

# On Polarization Properties of Light, Gaussian Beams and Spin-Orbit Interaction of Light

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*Submitted to*

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# Topics of discussion

Polarization properties of light

Gaussian Beam

Spin-orbit interaction of light

# Polarization properties of light

- ▶ Jones Formalism
- ▶ Stokes-Muller formalism

# Jones Vector

Electric field of *fully polarized* EM wave propagating along z-axis is given by

$$\mathbf{E}(\mathbf{r}, t) = \begin{bmatrix} A_x(\mathbf{r})e^{i\delta_x} \\ A_y(\mathbf{r})e^{i\delta_y} \\ 0 \end{bmatrix} e^{-i(kz - \omega t)}$$

Define normalized **Jones vector** *s.t.*  $\mathbf{J}^* \mathbf{J} = 1$  as

$$\mathbf{J}(\mathbf{r}, t) = \frac{1}{\sqrt{A_x^2 + A_y^2}} \begin{bmatrix} A_x(\mathbf{r})e^{i\delta_x} \\ A_y(\mathbf{r})e^{i\delta_y} \end{bmatrix}$$

Note that *intensity*,  $I = A_x^2 + A_y^2 = \mathbf{J}^* \mathbf{J}$

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Note that *intensity*,  $I = A_x^2 + A_y^2 = \mathbf{J}^* \mathbf{J}$

# Jones vector of usual polarization state

Polarization state	$\mathbf{J}$
$ H\rangle$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
$ V\rangle$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
$ P\rangle$	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$ M\rangle$	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
$ L\rangle$	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$
$ R\rangle$	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$

# Jones Matrix & evolution of Jones vector

Jones matrix for an optical element be  $\mathbf{M}$  s.t.

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

If a polarized light of Jones vector  $\mathbf{J}_{in}$  passes through that optical element then the Jones vector of output light is given by

$$\mathbf{J}_{out} = \mathbf{M} \mathbf{J}_{in}$$

- ▶ Composition rule:  $\mathbf{M} = \mathbf{M}_1 \mathbf{M}_2 \dots \mathbf{M}_n$
- ▶ Frame rotation by  $\theta$ :  $\mathbf{M}_\theta = \mathbf{R}(-\theta) \mathbf{M} \mathbf{R}(\theta)$  where  $\mathbf{R}(\theta)$  is *passive rotation matrix*.



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# Jones matrix of usual optical element

Optical element	$M$
Free space	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
x-Polariser	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Right circular polariser	$\frac{1}{2} \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
Linear di-attenuator	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
Half-wave plate with fast axis horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Quarter-wave plate with fast axis horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
General phase retarder	$\begin{bmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{bmatrix}$

# Polarization properties of light

- ▶ Jones Formalism

- ▶ Stokes-Muller formalism

# Coherency matrix

Coherency matrix,  $\mathbf{C}$  defined as

$$\mathbf{C} = \underbrace{\langle \mathbf{E} \otimes \mathbf{E}^\dagger \rangle = \langle \mathbf{E} \mathbf{E}^\dagger \rangle = \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix}}_{\text{only for fully polarized light}} = \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix}$$

- ▶  $\mathbf{C} = \mathbf{C}^\dagger$  (Hermitian).
- ▶ Time averaged intensity,  $I = \text{Tr}(\mathbf{C})$
- ▶

Let basis set,

$$\left\{ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{V}_0}, \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\mathbf{V}_1}, \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\mathbf{V}_2}, \underbrace{\begin{bmatrix} 0 & i \\ -i & 1 \end{bmatrix}}_{\mathbf{V}_3} \right\} \text{ s.t. } \mathbf{C} = \frac{1}{2} \sum_{i=0}^3 S_i \mathbf{V}_i$$

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- ▶  $\mathbf{C} = \mathbf{C}^\dagger$  (Hermitian).
- ▶ Time averaged intensity,  $I = \text{Tr}(\mathbf{C})$
- ▶ Evolution of coherency matrix as  $\mathbf{C}_{out} = \mathbf{M} \mathbf{C}_{in} \mathbf{M}^\dagger$

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# Stokes vector

$$\mathbf{C} = \frac{1}{2} \sum_{i=0}^3 S_i \mathbf{V}_i \longrightarrow \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \mathbf{S} \text{ (Stokes vector)}$$

Measurement of Stokes parameters by passing the light through

1. isotropic medium,  $I_0 = S_0$
2. x-polariser,  $I_1 = \frac{1}{2}(S_0 + S_1)$
3. 45°-polariser,  $I_2 = \frac{1}{2}(S_0 + S_2)$
4. circular polariser,  $I_3 = \frac{1}{2}(S_0 + S_3)$



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Measurement of Stokes parameters by passing the light through

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# Stokes vector of usual polarization state

Polarization state	$\mathbf{C}$	$\mathbf{S}$
$ H\rangle$	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$[1 \ 1 \ 0 \ 0]^T$
$ V\rangle$	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	$[1 \ -1 \ 0 \ 0]^T$
$ P\rangle$	$\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	$[1 \ 0 \ 1 \ 0]^T$
$ M\rangle$	$\frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$	$[1 \ 0 \ -1 \ 0]^T$
$ L\rangle$	$\frac{1}{2} \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$	$[1 \ 0 \ 0 \ 1]^T$
$ R\rangle$	$\frac{1}{2} \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$	$[1 \ 0 \ 0 \ -1]^T$
Un-polarized	$\frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$[1 \ 0 \ 0 \ 0]^T$

