HOMEWORK #2.

10/10

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1. Lesion's signal-to-noise ratio:
This is a measure that compares the level of a desired signal to the level of background noise.

$$SNR = \frac{Signal}{Noise}$$

$$SNR = \frac{N_b - N_l}{\cancel{N_b}\sigma} = \frac{N_b - N_l}{\sqrt{N_b}} = \frac{N_b - N_l}{N_b}\sqrt{N_b}.$$

multiplying $\sqrt{N_b}$ for Numerator & Denominator

$$= \frac{N_b - N_l}{N_b} \times \sqrt{N_b}, \text{ here } c = \frac{N_b - N_l}{N_b}$$

$$= c \times \sqrt{N_b}.$$

$$\approx c \times \sqrt{N_l}$$

ie, contrast $\rightarrow c$
$N_l, N_b \rightarrow$ No of photons.

~~[illegible]~~

$$SNR = c \times \sqrt{N_b}.$$

2. If the same object is imaged for twice as long (twice the photons),

$$SNR_{old} = C \times \sqrt{N_b} = C\sqrt{N_b}.$$

$$SNR_{NEW} = C \times \sqrt{2N_b} = C\sqrt{2 \times N_b} \text{ or } 1.414\, C\sqrt{N_b}.$$

3. $$PSF(x,y) = \frac{1}{2\pi} \cdot \frac{e^{-|x|}}{1+y^2} \quad \text{— (1)}$$

$$\int_{-\infty}^{\infty} \frac{e^{-i2\pi vy}}{1+y^2} \cdot dy = \pi \cdot e^{-|2\pi v|} \quad \text{— (2)}$$

Taking eqn (1).

$$MTF(u,v) = F_{2D}\{PSF(x,y)\} = \iint_{-\infty}^{\infty} PSF(x,y) \cdot e^{-i2\pi(ux+vy)} \cdot dx \cdot dy$$

$$= \iint_{-\infty}^{\infty} \frac{1}{2\pi} \frac{e^{-|x|}}{1+y^2} \cdot e^{-i2\pi(ux+vy)} \cdot dx \cdot dy.$$

$$= \frac{1}{2\pi} \left\{ \int_{-\infty}^{0} e^{+x} \cdot e^{-i2\pi ux} \cdot dx + \int_{0}^{\infty} e^{-x} \cdot e^{-i2\pi ux} \cdot dx \right\} \times \int_{-\infty}^{\infty} \frac{e^{-i2\pi vy}}{1+y^2} \cdot dy$$

$$= \frac{\pi \cdot e^{-|2\pi v|}}{2\pi} \left[\frac{1}{(1-i2\pi u)} \cdot e^{x(1-i2\pi u)} \Big|_{-\infty}^{0} + \frac{1}{(1+i2\pi u)} e^{x(1+i2\pi u)} \Big|_{0}^{\infty} \right]$$

$$= \frac{e^{-|2\pi v|}}{2} \left[\frac{1}{(1-i2\pi u)} \cdot \cancel{e^{x(1-i2\pi u)}} + \frac{1}{(1+i2\pi u)} \cancel{e^{x(1+i2\pi u)}} \right]$$

$$= \frac{e^{-|2\pi v|}}{2} \left[\frac{1}{1-i2\pi u} + \frac{1}{1+i2\pi u} \right]$$

$$= \frac{e^{-|2\pi v|}}{2} \left[\frac{1+i2\pi u + 1 - i2\pi u}{1-(i2\pi u)^2} \right] = \frac{e^{-|2\pi v|}}{\cancel{2}} \times \frac{\cancel{2}}{1-(i2\pi u)^2}$$

$$= \frac{e^{-|2\pi v|}}{1-(i2\pi u)^2}$$

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4. $\mathrm{rect}(x) = \begin{cases} 1 & \text{for } |x| \le \frac{1}{2} \\ 0 & \text{for } |x| > \frac{1}{2} \end{cases}$

$= FT(\mathrm{rect}(x))$

$= \mathrm{Sinc}(\pi u)$

$= \dfrac{\mathrm{Sin}\,\pi u}{\pi u}$.

5. $f(x,y) = \mathrm{rect}\left(\dfrac{x-x_0}{\Delta x_0}\right) \times \mathrm{rect}\left(\dfrac{y-y_0}{\Delta y_0}\right)$.

$f(u,v) = F.T\left(f(x,y)\right)$

$= F.T\ \mathrm{rect}\left(\dfrac{x-x_0}{\Delta x_0}\right) \times F.T\ \mathrm{rect}\left(\dfrac{y-y_0}{\Delta y_0}\right)$.

$= \Delta x_0 \cdot \mathrm{sinc}(\pi u \Delta x_0) \cdot e^{-i2\pi u x_0}$
$\times \Delta y_0 \cdot \mathrm{sinc}(\pi v \Delta y_0) \cdot e^{-i2\pi v y_0}$.

$= \Delta x_0 \cdot \mathrm{sinc}(\pi u \Delta x_0) \cdot e^{-i2\pi u x_0} \times \Delta y_0 \cdot \mathrm{sinc}(\pi v \Delta y_0) \cdot e^{-i2\pi v y_0}$.

6.

$$\delta(kx) * \delta\left(\frac{x}{k}\right) = \delta(x).$$

The property of δ function : $\delta(kx) = \frac{1}{|k|} \cdot \delta(x)$.

Then,

$$\delta(kx) * \delta\left(\frac{x}{k}\right) = \left[\frac{1}{k} \cdot \delta x\right] \times \left[\frac{1}{\frac{1}{k}} \cdot \delta x\right]$$

$$= \cancel{k} \times \frac{1}{\cancel{k}} \left[\delta(x) \times \delta(x)\right]$$

$$= \delta(x).$$

We get $\delta(kx) \cdot \delta\left(\frac{x}{k}\right) = \delta(x)$. //.

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