Home Work # 5

1. If the linear attenuation coefficient of air under a specific x-ray technique (KVp) is zero, ($\mu_{air} = 0\ cm^{-1}$), what is the CT number of air?

$$CT \quad number = 1000 \times \frac{(\mu - \mu_{water})}{\mu_{water}}$$

$$CT \quad number(air) = 1000 \times \frac{(0 - \mu_{water})}{\mu_{water}} = -1000$$

2. Assume a mono-energetic x-ray source for which $\mu_{water} = 0.2\ cm^{-1}$. If a lesion has a linear attenuation coefficient 10% higher than that of water, then what will be the lesion's CT number?

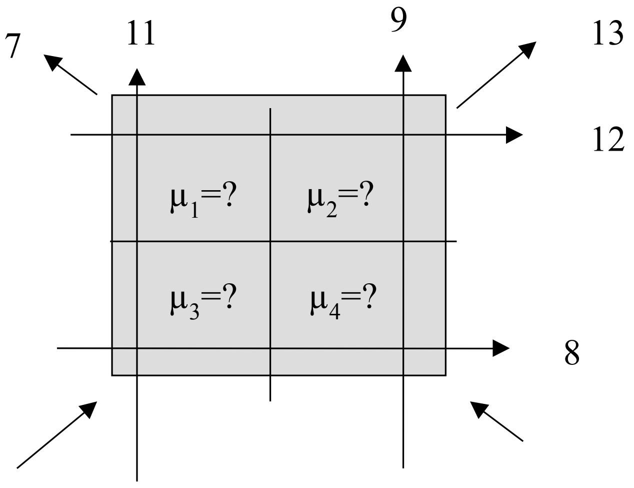
$$CT \quad number = 1000 \times \frac{(\mu - \mu_{water})}{\mu_{water}}$$

$$\mu_{water} = 0.2cm^{-1}$$

$$\mu_{lesion} = 0.2 \times (1 + 10\%) = 0.22cm^{-1}$$

$$CT \quad number(lesion) = 1000 \times \frac{(0.22cm^{-1} - 0.2cm^{-1})}{0.2cm^{-1}} = 100$$

3. Suppose we are imaging a body slice that consists of four voxels of unit dimension (Δx=1). The ray sums were measured and are given by the following diagram. Try to determine linear attenuation coefficient of each voxel with Algebraic Reconstruction Technique (ART). Please show each step of iterations.



Solution:
Recall Algebraic reconstruction technique (ART)

$$\mu_i^{(q+1)} = \mu_i^{(q)} + \frac{P - \sum \mu_i^{(q)}}{N}$$

First Iteration (Vertical)

Initial assumption: $\mu_1^{(0)} = \mu_2^{(0)} = \mu_3^{(0)} = \mu_4^{(0)} = 0$

$$P = 11 \Rightarrow \quad \mu_1^{(1)} = \mu_1^{(0)} + \frac{P - \left[\mu_1^{(0)} + \mu_3^{(0)}\right]}{2} = 0 + \frac{11 - [0+0]}{2} = 5.5$$

$$P = 9 \Rightarrow \quad \mu_2^{(1)} = \mu_2^{(0)} + \frac{P - \left[\mu_2^{(0)} + \mu_4^{(0)}\right]}{2} = 0 + \frac{9 - [0+0]}{2} = 4.5$$

$$P = 11 \Rightarrow \quad \mu_3^{(1)} = \mu_3^{(0)} + \frac{P - \left[\mu_1^{(0)} + \mu_3^{(0)}\right]}{2} = 0 + \frac{11 - [0+0]}{2} = 5.5$$

$$P = 9 \Rightarrow \quad \mu_4^{(1)} = \mu_4^{(0)} + \frac{P - \left[\mu_2^{(0)} + \mu_4^{(0)}\right]}{2} = 0 + \frac{9 - [0+0]}{2} = 4.5$$

Second Iteration (Horizontal)

