Topic 5: Formal Relational Query Languages (Chapter 6)

Database System Concepts, 6th Ed.

Topic 5: Formal Relational Query Languages

- Introduction
- Relational Algebra
- Tuple Relational Calculus
- Domain Relational Calculus

Introduction

- Database Query Languages
  - Procedural Language
  - Non-Procedural Language

Relational Algebra

- Procedural language
- Six basic operators
  - select: σ
  - project: Π
  - union: $\cup$
  - set difference: –
  - Cartesian product: x
  - rename: ρ
- The operators take one or two relations as inputs and produce a new relation as a result.

Relational Algebra (cont.) – Query Processing Flow

Example Relational Schemas Used in Example Queries

instructor (*ID*, *name*, *street*, *city*, *salary*)

course (*course_id*, *title*, *credits*)

section (*course_id*, *section_id*, *semester*, *year*)

teach (*ID*, *course_id*, *section_id*, *semester*, *year*)

Select Operation – Example

- Relation r

A	B	C	D
α	α	1	7
α	β	5	7
β	β	12	3
β	β	23	10

- $\sigma_{A=B \wedge D > 5}(r)$

A	B	C	D
α	α	1	7
β	β	23	10

Select Operation

- Notation: $\sigma_p(r)$
- p is called the **selection predicate**
- Defined as:

$$\sigma_p(\boldsymbol{r}) = \{t \mid t \in r \textbf{ and } p(t)\}$$

Where p is a formula in propositional calculus consisting of **terms** connected by : $\wedge$ (**and**), $\vee$ (**or**), $\neg$ (**not**)
Each **term** is one of:

<attribute> *op* <attribute> or <constant>

where *op* is one of: $=, \neq, >, \geq, <, \leq$

- Example of selection: find all instructions in the Physics department:

$$\sigma_{dept\_name=\text{“}Physics\text{”}}(instructor)$$

Project Operation – Example

- Relation *r*:

A	B	C
α	10	1
α	20	1
β	30	1
β	40	2

- $\Pi_{A,C}(r)$

A	C
α	1
α	1
β	1
β	2

=

A	C
α	1
β	1
β	2

Project Operation

- Notation:

$$\Pi_{A_1, A_2, \ldots, A_k}(r)$$

  where A_1, A_2 are attribute names and r is a relation name.

- The result is defined as the relation of k columns obtained by erasing the columns that are not listed
- Duplicate rows removed from result, since relations are sets
- Example: *find id, name and salary of all instructors:*

$$\Pi_{ID,\ name,\ salary}(instructor)$$

Union Operation – Example

- Relations *r, s:*

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- r ∪ s:

A	B
α	1
α	2
β	1
β	3

Union Operation

- Notation: $r \cup s$
- Defined as:

$$r \cup s = \{t \mid t \in r \text{ or } t \in s\}$$

- For $r \cup s$ to be valid.
  1. *r, s* must have the *same* **arity** (same number of attributes)
  2. The attribute domains must be **compatible** (example: 2nd column of *r* deals with the same type of values as does the 2nd column of *s*)
- Example: find all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or in both:

$$\Pi_{course\_id}(\sigma_{semester=\text{“Fall”} \wedge year=2009}(section)) \cup \Pi_{course\_id}(\sigma_{semester=\text{“Spring”} \wedge year=2010}(section))$$

Set difference of two relations

- Relations *r*, *s*:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- *r* – *s*:

A	B
α	1
β	1

Set Difference Operation

- Notation $r - s$
- Defined as:

$$r - s = \{t \mid t \in r \textbf{ and } t \notin s\}$$

- Set differences must be taken between **compatible** relations.
  - r and s must have the same arity
  - attribute domains of r and s must be compatible
- Example: find all courses taught in the Fall 2009 semester, but not in the Spring 2010 semester

$$\Pi_{course\_id}(\sigma_{semester=\text{“Fall”} \wedge year=2009}(section)) - \Pi_{course\_id}(\sigma_{semester=\text{“Spring”} \wedge year=2010}(section))$$

