On Polarization Properties of Light, Gaussian Beams and Spin-Orbit Interaction of Light

Pritish Karmakar

(21MS179, IISER Kolkata)

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Submitted to

Prof. Ayan Banerjee

(HOD, DPS, IISER Kolkata)

IISER KOLKATA

Topics of discussion

Polarization properties of light

Gaussian Beam

Spin-orbit interaction of light

Polarization properties of light

- Jones Formalism
- Stokes-Muller formalism

Jones Vector

Electric field of *fully polarized* EM wave propagating along z-axis is given by

$$\boldsymbol{E}(\boldsymbol{r}, t) = \begin{bmatrix} A_x(\boldsymbol{r})e^{i\delta_x} \\ A_y(\boldsymbol{r})e^{i\delta_y} \\ 0 \end{bmatrix} e^{-i(kz-\omega t)}$$

Define normalized **Jones vector** *s.t.* $\boldsymbol{J}^* \boldsymbol{J} = 1$ as

$$\boldsymbol{J}(\boldsymbol{r}, t) = \frac{1}{\sqrt{A_x^2 + A_y^2}} \begin{bmatrix} A_x(\boldsymbol{r})e^{i\delta_x} \\ A_y(\boldsymbol{r})e^{i\delta_y} \end{bmatrix}$$

Note that *intensity*, $I = A_x^2 + A_y^2 = \boldsymbol{J}^* \boldsymbol{J}$

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Wave Optics (2015)

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Note that *intensity*, $I = A_x^2 + A_y^2 = \boldsymbol{J}^* \boldsymbol{J}$

Jones vector of usual polarization state

Polarization state	$\boldsymbol{J}$
$\vert H\rangle$	$\begin{bmatrix}1\\0\end{bmatrix}$
$\vert V\rangle$	$\begin{bmatrix}0\\1\end{bmatrix}$
$\vert P\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$
$\vert M\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$
$\vert L\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$
$\vert R\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$

Jones Matrix & evolution of Jones vector

Jones matrix for an optical element be $\boldsymbol{M}$ *s.t.*

$$\boldsymbol{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

If a polarized light of Jones vector $\boldsymbol{J}_{in}$ passes through that optical element then the Jones vector of output light is given by

$$\boldsymbol{J}_{out} = \boldsymbol{M}\ \boldsymbol{J}_{in}$$

- Composition rule: $\boldsymbol{M} = \boldsymbol{M}_1\ \boldsymbol{M}_2 \ldots \boldsymbol{M}_n$
- Frame rotation by θ: $\boldsymbol{M}_\theta = R(-\theta)\ \boldsymbol{M}\ R(\theta)$ where $R(\theta)$ is *passive rotation matrix.*

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Jones matrix of usual optical element

Optical element	$\boldsymbol{M}$
Free space	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
x-Polariser	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Right circular polariser	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
Linear di-attenuator	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
Half-wave plate with fast axis horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Quarter-wave plate with fast axis horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
General phase retarder	$\begin{bmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{bmatrix}$

Polarization properties of light

- Jones Formalism
- Stokes-Muller formalism

Coherency matrix

Coherency matrix, $\boldsymbol{C}$ defined as

$$\boldsymbol{C} = \left\langle \boldsymbol{E} \otimes \boldsymbol{E}^{\dagger} \right\rangle = \begin{bmatrix} \langle E_x E_x^* \rangle & \left\langle E_x E_y^* \right\rangle \\ \langle E_y E_x^* \rangle & \left\langle E_y E_y^* \right\rangle \end{bmatrix} = \begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix}$$

- $\boldsymbol{C} = \boldsymbol{C}^{\dagger}$ (Hermitian).
- *Time averaged intensity*, $I = \mathrm{Tr}(\boldsymbol{C})$
- Evolution of coherency matrix as $\boldsymbol{C}_{out} = \boldsymbol{M}\,\boldsymbol{C}_{in}\,\boldsymbol{M}^{\dagger}$

Let basis set,

$$\left\{ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\boldsymbol{V}_0}, \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\boldsymbol{V}_1}, \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\boldsymbol{V}_2}, \underbrace{\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}}_{\boldsymbol{V}_3} \right\} \quad s.t.\ \boldsymbol{C} = \frac{1}{2}\sum_{i=0}^{3} S_i \boldsymbol{V}_i$$

Wave Optics (2015), Polarization lecture- YouTube

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Stokes vector

$$\boldsymbol{C} = \frac{1}{2}\sum_{i=0}^{3} S_i \boldsymbol{V_i} \longrightarrow \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \boldsymbol{S} \text{ (Stokes vector)}$$

Degree of Polarization is the measure of polarization of light.

