Handout #2
Image Quality

2.1 Contrast

(1) Subject contrast:

The difference in **x-ray transmission** between two adjacent areas in an object

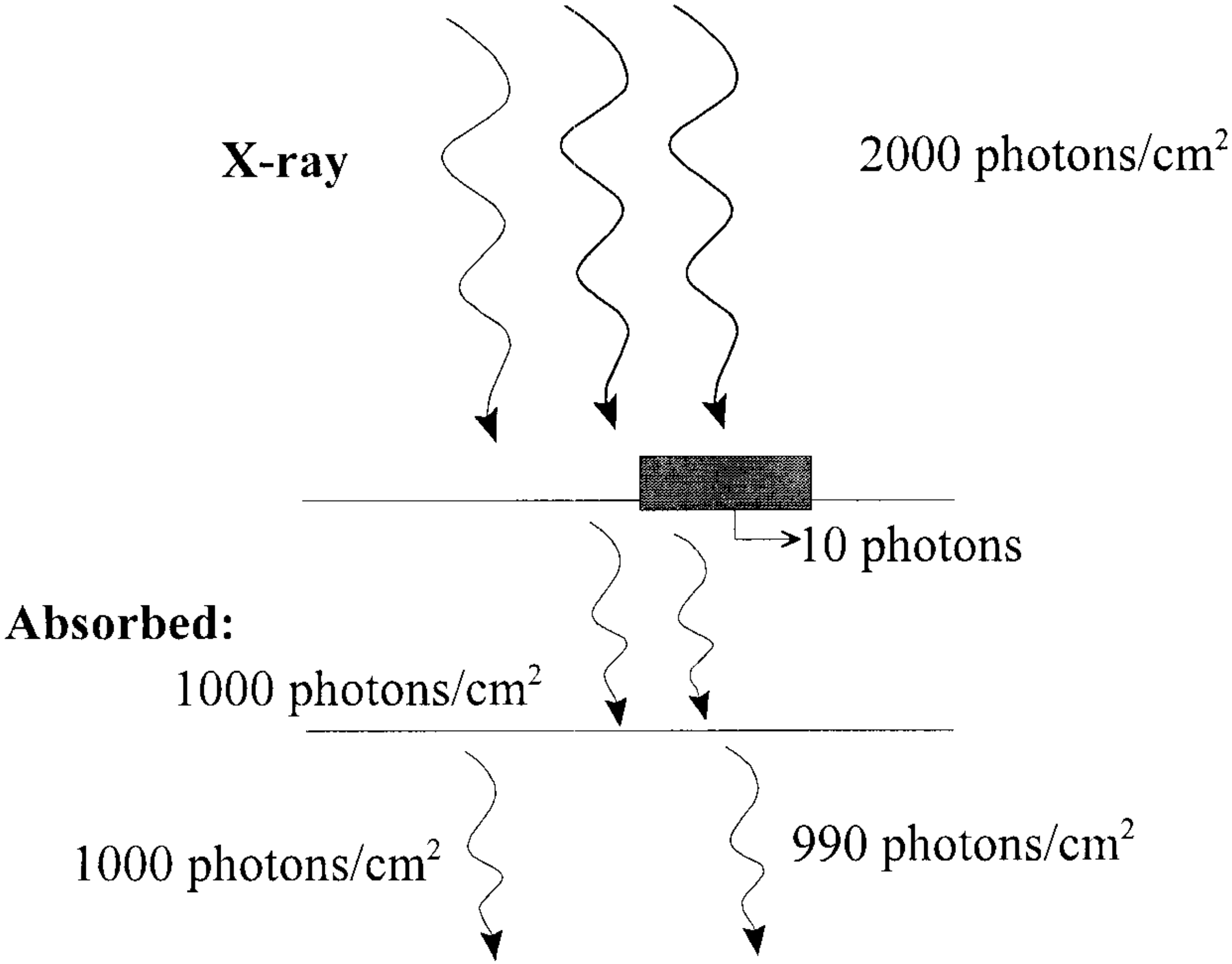
- In radiological imaging:

$$C = \frac{N_b - N_l}{N_b}$$

- More commonly:

$$C = \frac{N_b - N_l}{\frac{1}{2}(N_b + N_l)}$$

Where: N_b = Number of photons transmitted through background
N_l = Number of photons transmitted through the object
N_b - N_l = Signal

