1 GAUSSIAN and ITS FOURIER TRANSFORM

1.1 Standard normal

Standard normal curve is

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$$

```python
import matplotlib.pyplot as plt
import numpy as np
plt.style.use("classic")

def f(t):
    return np.exp(-(t)**2/2)/(np.sqrt(2*np.pi))

xv = np.linspace(-7,7,1000)
yv = f(xv)

plt.plot(xv, yv, lw=1)
plt.xlabel("$t$")
plt.ylabel("$f(t)$")
plt.grid(True)
```

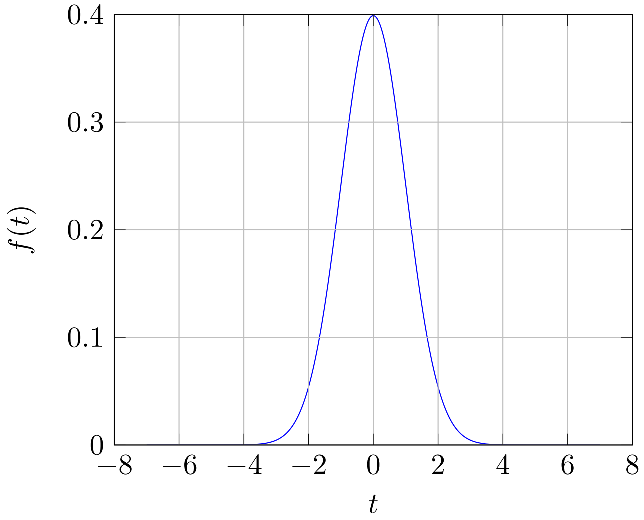


Figure 1: Standard Normal curve

1.2 Fourier transform of Standard normal

If $f(t) = \int_{-\infty}^{\infty} g(\omega) e^{i\omega t} d\omega$, then fourier transform of that is $g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$. Here, $f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$

$$\begin{aligned} g(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\cos(\omega t) dt - i\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\sin(\omega t) dt \end{aligned}$$

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import quad

def f(t):
    return np.exp(-(t)**2/2)/(np.sqrt(2*np.pi))

def ft(y):
    int_re = lambda t: f(t)*np.sin(y*t)
    int_im = lambda t: f(t)*np.cos(y*t)
    g_re = quad(int_re,-np.inf,np.inf)[0]/(2*np.pi)
    g_im = quad(int_im,-np.inf,np.inf)[0]/(2*np.pi)
    return g_re - 1j*g_im
g = np.frompyfunc(ft, 1, 1)

xv = np.linspace(-7,7,1000)
yv = np.abs(g(xv))
plt.plot(xv, yv, lw=1)
plt.xlabel("$\omega$")
plt.ylabel("$abs(g(\omega))$")
plt.grid(True)
```

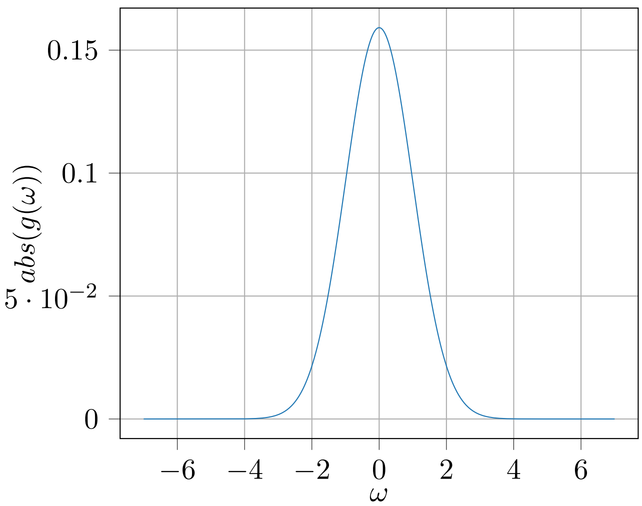


Figure 2: Fourier transform of Standard Normal

2 GAUSSIAN BEAMS

2.1 Intensity profile

Intensity of Gaussian beam is given by,

$$I(x,y,z) = \frac{c\epsilon}{2}|A|^2 \exp\left(-2(x^2+y^2)/w^2(z)\right)$$

where

$$w(z) = w_0\sqrt{1+z^2/z_02} \text{ and } z_0 = \pi w_0^2/\lambda$$

