

Summer Project 2023



Pritish Karmakar

21MS179

Contents

1	GAUSSIAN and ITS FOURIER TRANSFORM	1
1.1	Standard normal	1
1.2	Fourier transform of Standard normal	1
2	GAUSSIAN BEAMS	3
2.1	Intensity profile	3
3	HERMITE–GAUSSIAN BEAMS	5
3.1	Intensity profile	5
4	LAGUERRE–GAUSSIAN BEAMS	7
4.1	Intensity profile	7

List of Figures

1	Standard Normal curve	1
2	Fourier transform of Standard Normal	2
3	Intensity variation in a cross section ($z = 0, z_0 = 1, w_0 = 1$)	3
4	Intensity variation along z ($z_0 = 1, w_0 = 1$)	4
5	Intensity variation for different TEM in a cross section ($z = 0, z_0 = 1, w_0 = 1$)	6
6	Intensity variation for different modes in a cross section ($z = 0, z_0 = 1, w_0 = 1$)	8

1 GAUSSIAN and ITS FOURIER TRANSFORM

1.1 Standard normal

Standard normal curve is

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$$

```
1 import matplotlib.pyplot as plt
2 import numpy as np
3 plt.style.use("classic")
4
5 def f(t):
6     return np.exp(-(t)**2/2)/(np.sqrt(2*np.pi))
7
8 xv = np.linspace(-7,7,1000)
9 yv = f(xv)
10
11 plt.plot(xv, yv, lw=1)
12 plt.xlabel("$t$")
13 plt.ylabel("$f(t)$")
14 plt.grid(True)
```

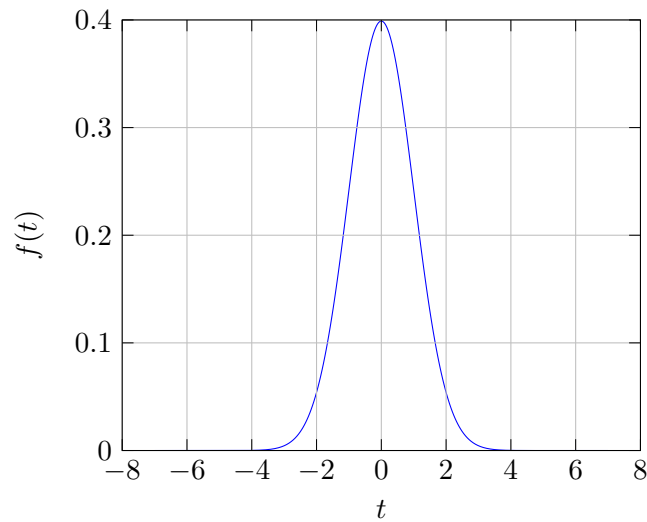


Figure 1: Standard Normal curve

1.2 Fourier transform of Standard normal

If $f(t) = \int_{-\infty}^{\infty} g(\omega) e^{i\omega t} d\omega$, then fourier transform of that is $g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$. Here,
 $f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$

$$\begin{aligned} g(\omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) \cos(\omega t) dt - i \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) \sin(\omega t) dt \end{aligned}$$

```

1 import numpy as np
2 import matplotlib.pyplot as plt
3 from scipy.integrate import quad
4
5 def f(t):
6     return np.exp(-(t)**2/2)/(np.sqrt(2*np.pi))
7
8 def ft(y):
9     int_re = lambda t: f(t)*np.sin(y*t)
10    int_im = lambda t: f(t)*np.cos(y*t)
11    g_re = quad(int_re,-np.inf,np.inf)[0]/(2*np.pi)
12    g_im = quad(int_im,-np.inf,np.inf)[0]/(2*np.pi)
13    return g_re - 1j*g_im
14 g = np.frompyfunc(ft, 1, 1)
15
16 xv = np.linspace(-7,7,1000)
17 yv = np.abs(g(xv))
18 plt.plot(xv, yv, lw=1)
19 plt.xlabel("$\omega$")
20 plt.ylabel("$abs(g(\omega))$")
21 plt.grid(True)

