International Journal of Modern Physics A
Vol. 33, No. 34 (2018) 1845006 (12 pages)

DOI: 10.1142/S0217751X18450069

Dark R^2 at low energy

Philippe Brax,* Patrick Valageas and Pierre Vanhove

Institut de Physique Théorique, Université Paris-Saclay,
CEA, CNRS, F-91191 Gif-sur-Yvette, Cedex, France
**Philippe.brax@ipht.fr*

Received 29 September 2018
Accepted 1 November 2018
Published 20 December 2018

We consider the consequences of the leading quartic corrections to the Einstein–Hilbert action of gravity at low energy. Using the equivalence between the scalar R^2 contribution and a scalar-tensor field theory, we analyze the possible ways of detecting the associated scalaron and suggest that short distance tests of gravity, and in particular future tests of Newton's law aboard satellites, would provide the best environment to detect such a modification of gravity. We also analyze the regimes for which the R^2 theory would result as a low energy manifestation of putative high energy UV completions involving extra dimensions. In the four-dimensional $N = 1$ supergravity limit of such extra-dimensional models, the R^2 models would emerge from the stabilization of a nearly no-scale superfield such as the ones associated to the Kähler modulus corresponding to the breathing mode of a six-dimensional compactification.

Keywords: Dark energy; supergravity.

1. Introduction

Einstein's General Relativity stands the test of time and remains unscathed after more than a century of intense investigations. In the last 20 years, the emergence of the acceleration of the expansion of the Universe[1–3] has led to various attempts to understand why gravity does not appear to be attractive on large cosmological scales. This could lead to intricate models of dark energy, although the recent observation of the neutron star merger by the LIGO/Virgo consortium[4,5] has reinforced the strength of the claim that, so far, the best candidate as an explanation for the cosmic acceleration is dark energy in the form of a constant vacuum energy, whose archetype is the original cosmological constant introduced by Einstein in 1917 and leading to the de Sitter space–time of 1919. Of course, this does not preclude the existence of corrections to the Einstein–Hilbert action coming solely from

*Corresponding author.

the gravitational sector. Moreover it is known that these corrections are generated when coupling gravity to matter.[6,7] In this paper, we study the physics associated to these corrections at low energy. We also surmise on the type of physics coming from compactifications of extra-dimensional models, which could lead to the corrections analyzed here and passing the Eöt-Wash tests of gravity.[8,9]

2. The Scenario

We take for granted both the existence of dark energy in the form of a constant and uniform vacuum energy together with the Einstein–Hilbert description of General Relativity. We also assume that such a setting is corrected at low energy, below a cutoff scale M, resulting in the action $S + \delta S$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} \mathcal{R} - \rho_\Lambda(\mu) + c_0(\mu) \mathcal{R}^2 + c_2(\mu) \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} \mathcal{R}^2 \right) \right] , \quad (1)$$

where δS contains all the higher-order terms in the curvature invariants

$$\delta S = \int d^4x \sqrt{-g} \frac{M_{\rm Pl}^2}{2} \sum_{n \geq 3} \alpha_n(\mu) M^2 \left(\frac{R}{M^2} \right)^n . \quad (2)$$

The $\alpha_n(\mu)$'s are dimensionless coefficients of order $\mathcal{O}(1)$ and R stands for the various Riemann tensor components $R_{\mu\nu\rho\sigma}$. The Ricci scalar is denoted $\mathcal{R} := R^{\mu\nu}{}_{\mu\nu}$ and the Ricci tensor $R_{\mu\nu} := R^{\lambda}{}_{\mu\lambda\nu}$. The scale μ is the renormalization sliding scale and M corresponds to the cutoff scale of the effective gravitational field theory, which is valid for $R \ll M^2 \ll M_{\rm Pl}^2$. We are interested in the low-energy effective action for dark energy describing the late time physics of the Universe after Big Bang Nucleosynthesis and below the electron mass $\mu \ll m_e$. In this regime, the typical curvature involved are tiny on cosmological scales.[10] As an effective field theory with higher order derivatives, there are new and dangerous degrees of freedom per power of the Riemann tensor which are generically ghost-like with masses of order of the ultraviolet cutoff scale $\mathcal{O}(M)$. As these masses are of the order of the ultraviolet cutoff, they have no influence below the scale M. Focusing on the quadratic terms in the curvature invariants, it is known that Ricci tensor term gives rise to a ghost[19] of mass squared of the order $M_{\rm Pl}^2/c_2(\mu)$. Using the results[11,12] that $c_2(\mu)$ is always asymptotically free since $dc_2(\mu)/d\log\mu^2 > 0$, whereas $c_0(\mu)$ is asymptotically safe $dc_0(\mu)/d\log\mu^2 < 0$, we infer that at low-energy when $\mu \ll m_e$ $c_2(\mu)$ tends to zero whereas $c_0(\mu)$ grows, leading to the hierarchy $c_0(\mu) \gg c_2(\mu)$ at very low energy. In this scenario, the Ricci tensor term decouples from our effective low energy and should be cured by the ultraviolet completion of the theory. We will then work with the low-energy effective action[13]

$$S_{R^2}(\mu) = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} \mathcal{R} - \rho_\Lambda(\mu) + c_0(\mu) \mathcal{R}^2 \right] + S_{\rm matter} , \quad (3)$$

obtained after integrating out all the massive particles of the Standard Model of mass above the electron mass.

