Practice Session 04: Energy Partitioning of Earthquake

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Stresses on the fault

The effective normal stress (σ_n) increases linearly with depth (z), and the approxiamte gradient is $\sim 17\ MPa$.

$$\sigma_n = 17z\ MPa \tag{1}$$

The static and dynamic shear strength depends on the assumed coefficient of friction (μ_s and μ_d respectively) and the effective normal stress as follows:

$$\tau_s = \mu_s \sigma_n$$
$$\tau_d = \mu_d \sigma_d \tag{2}$$

We use the values 0.6 and 0.4 for the static and dynamic coefficient of friction respectively.

Given a constant stress drop ($\Delta\sigma$) of 3 MPa, we can compute the shear stress before the earthquake (τ_0) as:

$$\tau_0 = \Delta\sigma + \tau_d \tag{3}$$

Seismic Moment and Slip Calculation

Given a moment magnitude (M_w) of 5, we can calculate the seismic moment (M_0) as follows:

$$M_0 = 10^{[\frac{3}{2}(10.7+M_w)-7]} \tag{4}$$

The rupture dimension (L) is approximated as:

$$L \sim \left(\frac{M_0}{\Delta\sigma}\right)^{\frac{1}{3}} \tag{5}$$

The rupture area would therefore be L^2, and the final slip (D_f) is given by $\Delta\sigma L/G$, where G is the shear modulus.

Based on these calculations and initial assumptions, we obtain the following values for a $M_w = 5$ earthquake:

$$
\begin{aligned}
M_w &= 5 \\
\mu_s &= 0.6 \\
\mu_d &= 0.2 \\
M_0 &= 3.55 \times 10^{16} \; Nm \\
L &= 2279 \; m \\
S &= L^2 = 5.2 \times 10^6 \; m^2 \\
D_f &= 0.23 \; m \\
D_c &= 0.1 \; m
\end{aligned}
\tag{6}
$$

where D_c is the critical slip distance for the transition from static to dynamic friction.

Energy Calculations

We use a simple linear slip-weakening model for calculations of energies. Fig. 1 shows the relation between stress and slip for the slip-weakening model. Based on this figure, the fracture energy (E_g) is estimated as the area $A'EC$ and the frictional/heat energy (E_f) is estimated as the area $OCBD$. The radiated energy

(E_r) can be approximated as the area ABC. Therefore, using our variables defined above, we can estimate the energies as:

$$E_g = \frac{1}{2} D_c (\tau_s - \tau_d) S$$

$$E_f = D_f \tau_d S \qquad (7)$$

$$E_r = \frac{1}{2} D_f (\tau_0 - \tau_d) S$$

Also, the total potential energy of the system is approximated as:

$$\Delta W = \frac{1}{2}(\tau_0 + \tau_f) S D_f \qquad (8)$$

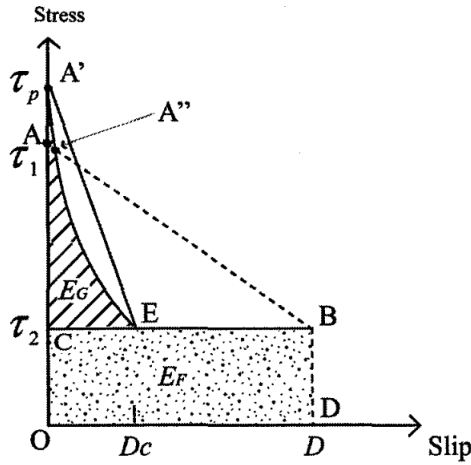**Figure 1:** Ideal slip weakening model (Kanamori and Rivera, 2006)

The upper bound of radiated energy can also be estimated by subtracting the frictoin and fracture energy from the total potential energy, but this method gives negative values for radiated energies for the given D_c and the moment magnitude. Therefore, we use the area of triangle to estimate the radiated energies for our calculations.