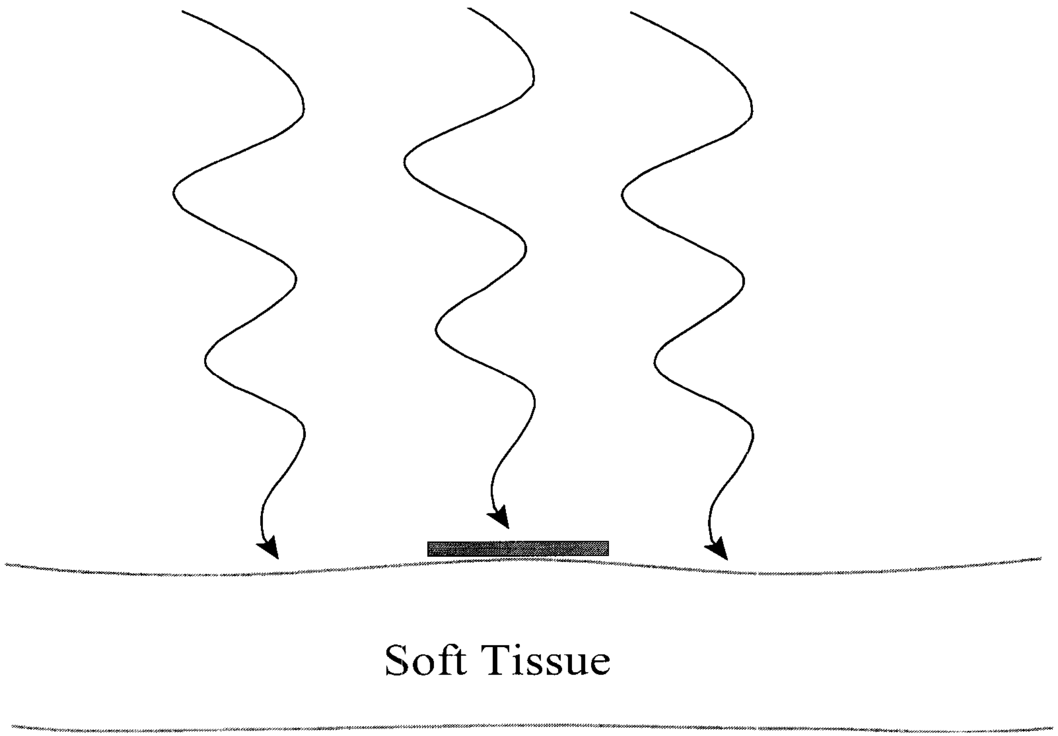


$$C = \frac{1000-990}{1000} = 1\%$$

(2) Displayed contrast

❖ If there is no subject contrast, the object will not be seen on the image;

Quiz: A small piece of lead sheet is placed on top of a large slab of soft tissue, what is the subject contrast under diagnostic x-ray beam?



Solution to the quiz:

$$C = \frac{N_b - N_l}{N_b}$$

$$N_l = 0$$

$$\therefore C = \frac{N_b - 0}{N_b} = 100\%$$

2.2 Noise

(1) Quantum noise

❖ Quantum noise is the primary noise source in most of the diagnostic x-ray imaging systems, it obeys **Poisson distribution**

$$P_K = \frac{N^K e^{-N}}{K!}$$

where: P_K is probability, in a given time interval, of emitting K photons from an x-ray source, and N is the average number of photons emitted during that interval.

Note: ***K!*** is the factorial of non-negative integer:

$$K! = K\times(K-1)\times(K-2)\ldots\times 1 = K\times(K-1)! = K\times(K-1)\times(K-2)!$$

$$0! \equiv 1$$

also in statistics:

$$N = \sum_{K=0}^{\infty} K \times P_K\ ; \qquad \sigma^2 = \sum_{K=0}^{\infty} (K-N)^2 \times P_K$$

❖ Accordingly, we can show that the X-ray quantum noise is given by:

Variance, σ^2 $\qquad \sigma^2 = N$

Standard deviation, σ $\qquad \sigma = \sqrt{N}$

(2) Other noise

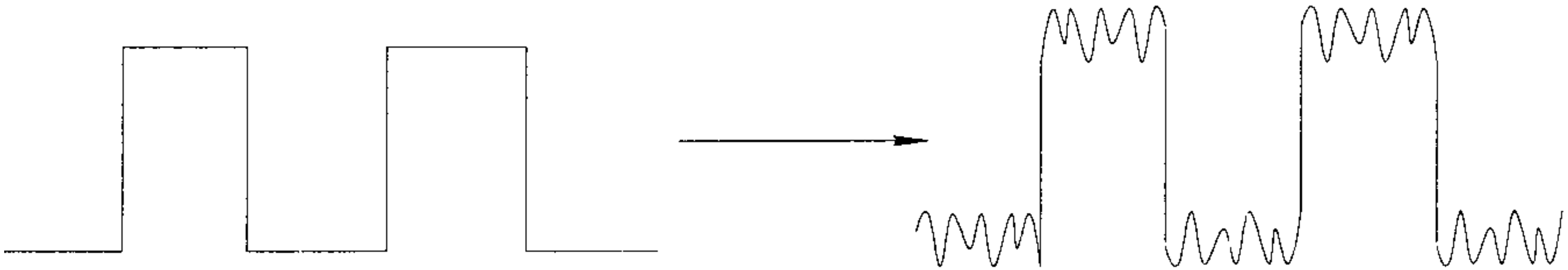
- Random noise: such as the granularity of the film emulsion
- Structure noise:
- Electronic noise:

All of above: additive noise

2.3 Signal-to-Noise Ratio (SNR)

❖ The ability to visualize a structure in a NOISE-FREE environment depends, among other factors, on the CONTRAST

❖ The ability to visualize a structure in a environment with NOISE depends, among other factors, on the signal-to-noise ratio (SNR)



❖ The signal-to-noise ratio (SNR) is a basic measure of visualization

$$SNR = \frac{Signal}{Noise}$$

$$SNR = \frac{N_b - N_l}{\sigma} = \frac{N_b - N_l}{\sqrt{N_b}} = \frac{(N_b - N_l)}{N_b}\sqrt{N_b} = C\sqrt{N_b} = C\sqrt{N_l}$$

Exercise:

A breast of a patient is imaged. In the middle of the breast, there is a lesion, that shows 10% contrast with the surrounding soft tissue under 30kVp beam quality, with 50mA, 1 second setting. If the number of x-ray photons transmitted through the area with the lesion is 10,000. What is the lesion's signal-to-noise ratio (SNR)?

Solution:

$$SNR = C\sqrt{N} = 0.1\times\sqrt{10000} = 0.1\times 100 = 10$$

How could we double the SNR?