Assumption from 1st iteration: $\mu_1^{(1)} = 5.5;\ \ \mu_2^{(1)} = 4.5;\ \ \mu_3^{(1)} = 5.5;\ \ \mu_4^{(1)} = 4.5$

$$P = 12 \Rightarrow \quad \mu_1^{(2)} = \mu_1^{(1)} + \frac{P - \left[\mu_1^{(1)} + \mu_2^{(1)}\right]}{2} = 5.5 + \frac{12 - [5.5 + 4.5]}{2} = 6.5$$

$$P = 12 \Rightarrow \quad \mu_2^{(2)} = \mu_2^{(1)} + \frac{P - \left[\mu_1^{(1)} + \mu_2^{(1)}\right]}{2} = 4.5 + \frac{12 - [5.5 + 4.5]}{2} = 5.5$$

$$P = 8 \Rightarrow \quad \mu_3^{(2)} = \mu_3^{(1)} + \frac{P - \left[\mu_3^{(1)} + \mu_4^{(1)}\right]}{2} = 5.5 + \frac{8 - [5.5 + 4.5]}{2} = 4.5$$

$$P = 8 \Rightarrow \quad \mu_4^{(2)} = \mu_4^{(1)} + \frac{P - \left[\mu_3^{(1)} + \mu_4^{(1)}\right]}{2} = 4.5 + \frac{8 - [5.5 + 4.5]}{2} = 3.5$$

Third Iteration (Diagonal)

Assumption from 2nd iteration: $\mu_1^{(2)} = 6.5;\ \ \mu_2^{(2)} = 5.5;\ \ \mu_3^{(2)} = 4.5;\ \ \mu_4^{(2)} = 3.5$

$$P = 7 \Rightarrow \quad \mu_1^{(3)} = \mu_1^{(2)} + \frac{P - \left[\mu_1^{(2)} + \mu_4^{(2)}\right]}{2} = 6.5 + \frac{7 - [6.5 + 3.5]}{2} = 5$$

$$P = 13 \Rightarrow \quad \mu_2^{(3)} = \mu_2^{(2)} + \frac{P - \left[\mu_2^{(2)} + \mu_3^{(2)}\right]}{2} = 5.5 + \frac{13 - [5.5 + 4.5]}{2} = 7$$

$$P = 13 \Rightarrow \quad \mu_3^{(3)} = \mu_3^{(2)} + \frac{P - \left[\mu_2^{(2)} + \mu_3^{(2)}\right]}{2} = 4.5 + \frac{13 - [5.5 + 4.5]}{2} = 6$$

$$P = 7 \Rightarrow \quad \mu_4^{(3)} = \mu_4^{(2)} + \frac{P - \left[\mu_1^{(2)} + \mu_4^{(2)}\right]}{2} = 3.5 + \frac{7 - [6.5 + 3.5]}{2} = 2$$

Therefore, we have: $\mu_1=5$; $\mu_2=7$; $\mu_3=6$; $\mu_4=2$

4. This problem is related to the CT reconstruction technique (Fourier Transform approach). Assuming that a series of projections from 0^0 to 180^0 at 1^0 increments; resulted a Sinogram that can be expressed as:

$$P_\theta(t) = 4 \times \sin\theta \times rect(3t)$$

Based on Fourier slice theorem, please determine the mathematical expression of the function in Fourier domain at $\theta=60^0$.

Solution:

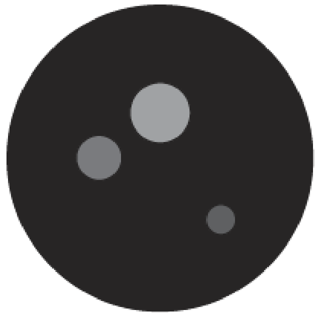
$$F(u,v)\big|_{\theta=60^0} = FT_{1D}[P_{\theta=60^0}(t)]$$

$$= FT_{1D}[4 \times \sin\theta \times rect(3t)]\big|_{\theta=60^0}$$

$$= FT_{1D}[4 \times \sin 60^0 \times rect(3t)] = 4 \times \frac{\sqrt{3}}{2} \times \frac{1}{3}\operatorname{sinc}(\frac{\pi\omega}{3})$$

$$= \frac{2\sqrt{3}}{3}\operatorname{sinc}(\frac{\pi\sqrt{u^2+v^2}}{3})$$

5. An exercise in CT projection and reconstruction with Matlab:
The cross-section of an object with three holes of different sizes and materials (different attenuation under x-ray) is shown by the right figure. The numerical value of each of the 256 by 256 voxels is given in an attached file named "cross-section.dat". The numerical values are related to the linear attenuation coefficients of the objet under a specific x-ray for the purpose of this exercise.