# Muller Matrix & evolution of Stokes vector

*Muller matrix* for an optical element  $\mathfrak{M}$  s.t.

$$\mathfrak{M} = \begin{bmatrix} \mu_{11} & \cdots & \mu_{14} \\ \vdots & \ddots & \vdots \\ \mu_{41} & \cdots & \mu_{44} \end{bmatrix}$$

Evolution of Stokes vector as,  $S_{out} = \mathfrak{M} S_{in}$

- ▶ Composition rule:  $\mathfrak{M} = \mathfrak{M}_1 \mathfrak{M}_2 \dots \mathfrak{M}_n$
- ▶ Frame rotation by  $\theta$ :  $\mathfrak{M}_\theta = T^{-1}(\theta) \mathfrak{M} T(\theta)$  where

$$T(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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# Poincare sphere representation

For elliptically polarized light, Stokes vector

$$\mathbf{S} = S_0 \begin{bmatrix} 1 \\ \cos 2\chi \cos 2\psi \\ \cos 2\chi \sin 2\psi \\ \sin 2\chi \end{bmatrix} \longrightarrow \underbrace{S_0 (\cos 2\chi \cos 2\psi, \cos 2\chi \sin 2\psi, \sin 2\psi)}_{\text{On Poincare sphere of radius } S_0}$$

where azimuth ( $\psi$ ) and ellipticity ( $\chi$ ) of polarization ellipse.

# Poincare sphere representation

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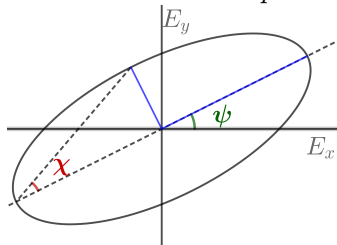
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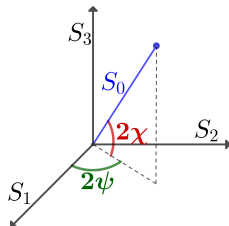
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where azimuth ( $\psi$ ) and ellipticity ( $\chi$ ) of polarization ellipse.

*Polarization ellipse*



*Poincare sphere*



# Poincare sphere representation

For un-polarized light,

$$\mathbf{S} = S_0 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \underbrace{(0, 0, 0)}_{\text{At Origin}}$$

For partially polarized light,

$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \longrightarrow \underbrace{(S_1, S_2, S_3)}_{\substack{\text{Inside sphere } s.t. \\ 0 < S_1^2 + S_2^2 + S_3^2 < S_0^2}}$$

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# Gaussian Beam and properties

- ▶ Paraxial wave
- ▶ Gaussian beam solution and properties
- ▶ Modes of Gaussian beam

# Paraxial wave

Maxwell's wave equation:

$$\nabla^2 \mathbf{E}(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E}(\mathbf{r}, t) = 0$$

Paraxial beam propagating predominantly in  $z$ -direction,

$$\mathbf{E}(x, y, z, t) = \boldsymbol{\psi}(x, y, z) e^{i(\omega t - kz)}$$

and taking slowly varying amplitude approx. *i.e.*

$$\left| \frac{\partial^2 \boldsymbol{\psi}}{\partial z^2} \right| \ll k \left| \frac{\partial \boldsymbol{\psi}}{\partial z} \right| \ll k^2 |\boldsymbol{\psi}|$$

Paraxial wave equation:

$$\frac{\partial^2 \boldsymbol{\psi}}{\partial x^2} + \frac{\partial^2 \boldsymbol{\psi}}{\partial y^2} - 2ik \frac{\partial \boldsymbol{\psi}}{\partial z} = 0$$

One of the solutions is **Gaussian beam**.

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One of the solutions is **Gaussian beam**.

# Gaussian Beam

- ▶ Paraxial wave
- ▶ Gaussian beam solution and properties
- ▶ Modes of Gaussian beam

# Gaussian beam solution

$$\text{Ansatz: } \psi(\mathbf{r}, z) = A \exp \left[ -i \left( p(z) + \frac{kr^2}{2q(z)} \right) \right]$$

$$\psi(\mathbf{r}, z) = A \left( \frac{w_0}{w(z)} \right) \exp \left( \tan^{-1} \left( \frac{z}{z_0} \right) - i \frac{kr^2}{2R(z)} - \frac{r^2}{w^2(z)} \right)$$

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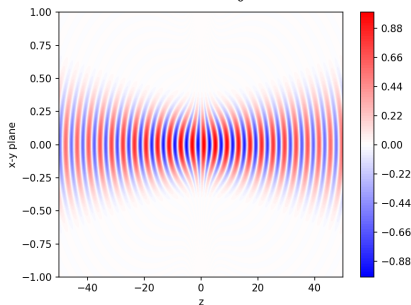
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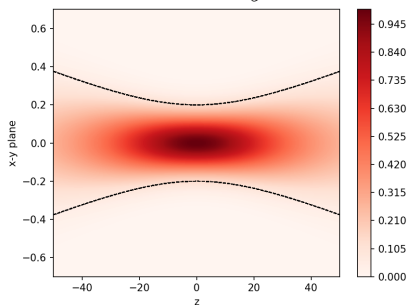
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*Electric field*



*Intensity*



# Gaussian beam properties

$$\psi(\mathbf{r}, z) = \underbrace{A\left(\frac{w_0}{w(z)}\right)}_{\text{term I}} \exp\left(i \tan^{-1}\left(\frac{z}{z_0}\right) - i \frac{kr^2}{2R(z)} - \frac{r^2}{w^2(z)}\right)$$

Term I related to **spreading of beam**.

