→ Neural network → network of neurons
→

COL351 New lecture

Polynomial time :-

toposort → $O(m+n)$
max flow → $O(n^3)$

Algo A has poly running time if running time of A is bounded by a poly. in the length of the input.

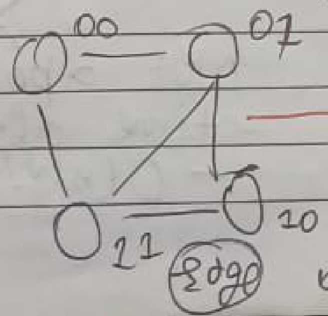
$\xrightarrow[\text{input}]{I} \boxed{A} \xrightarrow[\text{output}]{o}$

length of input # of bits required to encode the input

$|I|$ = Size of instance I
= # of bits required to encode instance I

Graph $G=(V,E) = \boxed{O(m \log n)}$
– n numbers # of bits then $\leq n \log U$ bits

$\boxed{U \text{ is the largest integer}}$



(00,01) (01,10) → $2\log n$ for each edge

for Graph we req. $O(2m \log n + n \log n)$

edge

vertices

$|I|^k$ is polynomial in $|I|$ for fixed k

max flow instance : $(\underbrace{m \log n}_{\text{Edge}} + n + \underbrace{m \log U}_{\text{capacity}})$ vertices

→ **Subset sum:-** $0 \le a_i \le W$, W
$1 \le i \le n$

Input size : $O(n \log W)$

$$\boxed{2^{|I|} \text{ is not a polynomial in } |I|, \text{ since } \nexists\, k \text{ s.t. } 2^{|I|} \le |I|^k \ \forall\, |I| \ge d}$$

Is $2^n \le n^{10^6}$ $\forall n$ ✗

$1001^n \le n^{106}$ $\forall n$? ✗

① **Toposort:-** ~~Is~~

Is $m+n \le (m \log n + n)^k$
$k = 1$
✓

② **Maxflow:-** Is $n^3 \le (\underbrace{m \log n}_{\text{Edge}} + \underbrace{m \log U}_{\text{capacity}} + \underbrace{n}_{\text{vertices}})^k$
$k = 3$
✓

③ **Subset sum:-** Is $nW \le (n \log W)^k$
✗

Not a polynomial time algorithm
$O(nW)$

Polynomial time

$$\log nW \leq k \log n + \log \log W$$

$$k \geq \frac{\log n + \log W}{\log n + \log \log W}$$

NO

$n^{\log n} \leq n^k$ no k exists

But $n^k < n^{\log n} \leq 2^n$

$$n^k \leq \underbrace{2^{\log^2 n}} \leq 2^n$$

It grows slower than exponential but faster then Polynomial

(4) $\xrightarrow{n}$ $\boxed{\text{Primality testing}}$ $\xrightarrow{\text{Yes}}$ / $\xrightarrow{\text{No}}$

$|I| = \log_2 n$

Trivial algorithm has a running time $O(\sqrt{n})$, $|I| = \log n$

Is there a k such that

$$\sqrt{n} \leq (\log n)^k$$

$$\frac{1}{2} \log n \leq k \log \log n$$

Not a Polynomial time

$$\frac{\log n}{2 \log \log n} \leq k$$

as n increases k will go to ∞

Polynomial time reduction

$\boxed{A} \longrightarrow$ Algorithm A to solve a Problem a

Problem b

$I_B \longrightarrow \boxed{\text{Converter}} \xrightarrow{I_A} \boxed{A} \longrightarrow \boxed{\text{Converter from } O_A \text{ to } O_B} \longrightarrow O_B$

$G = V, E$

$N : V \rightarrow \mathbb{R}^+$

max flow

s-t cut of min capacity

$G' = (V', E)$

s, t

$C : E \rightarrow \mathbb{R}^+$

Project corresponding to vertices in T

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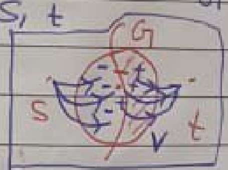
Project Selection < s-t max flow

Input Graph $G = (V, E)$ DAG, $\pi : V \rightarrow \mathbb{R}$	Input $G' = (V', E')$ Need not be DAG, $c : E \rightarrow \mathbb{R}^+$, s, t



$O(m)$ $\qquad$ $O(n^3)$

$I_{PS} \rightarrow \boxed{\text{Conv1}} \xrightarrow{I_{MF}} \boxed{\text{s-t min cut}} \rightarrow \boxed{\text{Conv2}} \rightarrow$ Solution to PS

