ECE 5843 MEDICAL IMAGING SYSTEMS

Handout #1 X-Ray

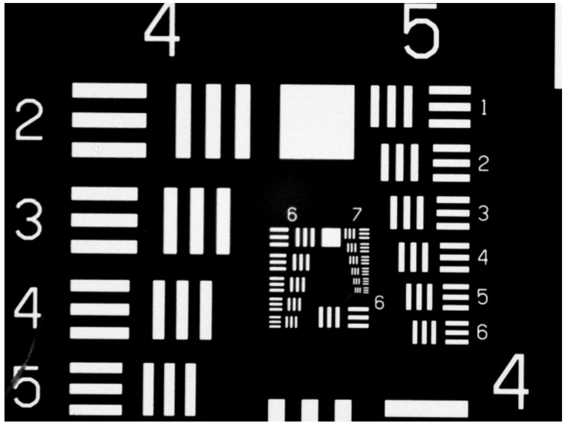
1.1 Background and Image Formation

(1) How an object is visualized?

(a) Light

(b) Reflection/transmission

(c) Contrast against background



(2) Image acquisition

Diagram: Eye perception

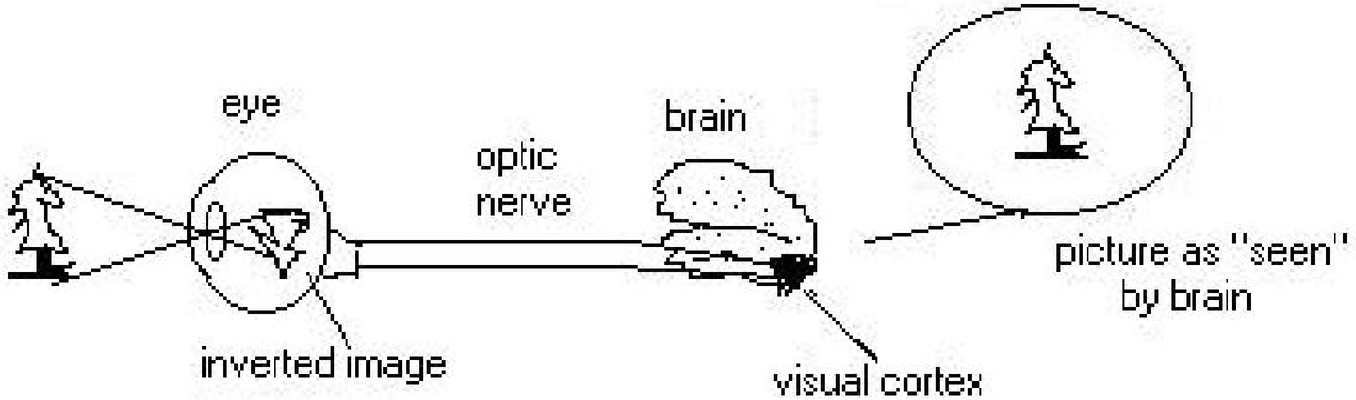
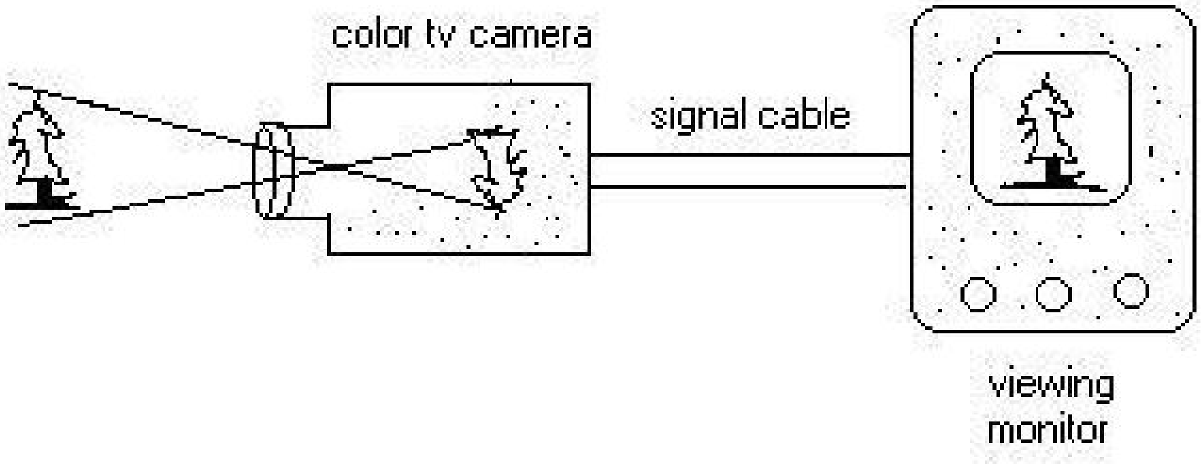


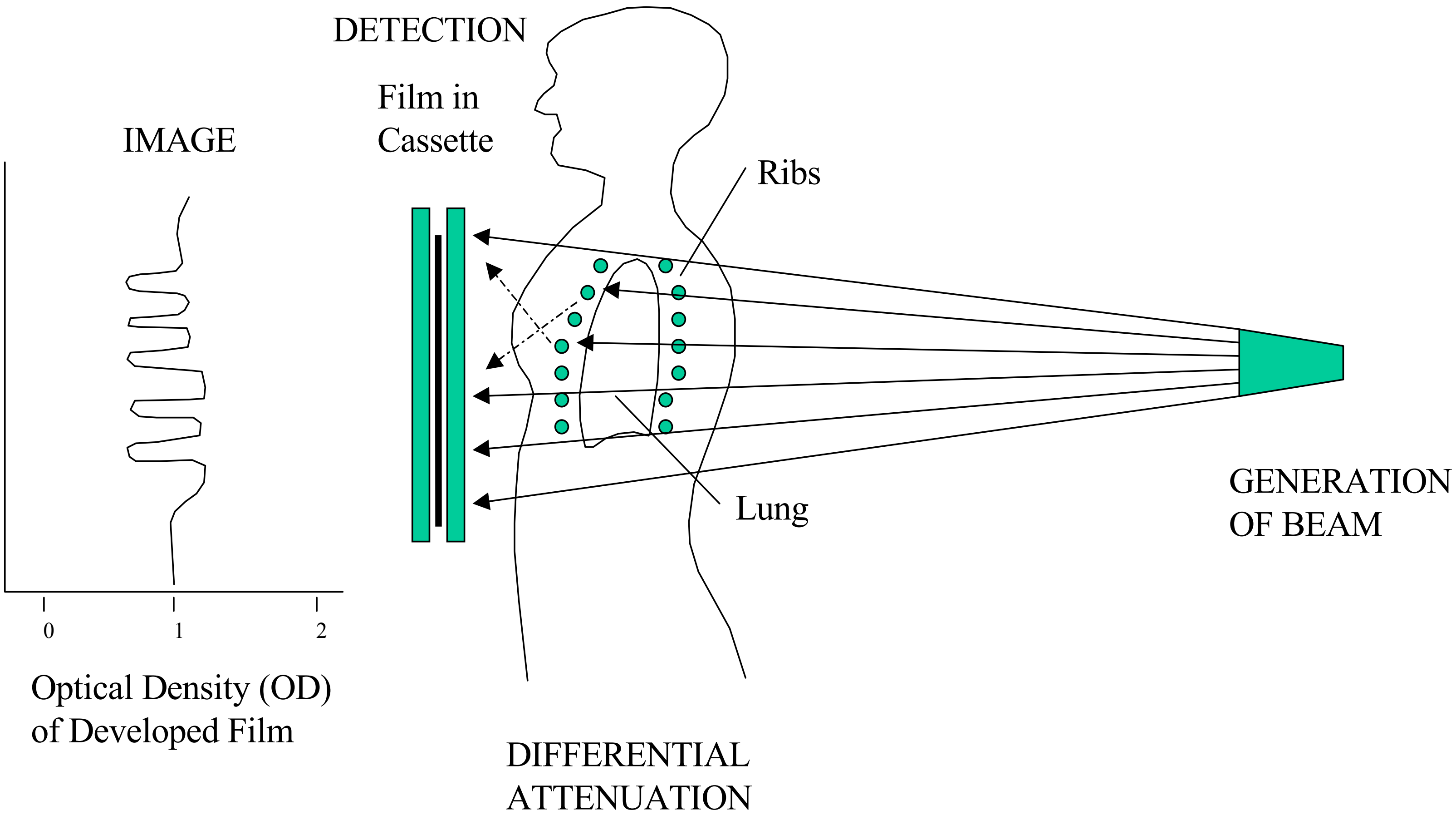
Diagram: Image formation with a lens



1.2 Radiographic Imaging and Source Considerations

(1) Photography and radiography

Radiography: “shadow” imaging



(2) There are four processes in radiographic imaging:

(a) **Generation** of x-ray beams:

- ❖ The beam is relatively uniform and travels in straight lines

(b) Differential **attenuation** of the x-ray beam by the tissues and organs of the patient:

- ❖ Different parts of the patient's body absorb and scatter different amount of x-ray, and thereby sculpt the primary x-ray image out of the formerly uniform beam.

