Second-Law Analysis of Heat Transfer in Swirling Flow Through a Cylindrical Duct

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A second-law analysis is made on a swirling flow in a cylindrical duct with constant wall temperature. A purely tangential entry of the fluid is considered and a simplified model, consisting of a central air core enclosed by a potential, free vortex region and a boundary layer, is assumed. The approximate hydrodynamic boundary layer equations, and the continuity equation, are set up and solved numerically for the velocity gradients in the boundary layer. Similarly, the temperature gradients within the thermal boundary layer are obtained from the energy equation. The local Nusselt number and rate of entropy generation are calculated and used to evaluate the rate of heat transfer and loss of available energy, respectively. A merit function, defined as the ratio of exergy transferred to the sum of exergy transferred and exergy destroyed, is evaluated for various values of Reynolds number, based on the inlet tangential velocity, and conclusions are drawn about the influence of inlet swirl on irreversibility.

Introduction

This investigation stems from the area of heat transfer augmentation by imparting a swirl to the fluid. Among a number of studies in the relevant field, Kreith and Sonju (1965) have discussed tape-induced turbulent flow through a pipe. Gutstein et al. (1970) have developed a theory based on solid body rotation for a helical vane insert. Bergles et al. (1980) reported a bibliography of different augmentation techniques. Nondecaying swirl flow has been the subject of many investigations (see Kreith and Margolis, 1959; Gambill and Bundy, 1963; Smithberg and Landis, 1964; Hong and Bergles, 1976). Blum and Oliver (1966) and Migay and Golubev (1970) showed that there was significant increase in heat transfer due to free swirling flow. Hay and West (1975) measured the local heat transfer coefficient along the axial direction for air flowing through a slot at the inlet to the pipe. Klepper (1972) used swirl generators at the pipe inlet and studied the performance of swirl flow using nitrogen gas as fluid. Zaherzadeh and Jagdish (1975) carried out experimental investigation of decaying swirl flow created by tangential vane swirls at the inlet of the test section. An analytical study of the heat transfer characteristics in decaying turbulent swirl flow generated by short twisted tapes placed at the entrance of the test section was carried out by Algifri and Bhardwaj (1985). They showed that the augmentation in the local heat transfer can be as high as 80 percent and an initial length of about 60 tube diameters is important to augmentation. Sparrow and Chaboki (1984) performed an experimental study on swirl-affected turbulent air flow and heat transfer in a tube. The swirling motion enhanced heat transfer substantially in the initial portion of the tube. Compared with the enhancements encountered in the conventional thermal entrance region in a nonswirling pipe flow, those associated with swirl were found to be remarkably greater. Junkhan et al. (1985) conducted experimental studies of three different turbulator inserts for fire tube boilers. Two commercial turbulators, consisting of narrow, thin metal strips bent and twisted in zig-zag fashion to allow a periodic contact with tube wall, displayed 135 and 175 percent increase in heat transfer coefficient at a high Reynolds number. A third turbulator consisting of a twisted strip, with width slightly less than the tube diameter, provided a 65 percent increase in the heat transfer coefficient while the increase in the friction factor was small as compared to the other two turbulator inserts.

Simultaneous release of hot air at one end and cold air at the other end due to a high-velocity swirling flow of compressible fluids is observed in Ranque–Hilsch vortex tubes. Refrigeration potential is obtained by creating a colder stream at the center through a low-pressure zone. Detailed analyses of vortex tube refrigerators have been discussed in many investigations. Among these the investigations of Hartnett and Eckert (1957), Merkulov (1960), Metenin (1960), and Parulekar (1961) are notable.

However, one aspect of swirling flow which is yet to be analyzed is the irreversibility associated with imparting a swirl. Bejan (1982) has shown that irreversibility, quantified by the rate of entropy generation, plays a very significant part in convective heat transfer processes. Irreversibility of a process is estimated from total production of entropy in the process. The method of second-law analysis seeks to evaluate the rate of entropy generation in various processes and then seeks to minimize the same by a suitable adjustment of flow parameters. Thus the analyses of heat transfer in ducts with constant heat flux (Bejan, 1978), flat plates, cylinders in cross flow and rectangular ducts (Bejan, 1979), heat exchangers (Bejan, 1977), and cryogenic apparatus (Bejan and Smith, 1975) are of importance. Sarangi and Chowdhury (1982) have analyzed counterflow heat exchangers to account for the entropy generated due to axial conduction and have derived an expression for optimum wall conductivity.