$w \rightarrow$  Physical radius

$w_0 \rightarrow$  Beam waist

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

$z_0 \rightarrow$  Rayleigh length

$$z_0 = \frac{\pi w_0^2}{\lambda}$$

# Gaussian beam properties

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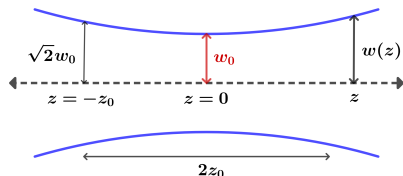
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$$z_0 = \frac{\pi w_0^2}{\lambda}$$



# Gaussian beam properties

$$\psi(\mathbf{r}, z) = \underbrace{A\left(\frac{w_0}{w(z)}\right)}_{\text{term I}} \exp\left(i \tan^{-1}\left(\frac{z}{z_0}\right) - i \frac{kr^2}{2R(z)} - \frac{r^2}{w^2(z)}\right)$$

Term I related to **spreading of beam**.

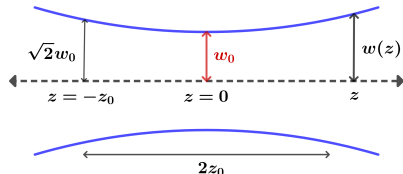
$w \rightarrow$  Physical radius

$w_0 \rightarrow$  Beam waist

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

$z_0 \rightarrow$  Rayleigh length

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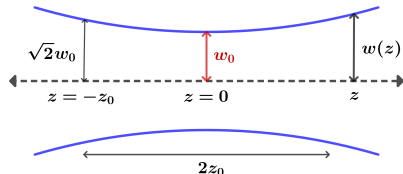
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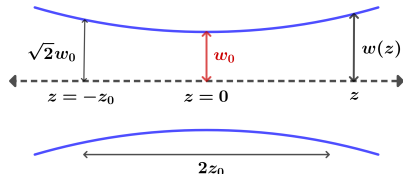
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Term II related to **Gouy phase** ( $\phi_G$ ).

$$\phi_G = \tan^{-1} \left( \frac{z}{z_0} \right)$$

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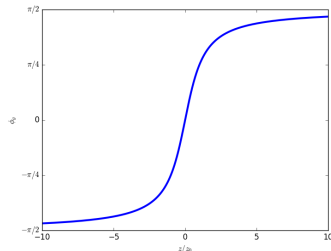
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Term III related to **radius of curvature** ( $R$ ) of beam wave-front.

$$R(z) = z \left[ 1 + \left( \frac{z_0}{z} \right)^2 \right]$$

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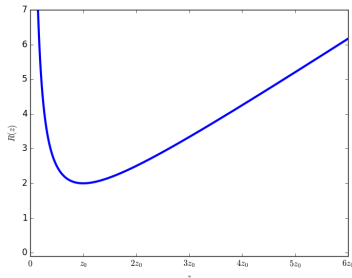
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Term IV related to **Gaussian intensity profile**.

$$I(r, z) \sim \exp \left( -\frac{2r^2}{w^2(z)} \right)$$

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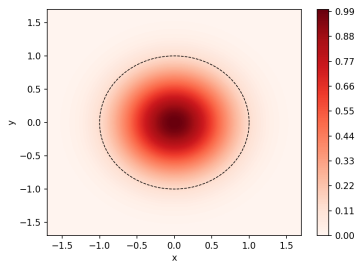
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## $q$ parameter and beam tracing

$$\psi(\mathbf{r}, z) = A \exp \left[ -i \left( p(z) + \frac{k r^2}{2 q(z)} \right) \right]$$

$q(z)$   $\longrightarrow$  characteristic of a beam if  $\lambda$  known.

$$q(z) = z + i z_0$$
$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda}{\pi w^2(z)}$$

$$q_{in} \longrightarrow \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{\text{Optical element}} \longrightarrow q_{out} = \frac{A q_{in} + B}{C q_{in} + D}$$

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# Gaussian Beam and properties

- ▶ Paraxial wave
- ▶ Gaussian beam solution and properties
- ▶ Modes of Gaussian beam

# Hermite-Gaussian mode

$$\psi_{m,n}(\mathbf{r}, z) = A \left( \frac{w_0}{w(z)} \right) H_m \left( \frac{\sqrt{2}x}{w(z)} \right) H_n \left( \frac{\sqrt{2}y}{w(z)} \right) \exp \left( -\frac{r^2}{w^2(z)} \right) \cdot \exp \left( i (m + n + 1) \tan^{-1} \left( \frac{z}{z_0} \right) - i \frac{kr^2}{2R(z)} \right)$$

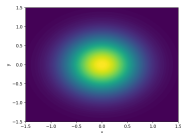
$m, n = 0 \Rightarrow \psi = \text{Gaussian}$

# Hermite-Gaussian mode

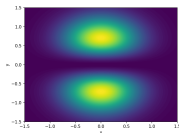
$$\psi_{m,n}(\mathbf{r}, z) = A \left( \frac{w_0}{w(z)} \right) H_m \left( \frac{\sqrt{2}x}{w(z)} \right) H_n \left( \frac{\sqrt{2}y}{w(z)} \right) \exp \left( -\frac{r^2}{w^2(z)} \right) \cdot \exp \left( i (m + n + 1) \tan^{-1} \left( \frac{z}{z_0} \right) - i \frac{kr^2}{2R(z)} \right)$$

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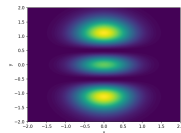
# Hermite-Gaussian Intensity profile