- Total degree of polarization $= \sqrt{S_1^2 + S_2^2 + S_3^2}/S_0$
- Degree of linear polarization $= \sqrt{S_1^2 + S_2^2}/S_0$
- Degree of circular polarization $= S_3/S_0$

$$0 \geq DOP \geq 1$$

Wave Optics (2015), Hecht, E. (1970)

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Stokes vector of usual polarization state

Polarization state	$\boldsymbol{C}$	$\boldsymbol{S}$
$\|H\rangle$	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}^T$
$\|V\rangle$	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix}^T$
$\|P\rangle$	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}^T$
$\|M\rangle$	$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix}^T$
$\|L\rangle$	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}^T$
$\|R\rangle$	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & -1 \end{bmatrix}^T$
Un-polarized	$\frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$

Muller Matrix & evolution of Stokes vector

Muller matrix for an optical element $\mathfrak{M}$ *s.t.*

$$\mathfrak{M} = \begin{bmatrix} \mu_{11} & \cdots & \mu_{14} \\ \vdots & \ddots & \vdots \\ \mu_{41} & \cdots & \mu_{44} \end{bmatrix}$$

Evolution of Stokes vector as, $\boldsymbol{S}_{out} = \mathfrak{M}\, \boldsymbol{S}_{in}$

- Composition rule: $\mathfrak{M} = \mathfrak{M}_1\, \mathfrak{M}_2 \ldots \mathfrak{M}_n$
- Frame rotation by θ: $\mathfrak{M}_\theta = T^{-1}(\theta)\, \mathfrak{M}\, T(\theta)$ where

$$T(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Wave Optics (2015)

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Poincare sphere representation

For elliptically polarized light, Stokes vector

$$\boldsymbol{S} = S_0 \begin{bmatrix} 1 \\ \cos 2\chi \cos 2\psi \\ \cos 2\chi \sin 2\psi \\ \sin 2\chi \end{bmatrix} \longrightarrow \underbrace{S_0(\cos 2\chi \cos 2\psi, \cos 2\chi \sin 2\psi, \sin 2\psi)}_{\text{On Poincare sphere of radius } S_0}$$

where azimuth (ψ) and ellipticity (χ) of polarization ellipse.

Wave Optics (2015), Born, M; Wolf, E (1999)

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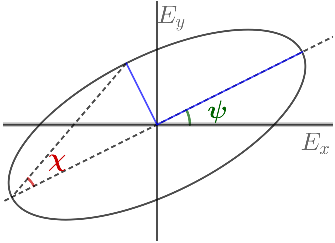
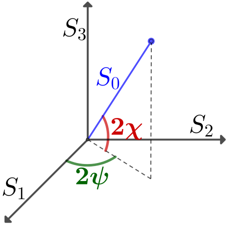
Polarization ellipse



Poincare sphere



Wave Optics (2015), Born, M; Wolf, E (1999)

Poincare sphere representation

For un-polarized light,

$$\boldsymbol{S} = S_0 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \underbrace{(0, 0, 0)}_{\text{At Origin}}$$

For partially polarized light,

$$\boldsymbol{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \longrightarrow \underbrace{(S_1, S_2, S_3)}_{\substack{\text{Inside sphere } s.t. \\ 0 < S_1^2 + S_2^2 + S_3^2 < S_0^2}}$$

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Gaussian Beam and properties

- Paraxial wave
- Gaussian beam solution and properties
- Modes of Gaussian beam

Paraxial wave

Maxwell's wave equation:

$$\nabla^2 \boldsymbol{E}(\boldsymbol{r}, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{E}(\boldsymbol{r}, t) = 0$$

Paraxial beam propagating predominantly in z-direction,

$$\boldsymbol{E}(x, y, z, t) = \boldsymbol{\psi}(x, y, z)e^{i(\omega t - kz)}$$

and taking slowly varying amplitude approx. *i.e.*

$$\left|\frac{\partial^2 \boldsymbol{\psi}}{\partial z^2}\right| \ll k\left|\frac{\partial \boldsymbol{\psi}}{\partial z}\right| \ll k^2\left|\boldsymbol{\psi}\right|$$

Paraxial wave equation:

$$\frac{\partial^2 \boldsymbol{\psi}}{\partial x^2} + \frac{\partial^2 \boldsymbol{\psi}}{\partial y^2} - 2ik\frac{\partial \boldsymbol{\psi}}{\partial r} = 0$$

One of the solutions is Gaussian beam.

Wave Optics (2015), Milonni, P.W ; Eberly, J.H.(2010)

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Gaussian Beam

- Paraxial wave
- Gaussian beam solution and properties
- Modes of Gaussian beam

Gaussian beam solution

$$\text{Ansatz: } \psi(\boldsymbol{r}, z) = A \exp\left[-i\left(p(z) + \frac{kr^2}{2q(z)}\right)\right]$$

$$\psi(\boldsymbol{r}, z) = A\left(\frac{w_0}{w(z)}\right)\exp\left(\tan^{-1}\left(\frac{z}{z_0}\right) - i\frac{kr^2}{2R(z)} - \frac{r^2}{w^2(z)}\right)$$

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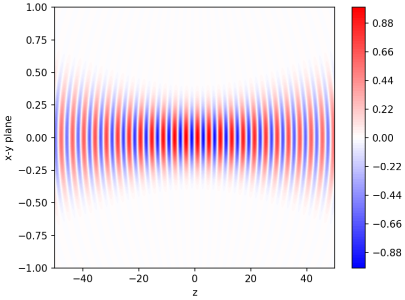
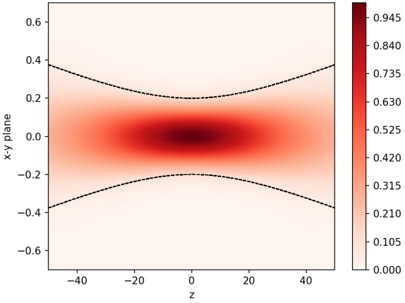
Electric field



Intensity



Milonni, P.W (2010), Kogelnik, H. (1966), ECE 4300- Cornell

Gaussian beam properties

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Term I related to **spreading of beam**.