This study presents a method of evaluating the rate of entropy generation for swirling flow in a cylindrical duct. Based on an evaluation of the irreversibility, this study draws some conclusions about the effect of swirl on the availability.

Flow Model

The simplified flow model assumes a swirling flow produced by tangential entry of the fluid into the cylindrical duct. The flow field has been considered to consist of the following three zones (Fig. 1):

(*a*) the central air core,

(*b*) the potential core of free vortex outside the central air core, and

(*c*) the zone of boundary layer near the wall of the duct.

The temperature distribution in the fluid flowing along the duct is as follows:

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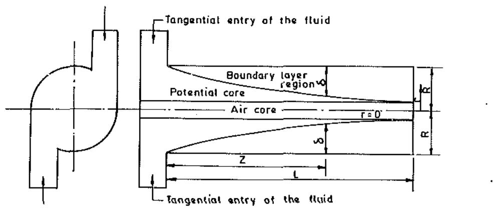

Fig. 1 Hydrodynamic picture inside the duct according to the theoretical model

(*a*) At the inlet, the fluid has the same initial temperature T_0 across the whole cross section.

(*b*) Downstream from the inlet, the fluid temperature at the wall is the same as the wall temperature T_w and decreases radially toward the core.

(*c*) Farther downstream, the thermal boundary layer grows in size.

(*d*) For a very long duct, the temperature across the cross section becomes almost equal and approaches the wall temperature T_w.

The approximate hydrodynamic boundary layer equations for laminar, steady-state flow of an incompressible, purely viscous Newtonian fluid, flowing symmetrically with respect to the axis $r = 0$ in cylindrical polar coordinates are given by Som and Biswas (1985) as

$$\frac{1}{\rho}\frac{\partial p}{\partial r}=\frac{V_\phi^2}{r} \tag{1}$$

$$V_r\frac{\partial V_\phi}{\partial r}+V_z\frac{\partial V_\phi}{\partial z}=\frac{\mu}{\rho}\frac{\partial}{\partial r}\left[\frac{\partial V_\phi}{\partial r}\right] \tag{2}$$

$$V_r\frac{\partial V_z}{\partial r}+V_z\frac{\partial V_z}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\frac{\partial}{\partial r}\left[\frac{\partial V_z}{\partial r}\right] \tag{3}$$

The equation of continuity is

$$\frac{\partial V_r}{\partial r}+\frac{\partial V_z}{\partial z}=0 \tag{4}$$

The thermal boundary layer equation is given by

$$V_r\frac{\partial \theta}{\partial r}+V_z\frac{\partial \theta}{\partial z}=\alpha\frac{\partial^2\theta}{\partial r^2} \tag{5}$$

where the viscous dissipation terms are neglected and θ (the excess temperature over the inlet temperature) is $T - T_0$.

The boundary layer momentum integrals are obtained by the usual Pohlhausen method of integrating equations (2) and (3) through a hydrodynamic boundary layer having thickness δ and equation (5) through a thermal boundary layer of thickness δ_t. The radial component of velocity is eliminated with the help of equation (4). Finally, the tangential and axial momentum integrals are obtained as

$$-\frac{\Omega}{(R-\delta)}\int_{R-\delta}^{R}\frac{\partial V_z}{\partial z}dr+\int_{R-\delta}^{R}\frac{\partial}{\partial z}(V_zV_\phi)dr=\frac{\mu}{\rho}\left[\frac{\partial V_\phi}{\partial r}\right]_{R-\delta}^{R} \tag{6}$$

$$2\int_{R-\delta}^{R}V_z\frac{\partial V_z}{\partial z}dr-\int_{R-\delta}^{R}\frac{\Omega^2}{(R-\delta)^3}\frac{d\delta}{dz}=\frac{\mu}{\rho}\left[\frac{\partial V_z}{\partial r}\right]_{R-\delta}^{R} \tag{7}$$

and the energy integral as

$$\int_{R-\delta_t}^{R}\frac{\partial}{\partial z}(V_z\theta)=\alpha\left[\frac{\partial\theta}{\partial r}\right]_{r=R} \tag{8}$$