Such $\mathcal{R}^2$ theories are equivalent to scalar field models[14] (our metric convention is $(-+++)$)

$$S_\phi(\mu) = \int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}\mathcal{R} - \rho_\Lambda(\mu) - \frac{(\partial\phi)^2}{2} - \frac{m(\mu)^2}{2}\phi^2\right) + S_{\rm matter}(\psi_i, e^{2\phi/(M_{\rm Pl}\sqrt{6})}g_{\mu\nu})\,, \tag{4}$$

where the mass of the scalaron is

$$m(\mu)^2 = \frac{M_{\rm Pl}^2}{12c_0(\mu)}\,, \tag{5}$$

and $S_{\rm matter}$ is the matter action depending on the matter fields ψ_i. The scalar potential has been expanded to lowest order in $\phi/M_{\rm Pl}$ as we are interested in the regime where $c_0\mathcal{R} \ll M_{\rm Pl}^2$ and the scalaron self-interactions are negligible, being suppressed by powers of $m(\mu)$, which is very small in Planck mass units. This effective field theory is valid at a scale μ much lower than the electron mass, and describes the late time acceleration of the expansion of the Universe. At this energy scale, the only matter fields which need to be taken into account are the three massive neutrinos. As befitting the cosmological constant problem,[3] the vacuum energy $\rho_\Lambda(\mu)$ combines the effect of all the quantum corrections associated to massive particles that have been integrated out down to the energy scale μ, the various phase transitions including the QCD and electroweak ones $\rho_\Lambda^{\rm SM}(\mu)$, and the bare cosmological constant ρ_Λ^0 seen as finite counterterm after renormalization. This decomposition stands in the minimal decoupling subtraction scheme $\overline{DS}$,[15] where the parameters of the Wilsonian action can be obtained by integrating the renormalization group equations by taking into account only the particles which have not been integrated out yet.

In the Jordan frame where the particles of the Standard Models are coupled to $e^{2\phi/(M_{\rm Pl}\sqrt{6})}g_{\mu\nu}$, the quantum fluctuations due to massive particles and the phase transitions in the matter sector are all scalar-independent. In the Einstein frame, the potential term of the $\mathcal{R}^2$ model is corrected by a term $e^{4\phi/(M_{\rm Pl}\sqrt{6})}\rho_\Lambda(\mu)$. This leads to correction to the scalaron mass

$$\frac{4}{3}\frac{\rho_\Lambda(\mu)}{M_{\rm Pl}^2} \ll \frac{M_{\rm Pl}^2}{c_0(\mu)}\,, \tag{6}$$

which is negligible compared to (5) when $\rho_\Lambda(\mu)$ is of the order of the dark energy scale now, i.e. 10^{-48} GeV^4, and the mass of the scalaron satisfies the Eöt-wash bound, as follows.

At low energy, $\max(m, m_f) < \mu < m_e$, before the scalaron and the neutrino decouple, the vacuum energy $\rho_\Lambda(\mu)$ is given by

$$\rho_\Lambda(\mu) = \rho_\Lambda(m_e) + \left(\frac{m^4}{64\pi^2} - 2\sum_{f=1}^{3}\frac{m_f^4}{64\pi^2}\right)\ln\frac{m_e^2}{\mu^2}\,. \tag{7}$$

A similar equation has already been considered[16] without the scalaron contribution. Here, we neglect the μ dependence of the masses and work in the leading log approximation. This expression matches with the vacuum energy $\rho_\Lambda(m_e)$ at the energy scale $\mu = m_e$, coming from the evolution of $\rho_\Lambda(\mu)$ at energies $\mu > m_e$ as required by the decoupling[17] at the scale the electron mass. On the other hand, when considering dark energy, we are interested in the very low energy Wilsonian action for μ much lower than the neutrino masses and the scalaron mass. In this regime, the neutrinos and the scalaron have decoupled and the vacuum energy becomes a constant corresponding to the one-particle irreducible vacuum energy associated with the classical equations of motion of the theory. In this paper, we identify the one-particle irreducible vacuum energy with the dark energy density $(\lambda M_{\text{Pl}})^4$ measured by cosmological probes,

$$\rho_{\text{vac}} := (\lambda M_{\text{Pl}})^4 \simeq 2.7 \times 10^{-11} \text{ eV}^4 \,, \tag{8}$$

implying that

$$\rho_{\text{vac}} = \rho_\Lambda(m_e) + \frac{m^4}{64\pi^2} \ln \frac{m^2}{m_e^2} - 2 \sum_{f=1}^{3} \frac{m_f^4}{64\pi^2} \ln \frac{m_f^2}{m_e^2} \,. \tag{9}$$

As is of course (in) famously well known, the energy density ρ_{vac} is much lower than the order of magnitude of particle physics scales. These contributions to the vacuum energy density have all been subsumed in $\rho_\Lambda(m_e)$, which contains all the physical effects at energies higher than m_e and contributing to the renormalized energy density, where the bare cosmological constant has been used as a counterterm in the renormalization process. Notice that in the $\overline{DS}$ renormalization scheme, massless fields such as photons and gluons do not contribute to the vacuum energy as their divergent contributions to the vacuum energy are canceled by the cosmological counterterm leaving only the contributions of massive fields in $\rho_\Lambda(\mu)$.

