166666666667%
Test accuracy: 87.33333333333334%


Epoch 140
Training loss: 0.8438952523635520031
Training accuracy: 88.0%
Test accuracy: 88.66666666666667%


Epoch 141
Training loss: 0.85058200704388999566
Training accuracy: 87.625%
Test accuracy: 89.99999999999999%


Epoch 142
Training loss: 0.8473580652430164179
Training accuracy: 87.95833333333334%
Test accuracy: 89.0%


Epoch 143
Training loss: 0.8288904536445277127
Training accuracy: 88.29166666666666%
Test accuracy: 88.33333333333333%


Epoch 144
Training loss: 0.84502554239888817776
Training accuracy: 88.20833333333333%
Test accuracy: 88.66666666666666%


Epoch 145
Training loss: 0.8249704041738632288
Training accuracy: 88.70833333333334%
Test accuracy: 86.83333333333334%


Epoch 146
Training loss: 0.8101243845468566538
Training accuracy: 88.87499999999999%
Test accuracy: 85.5%


Epoch 147
Training loss: 0.830451180332476665
Training accuracy: 88.16666666666667%
Test accuracy: 88.83333333333334%


Epoch 148
Training loss: 0.8144032512335116481
Training accuracy: 88.54166666666667%
Test accuracy: 87.33333333333333%
```

```
Epoch 149
Training loss: 0.8432930815210359316
Training accuracy: 87.87500000000001%
Test accuracy: 90.5%

Epoch 150
Training loss: 0.825801178390405318
Training accuracy: 88.375%
Test accuracy: 88.5%

Epoch 151
Training loss: 0.816428948408140164
Training accuracy: 88.41666666666666%
Test accuracy: 88.16666666666666%

Epoch 152
Training loss: 0.8285413194550985597
Training accuracy: 88.70833333333333%
Test accuracy: 88.16666666666666%

Epoch 153
Training loss: 0.8334283435540793537
Training accuracy: 88.16666666666666%
Test accuracy: 89.99999999999999%

Epoch 154
Training loss: 0.8176348516876234481
Training accuracy: 88.58333333333333%
Test accuracy: 88.33333333333334%

Epoch 155
Training loss: 0.80758688123256104613
Training accuracy: 88.625%
Test accuracy: 88.33333333333333%

Epoch 156
Training loss: 0.8152647293477368018
Training accuracy: 88.66666666666667%
Test accuracy: 88.16666666666666%

Epoch 157
Training loss: 0.8045939084748499149
Training accuracy: 88.875%
Test accuracy: 86.33333333333334%

Epoch 158
Training loss: 0.80871380920139343095
```

```
Training accuracy: 88.41666666666667%
Test accuracy: 89.00000000000001%


Epoch 159
Training loss: 0.80862215258834086297
Training accuracy: 88.54166666666667%
Test accuracy: 88.5%


Epoch 160
Training loss: 0.81109737952508566435
Training accuracy: 88.58333333333334%
Test accuracy: 88.5%


Epoch 161
Training loss: 0.7968302857733810207
Training accuracy: 89.08333333333333%
Test accuracy: 87.16666666666667%


Epoch 162
Training loss: 0.7887270445608660589
Training accuracy: 89.16666666666666%
Test accuracy: 86.16666666666666%


Epoch 163
Training loss: 0.81079918934649637744
Training accuracy: 88.41666666666666%
Test accuracy: 90.0%


Epoch 164
Training loss: 0.804321835729747886
Training accuracy: 88.75%
Test accuracy: 88.49999999999999%


Epoch 165
Training loss: 0.80519862423734845074
Training accuracy: 88.625%
Test accuracy: 89.16666666666667%


Epoch 166
Training loss: 0.79150341498594873225
Training accuracy: 88.83333333333334%
Test accuracy: 88.16666666666666%


Epoch 167
Training loss: 0.8054933571248680998
Training accuracy: 88.75%
Test accuracy: 88.83333333333334%
```

```
Epoch 168
Training loss: 0.8004371378537407327
Training accuracy: 88.74999999999999%
Test accuracy: 88.83333333333334%

Epoch 169
Training loss: 0.79403213159163128445
Training accuracy: 88.54166666666666%
Test accuracy: 89.33333333333333%

Epoch 170
Training loss: 0.8198124009414503929
Training accuracy: 88.41666666666667%
Test accuracy: 90.83333333333333%

Epoch 171
Training loss: 0.79965938213617195106
Training accuracy: 88.70833333333334%
Test accuracy: 89.33333333333333%

Epoch 172
Training loss: 0.7935774878280064536
Training accuracy: 89.16666666666667%
Test accuracy: 88.0%

Epoch 173
Training loss: 0.79964979396792001617
Training accuracy: 88.66666666666666%
Test accuracy: 89.66666666666666%

Epoch 174
Training loss: 0.7962230692132322766
Training accuracy: 88.875%
Test accuracy: 89.0%

Epoch 175
Training loss: 0.7759182197727546255
Training accuracy: 88.95833333333333%
Test accuracy: 88.5%

Epoch 176
Training loss: 0.7839940889231773586
Training accuracy: 89.04166666666666%
Test accuracy: 88.16666666666667%

Epoch 177
Training loss: 0.7790762599926182088
Training accuracy: 88.875%
```

Test accuracy: 88.5%

Epoch 178
Training loss: 0.7954524776762061021
Training accuracy: 88.75%
Test accuracy: 89.66666666666666%

Epoch 179
Training loss: 0.7894512102378247983
Training accuracy: 89.25%
Test accuracy: 88.33333333333334%

Epoch 180
Training loss: 0.7803963148911486146
Training accuracy: 88.75%
Test accuracy: 89.5%

Epoch 181
Training loss: 0.7864668859977220898
Training accuracy: 88.625%
Test accuracy: 90.33333333333334%

Epoch 182
Training loss: 0.7895783761356486484
Training accuracy: 88.74999999999999%
Test accuracy: 89.83333333333334%

Epoch 183
Training loss: 0.7855332497122383434
Training accuracy: 88.83333333333333%
Test accuracy: 89.33333333333334%

Epoch 184
Training loss: 0.7707840628993850167
Training accuracy: 88.91666666666666%
Test accuracy: 88.99999999999999%

Epoch 185
Training loss: 0.7867182039872418204
Training accuracy: 88.45833333333333%
Test accuracy: 90.66666666666666%

Epoch 186
Training loss: 0.7707891080309591755
Training accuracy: 89.00000000000001%
Test accuracy: 88.66666666666666%

Epoch 187

```
Training loss: 0.75172239125461242417
Training accuracy: 89.33333333333334%
Test accuracy: 87.66666666666666%


Epoch 188
Training loss: 0.77231962397597780896
Training accuracy: 88.79166666666667%
Test accuracy: 89.49999999999999%


Epoch 189
Training loss: 0.7669693061633796559
Training accuracy: 89.16666666666666%
Test accuracy: 88.33333333333333%


Epoch 190
Training loss: 0.76702278147236339814
Training accuracy: 89.20833333333334%
Test accuracy: 88.16666666666666%


Epoch 191
Training loss: 0.7727409459974207853
Training accuracy: 89.08333333333334%
Test accuracy: 89.16666666666667%


Epoch 192
Training loss: 0.7540243424422010283
Training accuracy: 89.20833333333333%
Test accuracy: 88.5%


Epoch 193
Training loss: 0.7655141737566350051
Training accuracy: 89.16666666666666%
Test accuracy: 89.16666666666667%


Epoch 194
Training loss: 0.7833166390186084041
Training accuracy: 88.20833333333334%
Test accuracy: 91.99999999999999%


Epoch 195
Training loss: 0.75936802310676597687
Training accuracy: 89.04166666666666%
Test accuracy: 89.0%


Epoch 196
Training loss: 0.77347086182156473777
Training accuracy: 88.70833333333333%
Test accuracy: 90.0%
```

```
Epoch 197
Training loss: 0.7599821046214927405
Training accuracy: 88.95833333333334%
Test accuracy: 89.0%

Epoch 198
Training loss: 0.7664173272979356108
Training accuracy: 89.0%
Test accuracy: 89.83333333333333%

Epoch 199
Training loss: 0.7583545119896498101
Training accuracy: 89.25%
Test accuracy: 88.16666666666667%

Epoch 200
Training loss: 0.7491805327490448665
Training accuracy: 89.5%
Test accuracy: 87.50000000000001%
```
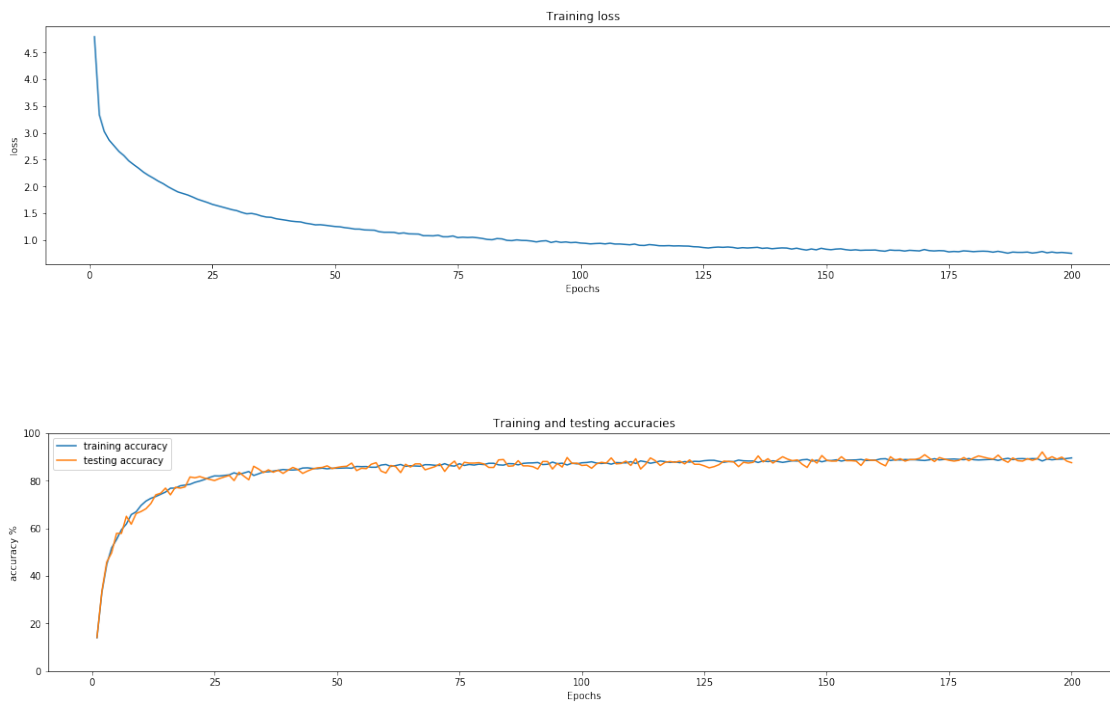

Here we see a slight increease in the accuracy reaching upto 90%

Finally let's try 2 hidden layers with 15 and 15 neurons in each respectively. We'll keep a regularization parameter to prevent overfitting.

