On Polarization Properties of Light, Gaussian Beams and Spin-Orbit Interaction of Light

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Submitted to

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Topics of discussion

Polarization properties of light

Gaussian Beam

Spin-orbit interaction of light

Polarization properties of light

- Jones Formalism
- Stokes-Muller formalism

Jones Vector

Electric field of *fully polarized* EM wave propagating along z-axis is given by

$$\boldsymbol{E}(\boldsymbol{r}, t) = \begin{bmatrix} A_x(\boldsymbol{r})e^{i\delta_x} \\ A_y(\boldsymbol{r})e^{i\delta_y} \\ 0 \end{bmatrix} e^{-i(kz-\omega t)}$$

Define normalized **Jones vector** *s.t.* $\boldsymbol{J}^* \boldsymbol{J} = 1$ as

$$\boldsymbol{J}(\boldsymbol{r}, t) = \frac{1}{\sqrt{A_x^2 + A_y^2}} \begin{bmatrix} A_x(\boldsymbol{r})e^{i\delta_x} \\ A_y(\boldsymbol{r})e^{i\delta_y} \end{bmatrix}$$

Note that *intensity*, $I = A_x^2 + A_y^2 = \boldsymbol{J}^* \boldsymbol{J}$

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Wave Optics (2015)

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Note that *intensity*, $I = A_x^2 + A_y^2 = \boldsymbol{J}^*\boldsymbol{J}$

Jones vector of usual polarization state

Polarization state	$\boldsymbol{J}$
$\vert H\rangle$	$\begin{bmatrix}1\\0\end{bmatrix}$
$\vert V\rangle$	$\begin{bmatrix}0\\1\end{bmatrix}$
$\vert P\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$
$\vert M\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$
$\vert L\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$
$\vert R\rangle$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$

Jones Matrix & evolution of Jones vector

Jones matrix for an optical element be $\boldsymbol{M}$ *s.t.*

$$\boldsymbol{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

If a polarized light of Jones vector $\boldsymbol{J}_{in}$ passes through that optical element then the Jones vector of output light is given by

$$\boldsymbol{J}_{out} = \boldsymbol{M}\ \boldsymbol{J}_{in}$$

- Composition rule: $\boldsymbol{M} = \boldsymbol{M}_1\ \boldsymbol{M}_2 \ldots \boldsymbol{M}_n$
- Frame rotation by θ: $\boldsymbol{M}_\theta = R(-\theta)\ \boldsymbol{M}\ R(\theta)$ where $R(\theta)$ is *passive rotation matrix.*

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Jones matrix of usual optical element

Optical element	$\boldsymbol{M}$
Free space	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
x-Polariser	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Right circular polariser	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
Linear di-attenuator	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
Half-wave plate with fast axis horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Quarter-wave plate with fast axis horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
General phase retarder	$\begin{bmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{bmatrix}$

Polarization properties of light

- Jones Formalism
- Stokes-Muller formalism

Coherency matrix

Coherency matrix, $\boldsymbol{C}$ defined as

$$\boldsymbol{C} = \left\langle \boldsymbol{E} \otimes \boldsymbol{E}^{\dagger} \right\rangle = \begin{bmatrix} \langle E_x E_x^* \rangle & \left\langle E_x E_y^* \right\rangle \\ \langle E_y E_x^* \rangle & \left\langle E_y E_y^* \right\rangle \end{bmatrix} = \begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix}$$

- $\boldsymbol{C} = \boldsymbol{C}^{\dagger}$ (Hermitian).
- *Time averaged intensity* $= \langle E_x E_x^* \rangle + \left\langle E_y E_y^* \right\rangle = \mathrm{Tr}(\boldsymbol{C})$
- Evolution of coherency matrix as $\boldsymbol{C}_{out} = \boldsymbol{M}\, \boldsymbol{C}_{in}\, \boldsymbol{M}^{\dagger}$

Let basis set,

$$\left\{ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\boldsymbol{V}_0}, \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\boldsymbol{V}_1}, \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{\boldsymbol{V}_2}, \underbrace{\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}}_{\boldsymbol{V}_3} \right\} \quad s.t.\ \boldsymbol{C} = \frac{1}{2} \sum_{i=0}^{3} S_i \boldsymbol{V}_i$$

Wave Optics (2015), Polarization lecture- YouTube

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Stokes vector

$$\boldsymbol{C} = \frac{1}{2}\sum_{i=0}^{3} S_i \boldsymbol{V_i} \longrightarrow \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \boldsymbol{S} \text{ (Stokes vector)}$$

$$S_1^2 + S_2^2 + S_3^2 \leq S_0^2$$

Degree of Polarization is the measure of polarization of light.

- Total degree of polarization $= \sqrt{S_1^2 + S_2^2 + S_3^2}/S_0$
- Degree of linear polarization $= \sqrt{S_1^2 + S_2^2}/S_0$
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$$0 \leq DOP \leq 1$$

Wave Optics (2015), Hecht, E. (1970)

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Stokes vector of usual polarization state

Polarization state	$\boldsymbol{C}$	$\boldsymbol{S}$
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$\lvert V\rangle$	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix}^T$
$\lvert P\rangle$	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}^T$
$\lvert M\rangle$	$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix}^T$
$\lvert L\rangle$	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}^T$
$\lvert R\rangle$	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & -1 \end{bmatrix}^T$
Un-polarized	$\frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$

Muller Matrix & evolution of Stokes vector

Muller matrix for an optical element $\mathfrak{M}$ *s.t.*

$$\mathfrak{M} = \begin{bmatrix} \mu_{11} & \cdots & \mu_{14} \\ \vdots & \ddots & \vdots \\ \mu_{41} & \cdots & \mu_{44} \end{bmatrix}$$

