Practice Sheet for COL351 midterm

1. Given a list of n natural numbers $d_1, d_2, \ldots d_n$ show how to decide in polynomial time whether there exists an undirected graph $G = (V, E)$ whose node degrees are precisely the numbers $d_1, d_2, \ldots d_n$. (That is, if $V = v_1, \ldots, v_n$, then the degree of v_i should be exactly d_i.) G should not contain multiple edges between the same pair of nodes, or loop edges with both endpoints equal to the same node.
2. A is an $n \times m$ matrix. The ith column of A has weight w_i. Design an algorithm to pick a subset of linearly independent columns of maximum total weight. Assume you have a subroutine to determine if a given subset of columns is linearly independent. How many calls will your algorithm make to this subroutine? Prove the correctness of your algorithm.
3. Given a string $S = "s_1 s_2 \cdots s_n"$, design an algorithm to find the minimum number of characters that need to be inserted to make the resulting string a palindrome. Consider the example $S = "abcbd"$. We can get palindrome "$adbcbda$" or palindrome "$dabcbad$" by inserting two characters (one a and one d). Moreover, we cannot get a palindrome by inserting just one character. Discuss the running time of your algorithm and prove its correctness.
4. An induced subgraph of a graph $G = (V, E)$ is a graph $H = (U, F)$ such that $U \subseteq V$, and $F = E \cap (U \times U)$. Given an undirected graph $G = (V, E)$ and an integer k, find the maximum induced subgraph H of G such that each vertex in H has degree at least k, or determine that it does not exist. The algorithm should run in time $O(|V| + |E|)$.
5. The girth of a graph is the length of the shortest cycle. Give an $O(mn)$ algorithm to find the girth of an undirected graph, where m, n are the number of edges and vertices in the graph.