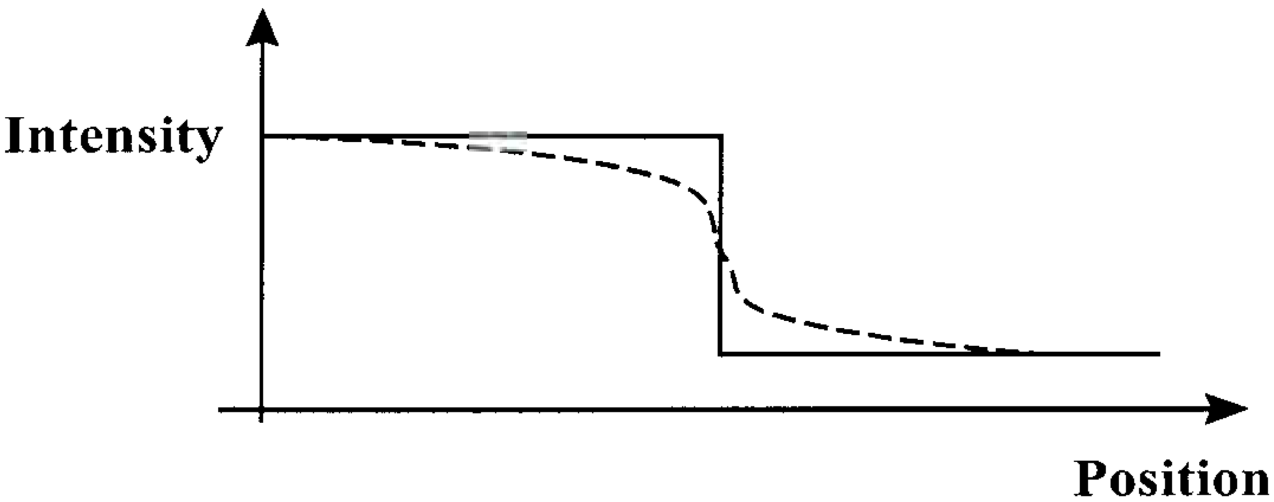
2.4 Spatial Resolution

Which is a measure of how good a system is in producing images of very small objects?

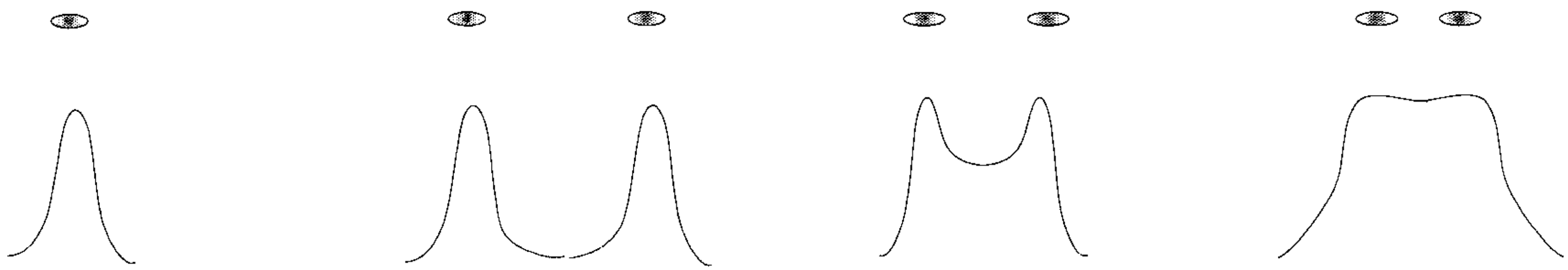
Unit: line pairs per mm (***lp/mm***)

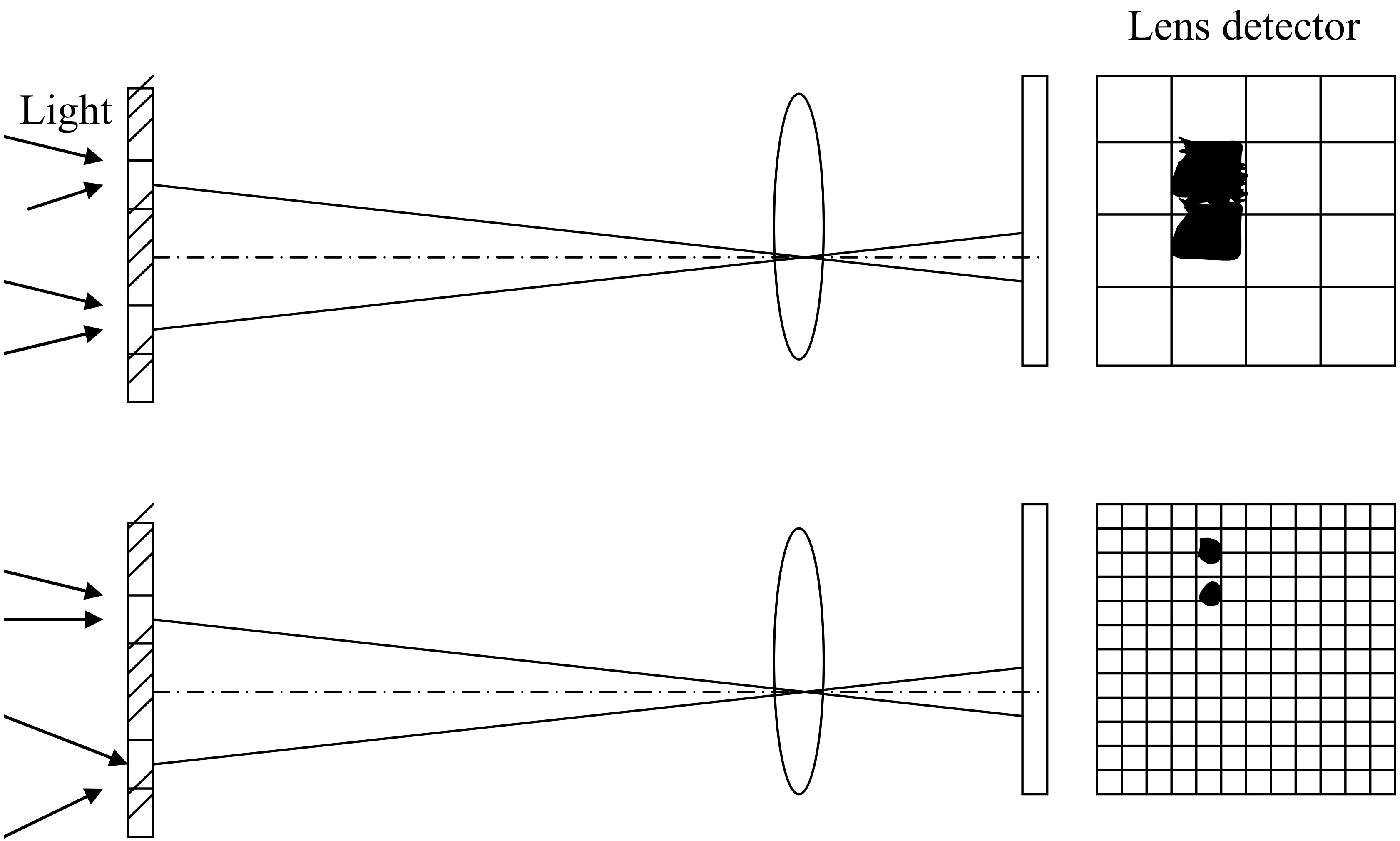
❖ Unsharpness:

Blurring at the edge



- Blurring affects the resolving power




Lens detector
Light

Pixel Pitch (*pixel size) : Δx (mm)

Smaller the pixel size, better the spatial resolution

$$\text{Spatial Resolution (Line pairs/mm)} = \frac{1}{2\,\Delta x}$$

Resolution pattern image, M=1.5

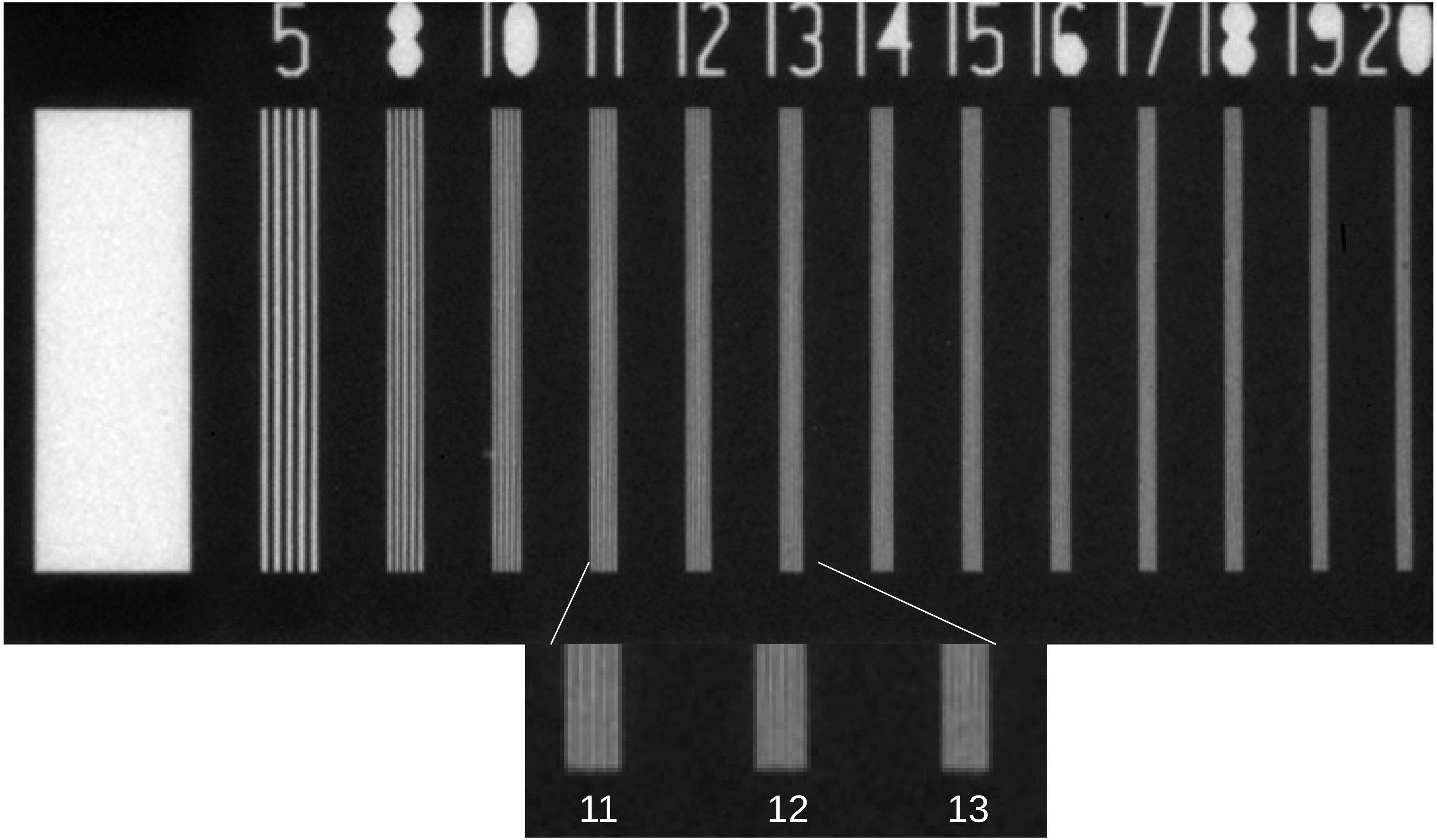
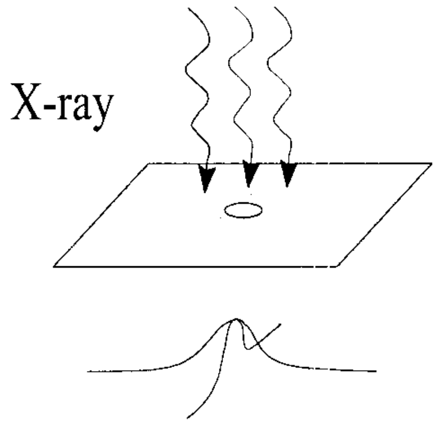
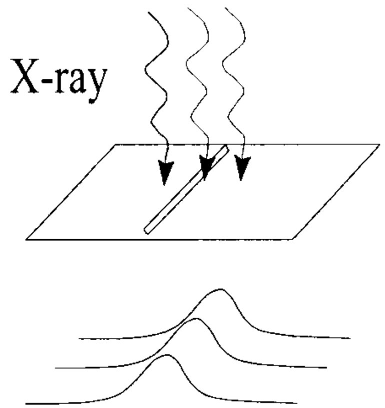


Figure: An image of line-pair targets acquired using x-ray imaging system under 26 kVp, 8s, Mag=1.5. The theoretical limiting resolution of x-ray imaging system is about 15 *lp/mm*.