Equations (6) and (7) are solved by taking the polynomial distribution of velocity components as

$$V_z=\frac{\Omega E}{R}(\eta-2\eta^2+\eta^3)=\frac{\Omega E}{R}f(\eta) \tag{9}$$

$$V_\phi=\frac{\Omega}{R}(2\eta-\eta^2)=\frac{\Omega}{R}\phi(\eta) \tag{10}$$

where

$$\eta=\frac{(R-r)}{\delta} \tag{11}$$

so that η varies from 0 to 1.0 through the boundary layer. With these simple assumptions for the functions $f(\eta)$ and $\phi(\eta)$ two variable quantities E and δ determine the boundary

Nomenclature

- c_p = specific heat at constant pressure, J/kg•K
- dV^* = nondimensional volume element, defined in equation (33)
- D = duct diameter, m
- E = velocity parameter, defined in equation (9)
- Ec = Eckert number, defined in equation (32)
- G = parameter defined in equation (16)
- h = heat transfer coefficient, W/m^2•K
- I = irreversibility, W
- K = thermal conductivity of fluid, W/m•K
- L = duct length, m
- M = merit function, defined in equation (40)
- Ns = total rate of entropy generation defined in equation (35)
- Ns_1 = local rate of entropy generation
- Ns_3 = rate of entropy generation per unit volume defined in equation (30)
- Nu_z = local Nusselt number = h_zD/K
- $\overline{\mathrm{Nu}}$ = average Nusselt number = h_cD/K
- P = fluid pressure, Pa
- Pr = Prandtl number = $\mu C_P/K$
- Q = heat transfer rate, W
- Q_a = exergy transfer accompanying energy transfer Q, W
- q = heat flux, W/m^2
- r = radial dimension, m
- R = duct radius, m
- Re = Reynolds number based on the inlet tangential velocity
- $\dot{S}$ = total rate of entropy generation, W/K
- $\dot{S}_3$ = rate of entropy generation per unit volume, W/m^3•K
- T = fluid temperature, K
- T_a = ambient temperature, K
- T_b = bulk temperature of fluid, K
- T_0 = inlet fluid temperature, K
- T_w = wall temperature, K
- V_r = radial velocity, m/s
- V_z = axial velocity, m/s
- V_ϕ = circumferential velocity, m/s
- z = axial dimension, m
- z_1 = z/D
- α = thermal diffusivity of fluid, m^2/s
- δ = hydrodynamic boundary layer thickness, m
- δ_t = thermal boundary layer thickness, m
- θ_b = $T_b - T_0$, K
- θ_w = $T_w - T_0$, K
- μ = fluid viscosity, N•s/m^2
- ρ = fluid density, kg/m^3
- σ = θ_w/T_w
- σ_a = T_a/T_w
- τ = shear stress of the fluid, N/m^2
- Ω = circulation constant, m^2/s

layer at any point, so that E and δ are functions of z only and can therefore be determined by using momentum integrals (6) and (7). The kinematic boundary conditions are: $V_z=0$, $V_\phi=0$ at $r=R$; $V_z=0$, $\partial V_z/\partial r = 0$, $V_\phi = \Omega/(R-\delta)$, $\partial V_\phi/\partial r = 0$ at $r=R-\delta$. As the analysis points out, the tangential velocity of equation (10) does not satisfy the kinematic condition at the boundary layer interface. But to get along harmoniously with the laminar boundary layer theory ($\delta/R << 1$), $(\Omega/R)/(1-(\delta/R))$ can be approximated as Ω/R. A similar treatment was discussed in the swirl problem of Taylor (1950). Equation (8) is solved by taking the polynomial distribution of temperature as

