Summer Project

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Vector form of electric field of fully polarized EM wave propagating along z-axis is given by

$$\mathbf{E}(\mathbf{x},t) = \begin{bmatrix} E_x(\mathbf{x},t) \\ E_y(\mathbf{x},t) \\ E_z(\mathbf{x},t) \end{bmatrix} = \begin{bmatrix} A_x(\mathbf{x})e^{-i(kz-\omega t-\delta_x)} \\ A_y(\mathbf{x})e^{-i(kz-\omega t-\delta_y)} \\ 0 \end{bmatrix} = \begin{bmatrix} A_x(\mathbf{x})e^{i\delta_x} \\ A_y(\mathbf{x})e^{i\delta_y} \\ 0 \end{bmatrix} e^{-i(kz-\omega t)} \tag{1.1}$$

We define normalized[1] *Jones vector* as

$$\mathbf{J}(\mathbf{x},t) = \frac{1}{\sqrt{A_x^2 + A_y^2}} \begin{bmatrix} A_x(\mathbf{x})e^{i\delta_x} \\ A_y(\mathbf{x})e^{i\delta_y} \end{bmatrix} \tag{1.2}$$

Such examples of usual polarization states are given below[1],

Polarization state	$\mathbf{J}$
$\lvert H\rangle$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
$\lvert V\rangle$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
$\lvert P\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\lvert M\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
$\lvert L\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$
$\lvert R\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$

Table 1: Jones vector of usual polarization state

Some properties of Jones vector are

1. The intensity of the EM wave is given by

$$I = \frac{1}{2}c\epsilon_0(A_x^2 + A_y^2) = \frac{1}{2}c\epsilon_0(E^*E) \tag{1.3}$$

2. For general elliptically polarized light we can measure the azimuth (α) ellipticity (ϵ) of the polarization ellipse by comparing Jones vector $\mathbf{J}$ with [2]

$$\begin{bmatrix} \cos\alpha\cos\epsilon - i\sin\alpha\sin\epsilon \\ \sin\alpha\cos\epsilon - i\cos\alpha\sin\epsilon \end{bmatrix}$$

[1] normalized as $\mathbf{J}\,\mathbf{J}^* = 1$

1.2.2 Jones Matrix & evolution of Jones vector

Jones matrix is a 2×2 matrix assigned for a particular optical element. let $\mathbf{M}$ be Jones matrix *s.t.*

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

then if a polarized light of Jones vector $\mathbf{J}_{in}$ passes through that optical element then the Jones vector of output light is given by

$$\mathbf{J}_{out} = \mathbf{M}\ \mathbf{J}_{in} \tag{1.4}$$
$$\Rightarrow \mathbf{E}_{out} = \mathbf{M}\ \mathbf{E}_{in} \tag{1.5}$$

To determine m_{ij} in $\mathbf{M}$,

1. Pass x-polarized light and determine $\mathbf{J}_{out}$, then
$$\mathbf{J}_{out} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} m_{11} \\ m_{21} \end{bmatrix}$$

2. Pass y-polarized light and determine $\mathbf{J}_{out}$, then
$$\mathbf{J}_{out} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} m_{12} \\ m_{22} \end{bmatrix}$$

Such examples of usual Jones matrix [2] are given below,[1]

Optical element	$\mathbf{M}$
Free space	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
x-Polariser	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
y-Polariser	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
Right circular polariser	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
Left circular polariser	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$
Linear di-attenuator	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
Half-wave plate with fast axis horizontal	$e^{-i\pi/2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Quarter-wave plate with fast axis horizontal	$e^{-i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
General phase retarder	$\begin{bmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{bmatrix}$

Table 2: Jones matrix related to usual optical element

[2] For polariser the Jones matrix $\mathbf{M} = \mathbf{J}\ \mathbf{J}^*$ where $\mathbf{J}$ is normalized Jones vector corresponding polarization state *s.t.* $\mathbf{J}_{out} = \mathbf{MJ} = (\mathbf{JJ}^*)\mathbf{J} = \mathbf{J}(\mathbf{J}^*\mathbf{J}) = \mathbf{J}$