$O(m+n)$

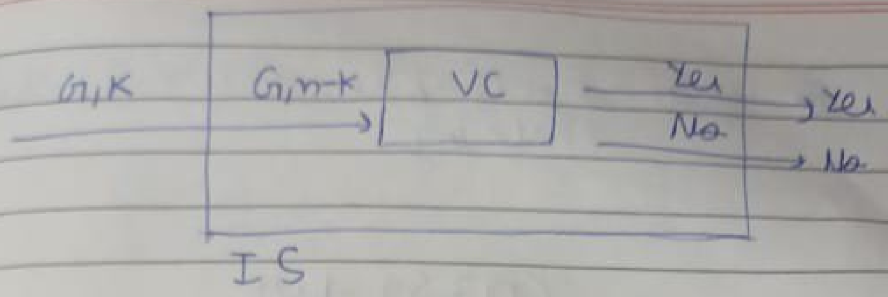
Conv1

$V' = V \cup \{s, t\}$

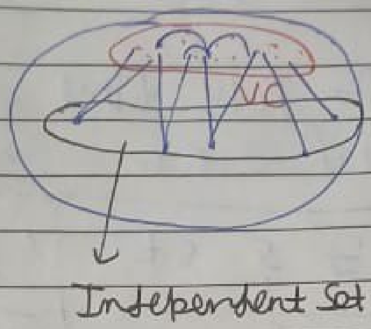
$E' = E \cup \{(s, v) \; v : \pi(v) < 0\} \cup \{(v, t) \; v : \pi(v) > 0\}$

Conv2

Pick t side of min cut



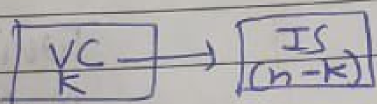
Independent set (G, K)
// Does G have an independent set of size $\geq K$



return (VC(G, n−k))

$IS \leq_P VC$

$VC \leq_P IS$ ⟶



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* COL351 :- (Lecture)

NP : Non- deterministic Poly. time → (TOC)

class of Problems for which there is a poly. time checkable Yes certificate

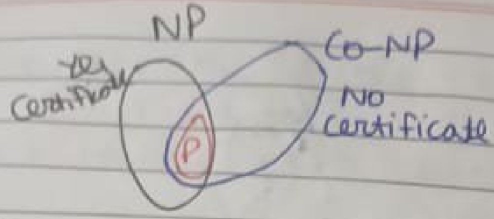
vertex cover :- Does G have a vertex cover of Size K.

$\xrightarrow[\text{certificate}]{G}$ $\boxed{\text{Checker}}$

[A Problem belongs to NP if given an sol^n to the Problem we can verify it in Polynomial time]

BiPartite matching : Does $G=(U,V,E)$ has a Perfect matching ?

YES certificate : A perfect Matching
NO certificate : A Hall Set



E.x :- Is N a Prime ?

NO certificate → Give a divisor (Easy)

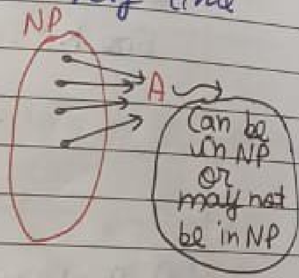
Yes certificate → "Primes is in NP"

NP-hard :- atleast as hard as any Problem in NP. A Problem A is NP-hard

⇒ for all Problems $B \in NP$, $B \leq A$

if A can be solved in Poly time then B can be solved in Poly time

$B \leq_P A \quad B \xrightarrow[P]{} A$



Cook - Levin :-

CNF - SAT is NP hard

given a boolean formula in CNF check if $\exists$ an assignment of values to variables so that the formula is satisfiable

$(x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_2} \vee x_3 \vee \overline{x_4}) \wedge (\overline{x_1} \vee x_4 \vee \overline{x_5}) \wedge (\overline{x_3} \vee \overline{x_4} \vee \overline{x_2})$

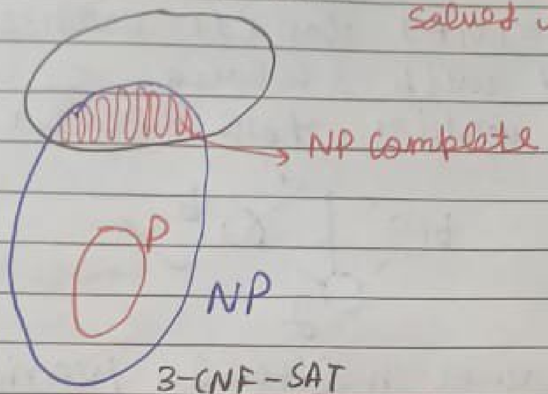
If I have to show a Problem A is NP Hard then i can reduce CNF-SAT to A