(c) **Detection** of the radiation exiting the body by film or other detectors:

- ❖ Any nonuniformities in the emerging beam recorded on film are revealed, on its development, as spatially varying shades of gray which forms the final visual image.

(d) **Analysis and interpretation** of the image:

- ❖ This depends on the quality of the final image, the viewing conditions, and the skills of the physician.

1.3 Fundamentals of X-Ray

(1) Electromagnetic radiation

❖ What is **x-ray**?

Electromagnetic (EM) radiation

X-ray is just like light, but invisible light.
(We can't see it, because it is out of the range of our eyes)

❖ EM **radiation**

Can be described as waves, and as particle-like unit of energy (photons or quanta)

Wave Characteristics:

Amplitude	A	
Wavelength	λ,	
Frequency	ν,	
Period	T,	$T = 1/\nu$

$$C = \lambda \times \nu$$

and C is propagation speed. In a vacuum, the light speed is a constant:

$$c = 3 \times 10^8 \text{ m/sec}$$

Different light has different wavelength:

Visible light: λ from 700nm (red) to about 400nm (violet)

Diagnostic X-ray: 20keV to 150keV (0.062 nm to 0.00827 nm)

Particle Characteristics

The amount of energy in a quantum:

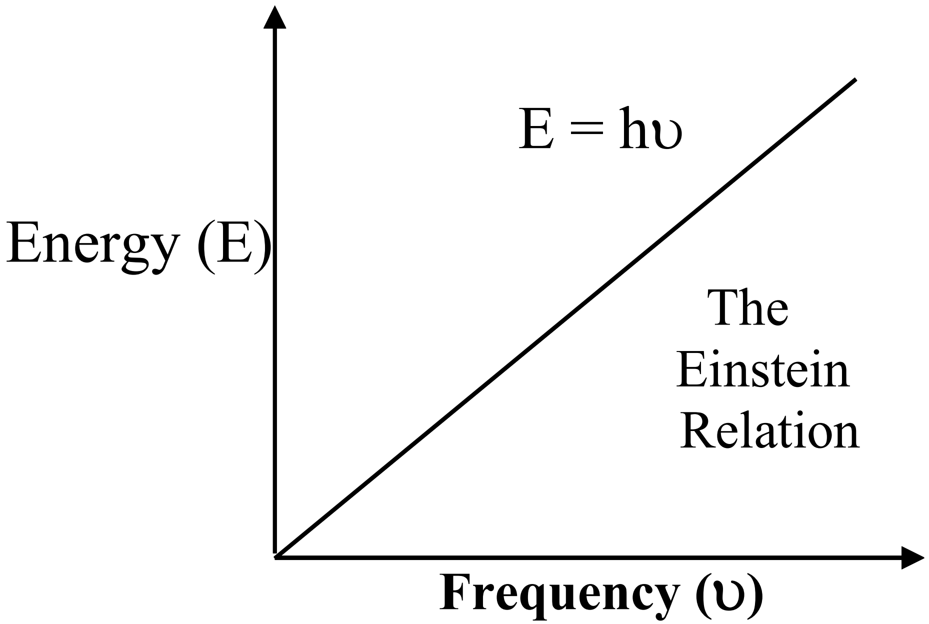
$$E = h \times \nu$$
$$= h\,C / \lambda$$

($h = 6.62 \times 10^{-34}$ J·s
$= 4.13 \times 10^{-18}$ keV·s **Planck constant**)

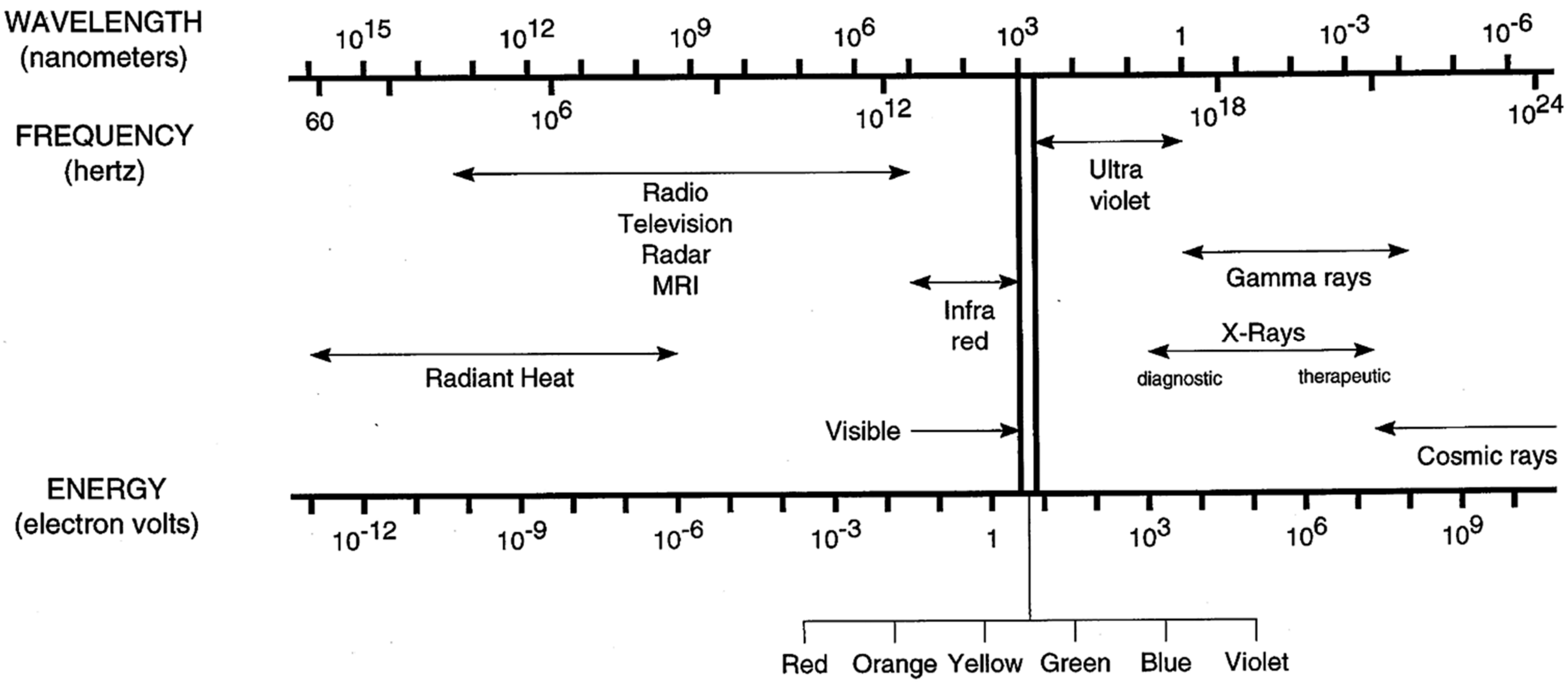
$E\ (\mathrm{KeV}) = 1.24\ /\ \lambda\ (\mathrm{nm})$

$1\ (\mathrm{eV}) = 1.6 \times 10^{-19}\ (\mathrm{J})$

Einstein Relation:



❖ The energy of a photon (a particle-like concept) is proportional to its (wavelike) frequency – a critically important idea sometimes known as the Einstein relation. Units were intentionally omitted from this diagram to emphasize the validity of relationship for all photon frequencies, in the radio frequency range (kilo- and megahertz) as well as for **x-ray and gamma ray photons** (10^{20} Hz and higher). Like the speed of light, **Planck's constant**, *h*, is one of the fundamental constants of nature. This expression also relates the frequency of the "probability wave" associated with a "particle", such as an electron, to its energy.



The Electromagnetic Spectrum

Diagnostic X-ray: E from **20keV to 150keV**

(2) Exercise

Visible light ranges from about 700nm to 400nm in wavelength. What are the frequency and energy (in eV) of orange light photons of 600nm ?

