NP-completeness

1. In the bin packing problem we are given a set of n objects which have sizes $a_1, a_2, ..., a_n$ where $0 < a_i \le 1$. A subset of these objects can fit into a bin if the sum of the sizes of the objects in this subset is at most 1. The objective is to minimize the number of bins required to pack all objects. Show that a polynomial time algorithm for bin-packing would imply a polynomial time algorithms for the subset sum problem.
2. In the machine scheduling problem we are given m machines and a set of n jobs with processing times $p_1, p_2, ..., p_n$. A job can be scheduled on any machine. The makespan of a schedule is defined as the time by which all jobs complete. Show that a polynomial-time algorithm for finding a schedule with minimum makespan would imply a polynomial-time algorithm for the bin-packing problem.
3. Given a graph with edge weights (may be negative) and two vertices s, t show that the problem of finding the shortest simple path between s and t is NP-hard by reducing the hamiltonian cycle problem to it. The Hamiltonian cycle problem is to determine if a given graph has a simple cycle that includes all vertices of the graph.