Summer Project

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1.2.1 Jones Vector

Vector form of electric field of fully polarized EM wave propagating along z-axis is given by

$$\mathbf{E}(\mathbf{x},t) = \begin{bmatrix} E_x(\mathbf{x},t) \\ E_y(\mathbf{x},t) \\ E_z(\mathbf{x},t) \end{bmatrix} = \begin{bmatrix} A_x(\mathbf{x})e^{-i(kz-\omega t-\delta_x)} \\ A_y(\mathbf{x})e^{-i(kz-\omega t-\delta_y)} \\ 0 \end{bmatrix} = \begin{bmatrix} A_x(\mathbf{x})e^{i\delta_x} \\ A_y(\mathbf{x})e^{i\delta_y} \\ 0 \end{bmatrix} e^{-i(kz-\omega t)} \tag{1.1}$$

We define normalized[1] *Jones vector* as

$$\mathbf{J}(\mathbf{x},t) = \frac{1}{\sqrt{A_x^2 + A_y^2}} \begin{bmatrix} A_x(\mathbf{x})e^{i\delta_x} \\ A_y(\mathbf{x})e^{i\delta_y} \end{bmatrix} \tag{1.2}$$

Such examples of usual polarization states are given below[2],

Polarization state	$\mathbf{J}$
$\lvert H\rangle$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
$\lvert V\rangle$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
$\lvert P\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\lvert M\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
$\lvert L\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$
$\lvert R\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$

Table 1: Jones vector of usual polarization state

Some properties of Jones vector are

1. The intensity of the EM wave is given by

$$I = \frac{1}{2}c\epsilon_0(A_x^2 + A_y^2) = \frac{1}{2}c\epsilon_0(E^*E) \tag{1.3}$$

2. For general elliptically polarized light we can measure the azimuth (α) ellipticity (ϵ) of the polarization ellipse by comparing Jones vector $\mathbf{J}$ with [1]

$$\begin{bmatrix} \cos\alpha\cos\epsilon - i\sin\alpha\sin\epsilon \\ \sin\alpha\cos\epsilon - i\cos\alpha\sin\epsilon \end{bmatrix}$$

[1] normalized as $\mathbf{J}\,\mathbf{J}^* = 1$

1.2.2 Jones Matrix & evolution of Jones vector

Jones matrix is a 2×2 matrix assigned for a particular optical element. let $\mathbf{M}$ be Jones matrix *s.t.*

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

then if a polarized light of Jones vector $\mathbf{J}_{in}$ passes through that optical element then the Jones vector of output light is given by

$$\mathbf{J}_{out} = \mathbf{M}\, \mathbf{J}_{in} \tag{1.4}$$

$$\Rightarrow \mathbf{E}_{out} = \mathbf{M}\, \mathbf{E}_{in} \tag{1.5}$$

To determine m_{ij} in $\mathbf{M}$,

1. Pass x-polarized light and determine $\mathbf{J}_{out}$, then

$$\mathbf{J}_{out} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} m_{11} \\ m_{21} \end{bmatrix}$$

2. Pass y-polarized light and determine $\mathbf{J}_{out}$, then

$$\mathbf{J}_{out} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} m_{12} \\ m_{22} \end{bmatrix}$$

Such examples of usual Jones matrix [2] are given below,[2]

Optical element	$\mathbf{M}$
Free space	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
x-Polariser	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
y-Polariser	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
Right circular polariser	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
Left circular polariser	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$
Linear di-attenuator	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
Half-wave plate with fast axis horizontal	$e^{-i\pi/2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Quarter-wave plate with fast axis horizontal	$e^{-i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
General phase retarder	$\begin{bmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{bmatrix}$

Table 2: Jones matrix related to usual optical element

[2] For polariser the Jones matrix $\mathbf{M} = \mathbf{J}\, \mathbf{J}^*$ where $\mathbf{J}$ is normalized Jones vector corresponding polarization state *s.t.* $\mathbf{J}_{out} = \mathbf{MJ} = (\mathbf{JJ}^*)\mathbf{J} = \mathbf{J}(\mathbf{J}^*\mathbf{J}) = \mathbf{J}$

Some properties of Jones matrix are

1. Resultant Jones matrix for composition of n optical element is given by
$$\mathbf{M} = \mathbf{M}_1\, \mathbf{M}_2 \dots \mathbf{M}_n \tag{1.6}$$
2. For an optical element when its optical axis aligned at an angle θ *w.r.t.* x-axis then resultant Jones matrix for this rotated optical element is given by
$$\mathbf{M}_\theta = R(-\theta)\, \mathbf{M}\, R(\theta) \tag{1.7}$$
where $R(\theta)$ is passive rotation matrix *s.t.*
$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{1.8}$$

1.2.3 Drawback of Jones formalism

Main drawback of Jones formalism is that its application is restricted in fully polarized light. This formalism cannot explain the partially polarized or unpolished light which we frequently observe in practical use.

