Homework #10_A

1. Please define sensitivity, specificity, and accuracy in medical image perception.

Sensitivity is the ability to detect a lesion when it is really there

Specificity is the ability to say that the lesion is absent when it is really not there

Accuracy = number of correct assessments / total number of cases
= (TP+TN) / (TP+TN+FP+FN)

2. A physician read 50 mammograms and sent 6 patients to surgical biopsy procedures (6 positive cases according to his/her judgment). The pathology report following the biopsy indicated that among these 6 cases, only 4 patients have breast cancer. An expert panel reviewed the mammograms of the rest 44 patients (those who were found negative by the physician and were not sent for biopsy) and found another 1 positive cases and the follow-up biopsy/pathology verified that these one patient have breast cancer. A 5-year follow-up indicated that no more new breast cancer cases were found among all patients.

(1) What is the sensitivity of the physician's diagnosis?

Solution:

Correct positive assessments (True positive): 4
Number of truly positive cases (TP+FN): 4+1=5
Sensitivity = 4/5 =80%

More Explanations:
Sensitivity is the ability to detect a lesion when it is really there
Sensitivity = number of correct positive assessments / number of truly positive cases
= TP / (TP+FN)
Sensitivity can be expressed as True Positive Fraction (TPF)

(2) What is the specificity of the physician's diagnosis?

Solution:

Number of correct negative assessments: 44-1 =43
Number of truly negative cases: 50-4-1= 45
Specificity = 43/45 = 95.5%

More explanation:
Specificity is the ability to say that the lesion is absent when it is really not there
Specificity = number of correct negative assessments / number of truly negative cases
= TN / (TN+FP)
Specificity can be expressed as True negative Fraction (TNF)

3. An observer study for comparing the performance of two medical imaging systems has result the following data:

For system 1

Test Positive if greater than	Sensitivity (TPF)	Specificity (TNF)
Threshold 1	85/100	55/100
Threshold 2	75/100	70/100
Threshold 3	70/100	80/100
Threshold 4	20/100	92/100

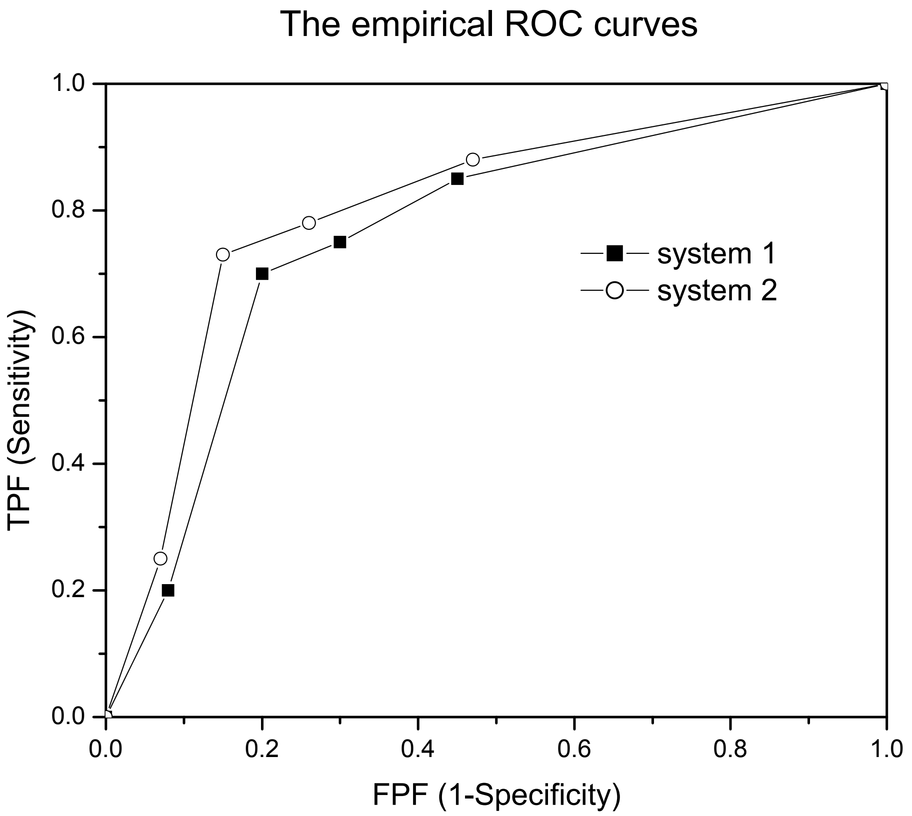
For system 2

Test Positive if greater than	Sensitivity (TPF)	Specificity (TNF)
Threshold 1	88/100	53/100
Threshold 2	78/100	74/100
Threshold 3	73/100	85/100
Threshold 4	25/100	93/100

(1) Please plot ROC curves for both of the systems;

(2) Which system is better?

(1):



(2): system 2 is better

Homework #10-B:

1. An observer study for comparing the performance of two medical imaging systems has resulted the following data:

For system A

Test Positive if greater than	Sensitivity (TPF)	Specificity (TNF)
Threshold 1	85/100	55/100
Threshold 2	75/100	70/100

For system B

Test Positive if greater than	Sensitivity (TPF)	Specificity (TNF)
Threshold 1	88/100	53/100
Threshold 2	78/100	74/100

(1) Please calculate a and b values, and plot ROC curves for both of the systems by using Matlab.
(2) Please calculate the area under the ROC curve (AUC) for both of the systems.
(3) Which system is better?

Solution:

(1) For System A.

❖ We choose also two pairs of FPF and TPF as:

Threshold 1 ($FPF_1=1-TNF_1=1-0.55=0.45$, $TPF_1=0.85$)

$\Phi^{-1}(FPF_1) = \Phi^{-1}(0.45) \approx -0.126$ =NORMSINV(0.45)=-0.1257

$\Phi^{-1}(TPF_1) = \Phi^{-1}(0.85) \approx 1.036$ =NORMSINV(0.85)=1.0364

Threshold 2 ($FPF_2=1-TNF_2=1-0.70=0.30$, $TPF_2=0.75$)

$\Phi^{-1}(FPF_2) = \Phi^{-1}(0.3) \approx -0.524$ =NORMSINV(0.3)=-0.5244

$\Phi^{-1}(TPF_2) = \Phi^{-1}(0.75) \approx 0.675$ =NORMSINV(0.75)=0.67449

Recall Eq. (11.18) and (11.19)

$$\begin{cases} a_1 + b_1 \times (-0.126) = 1.036 \\ a_1 + b_1 \times (-0.524) = 0.675 \end{cases}$$

Final: **a_1= 1.15, b_1=0.91**

For System B.

- We choose two pairs of FPF and TPF as:

  Threshold 1 (FPF_1=1-TNF_1=1-0.53=0.47, TPF_1=0.88)

$\Phi^{-1}(FPF_1) = \Phi^{-1}(0.47) \approx -0.075$ =NORMSINV(0.47)=-0.07527

$\Phi^{-1}(TPF_1) = \Phi^{-1}(0.88) \approx 1.175$ =NORMSINV(0.88)=1.175

Threshold 2 (FPF_2=1-TNF_2=1-0.74=0.26, TPF_2=0.78)

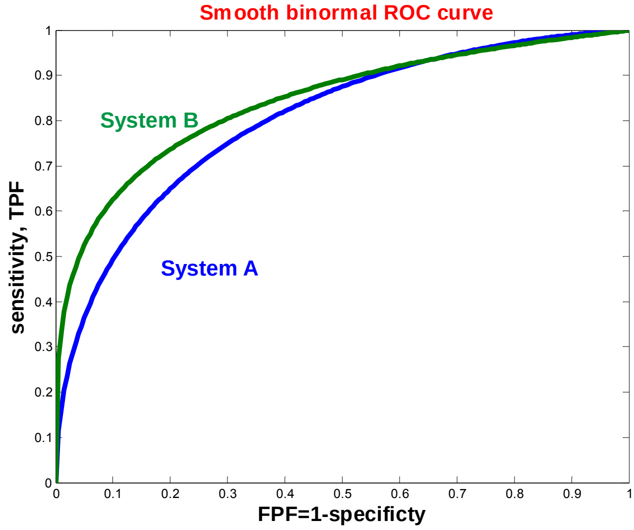
$\Phi^{-1}(FPF_2) = \Phi^{-1}(0.26) \approx -0.643$ =NORMSINV(0.26)=-0.64335

$\Phi^{-1}(TPF_2) = \Phi^{-1}(0.78) \approx 0.772$ =NORMSINV(0.78)=0.7722

Recall Eq. (11.18) & (11.19):

$$\begin{cases} a_2 + b_2 \times (-0.075) = 1.175 \\ a_2 + b_2 \times (-0.643) = 0.772 \end{cases}$$

Final: **a_2= 1.23, b_2=0.71**



(2) The area under the ROC curve (AUC)

For System A: **a_1= 1.15, b_1=0.91**

$$AUC_A = \Phi(\frac{a}{\sqrt{1+b^2}}) = \Phi\left(\frac{1.15}{\sqrt{1+0.91^2}}\right) = \Phi(0.85) = 0.8023$$

For System B: **a_2= 1.23, b_2=0.71**

$$AUC_B = \Phi(\frac{a}{\sqrt{1+b^2}}) = \Phi\left(\frac{1.23}{\sqrt{1+0.71^2}}\right) = \Phi(1.002) = 0.8418$$

(3) System B is better

2. The cumulative distribution function (CDF), Φ(X), of the standard normal distribution can be computed as:

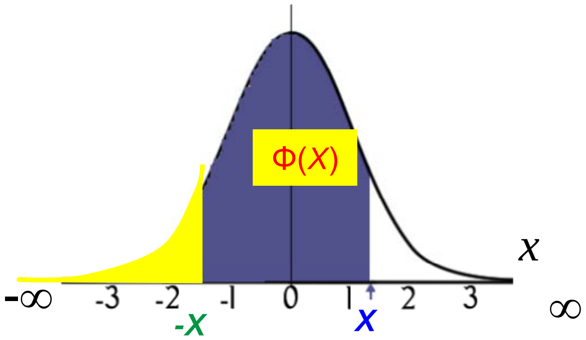
$$CDF = \Phi(X) = \int_{-\infty}^{X} \frac{1}{\sqrt{2\pi}} \times e^{-x^2/2} \times dx$$

Please prove that the symmetry property of Φ(X): i.e. for a real number ***X,*** we have

$$\Phi(-X) = 1 - \Phi(X)$$

Solution:

The **CDF**, Φ(X), can be computed as:



$$CDF = \Phi(X) = \int_{-\infty}^{X} \frac{1}{\sqrt{2\pi}} \times e^{-x^2/2} \times dx$$

For a real number X, we can get:

$$\Phi(-X) = \int_{-\infty}^{-X} \frac{1}{\sqrt{2\pi}} \times e^{-x^2/2} \times dx$$

Let: x=-t, then, -X→X, -∞→∞:

$$\Phi(-X) = \int_{\infty}^{X} \frac{1}{\sqrt{2\pi}} \times e^{-(-t)^2/2} \times d(-t) = -\int_{\infty}^{X} \frac{1}{\sqrt{2\pi}} \times e^{-t^2/2} \times dt$$
$$= \int_{X}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-t^2/2} \times dt = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-t^2/2} \times dt - \int_{-\infty}^{X} \frac{1}{\sqrt{2\pi}} \times e^{-t^2/2} \times dt$$

Recall Handout 11.6(2)f, the total area under the standard normal distribution is 1:

$$\Phi(\infty) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-t^2/2} \times dt = 1$$

Therefore:

$$\Phi(-X) = 1 - \int_{-\infty}^{X} \frac{1}{\sqrt{2\pi}} \times e^{-t^2/2} \times dt$$
$$= 1 - \Phi(X)$$