Evolution of Stokes vector as, $\boldsymbol{S}_{out} = \mathfrak{M}\, \boldsymbol{S}_{in}$

- Composition rule: $\mathfrak{M} = \mathfrak{M}_1\, \mathfrak{M}_2 \ldots \mathfrak{M}_n$
- Frame rotation by θ: $\mathfrak{M}_\theta = T^{-1}(\theta)\, \mathfrak{M}\, T(\theta)$ where

$$T(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Wave Optics (2015)

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Wave Optics (2015)

Poincare sphere representation

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \longrightarrow (S_1, S_2, S_3)$$

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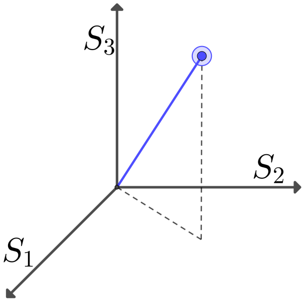

Poincare sphere representation

For elliptically polarized light, Stokes vector

$$\boldsymbol{S} = S_0 \begin{bmatrix} 1 \\ \cos 2\chi \cos 2\psi \\ \cos 2\chi \sin 2\psi \\ \sin 2\chi \end{bmatrix} \longrightarrow \underbrace{S_0(\cos 2\chi \cos 2\psi, \cos 2\chi \sin 2\psi, \sin 2\psi)}_{\text{On sphere of radius } S_0}$$

where azimuth (ψ) and ellipticity (χ) of polarization ellipse.

Wave Optics (2015), Born, M; Wolf, E (1999)

Poincare sphere representation

For elliptically polarized light, Stokes vector

$$\boldsymbol{S} = S_0 \begin{bmatrix} 1 \\ \cos 2\chi \cos 2\psi \\ \cos 2\chi \sin 2\psi \\ \sin 2\chi \end{bmatrix} \longrightarrow \underbrace{S_0(\cos 2\chi \cos 2\psi, \cos 2\chi \sin 2\psi, \sin 2\psi)}_{\text{On sphere of radius } S_0}$$

where azimuth (ψ) and ellipticity (χ) of polarization ellipse.

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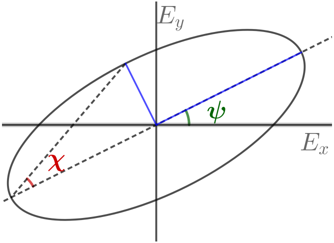
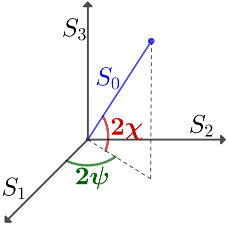
Polarization ellipse



Poincare sphere



Wave Optics (2015), Born, M; Wolf, E (1999)

Poincare sphere representation

For un-polarized light,

$$\boldsymbol{S} = S_0 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \underbrace{(0, 0, 0)}_{\text{At Origin}}$$

For partially polarized light,

$$\boldsymbol{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \longrightarrow \underbrace{(S_1, S_2, S_3)}_{\substack{\text{Inside sphere } s.t. \\ 0 < S_1^2 + S_2^2 + S_3^2 < S_0^2}}$$

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Wave Optics (2015), Born, M; Wolf, E (1999)

Gaussian Beam and properties

- Paraxial wave
- Gaussian beam solution and properties
- Modes of Gaussian beam

Paraxial wave

Maxwell's wave equation:

$$\nabla^2 \boldsymbol{E}(\boldsymbol{r}, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{E}(\boldsymbol{r}, t) = 0$$

Paraxial beam propagating predominantly in z-direction,

$$\boldsymbol{E}(x, y, z, t) = \boldsymbol{\psi}(x, y, z)e^{i(\omega t - kz)}$$

and taking slowly varying amplitude approx. *i.e.*

$$\left|\frac{\partial^2 \boldsymbol{\psi}}{\partial z^2}\right| \ll k \left|\frac{\partial \boldsymbol{\psi}}{\partial z}\right| \ll k^2 \left|\boldsymbol{\psi}\right|$$

Paraxial wave equation:

$$\frac{\partial^2 \boldsymbol{\psi}}{\partial x^2} + \frac{\partial^2 \boldsymbol{\psi}}{\partial y^2} - 2ik\frac{\partial \boldsymbol{\psi}}{\partial r} = 0$$

One of the solutions is Gaussian beam.

Wave Optics (2015), Milonni, P.W ; Eberly, J.H.(2010)

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Gaussian beam solution

$$\text{Ansatz: } \psi(\boldsymbol{r}, z) = A \exp\left[-i\left(p(z) + \frac{kr^2}{2q(z)}\right)\right]$$

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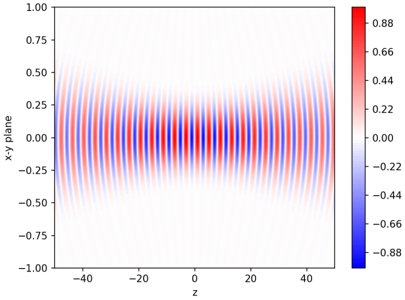
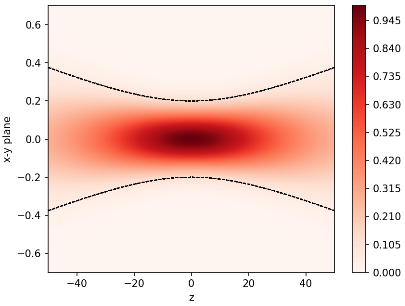
Electric field



Intensity



Milonni, P.W (2010), Kogelnik, H. (1966), ECE 4300- Cornell

Gaussian beam properties

$$\psi(\boldsymbol{r}, z) = A\underbrace{\left(\frac{w_0}{w(z)}\right)}_{\text{term I}}\exp\left(i\tan^{-1}\left(\frac{z}{z_0}\right) - i\frac{kr^2}{2R(z)} - \frac{r^2}{w^2(z)}\right)$$

Term I related to **spreading of beam**.