$\Rightarrow$ CNF-SAT $\longrightarrow$ A

Cook levin

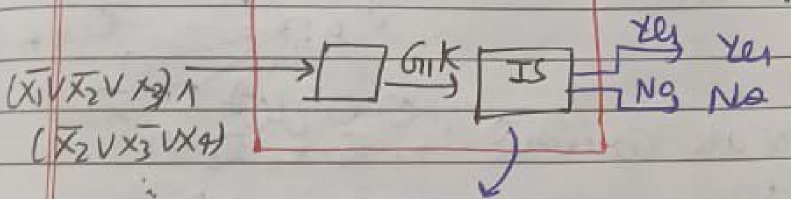
I can solve A in Poly time then CNF-SAT can solve in Poly time then all Problems can be solved in Poly time

NP-hard

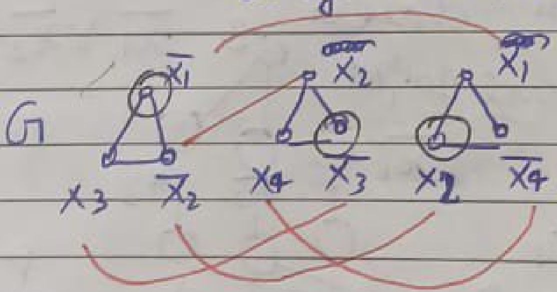


3-CNF-SAT
$\downarrow$
IS (independent set)
$\downarrow$
VC $\longrightarrow$ X

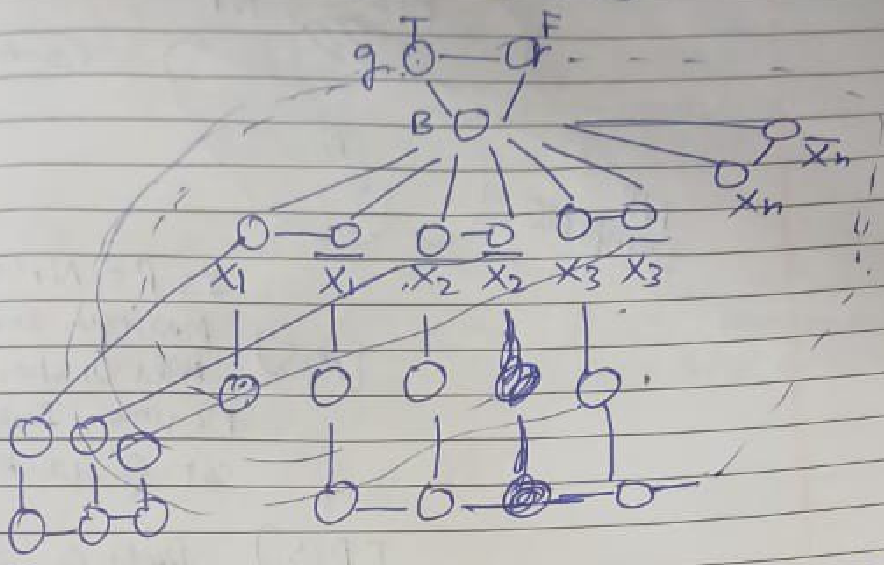
3-CNF-SAT



does this Graph G has IS of Size $\geq K$



One triangle for each clause which is labeled with its literals clause $(\bar{x}_1 \vee \bar{x}_2 \vee x_3)$



If there is an satisfying assignment there exist a three colorable in G

If G is 3-colorable then there is an satisfying assignment
$\Rightarrow \Phi$ is satisfying

We can build any Graph

NP

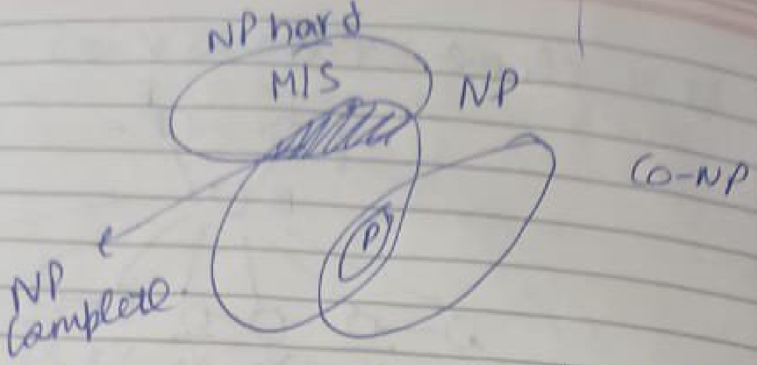
3-CNF - SAT → All Problems in NP can be reduced to it.

diff — 3-Colourable

diff → Subset Sum → Partition → Bin Packing

diff → Dir Hamiltonian Cycle → Hamiltonian Cycle undirected → longest path

diff → Independent set → VC



Is P = NP?

