LOCALIZED (SUPER)GRAVITY*

IGNATIOS ANTONIADIS[1†] AND PIERRE VANHOVE[1,2]

[1] *CERN Theory Division CH-1211 Geneva 23*

[2] *Service de Physique Théorique, CEA/DSM/PhT*

Unité de recherche associée au CNRS CEA/Saclay, 91191 Gif-sur-Yvette, France

Email: ignatios.antoniadis@cern.ch, pierre.vanhove@cern.ch

We discuss a string-theory-derived mechanism for localized gravity, which produces a deviation from Newton's law of gravitation at cosmological distances. This communication is based on the paper [1] by the Ruben Minasian and the present authors.

The acceleration of our Universe at very large distance [2,3] is usually interpreted as the signature of dark energy. But late time self-expanding cosmological solutions, which do not need the introduction of a cosmological constant, have been obtained by [4,5,6] for the DGP model [7]. In this model the four-dimensional metastable graviton can evaporate into the extra dimensions and the gravitational interactions are modified at cosmological scales. Of course this setup does not solve the cosmological constant problem, but it seems to separate this issue from the current cosmic acceleration of our Universe. In the following we will concentrate on the gravitational interactions in the absence of a cosmological constant. Once standard model interactions are introduced (by adding D3-branes, for instance) a non-zero (localized) cosmological constant can appear.

In this communication we discuss the main features of a localized gravity model derived in a string theory context [1]

*cern-th/2003-103, spht-t03/054

†on leave of absence from cpht ecole polytechnique, france

1. The induced gravity model

The DGP model and its generalization are specified by a bulk Einstein-Hilbert (EH) term and a four-dimensional (EH) term

$$M^{2+n} \int_{M_{4+n}} d^4x d^n y \sqrt{G}\, \mathcal{R}_{(4+n)} + M_P^2 \int_{M_4} d^4x \sqrt{g}\, \mathcal{R}_{(4)}\,, \tag{1}$$

with M and $M_P(=: \sqrt{r_c^n M^{2+n}})$ the (possibly independent) respective Planck scales. The scale $M \geq 1$ TeV would be related to the short-distance scale below which UV quantum gravity or stringy effects are seen. $M_P \sim 10^{19}$ GeV is our four-dimensional Planck mass. The four-dimensional metric is the restriction of the bulk metric $g_{\mu\nu} = G_{\mu\nu}|$ and we assume the WORLD[a] rigid, allowing the gauge $G_{i\mu}| = 0$ with $i \geq 5$. Finally only intrinsic curvature terms are omitted but no Gibbons–Hawking term is needed.

The effective potential between two test masses in four dimensions

$$\int d^3x\, e^{-ip\cdot x}\, V(x) = \frac{D(p)}{1 + r_c^n\, p^2\, D(p)} \left[\tilde{T}_{\mu\nu} T^{\mu\nu} - \frac{1}{2+n}\, \tilde{T}^{\mu}_{\mu}\, T^{\nu}_{\nu}\right] \tag{2}$$

$$D(p) = \int d^n q\, \frac{f_w(q)}{p^2 + q^2} \tag{3}$$

is a function of the bulk graviton retarded Green's function $G(x, 0; 0, 0) = \int d^4p\, e^{ip\cdot x}\, D(p)$ evaluated for two points localized on the WORLD ($y = y' = 0$). The integral (3) is UV-divergent for $n > 1$ unless there is a non-trivial brane thickness profile $f_w(q)$ of width w. If the four-dimensional WORLD has zero thickness, $f_w(q) \sim 1$, the bulk graviton does not have a normalizable wave function. It therefore cannot contribute to the induced potential, which always takes the form $V(p) \sim 1/p^2$ and Newton's law remains four-dimensional at all distances. For a non-zero thickness w, there is only one crossover length scale, R_c:

$$R_c = w \left(\frac{r_c}{w}\right)^{\frac{n}{2}}\,, \tag{4}$$

above which one obtains a higher-dimensional behaviour [8].[b] Therefore the effective potential presents two regimes: (i) at short distances ($w \ll r \ll R_c$) the gravitational interactions are mediated by the localized four-dimensional graviton and Newton's potential on the WORLD is

[a] We avoid calling M_4 a brane, since gravity localizes on singularities of orbifold fixed points.