$w \rightarrow$ Physical radius

$w_0 \rightarrow$ Beam waist

$$w(z) = w_0\sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

$z_0 \rightarrow$ Rayleigh length

$$z_0 = \frac{\pi w_0^2}{\lambda}$$

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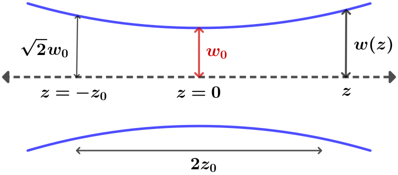

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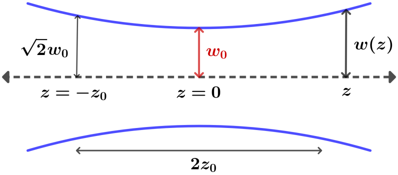

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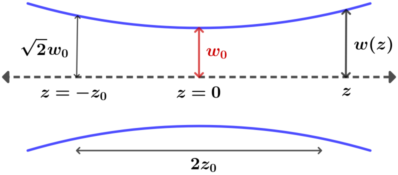

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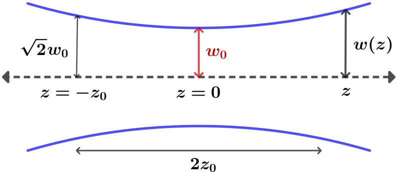

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Term II related to **Gouy phase** (ϕ_G).

$$\phi_G = \tan^{-1}\left(\frac{z}{z_0}\right)$$

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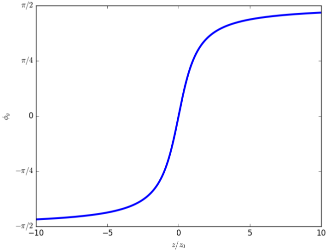

Milonni, P.W (2010), Kogelnik, H. (1966), ECE 4300- Cornell

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Term III related to **radius of curvature** (R) of beam wave-front.

$$R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right]$$

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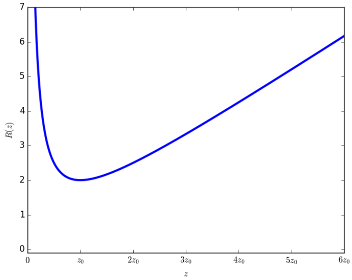

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Term IV related to **Gaussian intensity profile**.

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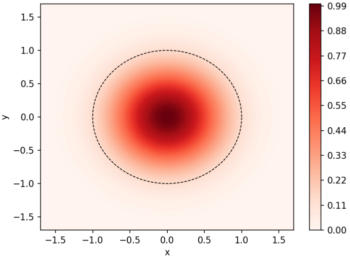

Milonni, P.W (2010), Kogelnik, H. (1966), ECE 4300- Cornell

q parameter and beam tracing

$$\psi(\boldsymbol{r}, z) = A \exp\left[-i\left(p(z) + \frac{kr^2}{2\,q(z)}\right)\right]$$

$q(z) \longrightarrow$ characteristic of a beam if λ known.

$$q(z) = z + i\,z_0$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\,\frac{\lambda}{\pi\,w^2(z)}$$

$$q_{in} \longrightarrow \underbrace{\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}}_{\text{Optical element}} \longrightarrow q_{out} = \frac{Aq_{in} + B}{Cq_{in} + D}$$

Haus, H. A. (1984), ECE 4300- Cornell

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Hermite-Gaussian mode

$$\psi_{m,n}(\boldsymbol{r}, z) = A \left(\frac{w_0}{w(z)}\right) H_m \left(\frac{\sqrt{2}x}{w(z)}\right) H_n \left(\frac{\sqrt{2}y}{w(z)}\right) \exp\left(-\frac{r^2}{w^2(z)}\right) \cdot \exp\left(i\,(m+n+1)\tan^{-1}\left(\frac{z}{z_0}\right) - i\frac{kr^2}{2R(z)}\right)$$

$m, n = 0 \;\Rightarrow\; \psi =$ Gaussian

Milonni, P.W (2010), Kogelnik, H. (1966)

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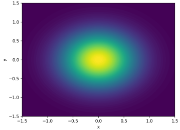
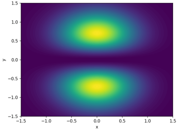
Milonni, P.W (2010), Kogelnik, H. (1966)

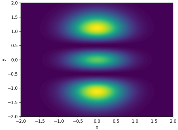
Hermite-Gaussian Intensity profile



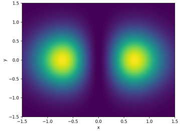
(a) TEM_{00}



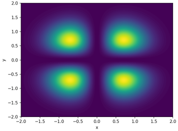
(b) TEM_{01}



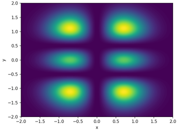
(c) TEM_{02}



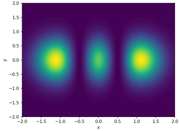
(d) TEM_{10}



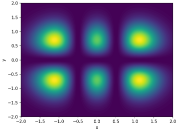
(e) TEM_{11}



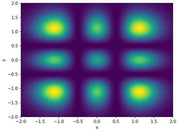
(f) TEM_{12}



(g) TEM_{20}



(h) TEM_{21}



(i) TEM_{22}

Laguerre-Gaussian mode

$$\psi_{p,l}(r,\phi,z) = A\,\frac{w_0}{w(z)}\left[\frac{r\sqrt{2}}{w(z)}\right]^{|l|} L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right)\exp\left(-\frac{r^2}{w^2(z)}\right)\cdot$$
$$\exp\left(-il\phi + i(2p+l+1)\tan^{-1}\left(\frac{z}{z_0}\right) - i\frac{kr^2}{2R(z)}\right)$$