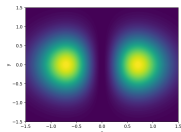
(a)  $TEM_{00}$



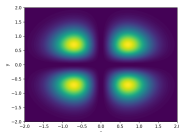
(b)  $TEM_{01}$



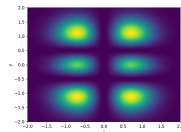
(c)  $TEM_{02}$



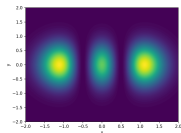
(d)  $TEM_{10}$



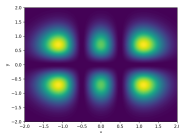
(e)  $TEM_{11}$



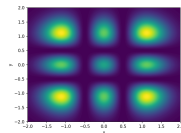
(f)  $TEM_{12}$



(g)  $TEM_{20}$



(h)  $TEM_{21}$



(i)  $TEM_{22}$

# Laguerre-Gaussian mode

$$\psi_{p,l}(r, \phi, z) = A \frac{w_0}{w(z)} \left[ \frac{r\sqrt{2}}{w(z)} \right]^{|l|} L_p^{|l|} \left( \frac{2r^2}{w^2(z)} \right) \exp \left( -\frac{r^2}{w^2(z)} \right) \cdot \exp \left( -il\phi + i(2p + l + 1) \tan^{-1} \left( \frac{z}{z_0} \right) - i \frac{kr^2}{2R(z)} \right)$$

$l, p = 0 \Rightarrow \psi = \text{Gaussian}$

$\exp(-il\phi) \longrightarrow$  Helical phase  
(carries OAM)

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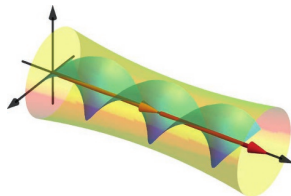
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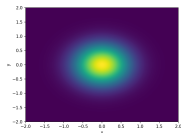
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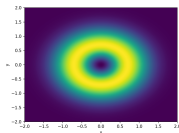
*Helical phase front*