```python
import numpy as np
import matplotlib.pyplot as plt

R=lambda r: np.exp(-2*r**2)
a1=np.linspace(-1.7,1.7,200)
xv,yv=np.meshgrid(a1,a1)
zv=R(np.sqrt(xv**2+yv**2))
plt.contourf(xv,yv,zv,levels=100,cmap='viridis')
plt.xlabel("x")
plt.ylabel("y")
plt.colorbar()

theta=np.linspace(0,2*np.pi,500)
x=np.cos(theta)
y=np.sin(theta)
plt.plot( x, y, "--", lw=0.8)
```

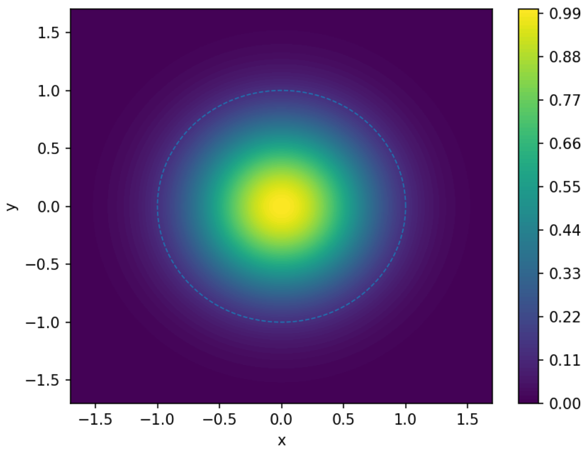


Figure 3: Intensity variation in a cross section $(z = 0, z_0 = 1, w_0 = 1)$

```python
import numpy as np
import matplotlib.pyplot as plt

R=lambda x,z: np.exp(-2*x**2/(1+z**2))/(1+z**2)
a1=np.linspace(-1.5,1.5,500)
a2=np.linspace(-3,3,500)
xv,zv=np.meshgrid(a2,a1)
I=R(xv,zv)
plt.contourf(zv,xv,I,levels=100,cmap='viridis')
plt.colorbar()
plt.xlabel("z")
plt.ylabel("x-y plane")

w=lambda z1: np.sqrt(1+z1**2)
plt.plot(zv,w(zv),"--",lw=0.7)
plt.plot(zv,-w(zv),"--",lw=0.7)
```

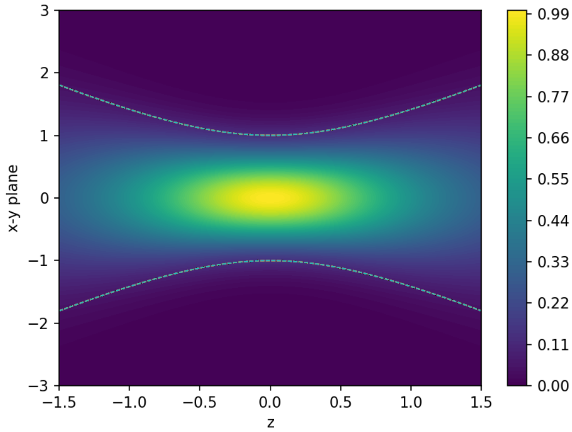


Figure 4: Intensity variation along z ($z_0 = 1, w_0 = 1$)

3 HERMITE–GAUSSIAN BEAMS

3.1 Intensity profile

Intensity of Hermite-Gaussian beam is given by,

$$I_{m,n}(x,y,z) = \frac{c\epsilon}{2}|A|^2 \left[H_m(\sqrt{2}x/w(z))\right]^2 \left[H_n(\sqrt{2}y/w(z))\right]^2 \exp\left(-2(x^2+y^2)/w^2(z)\right)$$

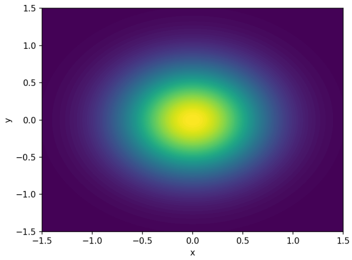
where

$$w(z) = w_0\sqrt{1+z^2/z_0 2} \tag{1}$$

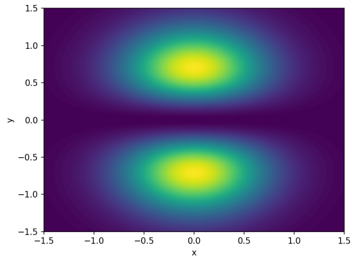
$$z_0 = \pi w_0^2/\lambda \tag{2}$$

$$H_n = \text{n-th order Hermite polynomial} \tag{3}$$

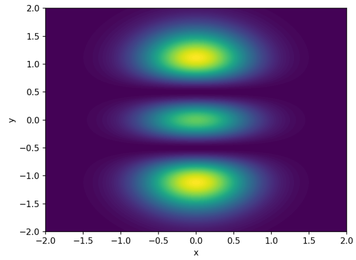
```
import numpy as np
import matplotlib.pyplot as plt
from scipy import special

