ECE5843, Medical Imaging Systems, Quiz #2
Fall 2018

Last Name: _________________ First Name: _______________

OUID: _____________________

Note:
1. Close book, no notes
2. Calculator is allowed

1. (5 points)
A liver is imaged by a CT system. Assume a mono-energetic x-ray source, for which the linear attenuation coefficients of water and liver are: $\mu_{\text{water}} = 0.2\ \text{cm}^{-1}$ and $\mu_{\text{liver}} = 0.215\ \text{cm}^{-1}$ respectively. What will be the CT number of the liver under the same mono-energetic x-ray source?

Solution:

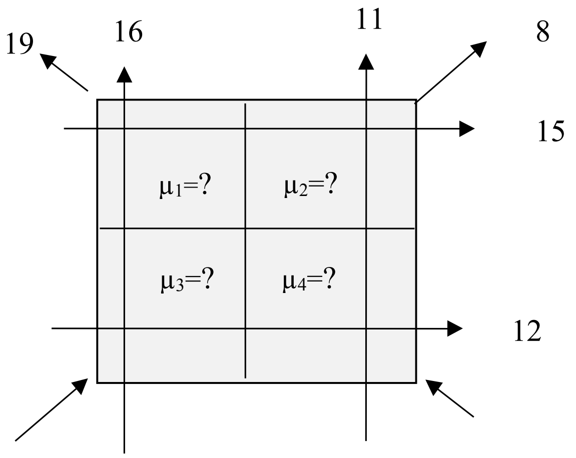
CT number = 1000 * (μ_{liver} - μ_{water}) / μ_{water}

$$CT\ number = 1000 \times \frac{\mu_{liver} - \mu_{water}}{\mu_{water}} = 1000 \times \frac{0.215 - 0.2}{0.2} = 75$$

2. (5 points)
Based on the Fourier Slice Theorem (for CT), the Fourier transform of a projection along the line at an angle ***115^0*** with respect to *x* axis (in spatial domain) corresponds to the radial line at an angle ***25^0*** with respect to *u* axis (in frequency domain).

3. (10 points)

Suppose we are imaging a body slice that consists of four voxels of unit dimension ($\Delta x=1$). The ray sums are measured and given by the following diagram. Try to determine linear attenuation coefficient of each voxel with Algebraic Reconstruction Technique (ART). Please show each step of iterations.



Solution:

Recall Algebraic reconstruction technique (ART)

$$\mu_i^{(q+1)} = \mu_i^{(q)} + \frac{P - \sum \mu_i^{(q)}}{N}$$

First Iteration (Vertical)

Initial assumption: $\mu_1^{(0)} = \mu_2^{(0)} = \mu_3^{(0)} = \mu_4^{(0)} = 0$

$$P = 16 \Rightarrow \quad \mu_1^{(1)} = \mu_1^{(0)} + \frac{P - \left[\mu_1^{(0)} + \mu_3^{(0)}\right]}{2} = 0 + \frac{16 - [0+0]}{2} = 8$$

$$P = 11 \Rightarrow \quad \mu_2^{(1)} = \mu_2^{(0)} + \frac{P - \left[\mu_2^{(0)} + \mu_4^{(0)}\right]}{2} = 0 + \frac{11 - [0+0]}{2} = 5.5$$

$$P = 16 \Rightarrow \quad \mu_3^{(1)} = \mu_3^{(0)} + \frac{P - \left[\mu_1^{(0)} + \mu_3^{(0)}\right]}{2} = 0 + \frac{16 - [0+0]}{2} = 8$$

$$P = 11 \Rightarrow \quad \mu_4^{(1)} = \mu_4^{(0)} + \frac{P - \left[\mu_2^{(0)} + \mu_4^{(0)}\right]}{2} = 0 + \frac{11 - [0+0]}{2} = 5.5$$

Second Iteration (Horizontal)

Assumption from 1st iteration: $\mu_1^{(1)} = 8;\ \ \mu_2^{(1)} = 5.5;\ \ \mu_3^{(1)} = 8;\ \ \mu_4^{(1)} = 5.5$

$$P = 15 \Rightarrow \quad \mu_1^{(2)} = \mu_1^{(1)} + \frac{P - \left[\mu_1^{(1)} + \mu_2^{(1)}\right]}{2} = 8 + \frac{15 - [8+5.5]}{2} = 8.75$$

$$P = 15 \Rightarrow \quad \mu_2^{(2)} = \mu_2^{(1)} + \frac{P - \left[\mu_1^{(1)} + \mu_2^{(1)}\right]}{2} = 5.5 + \frac{15 - [8+5.5]}{2} = 6.25$$

$$P = 12 \Rightarrow \quad \mu_3^{(2)} = \mu_3^{(1)} + \frac{P - \left[\mu_3^{(1)} + \mu_4^{(1)}\right]}{2} = 8 + \frac{12 - [8+5.5]}{2} = 7.25$$

$$P = 12 \Rightarrow \quad \mu_4^{(2)} = \mu_4^{(1)} + \frac{P - \left[\mu_3^{(1)} + \mu_4^{(1)}\right]}{2} = 5.5 + \frac{12 - [8+5.5]}{2} = 4.75$$

Third Iteration (Diagonal)

Assumption from 2nd iteration: $\mu_1^{(2)} = 8.75;\ \ \mu_2^{(2)} = 6.25;\ \ \mu_3^{(2)} = 7.25;\ \ \mu_4^{(2)} = 4.75$

$$P = 19 \Rightarrow \quad \mu_1^{(3)} = \mu_1^{(2)} + \frac{P - \left[\mu_1^{(2)} + \mu_4^{(2)}\right]}{2} = 8.75 + \frac{19 - [8.75+4.75]}{2} = 11.5$$

$$P = 8 \Rightarrow \quad \mu_2^{(3)} = \mu_2^{(2)} + \frac{P - \left[\mu_2^{(2)} + \mu_3^{(2)}\right]}{2} = 6.25 + \frac{8 - [6.25+7.25]}{2} = 3.5$$

$$P = 8 \Rightarrow \quad \mu_3^{(3)} = \mu_3^{(2)} + \frac{P - \left[\mu_2^{(2)} + \mu_3^{(2)}\right]}{2} = 7.25 + \frac{8 - [6.25+7.25]}{2} = 4.5$$

$$P = 19 \Rightarrow \quad \mu_4^{(3)} = \mu_4^{(2)} + \frac{P - \left[\mu_1^{(2)} + \mu_4^{(2)}\right]}{2} = 4.75 + \frac{19 - [8.75+4.75]}{2} = 7.5$$

Therefore, we have: $\mu_1=11.5$; $\mu_2=3.5$; $\mu_3=4.5$; $\mu_4=7.5$

4. (10 points)

A nuclear medicine technologist injects a patient with 500μCi of In-111 ($Tp_{1/2}$=2.82 days) labeled autologous platelets. Assuming none of the activity was biologically excreted. The technologist is supposed to image the patient when the activity is at 451μCi. When should the technologist image the patient (how many hours after the injection)?

Solution:

Given:

$A = A_o\, e^{-\lambda t}$

A_o =500 μCi

A =451 μCi

$Tp_{1/2}$=2.82 days = 67.68 (hours)

$\lambda = 0.693/(67.68 \text{ hours}) = 1.02\times10^{-2} (\text{hour}^{-1}) = 0.0102$ /hour

Therefore:

$451 = 500 \times e^{-(0.0102\,/\,\text{hour})\times t}$

t = ln(451/500) / (-0.0102) = 10 (hours)

5. (10 points)

The physical half-life of a radionuclide is 12 hours. It was used to label a pharmaceutical. If the effective half-life of the radiopharmaceutical is 4 hours for a clinical procedure, then what is the biological half-life of the patient?

Solution:

$T_{p1/2}$ = 12 hours

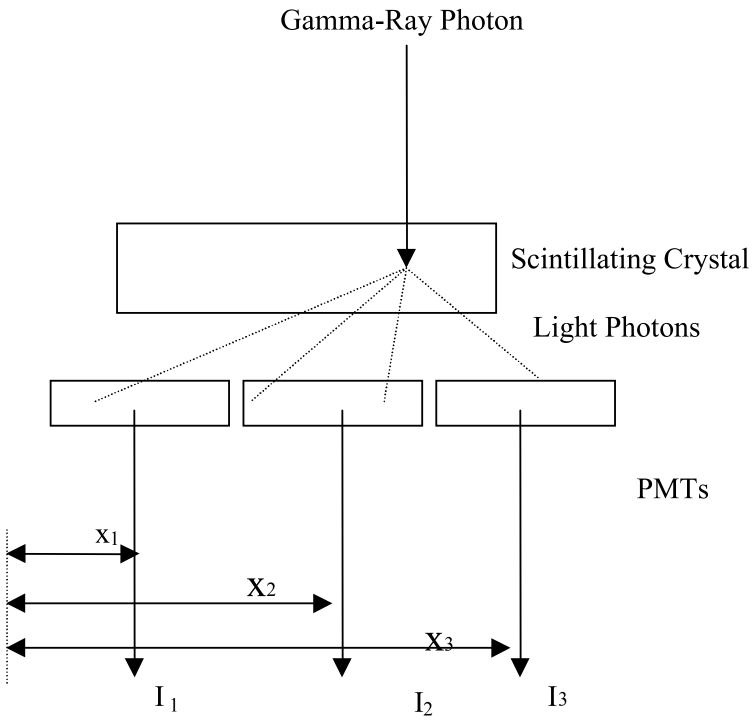
$T_{eff\,1/2}$ = 4 hours

$$\frac{1}{T_{eff\,1/2}} = \frac{1}{T_{b1/2}} + \frac{1}{T_{p1/2}}$$

$$\frac{1}{T_{b1/2}} = \frac{1}{T_{eff\,1/2}} - \frac{1}{T_{p1/2}} = \frac{1}{4} - \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$$

$T_{b1/2} = 6 \quad hours$

6. (10 points)
A Gamma camera is built with a scintillation crystal and three PMTs, as shown in the following diagram. When a gamma ray photon excites the scintillation crystal, the amplitude of the voltage pulse produced by any of the PMT tube is given in the following table. Please using centroid calculation formula to estimates the most likely position, within the crystal, of the gamma ray interaction.



Location of PMTs	PMT output
$X_1 = 1$cm	100 mV
$X_2 = 5$cm	1200 mV
$X_3 = 12$cm	500 mV

Solution:

$$X = \frac{I_1X_1 + I_2X_2 + I_3X_3}{I_1 + I_2 + I_3} = \frac{1\times100 + 5\times1200 + 12\times500}{100+1200+500} = 6.72cm$$

7. (5 points)
Two ultrasound signals differs in intensity by 20 dB, what is the ratio of their intensities?

Solution:

$$dB = 10\log_{10}\left(\frac{I_1}{I_0}\right)$$

So:

$$20dB = 10\log_{10}\left(\frac{I_1}{I_0}\right)$$

$$\Rightarrow \frac{I_1}{I_0} = 10^2 = 100$$

8. (5 points)

It is known that the wavelength of 2.5 MHz ultrasound in fat is 0.58 mm.

(a) What is the wavelength of 1 MHz ultrasound in fat?

(b) What is the wavelength of 10 MHz ultrasound in fat?

Solution:

$C = \lambda \times \nu = 0.58 \text{ mm} \times 2.5\times10^6 \text{ /s} = 1.45\times10^6 \text{ mm/s} = 1450 \text{ m/s}$

The speed of the 1 MHZ and 10 MHz ultrasound in soft tissue is same, 1450 m/s.

(a) The wavelength of 1 MHz ultrasound in soft tissue is:

$\lambda = C / \nu = 1450 \text{ (m/s)} / (1\times10^6 \text{/s}) = 1.45 \times10^{-3} \text{ (m)} = 1.45 \text{ mm}$

(b) The wavelength of 10 MHz ultrasound in soft tissue is shorter:

$\lambda = C / \nu = 1450 \text{ (m/s)} / (10\times10^6 \text{/s}) = 0.145 \times10^{-3} \text{ (m)} = 0.145 \text{ mm}$

9. (10 points)

The exponential attenuation with depth in a homogeneous medium for ultrasound beam can be expressed as:

$$I = I_0 e^{-2\mu x}$$

where: μ is attenuation coefficient, *x* is depth.

It is a common practice to characterize attenuation of ultrasound beam in terms of decibels of intensity loss per cm of tissue (dB/cm). Please mathematically prove that the relationship between μ (1/cm) and dB /cm is:

$$dB / cm = -8.686\mu$$

Solution:

The depth (thickness) of the tissue is *x, and* μ is attenuation coefficient

Therefore, total intensity loss per cm of tissue in terms of dB is:

$$dB = (dB / cm) \times x$$

$$dB = 10\,log_{10}\left(\frac{I}{I_0}\right) = 10\,log_{10}\left(e^{-2\mu x}\right)$$

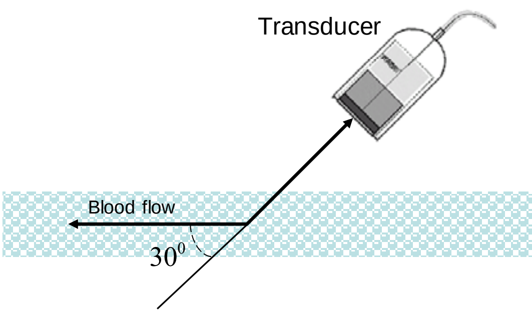
$$= -2 \times 10 \times \mu x \times log_{10} e$$

$$= -8.686\mu x$$

$$\therefore dB / cm = dB / x = -8.686\mu$$

10. (10 points)

An 8 MHz ultrasound pulse Doppler flow meter is used to monitor blood flow in a carotid artery. The probe is marking a 30^0 angle with respect to the direction of flow. Suppose the speed of the blood flow in the artery is uniform throughout the lumen and is approximately 45 cm/sec. Please calculate the Doppler frequency shift. Ultrasound speed in tissue is $1.54\times10^3 m/\sec$.



Solution:

$$\theta = 180^0 - 30^0 = 150^0$$

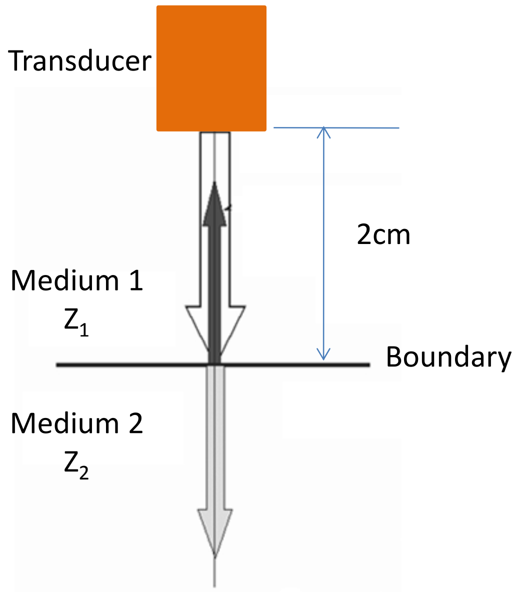
$$\Delta f = f_r - f_i = \frac{2 f_i V \cos\theta}{C} = \frac{2\times8\times10^6\times0.45\times\cos150^0}{1540} = -4.05\times10^3 (Hz)$$

11. (10 points)

As shown by the figure, two media are with Z values of $\mathbf{Z_1=1.48\times10^6\ kg/(m^2\,sec)}$ and $\mathbf{Z_2=0.18\times10^6\ kg/(m^2\,sec)}$ respectively.

A transducer emits an ultrasound beam of 2 MHz with an intensity of **10 mW/cm²**. Assume a **100%** beam transmittance when the beam enters the medium 1. It travels a distance of **2cm** to reach normally on the flat boundary between medium 1 and medium 2. The ultrasound beam has a **0.5 dB/cm per MHz** overall intensity attenuation while traveling through the medium 1.

What is the intensity transmitted into medium 2?



Solution:

The transducer emits an ultrasound beam intensity is **10mW/cm²**, the **100%** beam intensity transmittances when the beam enters the medium 1:

$$I_0=10\text{mW/cm}^2\times 100\% =10\text{mW/cm}^2,$$

The 2 MHz ultrasound beam for **0.5dB/cm per MHz** is:

$$-0.5\text{dB/cm per MHz}\times 2\text{ MHz}=-1\text{ dB/cm}$$

Therefore, the loss in dB when ultrasound beam travel **2cm** to reach a flat boundary

$$-1\text{dB/cm}\times 2\text{cm} =-2\text{ dB}$$

The incident intensity of ultrasound reaching boundary is:

$$-2dB=10\times\log_{10}\left(\frac{I_1}{I_0}\right)$$

$$\log_{10}\left(\frac{I_1}{I_0}\right)=-0.2$$

$$\Rightarrow I_1=I_0\times10^{-0.2}=10\times10^{-0.2}=6.3(mW/cm^2)$$

For normal incidence:

$\mathbf{R}=[(Z_2-Z_1)/(Z_2+Z_1)]^2$
$=(0.18\times10^6-1.48\times10^6)^2/(0.18\times10^6+1.48\times10^6)^2$
$=61.33\%$

$\mathbf{T}=1-\mathbf{R}=1-61.33\%=38.67\%$

The intensity transmitted into the medium 2, I_T is,

$$I_T=T\times I_1=6.3\times38.67\%=2.436(mW/cm^2)$$

12. (10 points)
This problem is related to the CT reconstruction technique (Fourier Transform approach). Assuming that a series of projections from 0^0 to 180^0 at 1^0 increments; resulted a Sinogram that can be expressed as:

$$P_\theta(t) = 2\Lambda(t) + rect(t)$$

Based on Fourier slice theorem, please determine the mathematical expression of the function in Fourier Domain at $\theta=15^0$.

Solution:

$$F(u,v)\big|_{\theta=15^0} = FT_{1D}[P_{\theta=15^0}(t)]$$

$$= FT_{1D}[2\Lambda(t) + rect(t)]$$

$$= FT_{1D}[2\Lambda(t)] + FT_{1D}[rect(t)]$$

$$= 2\mathrm{sinc}^2(\pi\omega) + \mathrm{sinc}(\pi\omega)$$

$$= 2\mathrm{sinc}^2(\pi\sqrt{u^2+v^2}) + \mathrm{sinc}(\pi\sqrt{u^2+v^2})$$

The following table may be helpful.

Frequently used functions and their Fourier transforms

Function	**Fourier Transform**
$\sin 2\pi x$	$\frac{1}{2i}\left[\delta(u-1)-\delta(u+1)\right]$
$\cos 2\pi x$	$\frac{1}{2}\left[\delta(u-1)+\delta(u+1)\right]$
$\exp^{[i\pi(x+y)]}$	$\delta(u-\frac{1}{2},v-\frac{1}{2})$
$\exp^{(-\pi r^2)}$	$\exp^{(-\pi \rho^2)}$
$\delta(x)$	1
$rect(x)=\begin{cases}1 & \lvert x\rvert \le \frac{1}{2}\\ 0 & otherwise\end{cases}$	$\sin c(\pi u)=\frac{\sin \pi u}{\pi u}$
$\Lambda(x)=\begin{cases}1-\lvert x\rvert & \lvert x\rvert \le 1\\ 0 & otherwise\end{cases}$	$\sin c^2(\pi u)=\left(\frac{\sin \pi u}{\pi u}\right)^2$
$comb(x)$	$comb(u)$