```
[7]: net = network.Linear_Network_CrossValidation([25,15,15,10])
```

```
[8]: net.SGD(training_data, 200, 10, 0.1, lmbda=1.0)
```

```
Epoch 1
Training loss: 4.4850064782964885078
Training accuracy: 9.916666666666668%
Test accuracy: 8.499999999999998%

Epoch 2
Training loss: 3.356805739666318296
Training accuracy: 14.666666666666668%
Test accuracy: 16.5%

Epoch 3
Training loss: 3.2432002048708097601
Training accuracy: 19.041666666666664%
Test accuracy: 19.499999999999996%

Epoch 4
Training loss: 3.216742481772191744
Training accuracy: 18.708333333333332%
Test accuracy: 19.833333333333332%

Epoch 5
Training loss: 3.2048485018208617062
Training accuracy: 22.333333333333336%
Test accuracy: 22.499999999999996%

Epoch 6
Training loss: 3.1970616176103365687
Training accuracy: 23.458333333333336%
Test accuracy: 22.333333333333336%

Epoch 7
Training loss: 3.1887496605536887408
Training accuracy: 22.375%
Test accuracy: 22.833333333333332%

Epoch 8
Training loss: 3.1825580041979554334
Training accuracy: 23.208333333333336%
Test accuracy: 22.499999999999996%

Epoch 9
Training loss: 3.170211927743147328
Training accuracy: 27.083333333333336%
```

```
Test accuracy: 22.333333333333332%

Epoch 10
Training loss: 3.160177211390351139
Training accuracy: 23.833333333333336%
Test accuracy: 21.333333333333336%

Epoch 11
Training loss: 3.1545633996040141993
Training accuracy: 30.458333333333336%
Test accuracy: 30.0%

Epoch 12
Training loss: 3.1468754136148765752
Training accuracy: 31.166666666666668%
Test accuracy: 30.833333333333332%

Epoch 13
Training loss: 3.138705717917055301
Training accuracy: 32.291666666666664%
Test accuracy: 33.333333333333336%

Epoch 14
Training loss: 3.1260554735067432575
Training accuracy: 34.54166666666667%
Test accuracy: 34.83333333333333%

Epoch 15
Training loss: 3.1148095681350496632
Training accuracy: 35.29166666666667%
Test accuracy: 37.66666666666667%

Epoch 16
Training loss: 3.0997341956324029333
Training accuracy: 34.25%
Test accuracy: 26.833333333333332%

Epoch 17
Training loss: 3.0928784680679185012
Training accuracy: 33.75%
Test accuracy: 32.5%

Epoch 18
Training loss: 3.0811709984576454067
Training accuracy: 35.458333333333336%
Test accuracy: 35.833333333333336%

Epoch 19
```

Training loss: 3.067233034350844062
Training accuracy: 35.70833333333333%
Test accuracy: 34.66666666666667%

Epoch 20
Training loss: 3.0557381413786215698
Training accuracy: 36.83333333333333%
Test accuracy: 39.5%

Epoch 21
Training loss: 3.0392835509045703535
Training accuracy: 39.20833333333333%
Test accuracy: 36.5%

Epoch 22
Training loss: 3.024857649851178058
Training accuracy: 39.16666666666667%
Test accuracy: 39.5%

Epoch 23
Training loss: 3.0095757197702931425
Training accuracy: 39.333333333333336%
Test accuracy: 38.66666666666667%

Epoch 24
Training loss: 2.9968851175881324619
Training accuracy: 40.33333333333333%
Test accuracy: 40.666666666666664%

Epoch 25
Training loss: 2.973875056258342664
Training accuracy: 41.291666666666664%
Test accuracy: 40.00000000000001%

Epoch 26
Training loss: 2.9609351706824136642
Training accuracy: 41.625%
Test accuracy: 43.66666666666667%

Epoch 27
Training loss: 2.9467658716130598624
Training accuracy: 40.83333333333333%
Test accuracy: 43.50000000000001%

Epoch 28
Training loss: 2.9211061901704385986
Training accuracy: 42.458333333333336%
Test accuracy: 39.666666666666664%

```
Epoch 29
Training loss: 2.9053420813347378279
Training accuracy: 41.99999999999999%
Test accuracy: 40.16666666666667%

Epoch 30
Training loss: 2.8872881228682418546
Training accuracy: 41.00000000000001%
Test accuracy: 43.66666666666667%

Epoch 31
Training loss: 2.8610202994739266263
Training accuracy: 42.95833333333333%
Test accuracy: 40.16666666666667%

Epoch 32
Training loss: 2.8441363051991943155
Training accuracy: 43.54166666666667%
Test accuracy: 42.5%

Epoch 33
Training loss: 2.8330324357810255972
Training accuracy: 43.95833333333333%
Test accuracy: 48.0%

Epoch 34
Training loss: 2.8056650006215426841
Training accuracy: 46.25000000000001%
Test accuracy: 45.833333333333336%

Epoch 35
Training loss: 2.7815924588915961059
Training accuracy: 46.83333333333333%
Test accuracy: 43.83333333333333%

Epoch 36
Training loss: 2.7573104345573065113
Training accuracy: 46.95833333333333%
Test accuracy: 45.833333333333336%

Epoch 37
Training loss: 2.7413646196354858639
Training accuracy: 46.99999999999999%
Test accuracy: 46.5%

Epoch 38
Training loss: 2.7271742149533882565
```

```
Training accuracy: 47.20833333333%
Test accuracy: 51.5%


Epoch 39
Training loss: 2.7001558407477445482
Training accuracy: 48.125%
Test accuracy: 46.0%


Epoch 40
Training loss: 2.6861866865012374993
Training accuracy: 49.0%
Test accuracy: 50.33333333333333%


Epoch 41
Training loss: 2.6583238320076925628
Training accuracy: 50.083333333333336%
Test accuracy: 48.83333333333333%


Epoch 42
Training loss: 2.6317074658853856822
Training accuracy: 51.04166666666667%
Test accuracy: 46.833333333333336%


Epoch 43
Training loss: 2.6138176654526768892
Training accuracy: 50.416666666666664%
Test accuracy: 49.83333333333333%


Epoch 44
Training loss: 2.6044999991294236416
Training accuracy: 50.33333333333333%
Test accuracy: 52.16666666666667%


Epoch 45
Training loss: 2.5793089454023069506
Training accuracy: 52.5%
Test accuracy: 51.833333333333336%


Epoch 46
Training loss: 2.5632994127822610584
Training accuracy: 52.125%
Test accuracy: 53.333333333333336%


Epoch 47
Training loss: 2.5347167003372911306
Training accuracy: 52.25%
Test accuracy: 50.66666666666667%
```

```
Epoch 48
Training loss: 2.5081880780601504175
Training accuracy: 53.125%
Test accuracy: 50.66666666666667%

Epoch 49
Training loss: 2.5019774462264822306
Training accuracy: 52.625%
Test accuracy: 55.333333333333336%

Epoch 50
Training loss: 2.4764848894662325287
Training accuracy: 54.041666666666664%
Test accuracy: 53.5%

Epoch 51
Training loss: 2.452226904106796056
Training accuracy: 55.45833333333333%
Test accuracy: 53.0%

Epoch 52
Training loss: 2.4271786882900317133
Training accuracy: 55.833333333333336%
Test accuracy: 53.166666666666664%

Epoch 53
Training loss: 2.4087328773039589653
Training accuracy: 56.95833333333333%
Test accuracy: 54.833333333333336%

Epoch 54
Training loss: 2.3978986053310927494
Training accuracy: 56.625%
Test accuracy: 58.0%

Epoch 55
Training loss: 2.3741816651786440753
Training accuracy: 57.791666666666664%
Test accuracy: 57.666666666666664%

Epoch 56
Training loss: 2.3542260355808065624
Training accuracy: 58.625%
Test accuracy: 56.16666666666667%

Epoch 57
Training loss: 2.335589776492270719
Training accuracy: 59.791666666666664%
```

```
Test accuracy: 56.33333333333333%


Epoch 58
Training loss: 2.3231360258998368397
Training accuracy: 59.125%
Test accuracy: 59.833333333333336%


Epoch 59
Training loss: 2.3041447095744746758
Training accuracy: 60.20833333333333%
Test accuracy: 59.333333333333336%


Epoch 60
Training loss: 2.2830765835297399668
Training accuracy: 60.125000000000014%
Test accuracy: 62.0%


Epoch 61
Training loss: 2.2635407150988852269
Training accuracy: 61.708333333333336%
Test accuracy: 61.16666666666667%


Epoch 62
Training loss: 2.243757019773092045
Training accuracy: 62.54166666666667%
Test accuracy: 61.666666666666664%


Epoch 63
Training loss: 2.2319958972046281365
Training accuracy: 62.70833333333333%
Test accuracy: 64.83333333333333%


Epoch 64
Training loss: 2.2069541656405040695
Training accuracy: 63.958333333333336%
Test accuracy: 63.166666666666664%


Epoch 65
Training loss: 2.1899232454828515661
Training accuracy: 64.0%
Test accuracy: 64.66666666666666%


Epoch 66
Training loss: 2.1752786555805750444
Training accuracy: 64.70833333333334%
Test accuracy: 64.5%


Epoch 67
```

Training loss: 2.1423056912061856105
Training accuracy: 65.12499999999999%
Test accuracy: 63.33333333333333%

Epoch 68
Training loss: 2.1407773707040331037
Training accuracy: 65.33333333333333%
Test accuracy: 65.49999999999999%

Epoch 69
Training loss: 2.1057280311128920547
Training accuracy: 66.70833333333334%
Test accuracy: 62.666666666666664%

Epoch 70
Training loss: 2.0927274522981452404
Training accuracy: 66.41666666666667%
Test accuracy: 66.83333333333333%

Epoch 71
Training loss: 2.0839770262945515422
Training accuracy: 66.29166666666666%
Test accuracy: 68.00000000000001%

Epoch 72
Training loss: 2.0677876537057070226
Training accuracy: 67.33333333333333%
Test accuracy: 67.16666666666666%

Epoch 73
Training loss: 2.0608752314827123479
Training accuracy: 67.25%
Test accuracy: 70.66666666666667%

Epoch 74
Training loss: 2.0311699091818183954
Training accuracy: 68.37500000000001%
Test accuracy: 69.5%

Epoch 75
Training loss: 2.0246517738084845745
Training accuracy: 68.75%
Test accuracy: 71.0%

Epoch 76
Training loss: 1.9988725003135568018
Training accuracy: 69.54166666666667%
Test accuracy: 69.66666666666666%