$w \rightarrow$ Physical radius

$w_0 \rightarrow$ Beam waist

$$w(z) = w_0\sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

$z_0 \rightarrow$ Rayleigh length

$$z_0 = \frac{\pi w_0^2}{\lambda}$$

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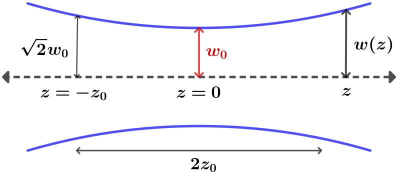

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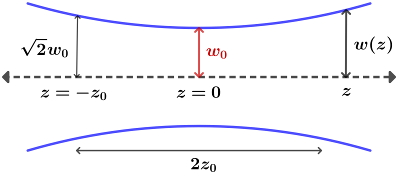

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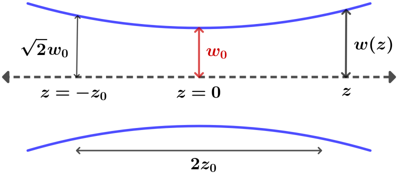

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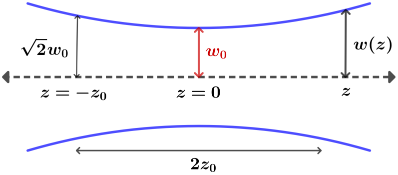

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$$\phi_G = \tan^{-1}\left(\frac{z}{z_0}\right)$$

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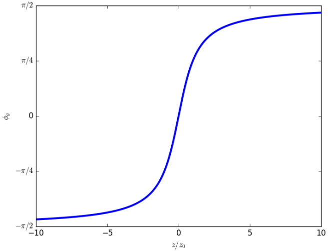

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Term III related to **radius of curvature** (R) of beam wave-front.

$$R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right]$$

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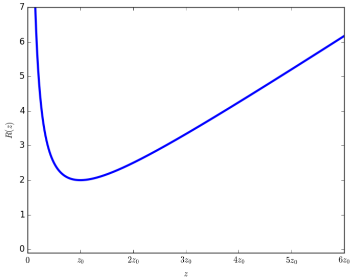

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Term IV related to **Gaussian intensity profile**.

$$I(r, z) \sim \exp\left(-\frac{2r^2}{w^2(z)}\right)$$

Milonni, P.W (2010), Kogelnik, H. (1966), ECE 4300- Cornell

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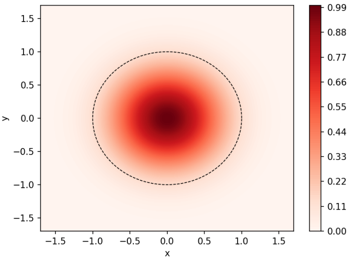

Milonni, P.W (2010), Kogelnik, H. (1966), ECE 4300- Cornell

q parameter and beam tracing

$$\psi(\boldsymbol{r}, z) = A \exp\left[-i\left(p(z) + \frac{kr^2}{2\,q(z)}\right)\right]$$

$q(z) \longrightarrow$ characteristic of a beam if λ known.

$$q(z) = z + i\,z_0$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\,\frac{\lambda}{\pi\,w^2(z)}$$

$$q_{in} \longrightarrow \underbrace{\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}}_{\text{Optical element}} \longrightarrow q_{out} = \frac{A q_{in} + B}{C q_{in} + D}$$

Haus, H. A. (1984), ECE 4300- Cornell

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Haus, H. A. (1984), ECE 4300- Cornell

Gaussian Beam and properties

- Paraxial wave
- Gaussian beam solution and properties
- Modes of Gaussian beam

Hermite-Gaussian mode

$$\psi_{m,n}(\boldsymbol{r}, z) = A\left(\frac{w_0}{w(z)}\right) H_m\left(\frac{\sqrt{2}x}{w(z)}\right) H_n\left(\frac{\sqrt{2}y}{w(z)}\right) \exp\left(-\frac{r^2}{w^2(z)}\right) \cdot \exp\left(i\,(m+n+1)\tan^{-1}\left(\frac{z}{z_0}\right) - i\frac{kr^2}{2R(z)}\right)$$

$m, n = 0 \;\Rightarrow\; \psi =$ Gaussian

Milonni, P.W (2010), Kogelnik, H. (1966)

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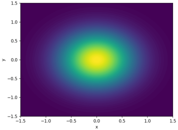
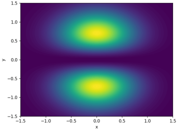
Milonni, P.W (2010), Kogelnik, H. (1966)

