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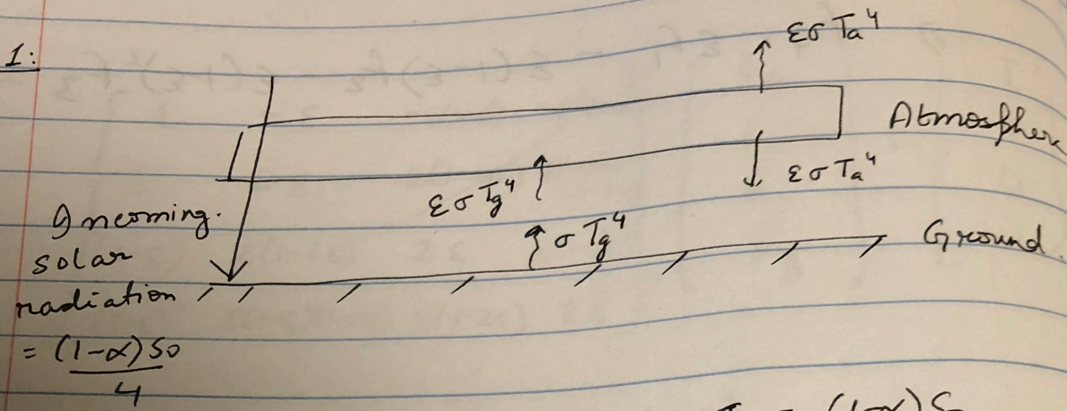
LAB 3: Energy balance models

PART 1:



Incoming radiation $I = \frac{(1-\alpha)S_0}{4}$

ε = Emissivity
T_a = atmosphere Temperature
T_g = ground temperature.
σ = steffan-Boltzmann constant.

Energy balance for the ground

$$\frac{(1-\alpha)S_0}{4} + \varepsilon\sigma T_a^4 = \sigma T_g^4$$

Energy balance for the atmosphere

$$\varepsilon\sigma T_g^4 = 2\varepsilon\sigma T_a^4$$

$$\Rightarrow \frac{(1-\alpha)S_0}{4} + \frac{1}{2}\varepsilon\sigma T_g^4 = \sigma T_g^4$$

$$\Rightarrow \frac{(1-\alpha)S_0}{4} = \sigma T_g^4\left[1 - \frac{\varepsilon}{2}\right]$$