Results

We show the energy partitioning at depth intervals of $1 \, km$ from $3 - 15 \, km$ and then their variation in depth. In our calculations, the radiated energy, as calculated from the area of the triangle, only depends on the stress drop and is constant with depth, but the relative contribution changes.
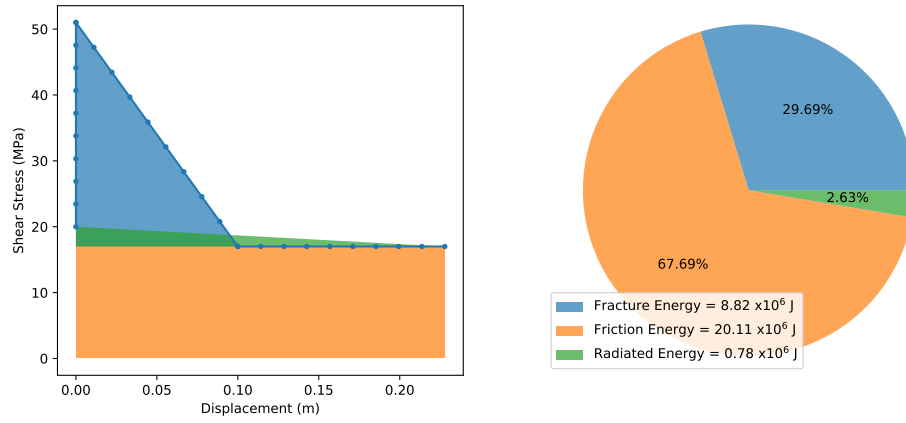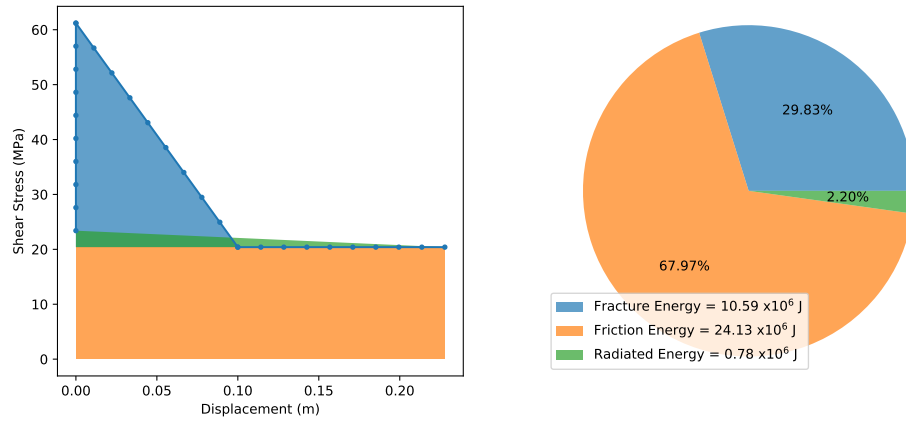