$l, p = 0 \;\Rightarrow\; \psi =$ Gaussian

$\exp(-il\phi) \longrightarrow$ Helical phase
(carries OAM)

Milonni, P.W (2010), Kogelnik, H. (1966), Bliokh, K. Y.; Rodríguez-Fortuño, F. J. (2015)

Laguerre-Gaussian mode

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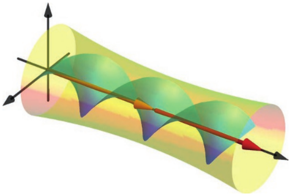
Milonni, P.W (2010), Kogelnik, H. (1966), Bliokh, K. Y.; Rodríguez-Fortuño, F. J. (2015)

Laguerre-Gaussian mode

$$\psi_{p,l}(r,\phi,z) = A\,\frac{w_0}{w(z)}\left[\frac{r\sqrt{2}}{w(z)}\right]^{|l|} L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right)\exp\left(-\frac{r^2}{w^2(z)}\right)\cdot$$
$$\exp\left(-il\phi + i(2p+l+1)\tan^{-1}\left(\frac{z}{z_0}\right) - i\frac{kr^2}{2R(z)}\right)$$

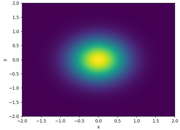
Helical phase front

$\exp(-il\phi) \longrightarrow$ Helical phase (carries OAM)

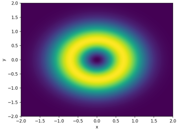


Milonni, P.W (2010), Kogelnik, H. (1966), Bliokh, K. Y.; Rodríguez-Fortuño, F. J. (2015)

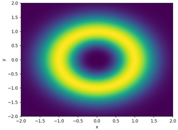
Laguerre-Gaussian Intensity profile



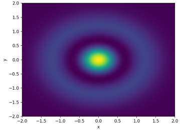
(a) $p = 0, |l| = 0$



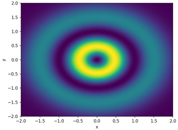
(b) $p = 0, |l| = 1$



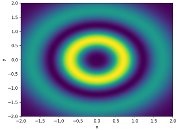
(c) $p = 0, |l| = 2$



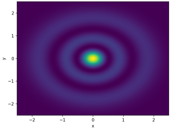
(d) $p = 1, |l| = 0$



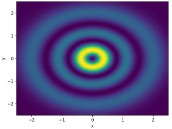
(e) $p = 1, |l| = 1$



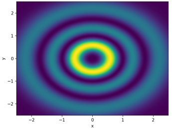
(f) $p = 1, |l| = 2$



(g) $p = 2, |l| = 0$

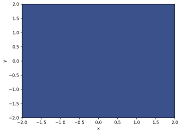


(h) $p = 2, |l| = 1$

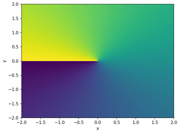


(i) $p = 2, |l| = 2$

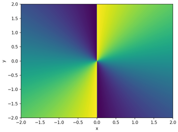
Laguerre-Gaussian Phase profile



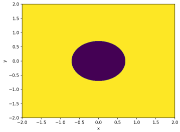
(a) $p = 0, l = 0$



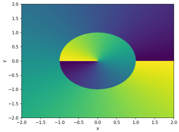
(b) $p = 0, l = 1$



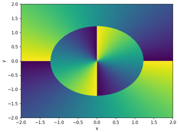
(c) $p = 0, l = 2$



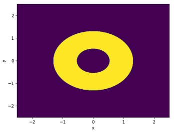
(d) $p = 1, l = 0$



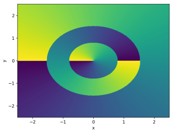
(e) $p = 1, l = 1$



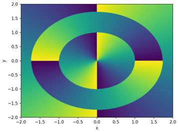
(f) $p = 1, l = 2$



(g) $p = 2, l = 0$



(h) $p = 2, l = 1$

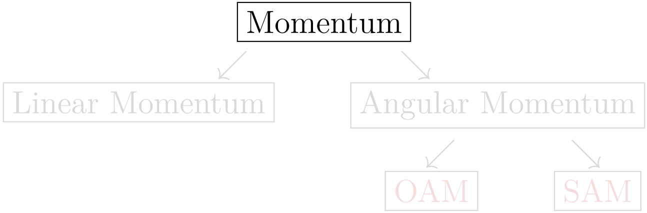


(i) $p = 2, l = 2$

Spin-orbit interaction

- Momentum of Light
- Geometric phase of light
- SOI in anisotropic medium

Momentum of light



For (paraxial beam only)

Momentum of light

Momentum

Linear Momentum

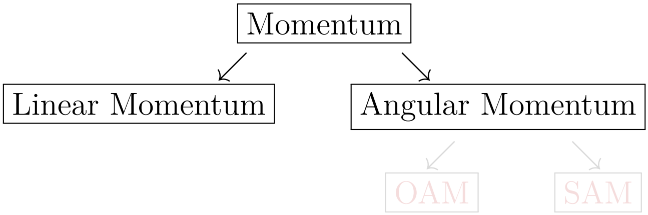
Angular Momentum

OAM

SAM

For (paraxial beam only)

Momentum of light



For (paraxial beam only)

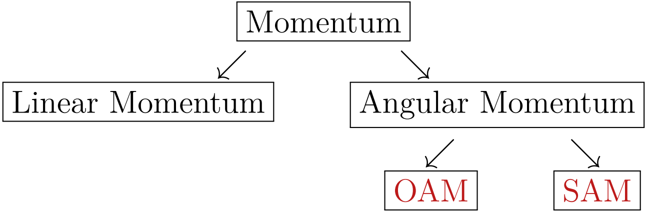
Momentum of light

Momentum

- Linear Momentum
- Angular Momentum
  - OAM
  - SAM

For (paraxial beam only)