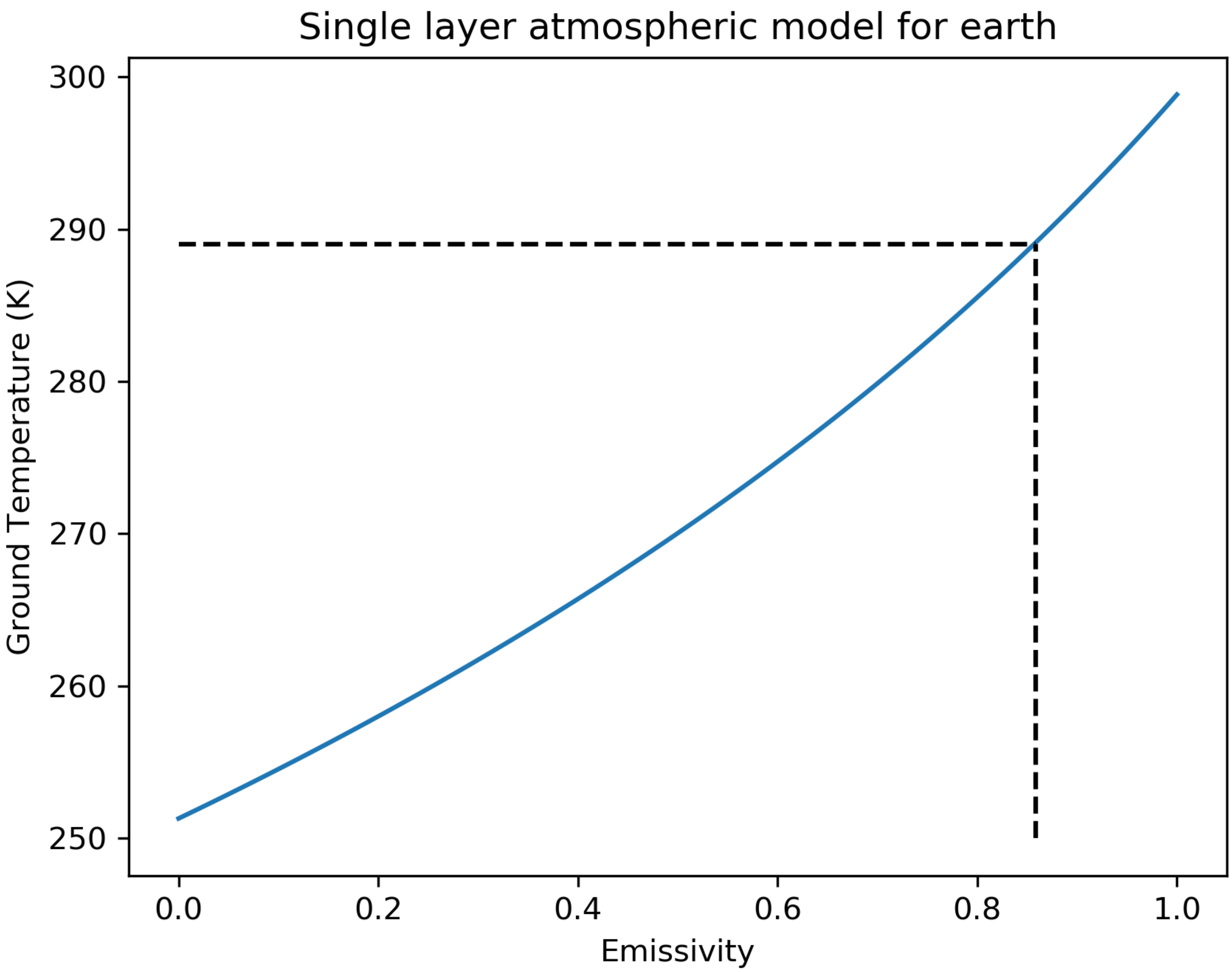
$$\Rightarrow \sigma T_g^4 (2-\varepsilon) = \frac{(1-\alpha)S_0}{2}$$

$$\Rightarrow T_g^4 = \frac{(1-\alpha)}{(2-\varepsilon)}\frac{S_0}{2\sigma}$$

Ans. $$\boxed{\Rightarrow T_g = \left[\frac{(1-\alpha)}{(2-\varepsilon)}\frac{S_0}{2\sigma}\right]^{1/4}}$$

PART2: Global avg. temperature = 289 K.

Best fit emissivity for the ~~[illegible]~~ global avg temperature = 0.86


Single layer atmospheric model for earth
Ground Temperature (K)
300
290
280
270
260
250
0.0
0.2
0.4
0.6
0.8
1.0
Emissivity

PART 3   Code submitted in python.

We need $N = 73$ layers to match the surface temperature of Venus at 700K.

PART 4   (I) = Incoming Radiation = $\frac{(1-\alpha) S_0}{4}$

Fluxes = F_g, F_1, F_2, F_3
$= \sigma T_g^4, \sigma T_1^4, \sigma T_2^4, \sigma T_3^4$
for each layer temperature respectively.
σ = steffan boltzman constant

ε = Emissivity of each layer ('e' in the diagram)