$$\theta=\theta_w(1-2\eta_t+\eta_t^2) \tag{12}$$

where

$$\eta_t=\frac{(R-r)}{\delta_t} \tag{13}$$

and the thermal boundary conditions are: $\theta=\theta_w$ at $r=R$, i.e., at the wall; and $\theta=0$, $\partial\theta/\partial r = 0$ at $r = R-\delta_t$, i.e., at the thermal boundary layer interface. The equations are nondimensionalized by writing $\delta_1 = \delta/R$, $\delta_{t_1} = \delta_t/R$, $z_1 = z/D$. $\partial\theta_w/\partial z$ is equated to zero to account for the constant wall temperature boundary condition. Finally the equations reduce to

$$\frac{d\delta_1}{\partial z_1}=\frac{225GE}{(E^2+105\delta_1)\delta_1} \tag{14}$$

$$\frac{dE}{dz_1}=\frac{60G}{\delta_1^2}-\frac{225GE^2}{(E^2+105\delta_1)\delta_1^2} \tag{15}$$

$$G=\frac{4}{\mathrm{Re}} \tag{16}$$

The Reynolds number Re is defined on the basis of the inlet tangential velocity and the diameter of the duct as

$$\mathrm{Re}=\frac{\rho V_{\phi_i}D}{\mu}$$

where $V_{\phi_i} = \Omega/R$

$$\frac{d\delta_{t_1}}{dz_1}=\frac{A_{11}}{E\left[\frac{\delta_R}{6}-\frac{\delta_R^2}{5}+\frac{\delta_R^3}{15}\right]} \tag{17a}$$

where

$$A_{11}=E\,\frac{d\delta_1}{dz_1}\left(\frac{1}{12\delta_R^2}-\frac{2}{15\delta_R^3}+\frac{1}{20\delta_R^4}\right)$$
$$-\frac{dE}{dz_1}\,\delta_{t_1}\left(\frac{1}{12\delta_R}-\frac{1}{15\delta_R^2}+\frac{1}{60\delta_R^3}\right)+\frac{8}{\delta_{t_1}\mathrm{Re\,Pr}}$$

for $\mathrm{Pr} > 1$, and

$$\frac{d\delta_{t_1}}{dz_1}=\frac{A_{22}}{E\left[\frac{\delta_R^2}{15}-\frac{\delta_R^3}{30}\right]} \tag{17b}$$

where

$$A_{22}=E\,\frac{d\delta_1}{dz_1}\left(-\frac{1}{12}+\frac{2\delta_R}{15}-\frac{\delta_R^2}{20}\right)$$
$$-\frac{dE}{dz_1}\,\delta_1\left(\frac{1}{12}-\frac{\delta_R}{15}+\frac{\delta_R^2}{60}\right)+\frac{8}{\delta_{t_1}\mathrm{Re\,Pr}}$$

for $\mathrm{Pr} < 1$, where

$$\mathrm{Pr}\sim\delta_R \quad \text{and} \quad \delta_R=\frac{\delta}{\delta_t} \tag{18}$$

In the case of $\mathrm{Pr} = 1$, equations (17a) and (17b) yield the same expression for the growth of the thermal boundary layer

$$\frac{d\delta_{t_1}}{dz_1}=-\frac{dE}{dz_1}\,\frac{\delta_1}{E}+\frac{240}{\delta_{t_1}E\,\mathrm{Re\,Pr}}$$

Equations (14) and (15) are first solved simultaneously using the fourth-order Runge–Kutta method to get the values of δ_1, E, $d\delta_1/dz_1$, and dE/dz_1 at different sections down the duct. Using these values, equation (17a) or (17b) is similarly solved to get the values of δ_{t_1} along the duct.