Cartesian-Product Operation – Example

- Relations *r, s*:

A	*B*
α	1
β	2

r

C	*D*	*E*
α	10	a
β	10	a
β	20	b
γ	10	b

s

- *r* x *s*:

A	*B*	*C*	*D*	*E*
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

Cartesian-Product Operation

- Notation $r \times s$
- Defined as:

$$r \times s = \{t\, q \mid t \in r \textbf{ and } q \in s\}$$

- Assume that attributes of r(R) and s(S) are disjoint. (That is, $R \cap S = \varnothing$).
- If attributes of *r(R)* and *s(S)* are not disjoint, then renaming must be used.

Composition of Operations

- Can build expressions using multiple operations
- Example: $\sigma_{A=C}(r \times s)$

- $r \times s$

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

- $\sigma_{A=C}(r \times s)$

A	B	C	D	E
α	1	α	10	a
β	2	β	10	a
β	2	β	20	b

Rename Operation

- Allows us to name, and therefore to refer to, the results of relational-algebra expressions.
- Allows us to refer to a relation by more than one name.
- Example:

$$\rho_x(E)$$

returns the expression E under the name X

- If a relational-algebra expression E has arity n, then

$$\rho_{x(A_1, A_2, \ldots, A_n)}(E)$$

returns the result of expression E under the name X, and with the attributes renamed to $A_1, A_2, \ldots, A_n$.

Example Query

- Find the largest salary in the university
  - Step 1: find instructor salaries that are less than some other instructor salary (i.e. not maximum)
    - using a copy of *instructor* under a new name *d*
    - $\Pi_{instructor.salary} (\sigma_{instructor.salary < d.salary} (instructor \times \rho_{d} (instructor)))$
  - Step 2: Find the largest salary
    - $\Pi_{salary} (instructor) - \Pi_{instructor.salary} (\sigma_{instructor.salary < d.salary} (instructor \times \rho_{d} (instructor)))$

Example Queries

- Find the names of all instructors in the Physics department, along with the *course_id* of all courses they have taught
  - Query 1

$$\Pi_{instructor.name,course\_id}\,(\sigma_{dept\_name=\text{“Physics”}}\,(\sigma_{instructor.ID=teaches.ID}\,(instructor \times teaches)))$$

  - Query 2

$$\Pi_{instructor.name,course\_id}\,(\sigma_{instructor.ID=teaches.ID}\,(\sigma_{dept\_name=\text{“Physics”}}\,(instructor) \times teaches))$$

Formal Definition

- A basic expression in the relational algebra consists of either one of the following:
  - A relation in the database
  - A constant relation
- Let E_1 and E_2 be relational-algebra expressions; the following are all relational-algebra expressions:
  - $E_1 \cup E_2$
  - $E_1 - E_2$
  - $E_1 \times E_2$
  - $\sigma_p (E_1)$, P is a predicate on attributes in E_1
  - $\Pi_S(E_1)$, S is a list consisting of some of the attributes in E_1
  - $\rho_x (E_1)$, x is the new name for the result of E_1

Additional Operations

We define additional operations that do not add any power to the relational algebra, but that simplify common queries.

- Set intersection
- Natural join
- Assignment
- Outer join

Set-Intersection Operation

- Notation: $r \cap s$
- Defined as:
- $r \cap s = \{ t \mid t \in r$ **and** $t \in s \}$
- Assume:
  - r, s have the *same arity*
  - attributes of r and s are compatible
- Note: $r \cap s = r - (r - s)$

Set-Intersection Operation – Example

- Relation *r, s*:

A	*B*
α	1
α	2
β	1

r

A	*B*
α	2
β	3

s

- $r \cap s$

A	*B*
α	2

Natural-Join Operation

- Notation: r ⋈ s
- Let *r* and *s* be relations on schemas *R* and *S* respectively.
  Then, r ⋈ s is a relation on schema $R \cup S$ obtained as follows:
  - Consider each pair of tuples t_r from *r* and t_s from *s*.
  - If t_r and t_s have the same value on each of the attributes in $R \cap S$, add a tuple *t* to the result, where
    - *t* has the same value as t_r on *r*
    - *t* has the same value as t_s on *s*
- Example:

  $R = (A, B, C, D)$

  $S = (E, B, D)$
  - Result schema = (A, B, C, D, E)
  - $r \bowtie s$ is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\ (\sigma_{r.B = s.B \wedge r.D = s.D}\ (r \times s))$$

Natural Join Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	ε

s

- r ⋈ s

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

Natural Join and Theta Join

- Find the names of all instructors in the Comp. Sci. department together with the course titles of all the courses that the instructors teach
  - $\Pi_{name,\ title}\ (\sigma_{dept\_name=\text{“Comp. Sci.”}}\ (instructor \bowtie teaches \bowtie course))$
- Natural join is associative
  - $(instructor \bowtie teaches) \bowtie course$ is equivalent to $instructor \bowtie (teaches \bowtie course)$
- Natural join is commutative
  - $Instructor \bowtie teaches$ is equivalent to $teaches \bowtie instructor$
- The **theta join** operation $r \bowtie_\theta s$ is defined as
  - $r \bowtie_\theta s = \sigma_\theta (r \times s)$

Assignment Operation

- The assignment operation ($\leftarrow$) provides a convenient way to express complex queries.
  - Write query as a sequential program consisting of
    - a series of assignments
    - followed by an expression the value of which is displayed as a result of the query.
  - Assignment must always be made to a temporary relation variable.
  - Example: find the largest instructor salary:

$$\Pi_{salary}(instructor) - \Pi_{instructor.salary}(\sigma_{instructor.salary < d.salary}(instructor \times \rho_{d}(instructor)))$$

Can be rewritten as:

$$Temp1 \leftarrow \Pi_{instructor.salary}(\sigma_{instructor.salary < d.salary}(instructor \times \rho_{d}(instructor)))$$

$$Result \leftarrow \Pi_{salary}(instructor) - Temp1$$

Outer Join

- An extension of the join operation that avoids loss of information.
- Computes the join and then adds tuples form one relation that does not match tuples in the other relation to the result of the join.
- Uses *null* values:
  - *null* signifies that the value is unknown or does not exist

Outer Join – Example

- Relation *instructor1*

ID	*name*	*dept_name*
10101	Srinivasan	Comp. Sci.
12121	Wu	Finance
15151	Mozart	Music

- Relation *teaches1*

ID	*course_id*
10101	CS-101
12121	FIN-201
76766	BIO-101

Outer Join – Example

- Join

  instructor ⋈ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201

- Left Outer Join

  instructor ⟕ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
15151	Mozart	Music	*null*

Outer Join – Example

- Right Outer Join

  instructor ⟖ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
76766	null	null	BIO-101

- Full Outer Join

  instructor ⟗ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
15151	Mozart	Music	*null*
76766	null	null	BIO-101

Outer Join using Joins

- Outer join can be expressed using basic operations
  - e.g. r ⟕ s can be written as

    $(r \bowtie s) \cup (r - \Pi_R(r \bowtie s) \times \{(null, \ldots, null)\}$

Division Operator

- Apply to “for all” or “for every” query
- Given relations r(R) and s(S), such that $S \subset R$, $r \div s$ is the largest relation t(R-S) such that

  $t \times s \subseteq r$

- E.g. let $r(ID, course\_id) = \prod_{ID,\ course\_id}(takes)$ and

  $s(course\_id) = \prod_{course\_id}(\sigma_{dept\_name=\text{“Biology”}}(course))$

then $r \div s$ gives us students who have taken all courses in the Biology department

Extended Relational-Algebra-Operations

- Generalized Projection
- Aggregate Functions

Generalized Projection

- Extends the projection operation by allowing arithmetic functions to be used in the projection list.

$$\Pi_{F_1, F_2, \ldots, F_n}(E)$$

- E is any relational-algebra expression
- Each of F_1, F_2, …, F_n are arithmetic expressions involving constants and attributes in the schema of E.
- Example: Given relation *instructor(ID, name, dept_name,* salary) where salary is annual salary, get the same information but with monthly salary

$$\Pi_{ID,\ name,\ dept\_name,\ salary/12}\ (instructor)$$

Aggregate Functions and Operations

- **Aggregation function** takes a collection of values and returns a single value as a result.

