Handout #9

Magnetic Resonance Imaging

Part (I) Nuclear Magnetic Resonance (NMR)

9.1 NMR and MRI

(1) NMR: Nuclear magnetic resonance is a spectroscopic study of the magnetic properties of the nucleus of the atom.

- The protons and neutrons of the nucleus have a magnetic field associated with their nuclear spin and charge distribution. Resonance is an energy coupling and causes the individual nuclei, when placed in a strong external magnetic field, selectively to absorb and at a later time release energy unique to the nuclei and their surrounding environment.
- NMR is not an imaging technique.

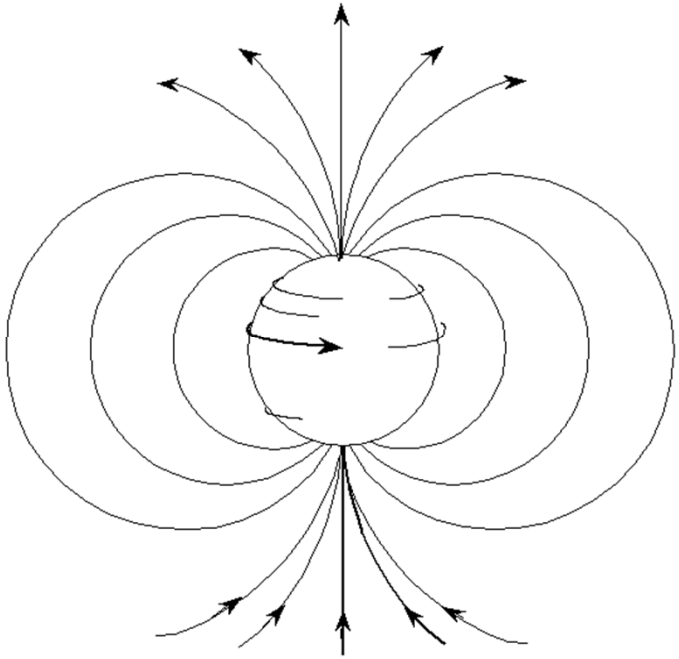
(2) MRI: Magnetic resonance imaging.

- Magnetic field gradients could be used to localize the NMR signal and to generate images that display nuclear properties reflecting clinically relevant information.
- **Magnetic field gradients** are precisely controlled magnetic fields that linearly vary over a predefined field of view (FOV) with positive and negative polarity.
- Positive gradient polarity means that the gradient values add to the main magnetic field; negative polarity implies a reduction in magnetic field (B_0).

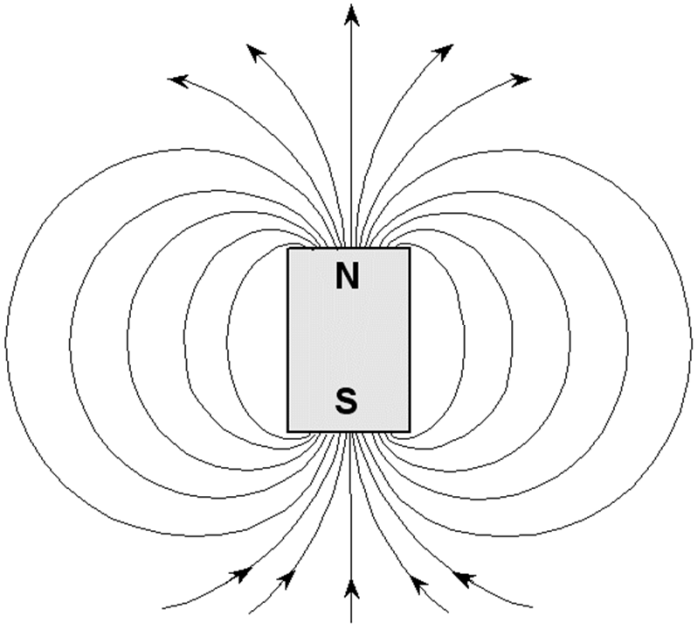
9.2 Physics Backgrounds

(1) Protons

- The **nucleus** of the atom is comprised of **protons** (with positive charge) and **neutrons** (zero charge).
- The positive charge of proton give rise to **a magnetic field** called **a magnetic dipole,** as a result of its **"spin"** and charge motion.



Spinning proton with dipole magnetic field



Bar magnet

(2) Magnetic moment

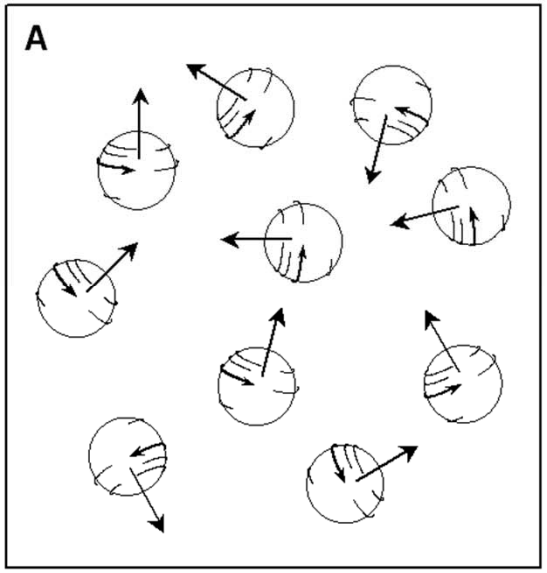
- The **magnetic moment** describes the strength and orientation of a magnetic dipole, and is represented as a vector since it possesses a magnitude and direction.

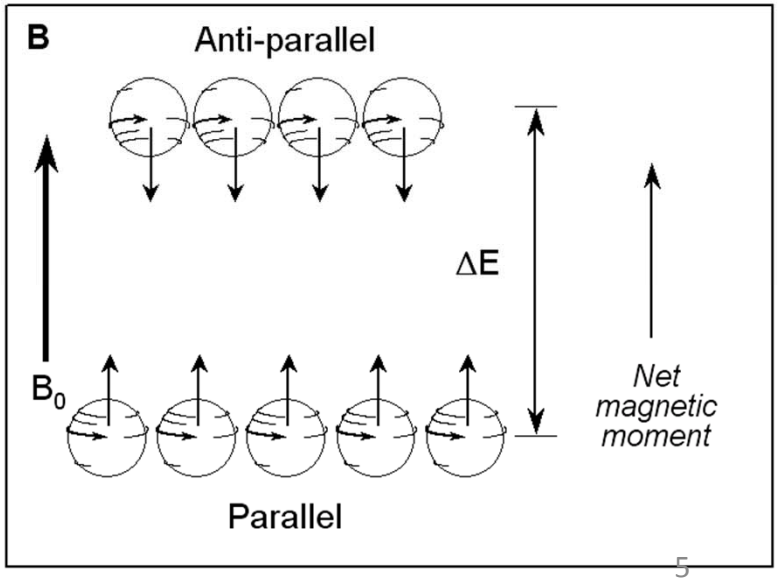
- **Longitudinal** magnetization is the vector component of the magnetic moment in **z** direction.
- **Transverse** magnetization is the vector component of the magnetic moment in **x-y** plane.

❖ There are large number (about 10^{21}) protons per cm^3 of tissue.

a) **Without an external magnetic field**, the protons assumes a random orientation of the individual magnetic moments, resulting an overall magnetic moment of zero (**no net magnetic moment exhibited by the tissue**).

b) **Under the influence of an externally applied magnetic field** (B_0), the protons become magnetized and align with the magnetic field. The protons assume a nonrandom alignment in two possible orientations: parallel and antiparallel to the direction of the applied magnetic field. A slightly greater number of protons exists in the parallel direction, and the vector addition results in a measurable **sample magnetic moment.**





(3) External magnetic field precession

- The **individual spins** (here spin and proton are considered synonymous) are also influenced by the external applied magnetic field.
- When the proton's magnetic field interacts with the external field, a force (torque) on the proton causes a **precession** about its axis, much the same way a spinning top wobbles or rotates about its axis due to the force of gravity acting upon it. This precessional motion occurs at an angular frequency (ω) proportional to **the magnetic field strength B_0**.

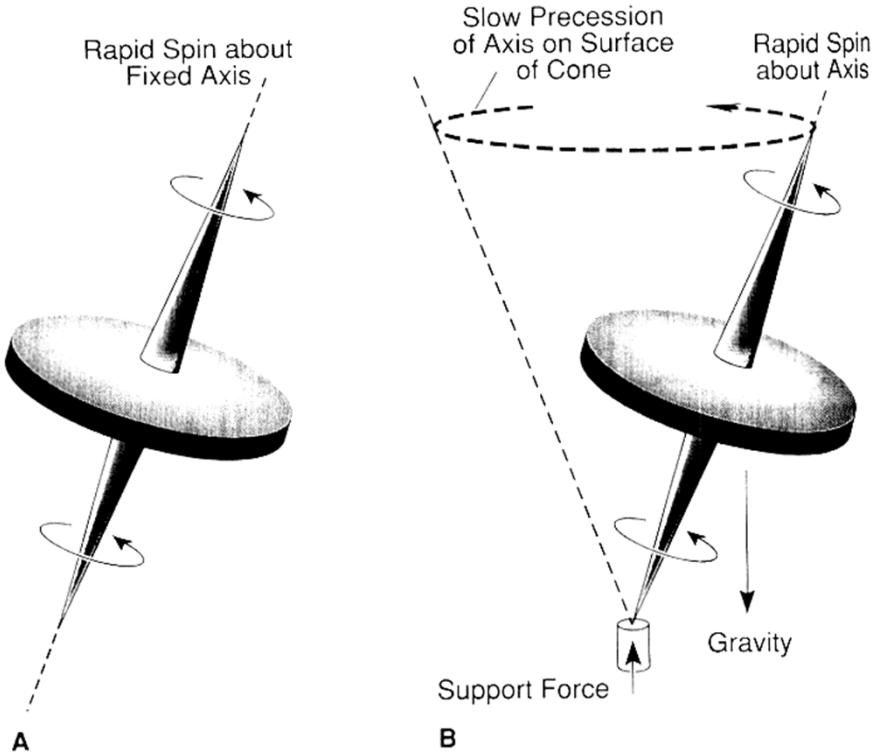


Figure: Frictionless top or gyroscope spinning rapidly.

(A) In outer space, the speed and orientation (i.e., the direction in which the axis points) remain constant.

(B) In a gravitational field, supported at the bottom, the resulting torque, or twisting force, causes the system to process at a constant, slow (relative to the rate of spin) rate and at a fixed angle of tilt from the vertical

(4) Larmor equation

The Larmor equation describes the dependency between the magnetic field and the precessional frequency:

$$\omega = \gamma B_0$$

or

$$f_0 = (\gamma / 2\pi) B_0$$

where ω: is the angular frequency of rotation,

γ: is the gyromagnetic ratio unique to each element

B_0: is the magnetic field strength in **Tesla (T),**

f_0: is the linear frequency in MHz

(γ /2π): is the gyromagnetic ratio expressed in MHz/T, it is unique to each element

1 T (SI unit) = 10,000 gauss (US unit)

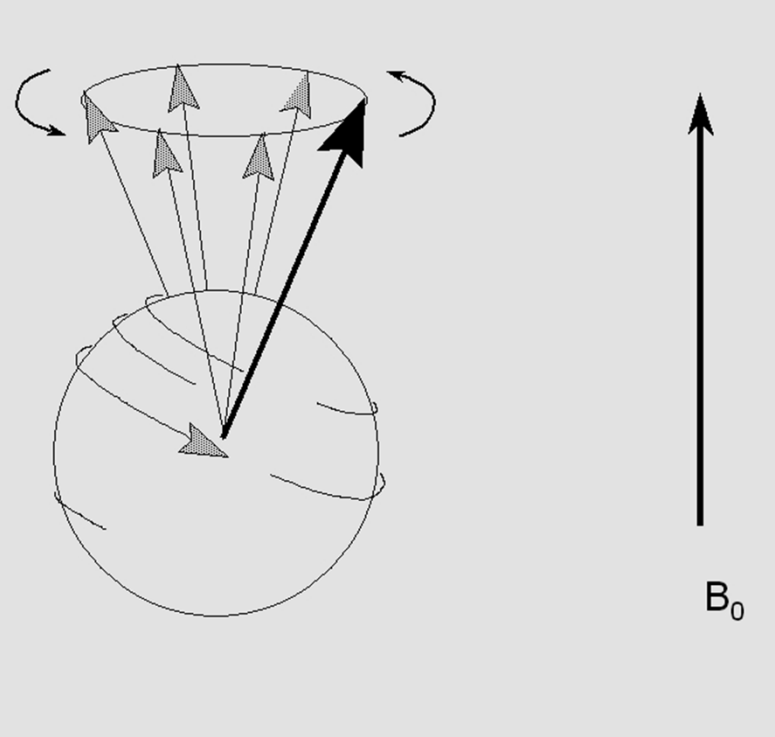


Figure: A single proton precesses about its axis with an angular frequency, **ω**, that is proportional to the externally applied magnetic field strength, according to the Larmor equation.

