DYNAMIC MODEL BASED CONTROL

LAB REPORT

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STATE SPACE APPROACH FOR LINEAR SYSTEMS

THE INVERTED PENDULUM

1. State Space Model of the System

1.1 The linearized model around the equilibrium point could be obtained by substituting $\theta = 0$, This is a design model for a control that tries to keep the pendulum in vertical position and the cart in the equilibrium position.

1.2 Given: m = 0.5kg, M = 5kg, L = 1m, g = 9.8m/s2.

The numerical values of the matrices A and B of the state space representation were obtained using MATLAB computation as the matrices are obtained as follows:

A =

0	1	0	0
10.780	0	0	0
0	0	0	1
-0.980	0	0	0

B =

0
-0.200
0
0.200

C=

1	0	0	0
0	0	1	0

1.3 The eigen values associated to this linear state space representation were also obtained using MATLAB as:

EigenVals =

0

0

3.283291

-3.283291

2. State Feedback Controller – Access to the whole state space

In this section we assume that we have access to the whole state as if sensors could give the angle θ, its derivative ˙θ, the position p and its derivative p

2.1 State Feedback

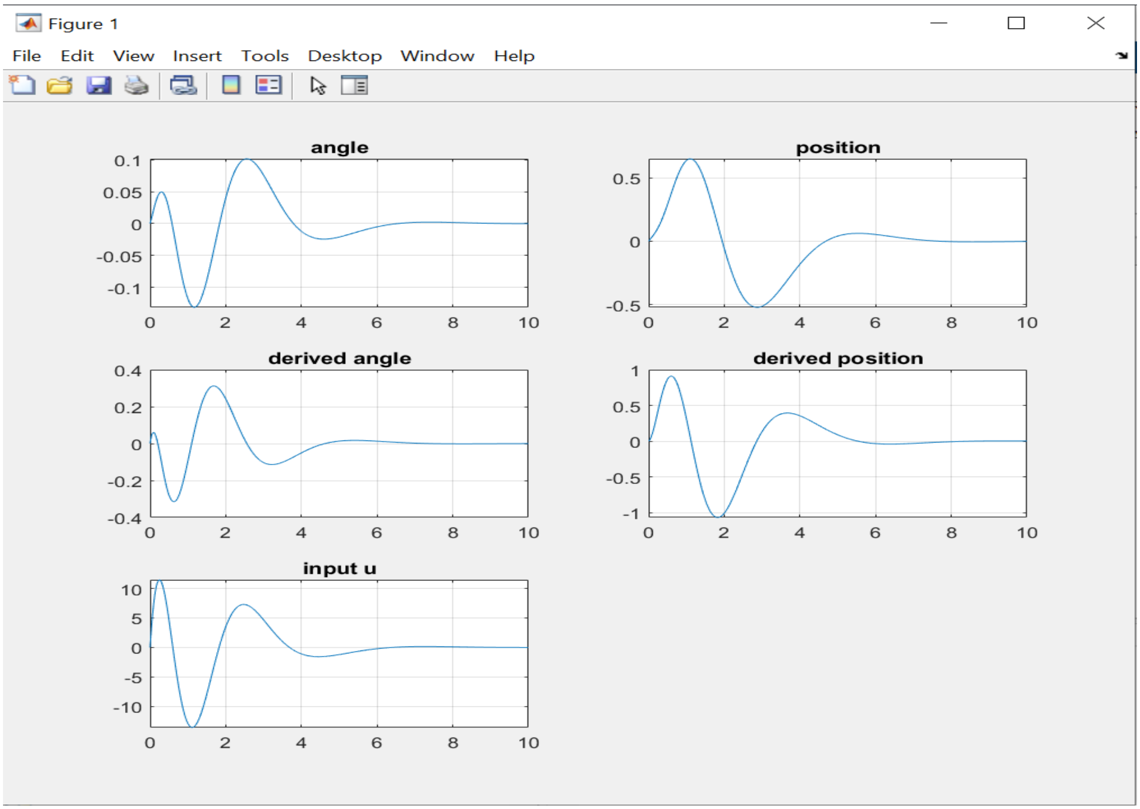
2.1.1 The controllability matrix gotten using MATLAB had a full rank this proves that the system is controllable. Therefore, it is possible to stabilize the pendulum and the cart around an equilibrium point with a static space feedback.