```
Epoch 77
Training loss: 1.9846538585974991995
Training accuracy: 69.58333333333333%
Test accuracy: 69.16666666666667%

Epoch 78
Training loss: 1.9519214609644449316
Training accuracy: 70.91666666666666%
Test accuracy: 67.16666666666666%

Epoch 79
Training loss: 1.9502124920417778286
Training accuracy: 70.5%
Test accuracy: 70.16666666666666%

Epoch 80
Training loss: 1.9353559834261043355
Training accuracy: 70.74999999999999%
Test accuracy: 72.83333333333333%

Epoch 81
Training loss: 1.9086712622242026586
Training accuracy: 71.91666666666667%
Test accuracy: 69.5%

Epoch 82
Training loss: 1.8981191522404278106
Training accuracy: 72.58333333333334%
Test accuracy: 70.33333333333333%

Epoch 83
Training loss: 1.8881939145386377953
Training accuracy: 72.625%
Test accuracy: 72.0%

Epoch 84
Training loss: 1.8706959282181593679
Training accuracy: 72.70833333333334%
Test accuracy: 71.16666666666667%

Epoch 85
Training loss: 1.8461974074922937936
Training accuracy: 73.79166666666666%
Test accuracy: 69.33333333333334%

Epoch 86
Training loss: 1.8439941128404285996
```

```
Training accuracy: 73.45833333333334%
Test accuracy: 73.5%


Epoch 87
Training loss: 1.8333436000222114443
Training accuracy: 73.83333333333334%
Test accuracy: 74.5%


Epoch 88
Training loss: 1.8197005523280204
Training accuracy: 74.16666666666666%
Test accuracy: 74.83333333333333%


Epoch 89
Training loss: 1.8105139833816417727
Training accuracy: 73.16666666666667%
Test accuracy: 78.66666666666667%


Epoch 90
Training loss: 1.792401468222083781
Training accuracy: 74.45833333333334%
Test accuracy: 75.5%


Epoch 91
Training loss: 1.7810047174499233268
Training accuracy: 74.625%
Test accuracy: 75.33333333333333%


Epoch 92
Training loss: 1.765299902122823938
Training accuracy: 74.37500000000001%
Test accuracy: 75.99999999999999%


Epoch 93
Training loss: 1.7498873781278681779
Training accuracy: 75.41666666666667%
Test accuracy: 73.0%


Epoch 94
Training loss: 1.726827081529860334
Training accuracy: 75.45833333333333%
Test accuracy: 75.0%


Epoch 95
Training loss: 1.7295224553217010096
Training accuracy: 75.29166666666667%
Test accuracy: 75.66666666666667%
```

```
Epoch 96
Training loss: 1.7246548127522062297
Training accuracy: 75.70833333333%
Test accuracy: 77.66666666666%

Epoch 97
Training loss: 1.7032964189077534928
Training accuracy: 75.875%
Test accuracy: 76.16666666666%

Epoch 98
Training loss: 1.6995911223933147227
Training accuracy: 76.29166666666%
Test accuracy: 76.49999999999%

Epoch 99
Training loss: 1.6651640978227772287
Training accuracy: 76.79166666666%
Test accuracy: 75.5%

Epoch 100
Training loss: 1.6808723451918177181
Training accuracy: 76.49999999999%
Test accuracy: 78.16666666666%

Epoch 101
Training loss: 1.6429320071485863347
Training accuracy: 77.04166666666%
Test accuracy: 76.16666666667%

Epoch 102
Training loss: 1.6394109113463755749
Training accuracy: 77.41666666666%
Test accuracy: 75.49999999999%

Epoch 103
Training loss: 1.6277586155694477767
Training accuracy: 77.75%
Test accuracy: 74.33333333333%

Epoch 104
Training loss: 1.6283481274764727302
Training accuracy: 77.54166666666%
Test accuracy: 77.66666666666%

Epoch 105
Training loss: 1.6203712566662611195
Training accuracy: 77.75000000000001%
```

Test accuracy: 78.16666666666667%

Epoch 106
Training loss: 1.6044070173546441207
Training accuracy: 78.54166666666667%
Test accuracy: 76.33333333333333%

Epoch 107
Training loss: 1.5839484847893517887
Training accuracy: 79.04166666666667%
Test accuracy: 75.33333333333333%

Epoch 108
Training loss: 1.5782264454868773425
Training accuracy: 78.75%
Test accuracy: 77.83333333333334%

Epoch 109
Training loss: 1.5807373960647599396
Training accuracy: 78.66666666666667%
Test accuracy: 78.33333333333334%

Epoch 110
Training loss: 1.5827687259254224922
Training accuracy: 78.375%
Test accuracy: 79.99999999999999%

Epoch 111
Training loss: 1.5682899960488519013
Training accuracy: 78.29166666666666%
Test accuracy: 81.33333333333334%

Epoch 112
Training loss: 1.5349081594944330439
Training accuracy: 79.70833333333334%
Test accuracy: 76.16666666666667%

Epoch 113
Training loss: 1.518288277049317607
Training accuracy: 79.83333333333333%
Test accuracy: 76.66666666666667%

Epoch 114
Training loss: 1.5281130021398682932
Training accuracy: 79.04166666666667%
Test accuracy: 79.0%

Epoch 115

```
Training loss: 1.522334181367994055
Training accuracy: 79.125%
Test accuracy: 79.83333333333333%


Epoch 116
Training loss: 1.5117069466936084875
Training accuracy: 78.87500000000001%
Test accuracy: 81.66666666666667%


Epoch 117
Training loss: 1.5090857267319704102
Training accuracy: 79.33333333333333%
Test accuracy: 81.66666666666666%


Epoch 118
Training loss: 1.498418350903087802
Training accuracy: 79.54166666666666%
Test accuracy: 81.66666666666667%


Epoch 119
Training loss: 1.4842821763492565848
Training accuracy: 80.41666666666667%
Test accuracy: 79.33333333333334%


Epoch 120
Training loss: 1.4834437734570311832
Training accuracy: 79.91666666666667%
Test accuracy: 81.83333333333334%


Epoch 121
Training loss: 1.4544172587766003447
Training accuracy: 80.58333333333333%
Test accuracy: 79.66666666666666%


Epoch 122
Training loss: 1.4503141223089414038
Training accuracy: 80.83333333333333%
Test accuracy: 80.83333333333334%


Epoch 123
Training loss: 1.4521998225408930161
Training accuracy: 80.5%
Test accuracy: 81.66666666666666%


Epoch 124
Training loss: 1.4330439050394409963
Training accuracy: 81.125%
Test accuracy: 80.33333333333334%
```

```
Epoch 125
Training loss: 1.4178019349950530134
Training accuracy: 81.25%
Test accuracy: 81.0%

Epoch 126
Training loss: 1.4063413248700011705
Training accuracy: 81.875%
Test accuracy: 78.33333333333333%

Epoch 127
Training loss: 1.4171777733106056664
Training accuracy: 81.66666666666667%
Test accuracy: 81.0%

Epoch 128
Training loss: 1.4024362252603030789
Training accuracy: 82.04166666666667%
Test accuracy: 80.5%

Epoch 129
Training loss: 1.3828547646318187213
Training accuracy: 81.87499999999999%
Test accuracy: 80.0%

Epoch 130
Training loss: 1.3835596830702955788
Training accuracy: 81.50000000000001%
Test accuracy: 82.5%

Epoch 131
Training loss: 1.3752081231646720192
Training accuracy: 82.29166666666667%
Test accuracy: 80.5%

Epoch 132
Training loss: 1.3814652758042693465
Training accuracy: 82.04166666666666%
Test accuracy: 82.66666666666667%

Epoch 133
Training loss: 1.3622488750226002483
Training accuracy: 82.25%
Test accuracy: 82.16666666666667%

Epoch 134
Training loss: 1.3708993073272228599
```

```
Training accuracy: 81.91666666666666%
Test accuracy: 83.66666666666667%


Epoch 135
Training loss: 1.3451353839415193901
Training accuracy: 82.875%
Test accuracy: 80.5%


Epoch 136
Training loss: 1.3326920537816569901
Training accuracy: 83.08333333333333%
Test accuracy: 79.83333333333333%


Epoch 137
Training loss: 1.3385968511041070021
Training accuracy: 83.08333333333334%
Test accuracy: 82.5%


Epoch 138
Training loss: 1.3337758253238553851
Training accuracy: 83.04166666666666%
Test accuracy: 82.33333333333334%


Epoch 139
Training loss: 1.3259104316468697047
Training accuracy: 82.91666666666667%
Test accuracy: 84.00000000000001%


Epoch 140
Training loss: 1.3087905092210158444
Training accuracy: 83.66666666666667%
Test accuracy: 81.0%


Epoch 141
Training loss: 1.319660184317914443
Training accuracy: 82.625%
Test accuracy: 86.49999999999999%


Epoch 142
Training loss: 1.3064673179315585598
Training accuracy: 83.125%
Test accuracy: 83.66666666666667%


Epoch 143
Training loss: 1.2809365515009170014
Training accuracy: 83.70833333333333%
Test accuracy: 81.66666666666667%
```

```
Epoch 144
Training loss: 1.2872712545188807299
Training accuracy: 83.50000000000001%
Test accuracy: 82.66666666666667%

Epoch 145
Training loss: 1.2958660797949415038
Training accuracy: 83.08333333333333%
Test accuracy: 85.83333333333333%

Epoch 146
Training loss: 1.2721159298840193667
Training accuracy: 83.87499999999999%
Test accuracy: 82.33333333333334%

Epoch 147
Training loss: 1.2859210391615178577
Training accuracy: 83.25%
Test accuracy: 84.83333333333334%

Epoch 148
Training loss: 1.2680142684812489846
Training accuracy: 83.95833333333333%
Test accuracy: 82.83333333333333%

Epoch 149
Training loss: 1.2413955859140263797
Training accuracy: 84.16666666666666%
Test accuracy: 82.16666666666667%

Epoch 150
Training loss: 1.2411662072628656659
Training accuracy: 84.45833333333333%
Test accuracy: 81.83333333333334%

Epoch 151
Training loss: 1.2405623698211687882
Training accuracy: 84.41666666666666%
Test accuracy: 82.83333333333334%

Epoch 152
Training loss: 1.2444146140355801715
Training accuracy: 84.0%
Test accuracy: 83.83333333333334%

Epoch 153
Training loss: 1.2325377299862857082
Training accuracy: 84.20833333333334%
```

```
Test accuracy: 83.66666666666667%


Epoch 154
Training loss: 1.22127167598089548
Training accuracy: 84.83333333333334%
Test accuracy: 82.16666666666666%


Epoch 155
Training loss: 1.2237970114797168009
Training accuracy: 84.16666666666666%
Test accuracy: 85.0%