1.3 Stokes-Muller formalism

1.3.1 Coherency matrix and coherency vector

Coherency matrix of a EM wave is defined as [1]
$$\mathbf{C} = \left\langle \mathbf{E} \otimes \mathbf{E}^\dagger \right\rangle = \left\langle \mathbf{E}\mathbf{E}^\dagger \right\rangle = \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix} = \begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix} \tag{1.9}$$
where $\otimes$ denotes Kronecker product, $\langle\cdot\rangle$ denotes the temporal avg of the corresponding quantity and $\delta = \delta_y - \delta_x$. And coherency vector is defined as

Examples of coherency matrix of usual polarization states are given below [3],

Polarization state	$\mathbf{J}$	$\mathbf{C}$
$\lvert H\rangle$	$\begin{bmatrix}1\\0\end{bmatrix}$	$\begin{bmatrix}1&0\\0&0\end{bmatrix}$
$\lvert V\rangle$	$\begin{bmatrix}0\\1\end{bmatrix}$	$\begin{bmatrix}0&0\\0&1\end{bmatrix}$
$\lvert P\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1&1\\1&1\end{bmatrix}$
$\lvert M\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1&-1\\-1&1\end{bmatrix}$
$\lvert L\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1&-i\\i&1\end{bmatrix}$
$\lvert R\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1&i\\-i&1\end{bmatrix}$
Un-polarized	–	$\frac{1}{2}\begin{bmatrix}1&0\\0&1\end{bmatrix}$

Table 3: coherency matrix of usual polarization state

Some properties of coherency matrix are

1. It is a hermitian matrix *i.e.* $\mathbf{C} = \mathbf{C}^\dagger$
2. Trace and determinant off the matrix are non-negative[3] *i.e.* $\mathrm{tr}(\mathbf{C}) > 0$ & $\det(\mathbf{C}) \geq 0$.
3. $\mathrm{Tr}(\mathbf{C}) = \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle$ is the time averaged intensity of input light.
4. let the polarized light (of electric field $\mathbf{E}_{in}$ & coherency matrix $\mathbf{C}_{in}$) passes through an optical element (of Jones matrix $\mathbf{M}$) then let output electric field be $\mathbf{E}_{out}$ by the equation 1.5, then output coherency matrix $\mathbf{C}_{out}$ is given by

$$\begin{aligned}
\mathbf{C}_{out} = \left\langle \mathbf{E}_{out}\mathbf{E}_{out}^\dagger \right\rangle &= \left\langle (\mathbf{M}\mathbf{E}_{in})\,(\mathbf{M}\mathbf{E}_{in})^\dagger \right\rangle \\
&= \left\langle (\mathbf{M}\mathbf{E}_{in}) \left(\mathbf{E}_{in}^\dagger \mathbf{M}^\dagger \right) \right\rangle \\
&= \mathbf{M} \left\langle \mathbf{E}_{in}\mathbf{E}_{in}^\dagger \right\rangle \mathbf{M}^\dagger \\
&= \mathbf{M}\,\mathbf{C}_{in}\,\mathbf{M}^\dagger
\end{aligned} \tag{1.10}$$

1.3.2 Stokes parameters and Stokes vector

Now we see that coherency matrix $\mathbf{C}$ of any polarization state in table 3 can be written in the linear combination of the 4 basis given below [4]

$$\beta = \left\{ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\boldsymbol{V_0}}, \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\boldsymbol{V_1}}, \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\boldsymbol{V_2}}, \underbrace{\begin{bmatrix} 0 & i \\ -i & 1 \end{bmatrix}}_{\boldsymbol{V_3}} \right\} \tag{1.11}$$

Now we can write any coherency matrix $\mathbf{C}$ as

$$\mathbf{C} = \frac{1}{2} \sum_{i=0}^{3} S_i \boldsymbol{V_i} \tag{1.12}$$

Note that all V_i's are Hermitian, so obviously is $\mathbf{C}$.

We call $\{S_0, S_1, S_2, S_3\}$ as a *Stokes parameter* and the values of S_i's are experimentally measurable.