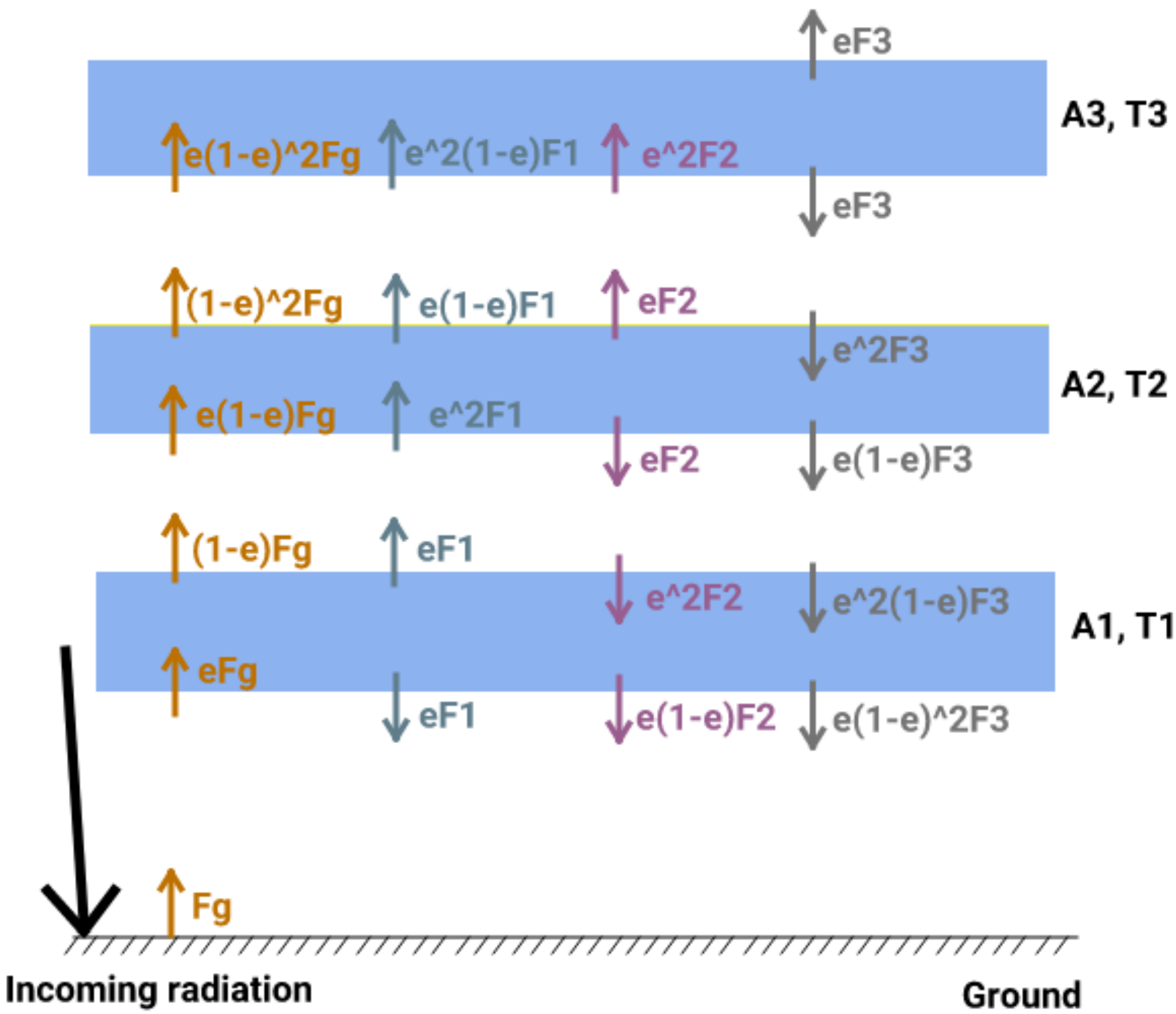
The only outgoing radiation for each atmospheric layer is $2 \varepsilon F_i$ (for i^{th} atmosphere)

Ground

A1

A2

A



eF3
A3, T3
e(1-e)^2Fg
e^2(1-e)F1
e^2F2
eF3
(1-e)^2Fg
e(1-e)F1
eF2
e^2F3
A2, T2
e(1-e)Fg
e^2F1
eF2
e(1-e)F3
(1-e)Fg
eF1
e^2F2
e^2(1-e)F3
A1, T1
eFg
eF1
e(1-e)F2
e(1-e)^2F3
Fg
Incoming radiation
Ground

Energy balance equation for:

Incoming = Outgoing.