```

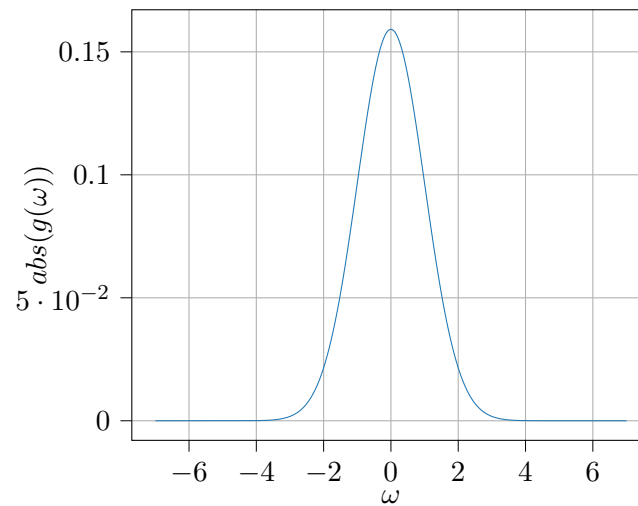


Figure 2: Fourier transform of Standard Normal

2 GAUSSIAN BEAMS

2.1 Intensity profile

Intensity of Gaussian beam is given by,

$$I(x, y, z) = \frac{c\epsilon}{2}|A|^2 \left[\frac{w_0}{w(z)} \right]^2 \exp(-2(x^2 + y^2)/w^2(z))$$

where

$$w(z) = w_0 \sqrt{1 + z^2/z_0^2} \text{ and } z_0 = \pi w_0^2/\lambda$$

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3
4 R=lambda r: np.exp(-2*r**2)
5 a1=np.linspace(-1.7,1.7,200)
6 xv,yv=np.meshgrid(a1,a1)
7 zv=R(np.sqrt(xv**2+yv**2))
8 plt.contourf(xv,yv,zv,levels=100,cmap='viridis')
9 plt.xlabel("x")
10 plt.ylabel("y")
11 plt.colorbar()
12
13 theta=np.linspace(0,2*np.pi,500)
14 x=np.cos(theta)
15 y=np.sin(theta)
16 plt.plot( x, y, "--", lw=0.8)
```

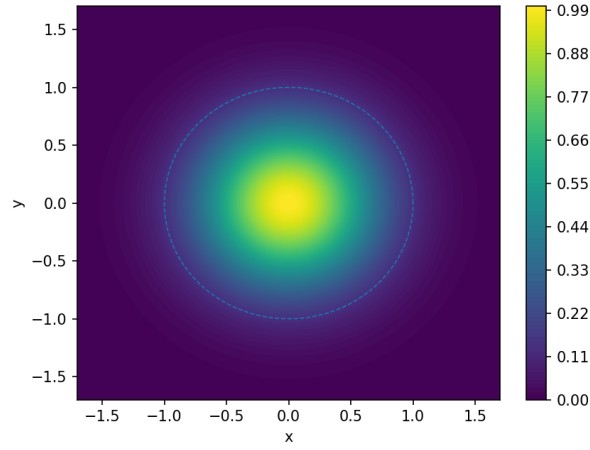


Figure 3: Intensity variation in a cross section ($z = 0, z_0 = 1, w_0 = 1$)

