Practice Session 05: Brune Spectral Ratio

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Synthetic Spectral Ratio

We construct a spectral ratio for for a frequency range of $3-200$ Hz and a moment ratio (M_{01}/M_{02}) of 100. The corner frequencies are $fc_1 = 20$ Hz and $fc_2 = 50$ Hz.

The spectral ratio is given by:

$$R = \frac{u_1(f)}{u_2(f)} = \frac{M_{01}}{M_{02}}\left(\frac{1+(f/fc_2^2)}{1+(f/fc_1^2)}\right) \tag{1}$$

Here R is the spectral ratio. The brune spectral ratio without noise is shown in Fig. 1. The synthetic spectral ratio with added uniformly distributed white noise with a peak amplitude of 5 is shown in Fig. 2

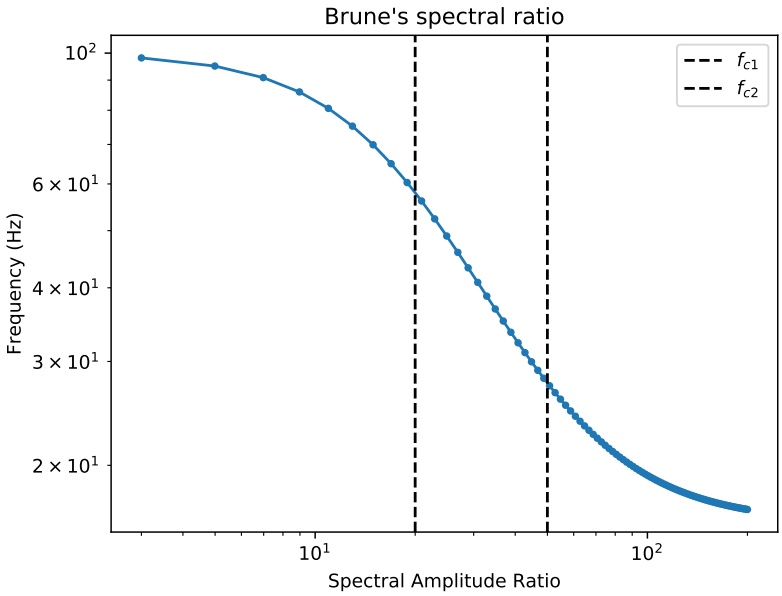

Figure 1: Brune's spectral ratio without noise

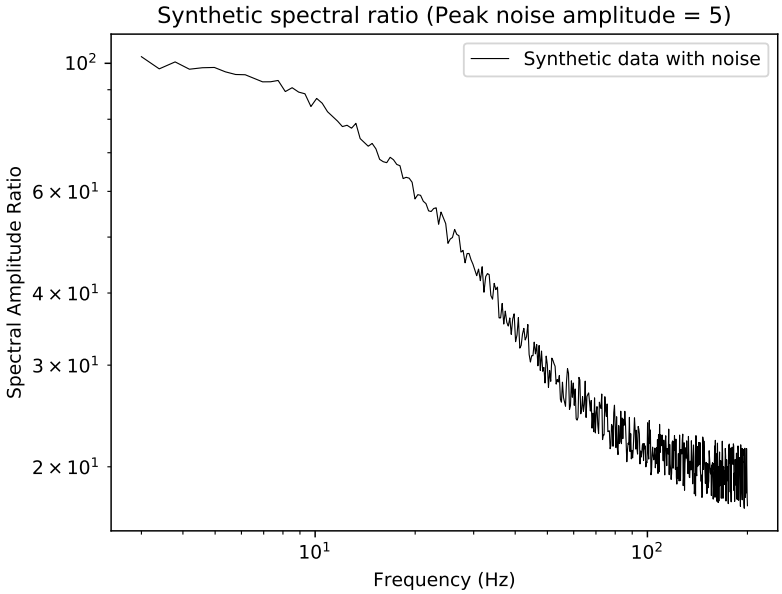

Figure 2: Synthetic spectral ratio with noise peak amplitude of 5.