Momentum of light



For (paraxial beam only)

Linear momentum of light

Monochromatic beam with angular frequency ω propagating in z-direction :

$$\boldsymbol{\mathcal{E}}(\boldsymbol{r},t) = \mathrm{Re}\Big\{\boldsymbol{E}(\boldsymbol{r})e^{-i(\omega t - kz)}\Big\}$$

$$\boldsymbol{\mathcal{B}}(\boldsymbol{r},t) = \mathrm{Re}\Big\{\boldsymbol{B}(\boldsymbol{r})e^{-i(\omega t - kz)}\Big\}$$

Maxwell-Faraday law:

$$\nabla \times \boldsymbol{E} = i\omega \boldsymbol{B}$$

Time-averaged linear momentum per length,

$$\mathcal{P}_z = \frac{1}{c^2}\int d\tau \, \langle \boldsymbol{S} \rangle_z = \frac{\epsilon_0}{2i\omega}\iint dx\, dy \, \langle \boldsymbol{E} \times (\nabla \times \boldsymbol{E}) \rangle_z$$

Haus, H. A. (1984)

Linear momentum of light

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Haus, H. A. (1984)

Angular momentum of light

Time-averaged AM per length :

$$\mathcal{J}_z = \frac{1}{c^2}\int d\tau[\boldsymbol{r}\times\langle\boldsymbol{S}\rangle]_z = \frac{\epsilon_0}{2i\omega}\iint dxdy\,[\boldsymbol{r}\times\langle\boldsymbol{E}\times(\nabla\times\boldsymbol{E})\rangle]_z$$

For paraxial beam,

$$\begin{aligned}\mathcal{J}_z =& \frac{\epsilon_0}{2i\omega}\iint dx\,dy\,\left[E_\xi^*\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)E_\xi\right]_{\xi=x,y}\\ &+\frac{\epsilon_0}{2i\omega}\iint dx\,dy\,(E_x^*E_y+E_y^*E_x)\end{aligned}$$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk; G. Nienhuis (1992), Cohen-Tannoudji, C., Dupont-Roc, J. (1989)

Angular momentum of light

Time-averaged AM per length :

$$\mathcal{J}_z = \frac{1}{c^2}\int d\tau [\boldsymbol{r}\times\langle\boldsymbol{S}\rangle]_z = \frac{\epsilon_0}{2i\omega}\iint dxdy\,[\boldsymbol{r}\times\langle\boldsymbol{E}\times(\nabla\times\boldsymbol{E})\rangle]_z$$

For paraxial beam,

$$\mathcal{J}_z = \frac{\epsilon_0}{2i\omega}\iint dx\,dy\,\left[E_\xi^*\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)E_\xi\right]_{\xi=x,y} + \frac{\epsilon_0}{2i\omega}\iint dx\,dy\,(E_x^* E_y + E_y^* E_x)$$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk; G. Nienhuis (1992), Cohen-Tannoudji, C., Dupont-Roc, J. (1989)

Angular momentum of light

Time-averaged AM per length :

$$\mathcal{J}_z = \frac{1}{c^2}\int d\tau [\boldsymbol{r}\times\langle\boldsymbol{S}\rangle]_z = \frac{\epsilon_0}{2i\omega}\iint dxdy\,[\boldsymbol{r}\times\langle\boldsymbol{E}\times(\nabla\times\boldsymbol{E})\rangle]_z$$

For paraxial beam,

$$\begin{aligned}\mathcal{J}_z =&\frac{\epsilon_0}{2i\omega}\iint dx\,dy\,\left[E_\xi^*\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)E_\xi\right]_{\xi=x,y}\\ &+\frac{\epsilon_0}{2i\omega}\iint dx\,dy\,(E_x^*E_y+E_y^*E_x)\end{aligned}$$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk; G. Nienhuis (1992), Cohen-Tannoudji, C., Dupont-Roc, J. (1989)

Angular momentum of light

Time-averaged AM per length :

$$\mathcal{J}_z = \frac{1}{c^2}\int d\tau[\boldsymbol{r}\times\langle\boldsymbol{S}\rangle]_z = \frac{\epsilon_0}{2i\omega}\iint dxdy\,[\boldsymbol{r}\times\langle\boldsymbol{E}\times(\nabla\times\boldsymbol{E})\rangle]_z$$

For paraxial beam,

$$\mathcal{J}_z = \frac{\epsilon_0}{2i\omega}\iint dx\,dy\,\left[E_\xi^*\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)E_\xi\right]_{\xi=x,y} + \frac{\epsilon_0}{2i\omega}\iint dx\,dy\,(E_x^*E_y+E_y^*E_x)$$

Orbital AM, $\mathcal{L}$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk; G. Nienhuis (1992), Cohen-Tannoudji, C., Dupont-Roc, J. (1989)

Angular momentum of light

Time-averaged AM per length :

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For paraxial beam,

$$\mathcal{J}_z = \frac{\epsilon_0}{2i\omega}\iint dx\,dy\,\left[E_\xi^*\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)E_\xi\right]_{\xi=x,y} + \frac{\epsilon_0}{2i\omega}\iint dx\,dy\,(E_x^*E_y+E_y^*E_x)$$

Spin AM, $\mathcal{S}$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk; G. Nienhuis (1992), Cohen-Tannoudji, C., Dupont-Roc, J. (1989)