The cross-section of the target object with three holes

(a) Using Matlab function “radon”, determine and print the sinograms resulted from the following sets of projection angles:

(i). 0, 1, 2, 3, ..., 179, with 1 degree increment
(ii). 0, 5, 10, 15, ..., 175; with 5 degree increment
(iii). 0, 30, 60, ..., 150; with 30 degree increment

(b) Using Matlab function “iradon”, reconstruct and print the CT slices from the sinograms obtained in (a), respectively.
Note: The following flow chart may be helpful. You can also refer to Matlab Help about “radon" and "iradon" for their respective usage.

Flowchart for CT projection and reconstruction exercise with Matlab

Clear the Matlab working environment:
```
clear;clc;close all;
```
↓

Load the provided cross-section data into Matlab using:
```
cross_section = load(‘cross-section.dat’, ‘-ASCII’);
```
↓

Set up the angle at which the projections are to be made, for example, for 36 projections from 0~179 degrees:
```
step = 5;
max_angle = 179;
theta = 0:step:max_angle;
```
↓

Obtain the sinogram by applying the projection along those angles using radon():
```
[R,xp] = radon(cross_section, theta);
```
↓

Plot the sinogram using imshow():
```
imshow([0 max_angle], xp, R', []);
```
↓

Conduct and display the reconstruction using iradon():
```
I = iradon(R,theta);
figure; imshow(I);
```

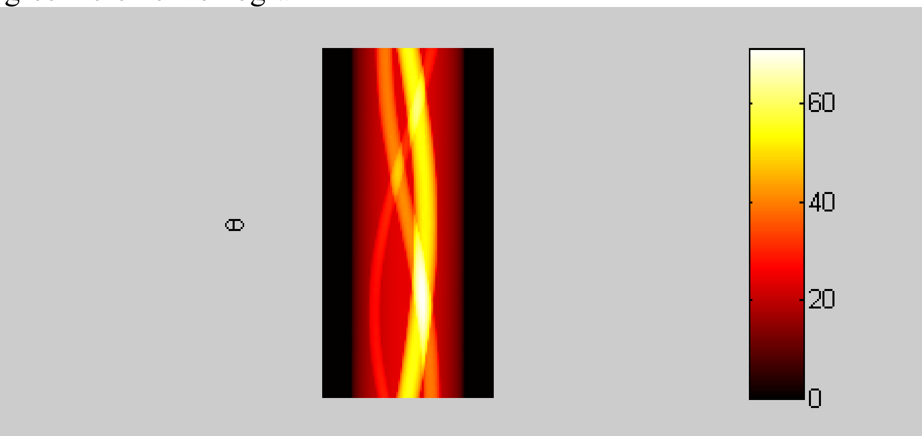
Matlab code

```
%% initialization step for the calculation
clear;
close all;
imtool close all;
clc;

max_angle = 179;

cross_section = load('cross_section.dat', '-ASCII');
%% projection and reconstruction on theta = 0:1:179 (degree)
step = 1;
theta = 0:step:max_angle;
[R,xp] = radon(cross_section,theta);
tmp = R;
figure;
imshow(R',[],'Xdata',[0 max_angle],'Ydata',xp,'InitialMagnification', 'fit');
ylabel('\theta');
xlabel('t');
colormap(hot), colorbar;

I = iradon(tmp,theta);
figure;imshow(I,'InitialMagnification', 'fit');


%% projection and reconstruction on theta = 0:5:175 (degree)
step = 5;
theta = 0:step:max_angle;
[R,xp] = radon(cross_section,theta);
tmp = R;
[r,c] = size(R);
R = zeros(r,max_angle+1);
for i = 1:length(theta)
   R(:,1+(i-1)*step) = tmp(:,i);
end
figure;
imshow(R',[],'Xdata',[0 max_angle],'Ydata',xp,'InitialMagnification', 'fit');
ylabel('\theta');
xlabel('t');
colormap(hot), colorbar;

I = iradon(tmp,theta);
figure;imshow(I,'InitialMagnification', 'fit');


%% projection and reconstruction on theta = 0:30:150 (degree)
step = 30;
theta = 0:step:max_angle;
[R,xp] = radon(cross_section,theta);
tmp = R;
```