Nusselt Number

The local Nusselt number is given by

$$\mathrm{Nu}_z=h_zD/K \tag{19}$$

where

$$h_z=\frac{-K\left[\frac{\partial\theta}{\partial r}\right]_{\text{wall}}}{(\theta_w-\theta_b)} \tag{20}$$

$$\theta_b=T_b-T_0$$
$$\theta_w=T_w-T_0$$

and T_b, T_w are the bulk temperature of the fluid and the wall temperature, respectively. Combining equations (12), (19), and (20), the expression for the local Nusselt number becomes

$$\mathrm{Nu}_z=\frac{4}{\delta_{t_1}(1-\theta_b/\theta_w)} \tag{21}$$

The bulk temperature in excess of the free-stream temperature is given by

$$\theta_b=\frac{\int_{R-\delta_t}^{R} V_z\cdot\theta\cdot r\,dr}{\int_{R-\delta_t}^{R} V_z\cdot r\cdot dr} \tag{22}$$

which leads to

$$\frac{\theta_b}{\theta_w}=\frac{1}{\delta_R^2}\left[1-\frac{4}{5\delta_R}+\frac{1}{5\delta_R^2}\right]\text{for Pr}>1 \tag{23a}$$

$$\frac{\theta_b}{\theta_w}=\left[1-\frac{4}{5}\,\delta_R+\frac{1}{5}\,\delta_R^2\right]\text{for Pr}<1 \tag{23b}$$

From a knowledge of δ_1, δ_{t_1} at various sections, local values of δ_R and hence θ_b/θ_w and Nu_z have been calculated. These values have been averaged to obtain the average Nusselt number $\overline{\mathrm{Nu}}$.

Entropy Generation

The rate of entropy generation per unit volume is given by Kirkwood and Crawford (1952) and Bird et al. (1960) as

$$\dot{S}_3=-\frac{1}{T^2}\,(q\cdot\nabla T)-\frac{1}{T}\,(\tau:\nabla V) \tag{24}$$

The first term on the right-hand side of equation (24) may be written as

$$-\frac{1}{T^2}\,(q\cdot\nabla T)=\frac{K}{T^2}\left[\left(\frac{\partial T}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial T}{\partial\phi}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2\right] \tag{25}$$

The second term of the right-hand side of equation (25) is zero for an axisymmetric case. As it was suggested by Bejan (1979), the axial conduction effect can be neglected for $\mathrm{Pe} >> 4$. Invoking the simplification $(\partial T/\partial z)^2 << (\partial T/\partial r)^2$, we get

$$-\frac{1}{T^2}\,(q\cdot\nabla T)=\frac{K}{T^2}\left(\frac{\partial T}{\partial r}\right)^2=\frac{K}{T^2}\left(\frac{\partial \theta}{\partial r}\right)^2 \quad (26)$$

The second term on the right-hand side of equation (24) is expanded as

$$(\tau:\nabla V)=\tau_{rr}\left(\frac{\partial V_r}{\partial r}\right)+\tau_{\phi\phi}\left(\frac{1}{r}\frac{\partial V_\phi}{\partial \phi}+\frac{V_r}{r}\right)$$
$$+\tau_{zz}\left(\frac{\partial V_z}{\partial z}\right)+\tau_{r\phi}\left[r\frac{\partial}{\partial r}\left(\frac{V_\phi}{r}\right)+\frac{1}{r}\frac{\partial V_r}{\partial \phi}\right]$$
$$+\tau_{\phi z}\left[\frac{1}{r}\frac{\partial V_z}{\partial \phi}+\frac{\partial V_\phi}{\partial z}\right]+\tau_{rz}\left[\frac{\partial V_z}{\partial r}+\frac{\partial V_r}{\partial z}\right] \quad (27)$$

Expressing the shear stresses in terms of the viscosity and the corresponding velocity gradient, equating gradients in the ϕ directions to zero, as the flow is axisymmetric, and neglecting gradients in the axial direction as they are small compared to the other terms, equation (27) can be written as

$$-\frac{1}{T}\,(\tau:\nabla V)=\frac{\mu}{T}\left[\left(\frac{\partial V_\phi}{\partial r}\right)^2+\left(\frac{\partial V_z}{\partial r}\right)^2\right] \quad (28)$$

Using equations (26) and (28), equation (24) becomes

$$\dot{S}_3=\frac{K}{T^2}\left[\left(\frac{\partial \theta}{\partial r}\right)^2\right]+\frac{\mu}{T}\left[\left(\frac{\partial V_\phi}{\partial r}\right)^2+\left(\frac{\partial V_z}{\partial r}\right)^2\right] \quad (29)$$

As expected, the irreversibility indicator $\dot{S}_3$ contains two additive parts, one due to conduction in the presence of a nonzero temperature gradient, and the other accounting for the viscous dissipation of mechanical power throughout the flow.