- The earth's magnetic field strength is about 0.5 guess (or 0.05 mT)
- Typical magnetic field strength used for MRI ranges from 0.1 to 3.0 T, (much higher for research MRI units)

Each element that exhibits a magnetic moment has a unique gyromagnetic ratio ($\gamma /2\pi$) that allows the discrimination of one element from another based upon precessional frequency in a given magnetic field strength.

Gyromagnetic ratio for elements of interest

Nucleus	$\gamma/2\pi$ (MHz/T)
^{1}H	42.58
^{13}C	10.7
^{19}F	40.1
^{23}Na	11.3
^{31}P	17.2
^{39}K	2.0

Exercise:

A sample containing ^1H is placed under the influence of an externally applied magnetic field, $B_0 = 3.0T$. Please find the precessional frequencies (linear frequency f_0 in MHz and angular frequency of rotation $\boldsymbol{\omega}$) of the protons.

Solution:

The gyromagnetic ratio $(\gamma/2\pi)$ for ^1H is 42.58MHz/T

$B_0 = 3.0T$

The linear frequency f_0:

$$f_0 = (\gamma/2\pi)\, B_0 = 42.58\text{MHz/T} \times 3.0\text{T} = 127.74\ \text{MHz}$$

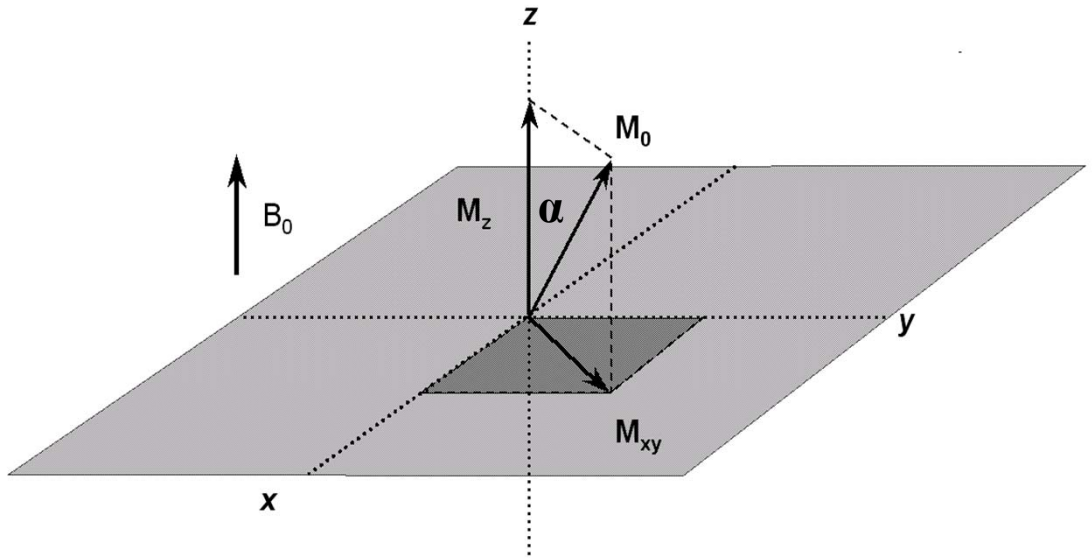
The angular frequency of rotation $\boldsymbol{\omega}$:

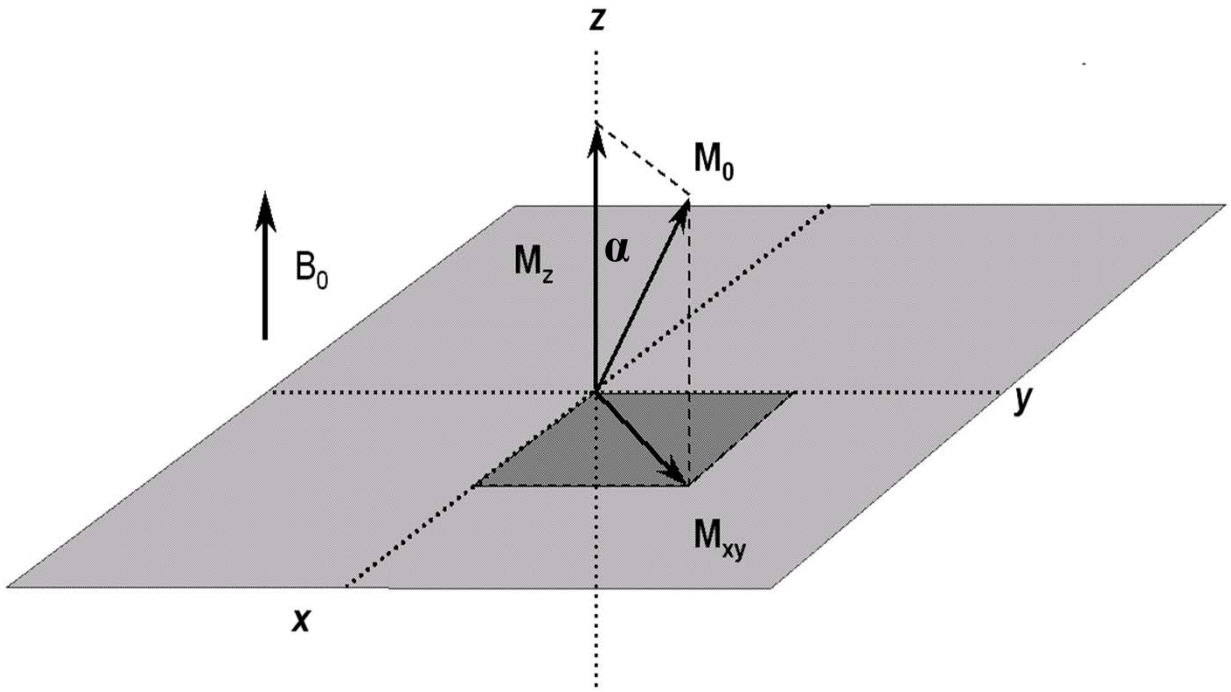
$$\omega = 2\pi f_0 = \gamma \times B_0 = 2\pi \times 42.58 \times 3.0 = 8.03 \times 10^8\ (/s)$$

- The precessional frequency allows the protons to be coupled to an external energy source (radio frequency electromagnetic waves). This coupling permits an absorption of energy and therefore **perturbation from equilibrium conditions.**
- The MR signal that is detected for imaging is produced as the perturbed system **goes back to its equilibrium state.**
- The above process is important for the generation and detection of the MR signal

9.3 More about Geometric Orientation and Equilibrium Condition

- Recall from the previous slide, that the net magnetization is represented as a vector.
- M_z is the component of the magnetic moment parallel to the applied magnetic field $\mathbf{B_0}$, and is known as **longitudinal magnetization.**
- M_{xy} is the component of the magnetic moment perpendicular to the applied magnetic field and is known as **transverse magnetization.**





- If the sample is left undisturbed in the field $\mathbf{B_0}$ for a long period of time, net magnetization will reach an equilibrium.
- At equilibrium, the longitudinal magnetization is maximal and is denoted as $\mathbf{M_0}$, the equilibrium magnetization .
- In the above illustration, $\mathbf{M_0}$ is shown displaced from the **z-axis.**

- At equilibrium, the transverse magnetization is zero, because the vector components of the spins are randomly oriented about 360^0 in the **x-y** plane and cancels each other.

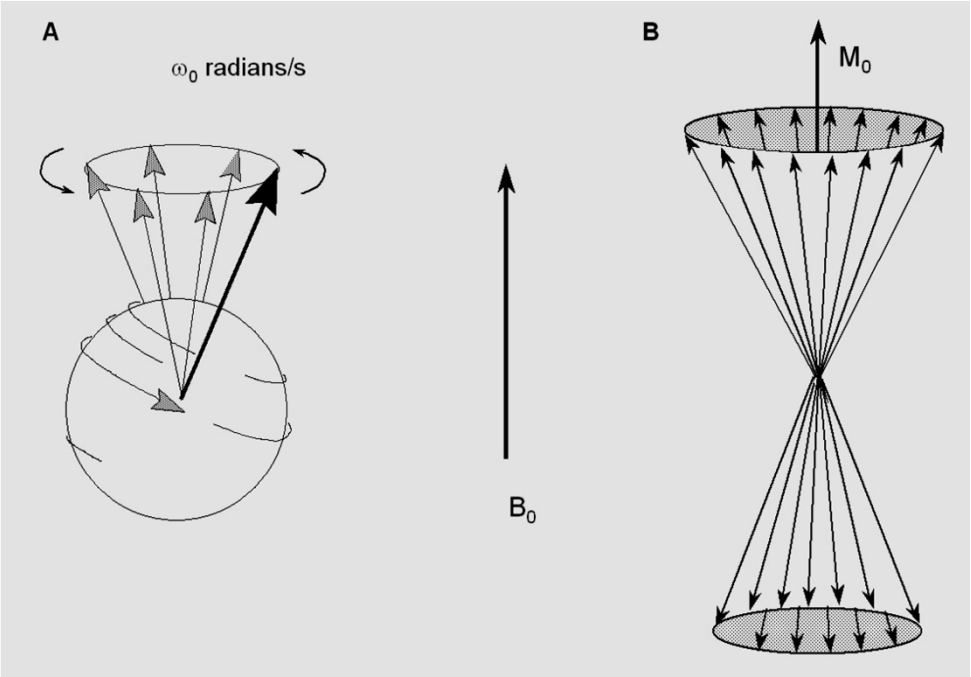


Figure: **A:** A single proton precesses about its axis with an angular frequency, ω, that is proportional to the externally applied magnetic field strength, according to the Larmor equation. **B:** A group of protons in the parallel and antiparallel energy states generates an equilibrium magnetization, M_0, in the direction of the applied magnetic field B_0. The protons are distributed randomly over the surface of the cone and produce no magnetization in the perpendicular direction.

9.4 Generation and Detection of MR Signal

(1) Basic concepts

- As described briefly earlier, the magnetic resonance signal is the result of excitation of the individual magnetized protons within the object by **irradiation with radio frequency (RF)** energy of a specific frequency.
- The displacement of the sample magnetization from equilibrium occurs when a RF pulse, also know as the B_1 field, interrogates the sample. From an RF transmitter antenna at a frequency equal to the precessional frequency of the protons. **Resonance** takes place when the energy of the **RF pulse** is precisely matched to the energy transitions between the protons in the parallel and antiparallel directions.
- This energy absorption causes the displacement of the magnetic moment from equilibrium, resulting in an **excited system.** As the system **returns to equilibrium,** MR signals are emitted in proportion to the number of the excited protons in the sample.