Step one:

$$\begin{aligned} \nu &= C/\lambda \\ &= (3 \times 10^{8}\ \text{m/sec}) \ / \ 600 \times 10^{-9}\ \text{m} \\ &= 5 \times 10^{14}\ \text{Hz} \end{aligned}$$

Step two:

$$\begin{aligned} E\ (\text{keV}) &= 1.24 \ / \ \lambda\ (\text{nm}) \\ &= 1.24 / 600 \\ &= 0.0021\ \text{keV} \\ &= 2.1\ \text{eV} \end{aligned}$$

Summary:

1. Photography and Radiography
2. Relationship between Wavelike concept and Particle-like concept

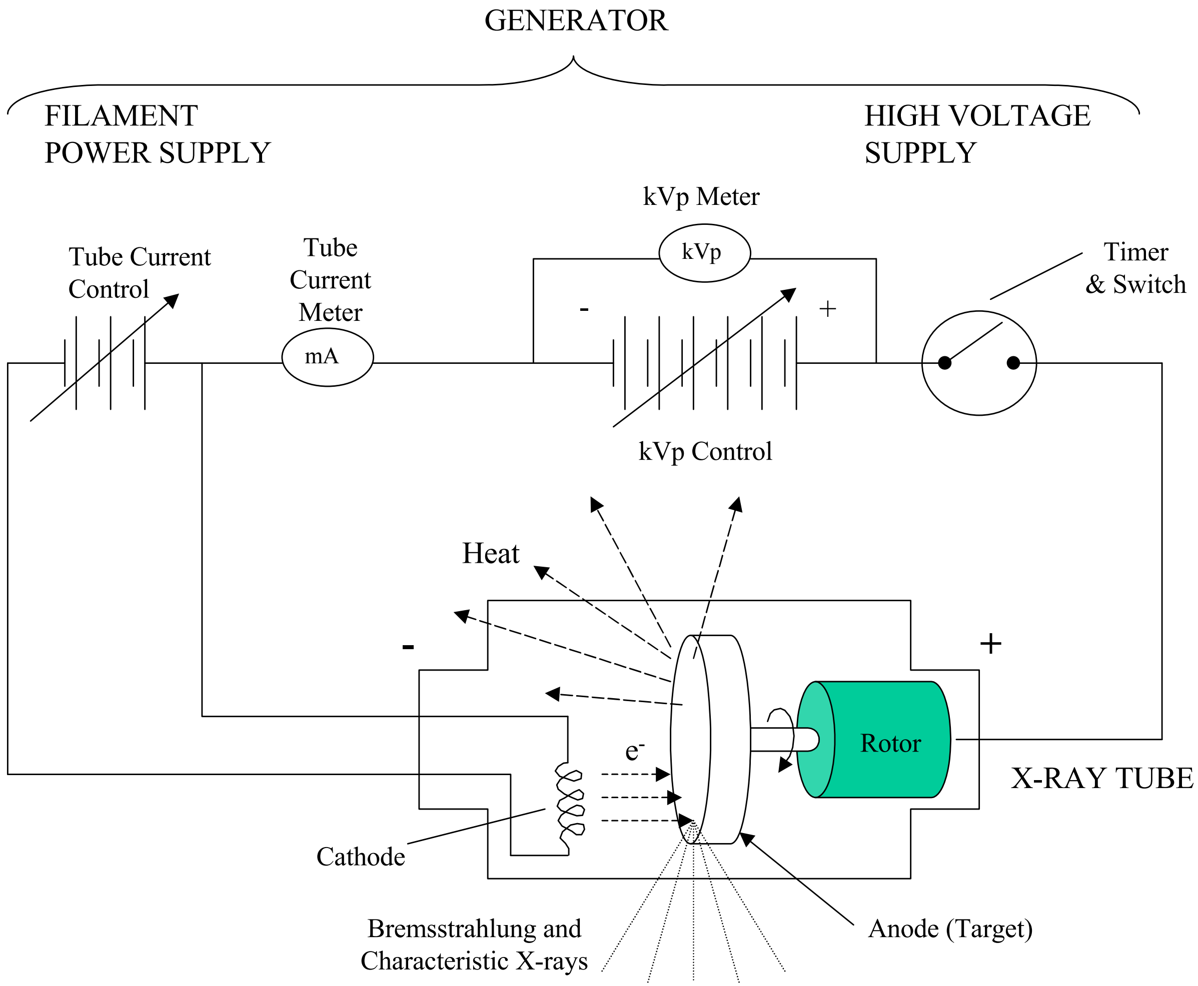
$$E = h \times \nu$$
$$= h\,C / \lambda$$

1.4 Production of X-Ray and Radiation

(1) X-ray tube

X-ray photons are generated by energy conversion when a fast moving stream of electrons is suddenly decelerated in the target anode of an x-ray tube.

The most common target materials are (**tungsten or molybdenum**)



X-ray Tube

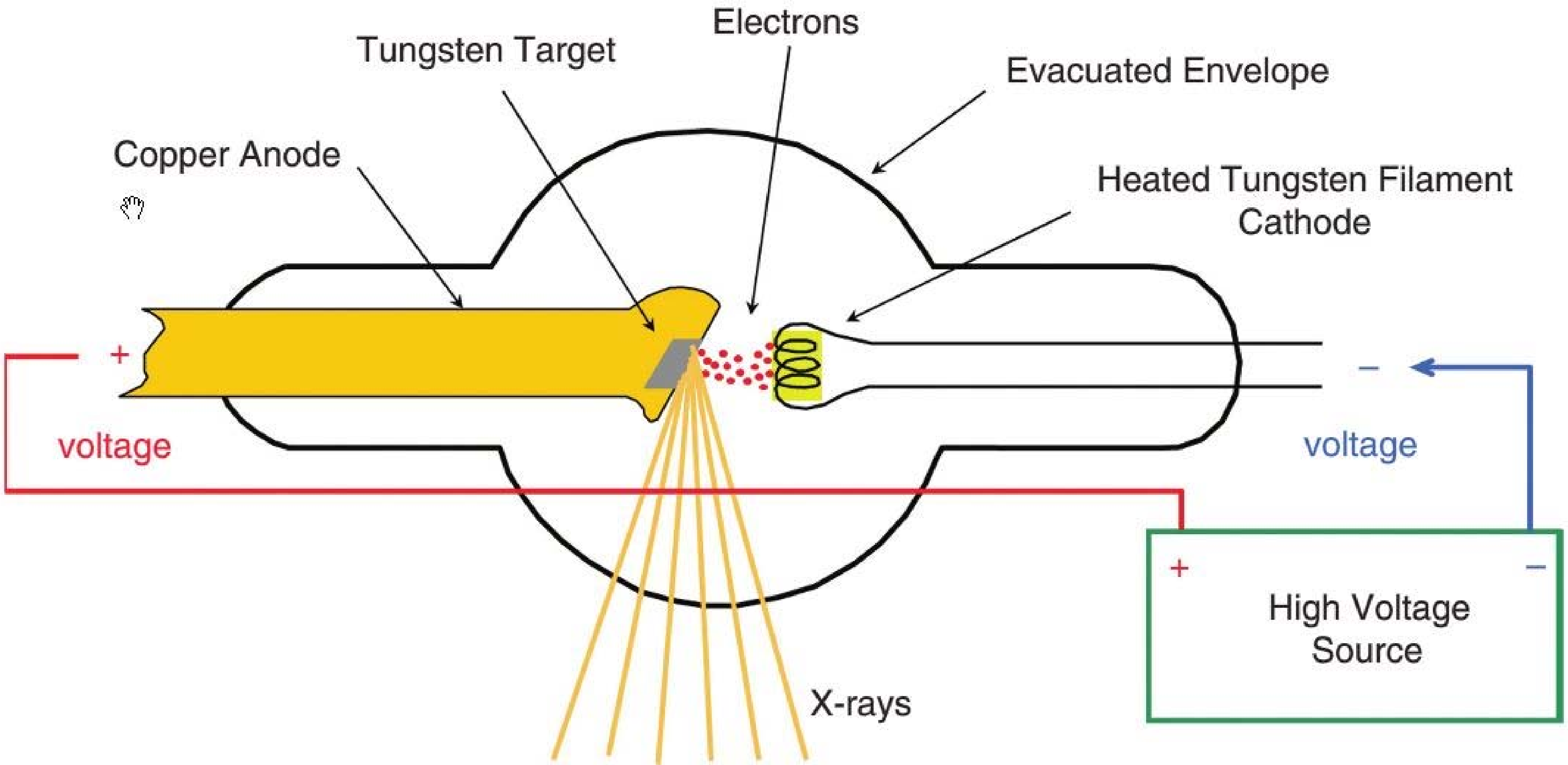


Fig 6-1 Minimum requirements for x-ray production include **a source and target of electrons**, an evacuated envelope, and connection of the electrodes to a high-voltage source. (page 172, The Essential Physics of Medical Imaging, Third Edition)