Equation (29) is evaluated by substituting expressions for θ, V_ϕ, and V_z from equations (12), (10), and (9). The temperature variation over the thermal boundary layer is negligible as compared with the absolute temperature at any radius. Hence T can be substituted by any characteristic reference temperature. Here the wall temperature has been considered as the reference temperature, hence $T \approx T_w$. A similar approach was suggested by Bejan (1979).

This finally leads to

$$Ns_3=\frac{\dot{S}_3R^2}{K}=\sigma^2\left[-\frac{1}{\delta_{t_1}}\,(-2+2\eta_t)\right]^2$$
$$+\mathrm{Ec}\ \mathrm{Pr}\left[\left[\frac{1}{\delta_1}\,(2-2\eta)\right]^2+\left[\frac{E}{\delta_1}\,(1-4n+3\eta^2)\right]^2\right] \quad (30)$$

where

$$\sigma=\theta_w/T_w \quad (31)$$

and

$$\mathrm{Ec}=(\Omega/R)^2/(c_pT_w) \quad (32)$$

and Ns_3 is a nondimensional rate of entropy generation per unit volume.

To evaluate the total rate of entropy generation, $\dot{S}_3$ is integrated to obtain

$$\dot{S}=\int\dot{S}_3dV$$

This integration is carried out by first defining a nondimensional volume element

$$dV^*=\frac{dV}{2R^3}=2\pi\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right)d\left(\frac{z}{D}\right) \quad (33)$$

which may also be expressed as

$$dV^*=2\pi(1-\eta\delta_1)(\delta_1d\eta)dz_1 \text{ for } \mathrm{Pr}>1 \quad (34a)$$

or

$$dV^*=2\pi(1-\eta_t\delta_{t_1})(\delta_{t_1}d\eta_t)dz_1 \text{ for } \mathrm{Pr}<1 \quad (34b)$$

Hence the integration is modified as follows

$$\dot{S}=\int\dot{S}_3dV=\int\frac{Ns_3K}{R^2}\,dV^*2R^3$$

or

$$Ns=\frac{\dot{S}}{2KR}=\int Ns_3dV^* \quad (35)$$

Substituting equations (30) and (34) into equation (35), the nondimensional rate of entropy generation Ns becomes

$$Ns=\int_0^{L/D}\int_0^1 Ns_3 2\pi(1-\eta\delta_1)\delta_1 d\eta\ dz_1 \quad (36a)$$

for $\mathrm{Pr}>1$

$$Ns=\int_0^{L/D}\int_0^1 Ns_3 2\pi(1-\eta_t\delta_{t_1})\delta_{t_1} d\eta_t\ dz_1 \quad (36b)$$

for $\mathrm{Pr}<1$

Local values of entropy generation Ns_1 can be obtained by carrying out the inner integral. This gives the entropy generated in a cross section.

Since the numerical values of all terms in the integrand are known over the prescribed range from previous analyses, the integration is carried out by a summation process to obtain a numerical value of Ns.

Merit Function

From this analysis, it is possible to evaluate both the rate of energy transferred usefully as well as the destruction of exergy due to irreversibilities.

If Q is the total rate of heat transfer, then

$$Q=h_c(T_w-T_b)2\pi RL$$
$$=\overline{\mathrm{Nu}}KR\theta_w\left(1-\frac{\theta_b}{\theta_w}\right)2\pi\,\frac{L}{D} \quad (37)$$