  avg: average value
  min: minimum value
  max: maximum value
  sum: sum of values
  count: number of values

- **Grouping: apply aggregate operation** on several groups in relational algebra:

$$_{G_1, G_2, \ldots, G_n}\mathcal{G}_{F_1(A_1), F_2(A_2), \ldots, F_n(A_n)}(E)$$

  E is any relational-algebra expression

  - G_1, G_2 …, G_n is a list of attributes on which to group (can be empty)
  - Each F_i is an aggregate function
  - Each A_i is an attribute name

- Note: Some books/articles use γ instead of $\mathcal{G}$ (Calligraphic G)

Aggregate Operation – Example

- Relation *r*:

A	*B*	*C*
α	α	7
α	β	7
β	β	3
β	β	10

- $\mathcal{G}_{\mathbf{sum(c)}}(\mathrm{r})$

sum(*c*)
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Aggregate Operation – Example

- Find the average salary in each department

  $dept\_name\ \mathcal{G}_{\mathbf{avg}(salary)}\ (instructor)$

ID	*name*	*dept_name*	*salary*
76766	Crick	Biology	72000
45565	Katz	Comp. Sci.	75000
10101	Srinivasan	Comp. Sci.	65000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
33456	Gold	Physics	87000
22222	Einstein	Physics	95000

dept_name	*avg_salary*
Biology	72000
Comp. Sci.	77333
Elec. Eng.	80000
Finance	85000
History	61000
Music	40000
Physics	91000

Aggregate Functions (Cont.)

- Result of aggregation does not have a name
  - Can use rename operation to give it a name
  - For convenience, we permit renaming as part of aggregate operation

$$dept\_name\ \mathcal{G}_{\textbf{avg}(salary)\ \textbf{as}\ avg\_sal}\ (instructor)$$

Modification of the Database

- The content of the database may be modified using the following operations:
  - Deletion
  - Insertion
  - Updating
- All these operations can be expressed using the assignment operator and relational algebra operations
- Example:

Multiset Relational Algebra

- Pure relational algebra removes all duplicates
  - e.g. after projection
- Multiset relational algebra retains duplicates, to match SQL semantics
  - SQL duplicate retention was initially for efficiency, but is now a feature
- Multiset relational algebra defined as follows
  - selection: has as many duplicates of a tuple as in the input, if the tuple satisfies the selection
  - projection: one tuple per input tuple, even if it is a duplicate
  - cross product: If there are *m* copies of *t1* in *r*, and *n* copies of *t2* in *s*, there are *m* x *n* copies of *t1.t2* in *r* x *s*
  - Other operators similarly defined
    - E.g. union: *m* + *n copies,* intersection: min(*m, n*) copies
      difference: min(0, *m* – *n*) copies

SQL and Relational Algebra

- **select** *A1, A2, .. An*
  from *r1, r2, …, rm*
  where P

  is equivalent to the following expression in multiset relational algebra

$$\Pi_{A1, .., An} (\sigma_{P} (r1 \times r2 \times .. \times rm))$$

- **select** *A1, A2,* **sum**(*A3*)
  from *r1, r2, …, rm*
  where P
  group by *A1, A2*

  is equivalent to the following expression in multiset relational algebra

$$A1, A2\,\mathcal{G}_{\textbf{sum}(A3)} (\sigma_{P} (r1 \times r2 \times .. \times rm)))$$

SQL and Relational Algebra

- More generally, the non-aggregated attributes in the **select** clause may be a subset of the **group by** attributes, in which case the equivalence is as follows:

  select *A1,* **sum**(*A3*)
  from *r1, r2, …, rm*
  where P
  group by *A1, A2*

  is equivalent to the following expression in multiset relational algebra

$$\Pi_{A1,sumA3}(\ {}_{A1,A2}\mathcal{G}_{\textbf{sum}(A3)\ \textbf{as}\ sumA3}(\sigma_{P}(r1 \times r2 \times .. \times rm)))$$

