Graph Search

1. A *cut-vertex* is a vertex whose removal disconnects the graph. A graph $G = (V, E)$ is 2-*vertex connected* (or *biconnected*) if it has no cut-vertex. Give a procedure that checks in linear time if a graph is biconnected.
2. Consider the following procedure to check if a directed graph $G = (V, E)$ is strongly connected and to prove its correctness.
    i. Pick a vertex $v \in V$ and run a DFS from v. If some vertex is not reached in the DFS then G is not strongly connected.
    ii. Reverse (replace edge (u, v) with the edge (v, u)) every edge of G and let the resulting graph be G^R.
    iii. Run a DFS on G^R from v. If some vertex is not reached in the DFS then G is not strongly connected.
    iv. Declare G is strongly connected.
3. A *strongly connected component* (SCC) is a maximal set of vertices that are strongly connected. Prove that any two strongly connected components of a graph are disjoint.
4. A directed graph $G = (V, E)$ is *weakly connected* if for every pair of vertices u, v, there is a path from u to v or from v to u. Give a linear time algorithm to check if G is weakly connected.
5. The *distance* between a pair of vertices u, v is the length of the shortest path between u and v. The *diameter* of a graph is the maximum distance between any pair of vertices. Give a linear-time algorithm to compute the diameter of a tree.