Best fit curve using the Brune's Model

We use damped least squares to solve the nonlinear curve fitting, analogous to robustfit in Matlab. The best fit curve is shown in Fig. 3. The values of moment ratio and the corner frequencies are as follows (the superscript represents the noise level):

$$R^5 = 102.212$$
$$fc_1^5 = 20.156Hz$$
$$fc_2^5 = 47.474Hz$$

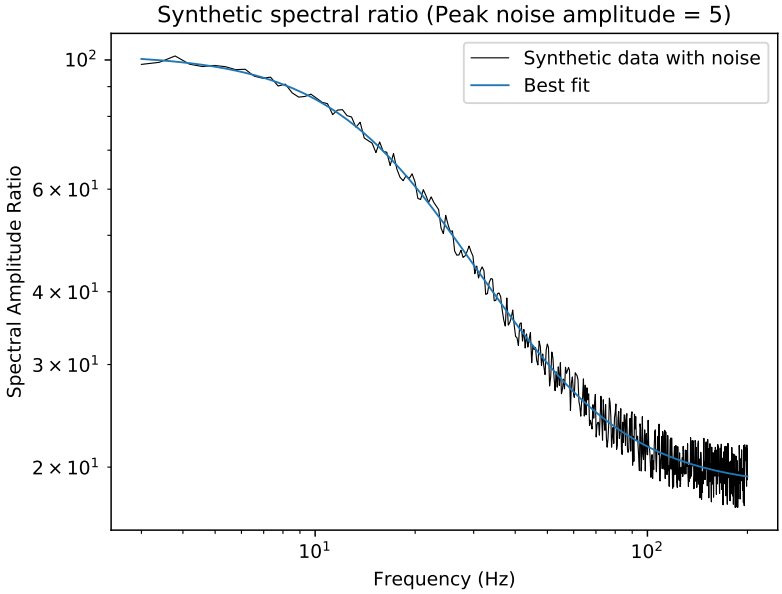

Figure 3: Synthetic spectral ratio with noise peak amplitude of 5.

We get values of moment ratio and corner frequencies very close to the true values from inversion.

Increasing the amplitude of white noise

We increase the peak noise amplitude and look at the inversion results. The results are shown in Fig. 4, 5, 6 and their values are shown below.

$$R^{10} = 104.444$$
$$fc_1^{10} = 20.354 Hz$$
$$fc_2^{10} = 45.680 Hz$$

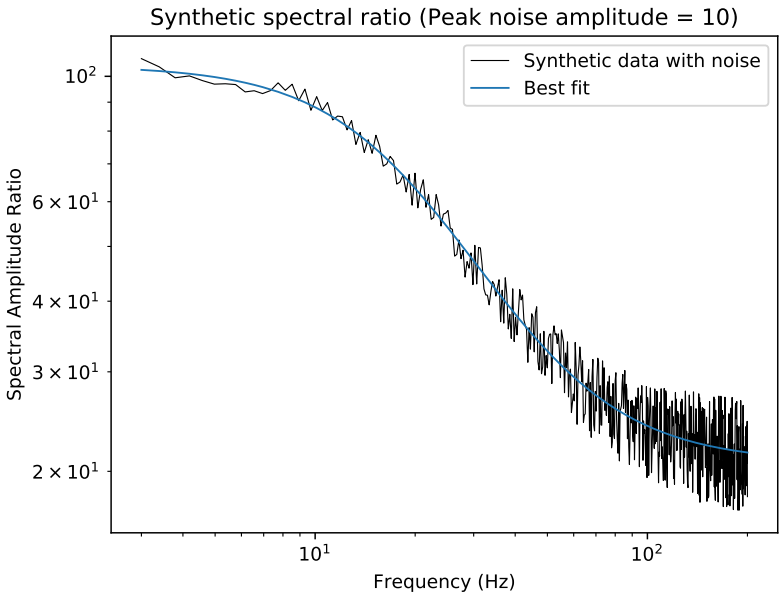

Figure 4: Synthetic spectral ratio with noise peak amplitude of 10.