[MIS] Maximum Independent set
Does G has a Maximum Independent set of size k?

[DIS] Does G has an Independent set of size $\geq k$

MIS
↓
Neither
Yes
Checker
Nor
No
Checker

DIS(K)

```
for i=1 to n do
    if MIS(i) then
        if i>=k return(true)
        else return(false)
        ~~else return (false)~~
```

$\forall x_1 \forall x_2 \ldots \forall x_n \; \phi(x_1 x_2 \ldots x_n)$

Approximation Algorithm

min-VC a poly time algo which computes a sol^n which is almost 2 times the OPT

Parameterized algorithm

min VC in time $O(2^k)$ where k is the size of the VC

G has an Independent set of size m (# of clauses) if ϕ is satisfiable

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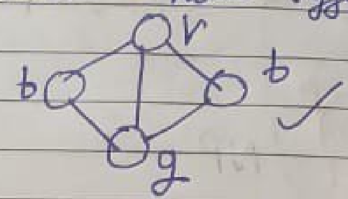
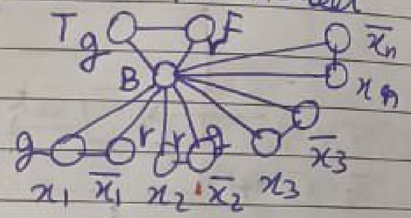
3-CNF-SAT $\longrightarrow$ 3-COLORING

GIVEN $G=(V,E)$ · Can the vertices be coloured with 3 colours so that adjacent vertices have different colours



ϕ m clauses n literals / variables

$G=(V,E)$



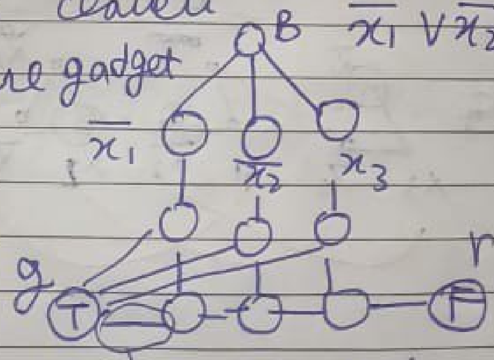
where are clauses?

$x_1 = T$ $x_2 = F$

for each clause $\overline{x_1} \vee \overline{x_2} \vee x_3$

clause gadget

introduce ~~7~~ 6 new vertices



→ This is our Graph

This edge is also there

$|I_R| = O(m+n)$

How much time conv1 takes $\rightarrow \boxed{O(m)}$

Total time $\rightarrow \boxed{O(n^3)}$

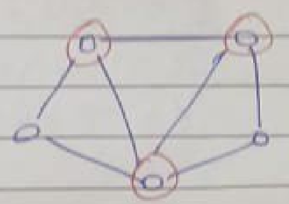
$I_1 \rightarrow \boxed{|I_1|^{k_1}} \; I_2 \rightarrow \boxed{|I_2|^{k_2}} \; I_3 \rightarrow \boxed{|I_3|^{k}}$

$$|I_2| \leq |I_1|^{k_1}$$
$$|I_3| \leq |I_2|^{k_2} \leq |I_1|^{k_1 k_2}$$

Total time taken by this Procedure

$$|I_1|^{k_1} + |I_2|^{k_2} + |I_3|^{k_3}$$
$$\leq |I_1|^{k_1} + |I_1|^{k_1 k_2} + |I_1|^{k_1 k_2 k_3}$$
$$\leq O(|I_1|^{k_1 k_2 k_3})$$

Vertex cover problem :-



$G=(V,E)$
$S \subseteq V$ is a vertex cover if all edges of E are incident to a vertex in S

Input VC Problem: Given $G=(V,E)$ find a VC of smallest size. optimization Problem

VC Problem:- Given $G=(V,E)$ does G have a VC of size $\leq K$.