2.1.2 Yes, the Eigen values can be placed freely as rank of the controllability matrix is 4 which makes all the modes controllable. Care should be exercised while placing freely to not de-stabilize the system.

2.1.3 The static state feedback F such that the desired eigen values are $-1 \pm j \ and \ -2 \pm 2j$ was obtained to be:

F = 152.0633   42.2449   8.1633   12.24489

2.1.4 The Simulink block diagram of the closed loop with the control law $u = Fx + v$ is constructed and the plot of state response $x(t)$ and the control response $u(t)$ when $v(t) = 0$ for the given initial conditions are plotted. (i.e. the plots of angle, derivative of angle, position, derivative of position and the control input are plotted against time in five different plots of the same figure are shown below)



The plots gives the information regarding the four states (i.e. angle, derivative of angle, position, derivative of position) when the system is provided with a static state feedback $'F'$ and by constructing a Simulink block diagram which corresponds to $u = Fx + v$. The fifth plot gives the control response $u(t)$ against time.

2.2 Observer

2.2.1 Yes, the system is completely observable because the observability matrix obtained from the MATLAB is having its full rank.

2.2.2 The two gains K_1 *and* K_2 corresponding to:

(a) Faster poles than the ones used for the controller: $-100 \pm j\ and\ -200 \pm 2j$

(b) Poles as fast as the older ones used for the controller: $-1 \pm j\ and\ -2 \pm 2j$

are obtained as follows:

K_fast =

260.203127591121	0.414580128339665
13903.9429914155	-97.4457045902020
604.595392047321	339.796872408786
121274.380280933	27946.2812973280

