Module 6
Image Denoising and Nonlinear Filtering

- Noise Models
- Morphological and Order Statistic Filters
- Bilateral Filter
- Anisotropic Diffusion
- Non-local (NL) Means
- Wavelet Soft Thresholding
- BM3D
- Homomorphic Filtering

IMAGE NOISE MODELS

- To proceed further in understanding and **designing** non-linear filters, we will need to deepen our statistical modeling of **noise**.

- Any **channel** (electrical wire, airwaves, optical fiber) is **imperfect**. Images sent over a channel always suffer a **degradation of information**:

  (a) **static** (high-frequency noise or thermal noise)

  (b) **bit errors**

  (c) **sensor noise**

- These errors degrade

  - visual interpretation

  - computer image analysis

White Noise is Uncorrelated Noise

- Assume the **additive white noise** model:

$$\mathbf{J} = \mathbf{I} + \mathbf{N}$$

  where $\mathbf{I}$ is an original image and $\mathbf{N}$ a **digital white noise image**.

- **White noise** suggests an (**on average**) a flat spectrum.

- An **equivalent meaning** is that the elements $N(m, n)$ of $\mathbf{N}$ are **uncorrelated**: any two elements $N(m, n)$, $N(p, q)$ of the noise matrix $\mathbf{N}$ (where $(m, n) \neq (p, q)$) affect one another **little or not at all**.

- Mathematically this is equivalent to the noise being **white**.

Range of Noise

- The set of values $\Re_N$ the noise can take. In digital images the noise must be **discrete integers.** However, we will use **approximate continuous models**.

- For **additive noise** the noise range is usually **symmetric:** $\Re_N = [-Q, Q]$ where possibly $Q = \infty$.

- For **multiplicative noise** assume the noise range is **non-negative**: $\Re_N = [0, Q]$ where possibly $Q = \infty$.

Probability Density Function of Noise

- The **probability density function** or **PDF** $f_N(q)$ is a **model** of the **probabilities** of $N(m, n)$:

$$\Pr\{a \leq N(m, n) \leq b\} = \int_a^b f_N(q)\, dq$$

where $[a, b] \subseteq \Re_N$.

- Assume that all elements $N(m, n)$ of $\mathbf{N}$ obey the **identical probability model** - the same PDF.

- So, $f_N(q)$ is not expressed as a function of (m, n).

Properties of PDF

- **Two** basic properties of the PDF

$$(1)\ f_N(q) \geq 0 \text{ for all } q \in \mathfrak{R}_N$$
$$(2)\ \int_{\mathfrak{R}_N} f_N(q)\,dq = 1$$

- Note: (1) and (2) imply that $f_N(q) \leq 1$ for all $q \in \mathfrak{R}_N$.

- For **symmetric noise**: $\mathfrak{R}_N = [-Q, Q]$ assume that $f_N(q)$ is a **symmetric function**

$$f_N(q) = f_N(-q)$$

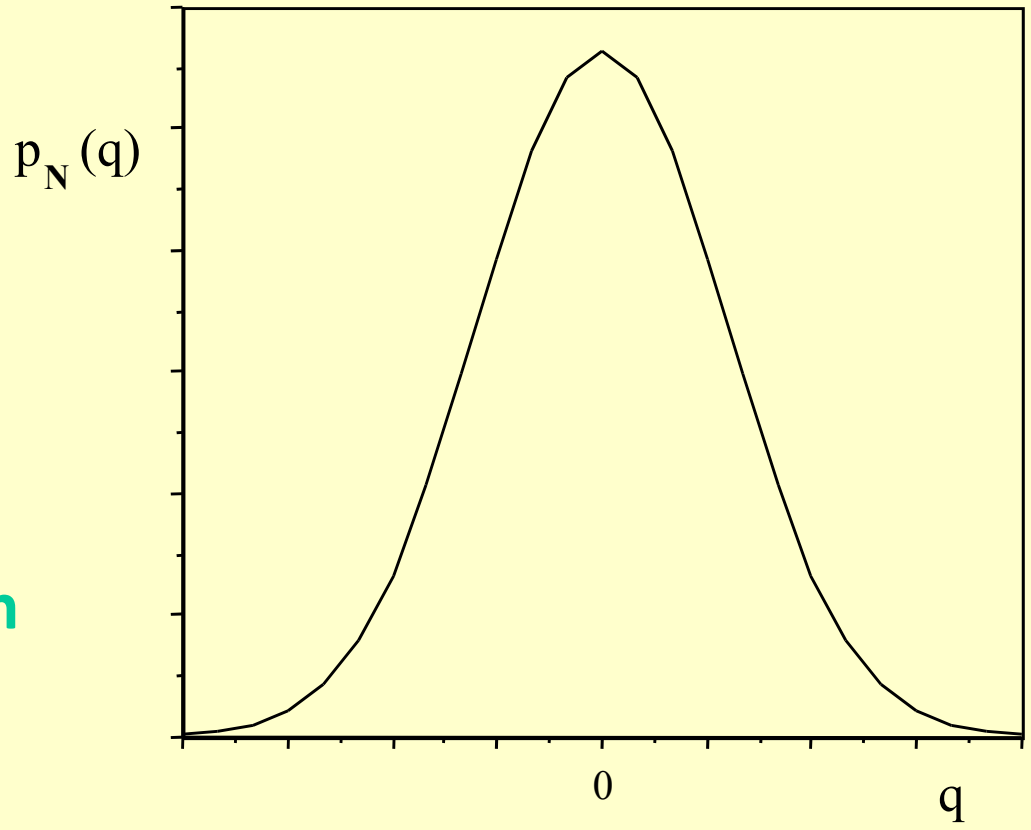
Gaussian PDF

- Here $\Re_N = [-\infty, \infty]$ and

$$f_N(q) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(q/\sigma)^2}$$

with **scale parameter** σ.

- The parameter σ^2 is the **variance** of the PDF
- **The most common** noise that occurs in applications … and **in nature**.
- The usual noise found in electrical circuits, most communication channels, etc.



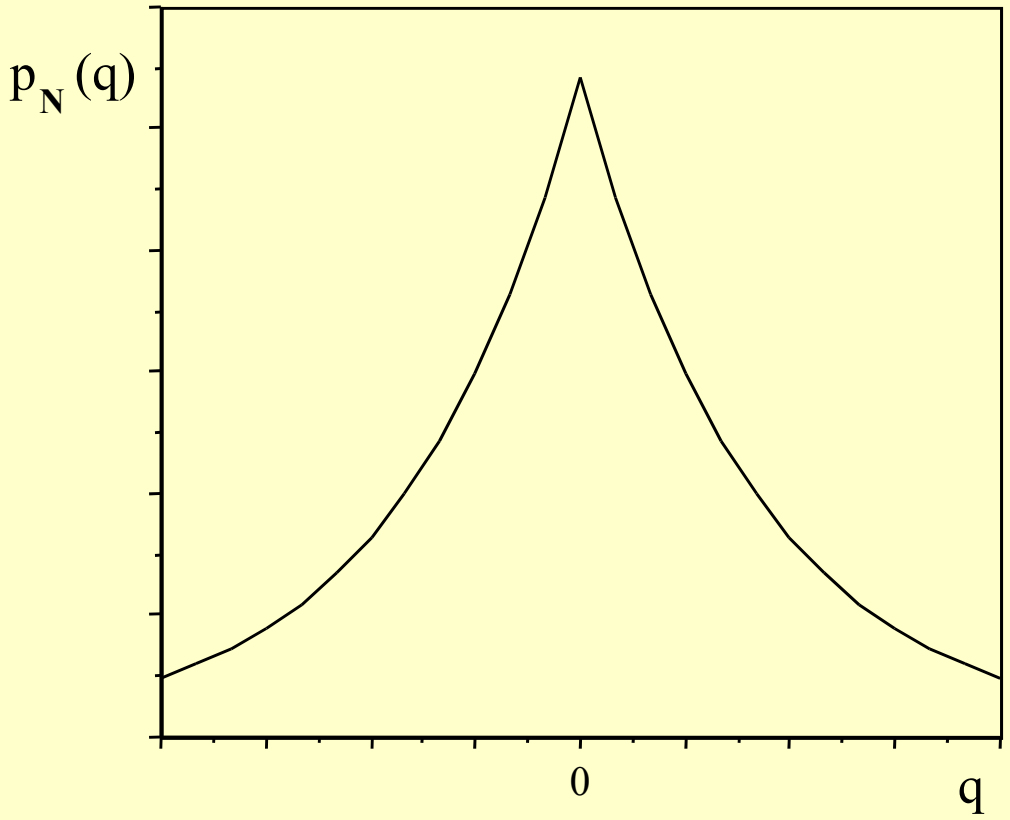
Laplacian PDF

- Here $\Re_N = [-\infty, \infty]$ and

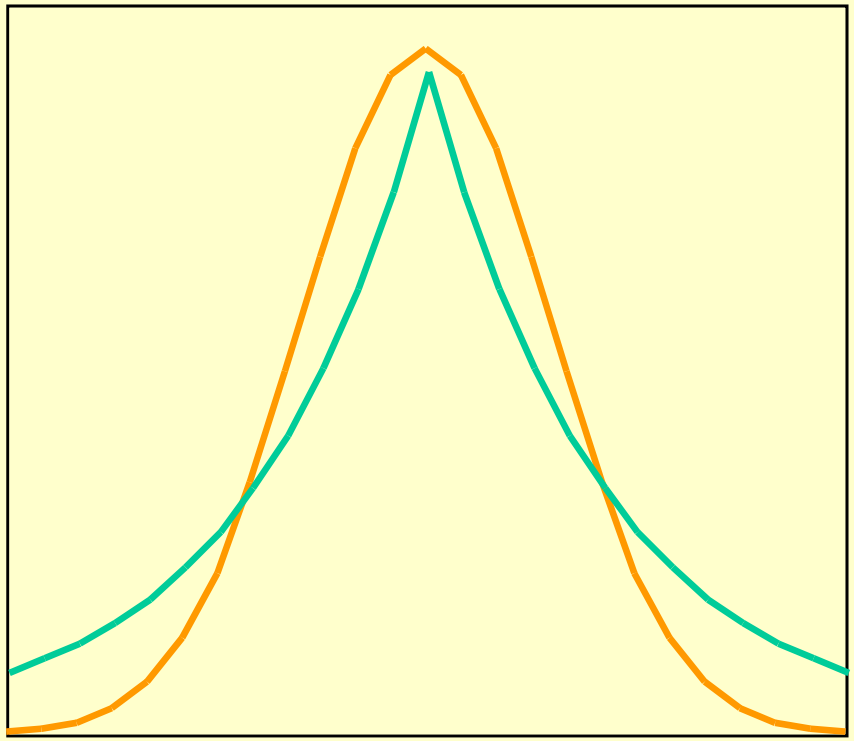
$$f_N(q) = \frac{1}{\sqrt{2}\sigma} e^{-\sqrt{2}|q|/\sigma}$$

with **scale parameter** σ.

- Parameter σ^2 is **again** the variance.
- Often models **impulse noise**.
- **Large number of small-amplitude values**, and a (relatively) large number of **high-amplitude values**.



Gaussian vs. Laplacian PDFs



Gaussian
Laplacian

- Laplacian noise is **impulsive** – a **heavy-tailed** PDF.

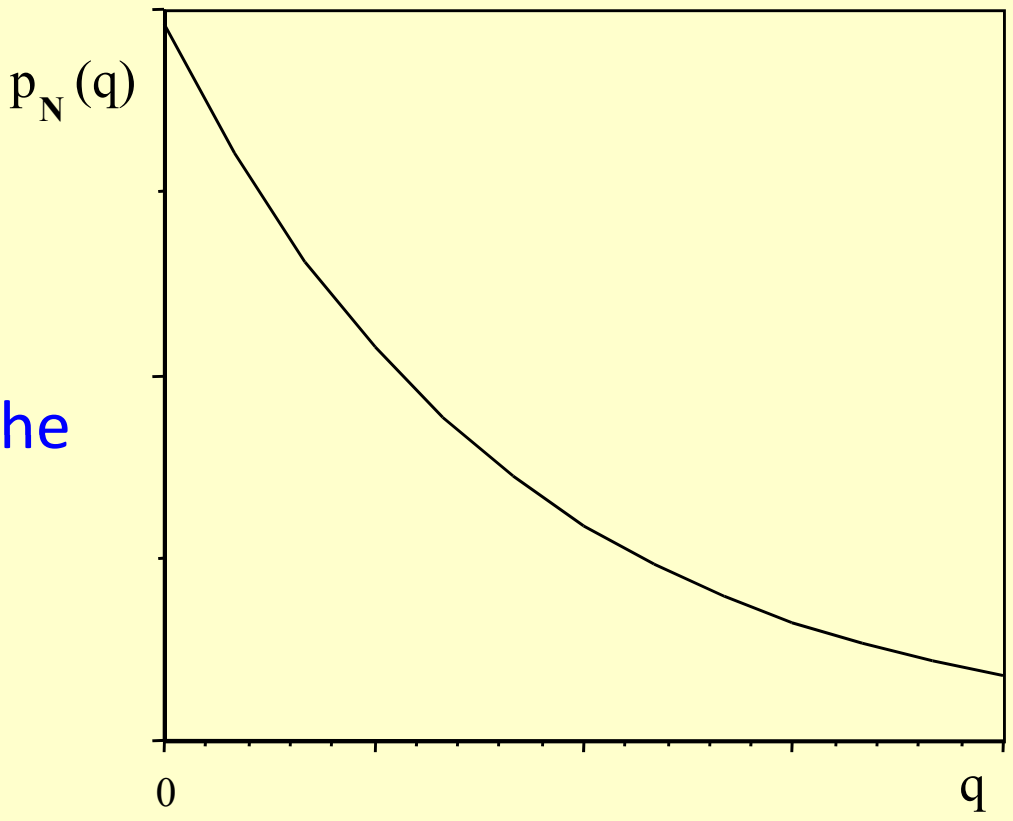
Exponential PDF

- Here $\Re_N = [0, \infty]$ and

$$f_N(q) = \frac{1}{\sigma} e^{-q/\sigma}$$

with **scale parameter** σ.

- Parameter σ^2 is again equal to the **variance** and s is the mean.
- Often models **multiplicative noise**.
- A coherent image noise model (radar, laser, electron microscopy, etc)



Salt-and-Pepper (S&P) Noise

- A model for **significant bit errors**.
- When transmitting digital image data, significant bit errors can occur:

  '1' → '0' (**'light' → 'dark'**)

  '0' → '1' (**'dark' → 'light'**)

- May be due to thermal effects or even gamma rays.
- Salt and pepper noise is **not additive**.

S&P Noise Model

- Let I_{max} and I_{min} be the brightest and darkest **allowable** gray levels in image $\mathbf{I}$ (e.g. 0 & 255)
- Define the simple probabilities

$$p_{s\text{-}p}(q) = \begin{cases} (1\text{-}p)/2 & ;\ q = -1 \\ p & ;\ q = 0 \\ (1\text{-}p)/2 & ;\ q = 1 \end{cases}$$

- If $\mathbf{J}$ is the noisy image, then

$$J(i,j) = \begin{cases} I_{min} & ;\ q = -1 \\ I(i,j) & ;\ q = 0 \\ I_{max} & ;\ q = 1 \end{cases}$$

- This is a **simple** model for the worst case type of bit errors.