```

1 import numpy as np
2 import matplotlib.pyplot as plt
3
4 R=lambda x,z: np.exp(-2*x**2/(1+z**2))/(1+z**2)
5 a1=np.linspace(-1.5,1.5,500)
6 a2=np.linspace(-3,3,500)
7 xv,zv=np.meshgrid(a2,a1)
8 I=R(xv,zv)
9 plt.contourf(zv,xv,I,levels=100,cmap='viridis')
10 plt.colorbar()
11 plt.xlabel("z")
12 plt.ylabel("x-y plane")
13
14 w=lambda z1: np.sqrt(1+z1**2)
15 plt.plot(zv,w(zv),"--",lw=0.7)
16 plt.plot(zv,-w(zv),"--",lw=0.7)

```

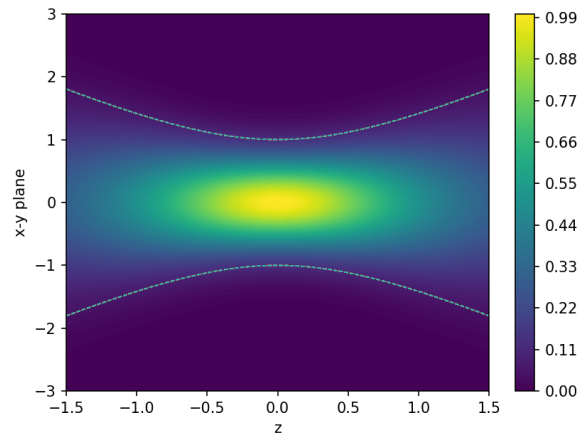


Figure 4: Intensity variation along z ($z_0 = 1, w_0 = 1$)

3 HERMITE-GAUSSIAN BEAMS

3.1 Intensity profile

Intensity of Hermite-Gaussian beam is given by,

$$I_{m,n}(x, y, z) = \frac{c\epsilon}{2} |A|^2 \left[H_m(\sqrt{2}x/w(z)) \right]^2 \left[H_n(\sqrt{2}y/w(z)) \right]^2 \exp(-2(x^2 + y^2)/w^2(z))$$

where

$$w(z) = w_0 \sqrt{1 + z^2/z_0^2} \quad (1)$$

$$z_0 = \pi w_0^2/\lambda \quad (2)$$

$$H_n = n\text{-th order Hermite polynomial} \quad (3)$$

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 from scipy import special
4
5 H=lambda n: special.hermite(n, monic=False)
6
7 m=int(input("m="))
8 n=int(input("n="))
9
10 R=lambda m,n,x,y: (H(m)(np.sqrt(2)*x)*H(n)(np.sqrt(2)*y))**2*np.exp(-2*(x**2+y
    **2))
11 l=2
12 a1=np.linspace(-l,l,200)
13 xv,yv=np.meshgrid(a1,a1)
14 zv=R(m,n,xv,yv)
15 plt.contourf(xv,yv,zv,levels=100,cmap='viridis')
16 plt.xlabel("x")
17 plt.ylabel("y")
18 #plt.colorbar()
```