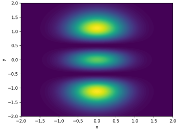
Hermite-Gaussian Intensity profile



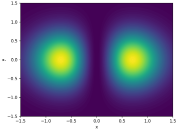
(a) TEM_{00}



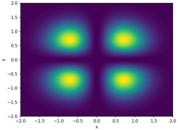
(b) TEM_{01}



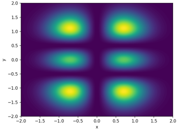
(c) TEM_{02}



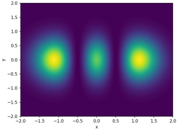
(d) TEM_{10}



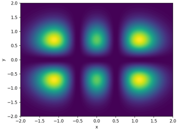
(e) TEM_{11}



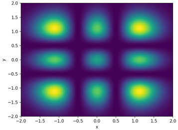
(f) TEM_{12}



(g) TEM_{20}



(h) TEM_{21}



(i) TEM_{22}

Laguerre-Gaussian mode

$$\psi_{p,l}(r,\phi,z) = A\,\frac{w_0}{w(z)}\left[\frac{r\sqrt{2}}{w(z)}\right]^{|l|} L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right)\exp\left(-\frac{r^2}{w^2(z)}\right)\cdot$$
$$\exp\left(-il\phi + i(2p+l+1)\tan^{-1}\left(\frac{z}{z_0}\right) - i\frac{kr^2}{2R(z)}\right)$$

$l, p = 0 \;\Rightarrow\; \psi =$ Gaussian

$\exp(-il\phi) \longrightarrow$ Helical phase
(carries OAM)

Milonni, P.W (2010), Kogelnik, H. (1966), Bliokh, K. Y.; Rodríguez-Fortuño, F. J. (2015)

Laguerre-Gaussian mode

$$\psi_{p,l}(r,\phi,z) = A\,\frac{w_0}{w(z)}\left[\frac{r\sqrt{2}}{w(z)}\right]^{|l|} L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right)\exp\left(-\frac{r^2}{w^2(z)}\right)\cdot$$
$$\exp\left(-il\phi + i(2p+l+1)\tan^{-1}\left(\frac{z}{z_0}\right) - i\frac{kr^2}{2R(z)}\right)$$

$l, p = 0 \;\Rightarrow\; \psi =$ Gaussian

$\exp(-il\phi) \longrightarrow$ Helical phase
(carries OAM)

Milonni, P.W (2010), Kogelnik, H. (1966), Bliokh, K. Y.; Rodríguez-Fortuño, F. J. (2015)

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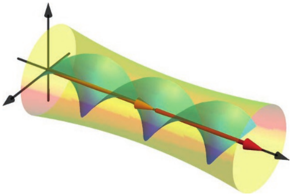
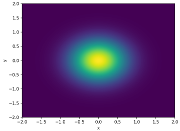
Helical phase front

$\exp(-il\phi) \longrightarrow$ Helical phase (carries OAM)

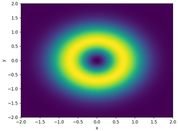


Milonni, P.W (2010), Kogelnik, H. (1966), Bliokh, K. Y.; Rodríguez-Fortuño, F. J. (2015)

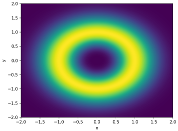
Laguerre-Gaussian Intensity profile



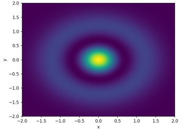
(a) $p = 0, |l| = 0$



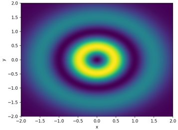
(b) $p = 0, |l| = 1$



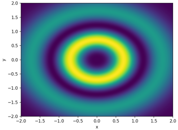
(c) $p = 0, |l| = 2$



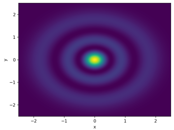
(d) $p = 1, |l| = 0$



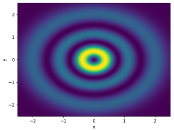
(e) $p = 1, |l| = 1$



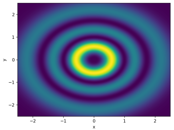
(f) $p = 1, |l| = 2$



(g) $p = 2, |l| = 0$

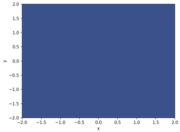


(h) $p = 2, |l| = 1$



(i) $p = 2, |l| = 2$

Laguerre-Gaussian Phase profile



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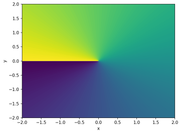
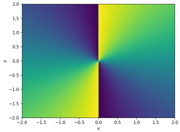
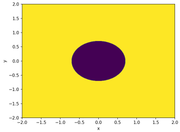


(b) $p = 0, l = 1$



(c) $p = 0, l = 2$



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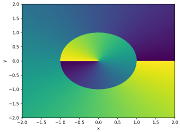

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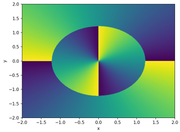

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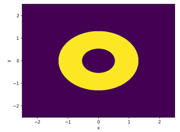

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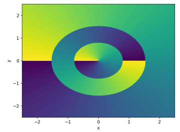

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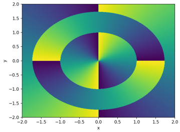

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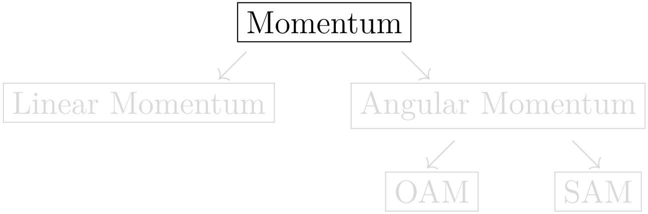
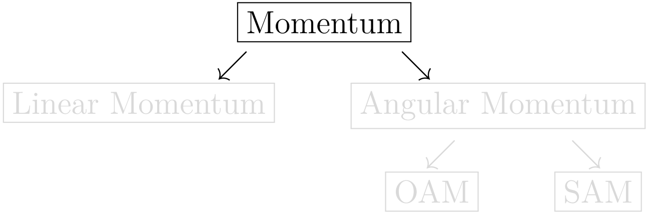
Spin-orbit interaction