Statistical Mean

- The **statistical mean** of $\mathbf{N}$ is

$$\mu_{\mathbf{N}} = \int_{\mathfrak{R}_{\mathbf{N}}} q \cdot f_{\mathbf{N}}(q)\, dq$$

- It measures the **center** about which the noise realizations cluster.
- For a **symmetric** PMF (Q can be ∞),

$$\mu_{\mathbf{N}} = \int_{-Q}^{Q} q \cdot f_{\mathbf{N}}(q)\, dq = 0 \quad \text{(why?)}$$

Statistical Variance

- The **variance** of $\mathbf{N}$ is

$$\sigma_{\mathbf{N}}^2 = \int_{\mathfrak{R}_{\mathbf{N}}} \left(q - \mu_{\mathbf{N}}\right)^2 \cdot f_{\mathbf{N}}(q)\, dq$$

- The **standard deviation** $\sigma_{\mathbf{N}}$ is the **average distance** from $\mu_{\mathbf{N}}$ that the noise realizations fall.
- For symmetric (zero-mean) noise, the variance is just

$$\sigma_{\mathbf{N}}^2 = \int_{\mathfrak{R}_{\mathbf{N}}} q^2 \cdot f_{\mathbf{N}}(q)\, dq$$

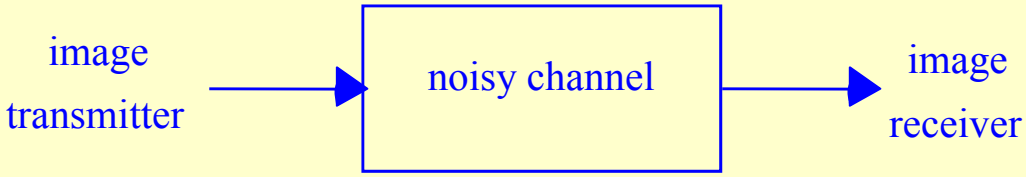
Mean & Variance of PDFs

- **Exercises**: Show that for Gaussian, Laplacian, and exponential PDFs, the **variance** is σ^2.

- Also show that for exponential PDF, the **mean** is equal to σ.

Denoising

- Consider **denoising** of an image contaminated by **white noise**:



- Two goals of filtering:
  - **Smoothing** - reduce noise (bit errors, channel, etc)
  - **Preservation** - of features: **edges** and **detail**

Smoothing vs. Preservation

- **Image denoising** contains **conflicting goals**:
  - Smoothing: **high noise frequencies** are attenuated
  - **Broadband images**: both low and high frequencies are important

- Linear filtering **cannot** differentiate between **desirable** high frequencies and **undesirable** high frequencies

- A **linear** low-pass denoising filter will always:
  - **Reduce** high frequency **noise**
  - **Blur** the image

WHY NONLINEAR FILTERING ?

- Linear filtering has **important** limitations.

- The basis of linear filtering is **frequency** or **spectrum shaping**:

  -**reduce** (attenuate) unwanted frequencies

  - **amplify** or **preserve** desired frequencies

Motivation for Nonlinear Filtering

(or nonlinear processing of linear filter responses)

- Generally, nonlinear filtering **cannot** be expressed by linear convolution **nor** by frequency shaping.
- Some approaches **nonlinearly modify linear responses**
- Others **combine filter bank responses** nonlinearly
- We will study these in the important context of **image denoising**.
- The common theme: **nonlinear filters** give you capabilities that linear filters **don't have**.

MORPHOLOGICAL FILTERS

MORPHOLOGICAL FILTERS

- Recall: For image $\mathbf{I}$, window $\mathbf{B}$, the **windowed set** at (m,n) is:

$$\mathbf{B}\circ\mathbf{I}(m,n) = \{I(m-p, n-q); (p, q) \in \mathbf{B}\} .$$

- A nonlinear filter F on a **windowed set** is a nonlinear function of the pixels covered by the window.

- Denote the **nonlinear filter** F on $\mathbf{B}\circ\mathbf{I}(m,n)$ by

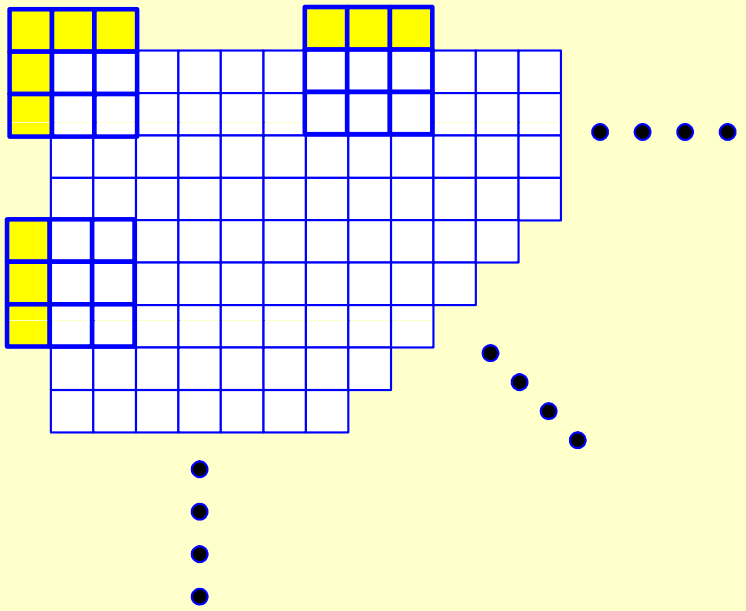
$$J(m,n) = F\{\mathbf{B}\circ\mathbf{I}(m,n)\} = F\{I(m-p, n-q); (p, q) \in \mathbf{B}\}$$

- Performing this at every pixel gives a **filtered image**:

$$\mathbf{J} = F[\mathbf{I}, \mathbf{B}] = [J(m,n); 0 \le m \le M-1, 0 \le n \le N-1]$$

Boundary-of-Image Processing

- As with binary morphological filters: when a window overlaps "empty space" …



- Pixel replication is assumed: fill the "empty" window slots by the nearest image pixel.

MEDIAN FILTER

- The **median filter** is a nonlinear filtering device that is related to the binary majority filter (Module 2).
- Despite being "automatic" (no design), it is **very effective for image denoising**.
- Despite its simplicity, it has an **interesting theory** that justifies its use.
- The median filter is a special member of several classes of filters, including two studied here: gray-level **morphological filters**, and **order statistic filters**.

Order Statistics

- **Definition**: Given a set of numbers

$$X = \{X_1, X_2, \ldots, X_{2P+1}\}$$

the **order statistics** (or **OS**) of the set are the **same elements** reordered from **smallest** to **largest**.

- The OS of X are denoted

$$X = \{X_{(1)}, X_{(2)}, \ldots, X_{(2P+1)}\}$$

such that

$$X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(2P+1)}$$

Special Order Statistics

- In particular:

$$\mathrm{MIN}\{X_1, X_2, \ldots, X_{2P+1}\} = X_{(1)}$$

$$\mathrm{MAX}\{X_1, X_2, \ldots, X_{2P+1}\} = X_{(2P+1)}$$

$$\mathrm{MED}\{X_1, X_2, \ldots, X_{2P+1}\} = X_{(P+1)}$$

- The **median** MED is the **middle value** in rank.

Median Filter

- Given image $\mathbf{I}$ and window $\mathbf{B}$, the **median filtered image** is:

$$\mathbf{J} = \mathrm{MED}[\mathbf{I}, \mathbf{B}]$$

- Each output is the **median** of the windowed set:

$$J(m,n) = \mathrm{MED}\{\mathbf{B} \circ \mathbf{I}(m,n)\}$$

$$= \mathrm{MED}\{I(m-p, n-q); (p, q) \in \mathbf{B}\}$$

Properties of the Median Filter

- The median filter **smooths** additive white noise.
- The median filter **does not degrade edges**.
- The median filter is **particularly effective** for **removing** large-amplitude noise **impulses**.

Example - 1-D Median Filter

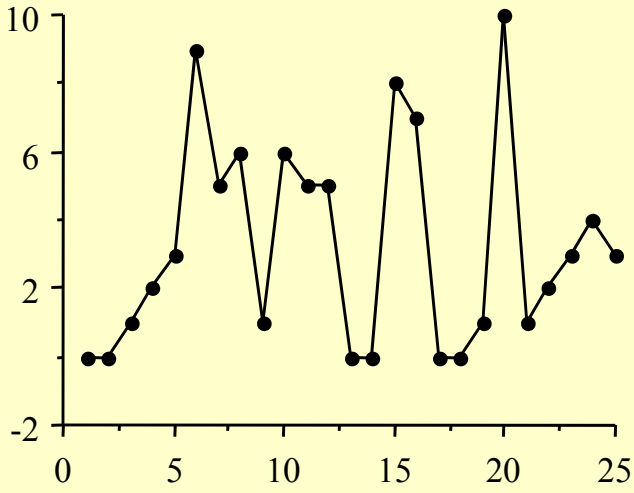
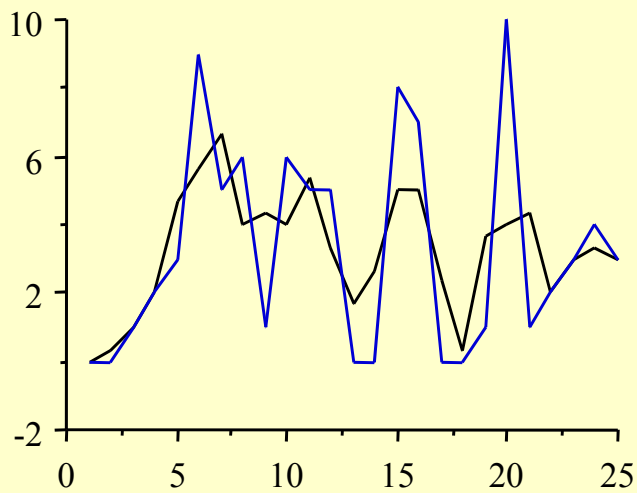
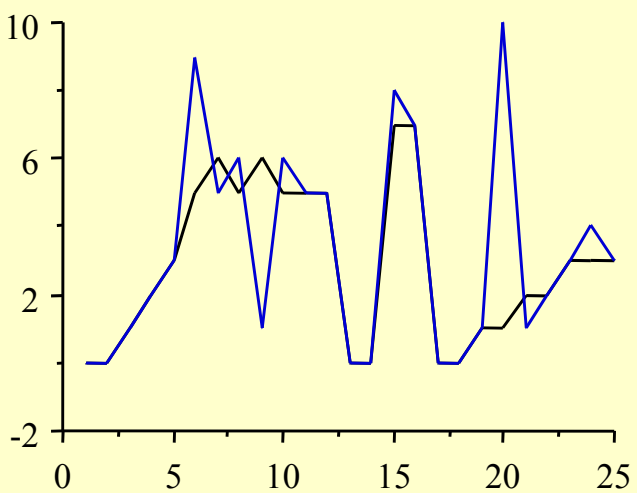


image scan line



AVE[**I**, **B**], **B** = ROW(3)

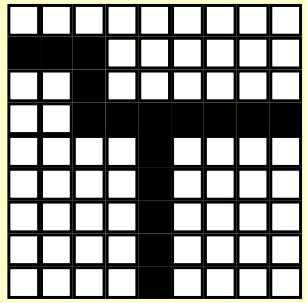


MED[**I**, **B**], **B** = ROW(3)

- Both filters smooth, but AVE **blurs** the structures.
- Using MED, the "noise" is **smoothed effectively - large noise spikes** are **eradicated**, rather than blurrrrrrrrred.
- MED maintains signal structure better - the edges are **sharp**.

Comments on Window Shape

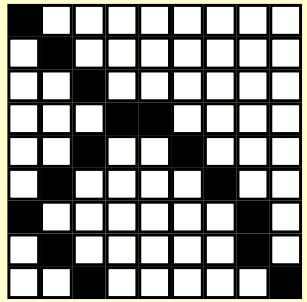
- $\mathbf{B}$ = SQUARE smooths noise strongly, but may damage image details of interest.



- $\mathbf{B}$ = CROSS reduces these effects, if known that the image contains horizontal/vertical features.

  Example: Images of dot-matrix characters.

- If the image contains curvilinear details that are diagonally oriented, then $\mathbf{B}$ = CROSS can perform poorly.



- **Variations:** combine the outputs of multiple CROSS, X-SHAPED, ROW, COL, etc. windows e.g. by taking their MED, AVE, etc. These **"detail-preserving filters"** work pretty well but the theory is **heuristic**.

Erosion and Dilation

- Given an image $\mathbf{I}$ and a window $\mathbf{B}$:

$$\mathbf{J} = \mathrm{DILATE}[\mathbf{I}, \mathbf{B}]$$

if

$$J(i, j) = \mathrm{MAX}\{\mathbf{B} \circ \mathbf{I}(i, j)\} \quad \text{(local max)}$$

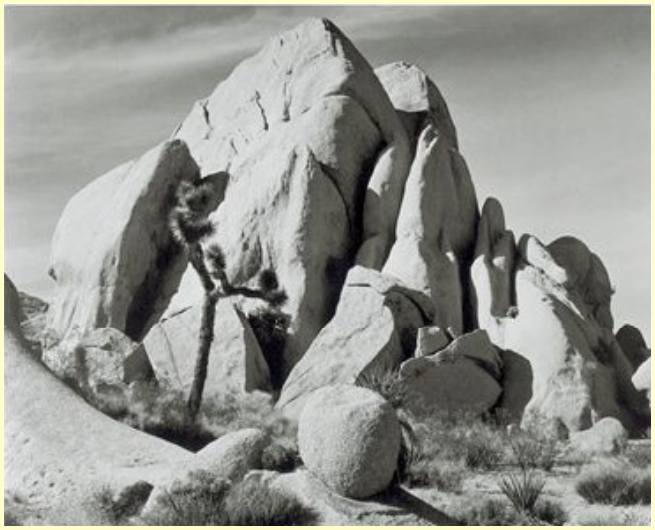
and

$$\mathbf{J} = \mathrm{ERODE}[\mathbf{I}, \mathbf{B}]$$

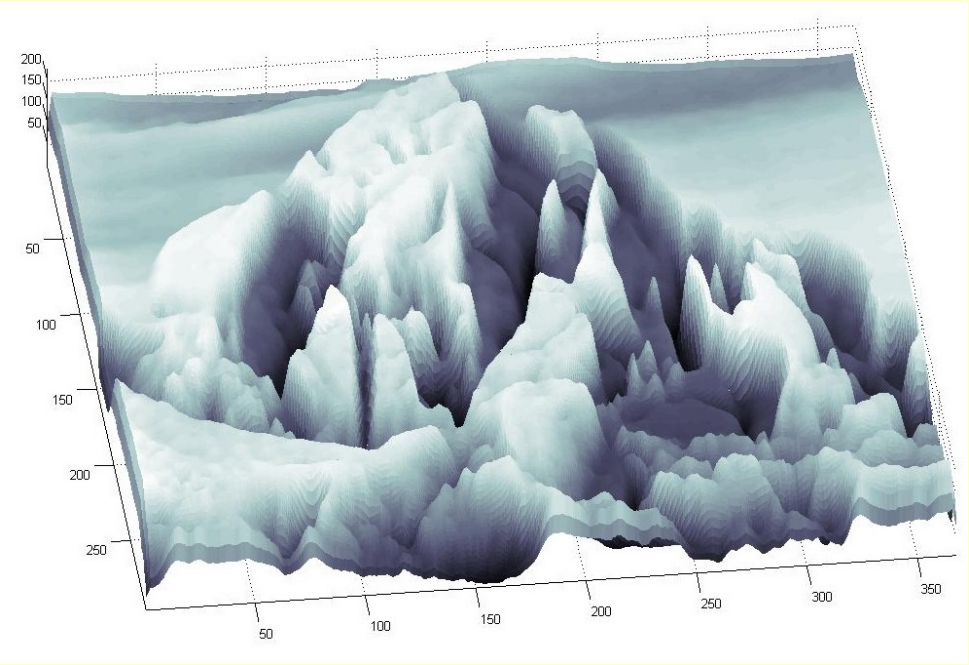
if

$$J(i, j) = \mathrm{MIN}\{\mathbf{B} \circ \mathbf{I}(i, j)\} \quad \text{(local min)}$$