(2) Relaxation times (T1 and T2) and pulse sequences

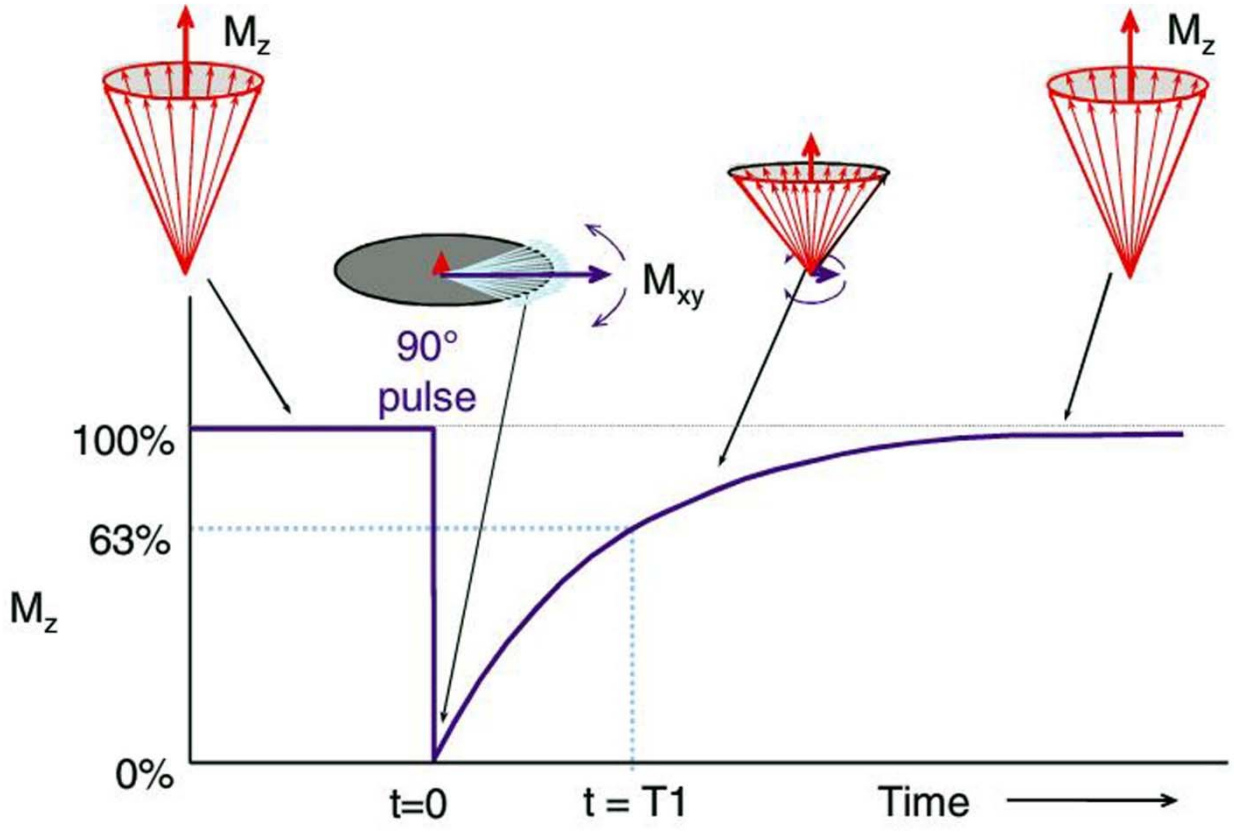
- **Longitudinal relaxation time: T1**
  The T1 is the time needed to recover 63% of the longitudinal magnetization, after a 90^0 pulse.

- **Transverse relaxation time: T2**
  Transverse relaxation is the loss of magnetization from the x-y plane. The T2 is the time over which the signal decays to 37% of the maximum transverse magnetization, after a 90^0 pulse.

- **90^0 pulse** : If we turn off the RF after magnetization has precessed down into the transverse plane, then the pulse is called a 90^0 pulse.
- **180^0 pulse :** An RF pulse twice as long as this is called a 180^0 pulse; it will place magnetization along the –z axis.

Longitudinal relaxation time: T1

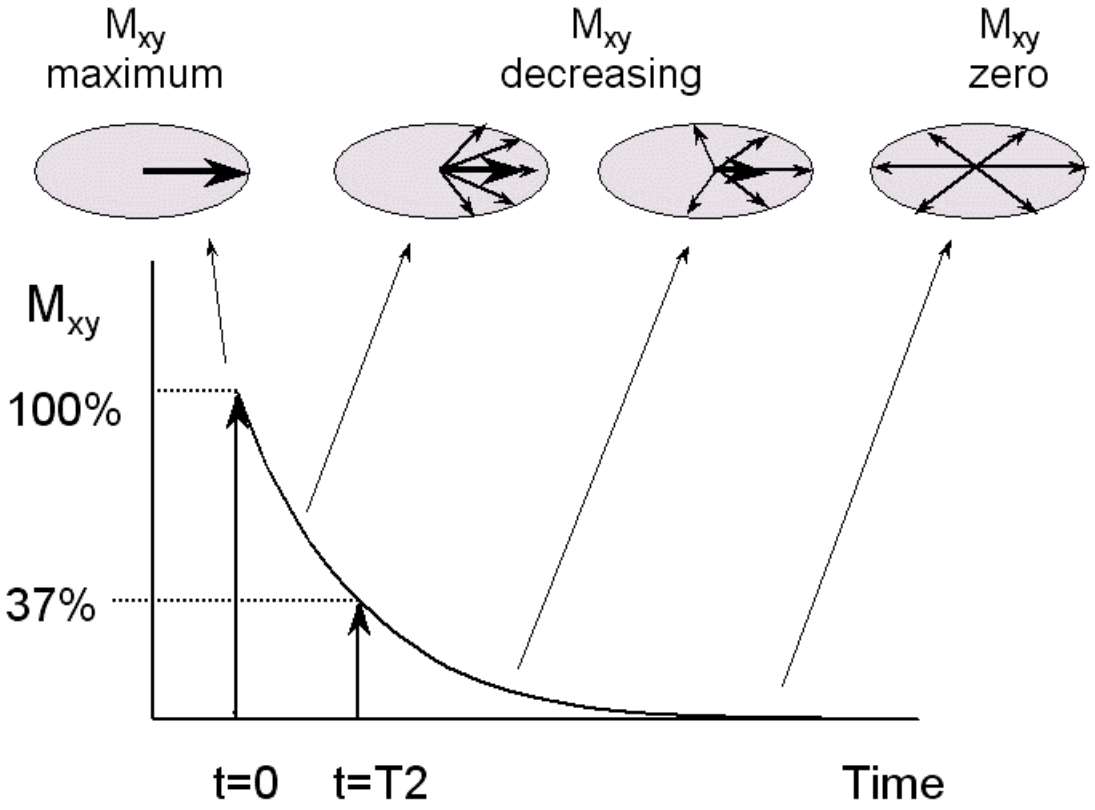
- After a 90^0 pulse, longitudinal magnetization (M_z) is converted from a maximum value at equilibrium to zero. Return of M_z to equilibrium occurs exponentially and is characterized by the spin-lattice T1 relaxation constant. After an elapsed time equal to T1, 63% of the longitudinal magnetization is recovered.



Transverse relaxation time: T2

- Transverse relaxation is the loss of magnetization from the x-y plane.

- Again, the exponential decay constant, T2, is the time over which the signal decays to 37% of the maximal transverse magnetization (e.g., after a 90^0 pulse).



❖ **Measuring T1 with the saturation-recovery pulse sequences (90^o-TR-90^o)**

- 90^o : Saturation pulse of Larmor frequency
- TR: the repetition time, during which the magnetization is allowed to relax part of the way back to its equilibrium state
- 90^o : Detection pulse at t-TR

❖ **Determining T2 with the Spin-echo-pulse sequence (90^o - 1/2 TE -180^o -1/2 TE)**

- 90^o excitation pulse
- TE: echo time
- 180^o refocusing pulse
- The NMR(echo) signal appears and is detected 1/2 TE later.

Table: Typical values of the relaxation times T1 and T2 for various Tissues and field strengths

Tissue	T1 (0.5T) (ms)	T1 (1.5T) (ms)	T2 (ms)
Adipose	210	260	80
Liver	350	500	40
Muscle	550	870	45
White matter	500	780	90
Gray matter	650	920	100
Cerebrospinal fluid	1800	2400	160

❖ **Proton density**, longitudinal relaxation time T1 and Transverse relaxation time T2 are primary NMR tissue parameters.

Part (II) Magnetic Resonance Imaging (MRI)

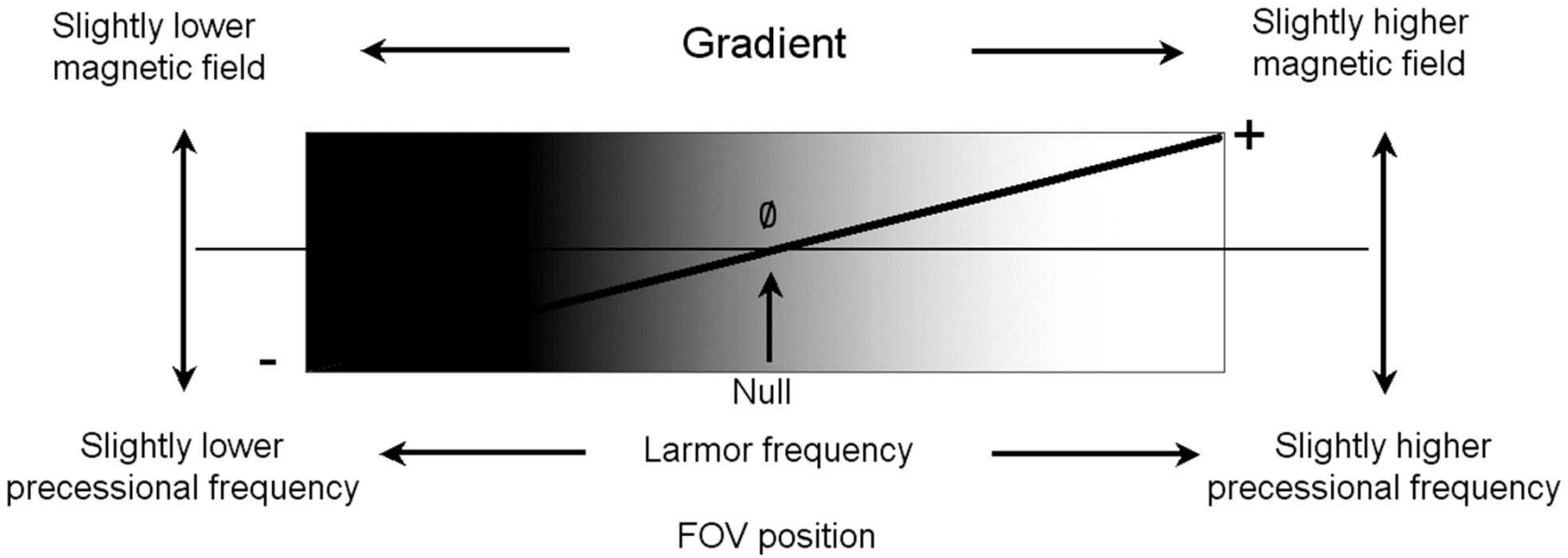
9.5 Imaging Concepts with Magnetic Field Gradients

- The importance of magnetic resonance imaging (MRI) in clinical imaging has exceeded even the most optimistic hopes of researchers from the 1980s.

As described earlier:

- Magnetic field gradients could be used to localize the NMR signal and to generate images that display nuclear properties reflecting clinically relevant information.
- **Magnetic field gradients** are precisely controlled magnetic fields that linearly vary over a predefined field of view (FOV) with positive and negative polarity.
- Positive gradient polarity means that the gradient values add to the main magnetic field; negative polarity implies a reduction in magnetic field (B_0).

- The gradient is a **linear,** position-dependent magnetic field applied across the FOV, and it causes protons to alter their precessional frequency corresponding to their position along the applied gradient in a known and predictable way.
- A linear increase (or decrease) in precessional frequency occurs with the variation of the local magnetic field strength away from the null.



*Reference: Jerry L. Prince and Jonathan M. Links, <<Medical Imaging Signals and Systems>>, 2006, p413

- Magnetic fields with predictable directionality and strength are produced in a coil wire energized with a direct electric current of specific polarity and amplitude.
- Magnetic field gradients are obtained by superimposing the magnetic fields of one or more coils with a precisely defined geometry.