Orbital AM

$$\mathcal{L} = \frac{\epsilon_0}{2i\omega} \iint dx\, dy\, \left[E_\xi^* \left(\boldsymbol{r} \times \nabla \right)_z E_\xi \right]_{\xi = x,y}$$

for vortex beam,

$$\boldsymbol{E}(r,\phi) = u(r)\, \exp(-il\phi)\, \hat{\boldsymbol{p}}$$

$$\mathcal{W}_z = \frac{\epsilon_0}{2} \iint dx\, dy\, \boldsymbol{E}^* \cdot \boldsymbol{E}$$

$$\frac{\mathcal{L}}{\mathcal{W}_z} = \frac{\text{OAM}}{\text{Total energy}} = \frac{l}{\omega}$$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk (1992),Berry, M. V.; Soskin, M. S. (1998)

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Allen, L. (1999), Wave Optics (2015), S.J. van Enk (1992),Berry, M. V.; Soskin, M. S. (1998)

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$$\mathcal{L} = \frac{\epsilon_0}{2i\omega} \iint dx\, dy\, \left[E_\xi^* \left(\boldsymbol{r} \times \nabla \right)_z E_\xi \right]_{\xi = x,y}$$

OAM depends on the choice of the axis.

$$\boldsymbol{r} \longrightarrow \boldsymbol{r}' = \boldsymbol{r} + \boldsymbol{r_0}$$

$$\mathcal{L} \longrightarrow \mathcal{L}' = \mathcal{L}_z + \Delta\mathcal{L}$$

$$\boxed{\text{OAM}}$$

$\swarrow$ $\searrow$

$\boxed{\text{Intrinsic}}$ $\boxed{\text{Extrinsic}}$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk (1992), Berry, M. V.; Soskin, M. S. (1998)

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$$\begin{array}{ccc} & \boxed{\text{OAM}} & \\ \swarrow & & \searrow \\ \boxed{\text{Intrinsic}} & & \boxed{\text{Extrinsic}} \end{array}$$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk (1992), Berry, M. V.; Soskin, M. S. (1998)

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$$\mathcal{L} = \frac{\epsilon_0}{2i\omega} \iint dx\, dy\, \left[E_\xi^* \left(\boldsymbol{r} \times \nabla \right)_z E_\xi \right]_{\xi = x, y}$$

OAM depends on the choice of the axis.

$$\boldsymbol{r} \longrightarrow \boldsymbol{r'} = \boldsymbol{r} + \boldsymbol{r_0}$$

$$\mathcal{L} \longrightarrow \mathcal{L}' = \mathcal{L}_z + \Delta\mathcal{L}$$

$$\boxed{\text{OAM}}$$

$$\swarrow \qquad \searrow$$

$$\boxed{\text{Intrinsic}} \qquad \boxed{\text{Extrinsic}}$$

Allen, L. (1999), Wave Optics (2015), S.J. van Enk (1992), Berry, M. V.; Soskin, M. S. (1998)

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Spin AM

$$\mathcal{S} = \frac{\epsilon_0}{2i\omega} \iint dx\, dy\, (E_x^* E_y + E_y^* E_x)$$

SAM is **intrinsic**.
For vortex beam,

$$\boldsymbol{E}(r,\phi) = u(r)\ \exp(-il\phi)\,\hat{\boldsymbol{p}}$$

$$\frac{\mathcal{S}}{\mathcal{W}_z} = \frac{\text{SAM}}{\text{Total energy}} = \frac{\sigma}{\omega}$$

$\sigma = 2\ \text{Im}(p_x^*\, p_y) \rightarrow$ helicity of beam

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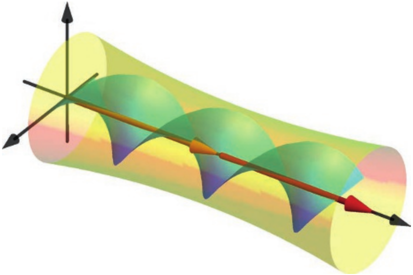
Visualisation of AM



Bliokh, K. Y. (2015)

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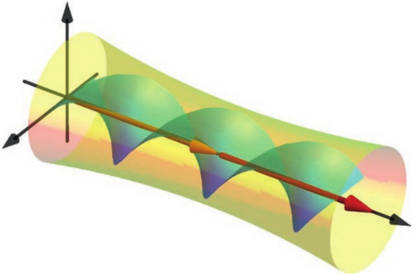

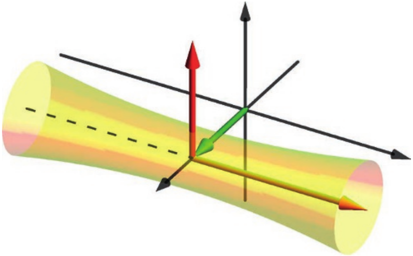

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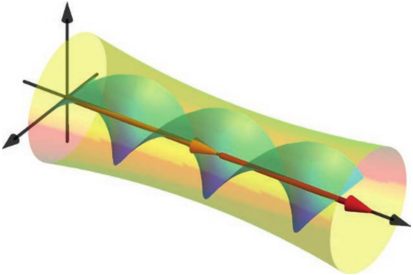

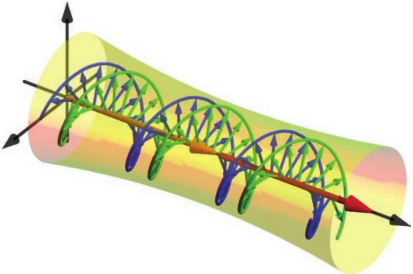

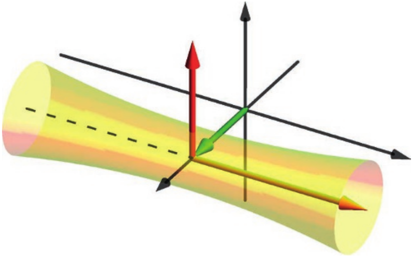

Bliokh, K. Y. (2015)

Spin-orbit interaction

- Momentum of Light
- Geometric phase of light
- SOI in anisotropic medium

Geometric phase

$$\begin{tikzcd}[column sep=small] & \boxed{\text{Phase}} \arrow[dl] \arrow[dr] & \\ \boxed{\text{Dynamical phase}} & & \boxed{\text{Geometric phase}} \end{tikzcd}$$

Wave Optics (2015), Bliokh, K. Y. (2015)

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Wave Optics (2015), Bliokh, K. Y. (2015)

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$$

Wave Optics (2015), Bliokh, K. Y. (2015)

Geometric phase

Phase

Dynamical phase

Geometric phase

Associated with optical path length.