Qualitative Properties of Morphological Filters

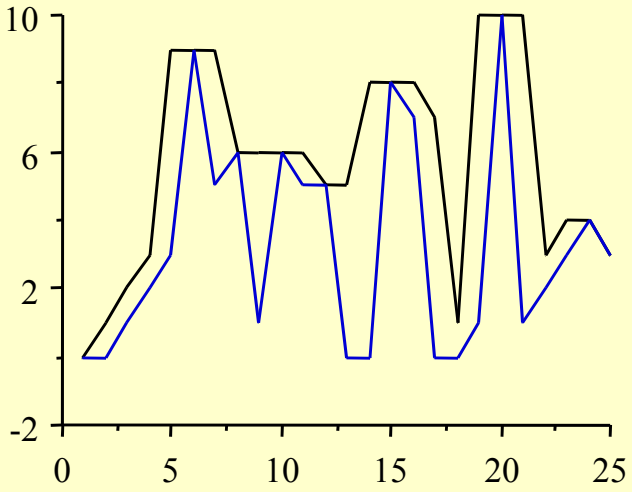


Ansel Adams' *Joshua Tree*

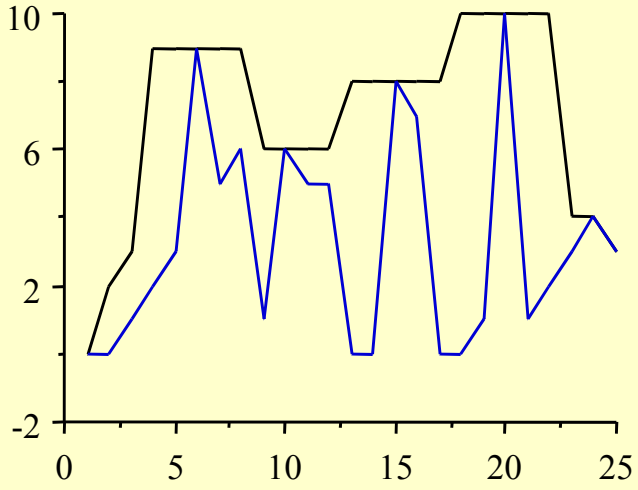


- **Gray-level morphological filters** effect shape – in this case, the shape of the intensity surface of the image.
- The DILATE (ERODE) filters have the **following properties**:
  - Increase the size of peaks (valleys)
  - Decrease the size of, or eliminate, valleys (peaks)

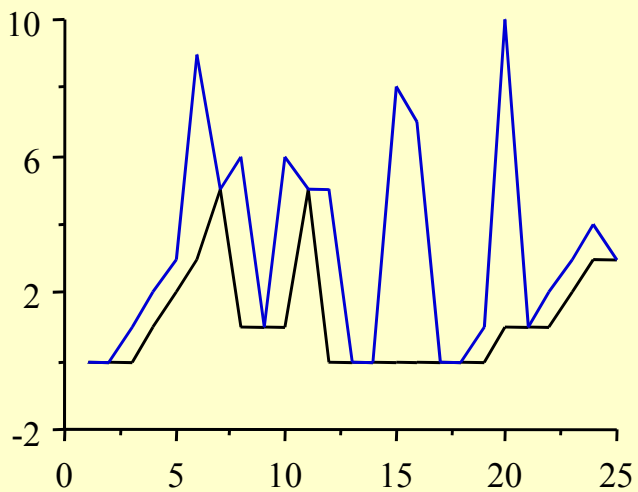
Dilation and Erosion Examples



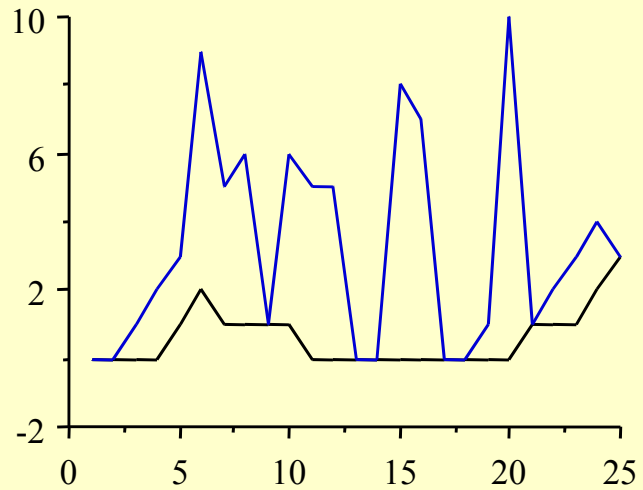
DILATE[**I**, **B**], **B** = ROW(3)



DILATE[DILATE[**I**, **B**], **B**], **B** = ROW(3)



ERODE[**I**, **B**], **B** = ROW(3)



ERODE[ERODE[**I**, **B**], **B**], **B** = ROW(3)

Most valleys gone after one pass. All valleys and negative-going impulses **gone** after two.

Most positive impulses reduced or gone after one pass. After two, all gone.

CLOSE and OPEN Filters

- The **CLOSE** and **OPEN** filters are defined by:

$$\mathbf{J} = \text{CLOSE}[\mathbf{I}, \mathbf{B}]$$
$$= \text{ERODE}\,[\text{DILATE}\,[\mathbf{I}, \mathbf{B}], \mathbf{B}]$$

and

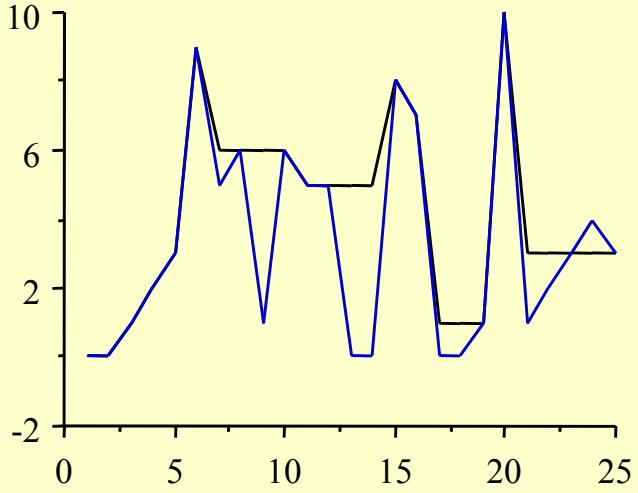
$$\mathbf{J} = \text{OPEN}[\mathbf{I}, \mathbf{B}]$$
$$= \text{DILATE}\,[\text{ERODE}\,[\mathbf{I}, \mathbf{B}], \mathbf{B}]$$

Properties of CLOSE and OPEN

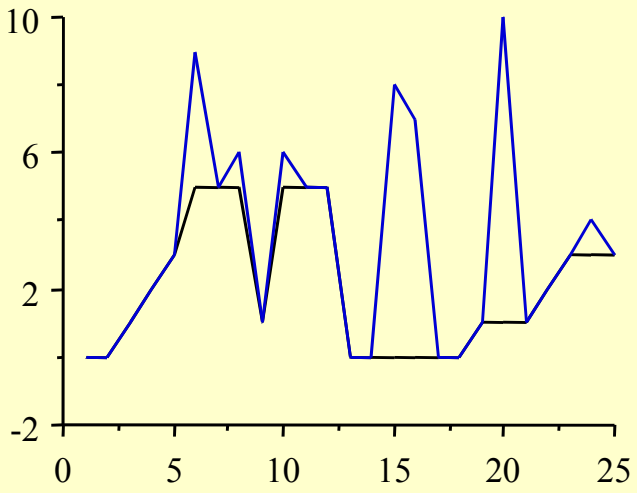
- Effective smoothing filters similar to the **median filter**.

- The CLOSE (OPEN) filter has the following properties:
  - **smooths noise**
  - preserves **edges**
  - **eradicates** negative- (positive-) going **impulses**
  - leaves the signal at approximately the same **level**

Close and Open Examples



CLOSE[**I**, **B**], **B** = ROW(3)



OPEN[**I**, **B**], **B** = ROW(3)

- CLOSE **preserves the signal** except that the negative-going impulses are **gone**.
- OPEN **preserves the signal** except the positive-going impulses are **gone**.

OPEN-CLOSE & CLOSE-OPEN Filters

- The **OPEN-CLOSE** and CLOSE-OPEN filters:

$$\mathbf{J} = \text{OPEN-CLOSE}\,[\mathbf{I}, \mathbf{B}]$$
$$= \text{OPEN}\,[\text{CLOSE}\,[\mathbf{I}, \mathbf{B}], \mathbf{B}]$$

and

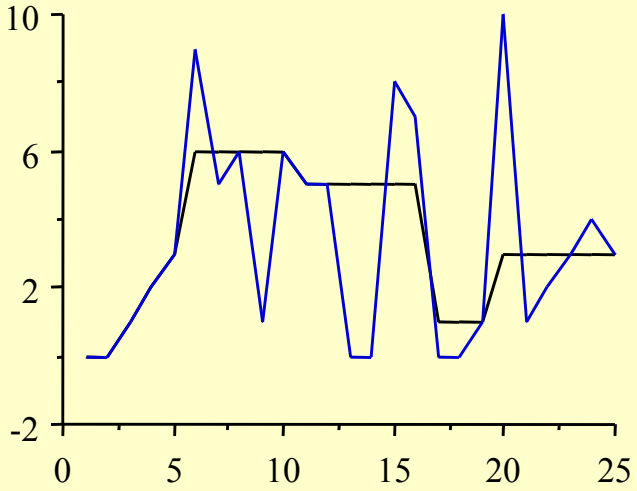
$$\mathbf{J} = \text{CLOSE-OPEN}\,[\mathbf{I}, \mathbf{B}]$$
$$= \text{CLOSE}\,[\text{OPEN}\,[\mathbf{I}, \mathbf{B}], \mathbf{B}]$$

OPEN-CLOSE & CLOSE-OPEN

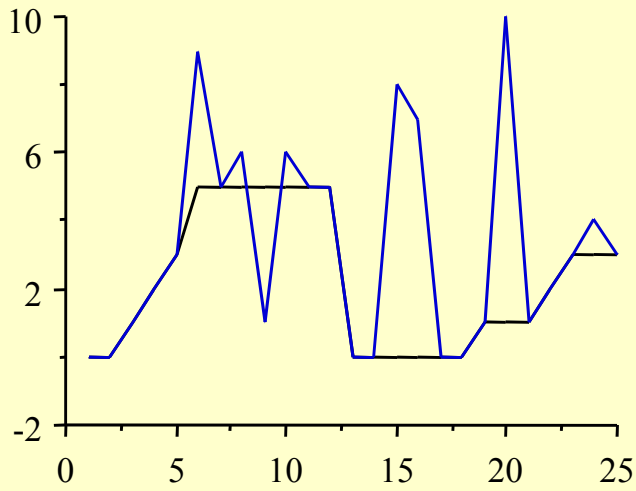
- Effective smoothing filters **very** similar to the median filter. Also very similar to **each another**.

- The CLOSE-OPEN and OPEN-CLOSE filters obey the following:
  - both **smooth noise**
  - both **preserve edges**
  - both **eradicate** positive & negative **impulses**

Close-Open & Open-Close Examples



OPEN-CLOSE[**I**, **B**], **B** = ROW(3)



CLOSE-OPEN[**I**, **B**], **B** = ROW(3)

- Note the **intense smoothing of noise** (especially impulses) and the retention of the global image structure.
- Note the **different possible interpretations** of "correct image structure" under CLOSE-OPEN and OPEN-CLOSE.

Application: Peak/Valley Detection

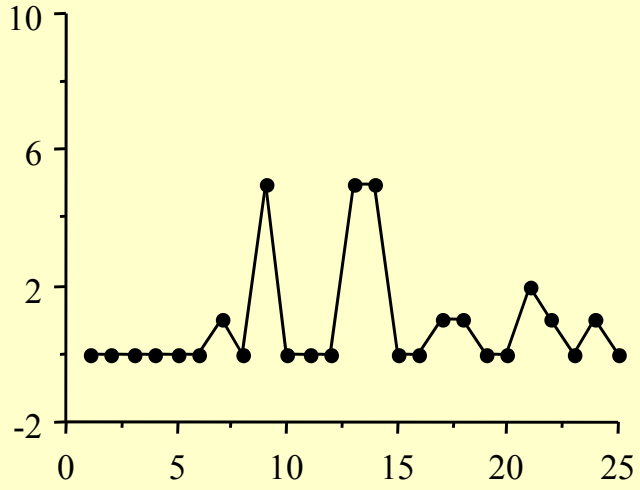
- Suppose we wish to find **bright targets** or **dark spots**.
- **Difference** the OPENed and CLOSEd image $\mathbf{I}$:

$$\mathbf{J}_{\text{peak}} = \mathbf{I} - \text{OPEN}[\mathbf{I}, \mathbf{B}]$$

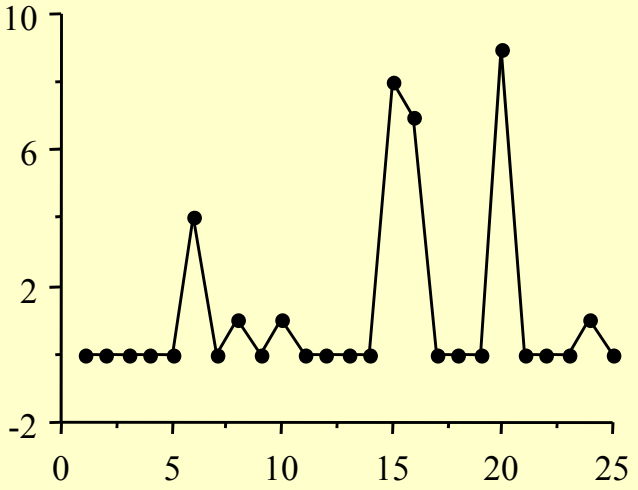
$$\mathbf{J}_{\text{valley}} = \text{CLOSE}[\mathbf{I}, \mathbf{B}] - \mathbf{I}$$

- As we'd expect, these operations highlight the **peaks** and **valleys** that occur.