$$R^{20} = 110.330$$
$$fc_1^{20} = 19.381 Hz$$
$$fc_2^{20} = 39.625 Hz$$

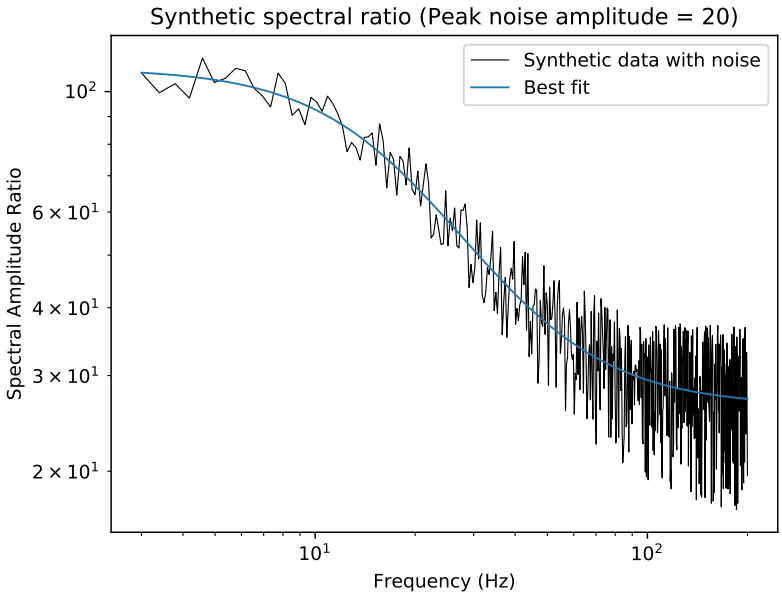

Figure 5: Synthetic spectral ratio with noise peak amplitude of 20.

$$R^{50} = 125.009$$
$$fc_1^{50} = 20.418Hz$$
$$fc_2^{50} = 35.746Hz$$

We see that we get increasing errors with increasing peak noise amplitude level but it scales linearly with increasing noise. The values of fc_2 are less robust to inversion than the moment ratio and fc_1. There is significant deviation in all the three values at peak noise amplitude of 50 and above.

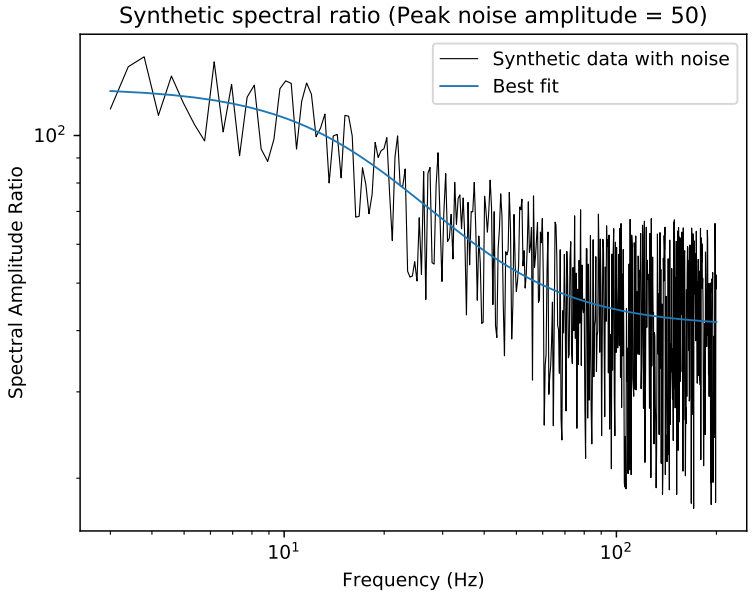

Figure 6: Synthetic spectral ratio with noise peak amplitude of 50.

Peak Noise Level	Moment Ratio	fc_1	fc_2
Frequency Range 3 – 200 Hz, Moment Ratio 100			
0	100	20	50
5	102.212	20.156	47.474
10	104.444	20.354	45.680
20	110.330	19.381	39.625
50	125.009	20.418	35.746
Frequency Range 3 – 35 Hz, Moment Ratio 100			
5	102.522	20.094	47.713
10	104.250	20.367	45.894
20	111.088	19.818	40.999
50	128.728	17.470	29.304
Frequency Range 3 – 35 Hz, Moment Ratio 10			
0	10	20	50
5	12.439	21.442	40.894
10	15.187	16.637	24.192
20	21.622	13.665	17.351
50	51.577	3.02	3.988

Table 1: Inverted values for all the cases

Synthetic spectral ratio with limited frequency

We cut the synthetic spectral ratio to a range of $3-35$ Hz and try the best fitting with peak noise levels of $5, 10, 20, 50$ and look at the inverted values below. The best fit curves are shown in Fig. 7, 8, 9, 10, and the values are listed in Table 1. We see that the moment ratio and the corner frequency fc_1 is better estimated than fc_2, because we have a limited frequency of data available.

