Exam 1 Spring 2017

1. a MILE LENGTH WEIGHT DDT
   b RIVER SPECIES
   c. LMBASS
   d CCATFISH
   e. aesthetics

2 a) $M_X(t) = E(e^{xt}) = \sum_x e^{xt} p(x) = e^{0t} p(0) + e^{1t} p(1) = q + pe^t$

b) $\mu_X = E(X) = \sum x\, p(x) = 0 \cdot p(0) + 1 \cdot p(1) = p$

c) $\sigma^2 = E((X-\mu)^2) = \sum (x-p)^2 p(x) = (0-p)^2 p(0) + (1-p)^2 p(1)$
$= p^2 q + q^2 p$
$= pq(p+q) = pq$

3. a) 0.0000 b) 1.0000 c) $E(Y) = 3E(Z) + 10 = 10$

D) $V(Y) = 9V(Z) = 9 \times 1 = 9$ E) $V(X) = 9V(Y) = 9 \times 4V(Z) = 36$

4. a) i) $P(X \le 20) = ppois(20, 10)$ ii) $P(X = x) = dpois(20, 10)$

iii) $P(X \ge 15) = 1 - P(X \le 14) = 1 - ppois(14, 10)$

b) $M_X'(t) = \underbrace{n(q+pe^t)^{n-1}}_{f} \underbrace{pe^t}_{g}$; $M_X'(t)\big|_{t=0} = n(1)^{n-1} p = np$

c) $M_X''(t) = f'g + g'f = n(n-1)(q+pe^t)^{n-2} pe^t \, pe^t + pe^t n(q+pe^t)^{n-1}$

$M_X''(t)\big|_{t=0} = n(n-1)(1)^{n-1} p \cdot p + pn(1)^{n-1}$

$\mu_2' = n(n-1)p^2 + np$

$\sigma^2 = \mu_2' - \mu_1'^2 = n(n-1)p^2 + np - (np)^2 = n^2p^2 - np^2 + np - n^2p^2$

$= np(1-p)$

$= npq$

5) See CANVAS

6) $P(u|+) = \dfrac{P(u)P(+|u)}{P(u)P(+|u) + P(\bar{u})P(+|\bar{u})} = \dfrac{0.5\% \; 0.99}{0.5 \times 0.99 + 99.5\% \times 0.06}$

≈ 0.07

$P(+|\bar{u}) + P(-|\bar{u}) = 1$

$P(+|\bar{u}) = 1 - P(-|\bar{u}) = 1 - 0.94 = 0.06$

Exam 1 Spring 2017 (b)

7. a) $X \sim$ Geometric

b) $P(Y=y)$; $Y \sim$ Neg Bin.

c) $P(Y=8) = \binom{8-1}{4-1} 0.8^3 (1-0.8)^7 = \frac{7!}{3!\,4!} 0.8^3 0.2^7 = \frac{7\times6\times5\times4!}{3!\,4!} 0.8^3 0.2^7$

$= 35 \times 0.8^3 \times 0.2^7$

d) $E(Y) = \frac{r}{p} = \frac{4}{0.8} = 5$

d) $V(Y) = \frac{rq}{p^2} = \frac{4\times0.2}{0.8^2} = \frac{0.8}{0.8^2} = \frac{1}{0.8} = \frac{10}{8} = 1.25$

8) a) $P(X=10) =$ dbinom(10, prob=0.7, size=10) $= 0.0282$

OR $P(Y=0) = 0.0282$.

b) $P(x<=2) =$ pbinom(2, 10, 0.7) $= 0.0016$

c) $P(X<2) = P(X\leq1) = 0.0001$

d) $P(X>5) = P(X\geq6) = P(Y\leq4) = 0.8497$

e) $P(8\leq Y\leq10) = P(Y\leq10) - P(Y\leq7) = 1 - 0.9984 = 0.0016$

$(= P(0\leq x\leq2) = 0.0016$

TRICK $X+Y=10$