Peak/Valley Detection Examples



I-OPEN[**I**, **B**], **B** = ROW(3)

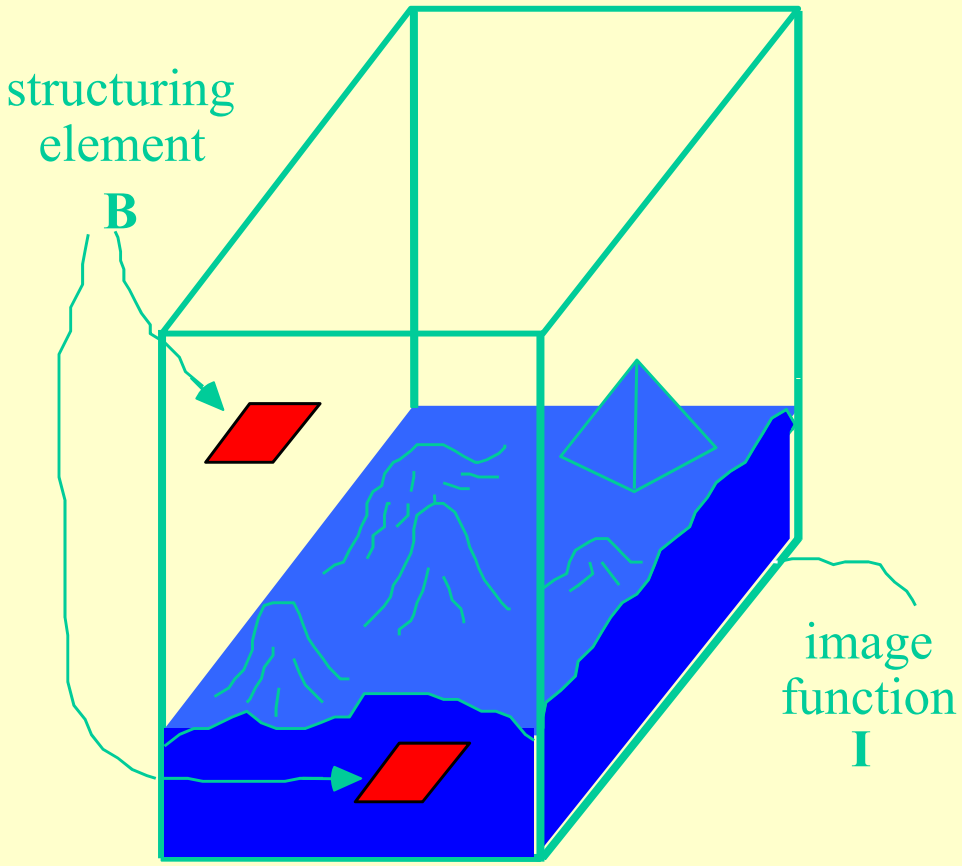


CLOS[**I**, **B**]-**I**, **B** = ROW(3)

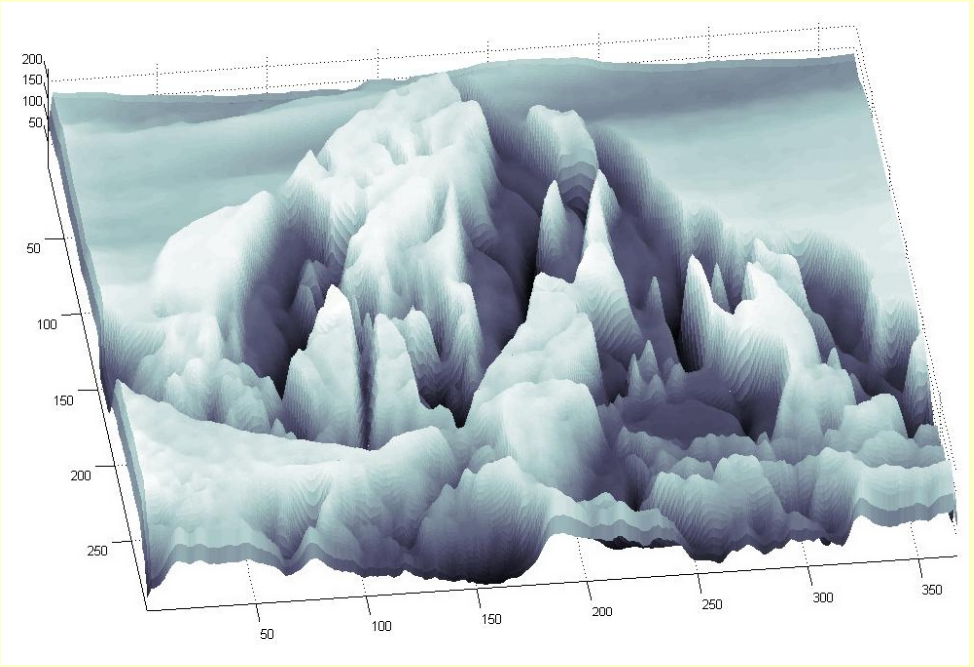
- Simple **thresholding** (binarization) of these could be used to determine the peak and valley **locations**.

Morphological Interpretation

- DILATE and ERODE are true morphological filters in the sense discussed in Module 2.
- As are CLOSE, OPEN, CLOSE-OPEN, and OPEN-CLOSE.
- They have the **identical interpretation** if we regard
  - **I** as a **3-D binary image** with value '1' below its "plot" and '0' above its "plot"
  - **B** as a floating **2-D structuring element**



Simulated image surface



Real image surface

- Leads to **fast** all-Boolean algorithms & architectures.

Morphological Interpretation

ERODE:

- When **B** lies **above** **I**, the AND of all it touches is '0'.
- When **B** lies **below** **I**, the AND of all it touches is '1'.
- Whenever **B** **crosses the boundary** of **I**, the AND of all it touches is '0'.

DILATE:

- When **B** lies **above** **I**, the OR of all it touches is '0'.
- When **B** lies **below** **I**, the OR of all it touches is '1'.
- Whenever **B** **crosses the boundary** of **I**, the OR of all it touches is '1'.

Relating Grayscale & Binary Morphological Filters

- For many morphological filters, there is a precise relationship between the grayscale and binary versions.
- As usual, we will consider the 1D case to develop an understanding of this.
- Here, the term **m-ary** is a synonym for **grayscale**.
  - Implies pixels/samples take one of m distinct values.
  - For B-bit pixels/samples, $m = 2^B$ ($= K$ in Modules 1-3).
- Several concepts involved:
  - Threshold decomposition and reconstruction.
  - The **stacking property**.
  - The notion of a **positive boolean function** or PBF.

Threshold Decomposition

- Let $x[n]$ be an m-ary signal.
- Define the binary threshold signal

$$x_{\geq\tau}[n] = \begin{cases} 1, & \text{for all } n \text{ s.t. } x[n] \geq \tau, \\ 0, & \text{for all } n \text{ s.t. } x[n] < \tau. \end{cases}$$

- The **threshold decomposition** of $x[n]$ is defined by the set of threshold signals $x_{\geq\tau}[n]$ for $\tau = [0, 1, 2, \ldots, \text{m-1}]$.

Threshold Decomposition Example

$x[n] =$	5	2	7	0	3	4
$x_{\geq 7}[n] =$	0	0	1	0	0	0
$x_{\geq 6}[n] =$	0	0	1	0	0	0
$x_{\geq 5}[n] =$	1	0	1	0	0	0
$x_{\geq 4}[n] =$	1	0	1	0	0	1
$x_{\geq 3}[n] =$	1	0	1	0	1	1
$x_{\geq 2}[n] =$	1	1	1	0	1	1
$x_{\geq 1}[n] =$	1	1	1	0	1	1
$x_{\geq 0}[n] =$	1	1	1	1	1	1

- Let $B=3$, so that
$$m = 2^B = 8.$$
- The pixels/samples take values in $[0, 7]$.
- Each threshold signal is **binary**.
- In each column, notice that there must be a **stack** of 0's on top of a **stack** of 1's.
- This is called the **stacking property**.

Threshold Reconstruction

$x[n] =$	5	2	7	0	3	4
$x_{\geq 7}[n] =$	0	0	1	0	0	0
$x_{\geq 6}[n] =$	0	0	1	0	0	0
$x_{\geq 5}[n] =$	1	0	1	0	0	0
$x_{\geq 4}[n] =$	1	0	1	0	0	1
$x_{\geq 3}[n] =$	1	0	1	0	1	1
$x_{\geq 2}[n] =$	1	1	1	0	1	1
$x_{\geq 1}[n] =$	1	1	1	0	1	1
$x_{\geq 0}[n] =$	1	1	1	1	1	1

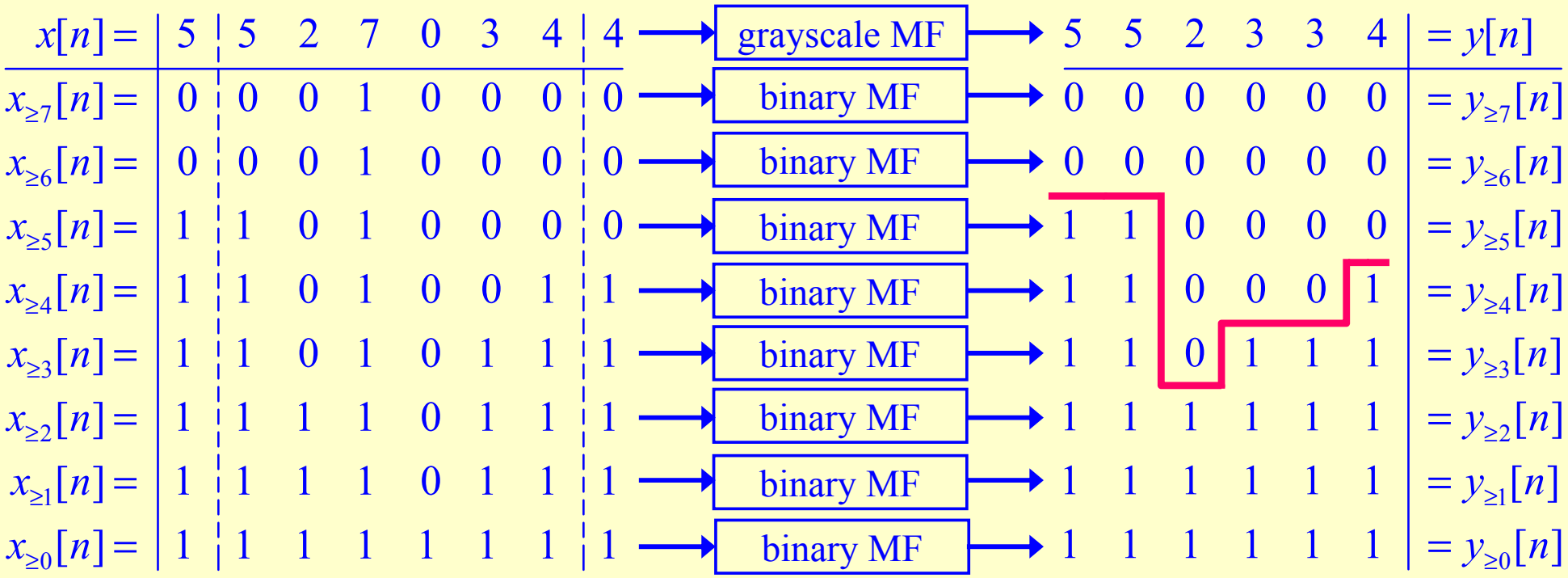
- Note that $x[n]$ can be reconstructed from any set of threshold signals that obey the stacking property:

$$x[n] = \max\left\{\tau \,\middle|\, x_{\geq\tau}[n] = 1\right\}.$$

- This is shown by the red line to the left.

Median Filtering by Threshold Decomposition

- On the next page, we will apply a **grayscale** 3-point median filter to $x[n]$.
- In parallel, we will apply **binary** 3-point median filers to each of the threshold signals $x_{\geq \tau}[n]$.
- As is typical for morphological and order statistic filters, we will handle the edge effects by replication.



- The result of applying the grayscale median filter to the m-ary signal is the **same** as the result of applying the binary median filters to the threshold signals!

When Does this Work?

- Amazing!
- Does this work for other types of filters?
- The answer is YES.
  - It works for any filter that preserves the stacking property at the outputs of the binary filters.
  - Filters that preserve the stacking property are called **stack filters**. They are a **very large** class of nonlinear filters.
- Fact: the stacking property is preserved if the binary version of the filter is a positive boolean function (PBF).
- Def: a PBF is a binary function where the minimum sum of products form contains no complemented variables.
- Ex: 3-pt binary MF: $y = x_1x_2 + x_1x_3 + x_2x_3$.

Stack Filters

- Stack filters are important for two main reasons:
  1. There is a theoretical framework for designing stack filters to minimize the mean absolute error (MAE) in the output signal.
  2. There are techniques for elegant fast hardware implementations.
- Examples of 3-point PBF's that define stack filters:
  - MED: $y = x_1x_2 + x_1x_3 + x_2x_3$
  - DILATE/MAX: $y = x_1 + x_2 + x_3$
  - ERODE/MIN: $y = x_1x_2x_3$

Morphological Filters as Stack Filters

- Because ERODE and DILATE are stack filters, so are OPEN, CLOSE, Close-Open, and Open-Close.
- This is the "detailed version" of what is meant by the figure on page 6.42.
- Ex: OPEN($x[n]$,**B**) with **B**=ROW(3):
  - Because OPEN($x[n]$,**B**) = DILATE[ERODE($x[n]$,**B**)], this is actually a 5-point filtering operation.
  - The PBF for the binary filters is given by

$$y = x_1x_2x_3 + x_2x_3x_4 + x_3x_4x_5$$

- It takes a lot more work to derive the PBFs for CLOSE, Close-Open, and Open-Close.

Fast Implementations

- Fast implementations of grayscale morphological and order statistic filters are challenging because they require **sorting** the pixels in the windowed set.
- But the binary versions of these filters can be implemented with boolean logic: AND, OR, XNOR, which do not require sorting.
- For stack filters, fast integrated circuits can be designed to:
  - perform the threshold decomposition,
  - filter the binary threshold signals,
  - reconstruct the output from the filtered threshold signals.

Fast Sorting

- Sorting is an expensive operation, especially for large window sizes.
- Let M be the number of pixels in the windowed set.
- Time complexity of common sorting algorithms:
  - $\mathcal{O}(\mathrm{M}^2)$: quicksort, insertion sort, selection sort…
  - $\mathcal{O}(\mathrm{M}\ \log\mathrm{M})$: merge sort, heapsort, introsort…
- In hardware, space complexity can often be traded one-for-one for time complexity.
- If the binary version of the filter is a PBF, then there are **very fast** bit-serial realizations that require only B clock cycles.

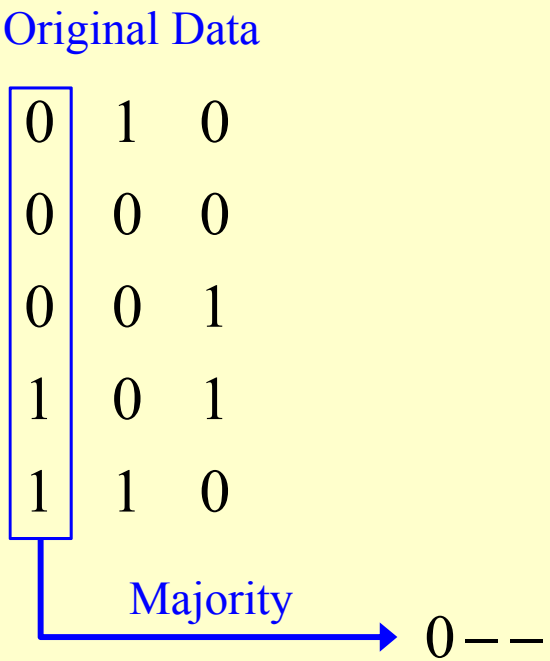
Bit-Serial Median Filtering

- This algorithm is due to Delman (Proc. SPIE, vol. 298, 1981, pp. 184-188).
- For an image $\mathbf{I}$, suppose we want to implement the filter $\mathbf{J} = \mathrm{MED}(\mathbf{I},\mathbf{B})$ with $\mathbf{B} = \mathrm{CROSS}(5)$.
- Suppose B=3-bit pixels and let the windowed set be

$$\mathbf{B} \circ \mathbf{I}(\mathrm{i}, \mathrm{j}) = \{2, 0, 1, 5, 6\}.$$

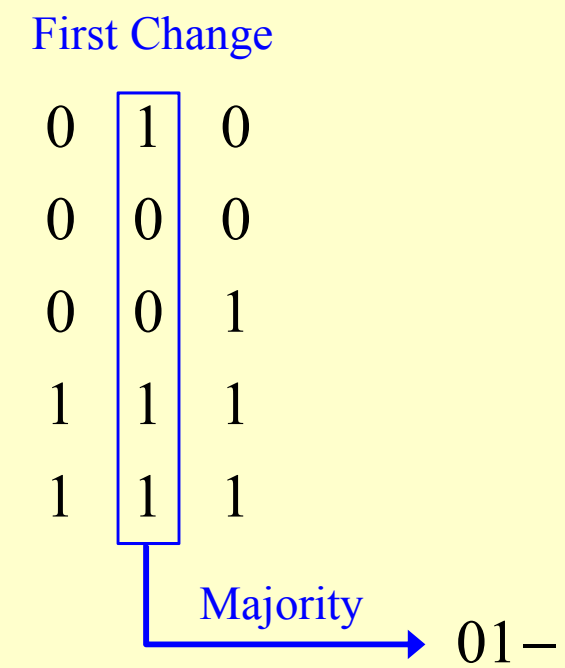
- The median is 2.
- With good sorting hardware,
  - insertion sort requires **at least** 25 clock cycles.
  - merge sort requires **at least** 15 clock cycles.
- The bit-serial algorithm finds the median in only three clock cycles!