Epoch 156
Training loss: 1.2202835807130927578
Training accuracy: 84.04166666666666%
Test accuracy: 85.33333333333333%


Epoch 157
Training loss: 1.2075148067370243655
Training accuracy: 84.54166666666666%
Test accuracy: 83.5%


Epoch 158
Training loss: 1.1986431462950857805
Training accuracy: 84.37499999999999%
Test accuracy: 84.16666666666666%


Epoch 159
Training loss: 1.2031189737939484987
Training accuracy: 84.875%
Test accuracy: 83.83333333333334%


Epoch 160
Training loss: 1.2032240720395311271
Training accuracy: 84.29166666666666%
Test accuracy: 85.33333333333334%


Epoch 161
Training loss: 1.1894628091619322769
Training accuracy: 84.375%
Test accuracy: 85.16666666666666%


Epoch 162
Training loss: 1.1763866716792061334
Training accuracy: 84.45833333333333%
Test accuracy: 84.83333333333333%


Epoch 163
```

Training loss: 1.1822431818465281286
Training accuracy: 84.91666666666667%
Test accuracy: 83.33333333333334%

Epoch 164
Training loss: 1.1836099314217062288
Training accuracy: 84.08333333333334%
Test accuracy: 86.5%

Epoch 165
Training loss: 1.1737219363981122391
Training accuracy: 85.04166666666666%
Test accuracy: 83.66666666666666%

Epoch 166
Training loss: 1.1658452517535298158
Training accuracy: 84.87499999999999%
Test accuracy: 84.0%

Epoch 167
Training loss: 1.1553331757552881924
Training accuracy: 85.0%
Test accuracy: 84.49999999999999%

Epoch 168
Training loss: 1.1557910241415638655
Training accuracy: 85.04166666666666%
Test accuracy: 84.0%

Epoch 169
Training loss: 1.1301753500787422925
Training accuracy: 85.99999999999999%
Test accuracy: 80.49999999999999%

Epoch 170
Training loss: 1.1343607785122263982
Training accuracy: 85.0%
Test accuracy: 83.83333333333334%

Epoch 171
Training loss: 1.1510424740460766448
Training accuracy: 84.79166666666667%
Test accuracy: 85.5%

Epoch 172
Training loss: 1.1249354224349255202
Training accuracy: 85.45833333333333%
Test accuracy: 83.49999999999999%

```
Epoch 173
Training loss: 1.1247047363009302611
Training accuracy: 85.54166666666666%
Test accuracy: 83.49999999999999%

Epoch 174
Training loss: 1.1237967223888397194
Training accuracy: 85.29166666666667%
Test accuracy: 84.83333333333334%

Epoch 175
Training loss: 1.1147545116480842052
Training accuracy: 85.625%
Test accuracy: 83.16666666666666%

Epoch 176
Training loss: 1.1047748684328695884
Training accuracy: 85.45833333333334%
Test accuracy: 84.99999999999999%

Epoch 177
Training loss: 1.1130921121547114152
Training accuracy: 85.16666666666666%
Test accuracy: 86.00000000000001%

Epoch 178
Training loss: 1.1109414043932233156
Training accuracy: 85.45833333333333%
Test accuracy: 85.00000000000001%

Epoch 179
Training loss: 1.1086664133032136249
Training accuracy: 85.37499999999999%
Test accuracy: 85.66666666666666%

Epoch 180
Training loss: 1.1016285063092241905
Training accuracy: 85.79166666666667%
Test accuracy: 84.83333333333334%

Epoch 181
Training loss: 1.0970076731483456799
Training accuracy: 85.41666666666666%
Test accuracy: 86.0%

Epoch 182
Training loss: 1.0886376006681835261
```

Training accuracy: 85.58333333333333%
Test accuracy: 85.33333333333333%

Epoch 183
Training loss: 1.076965967019693096
Training accuracy: 86.00000000000001%
Test accuracy: 83.83333333333334%

Epoch 184
Training loss: 1.0791030542458641371
Training accuracy: 85.79166666666667%
Test accuracy: 84.5%

Epoch 185
Training loss: 1.086553503845163421
Training accuracy: 85.5%
Test accuracy: 86.33333333333334%

Epoch 186
Training loss: 1.0757990159937518747
Training accuracy: 86.00000000000001%
Test accuracy: 85.5%

Epoch 187
Training loss: 1.0621987199240161012
Training accuracy: 86.16666666666666%
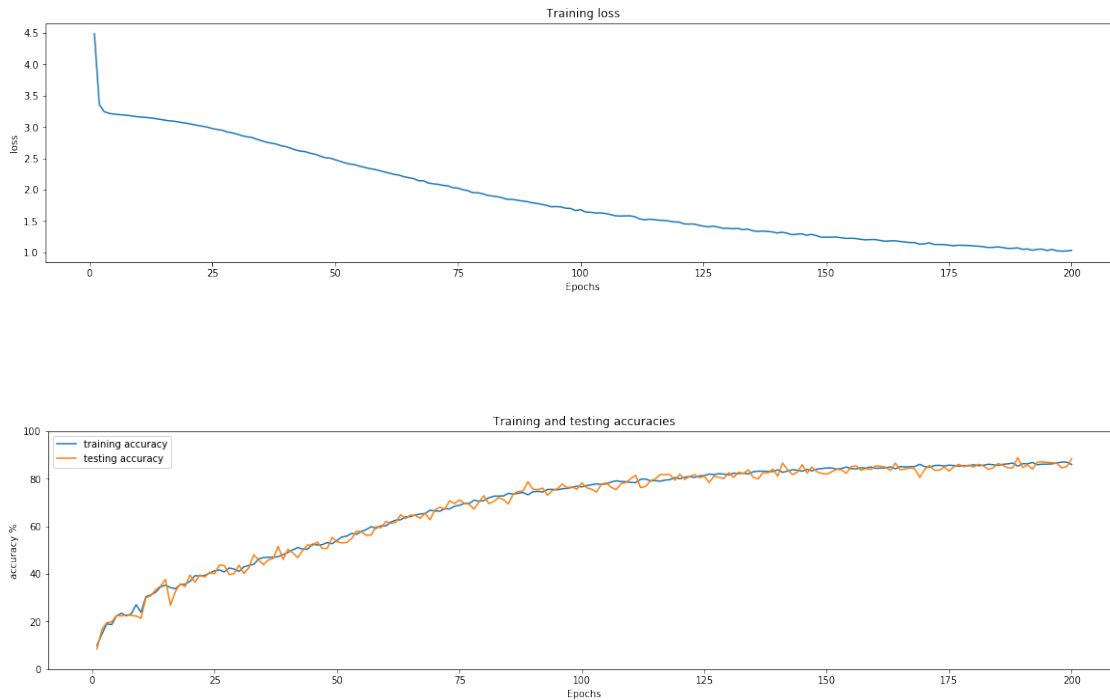Test accuracy: 84.5%

Epoch 188
Training loss: 1.0637489190452939322
Training accuracy: 86.5%
Test accuracy: 84.49999999999999%

Epoch 189
Training loss: 1.0705150586464930026
Training accuracy: 85.16666666666667%
Test accuracy: 88.83333333333334%

Epoch 190
Training loss: 1.0476088977868649868
Training accuracy: 86.25%
Test accuracy: 84.66666666666666%

Epoch 191
Training loss: 1.0530836098361415143
Training accuracy: 86.16666666666666%
Test accuracy: 85.66666666666667%

```
Epoch 192
Training loss: 1.0334311235185067992
Training accuracy: 86.74999999999999%
Test accuracy: 84.00000000000001%

Epoch 193
Training loss: 1.0476885268063559933
Training accuracy: 85.79166666666666%
Test accuracy: 86.83333333333334%

Epoch 194
Training loss: 1.0500284734024286511
Training accuracy: 86.04166666666667%
Test accuracy: 87.0%

Epoch 195
Training loss: 1.0287267488290544034
Training accuracy: 86.04166666666666%
Test accuracy: 86.83333333333334%

Epoch 196
Training loss: 1.0452435175130006009
Training accuracy: 86.16666666666666%
Test accuracy: 86.66666666666666%

Epoch 197
Training loss: 1.0254561796901198003
Training accuracy: 86.54166666666666%
Test accuracy: 86.33333333333333%

Epoch 198
Training loss: 1.018080162968010525
Training accuracy: 86.91666666666667%
Test accuracy: 84.49999999999999%

Epoch 199
Training loss: 1.0215660936076044658
Training accuracy: 86.87500000000001%
Test accuracy: 85.0%

Epoch 200
Training loss: 1.0314826435002884537
Training accuracy: 85.875%
Test accuracy: 88.33333333333333%
```

Training loss

Training and testing accuracies

We still don't get a result as good as the original images.

This might be because when reducing the dimensionality of the image, we may lose some spatial information of the image. This will make it tough for the neural network to learn some key spatial differences that separates one digit from another.

2.4 With this we end the 1st report. In the next report, we'll train a CNN using PyTorch to train on the entire MNIST dataset

[ ]:

Prithvi_Poddar_17191_report_2_CNN

June 23, 2020

1 Report 2

1.1 Prithvi Poddar 17191

1.1.1 In this report, we'll implement a simple Convolutional Neural Network and train it on the entire MNIST dataset so see the results.

P.S. This notebook was created entirely in google colab as it provides GPU facility, so that the training can be faster. Thus, I had to mention all the classes wwithin the notebook itself as I cannot import classes from my local machine. We will be using PyTorch for this notebook

Run the next line if your computer doesn't have pytorch installed. Otherwise you can skip it.