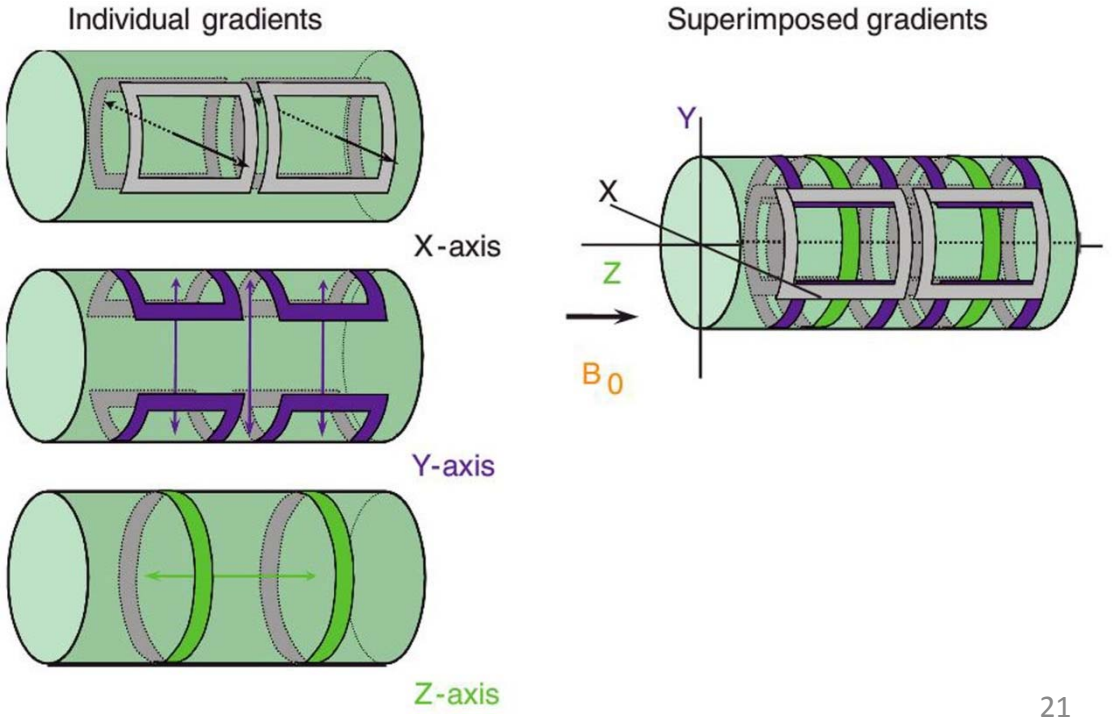


Figure: Within the large stationary magnetic field, field gradients are produced by three separate coil pairs placed within the central core of the magnet, along the *x, y,* or *z* directions.

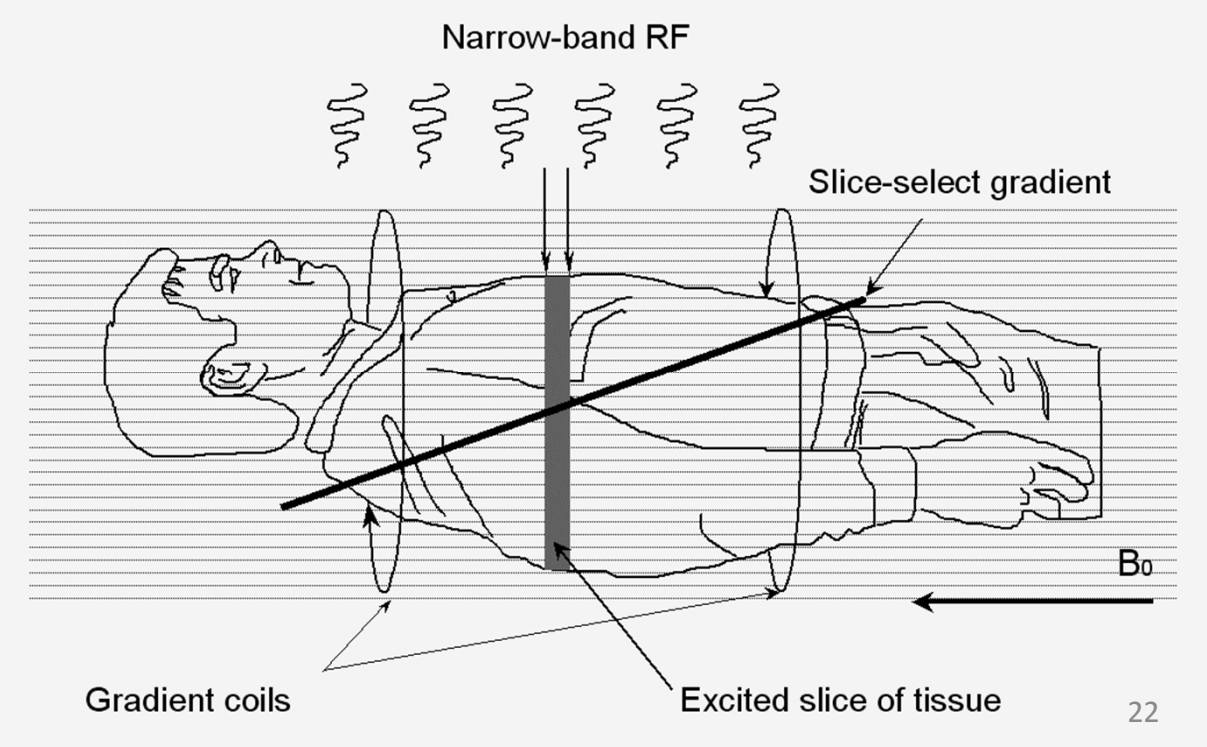
9.6 The Application of Magnetic Field Gradients

(1) Slice select gradient (SSG— Z-axis)

Principle of slice selection:

- The slice select gradient applied during the RF pulse results in proton excitation in a single plane and thus localizes the signal in the dimension orthogonal to the gradient. It is the first of three gradients applied to the sample volume.

- Slice thickness is determined by two parameters:
  1) The bandwidth (BW) of the RF pulse, and
  2) The gradient strength across the FOV.



- The gradient G_z which have units of Gauss per centimeter, yields a Larmor frequency *f(z)* that is a function of *z*:

$$f(z) = \frac{\gamma}{2\pi}(B_0 + G_z z)$$

The angular frequency $\omega(z)$:

$$\omega(z) = 2\pi f(z) = \gamma(B_0 + G_z z)$$

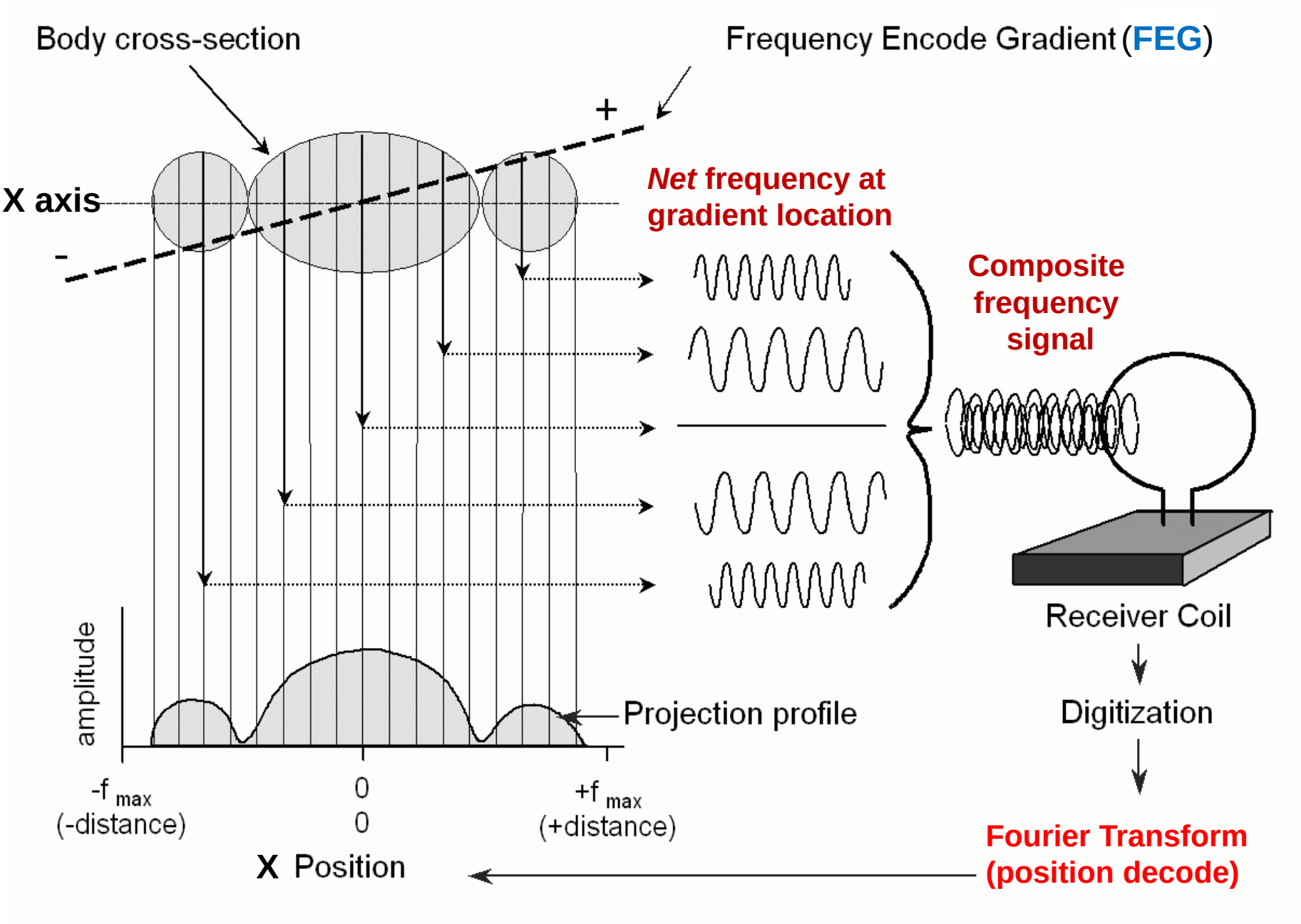
*Reference: Jerry L. Prince and Jonathan M. Links, <<Medical Imaging Signals and Systems>>, 2006, p419

(2) Frequency encode gradient (FEG — X-axis)

The frequency encode gradient (**FEG**), also known as the **readout gradient,** is applied in a direction perpendicular to the slice select gradient (**SSG**). The gradient G_x has unit of Gauss per centimeter.

- The **FEG** acts on through protons in a slab determined by the **SSG** excitation.
- The **signals** are frequency encoded depending on their position along the **FEG**.
- The Larmor frequency and angular frequency during a frequency encode gradient with the x-direction are given by:

$$\omega(x) = 2\pi f(x) = \gamma(B_0 + G_x x)$$
$$= \gamma B_0 + \gamma G_x x = \omega_0 + \gamma G_x x$$


Body cross-section
Frequency Encode Gradient (FEG)
+
X axis
-
Net frequency at gradient location
Composite frequency signal
Receiver Coil
Digitization
Fourier Transform (position decode)
amplitude
Projection profile
-f max (-distance)
0
0
+f max (+distance)
X Position

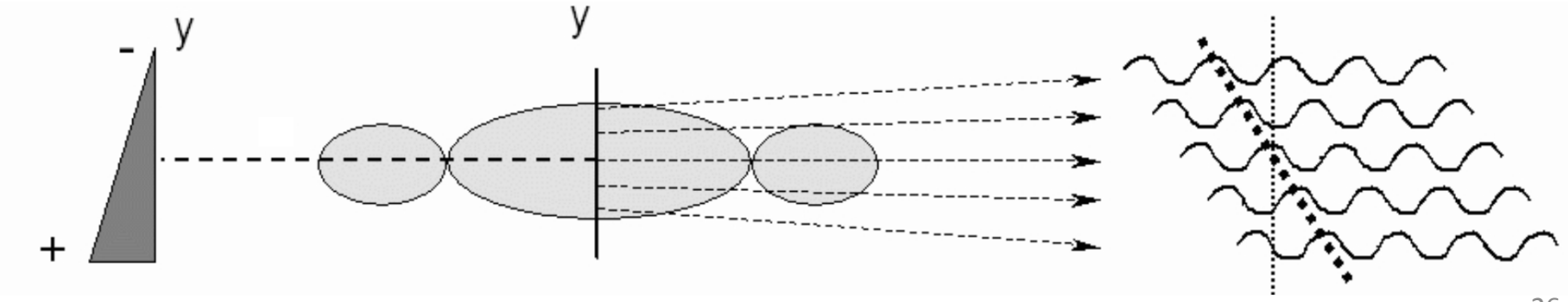
(3) Phase encode gradient (PEG— Y-axis)

Position of the spins in the third spatial dimension is determined with a phase encode gradient (**PEG**), along the third perpendicular axis. The gradient G_y has unit of Gauss per centimeter.