K_slow =

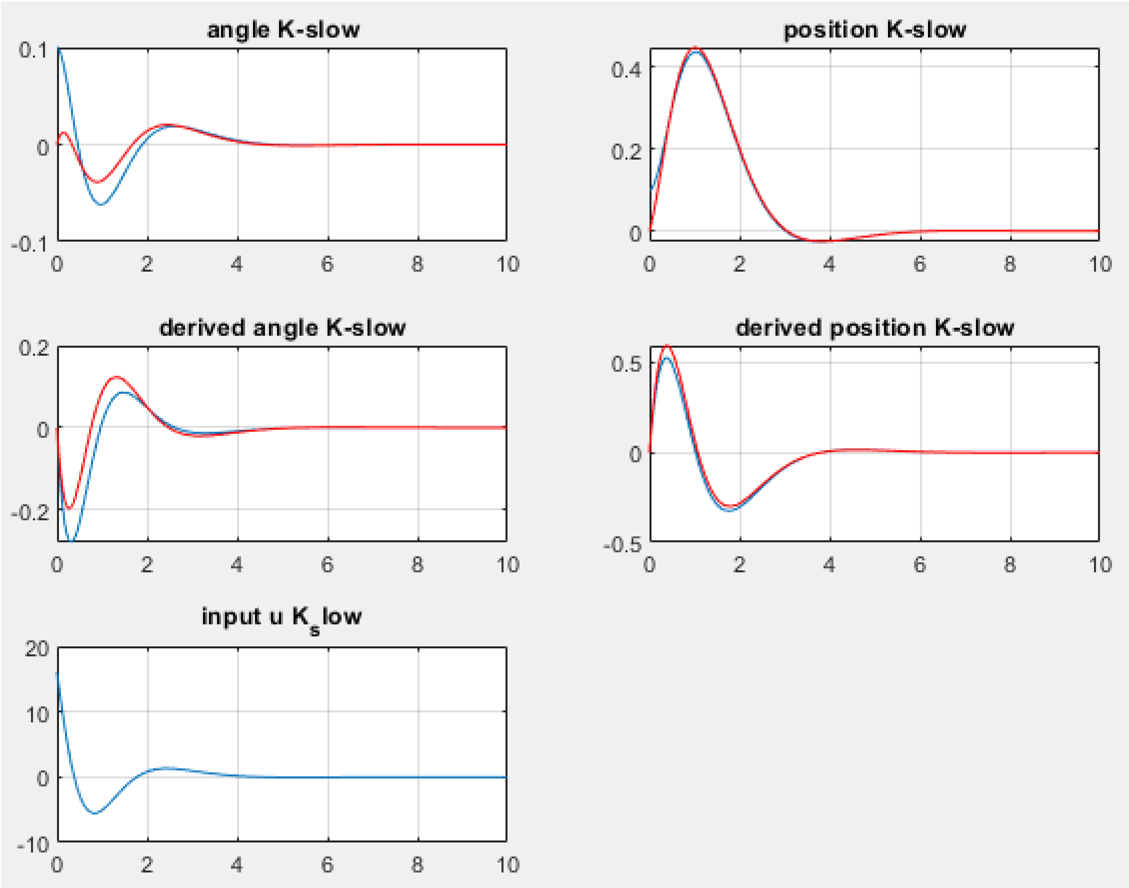
2.99980723398773 -0.998523874213042

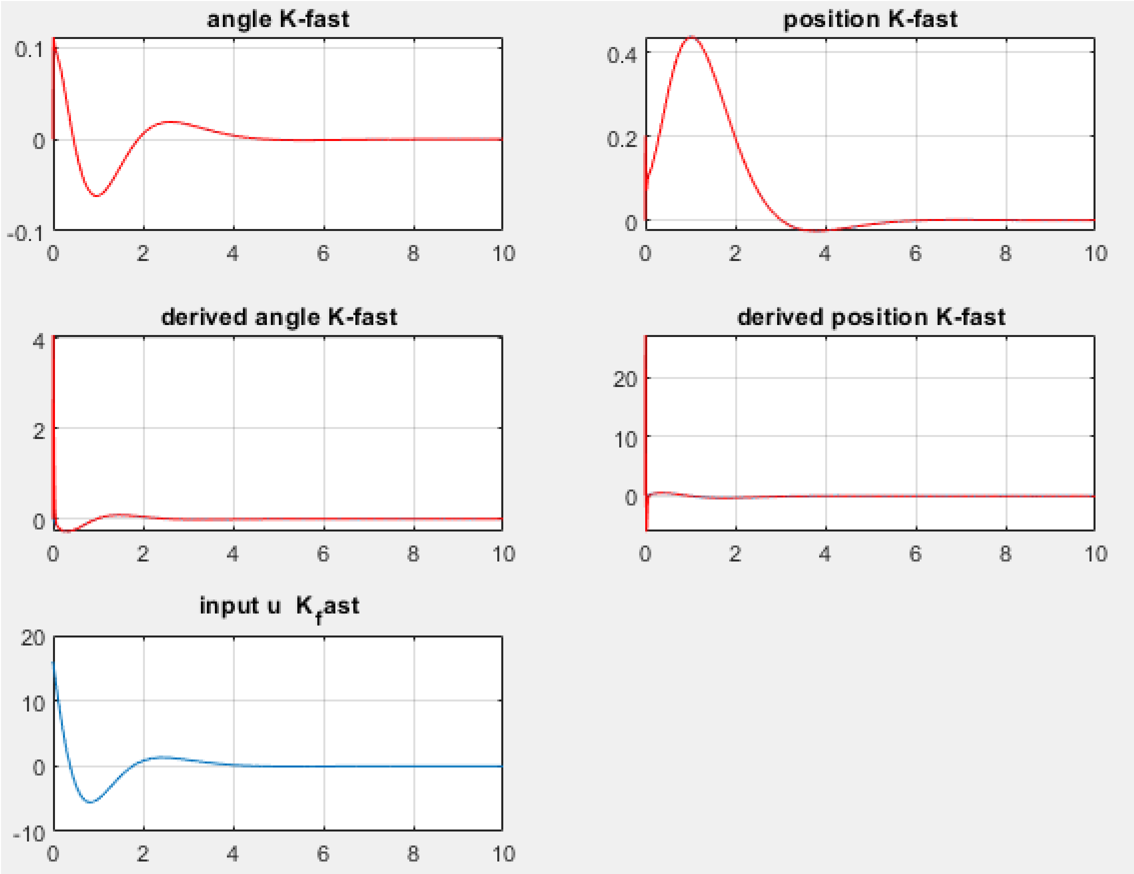
14.7800582668375 0.00428491327843668

1.00147374871623 3.00019276601227

-0.975716239048952 3.99994632283045

since we have two gains K_{slow} and K_{fast}, we have two state observers $\widehat{x_{slow}}$ and $\widehat{x_{fast}}$. The responses for $x(t)$ $with$ $\widehat{x_{slow}}$ and $\widehat{x_{fast}}$ are plotted on the same axis. Also, the response of $u(t)$ for the series of two series of observer poles is plotted. Red plot signifies observed states $\widehat{x_{slow}}$ and $\widehat{x_{fast}}$ respectively in the two graphs and blue signifies the real state x. All the plots are shown below:





We can observe that with fast poles, the derived states settle faster than with the slow poles. The observed states and the real states of angle and position converge faster for fast poles. The fast behaviour is not usually good as although the states of angle and position converge faster, the derivatives of these states take high values in short intervals of time which is not desirable in most of the real-world applications.

3. Estimated state feedback controller

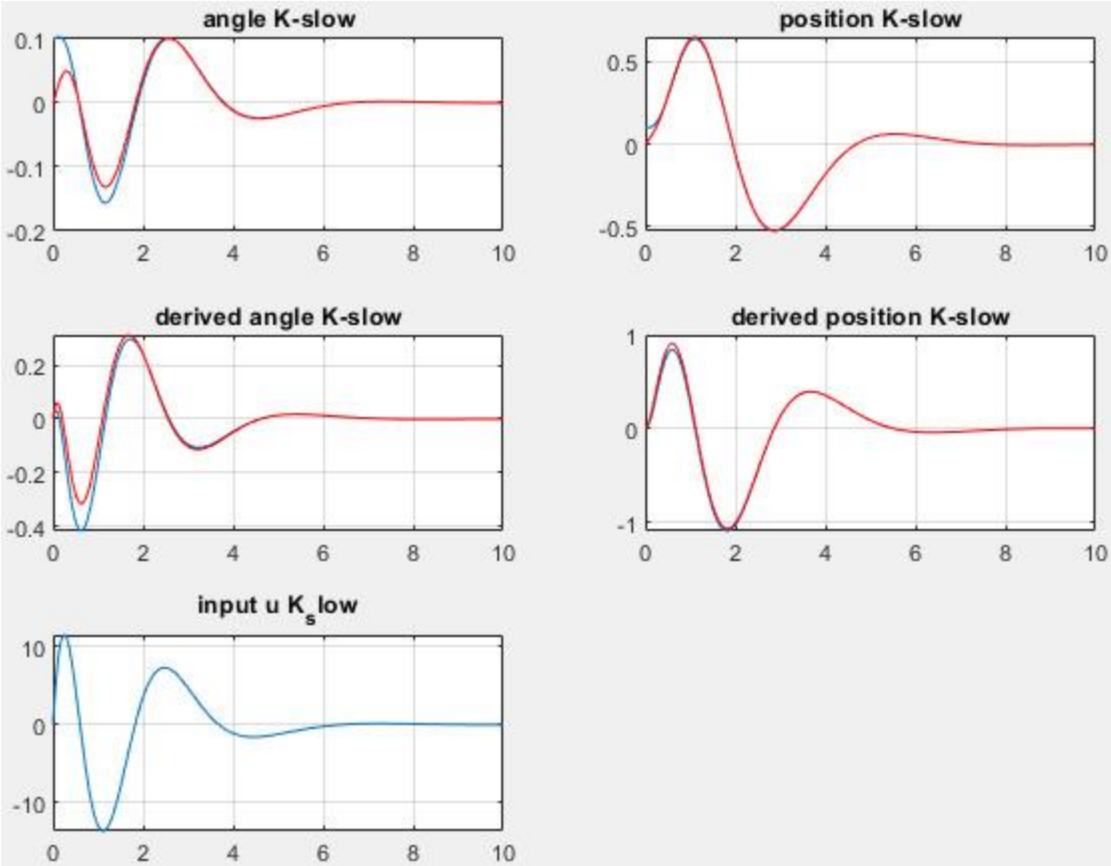
3.1 The state space equation for the observer is as follows: $\dot{\hat{x}} = (\boldsymbol{A} - \boldsymbol{KC})\hat{x} + Bu + Ky$