Energy Partitioning at 3.0 km depth
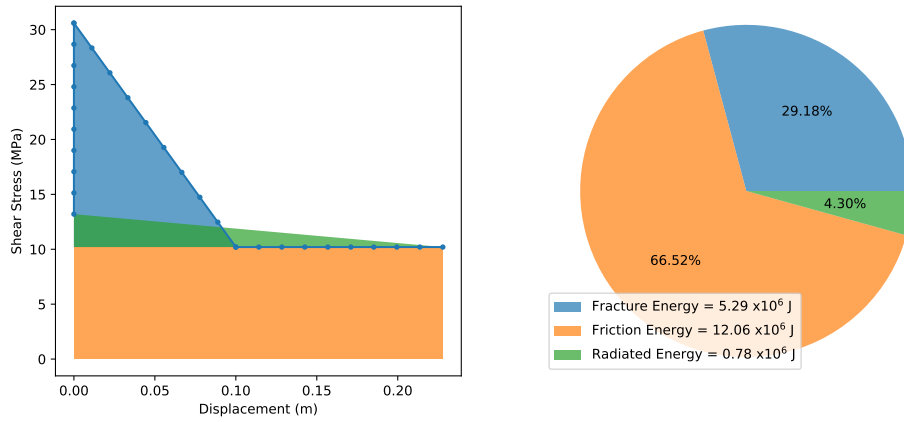
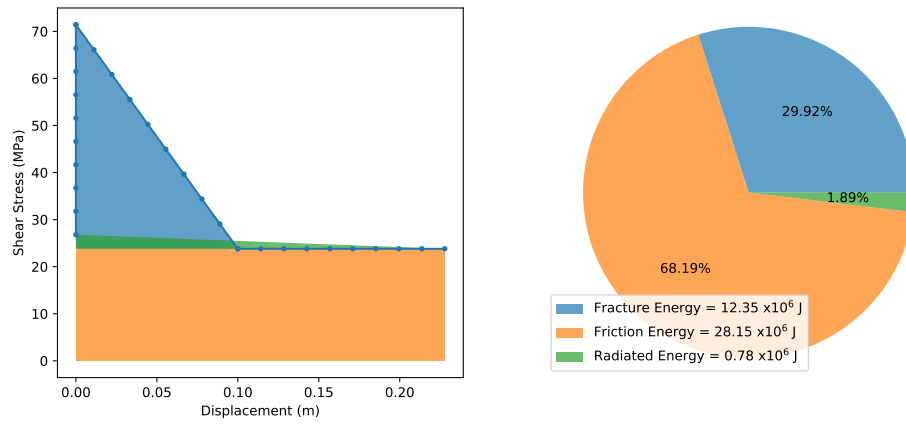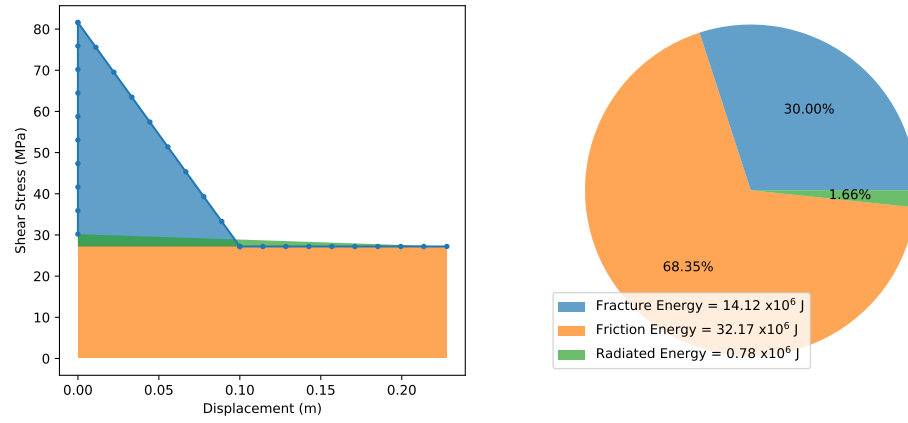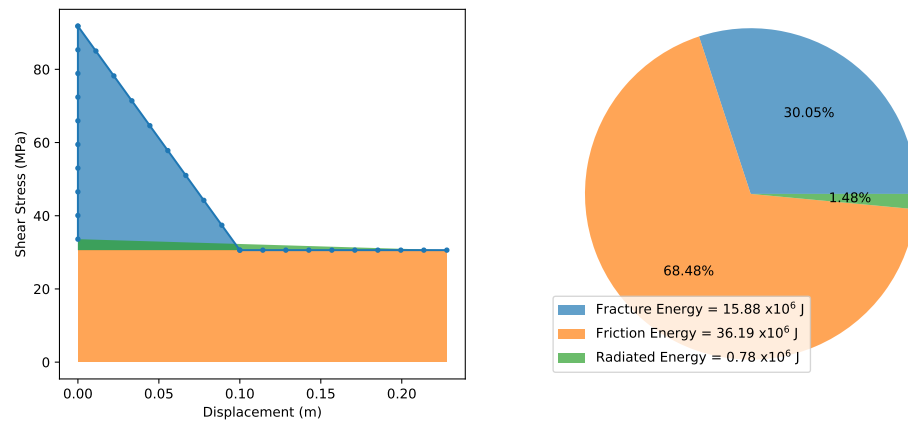