Ground: $I + \varepsilon F_1 + \varepsilon(1-\varepsilon)F_2 + \varepsilon(1-\varepsilon)^2 F_3 = F_G$

A1: $\varepsilon F_g + \varepsilon^2 F_2 + \varepsilon^2(1-\varepsilon)F_3 = 2\varepsilon F_1$

A2: $\varepsilon(1-\varepsilon)F_g + \varepsilon^2 F_1 + \varepsilon^2 F_3 = 2\varepsilon F_2$.

A3: $\varepsilon(1-\varepsilon)^2 F_g + \varepsilon^2(1-\varepsilon)F_1 + \varepsilon^2 F_2 = 2\varepsilon F_3$

Rearranging,

Ground: $-F_G + \varepsilon F_1 + \varepsilon(1-\varepsilon)F_2 + \varepsilon(1-\varepsilon)^2 F_3 = -I$

A1: $\varepsilon F_G - 2\varepsilon F_1 + \varepsilon^2 F_2 + \varepsilon^2(1-\varepsilon)F_3 = 0$

A_2: $\varepsilon(1-\varepsilon)F_G + \varepsilon^2 F_1 - 2\varepsilon F_2 + \varepsilon^2 F_3 = 0$

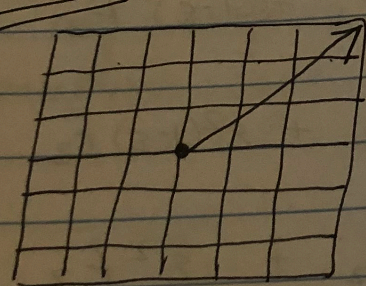
A_3: $\varepsilon(1-\varepsilon)^2 F_G + \varepsilon^2(1-\varepsilon)F_1 + \varepsilon^2 F_2 - 2\varepsilon F_3 = 0$

In matrix form,

$$
\begin{array}{c}
\text{Coefficient Matrix} \\
\begin{bmatrix}
-1 & \varepsilon & \varepsilon(1-\varepsilon) & \varepsilon(1-\varepsilon)^2 \\
\varepsilon & -2\varepsilon & \varepsilon^2 & \varepsilon^2(1-\varepsilon) \\
\varepsilon(1-\varepsilon) & \varepsilon^2 & -2\varepsilon & \varepsilon^2 \\
\varepsilon(1-\varepsilon)^2 & \varepsilon^2(1-\varepsilon) & \varepsilon^2 & -2\varepsilon
\end{bmatrix}
\end{array}
\begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{bmatrix} =
$$

PART 5 Computer program to solve the above matrix equation.

ROUGH WORK



Distance from centre to edge $= \frac{N}{\sqrt{2}}$.

N×N matrix.

Diagonal index = i, i

Subdiagonal index = $i+1, i$

superdiagonal index = $i, i+1$

Pseudocode & Algorithm for part 5

We have 2 matrices : Coefficient matrix and RHS.

RHS

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We can see a pattern in the coefficient matrix.

$$\begin{matrix} -1 & \varepsilon & \varepsilon(1-\varepsilon) & \varepsilon(1-\varepsilon)^2 & \varepsilon(1-\varepsilon)^3 & \cdots \\ \varepsilon & -2\varepsilon & \varepsilon^2 & \varepsilon^2(1-\varepsilon) & \varepsilon^2(1-\varepsilon)^2 & \cdots \\ \varepsilon(1-\varepsilon) & \varepsilon^2 & -2\varepsilon & \varepsilon^2 & \varepsilon^2(1-\varepsilon) & \cdots \\ \varepsilon(1-\varepsilon)^2 & \varepsilon^2(1-\varepsilon) & \varepsilon^2 & -2\varepsilon & \varepsilon^2 & \cdots \end{matrix}$$

- Firstly, the matrix is symmetric.
- Secondly, all the diagonal elements are same except first row & column.

We can loop only through diagonal

```
for i = 1:(N/√2) + 1      # no. of diagonals for
                          # N×N matrix.

    diagonal = ε^2 (1-ε)^i

end for.
```

For zero emissivity, we would get a singular matrix

If we solve the equations by hand, we get

$$F_g = I$$

$\Rightarrow$ The ground emits flux equal to the incoming radiation.

For emissivity $= 1$, I tested this with Venus parameters for 100 layers and I get the same answer.