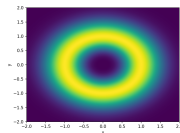
# Laguerre-Gaussian Intensity profile



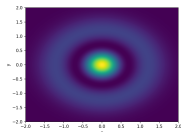
(a)  $p = 0, |l| = 0$



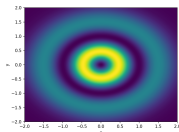
(b)  $p = 0, |l| = 1$



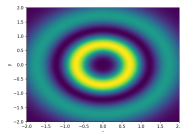
(c)  $p = 0, |l| = 2$



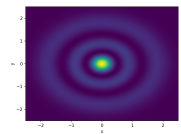
(d)  $p = 1, |l| = 0$



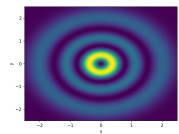
(e)  $p = 1, |l| = 1$



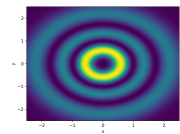
(f)  $p = 1, |l| = 2$



(g)  $p = 2, |l| = 0$



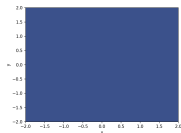
(h)  $p = 2, |l| = 1$



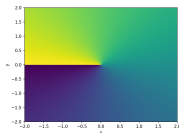
(i)  $p = 2, |l| = 2$



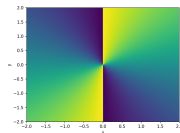
# Laguerre-Gaussian Phase profile



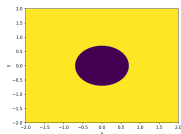
(a)  $p = 0, l = 0$



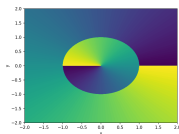
(b)  $p = 0, l = 1$



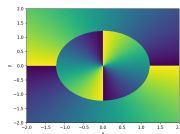
(c)  $p = 0, l = 2$



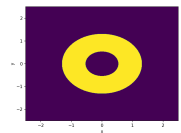
(d)  $p = 1, l = 0$



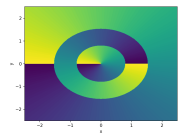
(e)  $p = 1, l = 1$



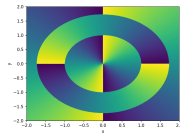
(f)  $p = 1, l = 2$



(g)  $p = 2, l = 0$



(h)  $p = 2, l = 1$

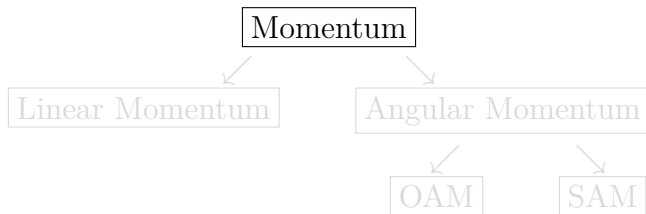


(i)  $p = 2, l = 2$

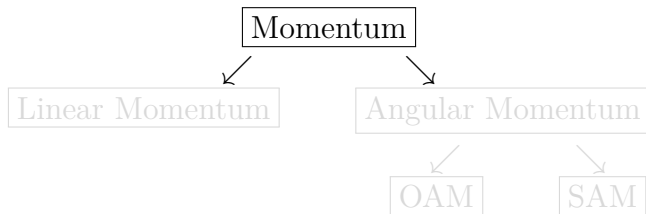
# Spin-orbit interaction

- ▶ Momentum of Light
- ▶ Geometric phase of light
- ▶ SOI in anisotropic medium

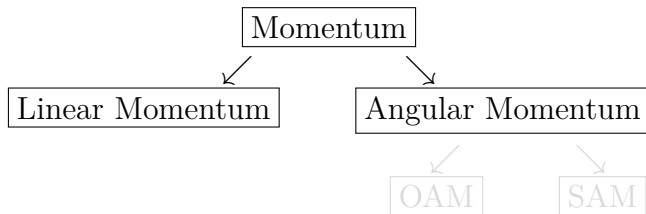
# Momentum of light



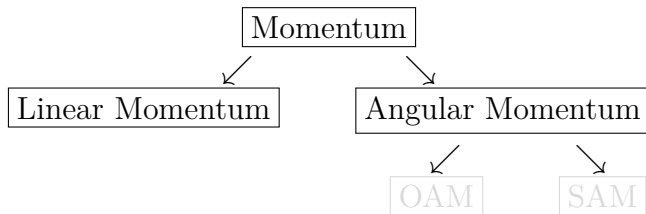
# Momentum of light



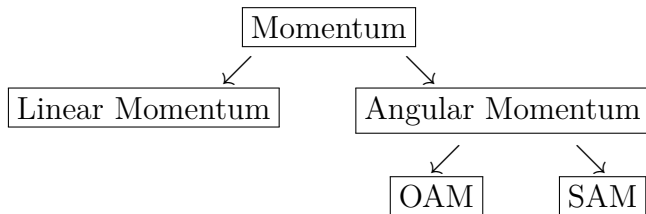
# Momentum of light



# Momentum of light



# Momentum of light



# Linear momentum of light

Monochromatic beam with angular frequency  $\omega$  propagating in z-direction :

$$\mathcal{E}(\mathbf{r}, t) = \text{Re}\{\mathbf{E}(\mathbf{r})e^{-i(\omega t - kz)}\}$$

$$\mathcal{B}(\mathbf{r}, t) = \text{Re}\{\mathbf{B}(\mathbf{r})e^{-i(\omega t - kz)}\}$$

Maxwell-Faraday law:

$$\nabla \times \mathbf{E} = i\omega \mathbf{B}$$

Time-averaged linear momentum per length,

$$\mathcal{P}_z = \frac{1}{c^2} \int d\tau \langle \mathbf{S} \rangle_z = \frac{\epsilon_0}{2i\omega} \iint dx dy \langle \mathbf{E} \times (\nabla \times \mathbf{E}) \rangle_z$$



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# Linear momentum of light

Monochromatic beam with angular frequency  $\omega$  propagating in z-direction :

$$\mathcal{E}(\mathbf{r}, t) = \text{Re}\{\mathbf{E}(\mathbf{r})e^{-(\omega t - kz)}\}$$

$$\mathcal{B}(\mathbf{r}, t) = \text{Re}\{\mathbf{B}(\mathbf{r})e^{-i(\omega t - kz)}\}$$

Maxwell-Faraday law:

$$\nabla \times \mathbf{E} = i\omega \mathbf{B}$$

Time-averaged linear momentum per length,

$$\mathcal{P}_z = \frac{1}{c^2} \int d\tau \langle \mathbf{S} \rangle_z = \frac{\epsilon_0}{2i\omega} \iint dx dy \langle \mathbf{E} \times (\nabla \times \mathbf{E}) \rangle_z$$

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Time-averaged AM per length :

$$\mathcal{J}_z = \frac{1}{c^2} \int d\tau [\mathbf{r} \times \langle \mathbf{S} \rangle]_z = \frac{\epsilon_0}{2i\omega} \iint dx dy [\mathbf{r} \times \langle \mathbf{E} \times (\nabla \times \mathbf{E}) \rangle]_z$$

For paraxial beam,

$$\begin{aligned} \mathcal{J}_z = & \frac{\epsilon_0}{2i\omega} \iint dx dy \left[ E_\xi^* \left( x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) E_\xi \right]_{\xi=x,y} \\ & + \frac{\epsilon_0}{2i\omega} \iint dx dy (E_x^* E_y + E_y^* E_x) \end{aligned}$$

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Spin AM,  $\mathcal{S}$



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$$\mathcal{L} = \frac{\epsilon_0}{2i\omega} \iint dx \, dy \left[ E_\xi^* (\mathbf{r} \times \nabla)_z E_\xi \right]_{\xi=x,y}$$

for vortex beam,

$$\mathbf{E}(r, \phi) = u(r) \exp(-il\phi) \hat{\mathbf{p}}$$

$$\mathcal{W}_z = \frac{\epsilon_0}{2} \iint dx \, dy \, \mathbf{E}^* \cdot \mathbf{E}$$

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$$\mathbf{r} \longrightarrow \mathbf{r}' = \mathbf{r} + \mathbf{r}_0$$

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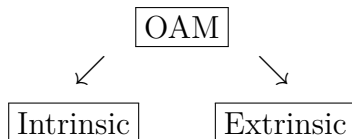
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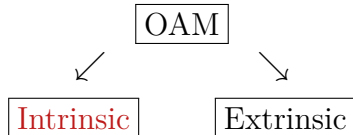
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$$\Delta\mathcal{J} = 0$$

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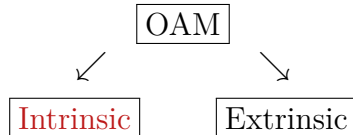
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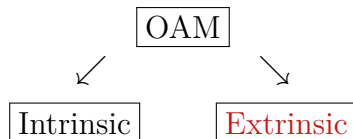
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$$\Delta\mathcal{J} \neq 0$$

# Spin AM

$$\mathcal{S} = \frac{\epsilon_0}{2i\omega} \iint dx dy (E_x^* E_y + E_y^* E_x)$$

SAM is **intrinsic**.