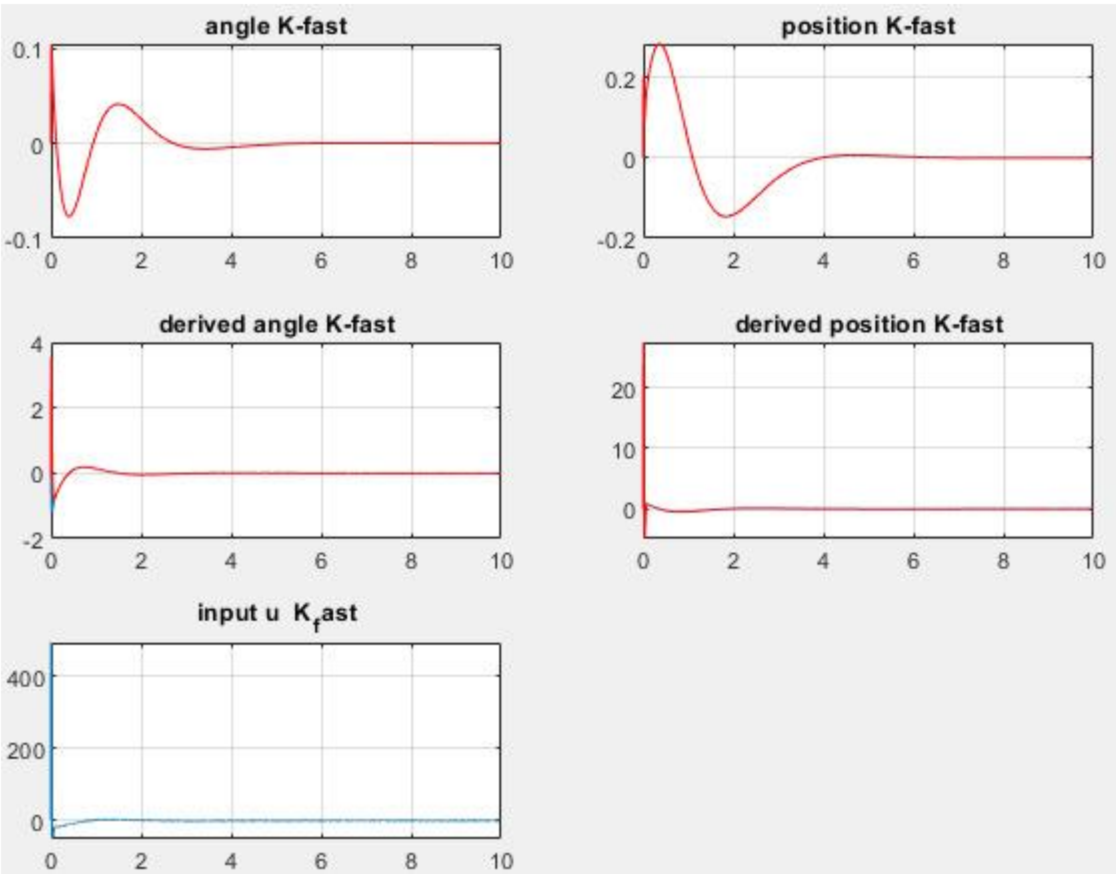
3.2 The output y can be multiplied by the following matrix to get angle and position:

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{matrix}$$

3.3 F matrix is constructed as before by using slow and fast Eigen values.

3.4 The plots are shown below and are in the same format as the previous results.


angle K-slow
position K-slow
derived angle K-slow
derived position K-slow
input u K_slow


angle K-fast
position K-fast
derived angle K-fast
derived position K-fast
input u K_fast

From the above plots we can observe that the estimated states and the real states converge faster with the presence of observer in the loop to control the input than in the previous case where the observer is not in the loop.

4. Animation

The results are animated with the given code files and the inverted pendulum settles in the equilibrium position in 8 seconds for slow Eigen values and in 4 seconds for fast Eigen values. The code files are all submitted for verification.