

# Emmy's $\mathcal{M}$ -Theory Recipes

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What ?

From non-perturbative string emerged  $\mathcal{M}$ -Theory.  
What are its complete degree of freedom?

How ?

Quantum deformations by  $\alpha'_M \propto (\ell_P)^2$  allowed by:

- ▷ the reparametrisation invariance (GR)
- ▷ supersymmetry algebra (SUGRA)
- ▷ unitarity of the theory (QGR)
- ▷  $\mathcal{U}$ -duality symmetries ( $\mathcal{M} \cdots$ )

Why ?

$\mathcal{M}$ -Theory is equivalent to a  
local quantum field theory of gravitation.

# Quantum Gravity

Classical General Relativity exists for every  $D \geq 4$ .

Topologically trivial in  $D = 2$  and non-dynamical in  $D = 3$ .

- ▷ Defined **perturbatively** by expanding around a vacuum

$$g_{\mu\nu} := \eta_{\mu\nu} + \kappa_{(d)} h_{\mu\nu}$$

dimensionfull

$$S_{\text{Einstein}} := \frac{1}{\kappa_{(d)}^2} \int d^D x \sqrt{-g} R$$

$$= -\frac{1}{2} \int d^D x (\partial_\alpha h_{\mu\nu})^2 + \sum_{n=1}^{\infty} \int d^D x (\partial h)^2 (\kappa_{(d)} h)^n$$

spin 2
vertices

Quantum General Relativity receives loop corrections:

- ▷  $\kappa_{(d)}^2 = (\ell_P)^{D-2} = (\alpha'_M)^{(D-2)/2}$  ( $\hbar = c = 1$ )
- ▷ Infinite set of counter-terms (non-renormalizability) [ 't Hooft, Veltman ]

$$\Delta S_{D=4} = \infty \times \sum_{n \geq 0} \int d^4 x \left( \kappa_{(d)}^2 R \right)^n R^2$$

- ▷ Corrections do *not* violate **unitarity** nor introduce **tachyons** or **ghosts**
- ▷ Field redefinitions or symmetries can **forbid** some terms  
[Goroff, Sagnotti], [van de Ven]
- ▷ **Ultra-violet** behaviour is background **independent**

# Eight Spoonfuls of SUGRA

Supergravity = GR + fermions with local grassmanian invariance.

▷ d.o.f. bosons = d.o.f. fermions

$$S \sim \frac{1}{\kappa_{(d)}^2} \int d^4x \sqrt{-g} \left( R - \frac{i}{2} \bar{\psi}^\alpha_\mu (\Gamma^{\mu\nu\rho})_{\alpha\beta} D_\nu(\omega) \psi^\beta_\rho \right)$$

spin 3/2

$$\delta^{(0)} e_\mu{}^r = \bar{\epsilon} \Gamma^r \psi_\mu, \quad \delta^{(0)} \psi_\mu = D_\mu(\omega) \epsilon$$

[Deser, Zumino], [Ferrara et al]

▷  $\alpha = 1, \dots, 4N$  with  $N \leq 8$  otherwise exist massless spin higher than 2 and more than one particle with spin 2. [Nahm]

▷ A “better” Ultra-violet behaviour [Grisaru], [Deser et al.], [Gates et al.], [Kallosh]

$$\Delta S_{D=4} = 0 \times \int d^4x R^2 + 0 \times \int d^4x \kappa_{(d)}^2 R^3 + \int d^4x \kappa_{(d)}^4 R^4 + \dots$$

Field Redefinition

Supersymmetry

Quantum correction

# Boil Everything to D=11

Classical  $N = 1$   $D = 11$  is

[Cremmer, Julia, Scherk]

$$S = \frac{1}{\kappa_{(d)}^2} \int d^{11}x e \left( R + \bar{\psi}_\mu \Gamma^{\mu\nu\rho} D_\nu \psi_\rho + G_{(4)}^2 \right) + \frac{\lambda}{6\kappa_{(d)}^2} \int C_{(3)} \wedge G_{(4)} \wedge G_{(4)}$$

▷ **Maximal:** no supergravity theories in  $D > 11$  [Nahm]

▷ **Minimal:** in its field content:  $(e_\mu^r, \psi_\mu^\alpha, C_{(3)})$ . [Nahm]

▷ **Unique:** no cosmological constant allowed [Bautier et al]

▷ **Non-trivial:** has exact maximally supersymmetric vacua:  $AdS_{4|7} \times S^{7|4}$

[Kallosh, Rajaraman]

▷ **Finite ?**  $\kappa_{(11)}^2 = (\ell_P)^9$  does not pair with powers of Riemann tensor  $R^n$

[Naive Argument]

# Incorporate Higher-order Derivative

**Finite** higher-derivative corrections are necessary:

▷ **Anomaly** considerations [Duff, Liu, Minasian], [Vafa, Witten], [Green, Schwarz]

$$S = (\alpha'_M)^3 \int C_{(3)} \wedge t_8 R^4.$$

versus

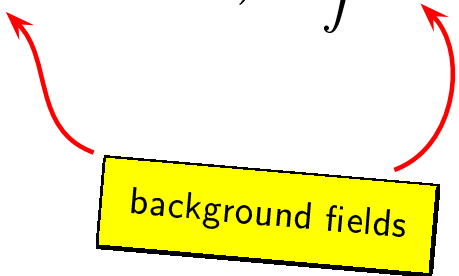
▷ **Strong coupling** limit of string theory amplitudes gives finite  $R^4$ ,  $\square^2 R^4$ ,  $\dots$

[Green, Kwon, Vanhove], [Green, Gutperle, Vanhove], [Green, Vanhove]

This is an **effective** theory of a **microscopic quantum theory of gravitation**.