In the next part, we change the moment ratio to 10 and see the response to increasing noise on the moment ratio and corner frequency inversion. Since the moment ratio is low, we see that any inversion with peak noise greater than 10 gives totally unreliable inversion results even for the corner frequencies (Fig, 11,

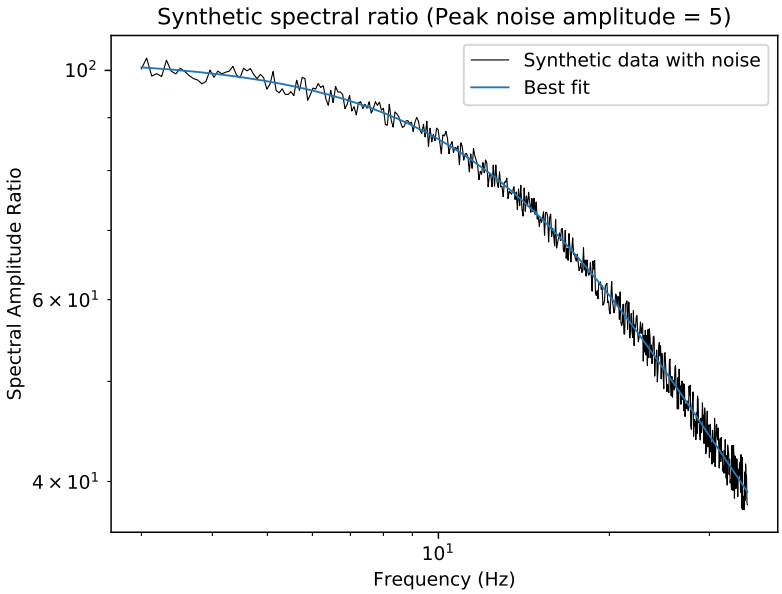

Figure 7: Synthetic spectral ratio with noise peak amplitude of 5.

12, 13, 14). The inversions give the peak noise level as the output rather than the moment ratio of synthetic data.

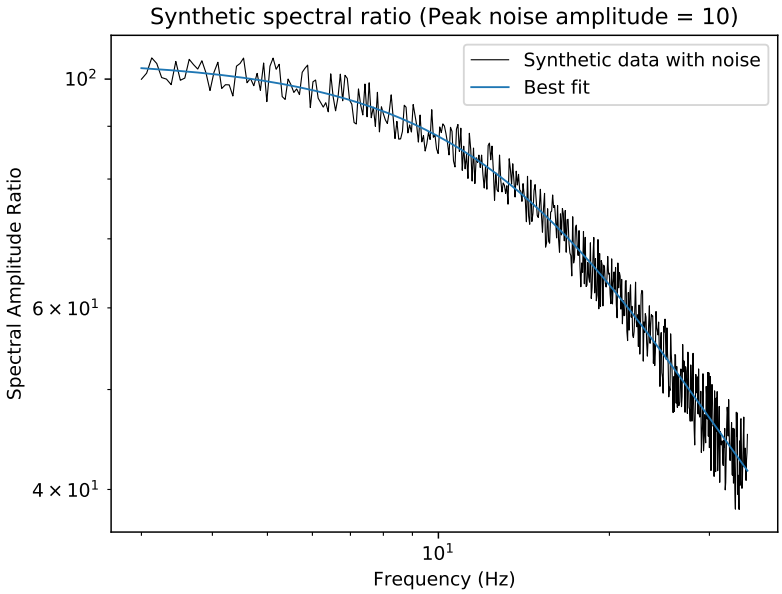

Figure 8: Synthetic spectral ratio with noise peak amplitude of 10.