Wave Optics (2015), Bliokh, K. Y. (2015)

Geometric phase

$$\boxed{\text{Phase}}$$

$$\swarrow \qquad \searrow$$

$$\boxed{\text{Dynamical phase}} \qquad \boxed{\text{Geometric phase}}$$

Associated with geometry of evolution.

Wave Optics (2015), Bliokh, K. Y. (2015)

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Associated with geometry of evolution.

- Spin-redirection Berry phase
- Pancharatnam-Berry Phase

Wave Optics (2015), Bliokh, K. Y. (2015)

Spin-redirection Berry phase

Associated with adiabatic evolution of wave-vector.

e.g., Polarized light through a helical optic fibre.

$$\boldsymbol{J} = \begin{bmatrix} 1 \\ i\sigma \end{bmatrix} \longrightarrow \boldsymbol{J'} = \boldsymbol{J} \exp(i\sigma\Theta)$$

$$\Theta = 2\pi(1 - \cos\theta)$$

$\Theta \rightarrow$ solid angle obtained at the apex of the cone.

$$|L\rangle \longrightarrow e^{i\Theta} |L\rangle$$

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Wave Optics (2015), Bliokh, K. Y. (2015), Parallel transport- Utah State

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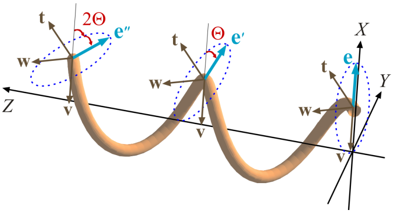

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Helicity-dependant geometric phase

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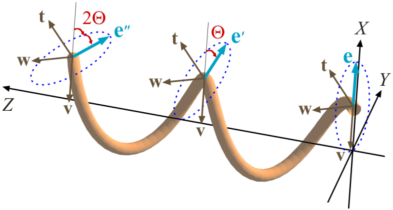

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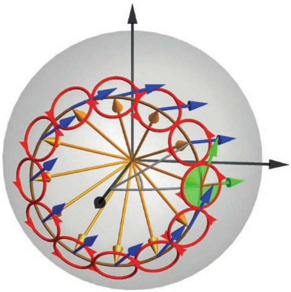

Parallel transport

Wave Optics (2015), Bliokh, K. Y. (2015), Parallel transport- Utah State

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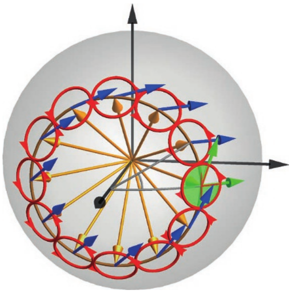

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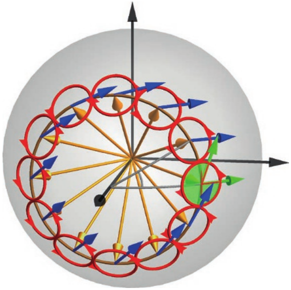

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Wave Optics (2015), Bliokh, K. Y. (2015), Parallel transport- Utah State

Pancharatnam-Berry Phase

Associated with cyclic evolution in Poincare sphere keeping wave-vector fixed.

e.g., Michelson interferometer.

QP1→fixed (aligned at $\pi/4$)

QP2→movable (aligned at β)

$\boldsymbol{J}_A = |x\rangle$

$\boldsymbol{J}_A \longrightarrow \boldsymbol{J}'_A$

$\boldsymbol{J}'_A = |x\rangle \exp(i\phi_d)\, \exp(-i\, 2\varphi)$

$$\varphi = \beta + \pi/4$$

Pancharatnam-Berry Phase

(φ depends on QP2 alignment)

Wave Optics (2015), Chyba, T. H.; Wang, L. J. (1988)

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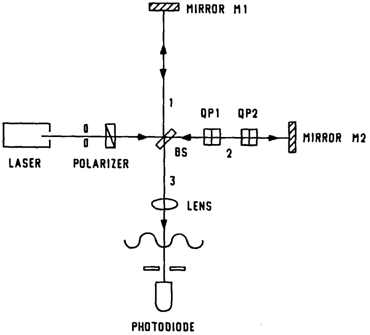

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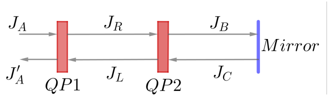

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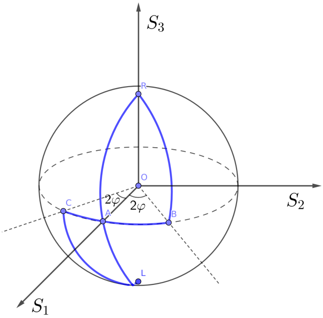

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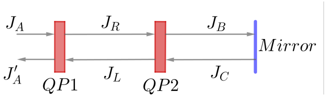

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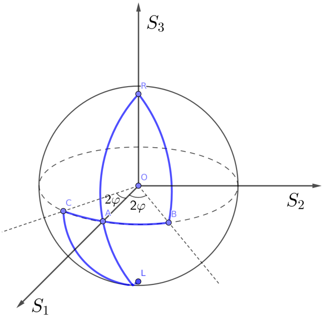