H=lambda n: special.hermite(n, monic=False)

m=int(input("m="))
n=int(input("n="))

R=lambda m,n,x,y: (H(m)(np.sqrt(2)*x)*H(n)(np.sqrt(2)*y))**2*np.exp(-2*(x**2+y
    **2))
l=2
a1=np.linspace(-l,l,200)
xv,yv=np.meshgrid(a1,a1)
zv=R(m,n,xv,yv)
plt.contourf(xv,yv,zv,levels=100,cmap='viridis')
plt.xlabel("x")
plt.ylabel("y")
#plt.colorbar()
```



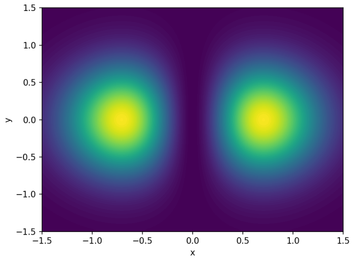
(a) TEM_{00}



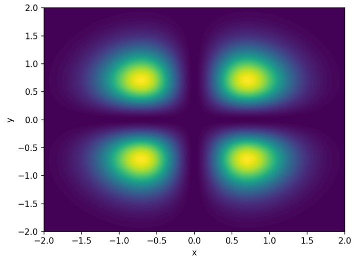
(b) TEM_{01}



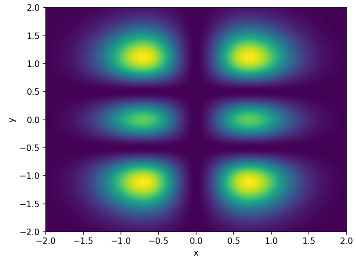
(c) TEM_{02}



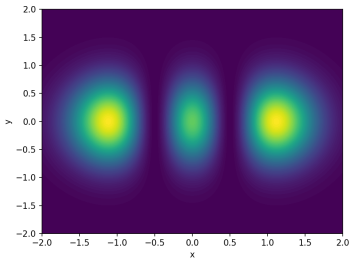
(d) TEM_{10}



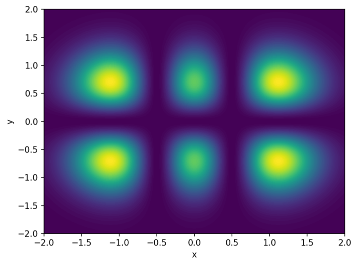
(e) TEM_{11}



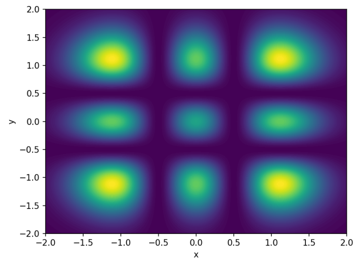
(f) TEM_{12}



(g) TEM_{20}



(h) TEM_{21}



(i) TEM_{22}

Figure 5: Intensity variation for different TEM in a cross section $(z = 0, z_0 = 1, w_0 = 1)$

4 LAGUERRE–GAUSSIAN BEAMS

4.1 Intensity profile

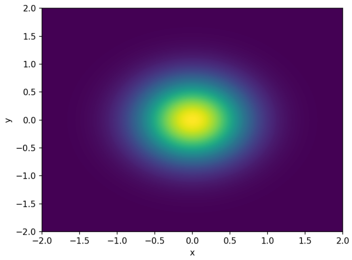
Intensity of Hermite-Gaussian beam is given by,

$$I_{p,l}(r,z) = \frac{c\epsilon}{2}|A|^2 \left[\frac{w_0}{w(z)}\right]^2 \left[\frac{r\sqrt{2}}{w(z)}\right]^{2|l|} \left[L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right)\right]^2 \exp\left(-2(x^2+y^2)/w^2(z)\right)$$

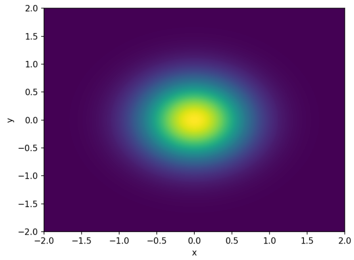
where

$$\begin{aligned} r^2 &= x^2 + y^2 \\ w(z) &= w_0\sqrt{1 + z^2/z_0 2} \\ z_0 &= \pi w_0^2/\lambda \\ L_p^{|l|} &= \text{Associated Laguerre polynomial} \end{aligned}$$

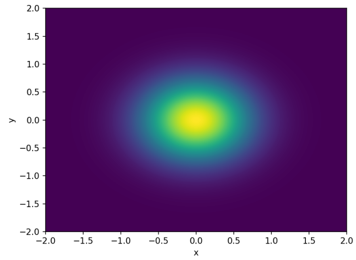
```python
import numpy as np
import matplotlib.pyplot as plt
from scipy import special