Basic concept:

- After the initial localization of the excited protons in the slab of tissue (by the **SSG**), all spins have the same phase.
- During the application of the **PEG**, a linear variation in the precessional frequency of the excited spins occurs across the tissue slab along the direction of the gradient.

$$\omega(y) = 2\pi f(y) = \gamma[B_0 + G_y y]$$



- After the **PEG** is turned off, spin precession reverts to the Larmor frequency, but now phase shifts are introduced, the magnitude of which are dependent on the spatial position relative to the **PEG** null and the **PEG** strength.

- Suppose the **PEG** is on for a time duration τ, the phase accumulated is given as:

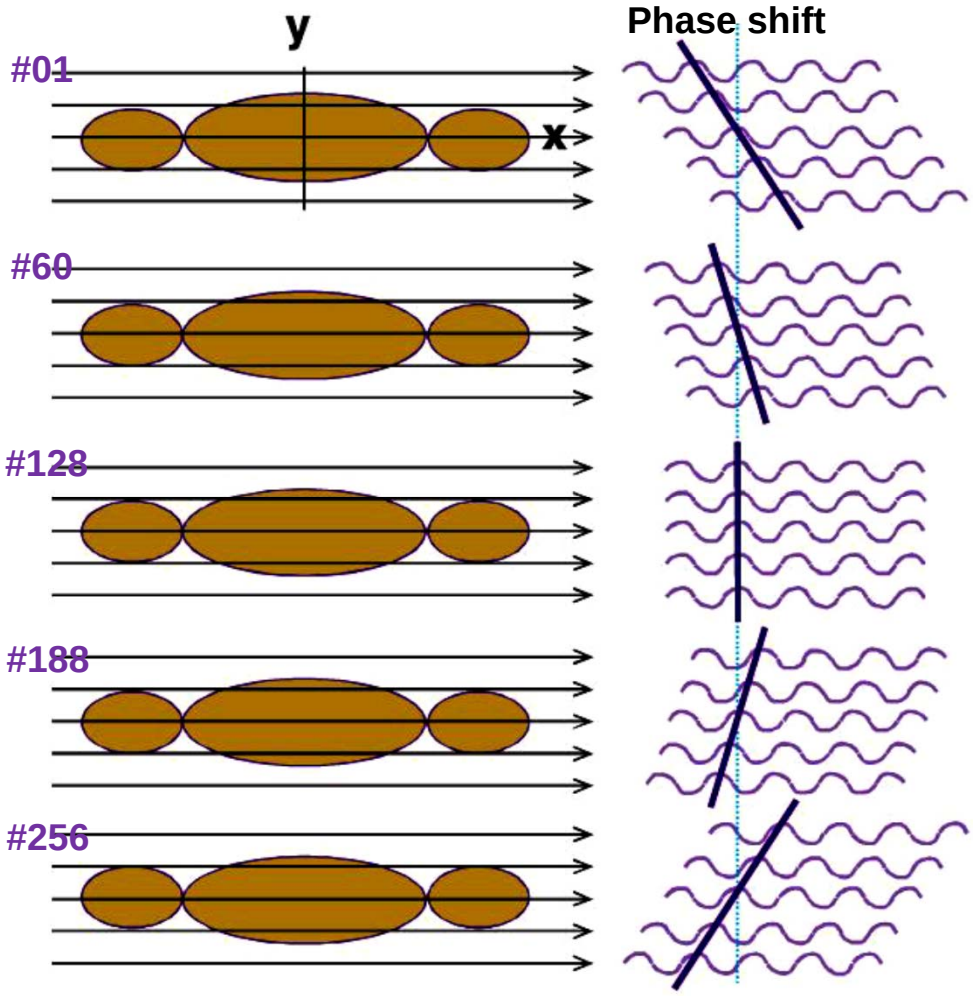
$$\Phi(y) = \omega(y)\tau = \gamma[B_0 + G_y y]\tau = \Phi_0 + y\gamma G_y \tau$$

$$where: \Phi_0 = \gamma B_0 \tau$$

- Thus, each location along the Y-axis is **spatially encoded** by the amount of phase shift.

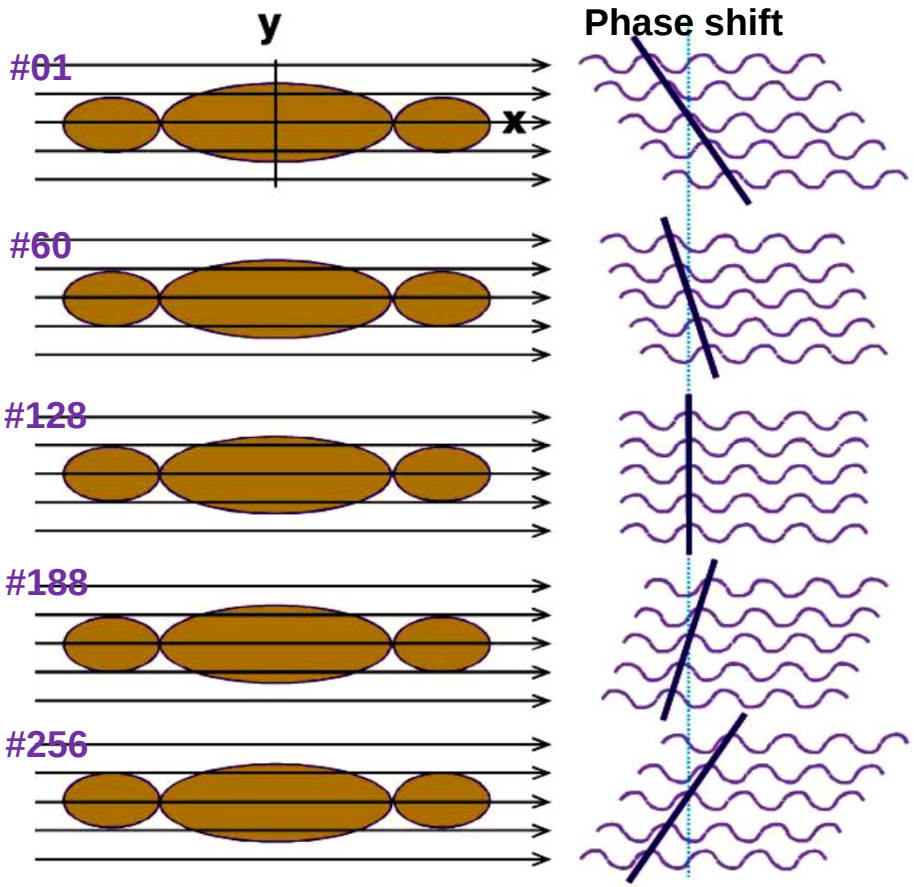
$$\Delta\Phi(y) = \Phi(y) - \Phi_0 = y\gamma G_y \tau$$

- All spins along the phase encoded Y-axis contributes to the MR signal.
- The phase of the individual spins at a given location along Y-direction cannot be derived from a single signal but only from a set of signals.
- In this respect, the **MR** signal is comparable to a mathematical equation with **many unknowns** (e.g.128, or 256…)
- To calculate the unknowns, one needs as many different equations as there are unknowns.



*Reference Book: D. Weishaupt, V. D. Kochli, B.Marincek, <<How Does MRI Work? : An Introduction to the Physics and Function of Magnetic Resonance Imaging>>, 2nd Edition, Springer, 2008

- This means that we must repeat the sequence many times with increasing or decreasing **PEG** strengths (G_y).
- Repeated measurements are performed with a specific temporal delay, time of repetition (**TR**).
- The number of phase-encoding steps performed depends on the desired image quality.



- More phase-encoding steps improve resolution and image quality but also prolong scan time.

- Here: **Time of repetition (TR):** the period of time between the beginning of a pulse sequence and the beginning of the succeeding (essentially identical) pulse sequence.

As a brief summary:

- The slice selection gradient (**SSG**)—Z-axis.

$$\omega(z) = 2\pi f(z) = \gamma(B_0 + G_z z) \tag{1}$$

- The phase encoding gradient (**PEG**)—Y-axis.

$$\omega(y) = 2\pi f(y) = \gamma[B_0 + G_y y] = \omega_0 + \gamma G_y y \tag{2}$$

$$\Phi(y) = \omega(y)\tau = \gamma[B_0 + G_y y]\tau = \Phi_0 + y\gamma G_y \tau$$

$$\Delta\Phi(y) = \Phi(y) - \Phi_0 = y\gamma G_y \tau \tag{3}$$

- The frequency encoding gradient (**FEG**)—X-axis.

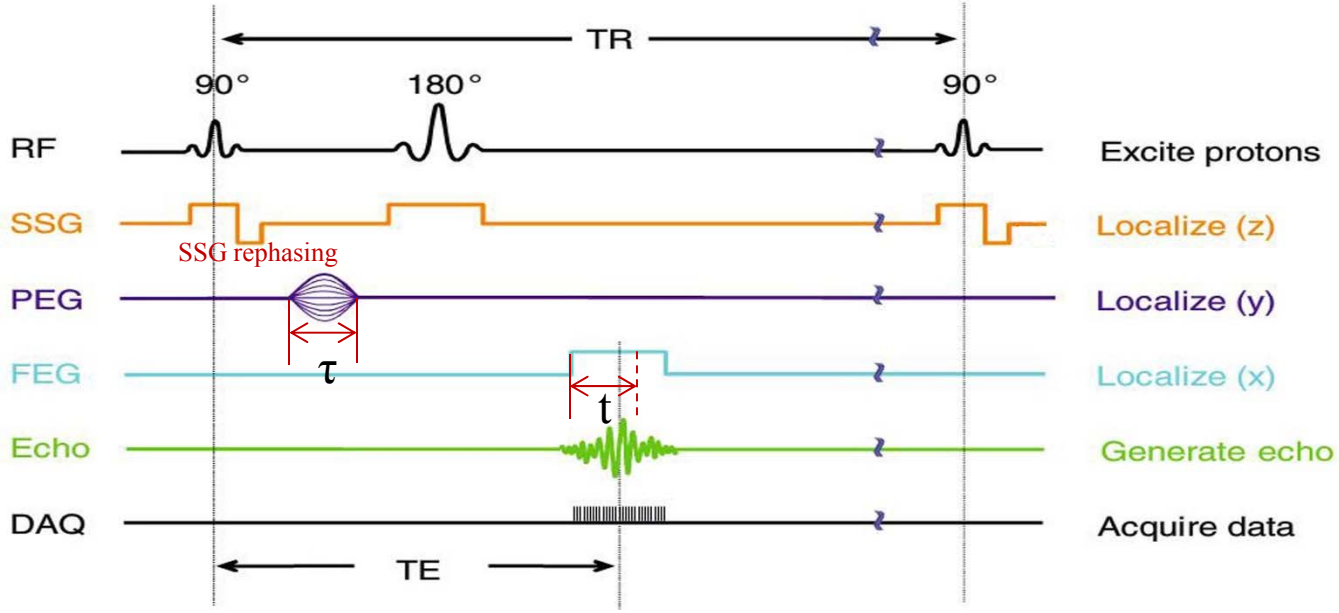
$$\omega(x) = 2\pi f(x) = \gamma(B_0 + G_x x) = \omega_0 + \gamma G_x x \tag{4}$$

(4) Spin-echo pulse sequence and timing of gradients: an example

❖ A typical spin-echo pulse sequence diagram, showing the timing of the **SSG**, **PEG**, and **FEG**

1) A narrow band RF excitation pulse simultaneously applied with the **SSG** causes a specific slab of tissues with protons at the same frequency to absorb energy. A 90-degree flip angle produces the largest M_{xy}.