X-ray Tube

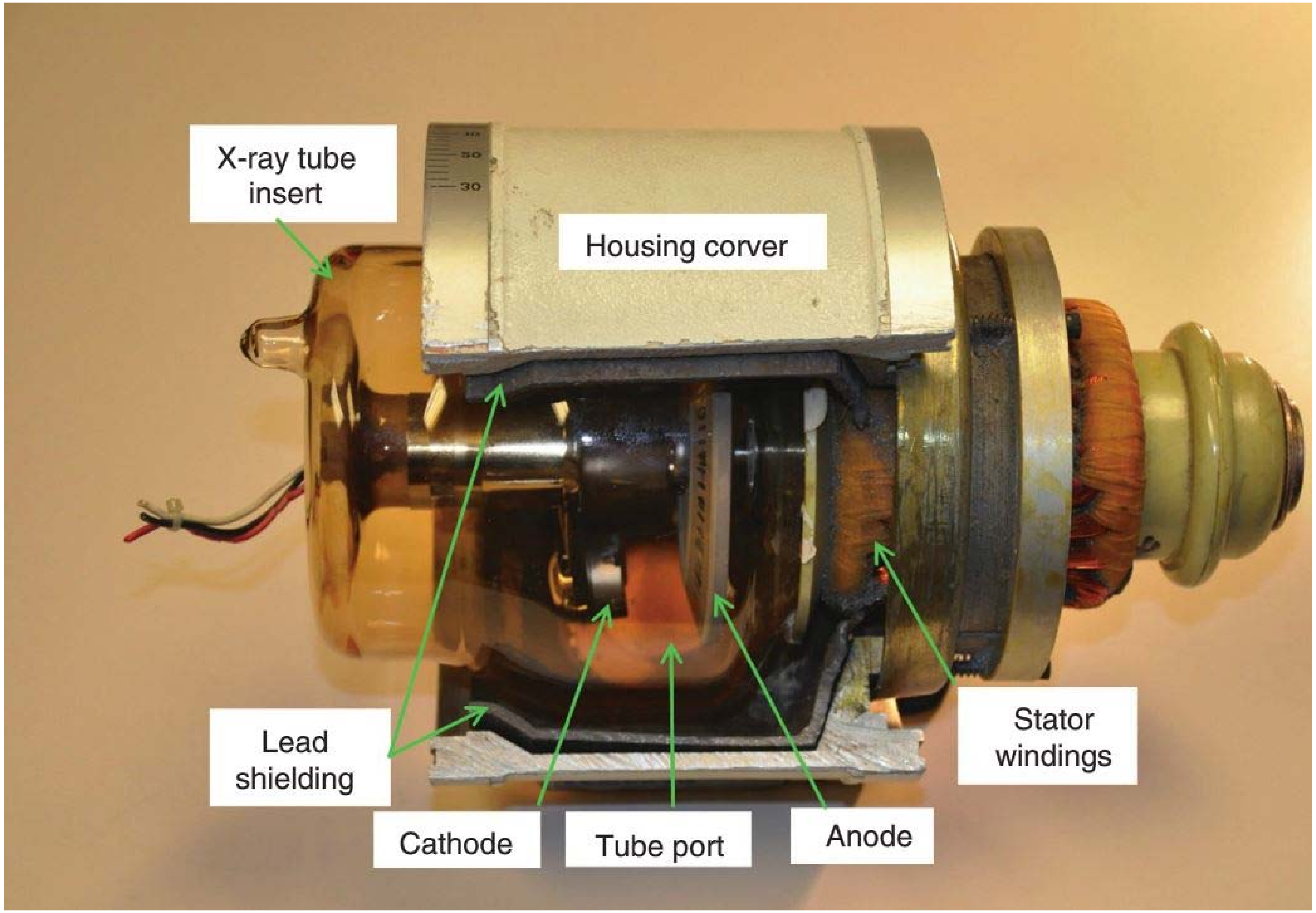


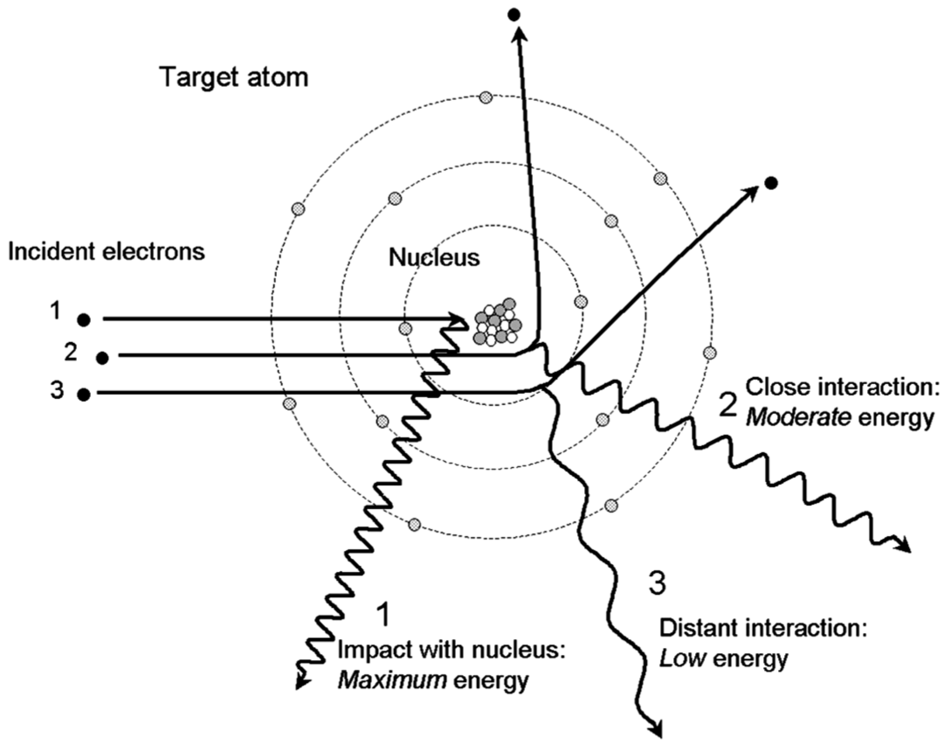
Fig 6-7 Picture of an x-ray tube insert and partially cut-away housing, shows the various components of the x-ray tube. For this housing, the lead shielding thickness is 2 mm. (page 177, The Essential Physics of Medical Imaging, Third Edition)

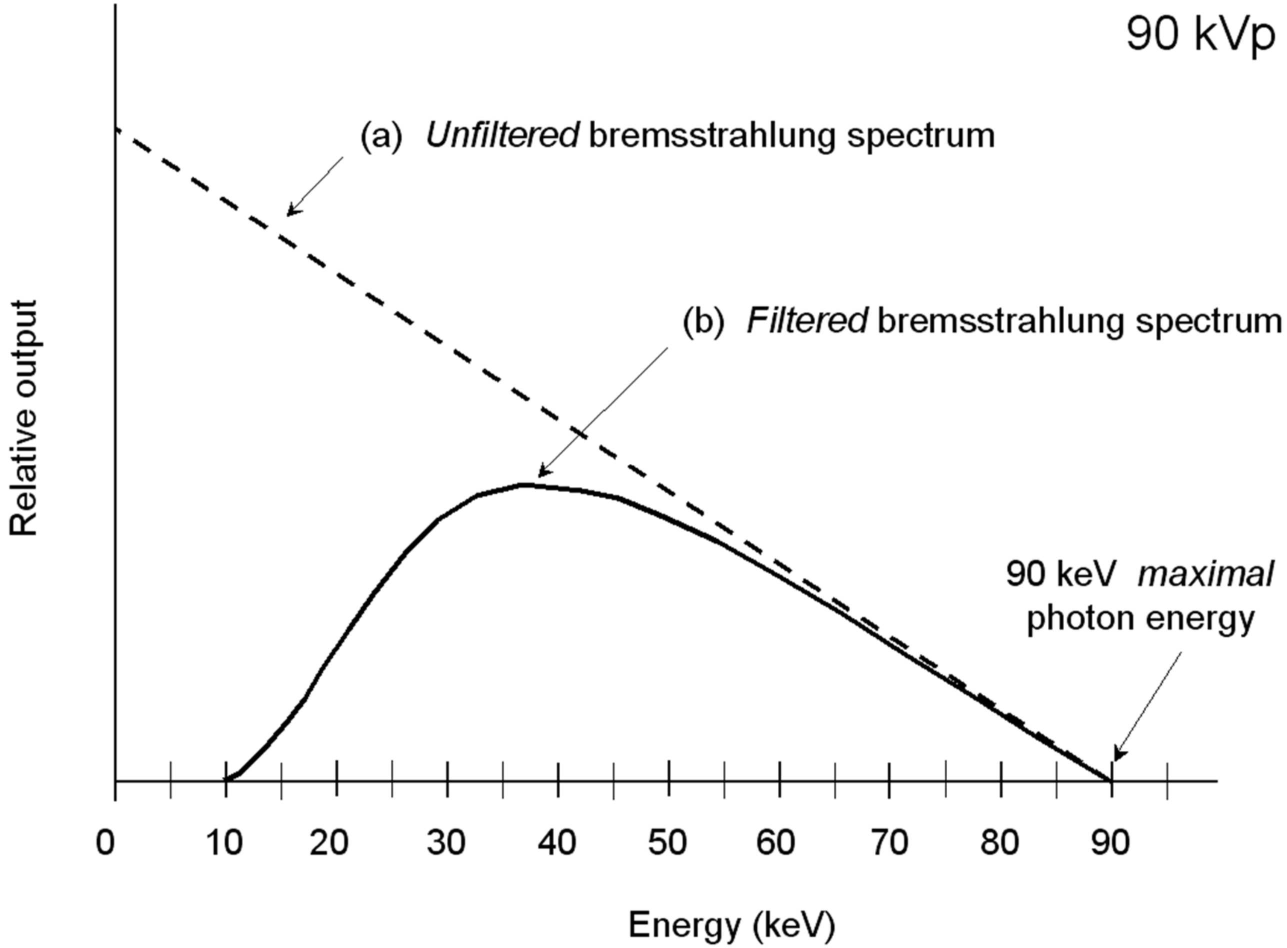
(2) X-rays are generated by two different processes:

(a) Bremsstrahlung

- ❖ Electrons are decelerated, resulting in loss of **Kinetic energy**. The lost energy is emitted in the form of x-ray photons. The spectrum of Bremsstrahlung is continuous.