Some properties of Jones matrix are

1. Resultant Jones matrix for composition of n optical element is given by
$$\mathbf{M} = \mathbf{M}_1\, \mathbf{M}_2 \ldots \mathbf{M}_n \tag{1.6}$$
2. For an optical element when its optical axis aligned at an angle θ *w.r.t.* x-axis then resultant Jones matrix for this rotated optical element is given by
$$\mathbf{M}_\theta = R(-\theta)\, \mathbf{M}\, R(\theta) \tag{1.7}$$
where $R(\theta)$ is passive rotation matrix *s.t.*
$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{1.8}$$

1.2.3 Drawback of Jones formalism

Main drawback of Jones formalism is that its application is restricted in fully polarized light. This formalism cannot explain the partially polarized or unpolished light which we frequently observe in practical use.

1.3 Stokes-Muller formalism

1.3.1 Coherency matrix and coherency vector

Coherency matrix of a EM wave is defined as [2]
$$\mathbf{C} = \left\langle \mathbf{E} \otimes \mathbf{E}^\dagger \right\rangle = \left\langle \mathbf{E}\mathbf{E}^\dagger \right\rangle = \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix} = \begin{bmatrix} A_x^2 & A_x A_y e^{-i\delta} \\ A_x A_y e^{i\delta} & A_y^2 \end{bmatrix} \tag{1.9}$$
where $\otimes$ denotes Kronecker product, $\langle\cdot\rangle$ denotes the temporal avg of the corresponding quantity and $\delta = \delta_y - \delta_x$. And coherency vector is defined as

Examples of coherency matrix of usual polarization states are given below [3],

Polarization state	$\mathbf{J}$	$\mathbf{C}$
$\lvert H\rangle$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
$\lvert V\rangle$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
$\lvert P\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
$\lvert M\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
$\lvert L\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$
$\lvert R\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
Un-polarized	–	$\frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Table 3: coherency matrix of usual polarization state

Some properties of coherency matrix are

1. It is a hermitian matrix *i.e.* $\mathbf{C} = \mathbf{C}^\dagger$
2. Trace and determinant off the matrix are non-negative[3] *i.e.* $\text{tr}(\mathbf{C}) > 0$ & $\det(\mathbf{C}) \geq 0$.
3. $\text{Tr}(\mathbf{C}) = \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle$ is the time averaged intensity of input light.
4. let the polarized light (of electric field $\mathbf{E}_{in}$ & coherency matrix $\mathbf{C}_{in}$) passes through an optical element (of Jones matrix $\mathbf{M}$) then let output electric field be $\mathbf{E}_{out}$ by the equation 1.5, then output coherency matrix $\mathbf{C}_{out}$ is given by

$$\begin{aligned}\mathbf{C}_{out} = \left\langle \mathbf{E}_{out}\mathbf{E}_{out}^\dagger \right\rangle &= \left\langle (\mathbf{M}\mathbf{E}_{in})\,(\mathbf{M}\mathbf{E}_{in})^\dagger \right\rangle \\ &= \left\langle (\mathbf{M}\mathbf{E}_{in})\left(\mathbf{E}_{in}^\dagger \mathbf{M}^\dagger\right) \right\rangle \\ &= \mathbf{M}\left\langle \mathbf{E}_{in}\mathbf{E}_{in}^\dagger \right\rangle \mathbf{M}^\dagger \\ &= \mathbf{M}\,\mathbf{C}_{in}\,\mathbf{M}^\dagger \end{aligned} \tag{1.10}$$