- Momentum of Light
- Geometric phase of light
- SOI in anisotropic medium

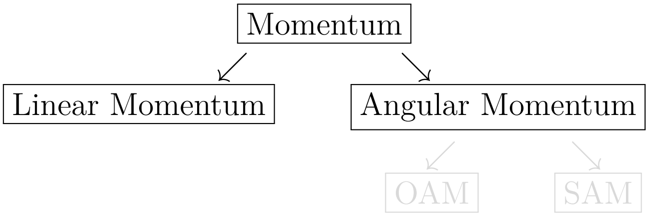
Momentum of light



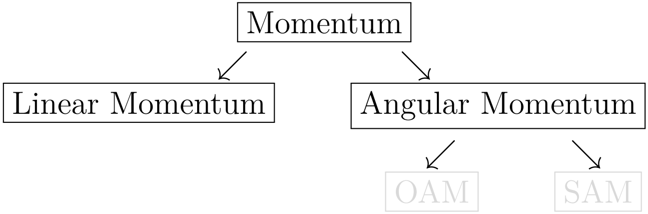
Momentum of light



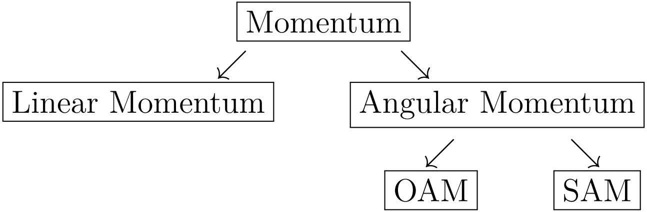
Momentum of light



Momentum of light



Momentum of light



Linear momentum of light

Monochromatic beam with angular frequency ω propagating in z-direction :

$$\boldsymbol{\mathcal{E}}(\boldsymbol{r}, t) = \mathrm{Re}\Big\{\boldsymbol{E}(\boldsymbol{r})e^{-(\omega t - kz)}\Big\}$$

$$\boldsymbol{\mathcal{B}}(\boldsymbol{r}, t) = \mathrm{Re}\Big\{\boldsymbol{B}(\boldsymbol{r})e^{-i(\omega t - kz)}\Big\}$$

Maxwell-Faraday law:

$$\nabla \times \boldsymbol{E} = i\omega \boldsymbol{B}$$

Time-averaged linear momentum per length,

$$\mathcal{P}_z = \frac{1}{c^2}\int d\tau \ \langle \boldsymbol{S} \rangle_z = \frac{\epsilon_0}{2i\omega}\iint dx\, dy \ \langle \boldsymbol{E} \times (\nabla \times \boldsymbol{E}) \rangle_z$$

Haus, H. A. (1984)

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Haus, H. A. (1984)

Angular momentum of light

Time-averaged AM per length :

$$\mathcal{J}_z = \frac{1}{c^2}\int d\tau[\boldsymbol{r}\times\langle\boldsymbol{S}\rangle]_z = \frac{\epsilon_0}{2i\omega}\iint dxdy\,[\boldsymbol{r}\times\langle\boldsymbol{E}\times(\nabla\times\boldsymbol{E})\rangle]_z$$

For paraxial beam,

$$\begin{aligned}\mathcal{J}_z =& \frac{\epsilon_0}{2i\omega}\iint dx\,dy\,\left[E_\xi^*\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)E_\xi\right]_{\xi=x,y}\\ &+\frac{\epsilon_0}{2i\omega}\iint dx\,dy\,(E_x^*E_y+E_y^*E_x)\end{aligned}$$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk; G. Nienhuis (1992), Cohen-Tannoudji, C., Dupont-Roc, J. (1989)

Angular momentum of light

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Orbital AM, $\mathcal{L}$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk; G. Nienhuis (1992), Cohen-Tannoudji, C., Dupont-Roc, J. (1989)

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Spin AM, $\mathcal{S}$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk; G. Nienhuis (1992), Cohen-Tannoudji, C., Dupont-Roc, J. (1989)

Orbital AM

$$\mathcal{L} = \frac{\epsilon_0}{2i\omega} \iint dx\, dy\, \left[E_\xi^* \left(\boldsymbol{r} \times \nabla \right)_z E_\xi \right]_{\xi = x,y}$$

for vortex beam,

$$\boldsymbol{E}(r, \phi) = u(r)\, \exp(-il\phi)\, \hat{\boldsymbol{p}}$$

$$\mathcal{W}_z = \frac{\epsilon_0}{2} \iint dx\, dy\, \boldsymbol{E}^* \cdot \boldsymbol{E}$$

$$\frac{\mathcal{L}}{\mathcal{W}_z} = \frac{\text{OAM}}{\text{Total energy}} = \frac{l}{\omega}$$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk (1992), Berry, M. V.; Soskin, M. S. (1998)