Tuple Relational Calculus

Tuple Relational Calculus

- A nonprocedural query language, where each query is of the form

$$\{t \mid P(t)\}$$

- It is the set of all tuples t such that predicate P is true for t
- t is a *tuple variable*, $t[A]$ denotes the value of tuple t on attribute A
- $t \in r$ denotes that tuple t is in relation r
- P is a *formula* built up out of *atoms*
- Forms of atoms:

Rule to Write Formula

Equivalence of Formulae

Example Queries

- Find the *ID, name, dept_name, salary* for instructors whose salary is greater than $80,000

$$\{t \mid t \in instructor \wedge t\,[salary\,] > 80000\}$$

- As in the previous query, but output only the *ID* attribute value

$$\{t \mid \exists\, s \in \text{instructor}\ (t\,[ID\,] = s\,[ID\,] \wedge s\,[salary\,] > 80000)\}$$

Notice that a relation on schema (*ID*) is implicitly defined by the query

Example Queries

- Find the names of all instructors whose department is in the Watson building

$$\{t \mid \exists s \in \textit{instructor}\ (t[\textit{name}] = s[\textit{name}] \wedge \exists u \in \textit{department}\ (u[\textit{dept\_name}] = s[\textit{dept\_name}] \wedge u[\textit{building}] = \text{“Watson”}))\}$$

- Find the set of all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or both

$$\{t \mid \exists s \in \textit{section}\ (t[\textit{course\_id}] = s[\textit{course\_id}] \wedge s[\textit{semester}] = \text{“Fall”} \wedge s[\text{year}] = 2009 \vee \exists u \in \textit{section}\ (t[\textit{course\_id}] = u[\textit{course\_id}] \wedge u[\textit{semester}] = \text{“Spring”} \wedge u[\text{year}] = 2010)\}$$

Example Queries

- Find the set of all courses taught in the Fall 2009 semester, and in the Spring 2010 semester

$$\{t \mid \exists s \in section\ (t[course\_id] = s[course\_id] \wedge s[semester] = \text{“Fall”} \wedge s[year] = 2009 \wedge \exists u \in section\ (t[course\_id] = u[course\_id] \wedge u[semester] = \text{“Spring”} \wedge u[year] = 2010)\}$$

- Find the set of all courses taught in the Fall 2009 semester, but not in the Spring 2010 semester

$$\{t \mid \exists s \in section\ (t[course\_id] = s[course\_id] \wedge s[semester] = \text{“Fall”} \wedge s[year] = 2009 \wedge \neg \exists u \in section\ (t[course\_id] = u[course\_id] \wedge u[semester] = \text{“Spring”} \wedge u[year] = 2010)\}$$

Example Queries

- Find all students who have taken all courses offered in the Biology department
  - $\{t \mid \exists\, r \in \mathit{student}\ (t\,[\mathit{ID}] = r\,[\mathit{ID}]) \wedge$
    $(\forall\, u \in \mathit{course}\ (u\,[\mathit{dept\_name}] = \text{“Biology”} \Rightarrow$
    $\exists\, s \in \mathit{takes}\ (t\,[\mathit{ID}] = s\,[\mathit{ID}\,] \wedge$
    $s\,[\mathit{course\_id}] = u\,[\mathit{course\_id}]))\}$

Safety of Expressions

- It is possible to write tuple calculus expressions that generate infinite relations.
- For example, $\{ t \mid \neg\, t \in r \}$ results in an infinite relation if the domain of any attribute of relation r is infinite

Another Example of Unsafe Tuple Relational Calculus Expression

Safe Tuple Relational Calculus Expressions

- To guard against the problem, we restrict the set of allowable expressions to safe expressions: producing a finite number of tuples
- An expression $\{t \mid P(t)\}$ in the tuple relational calculus is *safe* if it satisfies the following rules:

  1) Domain of P (dom(P)) has to be finite

  dom(P) = set of values that appear explicitly in P or appear in the relations mentioned in P.