1.3.2 Stokes parameters and Stokes vector

Now we see that coherency matrix $\mathbf{C}$ of any polarization state in table 3 can be written in the linear combination of the 4 basis given below

$$\beta = \left\{ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\boldsymbol{V_0}}, \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\boldsymbol{V_1}}, \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\boldsymbol{V_2}}, \underbrace{\begin{bmatrix} 0 & i \\ -i & 1 \end{bmatrix}}_{\boldsymbol{V_3}} \right\} \tag{1.11}$$

Now we can write any coherency matrix $\mathbf{C}$ as

$$\mathbf{C} = \frac{1}{2}\sum_{i=0}^{3} S_i \boldsymbol{V_i} \tag{1.12}$$

Note that all V_i's are Hermitian, so obviously is $\mathbf{C}$.

We call $\{S_0, S_1, S_2, S_3\}$ as a *Stokes parameter* and the values of S_i's are experimentally measurable.

A *Stokes vector* $\mathbf{S}$ is defined as[4]

$$\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \tag{1.13}$$

Examples of Stokes vector for different polarization states are given below

[3] Proofs to be done

[4] for intensity normalised Stokes vector, $\mathbf{S} = \begin{bmatrix} 1 & s_1 & s_2 & s_3 \end{bmatrix}$ where $s_i = S_i/S_0$

Polarization state	$\mathbf{C}$	$\mathbf{S}$
$\vert H\rangle$	$\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$	$\begin{bmatrix}1 & 1 & 0 & 0\end{bmatrix}^T$
$\vert V\rangle$	$\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & -1 & 0 & 0\end{bmatrix}^T$
$\vert P\rangle$	$\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0 & 1 & 0\end{bmatrix}^T$
$\vert M\rangle$	$\frac{1}{2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0 & -1 & 0\end{bmatrix}^T$
$\vert L\rangle$	$\frac{1}{2}\begin{bmatrix}1 & -i\\ i & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0 & 0 & 1\end{bmatrix}^T$
$\vert R\rangle$	$\frac{1}{2}\begin{bmatrix}1 & i\\ -i & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0 & 0 & -1\end{bmatrix}^T$
Un-polarized	$\frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}^T$

Table 4: coherency matrix of usual polarization state

Now we see from the equation 1.12

$$\begin{bmatrix}\langle E_xE_x^*\rangle & \langle E_xE_y^*\rangle\\ \langle E_yE_x^*\rangle & \langle E_yE_y^*\rangle\end{bmatrix} = \mathbf{C} = \frac{1}{2}\sum_{i=0}^{3} S_i\boldsymbol{V_i} = \frac{1}{2}\begin{bmatrix}S_0+S_1 & S_2+iS_3\\ S_2-iS_3 & S_0-iS_3\end{bmatrix} \tag{1.14}$$

From there we can write

$$\boldsymbol{S} = \begin{bmatrix}S_0\\ S_1\\ S_2\\ S_3\end{bmatrix} = \begin{bmatrix}\langle E_xE_x^*\rangle + \langle E_xE_y^*\rangle\\ \langle E_xE_x^*\rangle - \langle E_xE_y^*\rangle\\ \langle E_yE_x^*\rangle + \langle E_xE_y^*\rangle\\ i\left(\langle E_yE_x^*\rangle + \langle E_xE_y^*\rangle\right)\end{bmatrix} = \begin{bmatrix}A_x^2+A_y^2\\ A_x^2-A_y^2\\ 2A_yA_y\cos\delta\\ 2A_yA_y\sin\delta\end{bmatrix} \tag{1.15}$$

1.3.3 Measurement of Stokes parameters

1.3.4 Poincare sphere representation

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1.4.1 Jones vector of elliptically polarized light

1.4.2 Stokes vector and corresponding Poincare representation

References

[1] Jones Calculus.

[2] A. Banerjee, N. Ghosh, S. D. G. *Wave Optics*.

[3] Jizhong, W. A matrix method for describing unpolarized light and its applications.

2 GAUSSIAN BEAM

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