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OAM depends on the choice of the axis.

$$\boldsymbol{r} \longrightarrow \boldsymbol{r}' = \boldsymbol{r} + \boldsymbol{r_0}$$

$$\mathcal{L} \longrightarrow \mathcal{L}' = \mathcal{L}_z + \Delta\mathcal{L}$$

$$\begin{array}{ccc} & \boxed{\text{OAM}} & \\ \swarrow & & \searrow \\ \boxed{\text{Intrinsic}} & & \boxed{\text{Extrinsic}} \end{array}$$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk (1992), Berry, M. V.; Soskin, M. S. (1998)

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Orbital AM

$$\mathcal{L} = \frac{\epsilon_0}{2i\omega} \iint dx\, dy\, \left[E_\xi^* \left(\boldsymbol{r} \times \nabla \right)_z E_\xi \right]_{\xi = x,y}$$

OAM depends on the choice of the axis.

$$\boldsymbol{r} \longrightarrow \boldsymbol{r'} = \boldsymbol{r} + \boldsymbol{r_0}$$

$$\mathcal{L} \longrightarrow \mathcal{L}' = \mathcal{L}_z + \Delta\mathcal{L}$$

$$\begin{array}{ccc} & \boxed{\text{OAM}} & \\ \swarrow & & \searrow \\ \boxed{\text{Intrinsic}} & & \boxed{\text{Extrinsic}} \end{array}$$

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Spin AM

$$\mathcal{S} = \frac{\epsilon_0}{2i\omega} \iint dx\, dy\, (E_x^* E_y + E_y^* E_x)$$

SAM is **intrinsic**.
For vortex beam,

$$\boldsymbol{E}(r,\phi) = u(r)\ \exp(-il\phi)\, \hat{\boldsymbol{p}}$$

$$\frac{\mathcal{S}}{\mathcal{W}_z} = \frac{\text{SAM}}{\text{Total energy}} = \frac{\sigma}{\omega}$$

$\sigma = 2\ \mathrm{Im}(p_x^*\, p_y) \rightarrow$ helicity of beam

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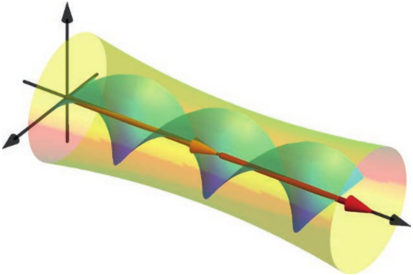
Visualisation of AM



Bliokh, K. Y. (2015)

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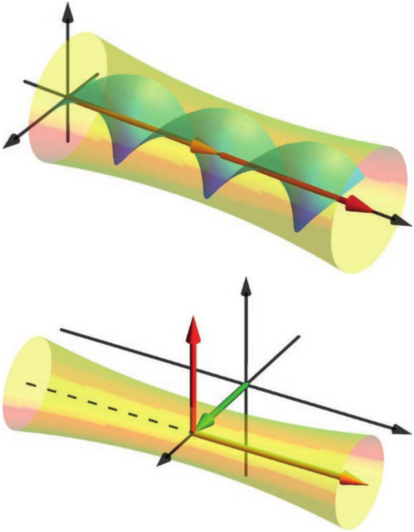

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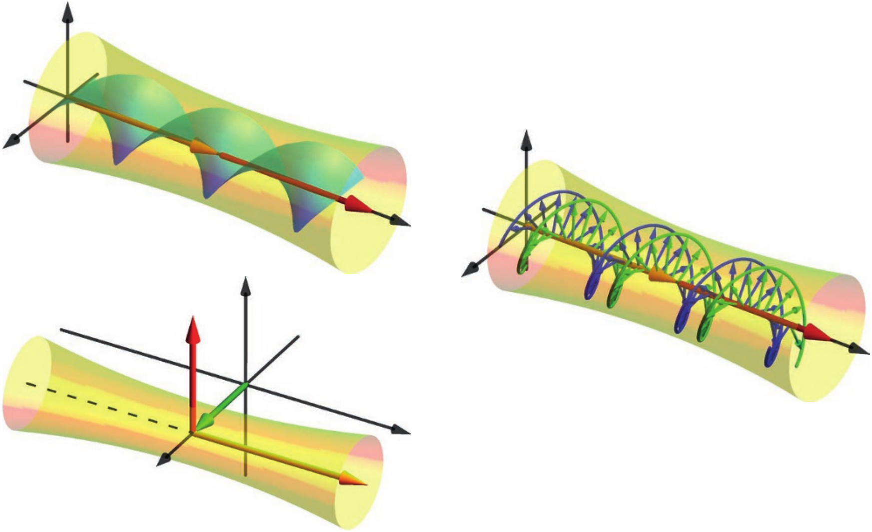

Bliokh, K. Y. (2015)

Spin-orbit interaction

- Momentum of Light
- Geometric phase of light
- SOI in anisotropic medium

Geometric phase

Phase

Dynamical phase

Geometric phase

Wave Optics (2015), Bliokh, K. Y. (2015)

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Associated with optical path length.

Wave Optics (2015), Bliokh, K. Y. (2015)

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Associated with geometry of evolution.

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Associated with geometry of evolution.