- ❖ Creation of bremsstrahlung radiation: Electrons interact with an atomic nucleus of the target atom via coulombic attraction, causing a conversion of kinetic energy to electromagnetic (x-ray) energy.



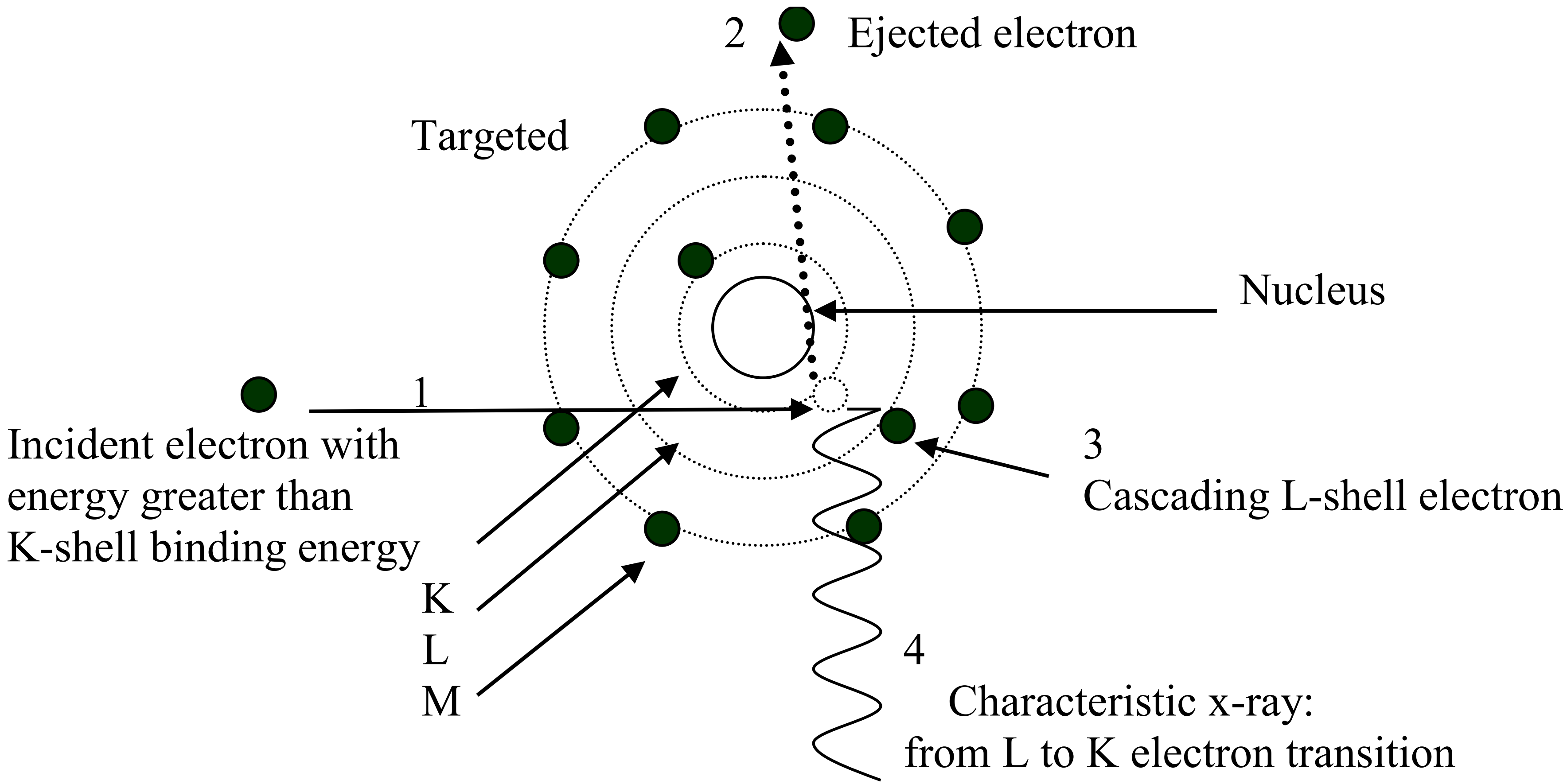


❖ The distribution of energies in x-ray spectrum produced by the interaction of 90 keV electrons with a target: (a) Unfiltered bremsstrahlung spectrum; (b) Typical bremsstrahlung spectrum with inherent and added x-ray tube filtration.

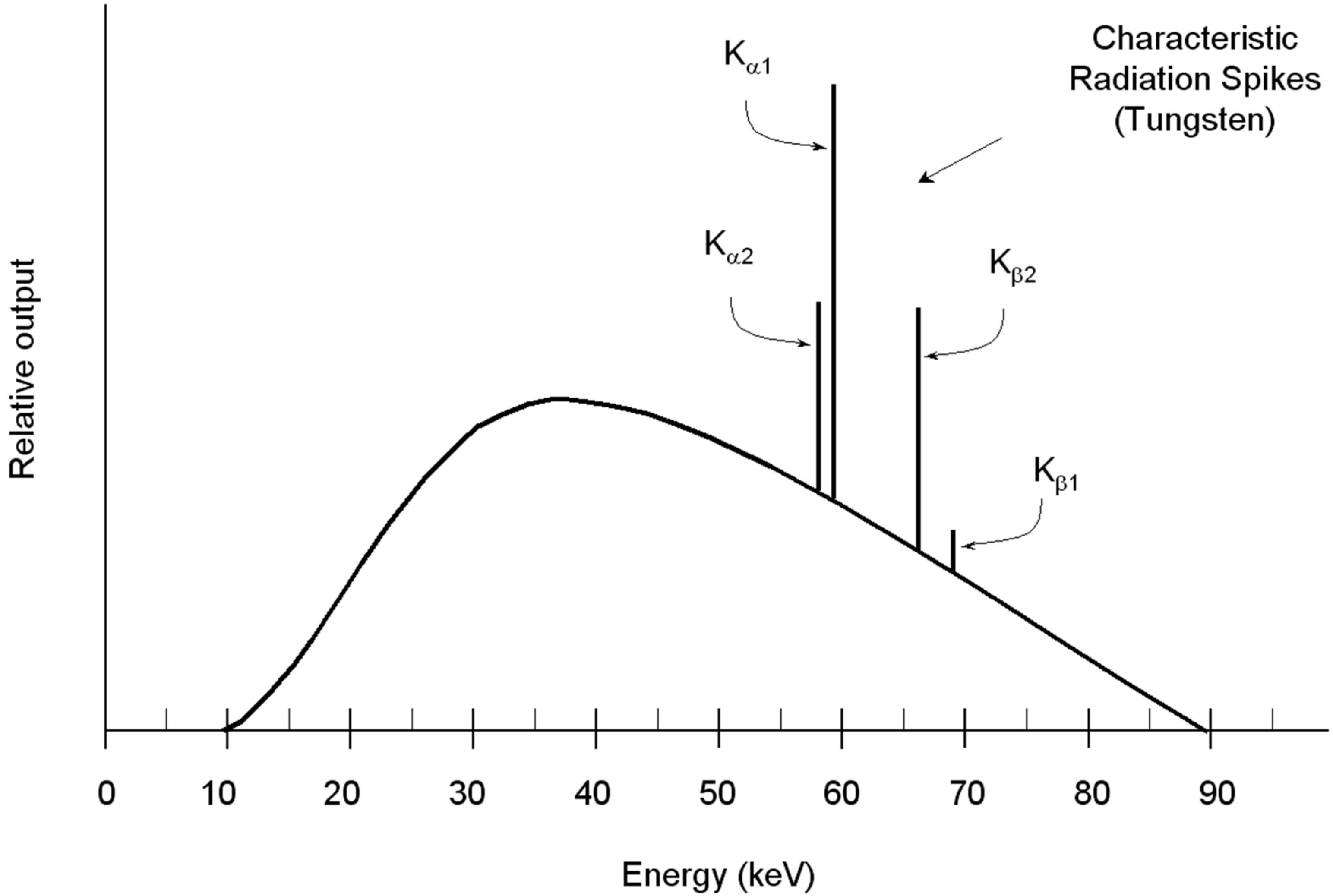
(b) Characteristics radiation

- Electron interacts with orbital electrons in inner shells, lose energy that is emitted in the form of an x-ray photon. The spectrum of characteristic radiation is discrete.