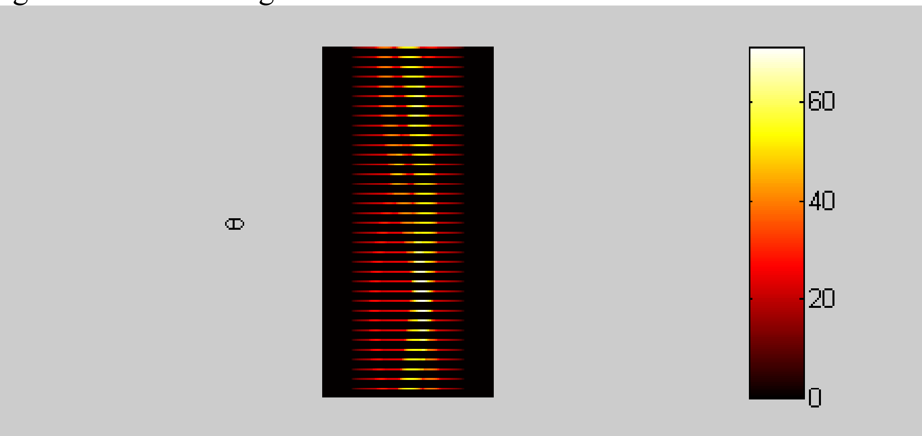
```
[r,c] = size(R);
R = zeros(r,max_angle+1);
for i = 1:length(theta)
    R(:,1+(i-1)*step) = tmp(:,i);
end
figure;
imshow(R',[],'Xdata',[0 max_angle],'Ydata',xp,'InitialMagnification', 'fit');
ylabel('\theta');
xlabel('t');
colormap(hot), colorbar;

I = iradon(tmp,theta);
figure;imshow(I,'InitialMagnification', 'fit');
```

Solution:

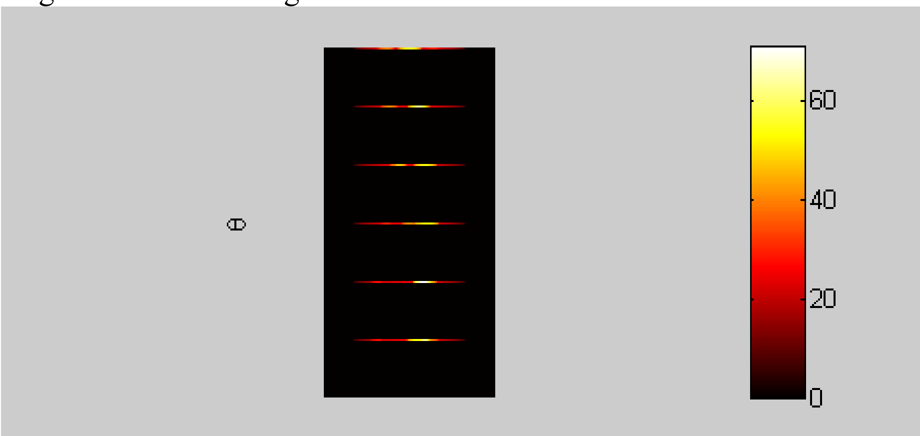
(a)--(i) 1 degree increment sinogram



(a)--(ii) 5 degree increment sinogram

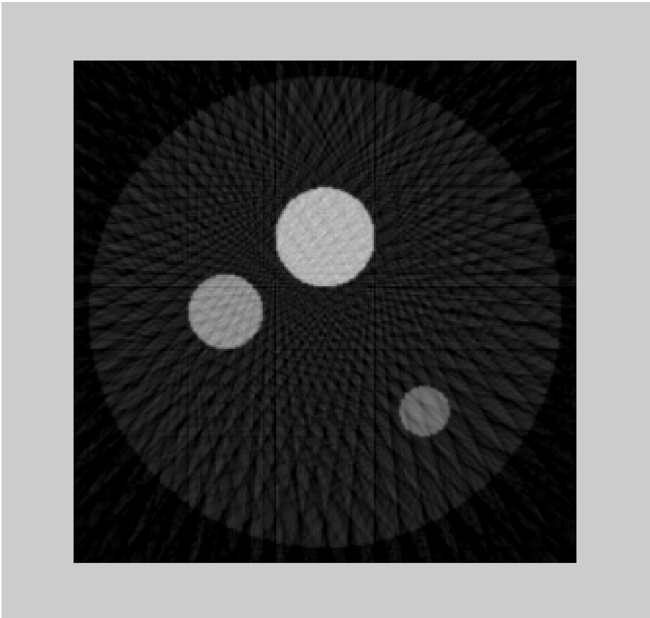


(a)--(iii) 30 degree increment sinogram



(b)-(i) Reconstructed image with 180 projections at1 degree increment

(b)-(ii) Reconstructed image with 36 projections at 5 degree increment



(b)-(iii) Reconstructed image with 6 projections at 30 degree increment