First Clock Cycle



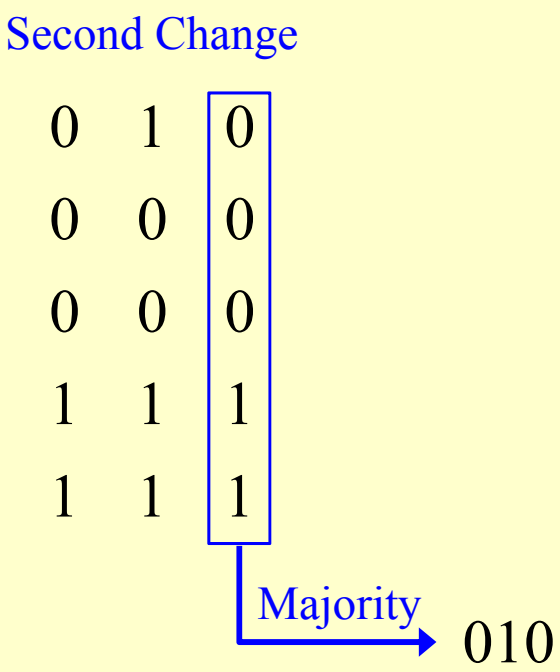
- Take the majority of the hi-order bits. This is the hi-order bit of the median (a zero in this case).
- Any pixel with a hi-order bit of one is **bigger** than the median.
  - Changing it to **all ones** will **not** change the median.
- This is the **First Change**, which is clocked into the registers by the rising edge of the second clock cycle.

Second Clock Cycle



- Take the majority of the second bits. This is the second bit of the median (a one in this case).
- Any pixel with a second bit of zero is **smaller** than the median.
  - Changing it to **all zeros** will **not** change the median.
- This is the **Second Change**, which is clocked into the registers by the rising edge of the third clock cycle.

Third Clock Cycle



- Take the majority of the lo-order bits. This is the lo-order bit of the median (a zero in this case).
- We are done! The median is found in only three clock cycles!
- This approach was generalized by Chen to develop fast bit-serial realizations for **any** stack filter (IEEE Trans. Circuits, Syst., vol. 36, no. 6, June 1989, pp. 785-794).

- For B-bit input pixels, each output pixel is computed in only B clock cycles.

ORDER STATISTIC FILTERS

ORDER STATISTIC FILTERS

- Recall the OS of a set $\mathbf{X} = \{X_1, \ldots, X_{2P+1}\}$:

$$\mathbf{X}_{OS} = \{X_{(1)}, \ldots, X_{(2P+1)}\}$$

such that

$$X_{(1)} \leq \cdots \leq X_{(2P+1)}.$$

- Define a **linear combination of order statistics**

$$\sum_{p=1}^{2P+1} A_p X_{(p)} = \mathbf{A}^T \mathbf{X}_{OS}$$

where $\mathbf{A} = [A_1, \ldots, A_{2P+1}]^T$. This defines a class of nonlinear filters called **order statistic (OS) filters**.

OS Filter Coefficients

- The **filter weights** are assumed to **sum to 1**:

$$\sum_{p=1}^{2P+1} A_p = \mathbf{A}^T\mathbf{e} = 1$$

where $\mathbf{e} = [1, 1, ..., 1]^T$. Then if $\mathbf{X} = c \cdot \mathbf{e}$ is **constant** then $\mathbf{A}^T\mathbf{X}_{OS} = c$, and the filter is then **level-preserving** – like a low-pass linear filter.

- The OS coefficients are assumed **symmetric**:

$$A_i = A_{2P+2-i} \text{ for } 1 \le i \le 2P+1$$

Defining OS filters

- Given an image $\mathbf{I}$, a window $\mathbf{B}$, and a coefficient set $\mathbf{A}$, the OS filter with coefficients $\mathbf{A}$ is defined by:

$$\mathbf{J} = \mathrm{OS}_{\mathbf{A}}[\mathbf{I}, \mathbf{B}]$$

if

$$\mathrm{J}(\mathrm{m},\mathrm{n}) = \mathbf{A}^{\mathrm{T}}\{\mathbf{B} \circ \mathbf{I}(\mathrm{m},\mathrm{n})\}_{\mathrm{OS}}$$

where

$$\{\mathbf{B} \circ \mathbf{I}(\mathrm{m},\mathrm{n})\}_{\mathrm{OS}}$$

are the OS of

$$\{\mathbf{B} \circ \mathbf{I}(\mathrm{m},\mathrm{n})\}.$$

- These are just the **ordered** values of the pixels covered by the window $\mathbf{B}$ when it's centered at (m,n).

General Properties of OS Filters

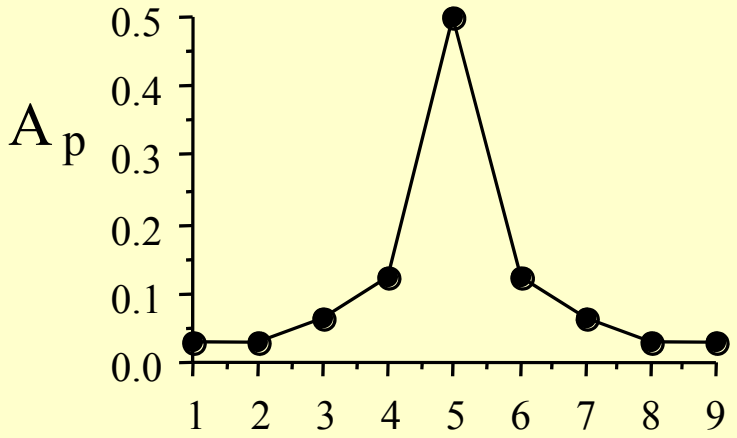
- The **behavior** of a filter $\mathrm{OS}_{\mathbf{A}}[\mathbf{I}, \mathbf{B}]$ is largely determined by its coefficients $\mathbf{A}$.
- Some important OS filters $\mathrm{OS}_{\mathbf{A}}[\mathbf{I}, \mathbf{B}]$ are:

- $\mathbf{A} = [0 \,,\ldots,\, 0,\, 1,\, 0 \,,\ldots,\, 0]^{\mathrm{T}}$ - **median filter**

- $\mathbf{A} = \left[\frac{1}{2\mathrm{P}+1},\ldots,\frac{1}{2\mathrm{P}+1}\right]^{\mathrm{T}}$ - **average filter**

Median-Like OS Filters

- **Generally**, if the coefficients $\mathbf{A}$ are concentrated near the middle:



then $OS_A[\mathbf{I}, \mathbf{B}]$ is much like a **median filter**:

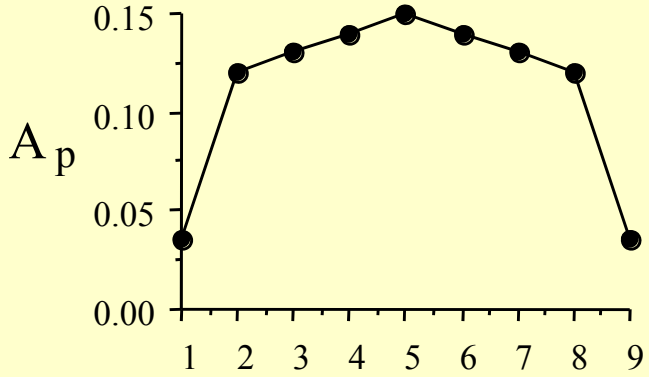
- preserve edges well
- smooth noise
- reduce impulses

BUT

- will not "streak" or "blotch" as much
- will suppress non-impulse noise better (more later)

Average-Like OS Filters

- **Generally**, if the coefficients are widely distributed:



then $\mathrm{OSA}[\mathbf{I}, \mathbf{B}]$ will act much like an **average filter**:

- preserve edges poorly (better than AVE)
- smooth noise
- blur impulses (not as badly as AVE)

- We will now explore the utility of these filters relative to some of the noise models we have looked at.

OS Filter Design Criterion

- The goal of a smoothing filter can be stated as **reduce the noise variance**.
- OS filters are best characterized by their ability to reduce noise variance.
- For certain well-known noise PMFs, specific choices of the coefficients $\mathbf{A}$ of $\mathrm{OS}_{\mathbf{A}}[\mathbf{J}, \mathbf{B}]$ result using the variance reduction criterion.

Gaussian Noise – Average Filter

- The average filter is **best** among all OS filters at reducing **Gaussian** noise variance. It is also the best linear filter. In fact, it is the best filter of any kind!

- If $\mathbf{B}$ contains $2P+1$ elements, then $\mathrm{AVE}[\mathbf{J}, \mathbf{B}]$ reduces (divides) the variance of Gaussian noise by a factor

$$2P+1$$

- In fact, $\mathrm{AVE}[\mathbf{J}, \mathbf{B}]$ reduces the variance of **any white noise** by a factor

$$2P+1$$

- This is pretty good, but for non-Gaussian PMFs other OS filters can do much better.

- **Major caveat**: The above **does not account** for the **effect on the signal.** Average filtering blurs the signal as it reduces noise, so there exists a **tradeoff** between **noise reduction** and **signal blurring**.

Laplacian Noise – Median Filter

- The median filter $\mathrm{MED}[\mathbf{J}, \mathbf{B}]$ is the asymptotically **best** OS filter for reducing the variance of **Laplacian** noise (as window size increases).

- If $\mathbf{B}$ contains $2P+1$ elements, then $\mathrm{MED}[\mathbf{J}, \mathbf{B}]$ reduces Laplacian noise variance by a factor of (approximately)

  $2 \cdot (2P+1)$ - quite an **improvement**!

- $\mathrm{MED}[\mathbf{J}, \mathbf{B}]$ is **fantastic** for removing salt-and-pepper noise. No better filter!

- If the noise is Gaussian, then $\mathrm{MED}[\mathbf{J}, \mathbf{B}]$ reduces the noise variance by a factor of (approximately) only

  $(0.75) \cdot (2P+1)$ - **not** an **improvement**!

- **Extended Caveat:** The median filter is noted to **not** blur images, even as the window size increases.

Median Filter vs. Average Filter

- Owing to its very important **edge-preserving property**, MED is generally regarded as **superior** to AVE (or any other linear filter!) for **reducing white noise in images**.

- However, as with any application, care should be taken in analyzing the situation rather than blindly applying a filter.

- Both MED and AVE are used **far too often** in situations where a design process might be **more advantageous**.

Robust OS Filter Coefficients

- A fairly sophisticated **statistical analysis** yields the **best** $\mathrm{OS}_{\mathbf{A}}$ filter for **specific** noise **and** image models.
- Fortunately, a great property of **certain** OS filters is the fact that they are ***robust.***
- A filter is **robust** if it gives **close to** optimal performance for a **wide variety** of noise types.
- OS filters $\mathrm{OS}_{\mathbf{A}}$ have been shown to be **remarkably robust** for coefficients $\mathbf{A}$ that resemble **a cross between** MED and AVE.

Trimmed Mean Filters

- A simple robust OS filter class is the **trimmed mean** OS filters $\mathrm{TM}_Q[\mathbf{J}, \mathbf{B}]$ with coefficients

$$\mathbf{A} = \Big[\underbrace{0,\ldots,0}_{Q},\ \underbrace{\frac{1}{2(P-Q)+1},\ldots,\frac{1}{2(P-Q)+1}}_{2(P-Q)+1},\ \underbrace{0,\ldots,0}_{Q}\Big]^T$$

  where $Q < P$ (usually $Q \approx P/2$).

- **Similar to AVE** since the innermost $2(Q\text{-}P)+1$ OS are averaged. If $Q = 0$, then $\mathrm{TM}_Q[\mathbf{J}, \mathbf{B}] = \mathrm{AVE}[\mathbf{J}, \mathbf{B}]$.

- **Similar to MED** since *only* inner OS are averaged, the rest of the OS being "trimmed." Indeed, if $Q = P$, then $\mathrm{TM}_Q[\mathbf{J}, \mathbf{B}] = \mathrm{MED}[\mathbf{J}, \mathbf{B}]$.

Trimmed Mean Filter Performance

- Trimmed mean filters **smooth noise well** (including impulse and Gaussian noise) and **preserve edges well**.

- For **both** Gaussian **and** Laplacian noise, $\mathrm{TM}_{P/2}$ reduces the variance by a factor

$$> (0.9) \cdot \mathbf{OPTIMAL}$$

where **OPTIMAL** is $(2P+1)$ for Gaussian noise and $2 \cdot (2P+1)$ for Laplacian noise.

Generalized Gaussian PDF

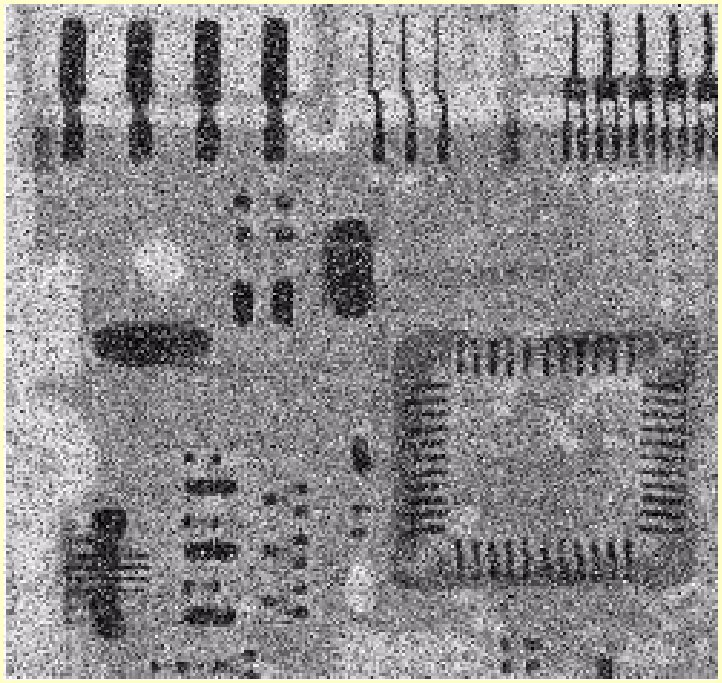
- The **generalized gaussian** PDF is used to model wide behaviors of noise:

$$f_\gamma(x) = K e^{-|x/\sigma|^\gamma} \quad -\infty < x < \infty$$

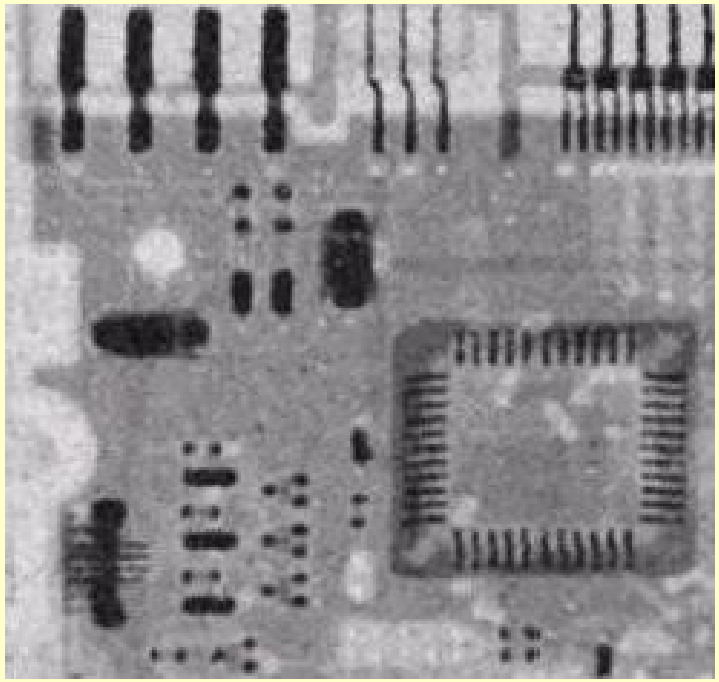
- For $\gamma < 1$, $f_\gamma(x)$ is **very heavy-tailed**.
- For $\gamma = 1$, $f_1(x)$ is **laplacian**.
- For $\gamma = 2$, $f_2(x)$ is **gaussian**.
- As $\gamma \rightarrow \infty$, $f_\infty(x)$ is **uniform**.
- For a wide range of values of γ

$$TM_{P/2} > (0.9) \cdot \text{OPTIMAL!}$$

Example – AWGN / S&P Noise



AWGN and S&P Noise



50% Trimmed Mean

Time-consuming!
Good examples:
N=3, SQUARE, P=2
N = 5, CROSS, P=3
AWGN with S&P

BILATERAL FILTER

BILATERAL FILTER

- Another principled approach to **image denoising**.
- Like OS filters, it **modifies linear filtering**.
- Like OS filters, it seeks to smooth while **retaining (edge) structures**.

Bilateral Filter Concept

- **Observation:** 2-D linear filtering is a method of **weighting** by **spatial distances** **[$\mathbf{i} = (i, j)$, $\mathbf{m} = (m, n)$]:**

$$J_G(\mathbf{i}) = \sum_{\mathbf{m}} I(\mathbf{m})\, G\left(\|\mathbf{m} - \mathbf{i}\|\right)$$

- If linear filter G is **isotropic**, e.g., a 2-D gaussian LPF*

$$G(i, j) = K_G \exp\left[-\left(i^2 + j^2\right) / \sigma_G^2\right]$$

then $\|\mathbf{m} - \mathbf{i}\| = \sqrt{(m-i)^2 + (n-j)^2}$ is Euclidean **distance**.

*The value of K_G is such that G has unit volume.

Bilateral Filter Concept

- **New idea:** Instead of weighting by spatial distance, weight by **luminance similarity**

$$J_H(\mathbf{i}) = \sum_{\mathbf{m}} I(\mathbf{m})\, H\left[I(\mathbf{m}) - I(\mathbf{i}) \right]$$

- **NOT** a linear filter operation. H could be a 1-D gaussian weighting function*

$$H(i) = K_H \exp\left[-i^2 / \sigma_H^2 \right]$$

- **Not very useful by itself.**

*The value of K_H is such that H has unit area.

Bilateral Filter Definition

- **Combine** spatial distance weighting with luminance similarity weighting:

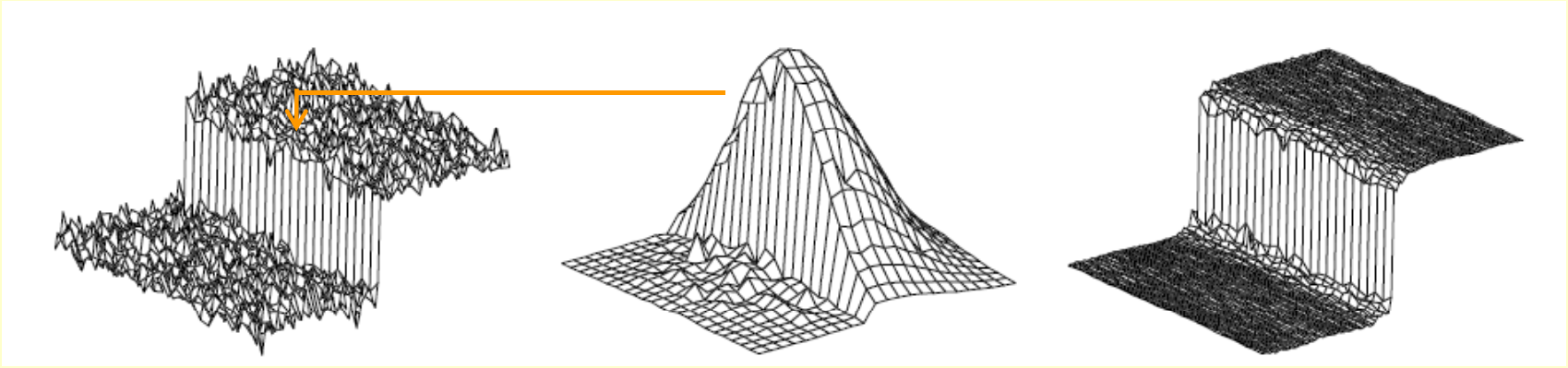
$$\mathrm{J}(\mathbf{i}) = \sum_{\mathbf{m}} \mathrm{I}(\mathbf{m})\, \mathrm{G}\left(\|\mathbf{m}-\mathbf{i}\|\right) \mathrm{H}\left[\mathrm{I}(\mathbf{m}) - \mathrm{I}(\mathbf{i})\right]$$

- **Computation** is **straightforward** and about twice as expensive as just spatial linear filtering.

- However, this can be sped up by pre-computing all values of H for every pair of luminances, and storing them in a **look-up table** (LUT).

Example

- **Noisy edge** filtered by bilateral filter (Tomasi, ICCV 1998).
- Uses gaussian LPF and **gaussian similarity**



Before

Bilateral weighting 2 pixels to right of edge

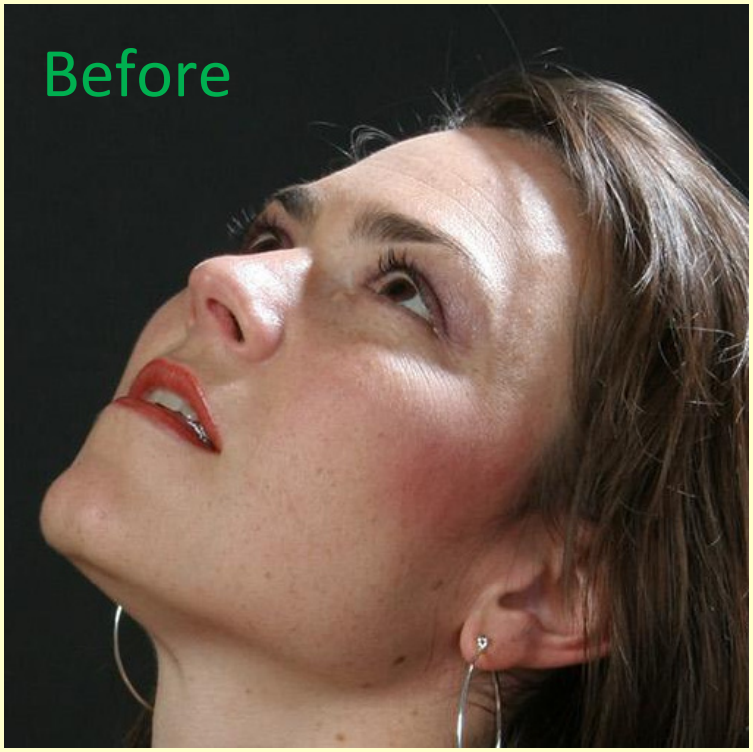
After

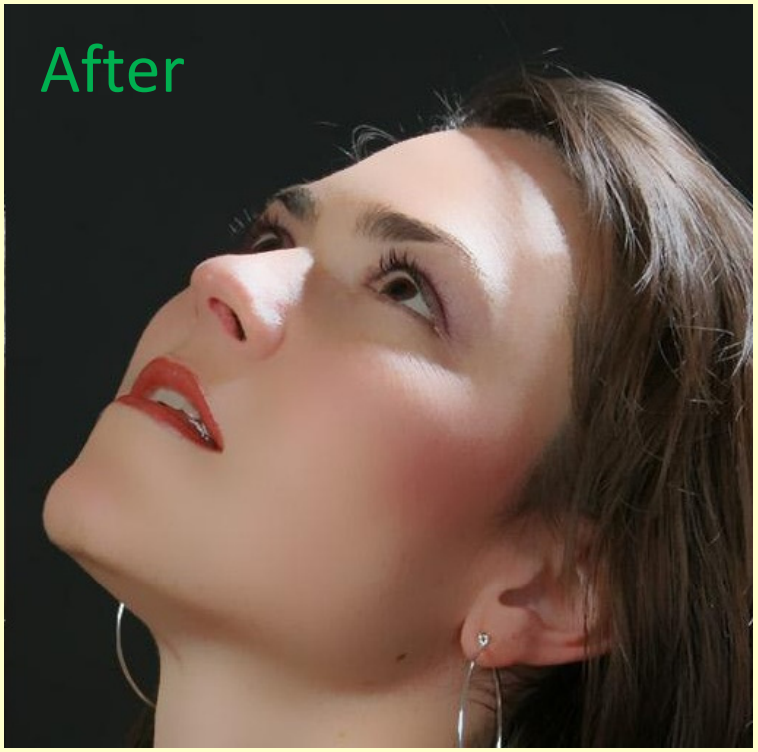
Color Example

- Can extend to **color** using **3-D similarity weighting**.
- If $\mathbf{I} = [\mathbf{R}, \mathbf{G}, \mathbf{B}]$, then use

$$H[\mathbf{R(m)}-\mathbf{R(i)}, \mathbf{G(m)}-\mathbf{G(i)}, \mathbf{B(m)}-\mathbf{B(i)}]$$

where $H(i, j, k) = K_H \exp\left[-\left(i^2 + j^2 + k^2\right) / \sigma_H^2\right]$





Comments on Bilateral Filter

- **Similar to median filter**, emphasizes luminances close in value to current pixel.
- Does not have a statistical (or other) design basis, so **best used with care**, e.g., **retouching** or **low-level noise smoothing**.
- The idea of doing **linear filtering that is decoupled near edges** is very powerful.
- This is also the theme of the **next method**.

NON-LOCAL (NL) MEANS

NL Means Concept

- **Idea:** Estimate the current pixel luminance as a weighted average of **all** pixel luminances
- The **weights** are decided by neighborhood **luminance similarity**.
- The **weight changes** from pixel to pixel.
- In all the following, $\mathbf{i} = (i, j)$, $\mathbf{m} = (m, n)$, $\mathbf{p} = (p, q)$.
- The method is **expensive** but **effective**.

NL Means Concept

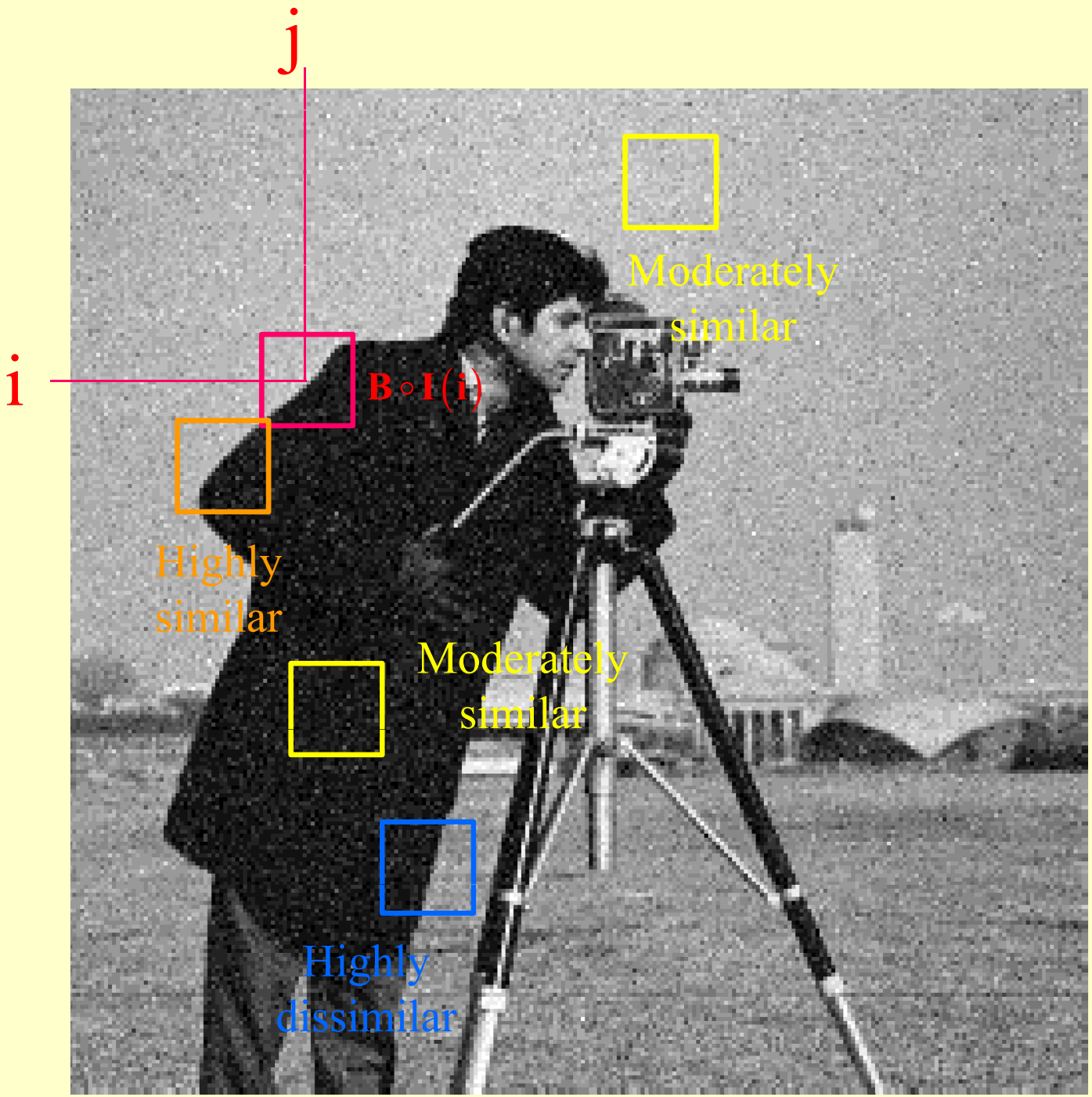
- Given a window $\mathbf{B}$, compute the **luminance similarity** of every windowed set $\mathbf{B}\circ\mathbf{I(i)}$ with **every other** windowed set $\mathbf{B}\circ\mathbf{I(m)}$:

$$W(\mathbf{i},\mathbf{m}) = K_W \exp\left[-\left\|\mathbf{B}\circ\mathbf{I}(\mathbf{i}) - \mathbf{B}\circ\mathbf{I}(\mathbf{m})\right\| / \sigma_W^2\right]$$

where

$$\left\|\mathbf{B}\circ\mathbf{I}(\mathbf{i}) - \mathbf{B}\circ\mathbf{I}(\mathbf{m})\right\| = \sum_{\mathbf{p}\in\mathbf{B}}\left[I(\mathbf{i}-\mathbf{p}) - I(\mathbf{m}-\mathbf{p})\right]^2$$

$$= \sum_{(p,q)\in\mathbf{B}}\left[I(i-p,j-q) - I(m-p,n-q)\right]^2$$


j
i
B ∘ I(i)
Moderately similar
Highly similar
Moderately similar
Highly dissimilar

NL Means Definition

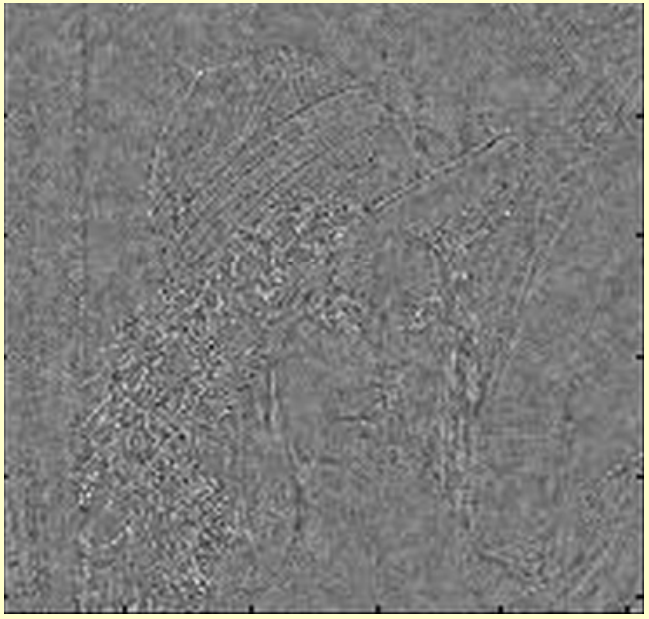
- Given a window $\mathbf{B}$, and the **weighting function** W just defined, the **NL-means filtered image** $\mathbf{I}$ is

$$\mathbf{J}(\mathbf{i}) = \sum_{\mathbf{m}} \mathrm{W}(\mathbf{m}, \mathbf{i}) \mathrm{I}(\mathbf{m})$$

- Thus each pixel value is replaced by a **weighted sum of all luminances**, where the **weight increases** with **neighborhood similarity.**

Example

Before



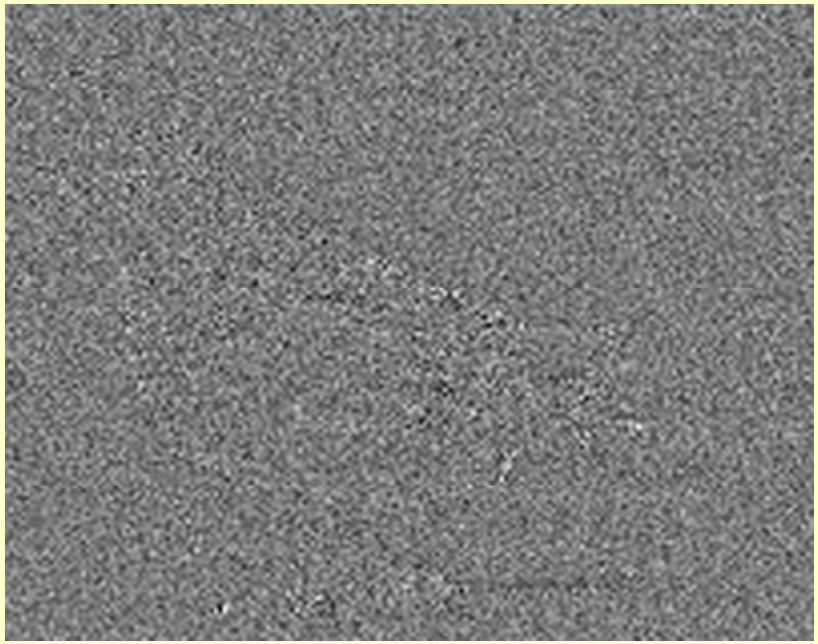
Difference

After

Example

Before

After



Difference

Comments

- **NL-Means** is very similar in spirit to **frame averaging.**
- The main drawback is the large computation / search.
- It can be modified to compare/search **less of the image**.
- It **works best** when the image contains a lot of **redundancy** (periodic, large smooth regions, textures).
- It can **fail** at **unique image patches**.

ANISOTROPIC DIFFUSION

ANISOTROPIC DIFFUSION

- A powerful approach to both image denoising and **edge detection**.
- Fact: Gaussian blur or smoothing is analogous to physical **diffusion** (e.g., of heat).
- In fact the gaussian function is a solution to a PDE called the **diffusion equation**.

Concept of Anisotropic Diffusion

- Diffuse (smooth image pixels) where the gradient magnitude is small.
- **Inhibit diffusion** where gradient magnitude is large.
- **Anisotropic** since smoothing **depends on direction**.
- Convolution with a Gaussian is **isotropic diffusion.**

- **Rationale:**

  - Smooth image without destroying image detail

  - Inhibit smoothing across edges (inter-region smoothing)

  - Encourage smoothing between edges (intra-region smoothing)

The Diffusion Equation

- **Continuous Formulation** - a system of partial differential equations:

$$\frac{\partial \mathbf{I}}{\partial t} = \mathrm{div}\left[\mathbf{c}\nabla \mathbf{I}\right] = c_{EW}\frac{\partial^2 I(x,y)}{\partial x^2} + c_{NS}\frac{\partial^2 I(x,y)}{\partial y^2} \qquad \mathbf{c} = \begin{bmatrix} c_{EW} & 0 \\ 0 & c_{NS} \end{bmatrix}$$

  where $\mathbf{I}$ is the image, $\mathbf{c}$ is a 2x2 matrix of **diffusion coefficients**, and div is the **divergence operator**

- The discrete formulation is much easier to understand...

Note: Recall

$$\mathrm{div}(\mathbf{F}) = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} \text{ where } \mathbf{F} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

Approximating Derivatives

$$\frac{\partial \mathbf{I}}{\partial t} \rightarrow \mathrm{I}_{t+1}(m,n) - \mathrm{I}_{t}(m,n)$$

where time t will be iteration number of the diffusion process.

$$\frac{\partial^2 \mathrm{I}(x,y)}{\partial x^2} \rightarrow \mathrm{I}(m, n+1) - 2\mathrm{I}(m, n) + \mathrm{I}(m, n-1)$$

$$\frac{\partial^2 \mathrm{I}(x,y)}{\partial y^2} \rightarrow \mathrm{I}(m+1, n) - 2\mathrm{I}(m, n) + \mathrm{I}(m-1, n)$$

which approximates twice derivatives by twice differences.

Diffusion Equation: Discrete Form

- Iterative approximation using **differences:**

$$I_{t+1}(m,n) = I_t(m,n) + \frac{1}{4}\left[c_{NS}(m,n)\nabla^{(2)}_{NS} I_t(m,n) + c_{EW}(m,n)\nabla^{(2)}_{EW} I_t(m,n) \right]$$

where ...

$$\nabla^{(2)}_{NS} I_t(m,n) = I(m,n+1) - 2I(m,n) + I(m,n-1)$$

$$\nabla^{(2)}_{EW} I_t(m,n) = I(m+1,n) - 2I(m,n) + I(m-1,n)$$

Note: If $c_{NS} = c_{EW} = c$, then the diffusion equation becomes the *heat equation* $\frac{\partial \mathbf{I}}{\partial t} = c\nabla^2 \mathbf{I}$

Diffusion Coefficients

- Selection of the diffusion coefficients c_{NS}, c_{EW} is important. Usually $c_{NS} = c_{EW}$.

- The idea is to **inhibit smoothing** over edges, **encourage smoothing** on non-edge image regions. So c_{NS}, c_{EW} should be **edge-sensitive**.

- At each pixel $I(i, j)$, diffusion proceeds in a given direction according to the diffusion coefficient corresponding to that direction and that location.

Diffusion Coefficients

- One popular form

$$c_d(m,n) = \exp\left\{-\left[\frac{\nabla_d I(m,n)}{k}\right]^2\right\} \qquad d = \text{NS, EW}$$

where k is an edge threshold – as the gradient increases, c_d rapidly falls.

- Properties of $c_d(m, n)$:
  - approaches 0 near edges – **disenabling diffusion**
  - approaches 1 over smooth areas –**enabling diffusion**

Anisotropic Diffusion Example

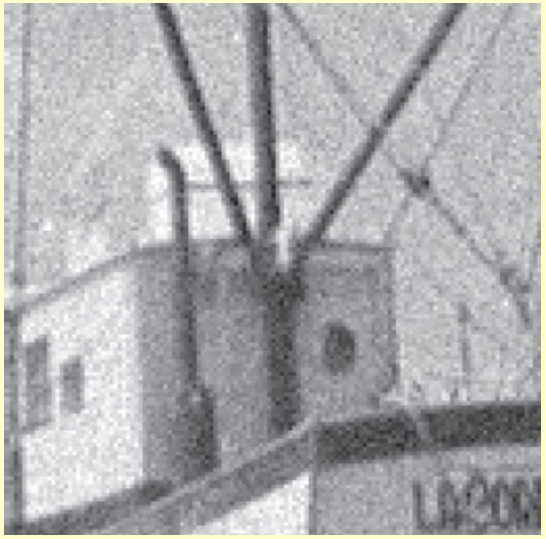
Laplacian Noise

8 iterations of
anisotropic diffusion

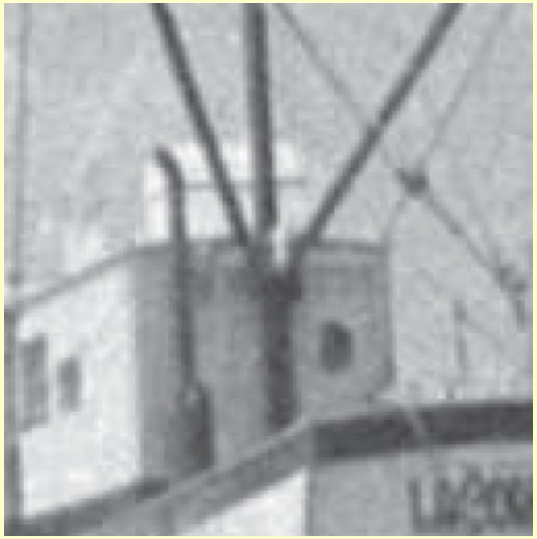
Comments on Anisotropic Diffusion

- Really a method of alternating edge detection with **edge-decoupled smoothing.**
- Number of iterations controls degree of smoothing (scale)
- Edge detection applied to the smoothed image can be quite effective.

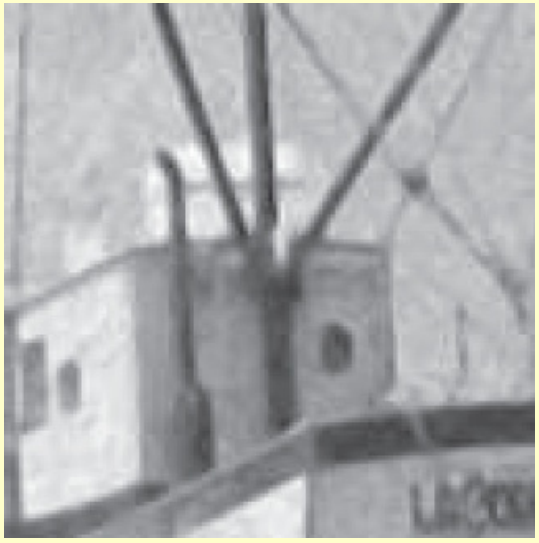
Comparison



AWGN



Gaussian filtered



Anisotropic diffusion

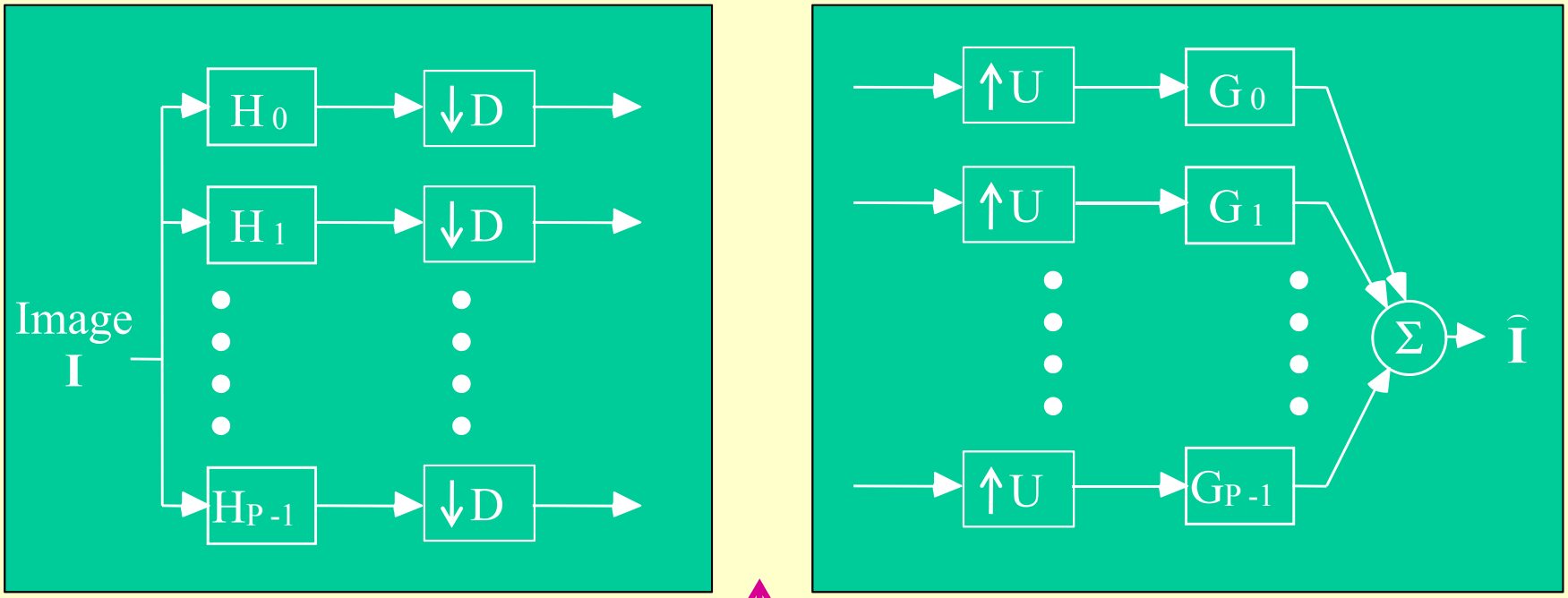
Bilateral filtered

NL-Means

WAVELET SHRINKAGE

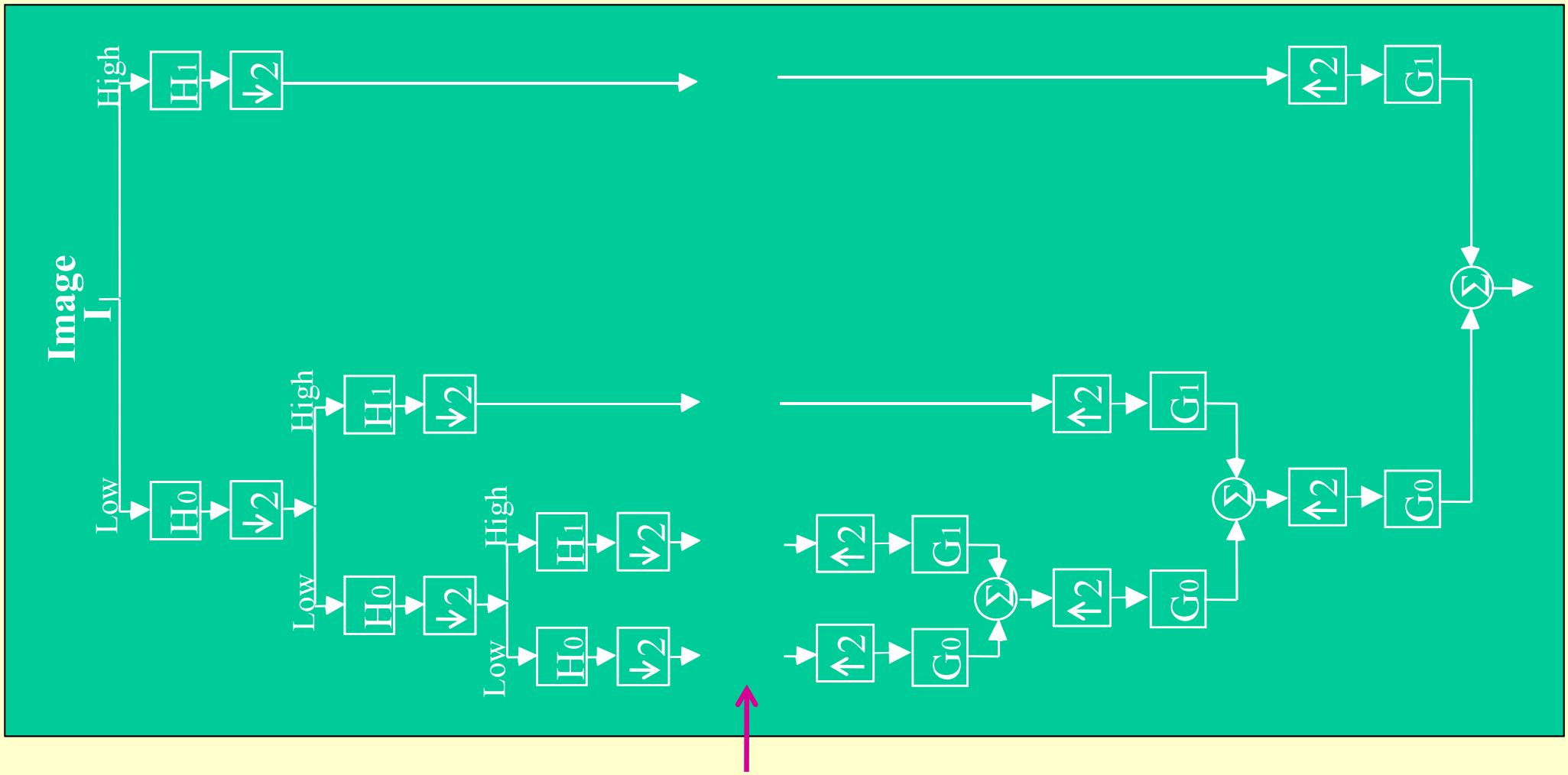
Wavelet Shrinkage

Perfect reconstruction analysis/synthesis filterbank or DWT



Denoise the DWT coefficients

Wavelet Shrinkage

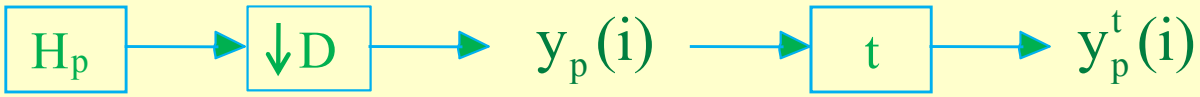


Denoise the DWT coefficients

Wavelet Shrinkage

- The process of **denoising** is accomplished simply by **thresholding the DWT coefficients** (filter responses).

- A **threshold is applied** ($p \neq 0$ since baseband signal is smooth already):



Wavelet Soft Thresholding

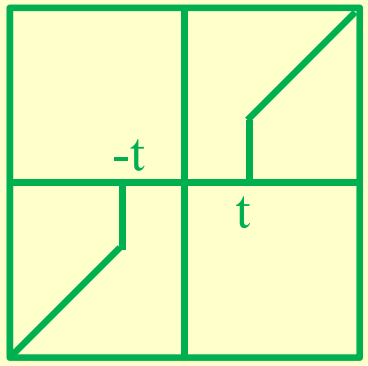
- Two methods of thresholding:
  - **Hard threshold**: signal zeroed if below threshold
  - **Soft threshold**, where in addition to hard threshold, the threshold is subtracted if above threshold.

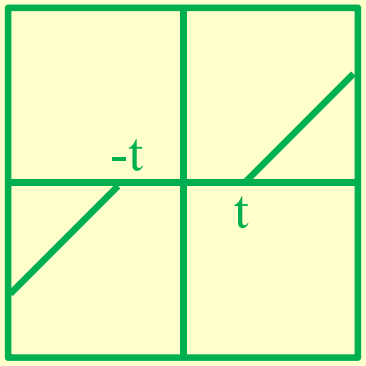
Hard threshold

$$y_p^t(i) = \begin{cases} y_p(i) \; ; \text{if } |y_p(i)| > t \\ 0 \quad ; \text{if } |y_p(i)| \le t \end{cases}$$

Soft threshold

$$y_p^t(i) = \text{sgn}\left[y_p(i) \right] \max\left\{ 0, |y_p(i)| - t \right\}$$





Wavelet Shrinkage

- **Soft thresholding** is much more effective. It "shrinks" the wavelet coefficients.

- Works since additive **noise increases coefficient energy**. Soft **thresholding reduces the energy**.

- **Threshold** can be chosen to **optimize** many criteria such as MSE, Bayesian risk, etc.

Wavelet Shrinkage

- **MMSE threshold ("SureShrink")**

$$t = \sigma\sqrt{2 \log NM}$$

where σ^2 = estimated noise variance, and NM = # pixels in the image.

- **Bayesian threshold** (an SNR) **("BayesShrink")**

$$t = \sigma_{signal} / \sigma_{noise}$$

is estimated for each filter response (band), hence **adaptive.** When SNR is large, little thresholding. When large, heavy thresholding.

Wavelet Shrinkage Examples

DWT Used

- There is no clear choice (yet) on wavelet filters to use.
- Here we used a 4-level DWT with the “Daubechies 10” orthogonal wavelet (D10)

Analysis LP

$H_0(0) = 0.1601023980$
$H_0(1) = 0.6038292698$
$H_0(2) = 0.7243085284$
$H_0(3) = 0.1384281459$
$H_0(4) = -0.2422948871$
$H_0(5) = -0.0322448696$
$H_0(6) = 0.0775714938$
$H_0(7) = -0.0062414902$
$H_0(8) = -0.0125807520$
$H_0(9) = 0.0033357253$

Analysis HP

$H_1(0) = 0.0033357253$
$H_1(1) = 0.0125807520$
$H_1(2) = -0.0062414902$
$H_1(3) = -0.0775714938$
$H_1(4) = -0.0322448696$
$H_1(5) = 0.2422948871$
$H_1(6) = 0.1384281459$
$H_1(7) = -0.7243085284$
$H_1(8) = 0.6038292698$
$H_1(9) = -0.1601023980$

Synthesis LP

$G_0(0) = 0.0033357253$
$G_0(1) = -0.0125807520$
$G_0(2) = -0.0062414902$
$G_0(3) = 0.0775714938$
$G_0(4) = -0.0322448696$
$G_0(5) = -0.2422948871$
$G_0(6) = 0.1384281459$
$G_0(7) = 0.7243085284$
$G_0(8) = 0.6038292698$
$G_0(9) = 0.1601023980$

Synthesis HP

$G_1(0) = -0.1601023980$
$G_1(1) = 0.6038292698$
$G_1(2) = -0.7243085284$
$G_1(3) = 0.1384281459$
$G_1(4) = 0.2422948871$
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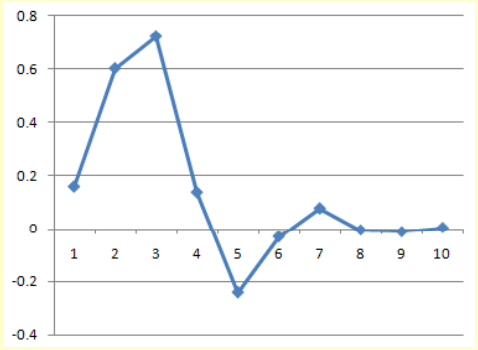
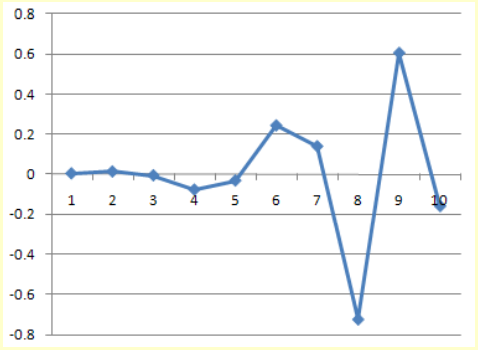
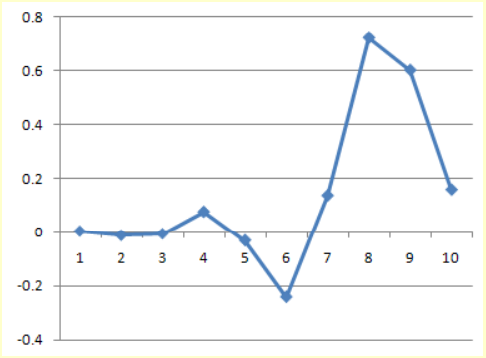
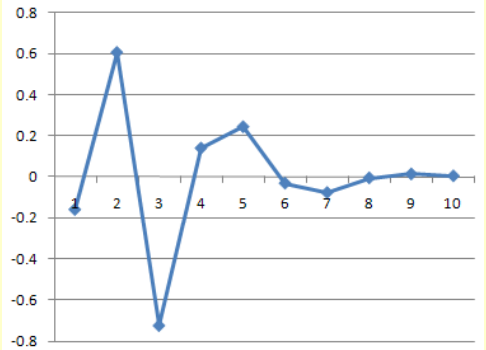








Comments

- Since DWTs can be computed **very fast**, so can **soft threshold based denoising**.

- The results are **optimal in wavelet space** and the appearance is **reasonably good**.

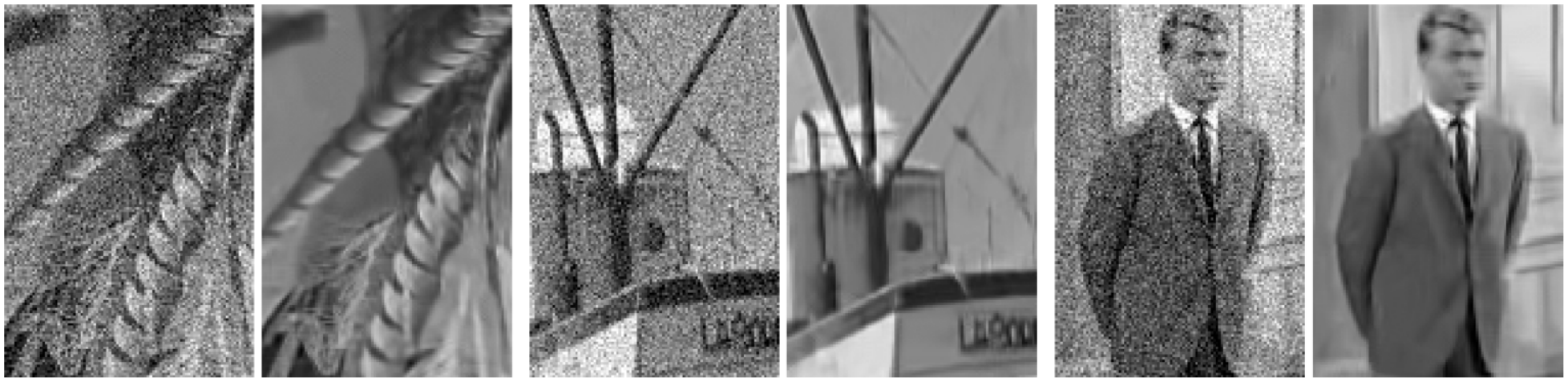
BM3D

Overview

- Quite a **complicated** algorithm – too much! Uses **many** of the ideas we have covered
- Image broken into **blocks**, blocks **thenmatched by similarity** into groups.
- Estimates are **collaboratively formed** for each **block** within each group.
- This is done by a **transformation** of the group and a **shrinkage process**.
- Perhaps the best **denoising algorithm extant**.

Dabov, Foi, Katkovnik, Egiazarian, IEEE Trans on Image Processing, Aug 2007.

Examples



HOMOMORPHIC FILTERING

- Assume a **multiplicative white noise** model:

$$\mathbf{J} = \mathbf{I} \otimes \mathbf{N}$$

where $\mathbf{I}$ is the original image and $\mathbf{N}$ is a **white noise image**.

- Thus

$$\mathrm{J}(m,n) = \mathrm{I}(m,n)\mathrm{N}(m,n) \text{ for } 0 \le m \le \mathrm{M}\text{-}1,\ 0 \le n \le \mathrm{N}\text{-}1.$$

- Direct linear filtering ineffective, since the noise / image spectra not **added**.

Multiplicative Noise

- Multiplicative noise is always **positive** since the sensed image intensities must be.
- Multiplicative noise often has an **exponential PMF** in practical applications, such as **radar**, **laser-based imaging**, **electron microscopy**, etc.
- **Synthetic aperture radar** images are an excellent example.
- Multiplicative noise appears **much worse** in bright image regions (since it multiplies the gray levels) and can be **hardly noticeable** in dark regions.

Homomorphic Approach

- Homomorphic filtering is succinctly expressed by the **following diagram**:

- We'll study each step:

Logarithmic Point Operation

- Idea is similar to log point operations used for histogram improvement: de-emphasize **dominant bright image pixels**.
- More than that! Since $\log(a \cdot b) = \log(a) + \log(b)$, then

$$\log[J(m,n)] = \log[I(m,n)] + \log[N(m,n)].$$

- So: the point operation **converts the problem** into that of smoothing

$$\mathbf{J}' = \mathbf{I}' + \mathbf{N}'$$

where

$I'(m,n) = \log[I(m,n)]$, $N'(m,n) = \log[N(m,n)]$, $J'(m,n) = \log[J(m,n)]$

- Importantly, $\mathbf{N}'$ is **still** a **white noise image**.
- This is just the additive white noise problem studied earlier!

Filtering

- The filtering problem here is very similar to the linear and nonlinear ones studied earlier in this Module and in Module 5.
- Depending on the noise statistics, AVE, MED, $\mathrm{OS}_{\mathbf{A}}$, OPEN-CLOSE, CLOSE-OPEN or other linear or nonlinear filter may be used.
- The **theoretical** performance of these various filters in homomorphic systems still remains an open question.
- Most will work reasonably well; in particular MED has been **observed** to be effective if the noise is exponential and the image contains edges.
- The objective is of course to produce a filtered result

$$\mathrm{F}[\mathbf{J}', \mathbf{B}] \approx \mathbf{I}'$$

Exponential Point Operation

- If the filtering operation **succeeds**:

$$F[\mathbf{J}', \mathbf{B}] \approx \mathbf{I}'$$

then the output image $\mathbf{K}$ will **approximate**

$$K(i, j) \approx \exp [I'(i, j)]$$
$$= I(i, j)$$

Comments

- We shall now study **some BIG topics – image quality and compression**… onward to **Module 7**..