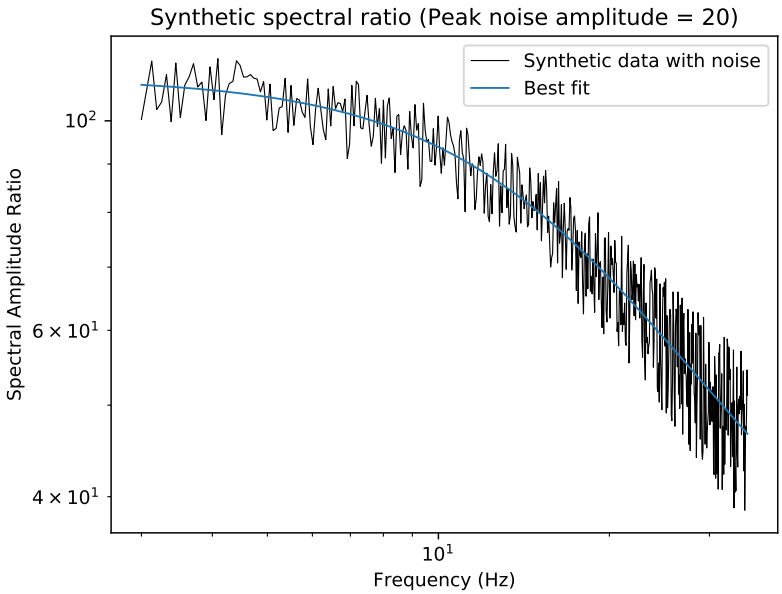

Figure 9: Synthetic spectral ratio with noise peak amplitude of 20.

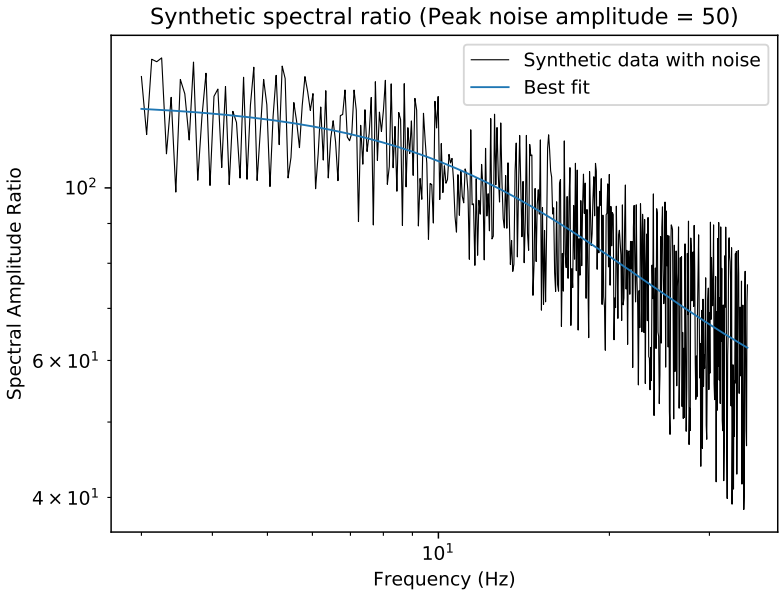

Figure 10: Synthetic spectral ratio with noise peak amplitude of 50.

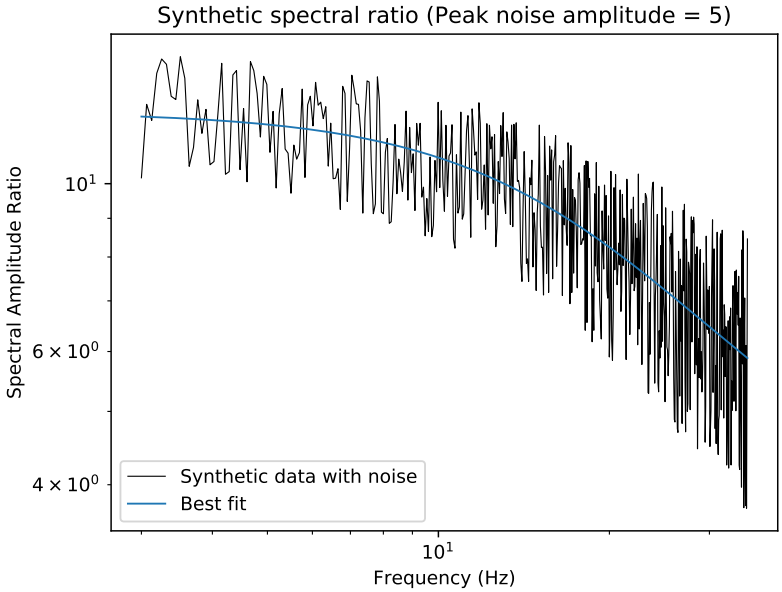

Figure 11: Synthetic spectral ratio with noise peak amplitude of 5.