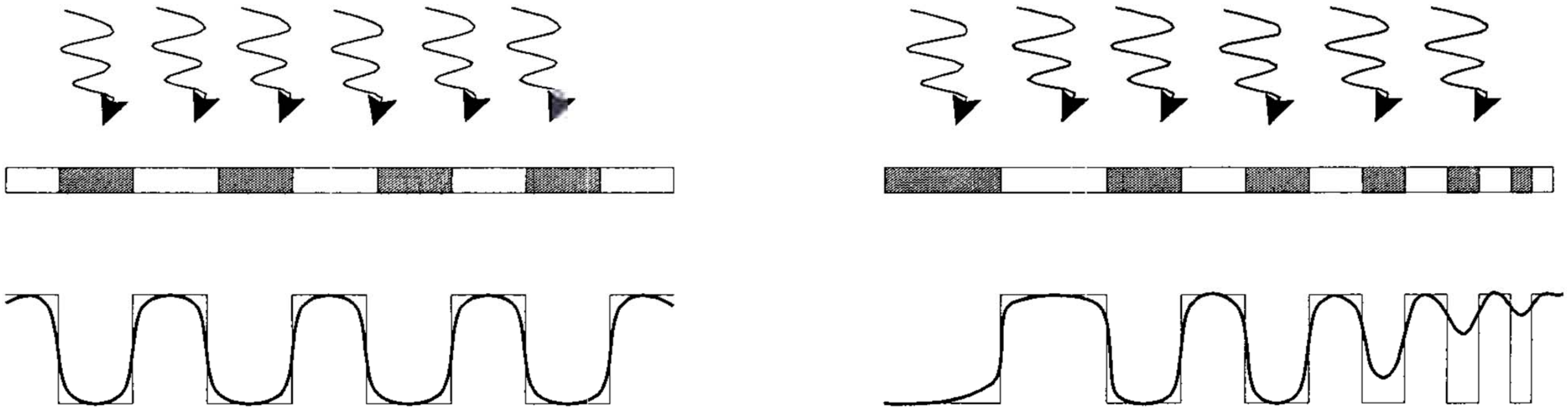
- ❖ Measuring the blurring effect: point spread function (PSF)
- ❖ How to measure PSF (x, y) : x-ray a pinhole in a lead sheet



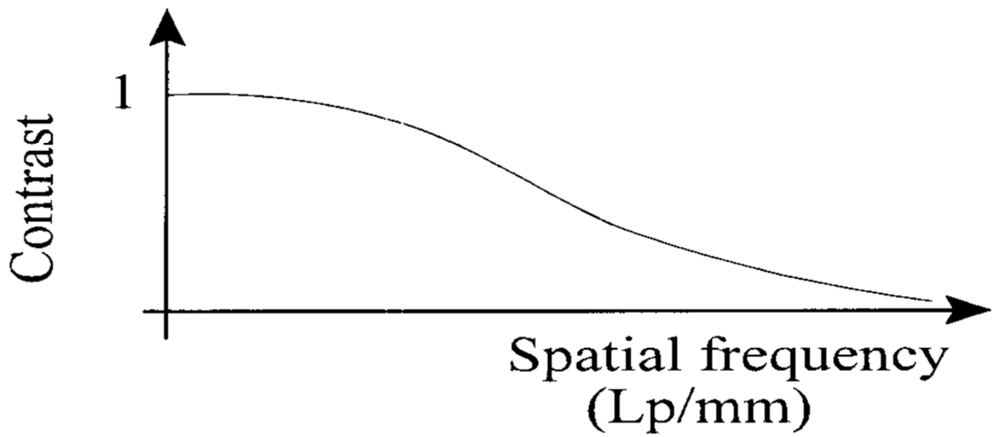
- ❖ Line spread function (LSF)



❖ Contrast transfer function (CTF)



$$C = \frac{\Delta N}{N_b}$$



2.5 Modulation Transfer Function (MTF)

❖ **MTF is equivalent to the normalized CTF**

Ways to measure MTF:

(1) From PSF

(A) Measure PSF (x, y) using a pinhole

Then apply two dimensional ***Fourier transform*** (2D FT) to the PSF (x, y)

(B) $$MTF(u,v) = \iint PSF(x,y)e^{-j2\pi(ux+vy)}dxdy$$

(2) From LSF (x)

(A) Measure LSF(x) using a slit,

Then apply one dimensional ***Fourier transform*** (1D FT) to the LSF (x)

(B) $$MTF(u) = \int LSF(x)e^{-j2\pi ux}dx$$

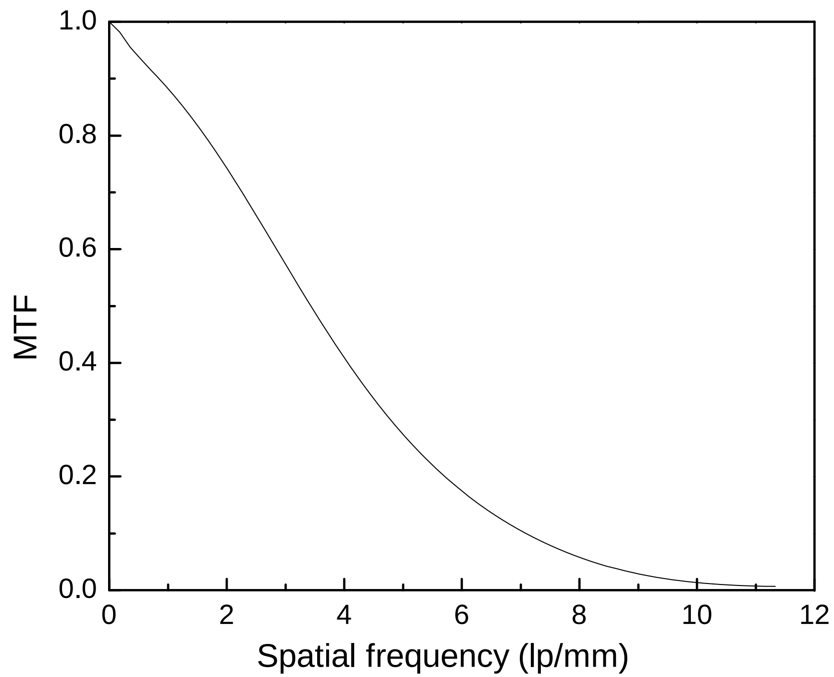
❖ Properties of MTF

(A) The value at 0 frequency is always equals to 1

(B) 1 should imply perfect reproduction of the object at the corresponding frequency.