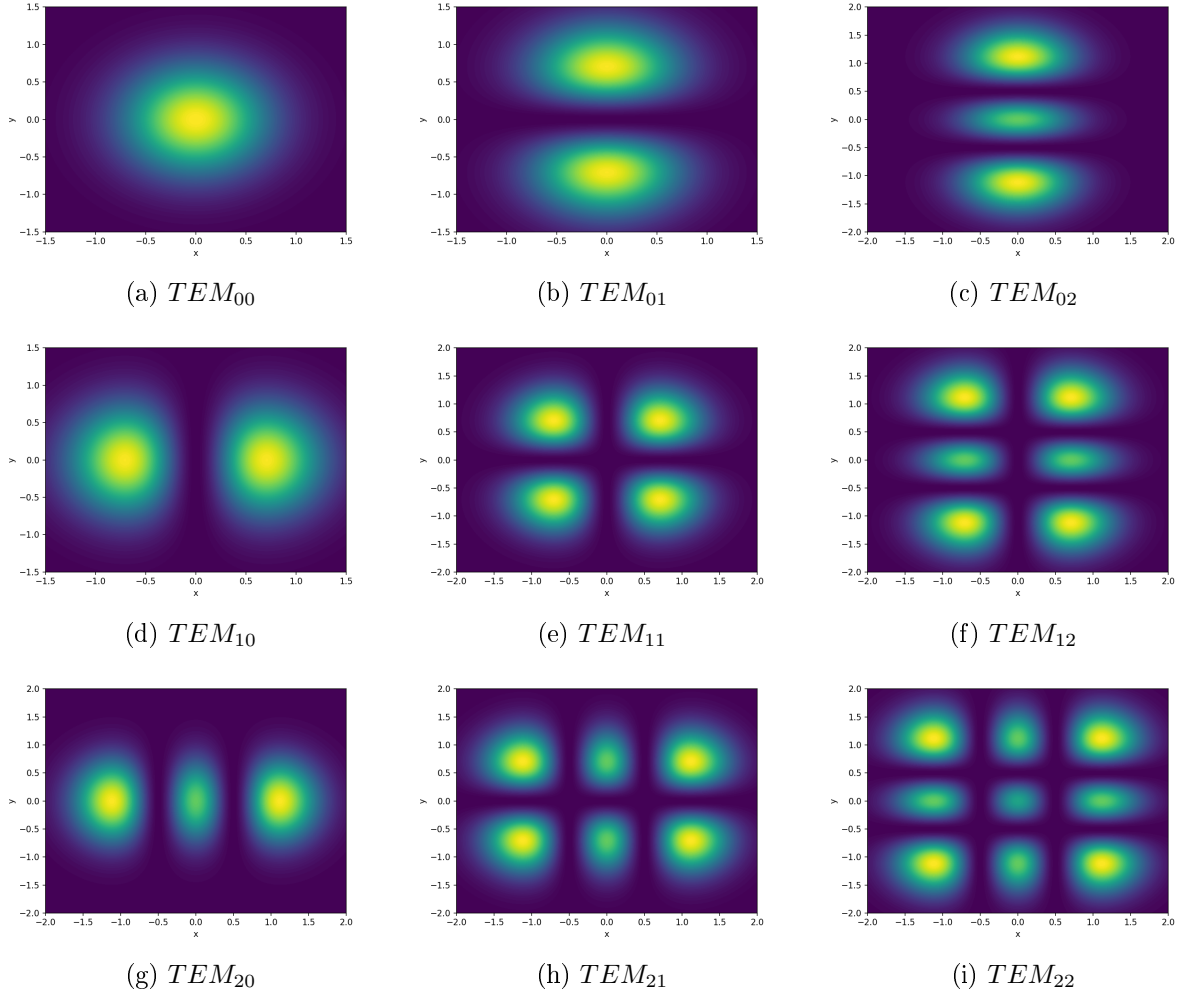



Figure 5: Intensity variation for different TEM in a cross section ($z = 0, z_0 = 1, w_0 = 1$)

4 LAGUERRE-GAUSSIAN BEAMS

4.1 Intensity profile

Intensity of Hermite-Gaussian beam is given by,

$$I_{p,l}(r,z) = \frac{c\epsilon}{2}|A|^2 \left[\frac{w_0}{w(z)} \right]^2 \left[\frac{r\sqrt{2}}{w(z)} \right]^{2|l|} \left[L_p^{|l|} \left(\frac{2r^2}{w^2(z)} \right) \right]^2 \exp(-2(x^2 + y^2)/w^2(z))$$

where

$$\begin{aligned} r^2 &= x^2 + y^2 \\ w(z) &= w_0 \sqrt{1 + z^2/z_0^2} \\ z_0 &= \pi w_0^2/\lambda \\ L_p^{|l|} &= \text{Associated Laguerre polynomial} \end{aligned}$$