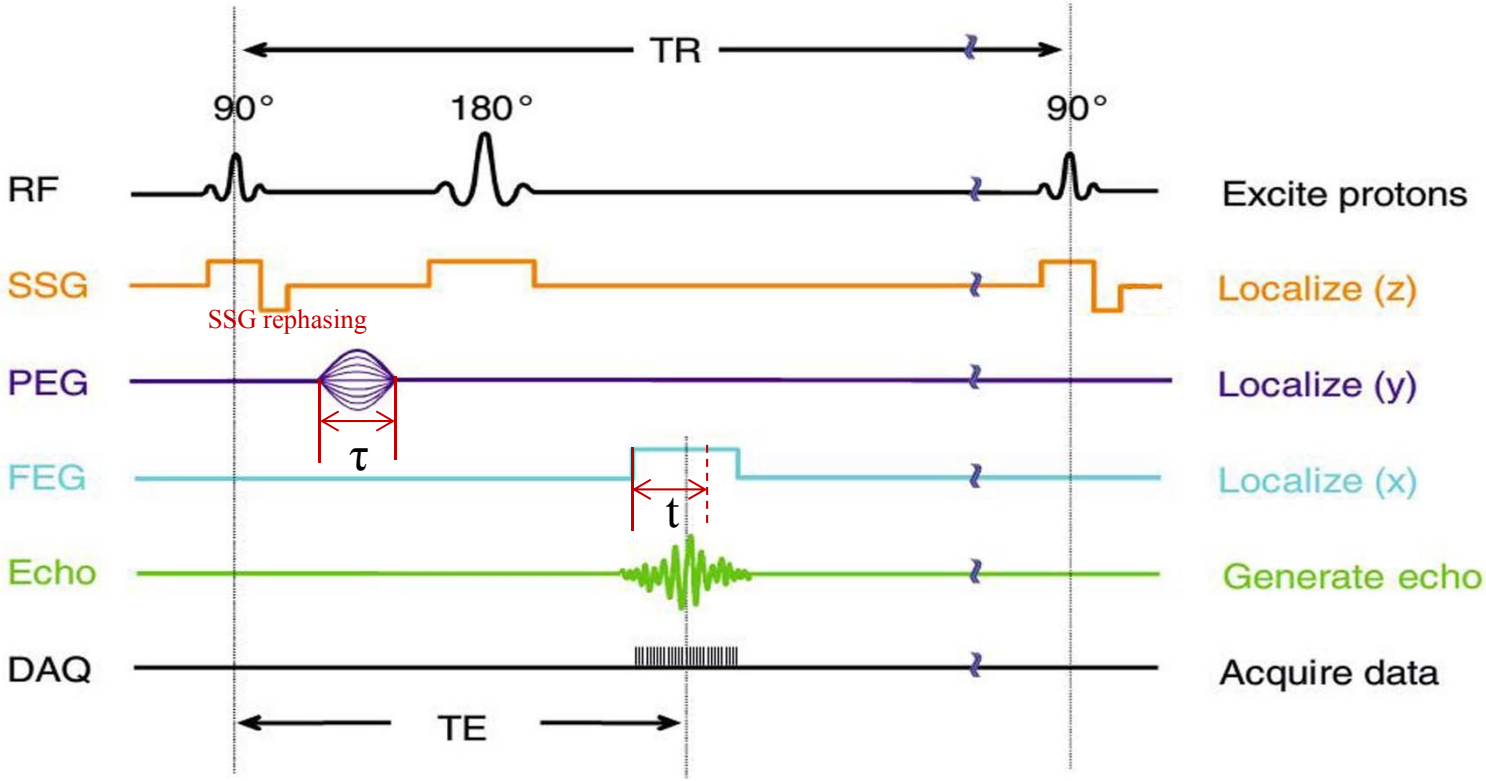
2) At the peak of the RF pulse, all protons in the slice are in phase, but the SSG causes the spins to become dephased after the gradient is turned off. To reestablish phase coherence, a reverse polarity gradient is applied.



3) A **PEG** is applied for a brief duration, which introduces a phase difference among the protons along the phase encode direction.

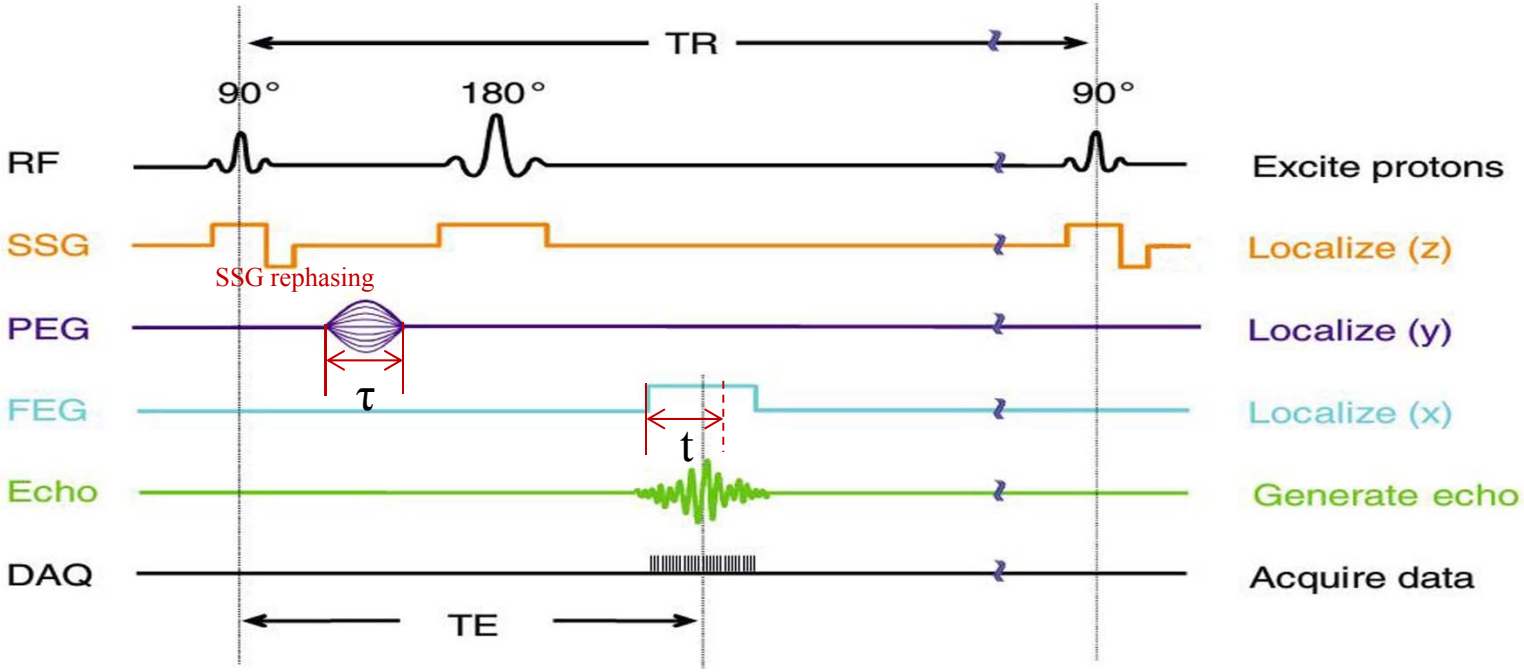
4) A refocusing 180-degree **RF** pulse is delivered at TE/2, while **SSG** is applied again.

Note: The time of echo (**TE**) is the time between the excitation pulse and the appearance of the peak amplitude of an induced echo.



5) During the evolution and decay of the echo signal, the **FEG** is applied orthogonal to both the **SSG** and **PEG** directions, generating spatially dependent changes in the precessional frequencies of the protons.

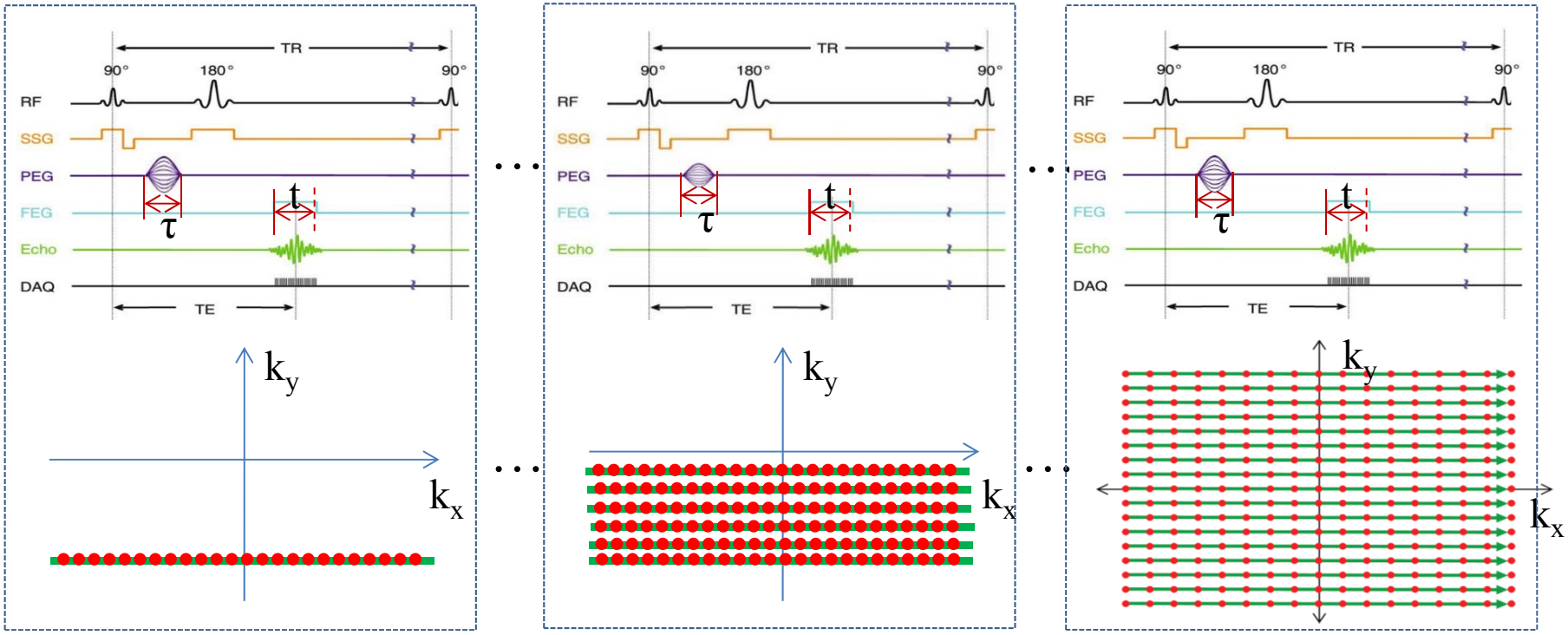
6) Data sampling and acquisition of the complex signal occurs simultaneous to the **FEG**.



7) Data are deposited in the k-space matrix at a row location specifically determined by the strength of the **PEG**. For each **TR**, an incremental variation of G_y sequentially fills each row.