```
[ ]: !pip3 install torch torchvision
```

```
[1]: import torch
     import numpy as np
     from torch import nn
     import torch.nn.functional as F
     from torchvision import datasets, transforms
     import matplotlib.pyplot as plt
```

```
[2]: device = torch.device("cuda:0" if torch.cuda.is_available() else "cpu")
```

```
[3]: device
```

```
[3]: device(type='cuda', index=0)
```

Creating the transform function to convert the images into tensors that are readable by pytorch

```
[4]: transform = transforms.Compose([transforms.ToTensor(), transforms.Normalize((0.
     ↪5,),(0.5))])
```

Downloading the MNIST dataset from torchvision

```
[5]: training_dataset = datasets.MNIST(root='./data', train=True, download=True,␣
     ↪transform=transform)
     validation_dataset = datasets.MNIST(root='./data', train=False, download=True,␣
     ↪transform=transform)
```

```
Downloading http://yann.lecun.com/exdb/mnist/train-images-idx3-ubyte.gz to
./data/MNIST/raw/train-images-idx3-ubyte.gz
```

1

```
HBox(children=(FloatProgress(value=1.0, bar_style='info', max=1.0), HTML(value='')))
```

```
Extracting ./data/MNIST/raw/train-images-idx3-ubyte.gz to ./data/MNIST/raw
Downloading http://yann.lecun.com/exdb/mnist/train-labels-idx1-ubyte.gz to
./data/MNIST/raw/train-labels-idx1-ubyte.gz
```

```
HBox(children=(FloatProgress(value=1.0, bar_style='info', max=1.0), HTML(value='')))
```

```
Extracting ./data/MNIST/raw/train-labels-idx1-ubyte.gz to ./data/MNIST/raw
Downloading http://yann.lecun.com/exdb/mnist/t10k-images-idx3-ubyte.gz to
./data/MNIST/raw/t10k-images-idx3-ubyte.gz
```

```
HBox(children=(FloatProgress(value=1.0, bar_style='info', max=1.0), HTML(value='')))
```

```
Extracting ./data/MNIST/raw/t10k-images-idx3-ubyte.gz to ./data/MNIST/raw
Downloading http://yann.lecun.com/exdb/mnist/t10k-labels-idx1-ubyte.gz to
./data/MNIST/raw/t10k-labels-idx1-ubyte.gz
```

```
HBox(children=(FloatProgress(value=1.0, bar_style='info', max=1.0), HTML(value='')))
```

```
Extracting ./data/MNIST/raw/t10k-labels-idx1-ubyte.gz to ./data/MNIST/raw
Processing...
Done!
```

```
/pytorch/torch/csrc/utils/tensor_numpy.cpp:141: UserWarning: The given NumPy
array is not writeable, and PyTorch does not support non-writeable tensors. This
means you can write to the underlying (supposedly non-writeable) NumPy array
using the tensor. You may want to copy the array to protect its data or make it
writeable before converting it to a tensor. This type of warning will be
suppressed for the rest of this program.
```

[6]:
```python
training_loader = torch.utils.data.DataLoader(training_dataset, batch_size=100,
 ↪shuffle=True)
validation_loader = torch.utils.data.DataLoader(validation_dataset, batch_size
 ↪= 100, shuffle=False)
```

Now we define the training function. We'll be using the Cross Entropy loss and the Adam optimizer.

[7]:
```python
def train(model, epochs, learning_rate):
    criterion = nn.CrossEntropyLoss()
    optimizer = torch.optim.Adam(model.parameters(), lr=learning_rate)

    training_losses=[]
    training_accuracies=[]
    test_losses=[]
```

```python
test_accuracies=[]

for e in range(epochs):
  training_loss = 0.0
  training_accuracy = 0.0
  test_loss = 0.0
  test_accuracy = 0.0

  for inputs, labels in training_loader:
    inputs = inputs.to(device)
    labels = labels.to(device)
    outputs = model.forward(inputs)
    loss = criterion(outputs, labels)
    optimizer.zero_grad()
    loss.backward()
    optimizer.step()

    _, preds = torch.max(outputs, 1) #returns the max value along axis 1␣
→along with its index
    training_loss += loss.item()
    training_accuracy += torch.sum(preds==labels.data)

  else:
    with torch.no_grad():
      for val_inputs, val_labels in validation_loader:
        val_inputs = val_inputs.to(device)
        val_labels = val_labels.to(device)
        val_outputs = model.forward(val_inputs)
        val_loss = criterion(val_outputs, val_labels)

        _, val_preds = torch.max(val_outputs, 1)
        test_loss += val_loss.item()
        test_accuracy += torch.sum(val_preds == val_labels.data)

    epoch_loss = training_loss/len(training_loader)
    epoch_acc = training_accuracy.float()/ len(training_loader)
    training_losses.append(epoch_loss)
    training_accuracies.append(epoch_acc)

    val_epoch_loss = test_loss/len(validation_loader)
    val_epoch_acc = test_accuracy.float()/ len(validation_loader)
    test_losses.append(val_epoch_loss)
    test_accuracies.append(val_epoch_acc)
    print('epoch :', (e+1))
    print('training loss: {:.4f}, acc {:.4f} '.format(epoch_loss, epoch_acc.
→item()))
```

```
        print('validation loss: {:.4f}, validation acc {:.4f} '.
   ↪format(val_epoch_loss, val_epoch_acc.item())))

   return training_losses, training_accuracies, test_losses ,test_accuracies
```

Defining the plotting function to plot the accuracies and losses later on

```
[8]: def plot_loss(training_losses, testing_losses):
       plt.plot(training_losses, label='training loss')
       plt.plot(testing_losses, label='testing loss')
       plt.legend()
```

```
[9]: def plot_accuracy(training_accuracies, testing_accuracies):
       plt.plot(training_accuracies, label='training accuracy')
       plt.plot(testing_accuracies, label='testing accuracy')
       plt.legend()
```

1.1.2   Having defined the helping functions, we can now start forming our CNN module.

Since, training takes substantial time, even when using a gpu, I'll be referring to the results from https://www.kaggle.com/cdeotte/how-to-choose-cnn-architecture-mnist

In the above mentioned link, experiments have already been done to determine the optimum parameters for training on the MNIST data set   For our model ,we'll have **2 convolutional layers**. The **first one with 32 filetrs and second one with 64 filters. Filter size will be 5 with stride=1** There will be **2 fully connected layers with 128 neurons in the first layer and 10 in the second layer** which will be the output layer. Activation function will be **RELU with softmax applied to the last layer**. Intermediate **max pooling will be done with kernel size=2 and strides=2. Dropout in the fully connected layer will be 0.4**

Important formulae that we'll need are:

Output height = ((height-kernel_size+2*padding)/stride)+1

Output width = ((width-kernel_size+2*padding)/stride)+1

We'll use max_pooling of size 2 and stride 2. This will simply half the size of the feature map, hence making our manual calculations easier

```
[10]: class Network_CNN(nn.Module):
          def __init__(self):
            super().__init__()
            self.conv1 = nn.Conv2d(1, 32, 5, 1)
            self.conv2 = nn.Conv2d(32, 64, 5, 1)
            self.fc1 = nn.Linear(4*4*64, 500)
            self.dropout1 = nn.Dropout(0.4)
            self.fc2 = nn.Linear(500, 10)
          def forward(self, x):
            x = F.relu(self.conv1(x))
            x = F.max_pool2d(x, 2, 2)
            x = F.relu(self.conv2(x))
            x = F.max_pool2d(x, 2, 2)
            x = x.view(-1, 4*4*64)
```

```
        x = F.relu(self.fc1(x))
        x = self.dropout1(x)
        x = F.softmax(self.fc2(x), dim=1)
        return x

    def visualize(self, x):
        a = F.relu(self.conv1(x))
        b = F.max_pool2d(a, 2, 2)
        c = F.relu(self.conv2(b))
        d = F.max_pool2d(c, 2, 2)
        return a, b, c, d
```

[11]:
```
model = Network_CNN().to(device)
model
```

[11]:
```
Network_CNN(
    (conv1): Conv2d(1, 32, kernel_size=(5, 5), stride=(1, 1))
    (conv2): Conv2d(32, 64, kernel_size=(5, 5), stride=(1, 1))
    (fc1): Linear(in_features=1024, out_features=500, bias=True)
    (dropout1): Dropout(p=0.4, inplace=False)
    (fc2): Linear(in_features=500, out_features=10, bias=True)
)
```

[12]:
```
training_losses, training_accuracies, test_losses, test_accuracies =␣
 ↪train(model, 50, 0.0001)
```

```
epoch : 1
training loss: 1.6740, acc 82.4383
validation loss: 1.5233, validation acc 94.5700
epoch : 2
training loss: 1.5132, acc 95.5450
validation loss: 1.4992, validation acc 96.7700
epoch : 3
training loss: 1.4975, acc 96.8550
validation loss: 1.4892, validation acc 97.6100
epoch : 4
training loss: 1.4887, acc 97.5900
validation loss: 1.4865, validation acc 97.7100
epoch : 5
training loss: 1.4851, acc 97.8933
validation loss: 1.4812, validation acc 98.3000
epoch : 6
training loss: 1.4818, acc 98.1717
validation loss: 1.4789, validation acc 98.4100
epoch : 7
training loss: 1.4793, acc 98.3817
validation loss: 1.4776, validation acc 98.5000
epoch : 8
training loss: 1.4776, acc 98.5467
```

```
validation loss: 1.4775, validation acc 98.4700
epoch : 9
training loss: 1.4760, acc 98.6900
validation loss: 1.4750, validation acc 98.8600
epoch : 10
training loss: 1.4743, acc 98.8600
validation loss: 1.4747, validation acc 98.7600
epoch : 11
training loss: 1.4739, acc 98.8900
validation loss: 1.4744, validation acc 98.8800
epoch : 12
training loss: 1.4727, acc 98.9783
validation loss: 1.4738, validation acc 98.8300
epoch : 13
training loss: 1.4723, acc 98.9950
validation loss: 1.4731, validation acc 98.9000
epoch : 14
training loss: 1.4711, acc 99.1150
validation loss: 1.4736, validation acc 98.8700
epoch : 15
training loss: 1.4704, acc 99.1850
validation loss: 1.4717, validation acc 99.0200
epoch : 16
training loss: 1.4700, acc 99.2300
validation loss: 1.4723, validation acc 98.9800
epoch : 17
training loss: 1.4689, acc 99.3133
validation loss: 1.4713, validation acc 99.0600
epoch : 18
training loss: 1.4693, acc 99.2733
validation loss: 1.4711, validation acc 99.1000
epoch : 19
training loss: 1.4683, acc 99.3767
validation loss: 1.4708, validation acc 99.0500
epoch : 20
training loss: 1.4682, acc 99.3850
validation loss: 1.4717, validation acc 99.0200
epoch : 21
training loss: 1.4679, acc 99.4067
validation loss: 1.4707, validation acc 99.1100
epoch : 22
training loss: 1.4675, acc 99.4367
validation loss: 1.4705, validation acc 99.1300
epoch : 23
training loss: 1.4671, acc 99.4983
validation loss: 1.4710, validation acc 99.0800
epoch : 24
training loss: 1.4668, acc 99.5150
```