For vortex beam,

$$\mathbf{E}(r, \phi) = u(r) \exp(-il\phi) \hat{\mathbf{p}}$$

$$\frac{\mathcal{S}}{\mathcal{W}_z} = \frac{\text{SAM}}{\text{Total energy}} = \frac{\sigma}{\omega}$$

$\sigma = 2 \operatorname{Im}(p_x^* p_y) \rightarrow$  helicity of beam

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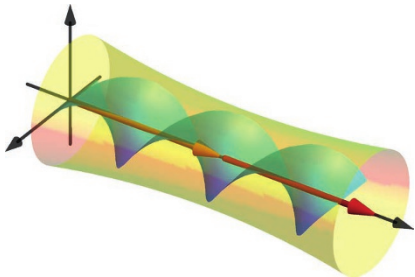
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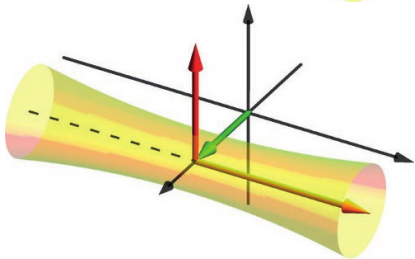
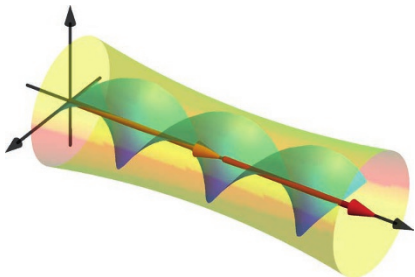
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# Visualisation of AM

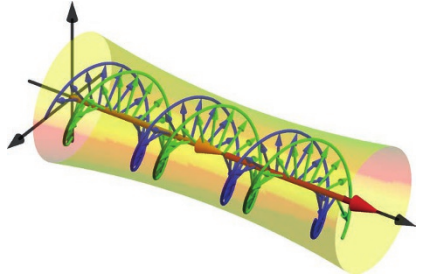
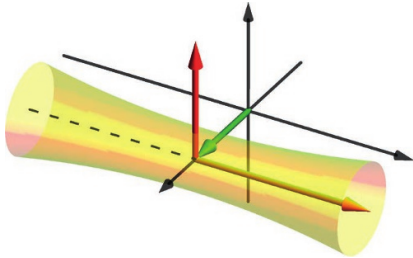
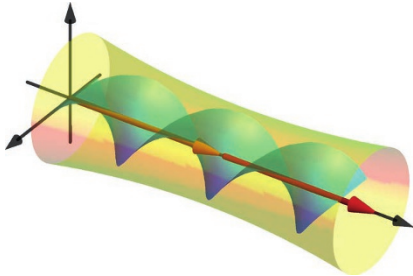




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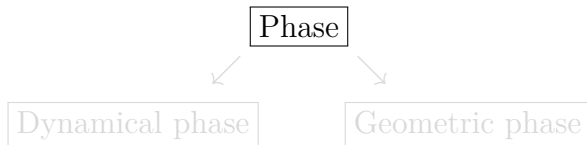
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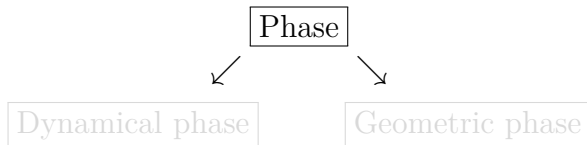
# Spin-orbit interaction

- ▶ Momentum of Light
- ▶ Geometric phase of light
- ▶ SOI in anisotropic medium

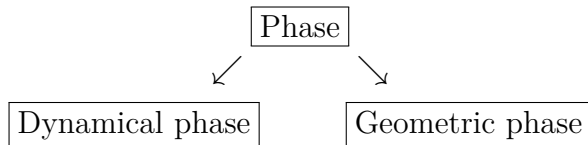
# Geometric phase



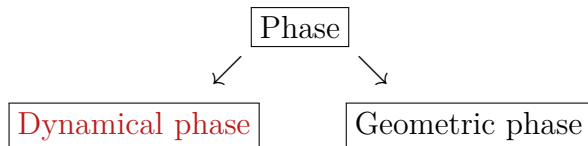
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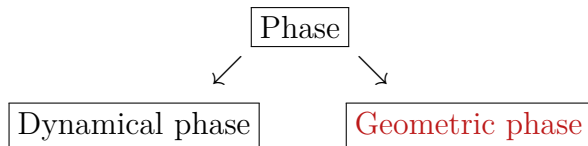


# Geometric phase



Associated with optical path length.

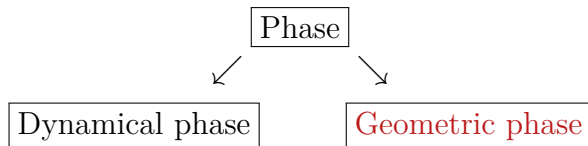
# Geometric phase



Associated with geometry of evolution.



# Geometric phase



Associated with geometry of evolution.