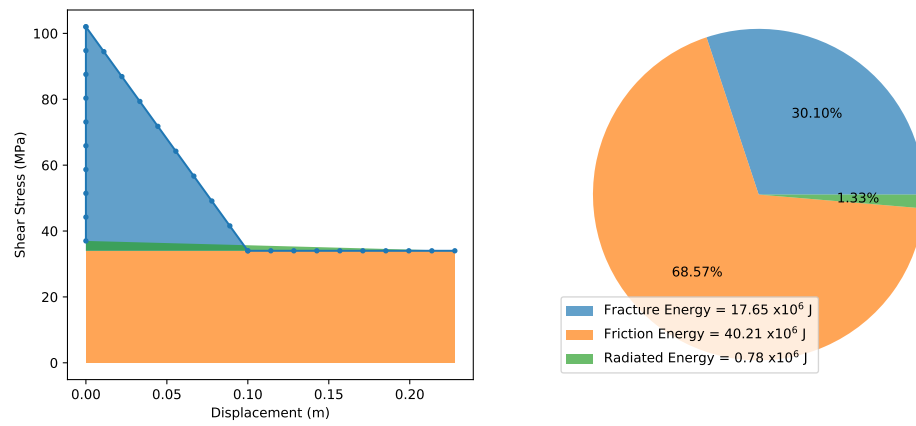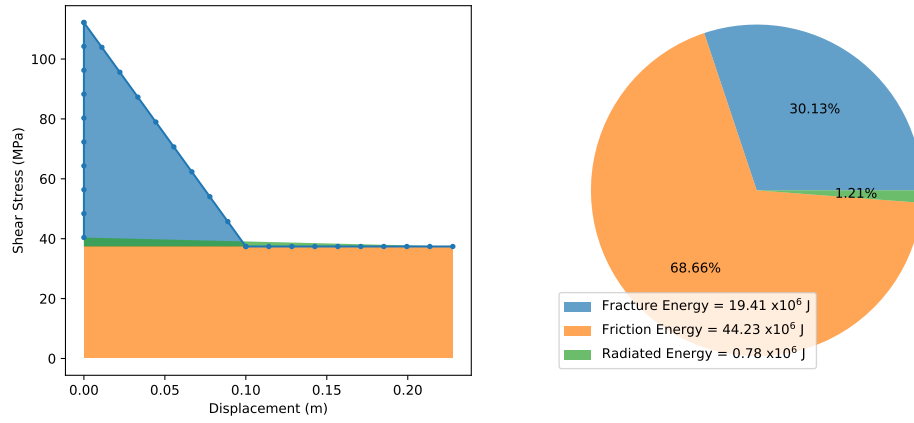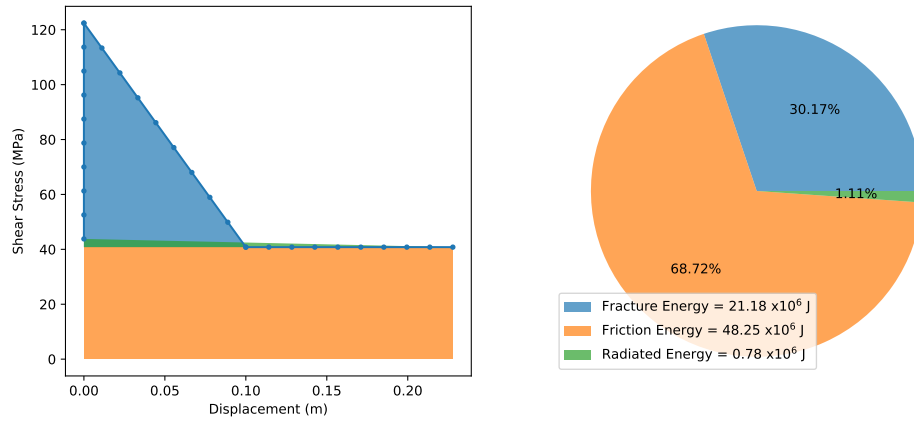Energy Partitioning at 4.0 km depth
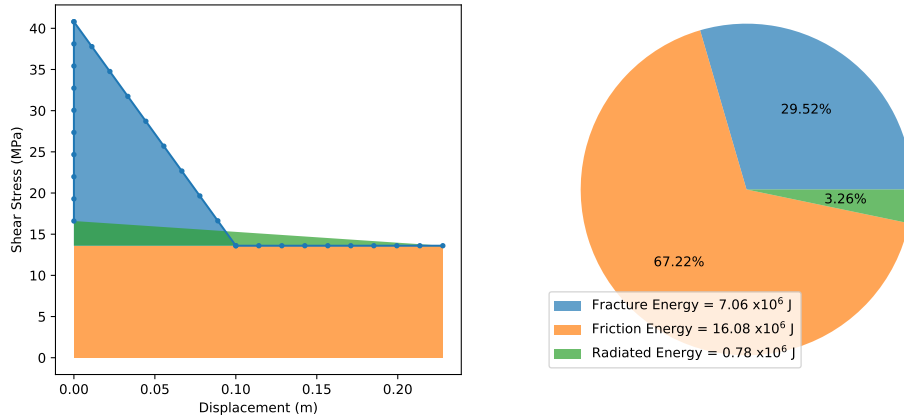
Energy Partitioning at 5.0 km depth