See a diagram in next slide



- ❖ Generation of **characteristic x-rays** in a target atom occurs in the following sequence:
  1) Incident electron impacts on K-shell orbital electron.
  2) Orbital electron is ejected with energy equal to the difference of incident electron energy from the binding energy of the K-shell electron.
  3) Cascading L shell electron fills vacancy.
  4) Emission of a characteristic K_{α} x-ray photon.



❖ Total spectrum of x-ray tube with tungsten target: At a potential difference of 90 kVp bremsstrahlung plus characteristic radiation.

(c) Filters and its impact to x-ray spectrum

- X-rays generated by x-ray tube are **polychromatic**. Only a portion of the energy spectrum is desirable. Therefore, the radiation dose to the patient can be substantially reduced by filtering out the undesired portion.
- **Aluminum** is an excellent absorber for low energy x-rays, while **copper** is useful for higher energy x-rays. They can be used in **combination.**

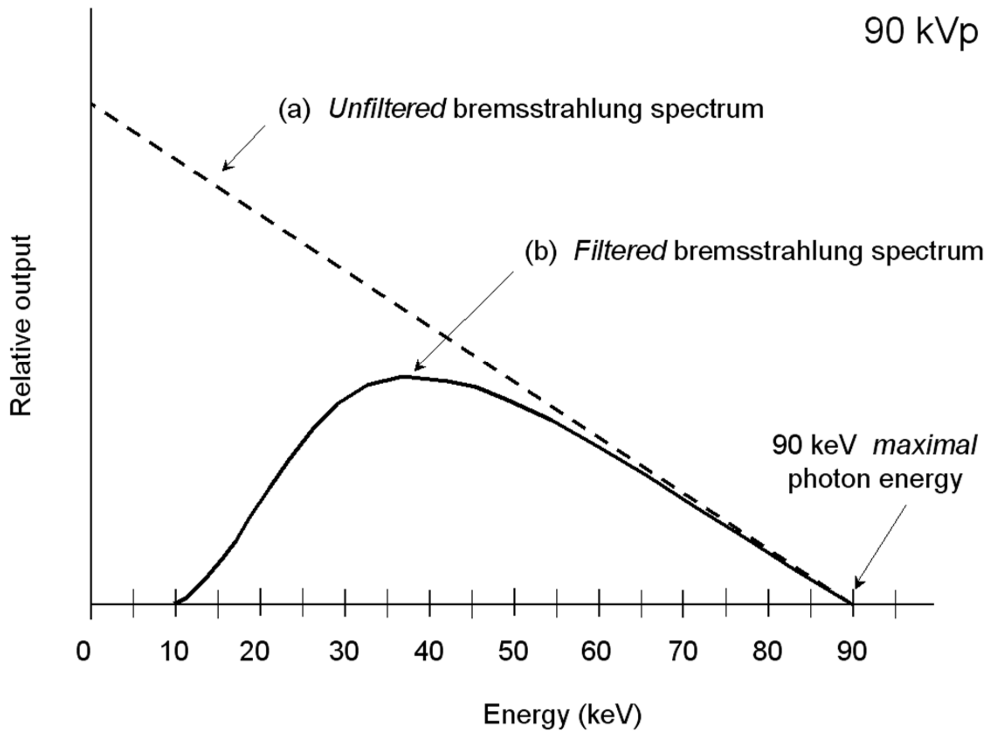


Figure: The distribution of energies in x-ray spectrum produced by the interaction of 90 keV electrons with a target: (a) Unfiltered bremsstrahlung spectrum; (b) Typical bremsstrahlung spectrum with inherent and added x-ray tube filtration.

(3) The quality and quantity of x-ray beam

Quality: Penetrability of x-ray beam (kVp - Target material)
Quantity: Number of photons (kVp, mA, exposure time, Target material)

(a) X-ray photon wavelength and the concepts of **kVp, keV**

The wavelength of x-ray photons is usually presented by its energy:

$$\lambda \ (\text{nm}) = 1.24 / E \ (\text{keV})$$

- ❖ The wavelength of x-ray photons produced when the electrons are braked by the tungsten nuclei in the target is related to the energy (**keV**) of the electron.
- ❖ The energy of the electron is related to the potential difference (**kVp**) applied across the x-ray tube
- ❖ **KVp** determines the maximum **keV** in the Bremsstrahlung spectrum
- ❖ The number of x-ray photons generated is proportional to the square of the kVp, $N \propto kVp^2$

(b) X-ray tube current

- ❖ The number of x-ray photons generated depends directly on the tube current used **(mA)**

(c) Duration of X-ray exposure (time in seconds)

- ❖ The number of x-ray photons generated depends directly on the duration of exposure (**s**)

(d) Concept of **mAs**

$$mAs = \text{tube current } (mA) \times \text{exposure time } (s)$$

(e) Number of x-ray photons vs **mAs** and **kVp**

$$N \propto mAs$$

$$N \propto kVp^{2}$$

Exercise:

An x-ray tube operating at 60kVp, 100mA, 0.1second, generates a x-ray beam, and the number of x-ray photons per unit area at a given location is determined as 100,000. What will be the number of x-ray photons per unit area at the same location when the x-ray tube is operated at 120kVp, 25mA, 0.1 seconds?

Solution:

At 60 kVp: $N_1 \propto (60\text{kVp})^2 \times 10\text{mAs}$

At 120kVp: $N_2 \propto (120\text{kVp})^2 \times 2.5\text{mAs}$

$N_1 = 100{,}000$

$N_1 / N_2 = (60/120)^2 \times (10/2.5) = 1$

$N_2 = N_1 = 100{,}000$

Exercise

Discuss the difference of kVp and keV ?

(4) Quantification of radiation

Radiation exposure: (Ionization in air)	**Roentgen (R)**	2.58×10^{-4} **C/Kg** of air
Radiation dose: (Radiation delivered to tissue)	**Rad**	1 **Rad** = 0.01 **Gray (Gy)**
Dose equivalent:	**Rem**	1 **Rem** = 0.01 **Sievert (Sv)**

Table: Quantification of Radiation

	US	**SI**	**Relationship**	**Note**
Exposure	R	C/Kg	2.58×10^{-4} C/Kg	A measure of the output of an x-ray tube
Dose	Rad	Gray	1Gray=100 Rad	A description of the deposition of ionization radiation in matter
Dose equivalent	Rem	Sievert	1 Sievert =100 Rem	Used in consideration of health risk associated with radiation

Summary:

(1) X-ray photons are generated by energy conversion when fast-moving electrons are suddenly decelerated in the target of an x-ray tube.

(2) X-rays are generate by two different processes, resulting in (a) the production of a continuous spectrum of x-rays (bremsstrahlung) and (b) characteristic radiation.

(3) X-ray beam quality -- **kVp** and target material

(4) $N \propto mAs$

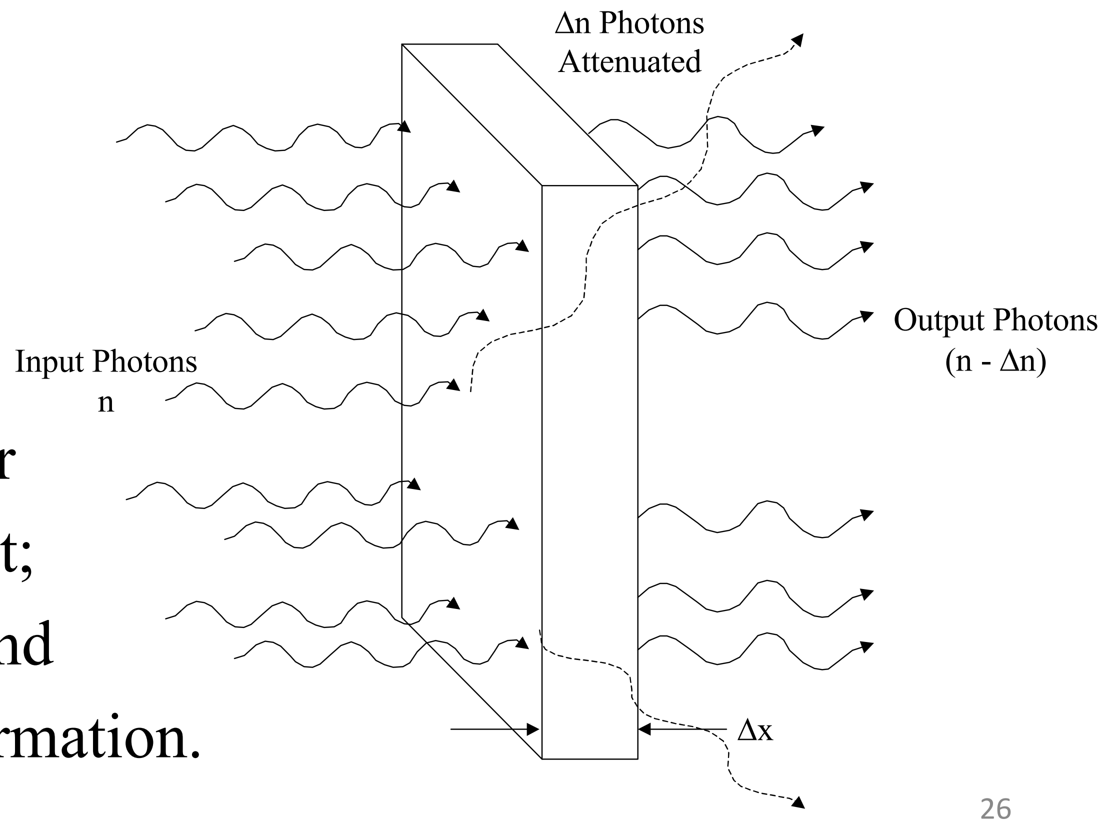
$N \propto kVp^{2}$

1.5 Interactions Between X-Ray and Matter

The x-ray beam incident on a patient is nearly uniform. The beam emerging from the patient is spatially modulated; the various parts of the beam pass through different types and thickness of tissues, and are attenuated by different amount.