L= lambda x,n,l: special.assoc_laguerre(x,n,l)

p=int(input("p="))
l=int(input("l="))

R=lambda p,l,x,y: ((np.sqrt(2*(x**2+y**2)))**(2*np.abs(l)))*(L(2*(x**2+y**2),p,
    np.abs(l)))**2*np.exp(-2*(x**2+y**2))
a=2
a1=np.linspace(-a,a,200)
xv,yv=np.meshgrid(a1,a1)
zv=R(p,l,xv,yv)
plt.contourf(xv,yv,zv,levels=300,cmap='viridis')
plt.xlabel("x")
plt.ylabel("y")
#plt.colorbar()
```



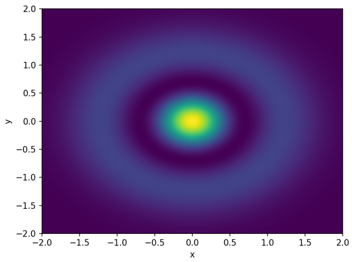
(a) $p = 0, |l| = 0$



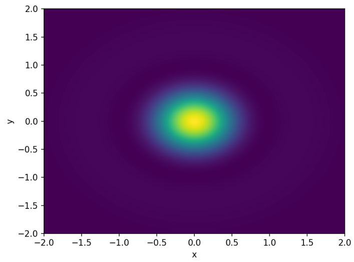
(b) $p = 0, |l| = 1$



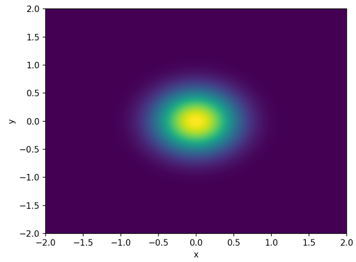
(c) $p = 0, |l| = 2$



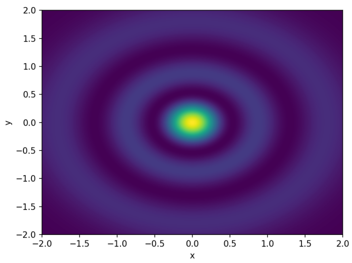
(d) $p = 1, |l| = 0$



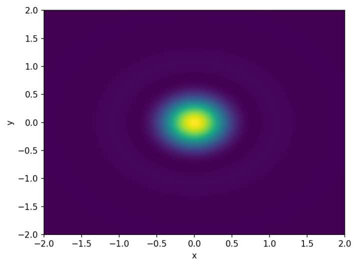
(e) $p = 1, |l| = 1$



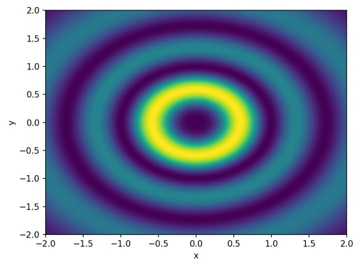
(f) $p = 1, |l| = 2$



(g) $p = 2, |l| = 0$



(h) $p = 2, |l| = 1$



(i) $p = 2, |l| = 2$

Figure 6: Intensity variation for different modes in a cross section $(z = 0, z_0 = 1, w_0 = 1)$