- Spin-redirection Berry phase
- Pancharatnam-Berry Phase

Wave Optics (2015), Bliokh, K. Y. (2015)

Spin-redirection Berry phase

Associated with adiabatic evolution of wave-vector.

e.g., Polarized light through a helical optic fibre.

$$\boldsymbol{J} \longrightarrow \boldsymbol{J}' = \boldsymbol{J} \exp(i\sigma\Theta)$$

$$\Theta = 2\pi(1 - \cos\theta)$$

$\Theta \rightarrow$ solid angle obtained at the apex of the cone.

$$|L\rangle \longrightarrow e^{i\Theta}\,|L\rangle$$
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Wave Optics (2015), Bliokh, K. Y. (2015), Parallel transport- Utah State

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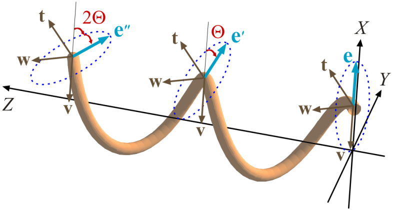

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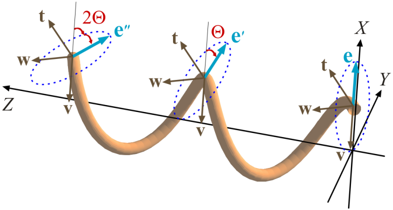
Helicity-dependant geometric phase



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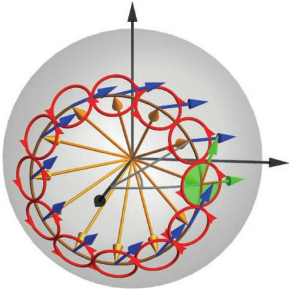

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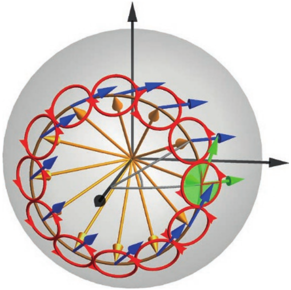

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Wave Optics (2015), Bliokh, K. Y. (2015), Parallel transport- Utah State

Pancharatnam-Berry Phase

Associated with cyclic evolution in Poincare sphere keeping wave-vector fixed.

e.g., Michelson interferometer.

QP1→fixed (aligned at $\pi/4$)

QP2→movable (aligned at β)

$\boldsymbol{J}_A = |x\rangle$

$\boldsymbol{J}_A \longrightarrow \boldsymbol{J}'_A$

$\boldsymbol{J}'_A = |x\rangle \exp(i\phi_d)\, \exp(-i\, 2\varphi)$

$$\varphi = \beta + \pi/4$$

Pancharatnam-Berry Phase

(φ depends on QP2 alignment)

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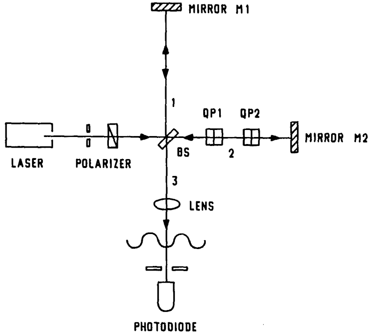

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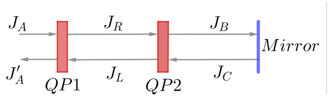

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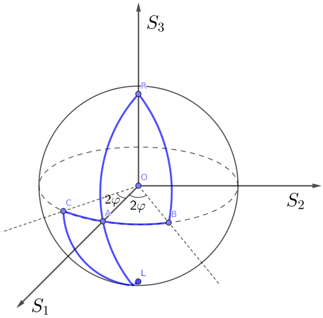

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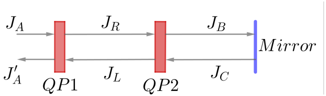

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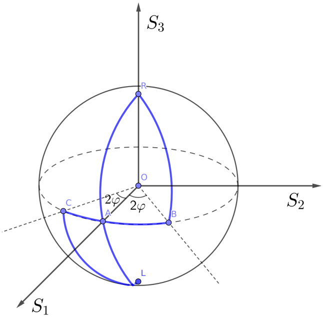

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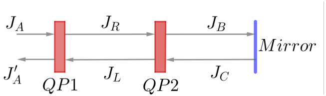

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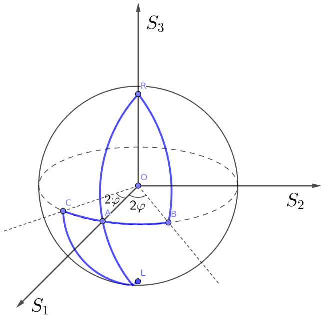

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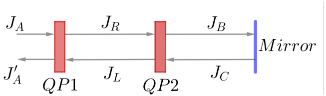

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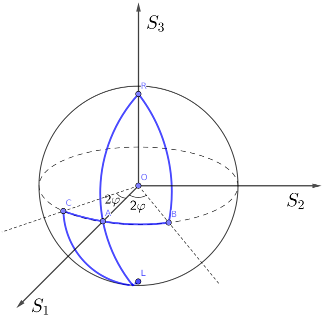

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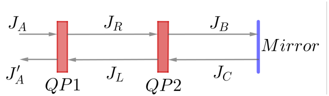