(1) Basic ways that an x-ray photon interacts with matter

- Coherent scattering
- **Photoelectric effect**
- **Compton scattering**
- Pair production
- Photo disintegration



The x-ray photons are either

ABSORBED: cease to exist;

SCATTERED: deflected and no longer carry useful information.

(2) Linear attenuation coefficient (μ)

$$N = N_0 \times e^{-\mu \times x}$$

N: Number of the transmitted photons
N_0: Number of incident photons
X: absorber thickness (cm)
μ: Linear attenuation coefficient (1/cm)

The above equation is known as **Lambert - Beer Law**

Exercise

The linear attenuation coefficient, μ, of a particular beam in a given material is 0.231 cm^{-1} . What fraction of x-rays (number of x-ray photons) remains after passage through 5cm?

Solution:

$$N / N_0 = e^{-(0.231\ cm^{-1}) \times (5cm)} = e^{-1.115} = 0.315$$

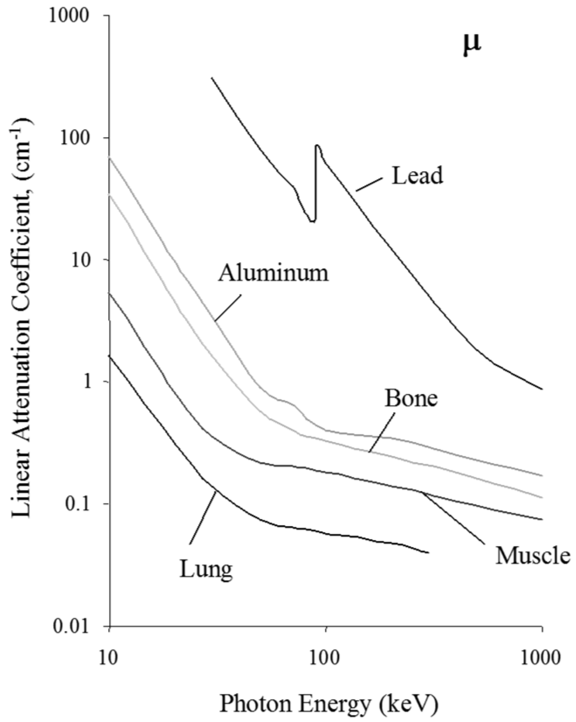
(3) Mass attenuation coefficient (μ/ρ)

ρ: the density of the material (gm/cm^3)

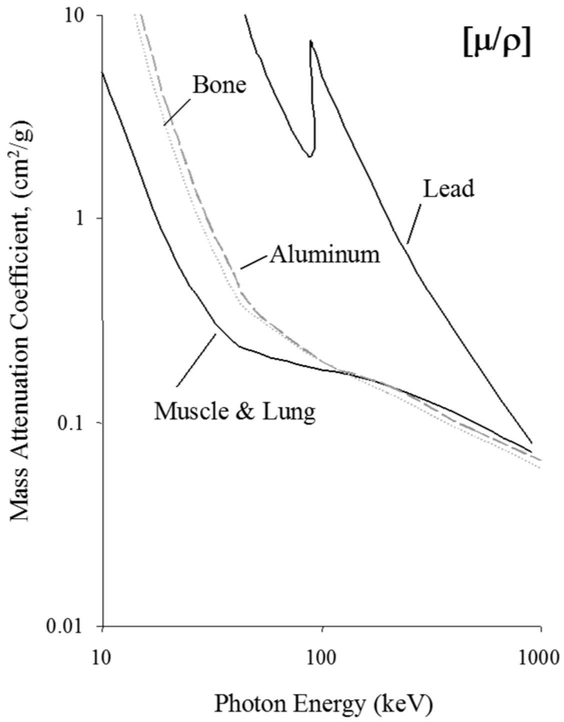
The unit of mass attenuation coefficient (**cm^2 / gm**)

- ❖ The linear attenuation coefficient is **dependent** on the density of the material.
- ❖ The mass attenuation coefficient is **independent** on the density of the material.

Linear Attenuation Coefficients vs Photon Energy



Mass Attenuation Coefficients vs Photon Energy



(4) Concept of half-value layer (HVL)

An indirect measure of x-ray beam quality (Photon energies).

Definition: The thickness of material required to reduce x-ray beam intensity to one-half, under certain conditions

Basic formula:

$$N = N_0 \times e^{-\mu \times x}$$

Let

$$N / N_0 = ½,$$

We have:

$$½ = e^{-\mu \times HVL}$$

$$\mathit{ln}\ (1/2) = -\mu \times HVL$$

Then:

$$HVL = 0.693 / \mu$$

SUMMARY:

This section covers:

1. Physics and clinical background of medical imaging
2. X-ray generation and interaction with matter

The key concepts are:

(1) X-rays are the source for radiography

(2) X-ray photons are generated by energy conversion when fast-moving electrons are suddenly decelerated in the target of an x-ray tube.

(3) X-rays are generate by two different processes, resulting in (a) the production of a continuous spectrum of x-rays (bremsstrahlung) and (b) characteristic radiation.

(4) Only two basic interactions between x-rays and matters are important in diagnostic radiology: Compton scattering and photoelectric effect.

(5) The amount of x-ray attenuation depends on the energy of the radiation (keV) and the characteristics of the tissue. Linear attenuation coefficient and mass attenuation coefficient is useful concept for calculations

(6) Radiation safety: The concepts of x-ray exposure (R) and absorbed dose (Rad)