(C) If the value is less than 1: less than perfect reproduction

(D) System MTF = The product of component MTFs



A typical MTF curve

Exercise:

(1) A system is consisted of:

A Scintillating screen: [MTF(5 lp/mm) = 0.5)]

A Film : [MTF(5 lp/mm)] = 0.8]

Focal spot of the x-ray tube: [MTF (5 lp/mm) = 0.8]

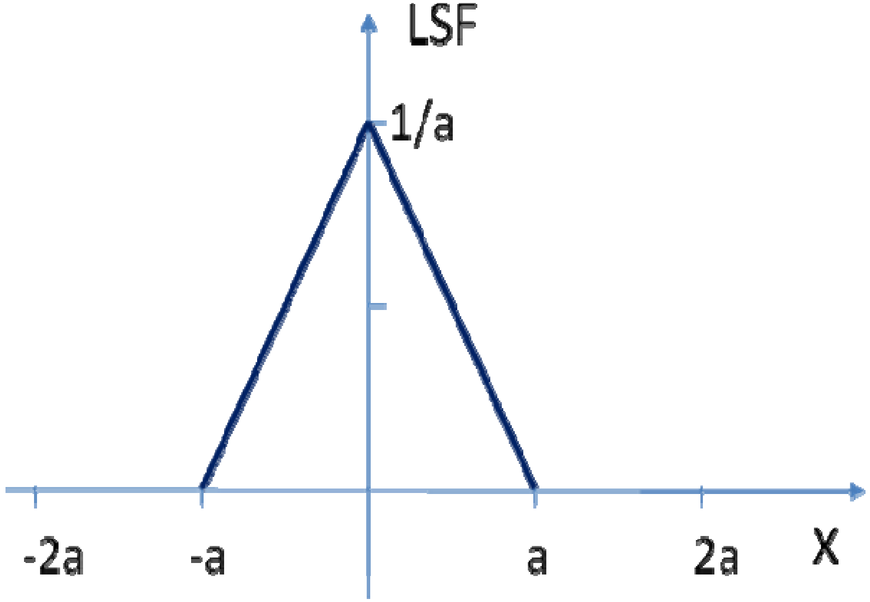
What is system MTF ?

Solution:

At 5 lp/mm, MTF = $0.5 \times 0.8 \times 0.8 = 0.32$

(2) Assume an x-ray imaging system is linear and stationary, it has a line spread function, (LSF) given as follows:

$$LSF(x) = \frac{1}{a} \times \Lambda\left(\frac{x}{a}\right) = \begin{cases} \frac{1}{a} - \left|\frac{x}{a^2}\right| & \left|\frac{x}{a}\right| \le 1 \\ 0 & otherwise \end{cases} \qquad a > 0$$



What is the modulation transfer function, of this system?

Method-(1):

$$\therefore MTF = \int LSF(x) e^{-i2\pi ux} dx = \frac{1}{a} \times \int_{-a}^{0} \left(1 + \frac{x}{a}\right) e^{-i2\pi ux} dx + \frac{1}{a} \times \int_{0}^{a} \left(1 - \frac{x}{a}\right) e^{-i2\pi ux} dx$$

$$= \frac{1}{a} \times \left[\frac{1}{a} \int_{-a}^{0} x e^{-i2\pi ux} dx + \int_{-a}^{a} e^{-i2\pi ux} dx - \frac{1}{a} \int_{0}^{a} x e^{-i2\pi ux} dx \right]$$

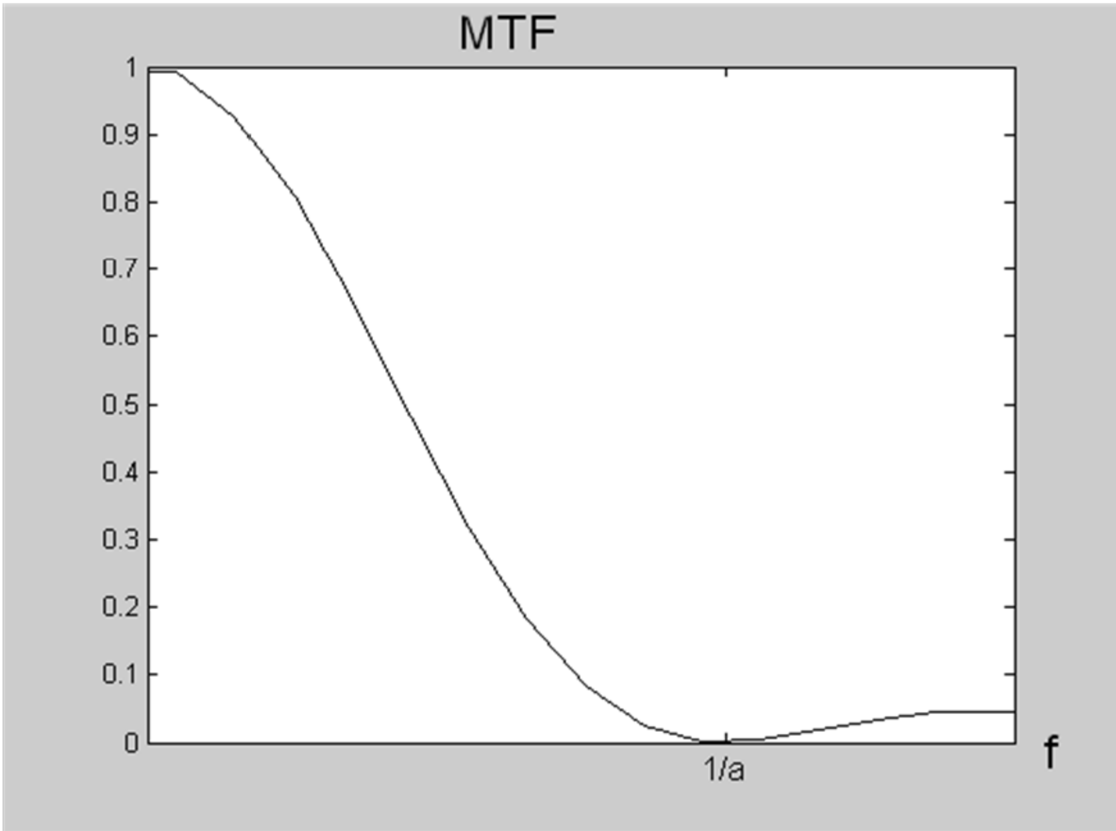
$$= \left(\frac{\sin(\pi au)}{\pi au} \right)^2$$