```
validation loss: 1.4720, validation acc 99.0000
epoch : 25
training loss: 1.4667, acc 99.5133
validation loss: 1.4701, validation acc 99.1800
epoch : 26
training loss: 1.4665, acc 99.5233
validation loss: 1.4692, validation acc 99.2800
epoch : 27
training loss: 1.4660, acc 99.5817
validation loss: 1.4706, validation acc 99.0500
epoch : 28
training loss: 1.4660, acc 99.5717
validation loss: 1.4701, validation acc 99.1400
epoch : 29
training loss: 1.4656, acc 99.6117
validation loss: 1.4700, validation acc 99.1600
epoch : 30
training loss: 1.4657, acc 99.6067
validation loss: 1.4705, validation acc 99.1000
epoch : 31
training loss: 1.4654, acc 99.6317
validation loss: 1.4702, validation acc 99.1200
epoch : 32
training loss: 1.4651, acc 99.6567
validation loss: 1.4702, validation acc 99.1500
epoch : 33
training loss: 1.4651, acc 99.6633
validation loss: 1.4704, validation acc 99.1200
epoch : 34
training loss: 1.4650, acc 99.6583
validation loss: 1.4693, validation acc 99.2600
epoch : 35
training loss: 1.4648, acc 99.6883
validation loss: 1.4692, validation acc 99.2200
epoch : 36
training loss: 1.4648, acc 99.6833
validation loss: 1.4696, validation acc 99.1900
epoch : 37
training loss: 1.4646, acc 99.7050
validation loss: 1.4693, validation acc 99.2000
epoch : 38
training loss: 1.4644, acc 99.7200
validation loss: 1.4692, validation acc 99.2300
epoch : 39
training loss: 1.4642, acc 99.7433
validation loss: 1.4692, validation acc 99.2300
epoch : 40
training loss: 1.4642, acc 99.7217
```

```
validation loss: 1.4706, validation acc 99.0600
epoch : 41
training loss: 1.4639, acc 99.7650
validation loss: 1.4693, validation acc 99.2600
epoch : 42
training loss: 1.4639, acc 99.7700
validation loss: 1.4696, validation acc 99.1700
epoch : 43
training loss: 1.4640, acc 99.7483
validation loss: 1.4698, validation acc 99.1200
epoch : 44
training loss: 1.4639, acc 99.7567
validation loss: 1.4695, validation acc 99.1600
epoch : 45
training loss: 1.4639, acc 99.7633
validation loss: 1.4691, validation acc 99.2300
epoch : 46
training loss: 1.4635, acc 99.7917
validation loss: 1.4695, validation acc 99.1600
epoch : 47
training loss: 1.4636, acc 99.7800
validation loss: 1.4695, validation acc 99.2400
epoch : 48
training loss: 1.4636, acc 99.7800
validation loss: 1.4700, validation acc 99.1300
epoch : 49
training loss: 1.4635, acc 99.7933
validation loss: 1.4699, validation acc 99.1700
epoch : 50
training loss: 1.4634, acc 99.7967
validation loss: 1.4692, validation acc 99.2000
```

We get an accuracy of 99%, much more than what we achieved earlier using just fully connected linear neural networks

[13]:
```
plot_loss(training_losses, test_losses)
```
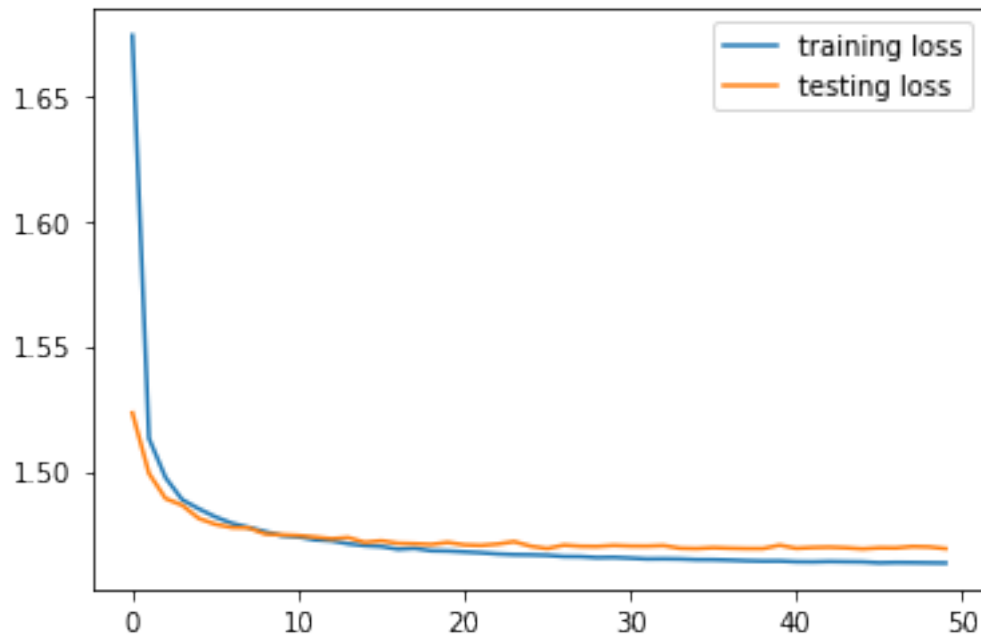
[14]: `plot_accuracy(training_accuracies, test_accuracies)`
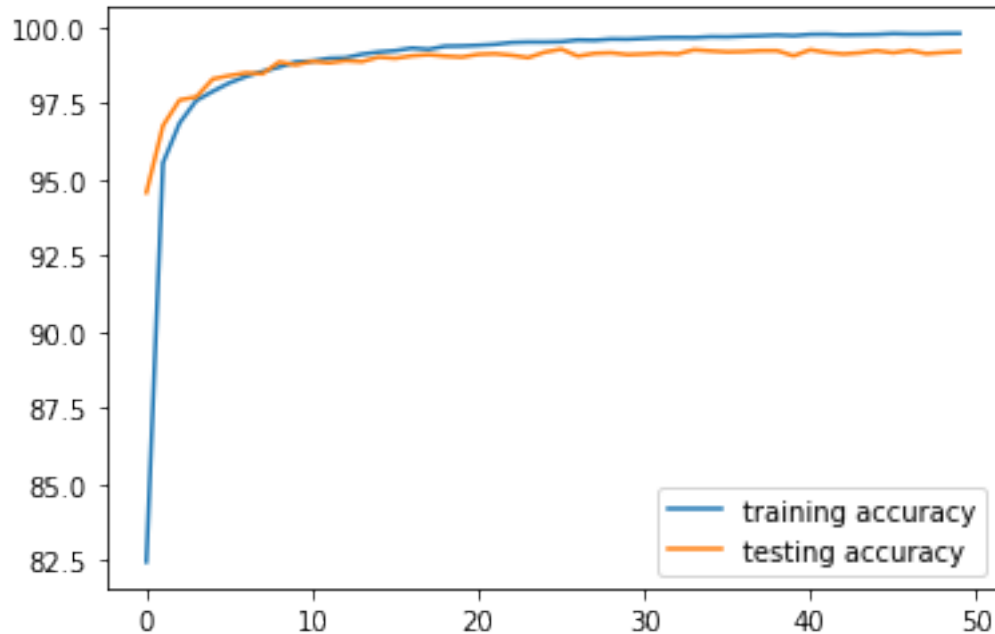
I had already done some pre-testing with the learning rate and it turned out that a slow learning rate provides a smoother accuracy curve as this network is quite complex in terms of the parameters to be learned

1.1.3 Now lets take a look at the indivisual concolutional layer outputs to try to visualize what the model is learning

```
[15]: data = iter(training_loader)
      images, labels = data.next()
```

```
[16]: def im_convert(tensor):
        image = tensor.cpu().clone().detach().numpy()
        image = image.transpose(1, 2, 0)
        image = image * np.array((0.5, 0.5, 0.5)) + np.array((0.5, 0.5, 0.5))
        image = image.clip(0, 1)
        return image
```

```
[31]: def filter_convert(tensor):
        image = tensor.cpu().clone().detach().numpy()
        image = image.clip(0, 1)
        return image
```

```
[23]: fig = plt.figure(figsize=(2, 2))
      plt.imshow(im_convert(images[3]))
      plt.title(str(labels[3].item()))
```
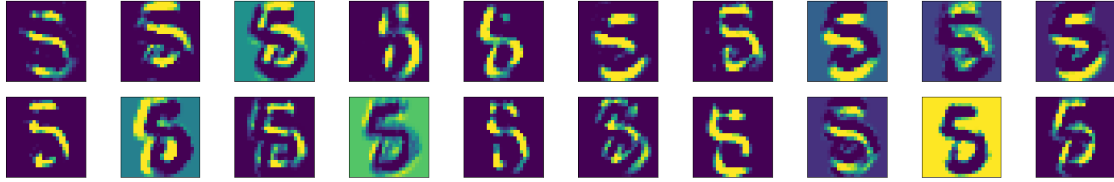
```
[23]: Text(0.5, 1.0, '5')
```
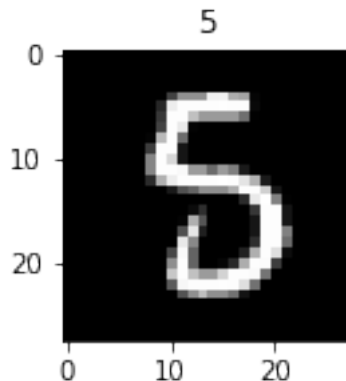

So this will be the input to our network i.e. the image of '5'

```
[19]: a, b, c, d = model.visualize(images.to(device))
```
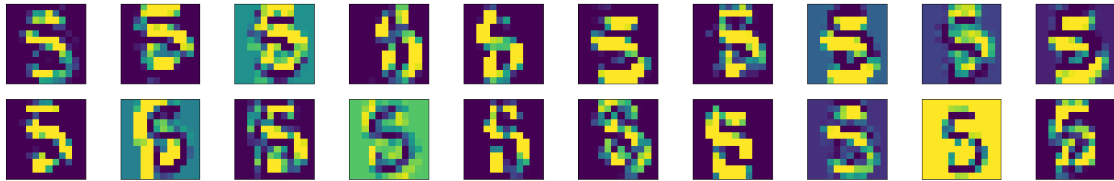
1.1.4 Looking at the output of Conv. Layer 1

```
[32]: fig = plt.figure(figsize=(25, 4))
      for idx in np.arange(20):
        ax = fig.add_subplot(2, 10, idx+1, xticks=[], yticks=[])
        plt.imshow(filter_convert(a[3][idx]))
```
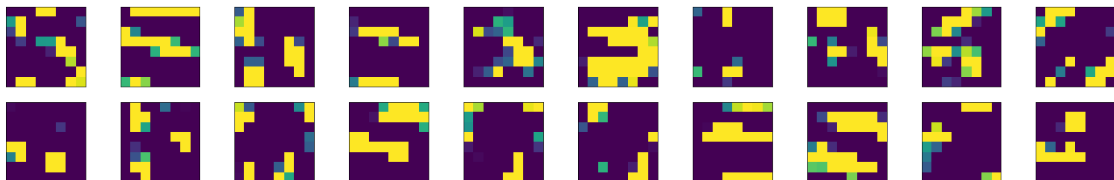
1.1.5 Output of the max pool of Conv Layer 1