A *Stokes vector* $\mathbf{S}$ is defined as[4]

$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \tag{1.13}$$

Examples of Stokes vector for different polarization states are given below

[3] Proofs to be done

[4] for intensity normalised Stokes vector, $\mathbf{S} = \begin{bmatrix} 1 & s_1 & s_2 & s_3 \end{bmatrix}$ where $s_i = S_i/S_0$

Polarization state	$\mathbf{C}$	$\mathbf{S}$
$\lvert H\rangle$	$\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$	$\begin{bmatrix}1 & 1 & 0 & 0\end{bmatrix}^T$
$\lvert V\rangle$	$\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & -1 & 0 & 0\end{bmatrix}^T$
$\lvert P\rangle$	$\frac{1}{2}\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0 & 1 & 0\end{bmatrix}^T$
$\lvert M\rangle$	$\frac{1}{2}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0 & -1 & 0\end{bmatrix}^T$
$\lvert L\rangle$	$\frac{1}{2}\begin{bmatrix}1 & -i\\i & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0 & 0 & 1\end{bmatrix}^T$
$\lvert R\rangle$	$\frac{1}{2}\begin{bmatrix}1 & i\\-i & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0 & 0 & -1\end{bmatrix}^T$
Un-polarized	$\frac{1}{2}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}^T$

Table 4: coherency matrix of usual polarization state

Now we see from the equation 1.12

$$\begin{bmatrix}\langle E_xE_x^*\rangle & \langle E_xE_y^*\rangle\\ \langle E_yE_x^*\rangle & \langle E_yE_y^*\rangle\end{bmatrix} = \mathbf{C} = \frac{1}{2}\sum_{i=0}^{3} S_i\boldsymbol{V_i} = \frac{1}{2}\begin{bmatrix}S_0+S_1 & S_2+iS_3\\ S_2-iS_3 & S_0-S_1\end{bmatrix} \tag{1.14}$$

From there we can write

$$\boldsymbol{S} = \begin{bmatrix}S_0\\S_1\\S_2\\S_3\end{bmatrix} = \begin{bmatrix}\langle E_xE_x^*\rangle + \langle E_xE_y^*\rangle\\ \langle E_xE_x^*\rangle - \langle E_xE_y^*\rangle\\ \langle E_yE_x^*\rangle + \langle E_xE_y^*\rangle\\ i\left(\langle E_yE_x^*\rangle + \langle E_xE_y^*\rangle\right)\end{bmatrix} \tag{1.15}$$

Now for a polarized light,

$$\mathbf{C} = \begin{bmatrix}\langle A_x^2\rangle & \langle A_xA_ye^{-i\delta}\rangle\\ \langle A_xA_ye^{i\delta}\rangle & \langle A_y^2\rangle\end{bmatrix} \text{ and } \boldsymbol{S} = \begin{bmatrix}S_0\\S_1\\S_2\\S_3\end{bmatrix} = \begin{bmatrix}\langle A_x^2+A_y^2\rangle\\ \langle A_x^2-A_y^2\rangle\\ \langle 2A_yA_y\cos\delta\rangle\\ \langle 2A_yA_y\sin\delta\rangle\end{bmatrix} \tag{1.16}$$

1.3.3 Measurement of Stokes parameters

To measure the 4 Stokes parameter of EM wave associated with, we have to do 4 steps experiment. In each case, we pass the light through various optical elements and measure the (time-averaged) intensity [5],

Step I Pass the light through homogenous isotropic medium (or, free space) and measure the intensity. From table 2 and eq. 1.10, we get,

$$\begin{aligned}\mathbf{C}_{out} &= \mathbf{M}\,\mathbf{C}_{in}\,\mathbf{M}^\dagger\\ &= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\frac{1}{2}\begin{bmatrix}S_0+S_1 & S_2+iS_3\\ S_2-iS_3 & S_0-S_1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\\ &= \frac{1}{2}\begin{bmatrix}S_0+S_1 & 0\\ 0 & S_0-S_3\end{bmatrix}\end{aligned} \tag{1.17}$$

So the measured intensity will be

$$I_0 = \mathrm{tr}(\mathbf{C}_{out}) = S_0 \tag{1.18}$$

Step II Pass the light through x-polariser and measure the intensity. From table 2 and eq. 1.10, we get,

$$\begin{aligned}
\mathbf{C}_{out} &= \mathbf{M}\,\mathbf{C}_{in}\,\mathbf{M}^\dagger \\
&= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{2} \begin{bmatrix} S_0 + S_1 & S_2 + iS_3 \\ S_2 - iS_3 & S_0 - S_1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\
&= \frac{1}{2} \begin{bmatrix} S_0 + S_1 & 0 \\ 0 & 0 \end{bmatrix}
\end{aligned} \tag{1.19}$$

So the measured intensity will be

$$I_1 = \mathrm{tr}(\mathbf{C}_{out}) = \frac{1}{2}(S_0 + S_1) \tag{1.20}$$

Step III Pass the light through the polariser with transmission axis is at 45° and measure the intensity. Then from eq. 1.7, **M** for this polariser will be

$$\mathbf{M} = R(-45°) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} R(45°) = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{1.21}$$