$$\therefore MTF = \mathrm{sinc}^2(\pi au)$$

Method-(2):

$$\because \int \Lambda(x) e^{-i2\pi ux} dx = \mathrm{sinc}^2(\pi u) = \left(\frac{\sin \pi u}{\pi u} \right)^2$$

$$\therefore MTF = \int \frac{1}{a} \times \Lambda\left(\frac{x}{a} \right) e^{-i2\pi ux} dx = \mathrm{sinc}^2(\pi a u)$$



(3) Assume an x-ray imaging system has a radial point spread function, (PSF) given as follows:

$$PSF(r,\theta) = \frac{1}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

where σ is a constant.

What is the 2D modulation transfer function, MTF(u,v) of this system?

[The flowing integral may help]

$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{c^2}} e^{-i2\pi ux} dx = \sqrt{\pi} c \times e^{-\pi^2 c^2 u^2}$$

where c is a constant

Solutions:

Let $c^2=2\sigma^2$, we have: $$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma^2}} e^{-i2\pi ux} dx = \sqrt{2\pi}\sigma \times e^{-2\pi^2\sigma^2 u^2}$$

$$\because \quad r = \sqrt{x^2 + y^2} \qquad PSF(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$MTF(u,v) = F_{2D}\left\{PSF(x,y)\right\} = \int \int_{-\infty}^{\infty} PSF(x,y) e^{-i2\pi(ux+vy)} dxdy$$

$$= \int \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}} e^{-i2\pi(ux+vy)} dxdy$$

$$= \frac{1}{2\pi\sigma^2} \int \int_{-\infty}^{\infty} e^{-\frac{x^2+y^2}{2\sigma^2}} e^{-i2\pi(ux+vy)} dxdy$$

$$= \frac{1}{2\pi\sigma^2} \times \left[\int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma^2}} e^{-i2\pi ux} dx \right] \left[\int_{-\infty}^{\infty} e^{-\frac{y^2}{2\sigma^2}} e^{-i2\pi vy} dy \right]$$

$$= \frac{1}{2\pi\sigma^2} \times \sqrt{2\pi}\sigma \times e^{-2\pi^2\sigma^2 u^2} \times \sqrt{2\pi}\sigma \times e^{-2\pi^2\sigma^2 v^2}$$

$$= e^{-2\pi^2\sigma^2(u^2+v^2)}$$

Therefore, $$MTF(u,v) = e^{-2\pi^2\sigma^2(u^2+v^2)}$$

Additional notes—a brief review of ***Fourier Transform***

2.6 Two Dimensional Fourier Transform*

(1) Definition

- The continuous 2-D Fourier transform G(u,v) of a function g(x,y) is defined as:

$$G(u,v) = F\left[g(x,y)\right] = \int_{-\infty}^{\infty}\int g(x,y)e^{-i2\pi(ux+vy)}dxdy$$

where *u* and *v* are referred to as spatial frequencies.

* Refer to Albert Macovski, *<<Medical Imaging Systems>>, Prentice Hall, New Jersey, 1983*

❖ The inverse Fourier transform is similarly defined as:

$$g(x,y) = F^{-1}\left[G(u,v)\right] = \int\limits_{-\infty}^{\infty}\int G(u,v)e^{i2\pi(ux+vy)}dudv$$

$$F^{-1}\left[G(u,v)\right] = \int\limits_{-\infty}^{+\infty}\int \left[\int\limits_{-\infty}^{\infty}\int g(x',y')e^{-i2\pi(ux'+vy')}dx'dy'\right]e^{i2\pi(ux+vy)}dudv$$

$$= \int\limits_{-\infty}^{+\infty}\int g(x',y')\left[\int\limits_{-\infty}^{\infty}\int e^{i2\pi[u(x-x')+v(y-y')]}dudv\right]dx'dy'$$

$$= \int\limits_{-\infty}^{\infty}\int g(x',y')\delta(x-x')\delta(y-y')dx'dy'$$

$$= g(x,y)$$

where $\int\limits_{-\infty}^{\infty}\int e^{i2\pi[u(x-x')+v(y-y')]}dudv = \delta(x-x')\delta(y-y')$

(2) Fourier transform relations

- δ - function response

$$F\left[\delta(x,y)\right]=\int_{-\infty}^{\infty}\int\delta(x,y)e^{-i2\pi(ux+vy)}dxdy=1$$

- Linearity

$$F\left[\alpha g+\beta h\right]=\alpha F[g]+\beta F[h]$$

The Fourier Transform operation is linear.