[b] For $n = 1$ the propagator (3) is not UV-divergent, but (4) predicts [9] a critical radius $R_c = \sqrt{wr_c} \ll r_c$ below which graviton's Kaluza–Klein excitations (induced by the cutoff) become massless, and the theory is five-dimensional.

given by $V(r) \sim 1/r$ and, (ii) at large distances ($r \gg R_c$) the modes of the bulk graviton dominate, changing the potential. For $n = 1$ the expressions (2) and (3) are finite and unambiguously give $V(r) \sim 1/r$ for $r \gg r_c$. For a co-dimension bigger than 1, the precise behaviour for large-distance interactions depends *crucially* on the UV completion of the theory.

At this point we stress a fundamental difference with the *finite extra dimensions* scenarios. In these cases Newton's law gets higher-dimensional at distances smaller than the characteristic size of the extradimensions.

2. String Theory realization

We explain following [1] how to realize (1) with $n \geq 6$ as the low-energy effective action of string theory on a non-compact six-dimensional manifold $\mathcal{M}_6$. We work in the context of $\mathcal{N} = 2$ supergravities in four dimensions but the mechanism for localizing gravity is independent of the number of supersymmetries. Of course for $\mathcal{N} \geq 3$ supersymmetries, there is no localization. We also start with a compact case and take the decompactification limit. The localized properties are then encoded in the different volume dependences.

In string perturbation, corrections to the four-dimensional Planck mass are in general very restrictive. In the heterotic string, they vanish to all orders in perturbation theory [10]; in type I theory, there are moduli-dependent corrections generated by open strings [11], but they vanish when the manifold $\mathcal{M}_6$ is decompactifiedg ; in type II theories, they are constant, independent of the moduli of the manifold $\mathcal{M}_6$, and receive contributions only from tree and one-loop levels (at least for supersymmetric backgrounds) [1,12].

The origin of the two EH terms in (1) can be traced back to the perturbative corrections to the eight-derivative effective action of type II strings in ten dimensions. These corrections include the tree-level and one-loop terms given by:[c]

$$\begin{aligned} \frac{1}{l_s^8} \quad & \int_{M_{10}} \frac{1}{g_s^2} \mathcal{R}_{(10)} + \frac{1}{l_s^2} \int_{M_{10}} \left(\frac{2\zeta(3)}{g_s^2} + 4\zeta(2) \right) t_8 t_8 R^4 \\ & - \frac{1}{l_s^2} \int_{M_{10}} \left(\frac{2\zeta(3)}{g_s^2} \mp 4\zeta(2) \right) R \wedge R \wedge R \wedge R \wedge e \wedge e + \cdots \end{aligned} \tag{5}$$

[c]The rank-eight tensor t_8 is defined as $t_8 M^4 \equiv -6(\mathrm{tr} M^2)^2 + 24 \mathrm{tr} M^4$, and the $\pm$ sign depends on the chirality (type IIA/B) of the theory. See [13] for more details.

where $l_s = M_s^{-1}$ is the string length scale and ϕ is the dilaton field determining the string coupling $g_s = e^{\langle\phi\rangle}$.