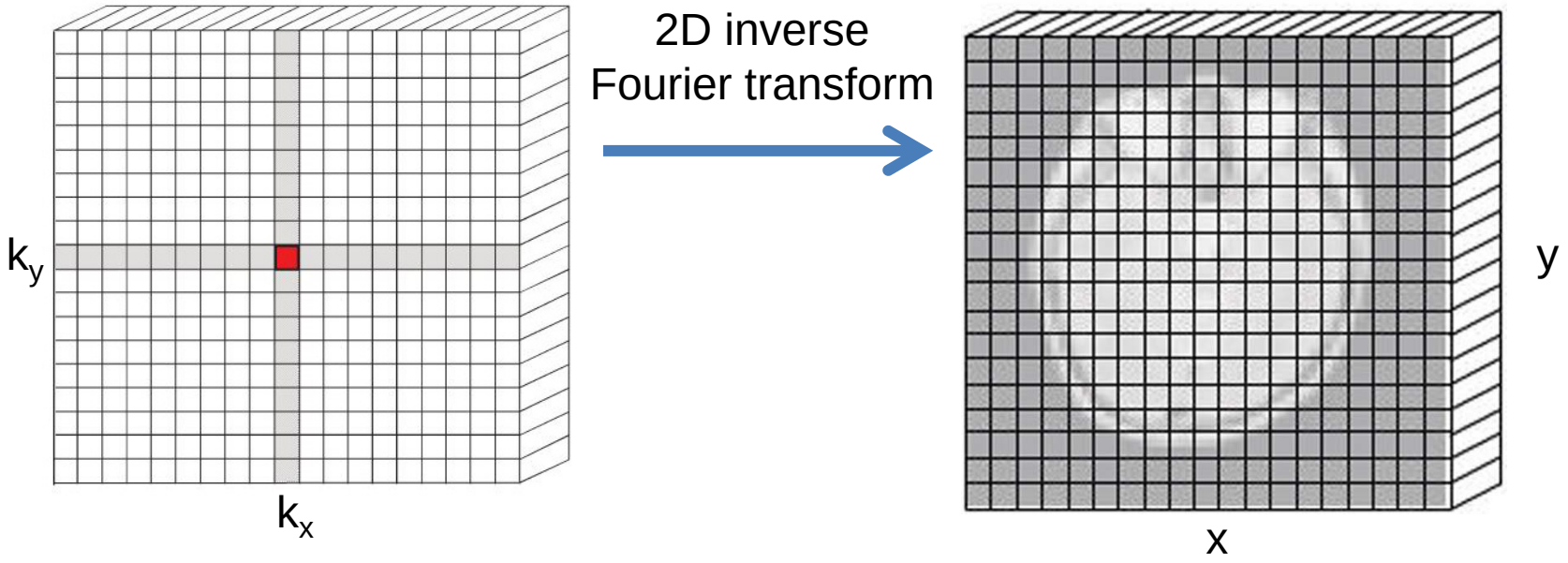
8) Once filled, the k-space matrix contain positionally dependent variations along the $\mathbf{k_y}$ and $\mathbf{k_x}$ directions respectively, and that $k_x = -\gamma G_x t$, $k_y = -\gamma G_y \tau$.

Here: **Time of repetition (TR):** the period of time between the beginning of a pulse sequence and the beginning of the succeeding (essentially identical) pulse sequence.



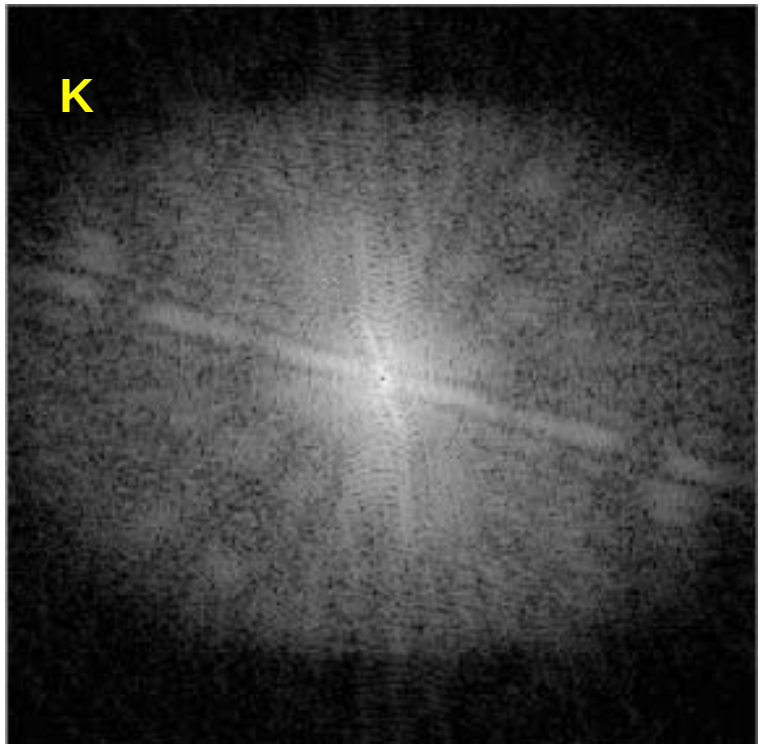
9) Then a 2D inverse Fourier transform applies to produce the spatial domain representation.

10) The final image is scaled and adjusted to represent the proton density, T1, T2, and flow characteristics of the tissues using a grayscale range, where each pixel represents a voxel.



Two Spaces

K-space

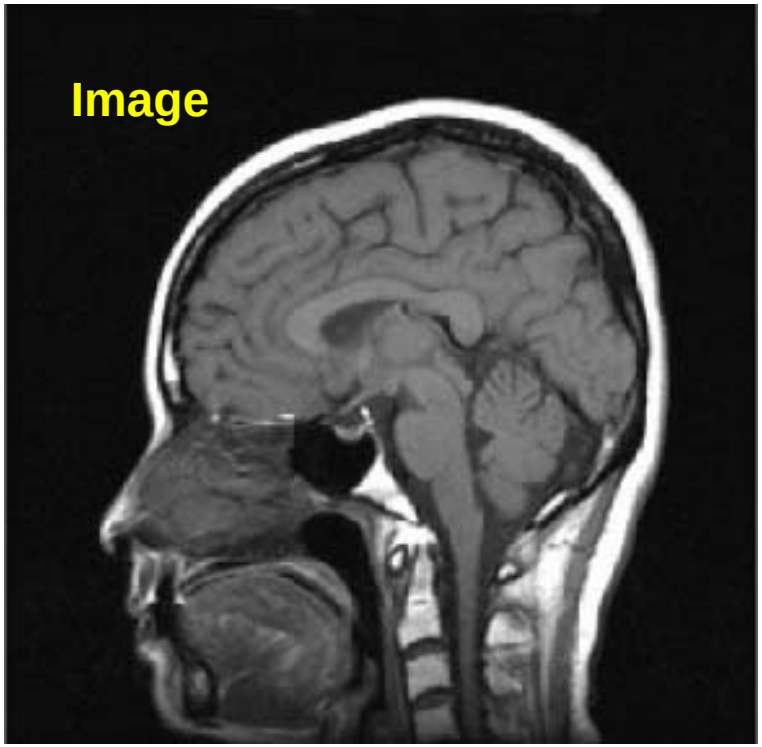


Acquired Data

IFT

FT

Image space



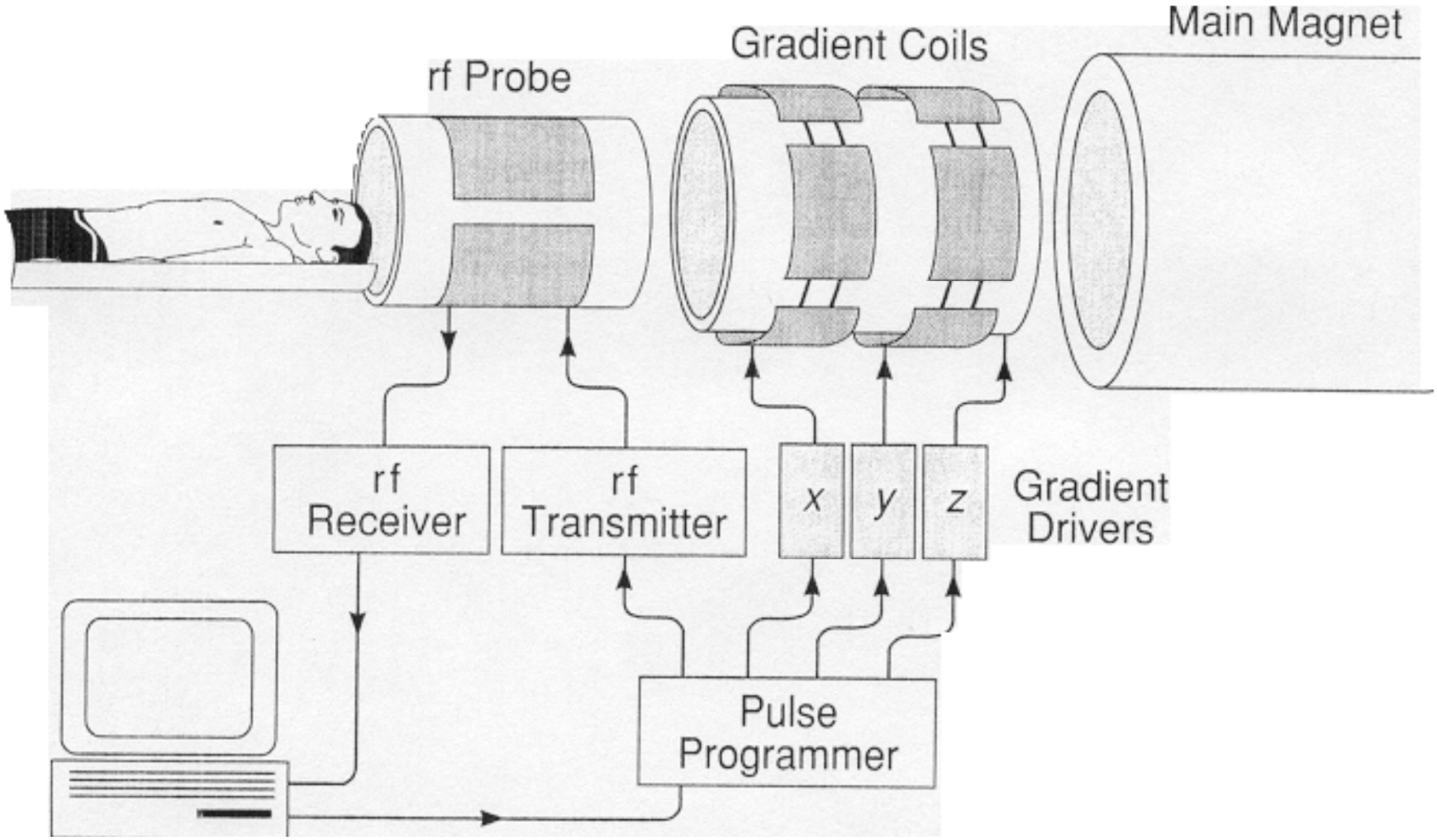
Final Image

9.7 Formation of MRI Images --MR Image Contrast

- An MR image is generally not a direct, voxel-by-voxel map of T1, T2 or proton density. It is, rather, a voxel-by-voxel map of the tissue net magnetization, that results at a specific instant (such as t - TR, or t = TE), and from a specific imaging pulse sequences.
- This map is obtained by determining, at that instant, the relative amount of proton NMR signal that originates in each voxel.
- The greater the magnetization in a voxel, the stronger the NMR signal coming from it at the time of signal detection, and the brighter the pixel in the corresponding MR image.

- The strength of NMR signal depends on T1, T2, proton density (three tissue parameters), and depends on the choice of pulse sequence and other imaging machine settings.
- Thus T1, T2, proton density has indirect but significant influence on **image contrast.**
- An image is produced in such a fashion that the contrast reflects differences primarily in tissue T1 is called T1 weighted. **T1 weighted, T2 weighted, and proton density weighted** imaging is adequate for many clinical purposes.
- Much of the skills and science of MRI involves the selection of pulse sequences and other MRI instrument characteristics that more effectively enhance image contrast.