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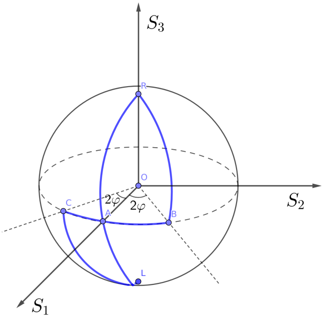

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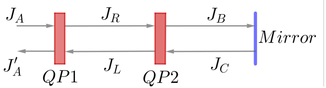

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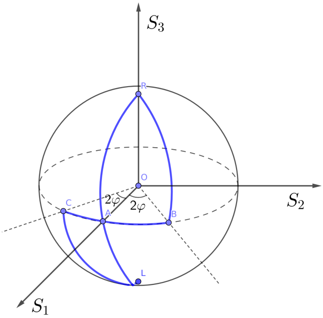

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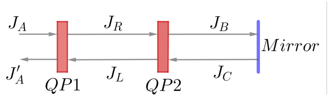

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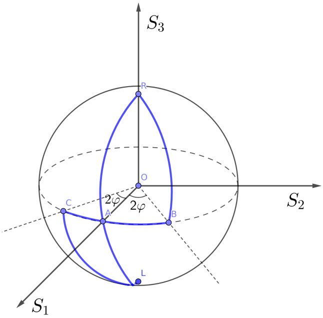

Pancharatnam-Berry Phase

(φ depends on QP2 alignment)

Wave Optics (2015), Chyba, T. H.; Wang, L. J. (1988)

Spin-orbit interaction

- Momentum of Light
- Geometric phase of light
- SOI in anisotropic medium

Spin-orbit interaction of light

Three types of AM:

- IOAM
- EOAM
- SAM

Inter-conversion between AM in a process represents **spin-orbit interaction** of light

Bliokh, K. Y. (2015)

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SOI in homogeneous-anisotropic media

e.g. **Quarter wave-plate**

$$\begin{array}{c}\text{Circularly}\\ \text{polarized}\\ (\sigma=\pm 1)\end{array} \longrightarrow \boxed{\begin{array}{c}\text{Q}\\ \text{W}\\ \text{P}\end{array}} \longrightarrow \begin{array}{c}\text{Linearly}\\ \text{polarized}\\ (\sigma=0)\end{array}$$

$\pm\hbar$ SAM per photon is transferred to the wave plate.

e.g. **Half wave-plate**

$$\begin{array}{c}\text{Circularly}\\ \text{polarized}\\ (\sigma=\pm 1)\end{array} \longrightarrow \boxed{\begin{array}{c}\text{H}\\ \text{W}\\ \text{P}\end{array}} \longrightarrow \begin{array}{c}\text{Circularly}\\ \text{polarized}\\ (\sigma=\mp 1)\end{array}$$

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e.g. **q-plate**

Inhomogeneous orientation of the fast axis varying with azimuth (ϕ).

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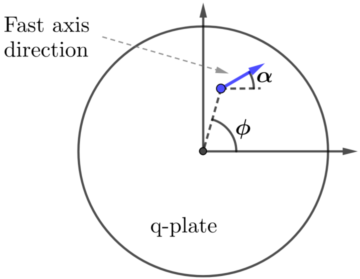
$$\alpha(\phi) = q\phi + \alpha_0$$



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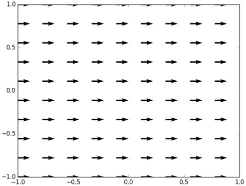
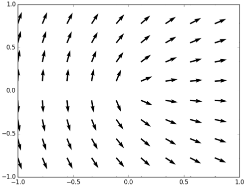
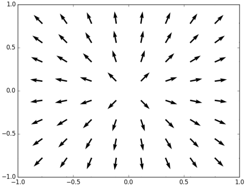


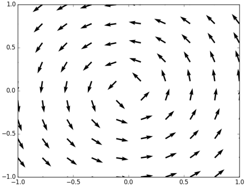
(a) $q = 0,\ \alpha_0 = 0$



(b) $q = 0.5,\ \alpha_0 = 0$



(a) $q = 1,\ \alpha_0 = 0$



(b) $q = 1,\ \alpha_0 = \pi/2$

SOI in q-plate

Q-plate of phase retardation of π

$$\begin{bmatrix} 1 \\ i\sigma \end{bmatrix} \longrightarrow \boxed{\mathbf{Q}_{\lambda/2}} \longrightarrow \begin{bmatrix} 1 \\ -i\sigma \end{bmatrix} \underbrace{\exp(i2\sigma q\phi)}_{\text{Vortex}} \exp(i2\sigma\alpha_0)$$

$$(\sigma = \pm 1, l = 0) \longrightarrow (\sigma = \mp 1, l = \pm 2q)$$

$q = 1 \rightarrow$ Angular momentum per photon is conserved.

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$$\begin{bmatrix} 1 \\ i\sigma \end{bmatrix} \longrightarrow \boxed{\mathbf{Q}_{\lambda/4}} \longrightarrow \begin{bmatrix} \cos(\alpha - \sigma\pi/4) \\ \sin(\alpha - \sigma\pi/4) \end{bmatrix} \underbrace{\exp(i\sigma q\phi)}_{\text{Vortex}} \exp(i\sigma\alpha_0)$$

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THANKS
TO ALL