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Pritish Karmakar

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1 Gaussian curve and its fourier transform

1.1 Standard normal

```
import matplotlib.pyplot as plt
import numpy as np
plt.style.use("classic")

def f(t):
    return np.exp(-(t)**2/2)/(np.sqrt(2*np.pi))

xv = np.linspace(-7,7,1000)
yv = f(xv)

plt.plot(xv, yv, lw=1)
plt.xlabel("$t$")
plt.ylabel("$f(t)$")
plt.grid(True)
```

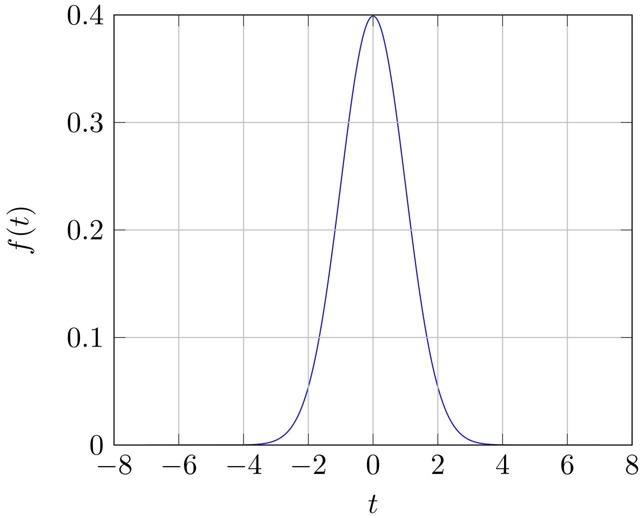

Figure 1: Standard Normal curve

1.2 Fourier transform of Standard normal

If $f(t) = \int_{-\infty}^{\infty} g(\omega)e^{i\omega t}d\omega$, then fourier transform of that is $g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$. Here, $f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$

$$\begin{aligned} g(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\cos(\omega t)dt - i\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\sin(\omega t)dt \end{aligned}$$

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import quad

def f(t):
    return np.exp(-(t)**2/2)/(np.sqrt(2*np.pi))

def ft(y):
    int_re = lambda t: f(t)*np.sin(y*t)
    int_im = lambda t: f(t)*np.cos(y*t)
    g_re = quad(int_re,-np.inf,np.inf)[0]/(2*np.pi)
    g_im = quad(int_im,-np.inf,np.inf)[0]/(2*np.pi)
    return g_re - 1j*g_im
g = np.frompyfunc(ft, 1, 1)

xv = np.linspace(-7,7,1000)
yv = np.abs(g(xv))
plt.plot(xv, yv, lw=1)
plt.xlabel("$\omega$")
plt.ylabel("$abs(g(\omega))$")
plt.grid(True)
```

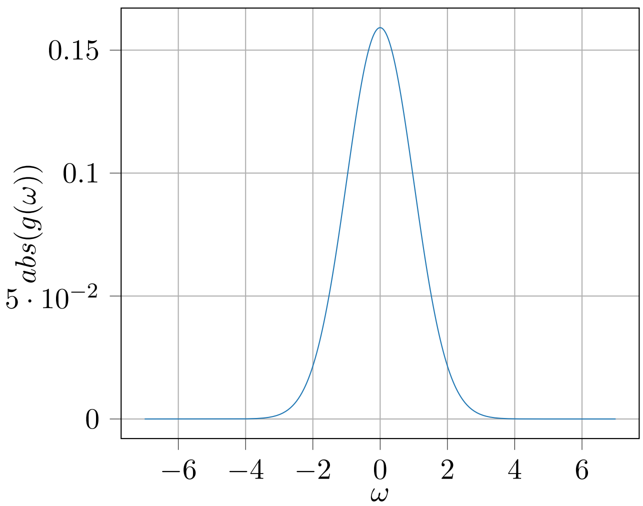

Figure 2: Fourier transform of Standard Normal

2 Gaussian beam

2.1 Intensity profile

```
import numpy as np
import matplotlib.pyplot as plt

R=lambda r: np.exp(-2*r**2)
a1=np.linspace(-1.7,1.7,200)
xv,yv=np.meshgrid(a1,a1)
zv=R(np.sqrt(xv**2+yv**2))
plt.contourf(xv,yv,zv,levels=100,cmap='viridis')
plt.xlabel("x")
plt.ylabel("y")
plt.colorbar()

theta=np.linspace(0,2*np.pi,500)
x=np.cos(theta)
y=np.sin(theta)
plt.plot( x, y, "--", lw=0.8)
```

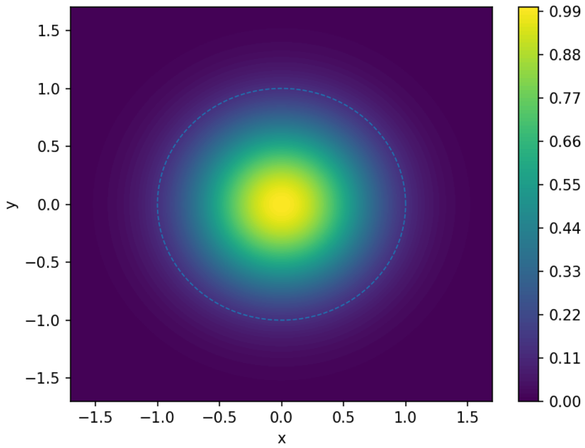

Figure 3: Intensity variation in a cross section ($z = 0, z_0 = 1$)

```
import numpy as np
import matplotlib.pyplot as plt

R=lambda x,z: np.exp(-2*x**2/(1+z**2))/(1+z**2)
a1=np.linspace(-1.5,1.5,500)
a2=np.linspace(-3,3,500)
xv,zv=np.meshgrid(a2,a1)
I=R(xv,zv)
plt.contourf(zv,xv,I,levels=100,cmap='viridis')
plt.colorbar()
plt.xlabel("z")
plt.ylabel("x-y plane")

w=lambda z1: np.sqrt(1+z1**2)
plt.plot(zv,w(zv),"--",lw=0.7)
plt.plot(zv,-w(zv),"--",lw=0.7)
```

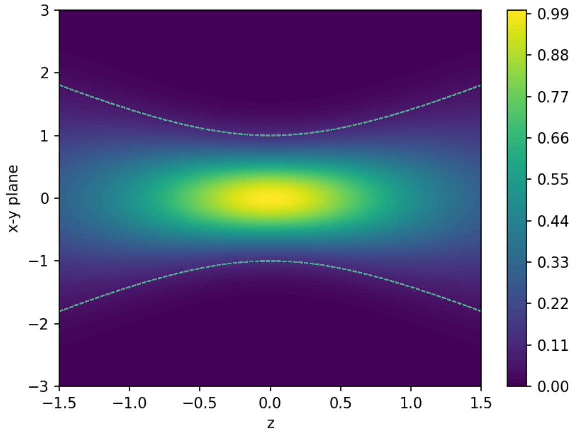

Figure 4: Intensity variation along z ($z_0 = 1$)