Handout #1 Appendix - Inverse Square Law

(1) Basic radiometric quantities

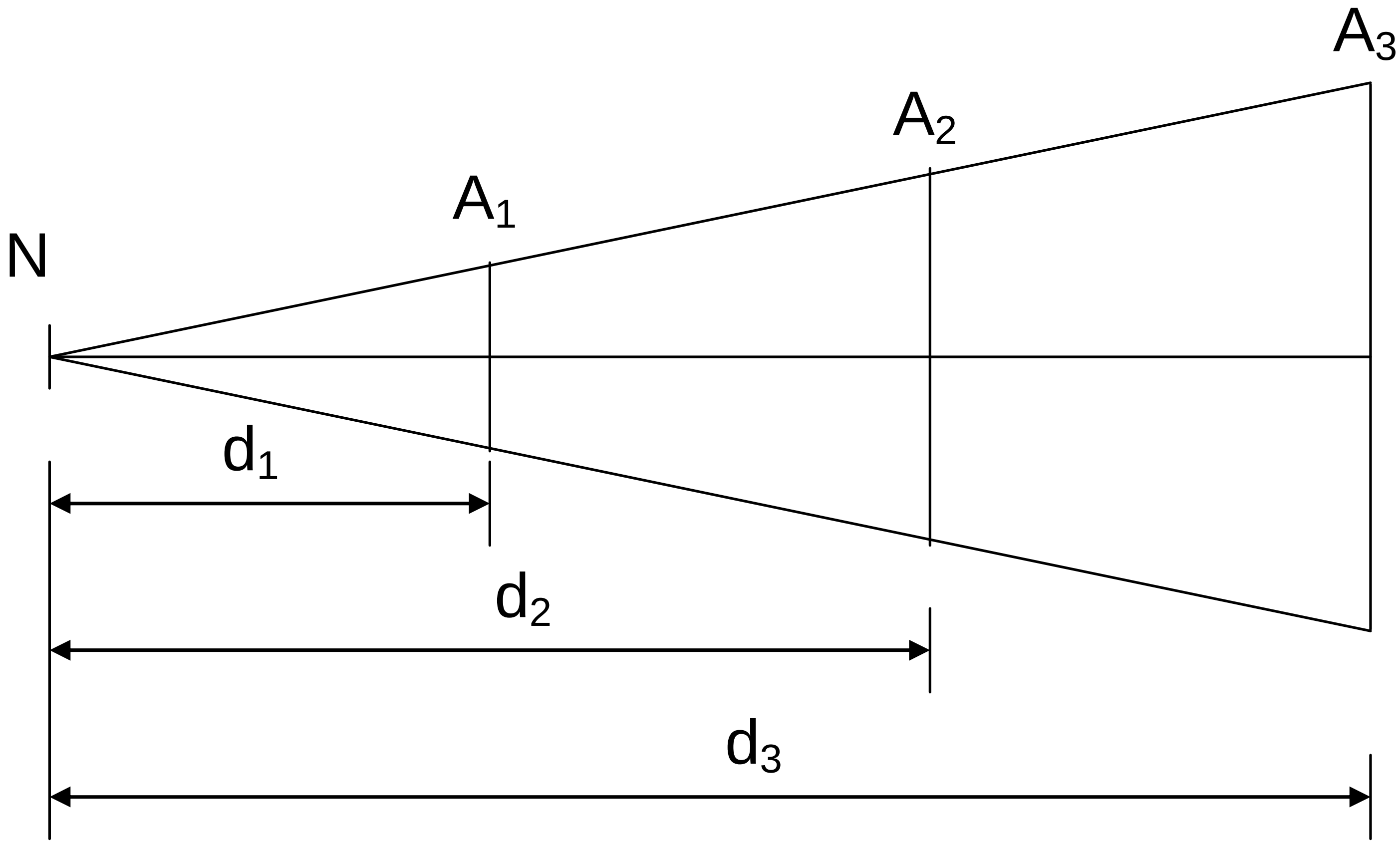
Table Basic radiometric quantities

Name	Symbol	Equation
Number of Photons	N	
Fluence	Φ	$\Phi = \dfrac{Photons}{Area} = \dfrac{N}{A}$
Flux	Φ/S	$\Phi / S = \dfrac{Photons}{Area \times Time} = \dfrac{N}{A \times T}$

- **Fluence:** the number of photons or particles passing through a unit cross-sectional area ***A***.
- **Flux:** the fluence rate (e.g., the rate at which photons or particles pass through a unit area per unit time).

(2) Inverse square law

Consider the transfer of radiation from a **small source** at the left in the following figure.



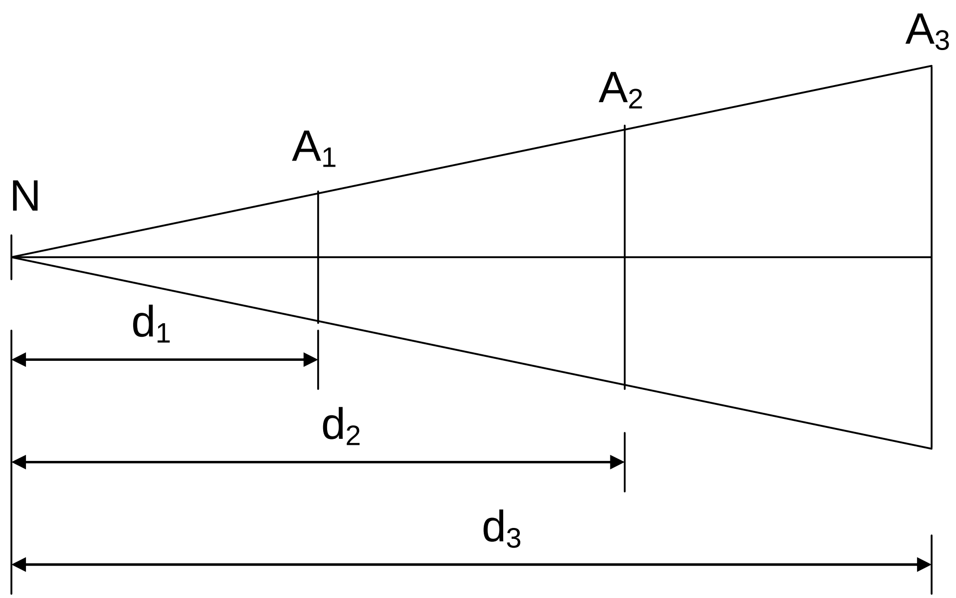
- The distance of the several areas are d_1, d_2 and d_3 etc.
- By the first law of thermodynamics, the number of photons ***N*** must be the **same** on area A_1, A_2 and A_3, as it is on all of the areas, only the **fluence Φ change**.
- The fluence Φ_1 can be determined on area A_1 by,

$$\Phi_1 = \frac{N}{A_1}$$

- By the geometry of similar triangles, if the fluence Φ_1 in A_1 position is known, then,

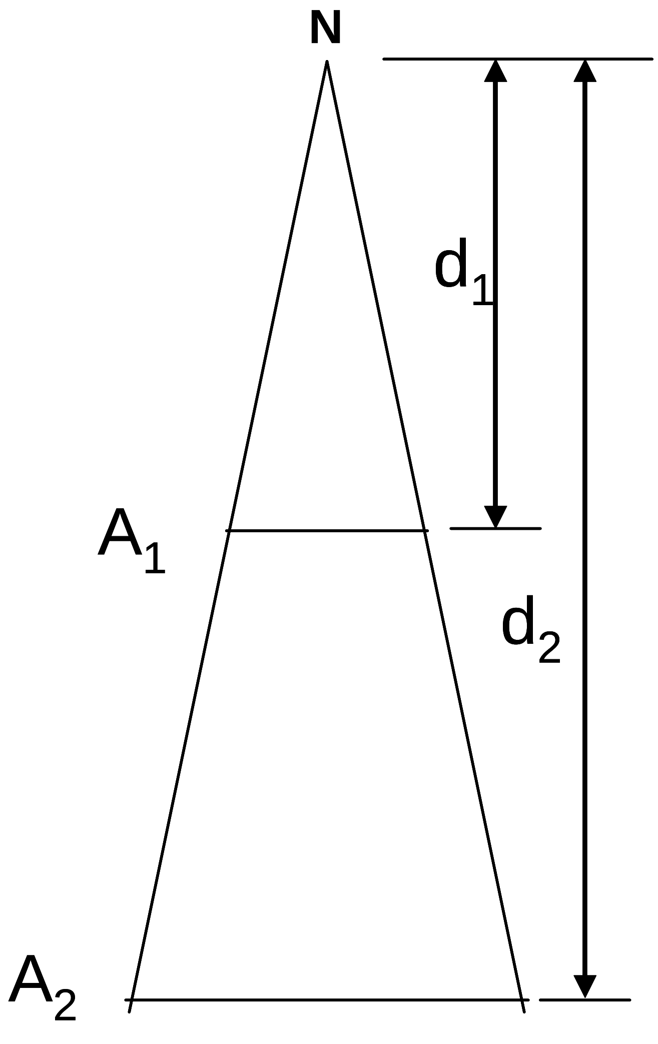
$$\Phi_2 = \frac{d_1^2}{d_2^2} \times \Phi_1$$

$$\Phi_3 = \frac{d_1^2}{d_3^2} \times \Phi_1$$



(3) Example:

The fluence of an x-ray exposure measured at position A_1 is 1.6×10^8 photons per m^2, what is the fluence at position A_2? (d_1=0.5m, d_2=1.0m)



Solution:

$$\Phi_1 = 1.6\times10^8\ photons/m^2$$

$$d_2 = 1.0m,\quad d_1 = 0.5m$$

$$\therefore \Phi_2 = \frac{d_1^2}{d_2^2}\times\Phi_1 = \frac{0.5^2}{1.0^2}\times1.6\times10^8 = 0.4\times10^8(photons/m^2)$$

Homework #1

1. Identify the rang of diagnostic x-ray in terms of wavelength (from ? nm to ? nm); frequency (from ? Hz to ? Hz); and energy (from ? keV to ? keV).

2. Mammography is an x-ray imaging procedure for breast cancer diagnosis and screening. Assume that 20keV x-ray is used in mammography. Also assume that the breast of a patient is 6 cm in thickness (soft tissue), and the propagation speed of x-ray photons in soft tissue is about the same as in vacuum. How long will it take for x-ray photons to travel through the breast?

3. Describe the nature of radiographic image formation (hint: what is the difference between radiography and photography?)

4. A material has a mass attenuation coefficient of 0.35 cm^2/g (for a given photon energy) and has a density of 1 g/cm^3. What is the thickness of the Half-Value Layer (HVL) of this material? (Note: the Half-Value Layer is the material thickness required to reduce an incident beam intensity by a factor of two).

5. Design a technique to measure the Half Value Layer (Hint: assume you have an x-ray Exposure Meter, and thin sheets of Aluminum).

6. An X-ray machine that generates x-ray beam at 32 kVp. You are given a meter that measures x-ray exposure in terms of mR; a ruler; and a slab of Lucite. Please design a procedure to measure the linear attenuation coefficient of the Lucite slab under the beam quality.

7. An x-ray tube operating at 60kVp, 120mA, 0.1second, generates a x-ray beam, and the number of x-ray photons per unit area at a given location is determined as 120,000. What will be the number of x-ray photons per unit area at the same location when the x-ray tube is operated at 100kVp, 50mA, 0.1 seconds?

8. Refer to the following schematic, the fluence of an x-ray exposure measured at position A_1 is 2.4×10^8 photons per m^2, what is the fluence at position A_2? (d_1=0.3m, d_2=1.0m)