Energy Partitioning at 6.0 km depth

Energy Partitioning at 7.0 km depth

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Energy Partitioning at 8.0 km depth

Energy Partitioning at 9.0 km depth

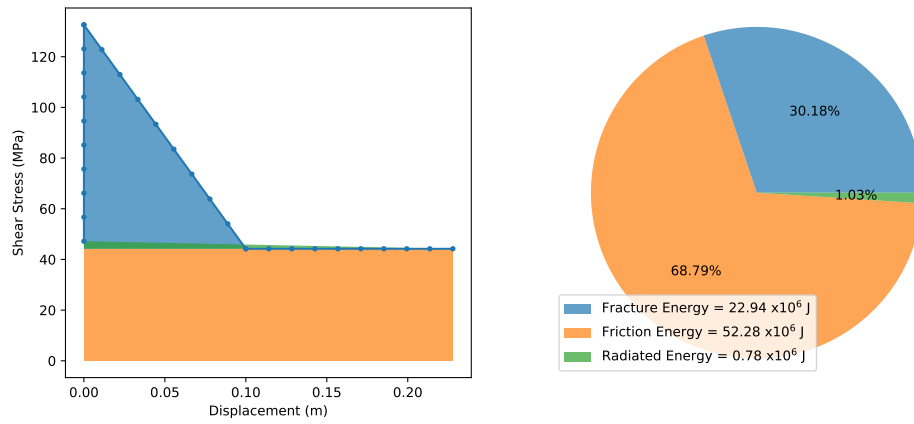## Energy Partitioning at 10.0 km depth

Energy Partitioning at 11.0 km depth

Energy Partitioning at 12.0 km depth
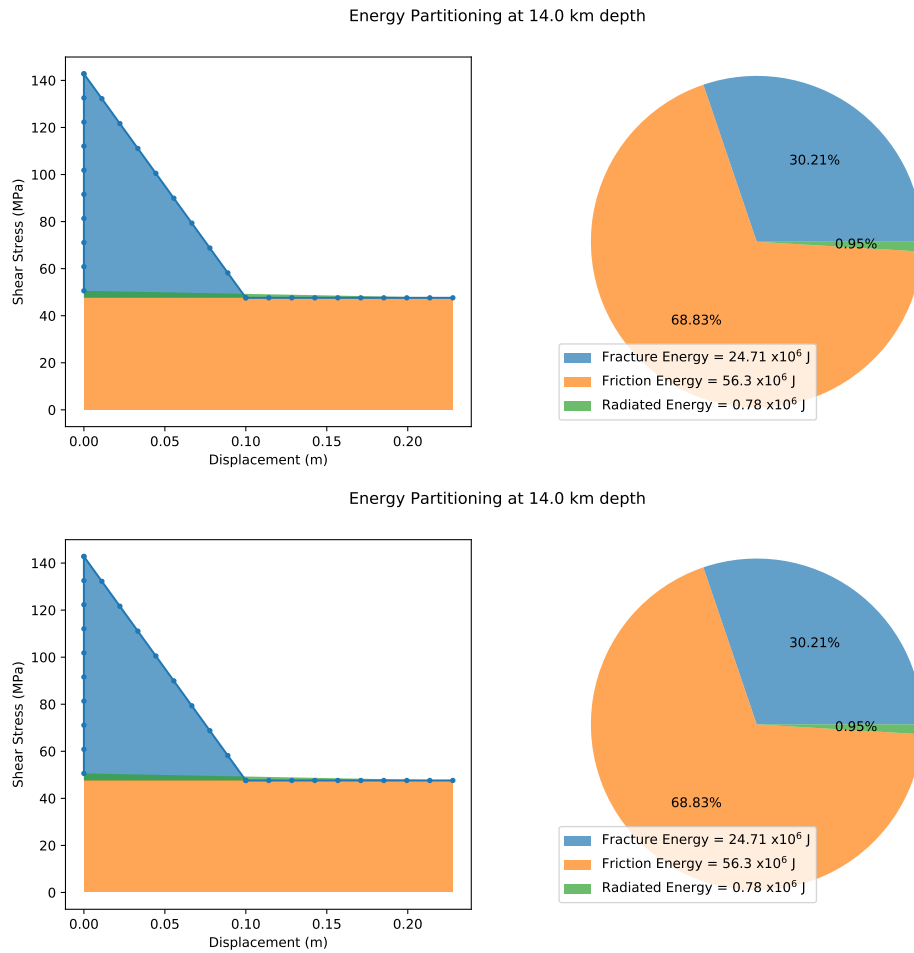

Energy Partitioning at 13.0 km depth

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Figure 2: Stress vs. Displacement and pie chart for energy partitioning. We use the area for calculation of radiated energies.

