# Applications of Max Flow

1. Given a directed graph $G = (V, E)$, we want to find a subgraph $H = (V, E_o)$ so that every vertex in $H$ has $k$ incoming and $k$ outgoing edges. Show how to do this.
2. Two paths are said to be edge (resp. vertex) disjoint if they do not share any edge (resp. vertex). You have to find the maximum number of edge (resp. vertex) disjoint paths between two given vertices $u, v$ in a directed graph $G = (V, E)$. Show how to do this.
3. Place a maximum number of rooks on a $m \times n$ chess-board with some squares cut out.
4. A graph is called $k$-regular if every vertex has degree $k$. Show that every $k$-regular bipartite graph has a perfect matching.
5. In sociology, one often studies a graph G in which nodes represent people, and edges represent those who are friends with each other. Let's assume for purposes of this question that friendship is symmetric, so we can consider an undirected graph. Now, suppose we want to study this graph G, looking for a close-knit group of people. One way to formalize this notion would be as follows. For a subset S of nodes let e(S) denote the number of edges in S, i.e., the number of edges that have both ends in S. We define the cohesiveness of S as e(S)/|S|. A natural thing to search for would be the set S of people achieving the maximum cohesiveness. Give a polynomial time algorithm that takes a rational number α and determines whether there exists a set S with cohesiveness at least α. Give a polynomial time algorithm to find a set S of nodes with maximum cohesiveness.
6. Consider a set of mobile computing clients in a certain town who each need to be connected to one of several possible base stations. Suppose there are $n$ clients, with the position of each client specified by its (x, y) coordinate in the plane. There are also k base stations; the position of each of these is also specified by its (x, y) coordinate in the plane. For each client, we wish to connect it to exactly one of the base stations. Our choice of connections is constrained in the following ways: (i) there is a range parameter r, and a client can only be connected to a base station that is within distance r, (ii) there is a load parameter L, and no more than L clients can be connected to any single base station. Your goal is to design a polynomial time algorithm for the following problem – given the above data, decide whether it is possible to assign every client to a base station subject to the two constraints.