```
[26]: fig = plt.figure(figsize=(25, 4))
      for idx in np.arange(20):
          ax = fig.add_subplot(2, 10, idx+1, xticks=[], yticks=[])
          plt.imshow(filter_convert(b[3][idx]))
```
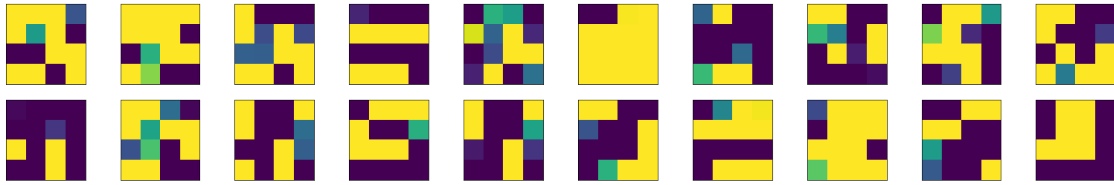

1.1.6 Output of Conv Layer 2

```
[27]: fig = plt.figure(figsize=(25, 4))
      for idx in np.arange(20):
          ax = fig.add_subplot(2, 10, idx+1, xticks=[], yticks=[])
          plt.imshow(filter_convert(c[3][idx]))
```


1.1.7 Output of max pool of Conv Layer 2

```
[28]: fig = plt.figure(figsize=(25, 4))
      for idx in np.arange(20):
          ax = fig.add_subplot(2, 10, idx+1, xticks=[], yticks=[])
```

```
plt.imshow(filter_convert(d[3][idx]))
```


1.2 Analysis of the above filter images

1.2.1 We can see that the 1st Conv Layer grabs the entire image of 5 as a whole along with the major curves and dashes

1.2.2 The second Conv Layer grabs the smaller curves and bends that are specific to only the number 5

1.2.3 At the end, the number 5 is just a superposition of these curves and dashes and hence the network is able to recognize it!!

[ ]:

Prithvi_Poddar_17191_Report_3_Autoencoders

June 24, 2020

1 Report 3: Autoencoders

1.1 Prithvi Poddar 17191

In this reort, we'll be developing Sparse autoencoder

1.2 Sparse Autoencoder

1.2.1 This autoencoder structure has been inspired by the note of Andrew NG on sparse autoencoders. We'll use the parameters mentioned in the notes itself

```
[1]: !pip3 install torch torchvision
```

```
Requirement already satisfied: torch in /usr/local/lib/python3.6/dist-packages
(1.5.1+cu101)
Requirement already satisfied: torchvision in /usr/local/lib/python3.6/dist-
packages (0.6.1+cu101)
Requirement already satisfied: future in /usr/local/lib/python3.6/dist-packages
(from torch) (0.16.0)
Requirement already satisfied: numpy in /usr/local/lib/python3.6/dist-packages
(from torch) (1.18.5)
Requirement already satisfied: pillow>=4.1.1 in /usr/local/lib/python3.6/dist-
packages (from torchvision) (7.0.0)
```

```
[39]: import torch
import matplotlib.pyplot as plt
import numpy as np
import torch.nn.functional as F
from torch import nn
from torchvision import datasets, transforms
```

```
[2]: device = torch.device("cuda:0" if torch.cuda.is_available() else "cpu")
device
```

```
[2]: device(type='cuda', index=0)
```

```
[3]: transform = transforms.Compose([transforms.ToTensor(), transforms.Normalize((0.
      ↪5,),(0.5))])
```

```
[4]: training_dataset = datasets.MNIST(root='./data', train=True, download=True,␣
     ↪transform=transform)
     validation_dataset = datasets.MNIST(root='./data', train=False, download=True,␣
     ↪transform=transform)
```

Downloading http://yann.lecun.com/exdb/mnist/train-images-idx3-ubyte.gz to
./data/MNIST/raw/train-images-idx3-ubyte.gz

HBox(children=(FloatProgress(value=1.0, bar_style='info', max=1.0), HTML(value='')))

Extracting ./data/MNIST/raw/train-images-idx3-ubyte.gz to ./data/MNIST/raw
Downloading http://yann.lecun.com/exdb/mnist/train-labels-idx1-ubyte.gz to
./data/MNIST/raw/train-labels-idx1-ubyte.gz

HBox(children=(FloatProgress(value=1.0, bar_style='info', max=1.0), HTML(value='')))

Extracting ./data/MNIST/raw/train-labels-idx1-ubyte.gz to ./data/MNIST/raw
Downloading http://yann.lecun.com/exdb/mnist/t10k-images-idx3-ubyte.gz to
./data/MNIST/raw/t10k-images-idx3-ubyte.gz

HBox(children=(FloatProgress(value=1.0, bar_style='info', max=1.0), HTML(value='')))

Extracting ./data/MNIST/raw/t10k-images-idx3-ubyte.gz to ./data/MNIST/raw
Downloading http://yann.lecun.com/exdb/mnist/t10k-labels-idx1-ubyte.gz to
./data/MNIST/raw/t10k-labels-idx1-ubyte.gz

HBox(children=(FloatProgress(value=1.0, bar_style='info', max=1.0), HTML(value='')))

Extracting ./data/MNIST/raw/t10k-labels-idx1-ubyte.gz to ./data/MNIST/raw
Processing...
Done!

/pytorch/torch/csrc/utils/tensor_numpy.cpp:141: UserWarning: The given NumPy
array is not writeable, and PyTorch does not support non-writeable tensors. This
means you can write to the underlying (supposedly non-writeable) NumPy array
using the tensor. You may want to copy the array to protect its data or make it
writeable before converting it to a tensor. This type of warning will be
suppressed for the rest of this program.

```
[40]: training_loader = torch.utils.data.DataLoader(training_dataset, batch_size=32,␣
      ↪shuffle=True, num_workers=0)
      validation_loader = torch.utils.data.DataLoader(validation_dataset, batch_size␣
      ↪= 32, shuffle=False, num_workers=0)
```

We start off by defining our autoencoder structure. It will have 5 linear enoding layers and 5
linear decoding layers as can be seen below

2

```
[6]: class SparseAutoencoder(nn.Module):

         def __init__(self):
             super(SparseAutoencoder, self).__init__()

             #encoder
             self.en1 = nn.Linear(784, 256)
             self.en2 = nn.Linear(256, 128)
             self.en3 = nn.Linear(128, 64)
             self.en4 = nn.Linear(64, 32)
             self.en5 = nn.Linear(32, 16)

             #decoder
             self.de1 = nn.Linear(16, 32)
             self.de2 = nn.Linear(32, 64)
             self.de3 = nn.Linear(64, 128)
             self.de4 = nn.Linear(128, 256)
             self.de5 = nn.Linear(256, 784)

         def forward(self, x):
             #encoding
             x = F.relu(self.en1(x))
             x = F.relu(self.en2(x))
             x = F.relu(self.en3(x))
             x = F.relu(self.en4(x))
             x = F.relu(self.en5(x))

             #decoding
             x = F.relu(self.de1(x))
             x = F.relu(self.de2(x))
             x = F.relu(self.de3(x))
             x = F.relu(self.de4(x))
             x = F.relu(self.de5(x))

             return x
```

```
[7]: model = SparseAutoencoder().to(device)
```

Nex we extract the layers in the autoencoder individually so that we can use them to calculate the sparse loss by passing the imput images separately through all the layers

```
[8]: model_children = list(model.children())
```

Defining the KL divergence and Sparse Loss

```
[9]: def kl_div(rho, rho_hat):
       rho_hat = torch.mean(torch.sigmoid(rho_hat), 1) #sigmoid to squash the output␣
       ↪between 0-1 to get probability distribution
       rho = torch.tensor([rho] * len(rho_hat)).to(device) # converting rho into a␣
       ↪tensor of same size as rho_hat, for further calculations
       return torch.sum(rho * torch.log(rho/rho_hat) + (1 - rho) * torch.log((1 -␣
       ↪rho)/(1 - rho_hat)))

     def sparse_loss(rho, images):
       values = images
       loss = 0
       for i in range(len(model_children)):
         values = model_children[i](values) #computing the activation after passing␣
       ↪through the layer i of the model
         loss+=kl_div(rho, values)
       return loss
```

Finally we define the functin to train our model:

```
[10]: def train(model, epochs, learning_rate, RHO, BETA):
       criterion = nn.MSELoss()
       optimizer = torch.optim.Adam(model.parameters(), lr = learning_rate)

       training_losses = []
       test_losses = []

       for e in range(epochs):
         training_loss = 0.0
         test_loss = 0.0
         for inputs, labels in training_loader:
           inputs = inputs.view(inputs.shape[0], -1)
           inputs = inputs.to(device)
           outputs = model.forward(inputs)
           mse_loss = criterion(outputs, inputs)
           sparsity = sparse_loss(RHO, inputs)
           loss = mse_loss + BETA*sparsity
           optimizer.zero_grad()
           loss.backward()
           optimizer.step()

           training_loss+=loss.item()

         else:
           with torch.no_grad():
             for val_inputs, val_labels in validation_loader:
               val_inputs = val_inputs.to(device)
               val_inputs = val_inputs.view(val_inputs.shape[0], -1)
               val_outputs = model.forward(val_inputs)
```

```
            val_mse_loss = criterion(val_outputs, val_inputs)
            val_sparsity = sparse_loss(RHO, val_inputs)
            val_loss = val_mse_loss + BETA*val_sparsity
            test_loss += val_loss.item()

        epoch_loss = training_loss/len(training_loader)
        training_losses.append(epoch_loss)
        val_epoch_loss = test_loss/len(validation_loader)
        test_losses.append(val_epoch_loss)
        print('epoch :', (e+1))
        print('training loss: {:.4f} '.format(epoch_loss))
        print('validation loss: {:.4f} '.format(val_epoch_loss))

    return training_losses, test_losses
```

```
[11]: training_losses, test_losses = train(model, 100, 0.0001, 0.05, 0.001)
```

```
epoch : 1
training loss: 0.9419
validation loss: 0.9367
epoch : 2
training loss: 0.9363
validation loss: 0.9367
epoch : 3
training loss: 0.9359
validation loss: 0.9341
epoch : 4
training loss: 0.9323
validation loss: 0.9275
epoch : 5
training loss: 0.9268
validation loss: 0.9254
epoch : 6
training loss: 0.9247
validation loss: 0.9229
epoch : 7
training loss: 0.9223
validation loss: 0.9198
epoch : 8
training loss: 0.9195
validation loss: 0.9178
epoch : 9
training loss: 0.9170
validation loss: 0.9152
epoch : 10
training loss: 0.9154
validation loss: 0.9138
epoch : 11
```