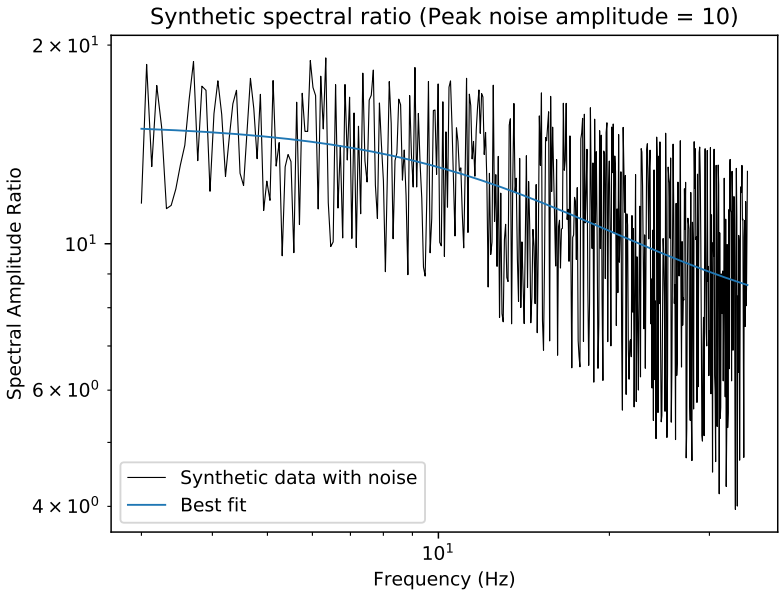

Figure 12: Synthetic spectral ratio with noise peak amplitude of 10.

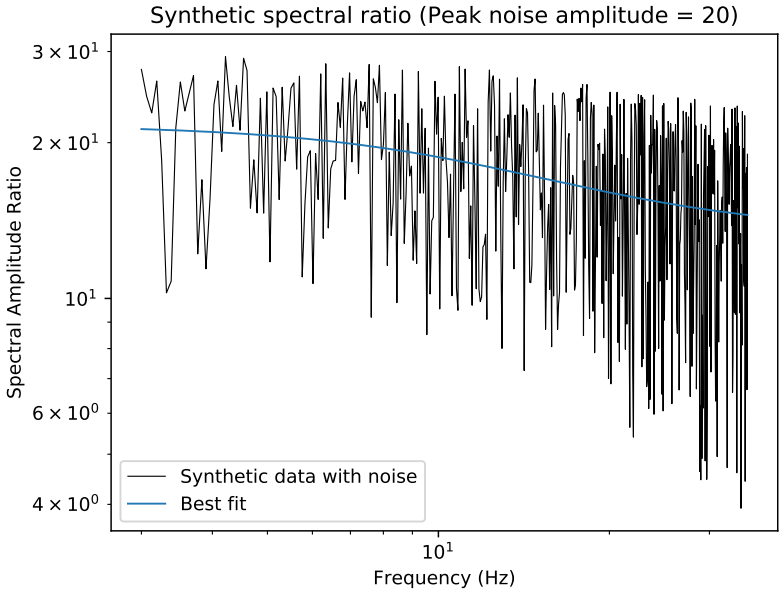

Figure 13: Synthetic spectral ratio with noise peak amplitude of 20.

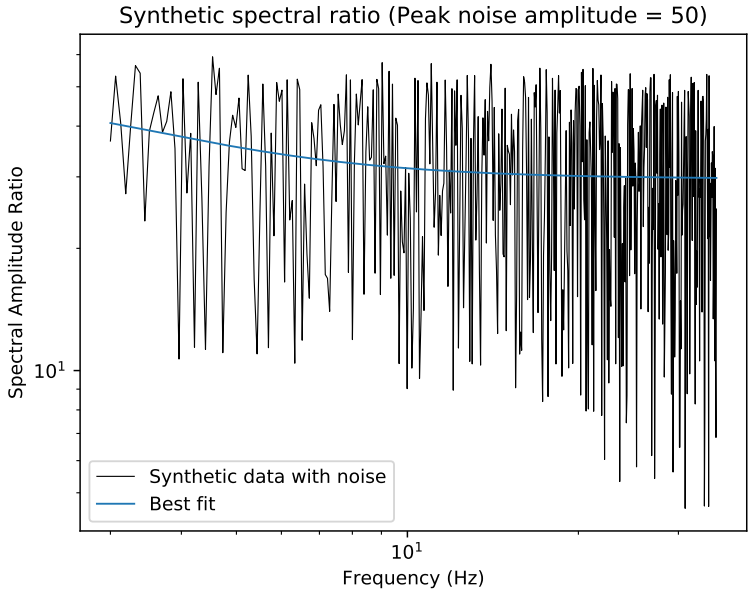

Figure 14: Synthetic spectral ratio with noise peak amplitude of 50.