On a direct product space-time $\mathcal{M}_6 \times \mathbb{R}^4$ the $t_8 t_8 R^4$ contribute in four dimensions to R^2 and R^4 terms [12] (and to a cosmological constant which is zero [1]). At the level of zero modes the second R^4 term splits as $\int_{M_6} R\wedge R\wedge R \times \int_{M_4} \mathcal{R}_{(4)} = \chi \int_{M_4} \mathcal{R}_{(4)}$, and we have

$$\frac{1}{l_s^8}\int_{M_4\times M_6} \frac{1}{g_s^2}\,\mathcal{R}_{(10)} + \frac{\chi}{l_s^2}\int_{M_4}\left(-\frac{2\zeta(3)}{g_s^2} \pm 4\zeta(2)\right)\mathcal{R}_{(4)}\,, \tag{6}$$

which gives the expressions for the Planck masses M and M_p. A number of conclusions (confirmed by string calculations in [1]) can be reached by looking closely at (6):

▷ $M_p \gg M$ requires a large non-zero Euler characteristic for M_6, and/or a weak string coupling constant $g_s \to 0$.

▷ The one-loop graviton amplitude for the supersymmetric orbifold $T^6/\mathbb{Z}_N$, takes the form of a sum of quasi-localized contributions at the positions of the fixed points x_f of the orbifold [1]:

$$\langle V_g^3\rangle \sim \frac{1}{N}\sum_{(h,g)}\sum_{x_f}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2^2}\int\prod_{i=1}^{3}\frac{d^2 z_i}{\tau_2}\,\frac{1}{F_{(h,g)}(\tau,z_i)^3}\,e^{-\frac{(y-x_f)^2}{\alpha' F_{(h,g)}(\tau,z_i)}}\ . \tag{7}$$

Focusing on one particular fixed point $x_f = 0$ and sending the radii to infinity, we obtain the effective action for the quasi-localized EH term

$$\chi\int d^4x d^6y\sqrt{g} f_w(y)\,\mathcal{R}_{(4)} \tag{8}$$

with a width given by the four-dimensional induced Planck mass

$$w \simeq l_P = l_s\,\left[\chi\,(2\zeta(3)/g_s^2 + 4\zeta(2))\right]^{-1/2}\ . \tag{9}$$

For a more general non-compact background, the Euler number can in general split in different singular points of the internal space, giving rise to different localized terms.

▷ Since χ is a topological invariant the localized $\mathcal{R}_{(4)}$ term coming from the closed string sector is universal, independent of the background geometry and dependent only on the internal topology[d]. It is a matter of simple

[d] In type IIA/B, χ counts the difference between the numbers of $\mathcal{N}=2$ vector multiplets and hypermultiplets: $\chi = \pm 4(n_V - n_H)$ (where the graviton multiplet counts as one vector). Field theory computations of [14] show that the Planck mass renormalization depends on the UV behaviour of the matter fields coupling to the external metric. But, even in the supersymmetric case, the corrections are not obviously given by an index.

inspection to see that if one wants to have a localized EH term in less than ten dimensions, namely something linear in curvature, with non-compact internal space in all directions, *the only possible dimension is four* (or five in the strong coupling limit).

The crossover radius of eq. (4) is given by the string parameters ($n = 6$)

$$R_c = \frac{r_c^3}{w^2} \sim g_s \frac{l_s^4}{l_P^3} \simeq g_s \times 10^{32}\ \text{cm}\,, \tag{10}$$

for $M_s \simeq 1$ TeV. Because R_c has to be of cosmological scale, the string coupling can be relatively small, and $|\chi| \simeq g_s^2\, l_P \sim g_s^2 \times 10^{32}$ must be very large. The hierarchy is obtained mainly thanks to the large value of χ, so that lowering the bound on R_c lowers the value of χ. Our actual knowledge[e] of gravity at very large distances indicates [15] that R_c should be of the order of the Hubble radius $R_c \simeq 10^{28}$ cm, which implies $g_s \geq 10^{-4}$ and $|\chi| \sim 10^{24}$. A large Euler number implies only a large number of closed string massless particles with no a-priori constraint on the observable gauge and matter sectors, which can be introduced for instance on D3-branes placed at the position where gravity localization occurs. All these particles are localized at the orbifold fixed points and should have sufficiently suppressed gravitational-type couplings, so that their presence with such a huge multiplicity does not contradict observations. Note that these results depend crucially on the scaling of the width w in terms of the Planck length: $w \sim l_P^\nu$, implies $R_c \sim 1/l_P^{2\nu+1}$ in string units. If there are models with $\nu > 1$, the required value of χ will be much lower, becoming $\mathcal{O}(1)$ for $\nu \geq 3/2$. In this case, the hierarchy will be determined by tuning the string coupling to infinitesimal values, $g_s \sim 10^{-16}$.

