**S-1**: The quantum noise in x-ray imaging obeys Poisson distribution:

$$P_K = \frac{N^K e^{-N}}{K!}$$

where: $P_K$ is probability, in a given time interval, of emitting K photons from an x-ray source, N is the average number of photons emitted during that interval, and K! is the factorial of non-negative integer.

Please prove that in Poisson process, the variance is given as: $\sigma^2 = N$.

**Hint: the following may be helpful:**

$$\sigma^2 = \sum_{K=0}^{\infty} (K-N)^2 \times P_K$$

$$\sum_{K=0}^{\infty} K^2 \times P_K = N^2 + N$$

$$N = \sum_{K=0}^{\infty} K \times P_K$$

**Solution:**

In Poisson process, the variance can be determined:

$$\begin{aligned}
\sigma^2 &= \sum_{K=0}^{\infty} (K-N)^2 \times P_K = \sum_{K=0}^{\infty} (K^2 - 2KN + N^2) \times P_K \\
&= \sum_{K=0}^{\infty} K^2 \times P_K - 2N \times \sum_{K=0}^{\infty} K \times P_K + N^2 \times \sum_{K=0}^{\infty} P_K \\
&= N^2 + N - 2N \times N + N^2 \\
&= N^2 + N - 2N^2 + N^2 \\
&= N
\end{aligned}$$

Note:

$$\sum_{K=0}^{\infty} P_K = 1$$

Therefore, we have shown that in Poisson process, the variance is given as $\sigma^2 = N$.