- ▶ Spin-redirection Berry phase
- ▶ Pancharatnam-Berry Phase

# Spin-redirection Berry phase

Associated with adiabatic evolution of wave-vector.  
*e.g.*, Polarized light through a helical optic fibre.

$$\mathbf{J} \longrightarrow \mathbf{J}' = \mathbf{J} \exp(i\sigma\Theta)$$

$$\Theta = 2\pi(1 - \cos \theta)$$

$\Theta \rightarrow$  solid angle obtained at  
the apex of the cone.

$$|L\rangle \longrightarrow e^{i\Theta} |L\rangle$$

$$|R\rangle \longrightarrow e^{-i\Theta} |R\rangle$$

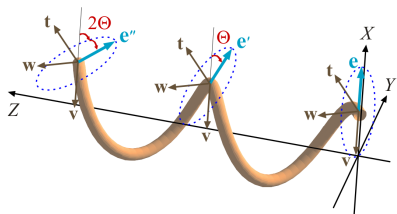
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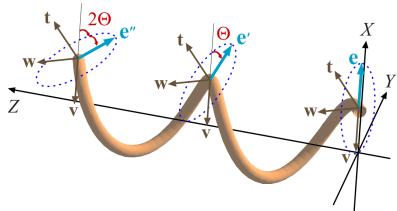
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Helicity-dependant geometric  
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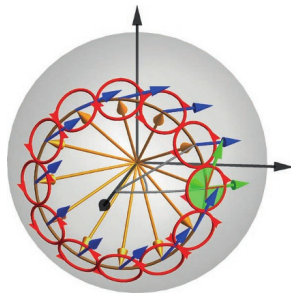
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*Parallel transport*

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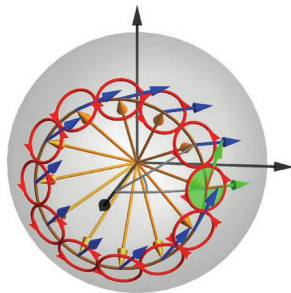
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$\Theta \rightarrow$  solid angle obtained at the apex of the cone.

$$|L\rangle \longrightarrow e^{i\Theta} |L\rangle$$

$$|R\rangle \longrightarrow e^{-i\Theta} |R\rangle$$



*Parallel transport*

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Associated with cyclic evolution in Poincare sphere  
keeping wave-vector fixed.

*e.g.*, Michelson interferometer.

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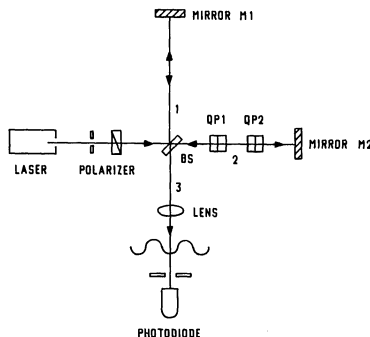
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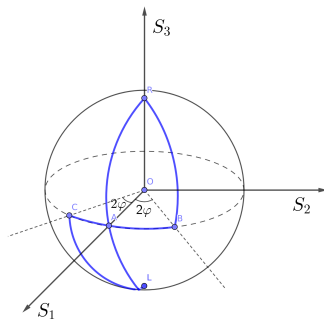
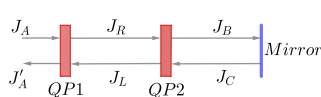
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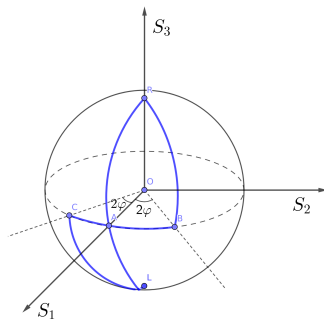
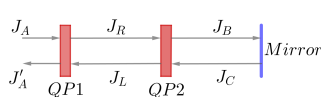
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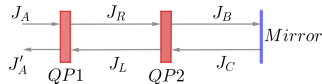
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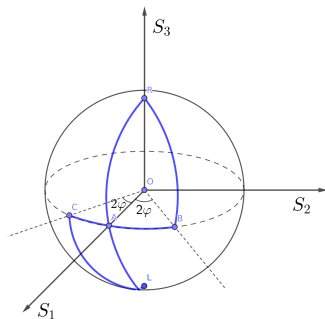
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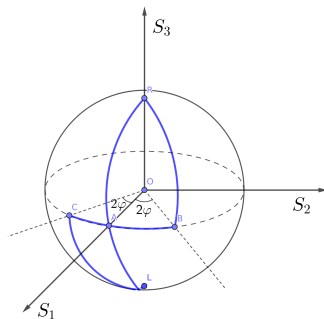
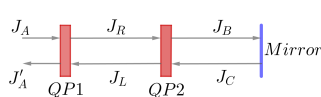
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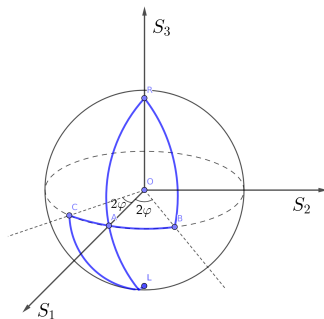
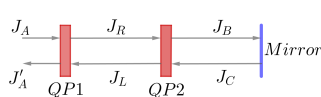
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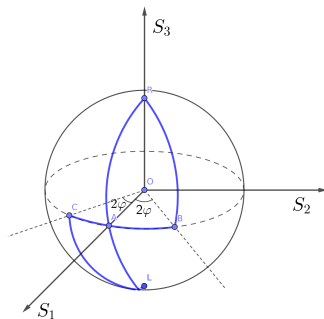
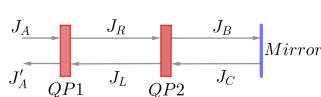
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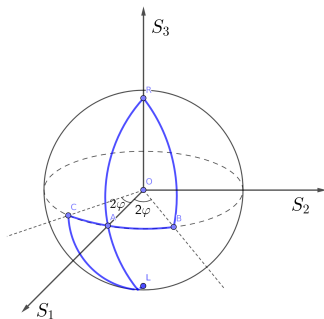
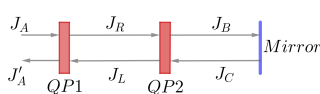
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# Spin-orbit interaction