3. Unitarity and strong coupling problems

Recent papers raised unitary [16] and strong coupling problems [17,18] with the DGP model and its higher co-dimension versions. All these problems depend crucially on the UV completion of the theory. A unitary UV regularization for the higher co-dimension version of the model has been proposed in [19]. It would be interesting to address these questions in the precise string theory context.

---

[e]We thank C. Deffayet for discussions on this point.

Acknowledgements

We thank C. Deffayet and G. Gabadadze for useful discussions. The work reported here has been supported in part by the European Commission under the RTN contract HPRN-CT-2000-00148.

References

1. I. Antoniadis, R. Minasian and P. Vanhove, Nucl. Phys. B **648**, 69 (2003) [arXiv:hep-th/0209030].
2. A. G. Riess *et al.* [Supernova Search Team Collaboration], Astron. J. **116** (1998) 1009 [arXiv:astro-ph/9805201].
   A. G. Riess *et al.*, Astrophys. J. **560** (2001) 49 [arXiv:astro-ph/0104455].
3. S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Astrophys. J. **517** (1999) 565 [arXiv:astro-ph/9812133].
4. C. Deffayet, Phys. Lett. B **502** (2001) 199 [arXiv:hep-th/0010186].
5. C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65** (2002) 044023 [arXiv:astro-ph/0105068].
6. G. Dvali, G. Gabadadze and M. Shifman, arXiv:hep-th/0208096.
   G. Dvali, G. Gabadadze and M. Shifman, Phys. Rev. D **67** (2003) 044020 [arXiv:hep-th/0202174].
7. G.R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000) [arXiv:hep-th/0005016].
8. G. R. Dvali and G. Gabadadze, Phys. Rev. D **63**, 065007 (2001) [arXiv:hep-th/0008054];
   G. R. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, Phys. Rev. D **64**, 084004 (2001) [arXiv:hep-ph/0102216].
9. E. Kiritsis, N. Tetradis and T. N. Tomaras, JHEP **0108** (2001) 012 [arXiv:hep-th/0106050].
10. I. Antoniadis, E. Gava and K. S. Narain, Phys. Lett. B **283** (1992) 209 [arXiv:hep-th/9203071].
11. I. Antoniadis, C. Bachas, C. Fabre, H. Partouche and T. R. Taylor, Nucl. Phys. B **489**, 160 (1997), [arXiv:hep-th/9608012].
    I. Antoniadis, H. Partouche and T. R. Taylor, Nucl. Phys. B **499**, 29 (1997), [arXiv:hep-th/9703076].
12. I. Antoniadis, S. Ferrara, R. Minasian and K. S. Narain, Nucl. Phys. B **507**, 571 (1997), [arXiv:hep-th/9707013].
13. K. Peeters, P. Vanhove and A. Westerberg, Class. Quant. Grav. **18** (2001) 843 [arXiv:hep-th/0010167].
14. S. L. Adler, Rev. Mod. Phys. **54** (1982) 729 [Erratum-ibid. **55** (1983) 837].
15. A. Lue and G. Starkman, Phys. Rev. D **67** (2003) 064002 [arXiv:astro-ph/0212083].
16. S. L. Dubovsky and V. A. Rubakov, arXiv:hep-th/0212222.
17. M. A. Luty, M. Porrati and R. Rattazzi, arXiv:hep-th/0303116.
18. V. A. Rubakov, arXiv:hep-th/0303125.
19. M. Kolanovic, M. Porrati and J. W. Rombouts, arXiv:hep-th/0304148.