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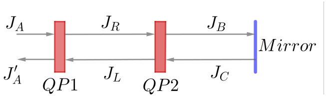

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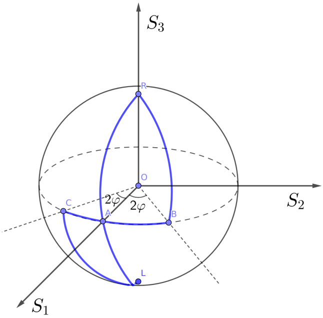
$$\varphi = \beta + \pi/4$$

Pancharatnam-Berry Phase (φ depends on QP2 alignment)



Wave Optics (2015), Chyba, T. H.; Wang, L. J. (1988)

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Associated with cyclic evolution in Poincare sphere keeping wave-vector fixed.

e.g., Michelson interferometer.

QP1→fixed (aligned at $\pi/4$)

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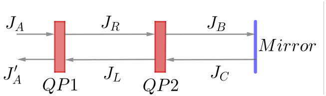

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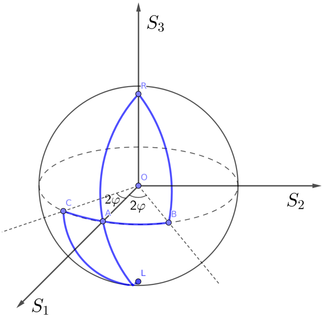

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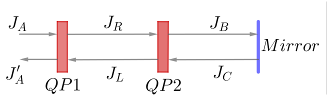

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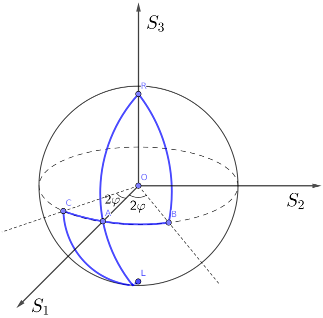

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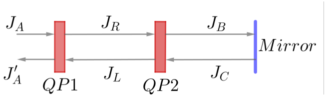

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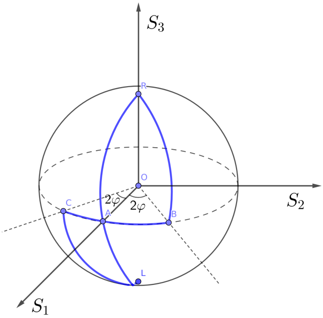

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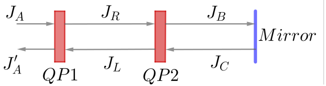

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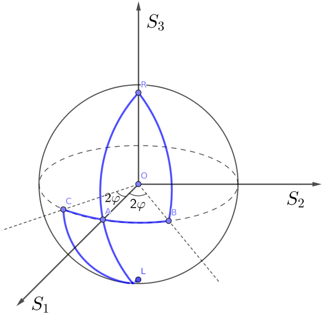

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Spin-orbit interaction

- Momentum of Light
- Geometric phase of light
- SOI in anisotropic medium

Spin-orbit interaction of light

Three types of AM:

- IOAM
- EOAM
- SAM

Inter-conversion between AM in a process represents **spin-orbit interaction** of light

Bliokh, K. Y. (2015)

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SOI in homogeneous-anisotropic media

e.g. **Quarter wave-plate**

Circularly polarized ($\sigma = \pm 1$) $\longrightarrow$ $\boxed{\text{QWP}}$ $\longrightarrow$ Linearly polarized ($\sigma = 0$)

$\pm\hbar$ SAM per photon is transferred to the wave plate.

e.g. **Half wave-plate**

Circularly polarized ($\sigma = \pm 1$) $\longrightarrow$ $\boxed{\text{HWP}}$ $\longrightarrow$ Circularly polarized ($\sigma = \mp 1$)

$\pm 2\hbar$ SAM per photon is transferred to the wave plate and spin is flipped.

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Inhomogeneous orientation of the fast axis varying with azimuth (ϕ).

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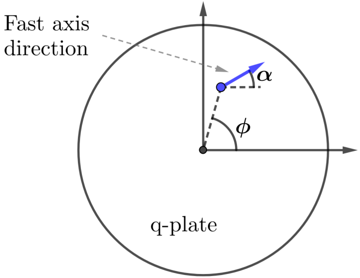
$$\alpha(\phi) = q\phi + \alpha_0$$



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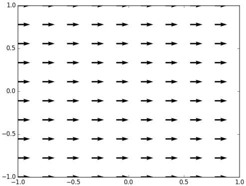
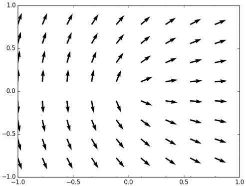


(a) $q = 0$, $\alpha_0 = 0$



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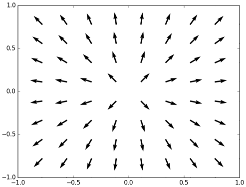

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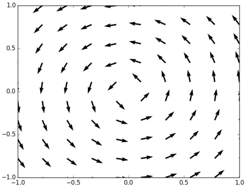

(b) $q = 1$, $\alpha_0 = \pi/2$

SOI in q-plate

Q-plate of phase retardation of π

$$\begin{bmatrix} 1 \\ i\sigma \end{bmatrix} \longrightarrow \boxed{\mathbf{Q}_{\lambda/2}} \longrightarrow \begin{bmatrix} 1 \\ -i\sigma \end{bmatrix} \underbrace{\exp(i2\sigma q\phi)}_{\text{Vortex}} \exp(i2\sigma\alpha_0)$$

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