

# Dynamic Programming

1. **Maximum Value Contiguous Subsequence.** Given a sequence of  $n$  real numbers  $A_1, A_2, \dots, A_n$ , determine a contiguous subsequence  $A_i \dots A_j$  for which the sum of elements in the subsequence is maximized.
2. **Making Change.** You are given  $n$  types of coin denominations of values  $v_1 < v_2 < \dots < v_n$  (all integers). Assume  $v_1 = 1$ , so you can always make change for any amount of money  $C$ . Give an algorithm which makes change for an amount of money  $C$  with as few coins as possible.
3. **The Integer Knapsack Problem (Duplicate Items Permitted).** You have  $n$  types of items, where the  $i$ th item type has an integer size  $s_i$  and a real value  $v_i$ . You are trying to fill a knapsack of total capacity  $C$  with a selection of items of maximum value. You can add multiple items of the same type to the knapsack.
4. **Box Stacking.** You are given a set of  $n$  types of rectangular 3-D boxes, where the  $i$ th box has height  $h_i$ , width  $w_i$  and depth  $d_i$  (all real numbers). You want to create a stack of boxes which is as tall as possible, but you can only stack a box on top of another box if the dimensions of the 2-D base of the lower box are each strictly larger than those of the 2-D base of the higher box. Of course, you can rotate a box so that any side functions as its base. It is also allowable to use multiple instances of the same type of box.
5. **Building Bridges.** Consider a 2-D map with a horizontal river passing through its center. There are  $n$  cities on the southern bank with x-coordinates  $a_1 \dots a_n$  and  $n$  cities on the northern bank with x-coordinates  $b_1 \dots b_n$ . You want to connect as many north-south pairs of cities as possible with bridges such that no two bridges cross. When connecting cities, you are only allowed to connect the  $i$ th city on the northern bank to the  $i$ th city on the southern bank.
6. **Balanced Partition.** You have a set of  $n$  integers each in the range  $0 \dots K$ . Partition these integers into two subsets such that you minimize  $|S_1 - S_2|$ , where  $S_1$  and  $S_2$  denote the sums of the elements in each 1 of the two subsets.
7. **Edit Distance.** Given two text strings  $A$  of length  $n$  and  $B$  of length  $m$ , you want to transform  $A$  into  $B$  with a minimum number of operations of the following types: delete a character from  $A$ , insert a character into  $A$ , or change some character in  $A$  into a new character. The minimal number of such operations required to transform  $A$  into  $B$  is called the edit distance between  $A$  and  $B$ .