```
training loss: 0.9143
validation loss: 0.9127
epoch : 12
training loss: 0.9129
validation loss: 0.9109
epoch : 13
training loss: 0.9106
validation loss: 0.9079
epoch : 14
training loss: 0.9080
validation loss: 0.9058
epoch : 15
training loss: 0.9063
validation loss: 0.9044
epoch : 16
training loss: 0.9050
validation loss: 0.9033
epoch : 17
training loss: 0.9039
validation loss: 0.9021
epoch : 18
training loss: 0.9024
validation loss: 0.8999
epoch : 19
training loss: 0.9008
validation loss: 0.8991
epoch : 20
training loss: 0.8998
validation loss: 0.8979
epoch : 21
training loss: 0.8990
validation loss: 0.8957
epoch : 22
training loss: 0.8966
validation loss: 0.8948
epoch : 23
training loss: 0.8958
validation loss: 0.8939
epoch : 24
training loss: 0.8950
validation loss: 0.8930
epoch : 25
training loss: 0.8941
validation loss: 0.8921
epoch : 26
training loss: 0.8931
validation loss: 0.8911
epoch : 27
```

```
training loss: 0.8924
validation loss: 0.8906
epoch : 28
training loss: 0.8918
validation loss: 0.8900
epoch : 29
training loss: 0.8909
validation loss: 0.8890
epoch : 30
training loss: 0.8903
validation loss: 0.8885
epoch : 31
training loss: 0.8897
validation loss: 0.8877
epoch : 32
training loss: 0.8884
validation loss: 0.8860
epoch : 33
training loss: 0.8874
validation loss: 0.8856
epoch : 34
training loss: 0.8869
validation loss: 0.8851
epoch : 35
training loss: 0.8865
validation loss: 0.8847
epoch : 36
training loss: 0.8859
validation loss: 0.8840
epoch : 37
training loss: 0.8853
validation loss: 0.8836
epoch : 38
training loss: 0.8843
validation loss: 0.8823
epoch : 39
training loss: 0.8835
validation loss: 0.8815
epoch : 40
training loss: 0.8828
validation loss: 0.8809
epoch : 41
training loss: 0.8822
validation loss: 0.8803
epoch : 42
training loss: 0.8817
validation loss: 0.8799
epoch : 43
```

```
training loss: 0.8811
validation loss: 0.8792
epoch : 44
training loss: 0.8806
validation loss: 0.8787
epoch : 45
training loss: 0.8801
validation loss: 0.8782
epoch : 46
training loss: 0.8795
validation loss: 0.8775
epoch : 47
training loss: 0.8790
validation loss: 0.8770
epoch : 48
training loss: 0.8786
validation loss: 0.8767
epoch : 49
training loss: 0.8782
validation loss: 0.8764
epoch : 50
training loss: 0.8779
validation loss: 0.8761
epoch : 51
training loss: 0.8776
validation loss: 0.8750
epoch : 52
training loss: 0.8766
validation loss: 0.8748
epoch : 53
training loss: 0.8763
validation loss: 0.8745
epoch : 54
training loss: 0.8759
validation loss: 0.8741
epoch : 55
training loss: 0.8757
validation loss: 0.8740
epoch : 56
training loss: 0.8754
validation loss: 0.8737
epoch : 57
training loss: 0.8751
validation loss: 0.8733
epoch : 58
training loss: 0.8748
validation loss: 0.8731
epoch : 59
```

```
training loss: 0.8745
validation loss: 0.8727
epoch : 60
training loss: 0.8743
validation loss: 0.8727
epoch : 61
training loss: 0.8740
validation loss: 0.8725
epoch : 62
training loss: 0.8737
validation loss: 0.8721
epoch : 63
training loss: 0.8735
validation loss: 0.8719
epoch : 64
training loss: 0.8733
validation loss: 0.8718
epoch : 65
training loss: 0.8731
validation loss: 0.8714
epoch : 66
training loss: 0.8728
validation loss: 0.8712
epoch : 67
training loss: 0.8726
validation loss: 0.8710
epoch : 68
training loss: 0.8724
validation loss: 0.8708
epoch : 69
training loss: 0.8722
validation loss: 0.8706
epoch : 70
training loss: 0.8720
validation loss: 0.8703
epoch : 71
training loss: 0.8718
validation loss: 0.8702
epoch : 72
training loss: 0.8716
validation loss: 0.8701
epoch : 73
training loss: 0.8714
validation loss: 0.8699
epoch : 74
training loss: 0.8713
validation loss: 0.8699
epoch : 75
```

```
training loss: 0.8711
validation loss: 0.8697
epoch : 76
training loss: 0.8709
validation loss: 0.8694
epoch : 77
training loss: 0.8708
validation loss: 0.8694
epoch : 78
training loss: 0.8706
validation loss: 0.8692
epoch : 79
training loss: 0.8705
validation loss: 0.8690
epoch : 80
training loss: 0.8703
validation loss: 0.8689
epoch : 81
training loss: 0.8701
validation loss: 0.8686
epoch : 82
training loss: 0.8700
validation loss: 0.8685
epoch : 83
training loss: 0.8699
validation loss: 0.8685
epoch : 84
training loss: 0.8697
validation loss: 0.8682
epoch : 85
training loss: 0.8696
validation loss: 0.8682
epoch : 86
training loss: 0.8695
validation loss: 0.8683
epoch : 87
training loss: 0.8693
validation loss: 0.8678
epoch : 88
training loss: 0.8692
validation loss: 0.8678
epoch : 89
training loss: 0.8691
validation loss: 0.8676
epoch : 90
training loss: 0.8689
validation loss: 0.8674
epoch : 91
```

```
training loss: 0.8688
validation loss: 0.8673
epoch : 92
training loss: 0.8687
validation loss: 0.8672
epoch : 94
training loss: 0.8685
validation loss: 0.8671
epoch : 95
training loss: 0.8684
validation loss: 0.8670
epoch : 96
training loss: 0.8683
validation loss: 0.8670
epoch : 97
training loss: 0.8682
validation loss: 0.8670
epoch : 98
training loss: 0.8681
validation loss: 0.8668
epoch : 99
training loss: 0.8680
validation loss: 0.8668
epoch : 100
training loss: 0.8680
validation loss: 0.8666
```

[12]:
```python
def plot_loss(training_losses, testing_losses):
    plt.plot(training_losses, label='training loss')
    plt.plot(testing_losses, label='testing loss')
    plt.legend()
```

[13]:
```python
plot_loss(training_losses, test_losses)
```
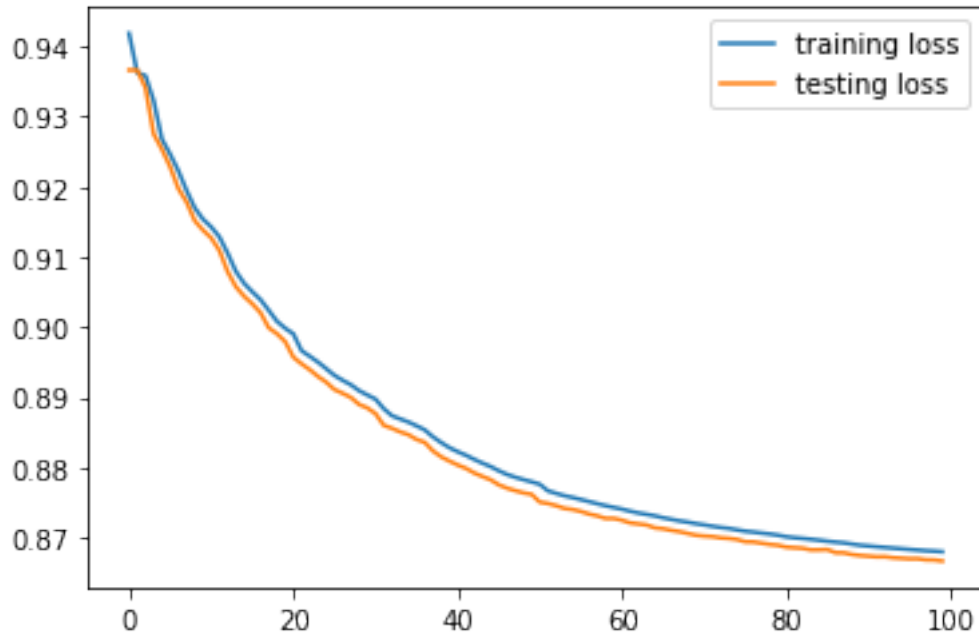
1.2.2 Now let's see how well this autoencoder regenerates the images in the MNIST validation dataset

We'll input 20 images from the validation set and see the outputs

```python
[14]: def im_convert(tensor):
    image = tensor.cpu().clone().detach().numpy()
    image = image.transpose(1, 2, 0)
    image = image * np.array((0.5, 0.5, 0.5)) + np.array((0.5, 0.5, 0.5))
    image = image.clip(0, 1)
    return image
```

```python
[15]: def output_convert(image):
    img = image.view(28, 28)
    img = img.cpu().clone().detach().numpy()
    return img
```
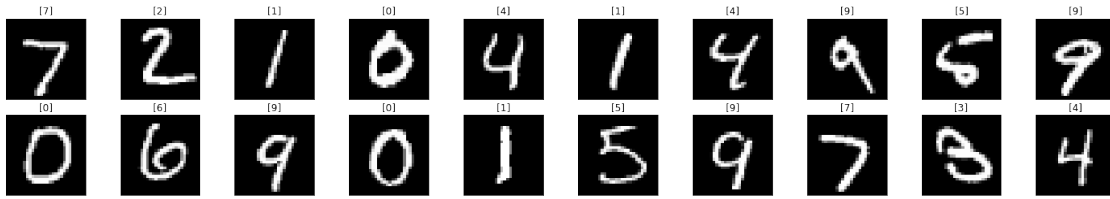
```python
[16]: data = iter(validation_loader)
images, labels = data.next()
```

The input images

```python
[17]: fig = plt.figure(figsize=(25, 4))

for idx in np.arange(20):
    ax = fig.add_subplot(2, 10, idx+1, xticks=[], yticks=[])
    plt.imshow(im_convert(images[idx]))
```
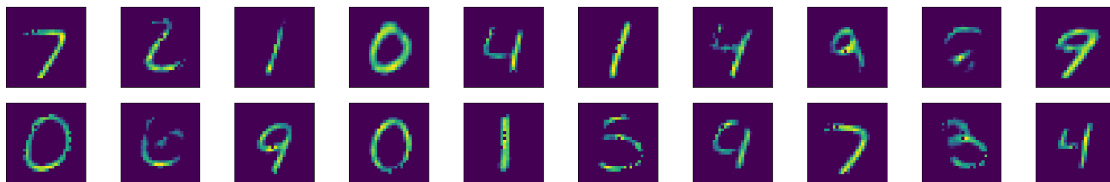
```
ax.set_title([labels[idx].item()])
```


[18]:
```
imgs = images.to(device)
imgs = imgs.view(imgs.shape[0], -1)
reconstructed = model.forward(imgs)
```

The output images

[19]:
```
fig = plt.figure(figsize=(25, 4))

for idx in np.arange(20):
  ax = fig.add_subplot(2, 10, idx+1, xticks=[], yticks=[])
  plt.imshow(output_convert(reconstructed[idx]))
```


[ ]: