ECE5843, Medical Imaging Systems, Quiz #1
Fall 2018

Last Name: ___________________ First Name: ________________

OUID ________________________

Note:

(1) Close book, no notes
(2) Calculator is allowed

1. (10 points)
An object was imaged twice using the same x-ray machine. The first image was taken under: 100mA, 30 KVp, and 1 second exposure time. The lesion's signal-to-noise ratio was determined as SNR_1=12. (Assume that the subject contrast is 10% under 30 KVp). The second image was taken under: 25mA, 60 KVp, 4 seconds exposure time. What would be the SNR_2? (Assume that the subject contrast is 7% under 60 KVp).

Solution:

The $SNR = C\sqrt{N}$
Then $SNR_1 = 0.1 \times \sqrt{N_1}$ and $SNR_2 = 0.07 \times \sqrt{N_2}$

Since: $N \propto (\text{Exposure time}) \times (\text{Current}) \times (\text{KV}_p)^2$
We have:
First exposure: $N_1 \propto (1s) \times (100\text{mA}) \times (30)^2$
Second exposure: $N_2 \propto (4s) \times (25\text{mA}) \times (60)^2$
So: $N_2 = 4 N_1$

$$\frac{SNR_2}{SNR_1} = \frac{0.07 \times \sqrt{N_2}}{0.1 \times \sqrt{N_1}} = \frac{0.07 \times \sqrt{4N_1}}{0.1 \times \sqrt{N_1}} = 1.4$$

$SNR_2 = 1.4 \times SNR_1 = 1.4 \times 12 = 16.8$

2. (5 points)
The MTF curve of an X-ray imaging system can be obtained by (Select all that apply):

(A) Applying two dimensional Fourier transform to the line spread function (LSF);
(B) Applying one dimensional Fourier transform to the line spread function (LSF);
(C) Applying two dimensional Fourier transform to the point spread function (PSF)
(D) Applying one dimensional Fourier transform to the point spread function (PSF)

3. (5 points)
Under a specific x-ray beam, the HVL measurement shows a 1.5 mm Aluminum. Please calculate the percentage transmittance of 4.0 mm thick Aluminum under the same x-ray quality.

Solution:

$$N = N_0 e^{-\mu x}$$

Step one: when x=HVL=1.5mm, $N=0.5\times N_0$, $N_0/N=2$, determine μ

$$\mu = \frac{\ln(N_0 / N)}{x} = \frac{\ln(2)}{HVL} = \frac{0.693}{1.5mm} = 0.462\,(mm^{-1})$$

Step two: the percentage transmittance at a thickness of 4.0mm

$$T = \frac{N}{N_0} = e^{-0.462\times 4.0} = 0.158 = 15.8\%$$

4. (5 points)
The inverse square law has a very practical use in radiography. Suppose an X-ray image was taken using 15mAs at 80 KVp with a 1.0 m source to object distance (SID_1=1.0m). Now you are asked to image the object again but with a 2.0 m source to object distance (SID_2=2.0 m), also at 80 KVp. What mAs setting should be used in order to yield the same exposure to the object?

Solution:
The intensity at the detector should remain constant in order to have the same exposure. We determine that:

$$\frac{mAs(new)}{SID_2^2} = \frac{mAs(old)}{SID_1^2}$$

$$mAs(new) = \frac{SID_2^2}{SID_1^2} \times mAs(old)$$

$$= 15mAs \times \frac{2^2}{1^2} = 60mAs$$

5. (5 points)
A system is consisted by three components: a scintillating screen, a film and an x-ray tube. At 6 *lp*/mm, the MTF of the scintillating screen equals to 0.5; the MTF of the film equals to 0.8, and the MFT of the overall system is 0.32. What is the MTF of the X-ray tube at 6 *lp*/mm?

Solution:

Overall MTF (6 lp/mm) = 0.5 × 0.8 × ? = 0.32,

So, the MTF of the X-ray tube is 0.8 at 6 *lp*/mm

6. (5 points)
The spatial resolution of a screen-film based radiography system can be improved by changing the following technological parameter (Select only one answer):

(A) Lower tube voltage

(B) Use thicker screen

(C) Use thinner screen

(D) Use higher mAs

7. (5 points)
The advantages of digital radiography as compared with the screen-film based radiography is: (Select only one answer)

(A) Higher spatial resolution

(B) Less scattered radiation

(C) Larger fields of view

(D) Wider dynamic range

(E) Lower equipment cost

8. (7.5 points)
Assuming that 1000 x-ray photons, each with an energy of 100 KeV, enter to an intensifying screen. The screen is with an absorption efficiency (quantum efficiency) of 50% and an intrinsic efficiency of 20%.

(a) How many x-rays photons will contribute to the image?

Solution:

$$1000\,(\text{x - ray photons})\ \times 50\% = 500\,(\text{x - ray photons})$$

(b) Assuming that the average energy of a blue light photon is 3eV, how many blue light photons will be produced for each x-ray photon absorbed?

Solution:

$$\frac{100KeV}{3eV} \times 20\% = 33333 \times 20\% \approx 6667\,(\text{light photons})$$

9. (7.5 point)
A direct flat panel detector system was built with 60% overall quantum efficiency for x-ray imaging in the diagnostic energy range. It has a 75% fill factor. The same type of the direct flat panel detector system was re-designed to offer higher spatial resolution. It now has a 50% fill factor. Please determine the overall quantum efficiency of the second system for x-ray imaging in the same diagnostic energy range.

Solution:

The quantum efficiency of the detector material:

= the overall quantum efficiency / detector fill factor

= (60%) / (75%)

= 80%

For the detector system with 50% fill factor,

The overall quantum efficiency = 80% × 50%

= 40%

10. (10 points)
The diameter of the input screen of an image intensifier tube is 36 cm, and the diameter of the output screen is 2.5 cm. Assume that a 50KeV x-ray photon is absorbed by the input screen and some 20% of the absorbed photon energy is re-emitted from the input screen in the form of 4000 visible light photons. These together may cause the ejection of a total of about 120 electrons from the adjacent photocathode. Each such electron acquires enough kinetic energy on the way to the anode, and produces 1500 or so visible light photons at the output screen.

(a) What is the minification gain of the tube?

Solution:

$$\text{Minification gain} = \left(\frac{d_i}{d_0}\right)^2 = (36 / 2.5)^2 = 207.36$$

(b) What is the electronic gain of the tube?

Solution:

The # of light photons from input screen is : 4000 light photons

The # of light photons from output screen is: 120 (electrons) × 1500 (light photons / electron)

=120 × 1500= 180,000 (light photons)

Electronic gain =180,000 / 4000 = 45

(c) What is the overall brightness gain of the tube?

Solution:

The overall brightness gain = (electronic gain) × (minification gain)
= 207.36 × 45 = 9331.2

11. (7.5 points)
A specific type of fluid is flowing through a plastic pipe. The diameter of the pipe is known.

(a) Please propose a method to monitor the "quality" of the fluid (assuming that the linear attenuation coefficient can be used as an indicator of the quality of the fluid).

Solution:

Using digital subtraction radiography technique:

First, take an x-ray measurement while no fluid in pipe, record the result as $I_{o;}$
Second, take an x-ray exposure while fluid in pipe, record the result as I_f
Calculate the linear attenuation coefficient with following formula:

$$\mu = \frac{\ln(I_0) - \ln(I_f)}{x}$$

(b) Validate your method mathematically.

Solution:

(1) while no fluid in pipe: $I_0 = I_0$

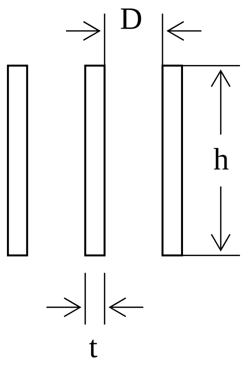
(2) while fluid in pipe: $I_f = I_0 e^{-\mu x}$

where: μ : linear attenuation coefficient of fluid; x : thickness of pipe that fluid is in.

$$\ln(I_0) - \ln(I_f) = \ln(I_0) - \ln(I_0 e^{-\mu x})$$
$$= \ln I_0 - \ln I_0 - \ln(e^{-\mu x}) = \mu x$$

$$\therefore \ln(I_0) - \ln(I_f) = \mu x$$
$$\mu = \frac{\ln(I_0) - \ln(I_f)}{x}$$

12. (7.5 points)
The dimensions of a linear anti-scatter grid is given as follows: *D* is 0.10mm, *h* is 0.8mm.



(a) What is the grid ratio?

Solution:

$$Grid\ Ratio = \frac{h}{D} = \frac{0.8mm}{0.1mm} = 8:1$$

(b) For a specific clinical procedure, a 120KVp, 4mAs exposure provides an acceptable signal with no grid. What mAs setting should be used when the above grid is employed in the procedure at the same 120KVp? (hint: the following table may be helpful):

Grid Ratio	No Grid	2:1	4:1	8:1	12:1	16:1
Bucky factor (70KVp)	1	1.1	2.7	3.5	4.0	4.5
Bucky factor (95KVp)	1	1.1	2.7	3.8	4.3	5.0
Bucky factor (120KVp)	1	1.1	2.7	4.0	5.0	6.0

Solution:

At 120KVp, with 8:1 grid ratio, the Bucky factor is 4.0 according to the above table, therefore, the needed mAs setting is determined as follows:

$$4.0 \times 4mAs = 16.0mAs$$

13. (10 points)

The quantum noise in x-ray imaging obeys Poisson distribution:

$$P_K = \frac{N^K e^{-N}}{K!}$$

where: $\boldsymbol{P_K}$ is probability, in a given time interval, of emitting $\boldsymbol{K}$ photons from an x-ray source, $\boldsymbol{N}$ is the average number of photons emitted during that interval, and $\boldsymbol{K!}$ is the factorial of non-negative integer.

Please prove that in Poisson process, the variance is given as: $\boldsymbol{\sigma^2 = N}$.

Hint: the following may be helpful:

$$\sum_{K=0}^{\infty} K^2 \times P_K = N^2 + N$$

$$N = \sum_{K=0}^{\infty} K \times P_K$$

$$\sum_{K=0}^{\infty} P_K = 1$$

Solution:

In Poisson process, the variance can be determined:

$$\begin{aligned}
\sigma^2 &= \sum_{K=0}^{\infty} (K - N)^2 \times P_K = \sum_{K=0}^{\infty} (K^2 - 2KN + N^2) \times P_K \\
&= \sum_{K=0}^{\infty} K^2 \times P_K - 2N \times \sum_{K=0}^{\infty} K \times P_K + N^2 \times \sum_{K=0}^{\infty} P_K \\
&= N^2 + N - 2N \times N + N^2 \\
&= N^2 + N - 2N^2 + N^2 \\
&= N
\end{aligned}$$

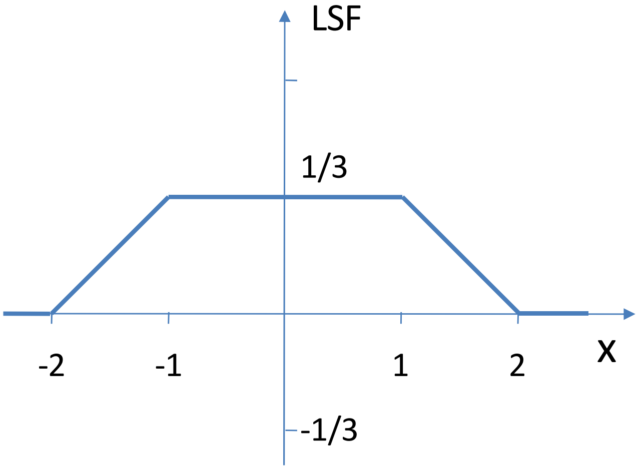
Therefore, we have shown that in Poisson process, the variance is given as $\boldsymbol{\sigma^2 = N}$.

14. (10 points)

An x-ray imaging system is linear and stationary, it has a line spread function (LSF) given as follows:

$$LSF(x) = \frac{1}{3} \times \left[2\Lambda(\frac{x}{2}) - \Lambda(x) \right]$$

What is the modulation transfer function, **MTF(*u*)** of this system?



Solution:

$$LSF(x) = \frac{1}{3} \times \left[2\Lambda(\frac{x}{2}) - \Lambda(x) \right]$$

$$\int \Lambda(x) e^{-i2\pi ux} dx = \mathrm{sinc}^2(\pi u) = \left(\frac{\sin \pi u}{\pi u} \right)^2$$

$$MTF(u) = \int LSF(x) e^{-i2\pi ux} dx = \frac{1}{3} \times \int \left[2\Lambda(\frac{x}{2}) - \Lambda(x) \right] e^{-i2\pi ux} dx$$

$$= \frac{1}{3} \times \left[4\mathrm{sinc}^2(2\pi u) - \mathrm{sinc}^2(\pi u) \right]$$

The following table may be helpful.

Frequently used functions and their Fourier transforms

Function	**Fourier Transform**
$\sin 2\pi x$	$\frac{1}{2i}\left[\delta(u-1)-\delta(u+1)\right]$
$\cos 2\pi x$	$\frac{1}{2}\left[\delta(u-1)+\delta(u+1)\right]$
$\exp^{[i\pi(x+y)]}$	$\delta(u-\frac{1}{2},v-\frac{1}{2})$
$\exp^{(-\pi r^2)}$	$\exp^{(-\pi \rho^2)}$
$\delta(x)$	1
$rect(x)=\begin{cases}1 & \lvert x\rvert \le \frac{1}{2} \\ 0 & otherwise\end{cases}$	$\sin c(\pi u)=\frac{\sin \pi u}{\pi u}$
$\Lambda(x)=\begin{cases}1-\lvert x\rvert & \lvert x\rvert \le 1 \\ 0 & otherwise\end{cases}$	$\sin c^2(\pi u)=\left(\frac{\sin \pi u}{\pi u}\right)^2$
$comb(x)$	$comb(u)$