- Magnification

$$F\left[g(ax,by)\right]=\frac{1}{|ab|}G(\frac{u}{a},\frac{v}{b})$$

- Shift

$$F\left[g(x-a, y-b)\right] = G(u,v)e^{-i2\pi(ua+vb)}$$

- Convolution

$$F\left[\int\int_{-\infty}^{\infty} g(\xi,\eta)h(x-\xi, y-\eta)d\xi d\eta\right] = G(u,v)H(u,v)$$

The convolution of two functions in space can be represented by simply multiplying their frequency spectra.

- Cross Correlation

$$F\left[\int\int_{-\infty}^{\infty} g(\xi,\eta)\times h^*(x+\xi, y+\eta)d\xi d\eta\right] = G(u,v)H^*(u,v)$$

where: h^* and H^* denote the complex conjugate of h and H.

❖ Separability (Cartesian)

If $g(x,y)=g_x(x) \times g_y(y)$, then

$$F[g(x,y)] = F_x[g_x(x)] \times F_y[g_y(y)]$$

where F_x and F_y are one-dimensional operators.

(3) Frequently occurring function and their Fourier transform

We present the transforms of a number of well-known continuous functions as,

Table: Frequently used functions and their Fourier transforms

Function	**Fourier Transform**
$\sin 2\pi x$	$\frac{1}{2i}\left[\delta(u-1)-\delta(u+1)\right]$
$\cos 2\pi x$	$\frac{1}{2}\left[\delta(u-1)+\delta(u+1)\right]$
$\exp^{[i\pi(x+y)]}$	$\delta(u-\frac{1}{2}, v-\frac{1}{2})$
$\exp^{(-\pi r^2)}$	$\exp^{(-\pi \rho^2)}$
$\delta(x,y)$	1
$rect(x) = \begin{cases} 1 & \lvert x\rvert \le \frac{1}{2} \\ 0 & otherwise \end{cases}$	$\sin c(\pi u) = \frac{\sin \pi u}{\pi u}$
$\Lambda(x) = \begin{cases} 1-\lvert x\rvert & \lvert x\rvert \le 1 \\ 0 & otherwise \end{cases}$	$\sin c^2(\pi u) = \left(\frac{\sin \pi u}{\pi u}\right)^2$
$comb(x)$	$comb(u)$

Exercise:

Please prove the following:

$$(a) \quad \delta(x) * \delta(x) = \delta(x)$$

$$(b) \quad \delta(ax) * \delta(bx) = \delta(abx)$$

where $*$ is convolution operation.

Solution:

(a) δ function has the property: $\begin{cases} F[\delta(x)] = 1 \\ F^{-1}[1] = \delta(x) \end{cases}$

Let:

$$f(x) = \delta(x) * \delta(x)$$

$$\because F(u) = F[f(x)] = F[\delta(x) * \delta(x)] = F[\delta(x)] \times F[\delta(x)] = 1 \times 1 = 1$$

Taking the inverse **Fourier transform,** we get:

$$\therefore f(x) = F^{-1}[F(u)] = F^{-1}(1) = \delta(x)$$

$$\delta(x) * \delta(x) = \delta(x)$$

(b) It can be proved as follows:

The property of the δ function: $\delta(ax) = \frac{1}{|a|}\delta(x)$

Then:

$$\delta(ax) * \delta(bx) = \left[\frac{1}{|a|}\delta(x)\right] * \left[\frac{1}{|b|}\delta(x)\right]$$

$$= \frac{1}{|ab|}[\delta(x) * \delta(x)]$$

$$= \frac{1}{|ab|}\delta(x) = \delta(abx)$$

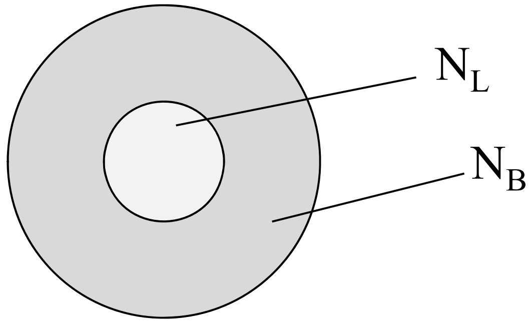
We get: $\delta(ax) * \delta(bx) = \delta(abx)$

SUMMARY

1) Contrast
2) Spatial resolution: pixel size (mm) and line pairs per mm
3) FT and MTF
4) Quantum noise in x-ray imaging,
5) Signal to noise ratio (SNR) and lesion detectability

Homework #2

Questions 1 and 2 refer to the figure which represents an x-ray image of a uniform, disk-shaped lesion with an average of N_L photons, where N_L is just slightly smaller than the average background count level, N_B photons.



1. What is the lesion's signal-to-noise ratio (SNR)?
2. If the same object is imaged for twice as long (twice the photons), how is the new SNR related to the old SNR?

3. Assume an imaging system has a point spread function (PSF) given as follows:

$$PSF(x,y) = \frac{1}{2\pi}\frac{e^{-|x|}}{1+y^2}$$

What is the 2D modulation transfer function *MTF(u,v)* of this system?

[The flowing integral may help]

$$\int_{-\infty}^{\infty} \frac{e^{-i2\pi v y}}{1+y^2}\,dy = \pi\, e^{-|2\pi v|}$$

4. Consider the 1-D ***rect*** function:

$$rect(x) = \begin{cases} 1 & \text{for } |x| \le \dfrac{1}{2} \\ 0 & \text{for } |x| > \dfrac{1}{2} \end{cases}$$

What is its Fourier transform?

5. Detectors of some medical imaging systems can be modeled as ***rect*** functions of different sizes and locations. Compute the Fourier transform of the following scaled and translated ***rect*** function:

$$f(x, y) = rect(\frac{x - x_0}{\Delta x_0}) \times rect(\frac{y - y_0}{\Delta y_0})$$

$$\text{where} : \mathrm{x_0, y_0, \Delta x_0} \text{ and } \mathrm{\Delta y_0} \text{ are constant.}$$

6. Please prove the following:

$$\delta(kx) * \delta(\frac{x}{k}) = \delta(x)$$

where $*$ is convolution operation, and k is a constant.