# Add some Brane

Branes are **solitonic** extended solutions of supergravity theories

$$S = - \int d^{p+1} \xi \sqrt{-\det(g + dA - B)} + \int C \wedge e^{dA - B}$$


**Consistency** of the world-sheet theory **constraints** the background fields

- ▷ higher-loop  $\sigma$ -model  $\beta$ -functions [Grisaru et al]

$$S = \frac{1}{\alpha'} \int d^2 \sigma \partial_i X^\mu \partial^i X^\nu G_{\mu\nu}(X)$$

- ▷  $\kappa$ -symmetry invariance

$$\delta_\kappa(X^\mu(\xi), \theta^\alpha(\xi)) = \kappa^a E_a^M, \quad \delta_\kappa A = i_\kappa B, \quad \kappa^\alpha(\xi) = P_+ \zeta$$

only if the **backgrounds**  $E_M^A$ ,  $B_{MN}$  and  $C_{M_1 \dots M_k}$  satisfy the **on-shell constraints** that **imply** their equation-of-motions

Quantum corrections to the effective brane world-volume theory  $\iff$  corrections to  $\mathcal{M}$ -Theory.

# Serve with Strings

Noether's deformations of the ( $D = 11$ ) supersymmetry algebra

$$\left( \delta_{\epsilon}^{(0)} + \sum_{n \geq 3} a_n (\alpha'_M)^n \delta_{\epsilon}^{(n)} \right) \left( S^{(0)} + \sum_{n \geq 3} a_n (\alpha'_M)^n S^{(n)} \right) = 0$$

The super-algebra closes on-shell:

$$\begin{aligned} [\delta_{1,\text{sg}}, \delta_{2,\text{sg}}] = & \delta^{\text{translation}}(\xi^{\nu}) \\ & + \delta^{\text{susy}}(-\xi^{\nu} \psi_{\nu}) \\ & + \delta^{\text{gauge}}(-\xi^{\sigma} C_{\sigma\nu\rho} - 2\bar{\epsilon}_2 \Gamma_{\nu\rho} \epsilon_1) \\ & + \delta^{\text{Lorentz}}(\xi^{\nu} \hat{\omega}_{\nu}^r{}_t + 4\bar{\epsilon}_2 S^{r\nu\rho\sigma\kappa}{}_{t\epsilon_1} \hat{F}_{\nu\rho\sigma\kappa}), \end{aligned}$$

field dependent  
parameters

$\xi^{\mu}$  receives quantum corrections

$$\xi^{\mu} = \bar{\epsilon}_1 \Gamma^{\mu} \epsilon_2 + a_3 \times 0 \times (\alpha'_M)^3 t_8^{\mu t s_1 \dots s_6} R_{[r_1 r_2}^{s_1 s_2} R_{r_3 r_4}^{s_3 s_4} R_{r_5] t}^{s_5 s_6}$$

[Peeters, Vanhove, Westerberg]

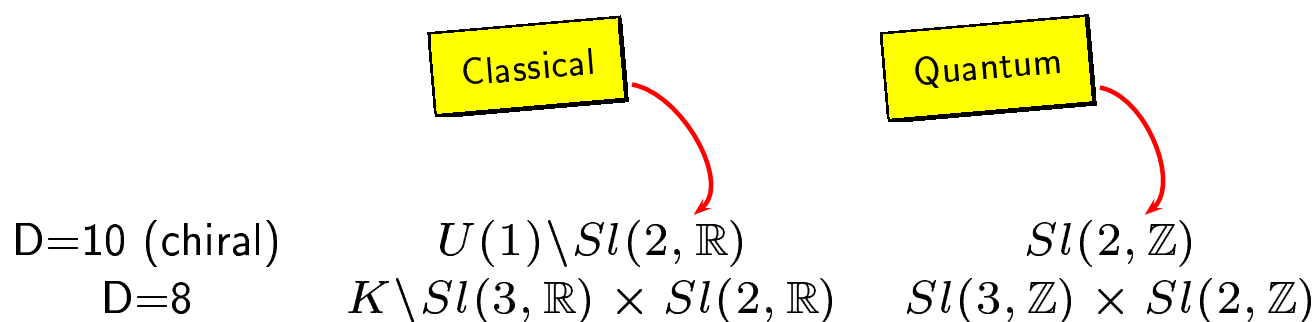
# Improve with $\mathcal{U}$ -duality toppings

Is the procedure unique ? Field theory reasoning are not enough!

▷  $a_n$  not fixed by supersymmetry alone

All extended supergravity theories in  $D \leq 10$  presents non-linear symmetries

$\mathcal{U}$ -duality groups



- ▷ Invariance under the quantum symmetry fixes the  $a_n$  by constraining the
  - ▷ Supersymmetry algebra [Green, Sethi]
  - ▷ S-matrix elements [Green, Kwon, Vanhove], [Green, Gutperle, Vanhove], [Green, Vanhove]

More recipes:

how to accomodate the Membrane with these higher-derivative corrections?