From eq. 1.10, we get,

$$\begin{aligned}
\mathbf{C}_{out} &= \mathbf{M}\,\mathbf{C}_{in}\,\mathbf{M}^\dagger \\
&= \mathbf{M}\,\mathbf{C}_{in}\,\mathbf{M}^\dagger \\
&= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} S_0 + S_1 & S_2 + iS_3 \\ S_2 - iS_3 & S_0 - S_1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \\
&= \frac{1}{4} \begin{bmatrix} S_0 + S_2 & S_0 + S_2 \\ S_0 + S_2 & S_0 + S_2 \end{bmatrix}
\end{aligned} \tag{1.22}$$

So the measured intensity will be

$$I_1 = \mathrm{tr}(\mathbf{C}_{out}) = \frac{1}{2}(S_0 + S_2) \tag{1.23}$$

Step IV Pass the light through right circular polariser and measure the intensity. From table 2 and eq. 1.10, we get,

$$\begin{aligned}
\mathbf{C}_{out} &= \mathbf{M}\,\mathbf{C}_{in}\,\mathbf{M}^\dagger \\
&= \frac{1}{2} \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} S_0 + S_1 & S_2 + iS_3 \\ S_2 - iS_3 & S_0 - S_1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}
\end{aligned} \tag{1.24}$$

So the measured intensity will be

$$I_1 = \mathrm{tr}(\mathbf{C}_{out}) = \frac{1}{2}(S_0 + S_3) \tag{1.25}$$

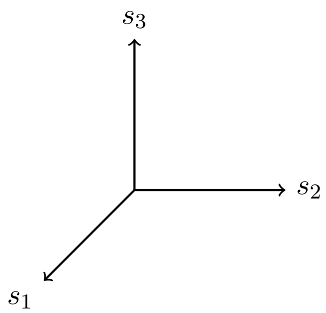
from equations 1.18, 1.20, 1.23 and 1.25, we can get the values of all S_i's.

1.3.4 Poincare sphere representation

For total intensity normalised Stokes vector is $\mathbf{S} = \begin{bmatrix} 1 & s_1 & s_2 & s_3 \end{bmatrix}$ where $s_i = S_i/S_0$. Observe that $\mathbf{S}$ is a 3-dimensional quantity. Therefore we can write,

$$\begin{bmatrix} 1 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} \rightarrow \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}$$

Poincare sphere representation is a coordinate system to define the state of polarization of light where the mutually orthogonal coordinate axes are $\{s_1, s_2, s_3\}$.



Example of special cases are

Case I For fully polarized light

$$s_1 = \frac{A_x^2 - A_y^2}{A_x^2 + A_y^2} \tag{1.26}$$

$$s_2 = \frac{2A_yA_y \cos\delta}{A_x^2 + A_y^2} \tag{1.27}$$

$$s_3 = \frac{2A_yA_y \sin\delta}{A_x^2 + A_y^2} \tag{1.28}$$

from there we can see

$$s_1^2 + s_2^2 + s_3^2 = 1 \tag{1.29}$$

which implies that fully polarized has the locus at any point in the sphere of radius 1 in Poincare sphere representation.

Case II For fully un-polarized light

$$s_1 = s_2 = s_3 = 0 \tag{1.30}$$

which implies that fully un-polarized has the locus at any the centre $(0, 0, 0)$ in the sphere of radius 1 in Poincare sphere representation.

1.3.5 Degree of Polarization

Degree of Polarization is the measure of polarisation of light.
We define

- Total degree of polarization, $DOP = \sqrt{s_1^2 + s_2^2 + s_3^2}$
- Degree of linear polarization $= \sqrt{s_1^2 + s_2^2}$
- Degree of circular polarization $= \sqrt{s_1^2 + s_2^2 + s_3^2}$

For any mixed polarization state we can decompose the Stokes vector into fullu polarized and un polarized components,

$$\begin{bmatrix} 1 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \sqrt{s_1^2 + s_2^2 + s_3^2} \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}}_{\text{fully polarised, } DOP=1} + \underbrace{\begin{bmatrix} 1 - \sqrt{s_1^2 + s_2^2 + s_3^2} \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\text{un-polarised}} \tag{1.31}$$

1.3.6 Muller matrix

1.3.7 Muller Matrix & evolution of Stokes vector

1.3.8 Relationship between Jones & Muller matrix

1.4 More on Elliptically polarized light

1.4.1 Jones vector of elliptically polarized light

1.4.2 Stokes vector and corresponding Poincare representation

References

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2 GAUSSIAN BEAM

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