MATH 4753 Laboratory 13:
1. One sided testing 2. Bayesian inference

In this lab we will investigate one sided hypothesis testing and Bayesian inference. You can find additional material on these topics in Chapter 8 of MS especially near the end of the chapter.

One sided hypothesis testing

One sided hypothesis testing is just a slight modification to that which you are accustomed (two sided). To determine whether a one or two sided hypothesis test should be conducted you need to know what the alternate hypothesis (H_1:) is.
Suppose we are looking at the following hypotheses:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_1: \mu_1 - \mu_2 \neq 0$$

Because there are two possibilities for the alternate, namely $H_1: \mu_1 - \mu_2 < 0 \; or \; \mu_1 - \mu_2 > 0$ we know that this is a two sided test. This means that we have no prior reason to believe that $\mu_1 > \mu_2$ or that $\mu_2 > \mu_1$, the researcher has no hunch as to which should be true if H_0 is false. If H_0 were false and therefore that H_1 true we would expect that $t_{calc} > t_{\frac{\alpha}{2}}$ or $t_{calc} < t_{1-\frac{\alpha}{2}}$.
If on the other hand we knew from prior information that $H_1: \mu_1 > \mu_2$ was Truth if H_0 was false, that is $H_1: \mu_1 - \mu_2 > 0$, then we could express the hypotheses as:

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_1: \mu_1 - \mu_2 > 0$$

This would mean that we would expect $t_{calc} > t_\alpha$ if H_1 were true. The only place where a type 1 error could occur would be in the upper tail since we contemplate rejecting the NULL only if $t_{calc} = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ is big enough i.e $\bar{y}_1 > \bar{y}_2$ in relation to the $se(\bar{Y}_1 - \bar{Y}_2)$.
A similar argument proceeds if $H_1: \mu_1 - \mu_2 < 0$.
In such cases we do a one-sided test or one tailed test.

Bayesian inference.

The other major paradigm of statistics is what is called the Bayesian view. This relies on Bayes' formula

$$p(\theta|x) \propto p(\theta) f(x|\theta)$$

The left hand side ($p(\theta|x)$) is called the posterior and expresses our latest knowledge of the parameter. The prior ($p(\theta)$) expresses our prior beliefs about the parameter prior to collecting the latest data concerning the parameter. The likelihood ($f(x|\theta)$) is the likelihood of the parameter for this data and will contain the information from the data necessary to update the prior and form the posterior. In this way the posterior is seen as an updated prior.
If we allow θ to change through its possible values, $f(x|\theta)$ will be large when θ approaches the max. lik. Value and become smaller as it moves away from the max. lik val. However this will be moderated by the prior, it may be that these two functions will be slightly out of sync so that at the max. lik value, $p(\theta)$ is nowhere near its max value and at the max value of the prior the likelihood is away from its max value. The combination of these two using Bayes' rule forms the posterior.
We will examine only one simple example of Bayesian inference, namely a binomial experiment using a Beta prior.

Tasks

All output is to be made using RMD.

- Knit html (make sure there is a toc)
- Upload rmd and html documents

Note: All plots you are asked to make should be created through RMD

You are expected to adjust the functions as needed to answer the questions within the tasks below.

- Task 1
  - Make a folder LAB13
  - Download the file "lab13.r"
  - Place this file with the others in LAB13.
  - Start Rstudio
  - Open "lab13.r" from within Rstudio.
  - Go to the "session" menu within Rstudio and "set working directory" to where the source files are located.
  - Issue the function `getwd()` and copy the output here.
  - Create your own R file and record the R code you used to complete the lab.

- Task 2
  - Suppose :

```
set.seed(13); x=rnorm(40,mean=15,sd=5)# 1
set.seed(20); y=rnorm(35,mean=10,sd=4)# 2

var.test(x,y)# 3
```

  - Are the population variances equal?
  - What do you conclude from the var.test()?
  - Suppose that you are told beforehand that there is evidence from past experimentation that $\mu_x > \mu_y$
    - What is the NULL and alternate hypotheses?
    - Use the appropriate test to determine if the NULL is rejected?
    - What is the 95% ci?
    - Interpret the interval.
  - Using the same x, adjust the mean value in the second line above until you obtain

```
> t.test(x,y,alt="greater")

        Welch Two Sample t-test

data:  x and y
t = 1.3181, df = 72.62, p-value = 0.09581
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 -0.3605553        Inf
sample estimates:
```

```
mean of x mean of y
 14.65977  13.29429
```

- Interpret the above p-Value.
- Interpret the above confidence interval.

- Task 3
    - Suppose we have the following data and tests:

```
set.seed(50); x=rnorm(30,mean=50, sd=10)
set.seed(40); y=rnorm(40,mean=55, sd=20)

var.test(x,y) #1
var.test(y,x) #2

var.test(x,y,alt="less")#3
var.test(y,x,alt="greater")#4
```

- Show that the first two tests are equivalent in terms of
    - Their F values
    - P-Values
    - Confidence intervals
- Give the output of tests 3 and 4.
- Interpret them
- What can be said about the relationship between
    - Their F values
    - P-Values
    - Confidence intervals

- Task 4
    - Go to http://en.wikipedia.org/wiki/Conjugate_prior
    - In the context of Bayesian theory what are conjugate distributions?
    - If the prior is a Beta and the likelihood comes from a Binomial what distribution will the posterior take?
    - If $\theta \sim Beta(\alpha, \beta)$ we call α and β hyper-parameters because θ, THE parameter, is explained in terms of other parameters α and β
    - Suppose we perform a Binomial experiment, $Y \sim Bin(n, \theta)$, this can be viewed as n Bernoulli trials with n=10, y=4 successes. If we use a Beta prior with hyper parameters (1,1) what is the distribution of posterior? Give its hyper-parameters.
    - What is the moment estimate for θ?
    - In a Bayesian analysis the point estimate is the mean of the posterior. What is the mean value of the above posterior? N.B. If $X \sim Beta(\alpha, \beta)$ then $E(X) = \frac{\alpha}{\alpha+\beta}$
    - Compare these two point estimates.
    - Find the central 95% posterior interval using qbeta()
    - What is the 95% confidence interval for θ? (Use $\hat{\theta} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}$)
    - Compare the two intervals.

################### LAB FINISHES HERE ################################

- Task 5: Extra for experts!
    - Derive the posterior distribution analytically when $\theta \sim Beta(\alpha, \beta); Y \sim Bin(n, \theta)$