- ▶ Momentum of Light
- ▶ Geometric phase of light
- ▶ SOI in anisotropic medium



# Spin-orbit interaction of light

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- ▶ IOAM
- ▶ EOAM
- ▶ SAM

Inter-conversion between AM in a process represents  
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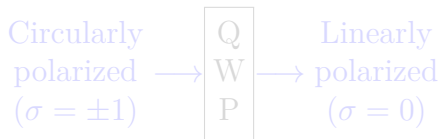
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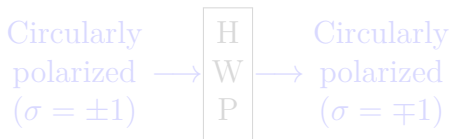
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*e.g.* **Quarter wave-plate**



$\pm \hbar$  SAM per photon is transferred to the wave plate.

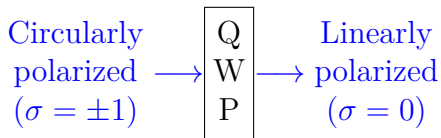
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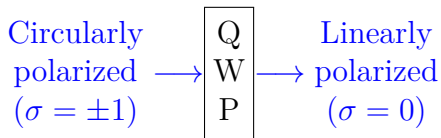
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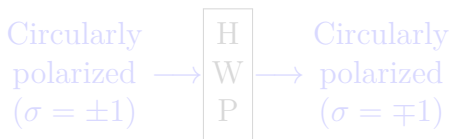
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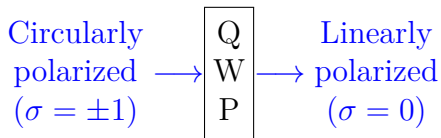


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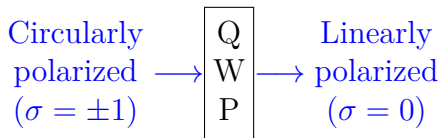
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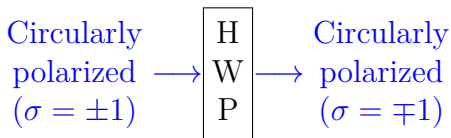
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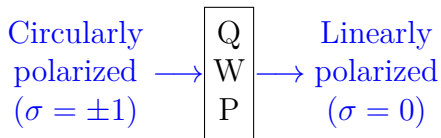
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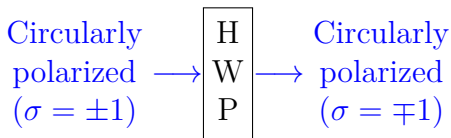
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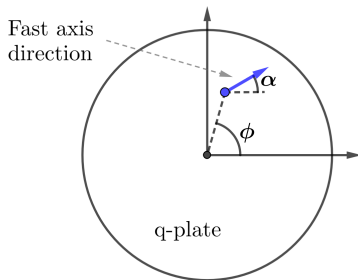
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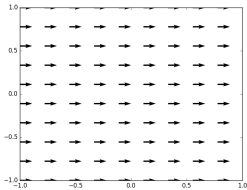
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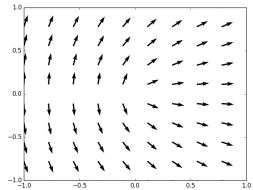
$$\alpha(\phi) = q\phi + \alpha_0$$



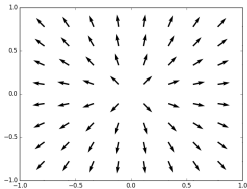
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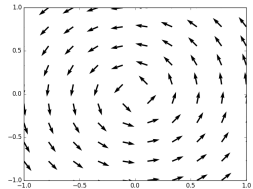
(a)  $q = 0, \alpha_0 = 0$



(b)  $q = 0.5, \alpha_0 = 0$



(a)  $q = 1, \alpha_0 = 0$



(b)  $q = 1, \alpha_0 = \pi/2$

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## Q-plate of phase retardation of $\pi$

$$\begin{bmatrix} 1 \\ i\sigma \end{bmatrix} \longrightarrow \boxed{\mathbf{Q}_{\lambda/2}} \longrightarrow \begin{bmatrix} 1 \\ -i\sigma \end{bmatrix} \underbrace{\exp(i2\sigma q\phi)}_{\text{Vortex}} \exp(i2\sigma\alpha_0)$$

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## Q-plate of phase retardation of $\pi/2$

$$\begin{bmatrix} 1 \\ i\sigma \end{bmatrix} \longrightarrow \boxed{\mathbf{Q}_{\lambda/4}} \longrightarrow \begin{bmatrix} \cos(\alpha - \sigma\pi/4) \\ \sin(\alpha - \sigma\pi/4) \end{bmatrix} \underbrace{\exp(i\sigma q\phi)}_{\text{Vortex}} \exp(i\sigma\alpha_0)$$

$$(\sigma = \pm 1, l = 0) \longrightarrow (\sigma = 0, l = \pm q)$$

$q = 1 \rightarrow$  Angular momentum per photon is conserved.

THANKS  
TO ALL