The rate of exergy transfer accompanying energy transfer at the rate of Q is given by Moran (1982) as

$$Q_a=Q\left[1-\frac{T_a}{T_w}\right]$$
$$=Q[1-\sigma_a] \quad (38)$$

where T_a, the ambient temperature, has been considered as the exergy reference environment temperature and T_w, the wall temperature, has been considered as a suitable temperature at the surface where the heat transfer takes place. If $\dot{S}$ is the total rate of entropy generation, the destruction of exergy is

$$I=T_a\dot{S}$$
$$=(T_a/T_w)T_w\dot{S}$$
$$=\sigma_aT_wNs2KR \quad (39)$$

A merit function is defined as the ratio of exergy transferred to the sum of exergy transferred and exergy destroyed

$$M=\frac{Q_a}{Q_a+I} \quad (40)$$

Introducing equations (37)–(39), equation (40) becomes

$$M=\frac{\overline{\mathrm{Nu}}\left(1-\frac{\theta_b}{\theta_w}\right)(1-\sigma_a)}{\overline{\mathrm{Nu}}\left(1-\frac{\theta_b}{\theta_w}\right)(1-\sigma_a)+\frac{1}{\pi}\left(\frac{D}{L}\right)\frac{\sigma_a}{\sigma}\,Ns} \quad (41)$$

This merit function is now evaluated for various flow parameters. The merit of the merit function lies in its simultaneous accountability of exergy and its destruction which is caused by irreversibilities associated with energy transport and momentum transport. Irreversibilities due to external interaction and internal dissipative effects are together taken care of by this parameter. Another widely used

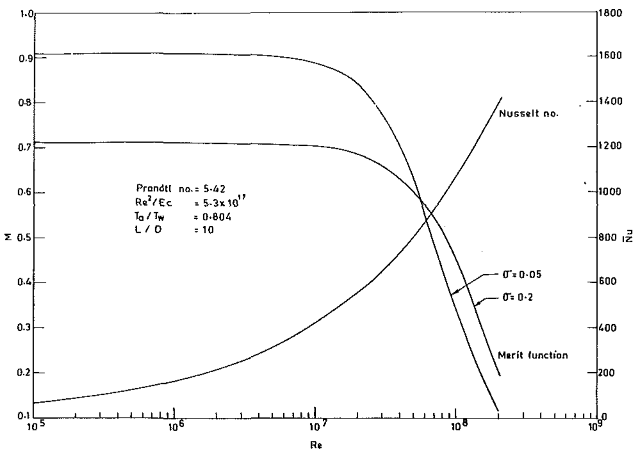

Fig. 2 Variation of $\overline{\mathrm{Nu}}$ and M with Reynolds number (based on inlet tangential velocity)

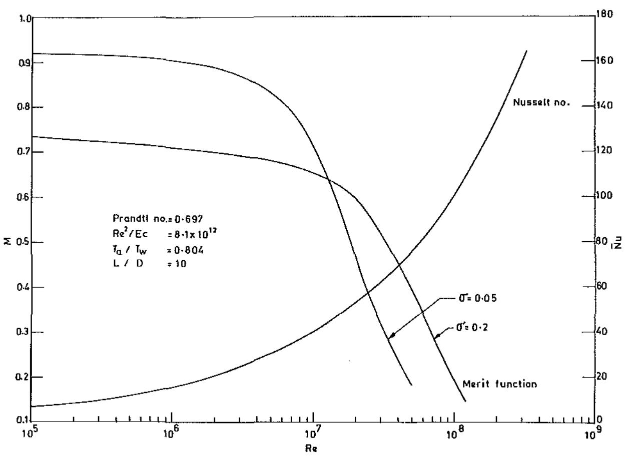

Fig. 3 Variation of $\overline{\mathrm{Nu}}$ and M with Reynolds number (based on inlet tangential velocity)

