Homework #8

(1) What will happen to a patient, if the patient is placed under the influence of a strong externally applied magnetic field? (hint: consider the protons of the tissue)

Solution:

The protons become magnetized and align with the magnetic field and precessional spin.

(2) Will individual spins (here spin and proton are considered synonymous) be also influenced by the external applied magnetic field? Explain in words as well as with a proper mathematical formula.

Solution:

When the proton's magnetic field interacts with the external field, a force (torque) on the proton causes a precession about its axis, much the same way a spinning top wobbles or rotates about its axis due to the force of gravity acting upon it. This precessional motion occurs at an angular frequency (ω) proportional to the magnetic field strength *Bo*.

The Larmor equation describes the dependency between the magnetic field and the precessional frequency

$$\omega = \gamma \times B_0$$

or

$$f = \left(\frac{\gamma}{2\pi} \right) \times B_0$$

Where ω is the angular frequenncy of rotation,

γ is the gyromagnetic ratio unique to each element

B_0 is the magnetic field strength in Tesla (T),

f is the linear frequency in MHz

(γ /2π) is the gyromagnetic ratio expressed in MHz/T, its unique

(3) The sample containing ^{13}C was placed under the influence of an externally applied magnetic field, B_0 = 2.5 tesla. The corresponding precessional frequency of the element was determined as 26.76 MHz. What is the gyromagnetic ratio for the element (^{13}C)?

Solution:

The externally applied magnetic field B_0=2.5 tesla and
the precessional frequency f=26.76 MHz.

Gyromagnetic Ratio (^{13}C) = f / B_0 = 26.76 / 2.5 = 10.7 MHz/T

(4) A sample containing ^1H is under a magnetic field, B_0 = 1.5 tesla, and a z gradient of G_z =3 gauss/cm. The sample is 0.5m in length along **z** axis, centered at $z=0$. (a) What is the range of linear precessional frequency of the protons in the sample? (b) What is the linear precessional frequency for the protons at $z = 0$? Note that the gyromagnetic ratio of ^1H is $\gamma/2\pi$ = 42.58 MHz/tesla.

Solution:

(a)

$$f(z) = \frac{\gamma}{2\pi}(B_0 + G_z z)$$

In this case, G_z=3gauss/cm=3×10^{-4}Tesla/10^{-2}m=3×10^{-2}Tesla/m.

The ranges of precessional frequency within the slab of sample is between:

$$f_{\min} = \frac{\gamma}{2\pi}(B_0 - G_z z_1) = 42.58\times(1.5T - 0.03T/m\times0.25m) = 63.55(MHz)$$

$$f_{\max} = \frac{\gamma}{2\pi}(B_0 + G_z z_2) = 42.58\times(1.5T + 0.03T/m\times0.25m) = 64.19(MHz)$$

(b)

The gyromagnetic ratio of ^1H is $\gamma/2\pi$ = 42.58 MHz/tesla.

So at z=0, linear precessional frequency is

$$f(z) = \frac{\gamma}{2\pi}(B_0 + G_z z) = \frac{\gamma}{2\pi}B_0 = 42.58\times1.5 = 63.87(MHz)$$