MRI instrumentation (Magnet and RF transmitter and receiver)



MRI – a T1 weighted image:

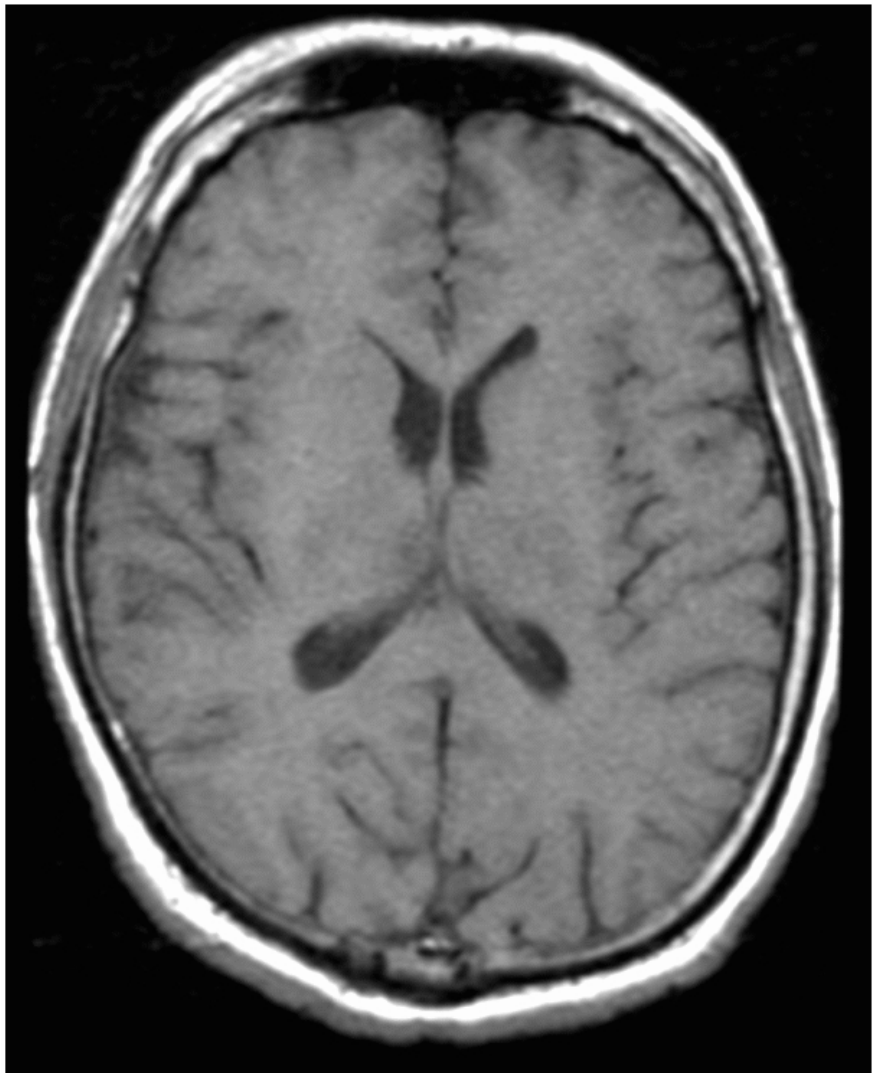


Figure: The T1-weighted spin echo axial brain image obtained with TR= 549 msec and TE= 11 msec demonstrates bright image intensity for short-T1 tissues (white matter and fat) and dark intensity for long-T1 tissues (cerebrospinal fluid).

MRI – a T2 weighted image:

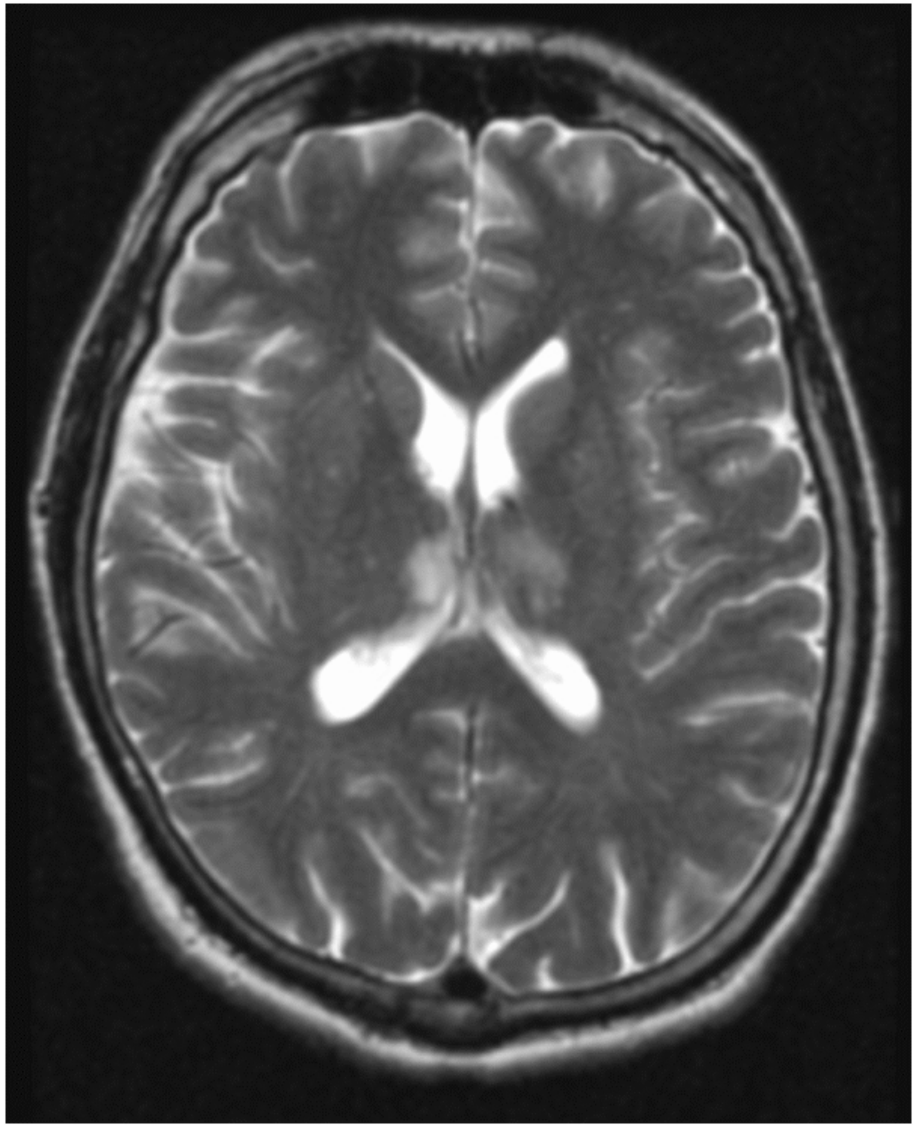


Figure: The T2-weighted spin echo axial brain image, obtained with TR= 2400 msec and TE= 90 msec, has bright image intensity for long-T2 tissues such as cerebrospinal fluid and dark intensity for short-T2 tissues such as white matter and fat.

MRI – a spin (proton) density weighted image:

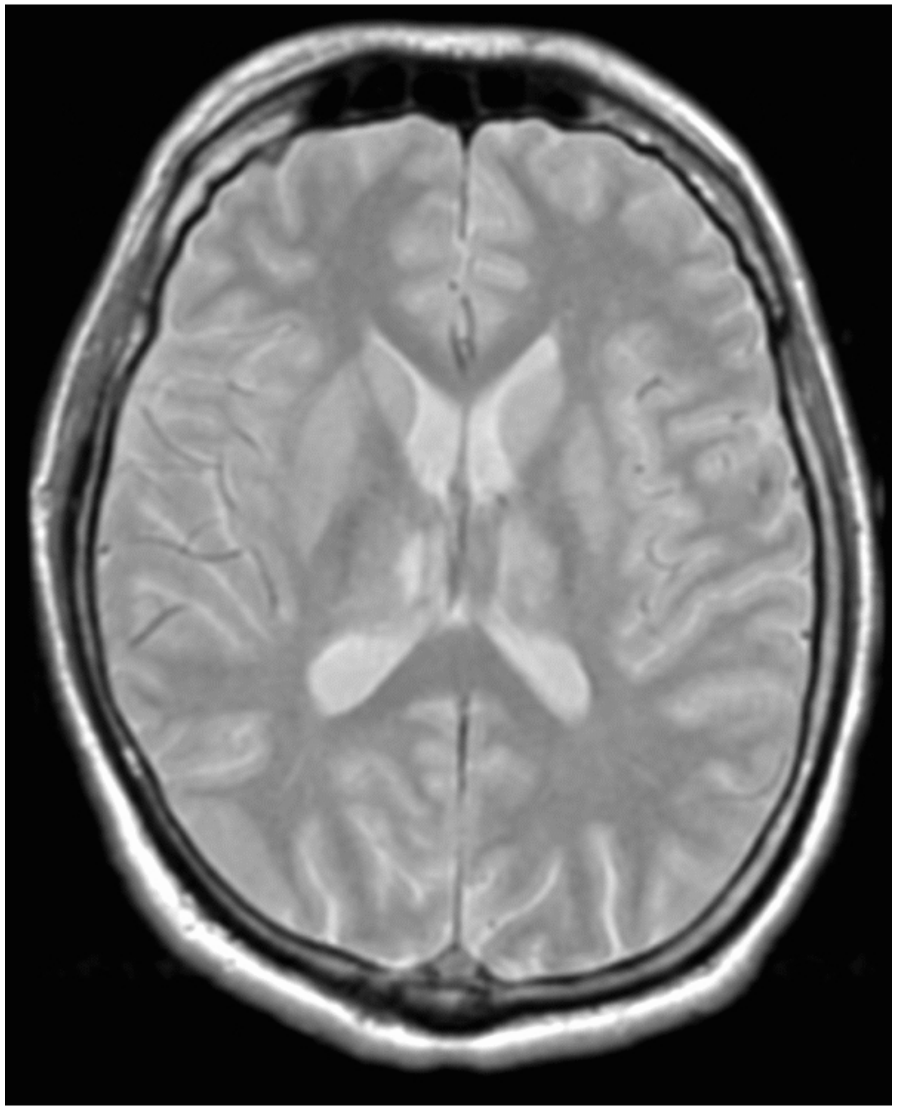


Figure: The proton density-weighted axial spin echo brain image, obtained with TR= 2400 msec and TE= 30 msec, shows reduced contrast compared with the T1-weighted images (but an overall higher signal amplitude). Tissues with higher spin density (e.g., fat, cerebrospinal fluid (CSF)) have higher image intensity.

Summary

1) There is no net magnetic moment exhibited by the tissue
2) Under the influence of a strong **externally applied magnetic field,** the protons become magnetized and align with the magnetic field
3) The **Larmor equation** describes the dependency between the external magnetic field and the precessional frequency: $\omega = \gamma B_0$ or $f = (\gamma / 2\pi) B_0$
4) The magnetic **resonance** signal is the result of excitation of the individual magnetized protons within the object by irradiation with radio frequency (**RF**) energy of a specific frequency (at a frequency equal to the precessional frequency of the protons). As the excited system **returns to equilibrium**, MR signals are emitted in proportion to the **number of the excited protons** in the sample.
5) Magnetic field gradients could be used to localize the NMR signal and to generate images that display nuclear properties reflecting clinically relevant information.
6) The strength of NMR signal depends on T1, T2, proton density (three tissue parameters), and depends on the choice of pulse sequence and other imaging machine settings.

References:

- J.T. Bushberg, J.A. Seibert, E.M. Leidholdt, Jr., J.M. Boone, <<The Essential Physics of Medical Imaging>>, Editions 3, Williams & Wilkins, Baltimore, 2012
- Jerry L. Prince and Jonathan M. Links, <<Medical Imaging Signals and Systems>>, Pearson Prentice Hall, 2006
- D. Weishaupt, V. D. Kochli, B.Marincek, <<How Does MRI Work? : An Introduction to the Physics and Function of Magnetic Resonance Imaging>>, 2nd Edition, Springer, 2008
- James H. McClellan et. al., <<DSP First – A Multimedia Approach>>, Prentice Hall, 1998

Homework #8

1. What will happen to a patient, if the patient is placed under the influence of a strong externally applied magnetic field? (hint: consider the protons of the tissue)
2. Will individual spins (here spin and proton are considered synonymous) be also influenced by the external applied magnetic field? Explain in words as well as with a proper mathematical formula.
3. The sample containing ^{13}C was placed under the influence of an externally applied magnetic field, $B_0 = 2.5$ tesla. The corresponding precessional frequency of the element was determined as 26.76 MHz. What is the gyromagnetic ratio for the element (^{13}C)?
4. A sample containing ^{1}H is placed under a magnetic field, $B_0 = 1.5$ tesla, and a **z** gradient of $G_z = 3$ gauss/cm. The sample is 0.5m in length along **z** axis, centered at z = 0. (a) What is the range of precessional frequency of the protons in the sample? (b) What is the precessional frequency for the protons at z = 0? Note that the gyromagnetic ratio of ^{1}H is $\gamma/2\pi = 42.58$ MHz / tesla.