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 from scipy import special
4
5 L= lambda x,p,l: special.assoc_laguerre(x,p,l)
6
7 p=int(input("p="))
8 l=int(input("l="))
9
10 R=lambda p,l,x,y: ((np.sqrt(2*(x**2+y**2)))**(2*np.abs(l)))*(L(2*(x**2+y**2),p,
    np.abs(l))**2*np.exp(-2*(x**2+y**2)))
11 a=2.5
12 a1=np.linspace(-a,a,200)
13 xv,yv=np.meshgrid(a1,a1)
14 zv=R(p,l,xv,yv)
15 plt.contourf(xv,yv,zv,levels=300,cmap='viridis')
16 plt.xlabel("x")
17 plt.ylabel("y")
18 #plt.colorbar()
```

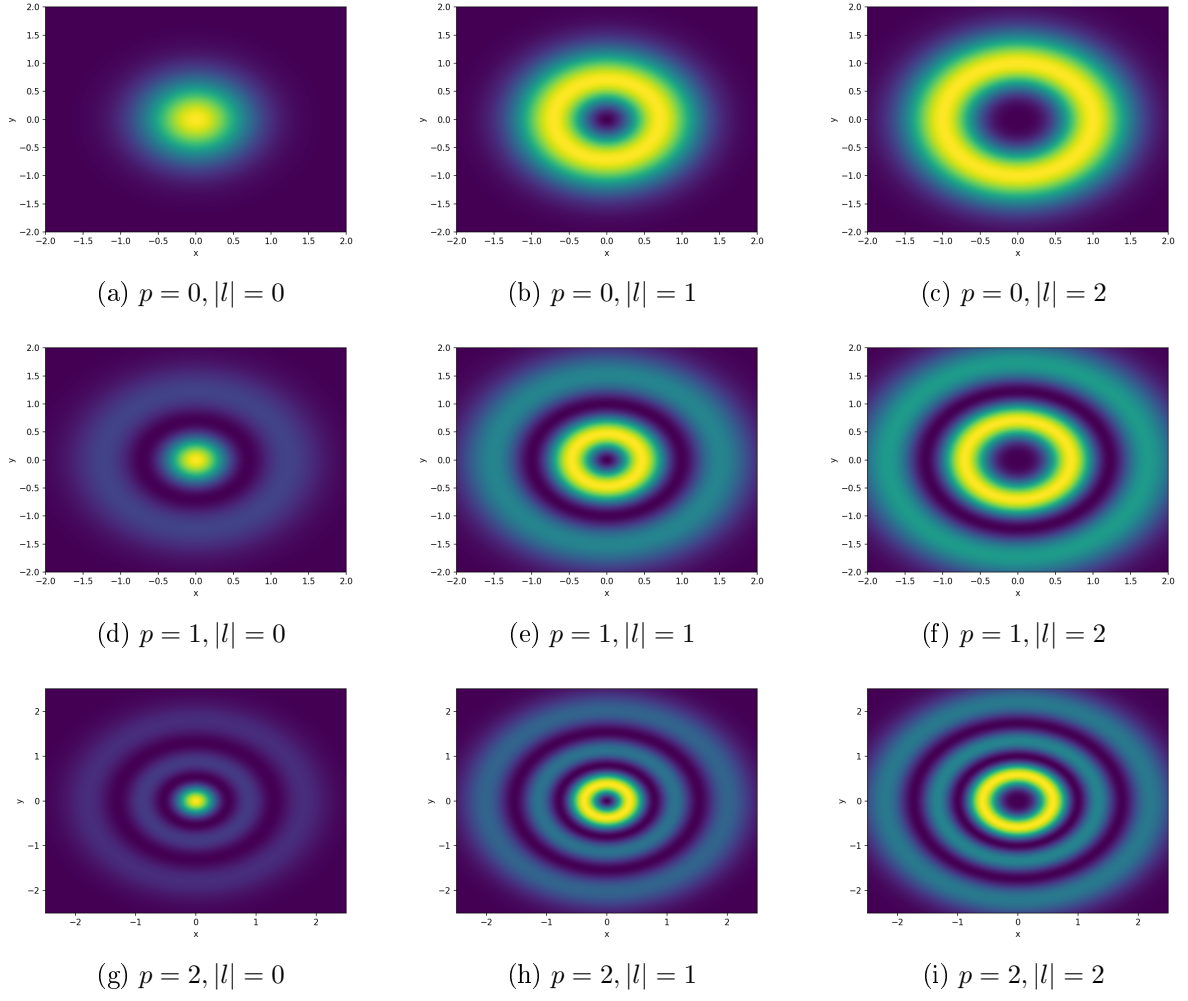


Figure 6: Intensity variation for different modes in a cross section ($z = 0, z_0 = 1, w_0 = 1$)