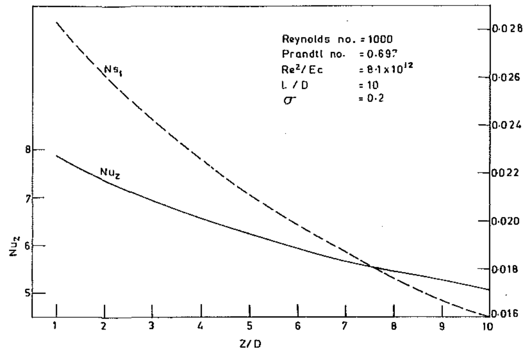

Fig. 4 Variation of Nu_z and Ns_1 along axial direction

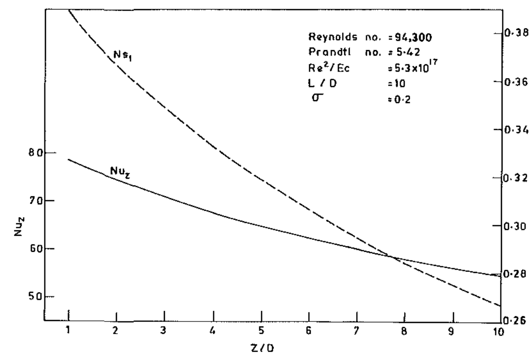

Fig. 5 Variation of Nu_z and Ns_1 along axial direction

parameter in the study of the second law is the dimensionless irreversibility (I/Q). The dimensionless irreversibility also can summarize the second-law analysis in a nice way. However, as we are interested in finding out energy transferred usefully, i.e., available energy as a function of total energy interaction in a particular energy transfer process, it would be quite meaningful to express the results in terms of M. Further it may be pointed out that I/Q is obtainable by reorganizing the expression for M. Therefore depiction of the study in terms of the merit function is not an entirely new idea but a different way to express the same physical laws of nature. However, it might be noted that equation (40) is a type of second-law efficiency (Moran, 1982).

Results and Discussion

Numerical evaluation of the Nusselt number, as well as the merit function M, is carried out for two different cases: (*i*) $\delta < \delta_t$, i.e., Pr < 1; and (*ii*) $\delta > \delta_t$, i.e., Pr > 1. The results are shown in Figs. 2 and 3. In each case, the effects of inlet swirl velocity, i.e., the Reynolds number, on these two variables are shown. To ensure that the variation in Reynolds number is due to change of the swirl velocity only and not due to changes in fluid properties or duct diameter, the ratio $\mathrm{Re}^2/\mathrm{Ec}$ is kept constant.

From these graphs, it is evident that the Nusselt number definitely increases with swirl, i.e., Reynolds number. However, it is also evident that beyond a certain value of swirl, the merit function M takes a sharp plunge. Close observation depicts that up to a certain value of Reynolds number there is a steady and slow rise in the Nusselt number. Here, the increase in irreversibility associated with the increase in exergy transfer occurs at a relatively lower rate; hence, merit function becomes independent of Reynolds number. Beyond a certain range of Reynolds number (Re ≈ 10^7 in Fig. 2 and Re ≈ 10^6 in Fig. 3), there is a sharp increase in Nusselt number at the price of high irreversibility with a drastic decrease in available energy. Below this critical range of Reynolds number, M is also a strong function of σ. The greater the temperature difference, the greater will be the unavailable energy. This is quite obvious and follows the law of degradation of energy. Figures 4 and 5 show the variation of local Nusselt number as well as the local entropy generation number at various cross sections along the duct for two different cases with Prandtl number lower and greater than one. It appears from the figures and computed data that the flow regime (z/D = 10), shown here, is not fully developed. It is well known that augmentation in heat transfer is generally dominant in the developing region. The local Nusselt number follows a decaying trend of variation from a very high to an asymptotic constant value along the length of the duct. Decrease in local Nusselt number along the axial direction in any developing flow is attributed to the growth of thermal boundary layer. When thermal boundary layer grows progressively, the temperature gradient at the wall decreases. Development of hydrodynamic boundary layer also gives rise to a decrease in the velocity gradient at the wall. These two phenomena exert influence on the local entropy generation number, which decreases rapidly to a smaller value along the tube length in a developing flow.

Concluding Remarks

This method of second-law analysis provides an insight into the irreversibilities associated with swirling flow. Besides providing an upper limit on the Reynolds number, it can also be used to evaluate the total losses incurred in heat transfer of swirl duct flow. The study forms a basis for calculating and minimizing the irreversibility in thermal design involving a variety of heat transfer processes pertaining to efficient energy conversion. The analysis can be extended to other convective heat transfer configurations associated with different types of internal turbulence promoters for selection of best fitted insert for a particular process. A similar type of investigation can also be conducted to optimize the performance of vortex tube refrigeration devices.

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