  - Example:

  2) All values that appear in the result of $\{t \mid P(t)\}$ are the values from dom(P).

Examples of Safe/Unsafe Tuple Relational Calculus Expressions

Domain Relational Calculus

Domain Relational Calculus

- A nonprocedural query language equivalent in power to the tuple relational calculus
- The theoretical basis of the widely used QBE (Query By Example) language.
- Each query is an expression of the form:

$$\{ < x_1, x_2, \ldots, x_n > \mid P\ (x_1, x_2, \ldots, x_n)\}$$

  - $x_1, x_2, \ldots, x_n$ represent domain variables
  - P represents a formula
- Domain variables form a relation of n attributes
- P is composed of atoms
- READ forms of atoms and rules to construct P from the textbook

Example Queries

- Find the *ID, name, dept_name, salary* for instructors whose salary is greater than $80,000
  - $\{< i, n, d, s > \mid < i, n, d, s > \in instructor \wedge s > 80000\}$
- As in the previous query, but output only the *ID* attribute value
  - $\{< i > \mid \exists\, n, d, s\ (< i, n, d, s > \in instructor \wedge s > 80000)\}$
- Find the names of all instructors whose department is in the Watson building

$$\{< n > \mid \exists\, i, d, s\ (< i, n, d, s > \in instructor \wedge \exists\, b, a\ (< d, b, a > \in department \wedge b = \text{“Watson”}))\}$$

Example Queries

- Find the set of all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or both

$$\{<c> \mid \exists\, a, s, y, b, r, t \ (<c, a, s, y, b, t> \in section \ \wedge \ s = \text{“Fall”} \wedge y = 2009) \ \vee \exists\, a, s, y, b, r, t \ (<c, a, s, y, b, t> \in section] \wedge \ s = \text{“Spring”} \wedge y = 2010)\}$$

This case can also be written as

$$\{<c> \mid \exists\, a, s, y, b, r, t \ (<c, a, s, y, b, t> \in section \ \wedge \ ((s = \text{“Fall”} \wedge y = 2009) \ \vee (s = \text{“Spring”} \wedge y = 2010))\}$$

- Find the set of all courses taught in the Fall 2009 semester, and in the Spring 2010 semester

$$\{<c> \mid \exists\, a, s, y, b, r, t \ (<c, a, s, y, b, t> \in section \ \wedge \ s = \text{“Fall”} \wedge y = 2009) \ \wedge \exists\, a, s, y, b, r, t \ (<c, a, s, y, b, t> \in section] \wedge \ s = \text{“Spring”} \wedge y = 2010)\}$$

Example Queries

- Find all students who have taken all courses offered in the Biology department
  - $\{< i > \mid \exists\, n, d, tc\ (< i, n, d, tc > \in student\ \wedge$
    $(\forall\, ci, ti, dn, cr\ (< ci, ti, dn, cr > \in course \wedge dn =$"Biology"
    $\Rightarrow\ \exists\, si, se, y, g\ (<i, ci, si, se, y, g> \in takes\ ))\}$

* The above query fixes bug in page 246, last query

Safety of Expressions

The expression:

$$\{ < x_1, x_2, \ldots, x_n > \mid P(x_1, x_2, \ldots, x_n) \}$$

is safe if all of the following hold:

1. All values that appear in tuples of the expression are values from *dom* (P) (that is, the values appear either in P or in a tuple of a relation mentioned in P).
2. For every "there exists" subformula of the form $\exists x (P_1(x))$, the subformula is true if and only if there is a value of x in *dom* (P_1) such that $P_1(x)$ is true.
3. For every "for all" subformula of the form $\forall_x (P_1 (x))$, the subformula is true if and only if $P_1(x)$ is true for all values x from *dom* (P_1).

Expressive Power of Languages

- All three of the following are equivalent:
  - The basic relational algebra (without the extended relational-algebra operations)
  - The tuple relational calculus restricted to safe expressions
  - The domain relational calculus restricted to safe expressions

End of Topic 5