Next, we show the variation of the energies with depth. The Fig. 3 is plotted using the given values of input parameters D_c and M_0. In Fig. 4, we reduce the D_c in order to obtain positive values of radiated energies using the total potential energy. The radiated energy calculation from the area of the triangle does not change. Fig. 5 shows the energy partitioning for M_w 7 earthquake.
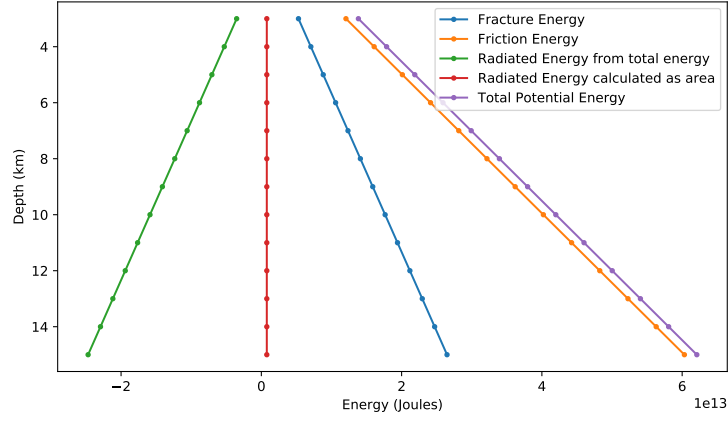

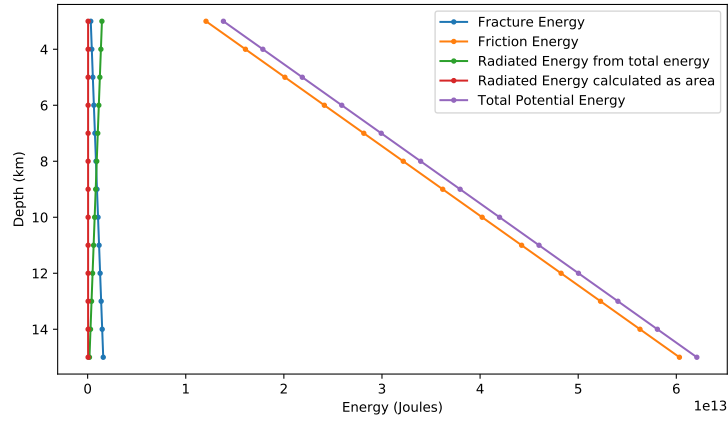**Figure 3:** Energy partitioning for M_w 5 earthquake with $D_c = 0.1$ m.

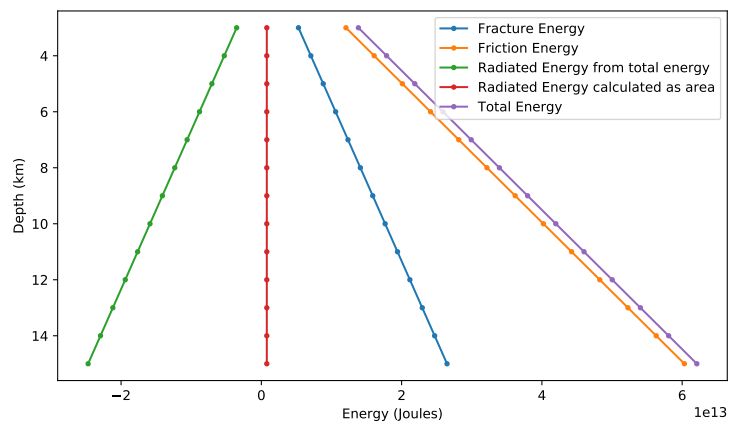**Figure 4:** Energy partitioning for M_w 5 earthquake with $D_c = 0.006$ m.

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Figure 5: Energy partitioning for M_w 7 earthquake with $D_c = 1$ m.