The neutrino contributions to ρ_{vac} are strongly constrained by cosmological and astrophysical measurements. The Planck results[18] for the cosmic microwave background give the upper bound $m_1 + m_2 + m_3 < 0.23$ eV for the sum of the neutrino masses. The oscillations of the solar neutrinos yield the squared mass difference $m_2^2 - m_1^2 = 7.5 \times 10^{-5}$ eV^2. As a result, the neutrino contributions to the vacuum energy are bounded

$$2 \times 10^4 \rho_{\text{vac}} \leq -\sum_{f=1}^{3} \frac{m_f^4}{64\pi^2} \log\left(\frac{m_f^2}{m_e^2}\right) \leq 2 \times 10^5 \rho_{\text{vac}} \,. \tag{10}$$

This is of course one aspect of the usual cosmological constant problem as these contributions potentially exceed the measured value of the dark energy. In the following, we shall address the question: what can be reasonably said about the scalaron sector and in particular what is the plausible order of magnitude of the scalaron mass and its contribution to the dark energy?

3. Nailing Down the Scalaron Mass

At low energy, the scalaron induces a modification to Newton's law with a strength 4/3 within its Compton wavelength. This follows from the modification of the large distance gravitational potential[19]

$$V(r) = -\frac{GM}{r}\left(1 + \frac{1}{3}e^{-mr}\right). \tag{11}$$

Of course, such deviations from Newton's law are severely constrained and the apparent absence of evidence for short-range forces in the Eöt-Wash experiment[8,9] provide the strong lower bound

$$m \gtrsim 2.8 \times 10^{-3}\ \text{eV} \simeq 1.22\lambda M_{\text{Pl}}\,, \tag{12}$$

which happens to be of the same order as the dark energy scale. This could be a numerical coincidence or in fact gives us a hint about some underlying physical properties of the vacuum energy. In any case, this implies that the contribution of the scalaron to the vacuum energy is bounded from above by

$$\frac{m^4}{64\pi^2}\log\frac{m^2}{m_e^2} \lesssim -\rho_{\text{vac}}\,. \tag{13}$$

Can we actually do better and obtain a lower bound on this contribution? This requires to analyze systems such as galaxy clusters.[20]

The gas of a galaxy cluster has a typical temperature of $T_{\text{X}} \sim 1$ keV, in regions of total matter density of about 500 times the mean density of the Universe. These systems typically appeared at a redshift $z \gtrsim 0.1$ and have a lifetime of at least the order of the age of the Universe. In such clusters, the scalaron and the neutrinos, coming either from the early Universe with an energy of order 10^{-4} eV or from astrophysical processes such as the burning of stars with an energy around 100 keV, have a very small cross-section with ambient matter $\sigma \simeq \beta^2/M_{\text{Pl}}^2$ and $\sigma \simeq m_e^2/M_W^4$, where $M_Z \simeq 10^2$ GeV is the mass of the Z boson, respectively. From the particle physics point of view, both the neutrinos and the scalaron decouple from the physics inside the clusters. Hence, the physics of the gas can be described by an effective action which takes into account the coupling between nonrelativistic matter particles (such as electrons and protons) and General Relativity augmented with a vacuum energy. As all the matter fields in the gas, such as the electrons, are nonrelativistic, the vacuum energy in the cluster gas is the one of all the physical fields of masses greater than T_X, i.e. $\rho_\Lambda(m_e)$. We make the strong assumption that the effective decoupling of the neutrinos and the scalaron implies that the vacuum energy in the plasma excludes their quantum fluctuations. Under this strong hypothesis, whose verification is beyond the present paper, we can deduce that if the vacuum energy $\rho_\Lambda(m_e)$ were extremely negative, the galaxy cluster would have gravitationally collapsed in a time $\frac{\pi}{\sqrt{3}}\frac{M_{\text{Pl}}}{\sqrt{-\rho_\Lambda(m_e)}}$ which must be larger than the age of the Universe. This implies that

$$\rho_\Lambda(m_e) \gtrsim -\rho_{\text{vac}}\,. \tag{14}$$

Using the neutrino bounds (10) and (14), this leads to a lower bound for the contribution of the scalaron to the dark energy

$$\frac{m^4}{64\pi^2}\log\frac{m^2}{m_e^2}\gtrsim\sum_{f=1}^{3}\frac{m_f^4}{64\pi^2}\log\frac{m_f^2}{m_e^2}\,. \tag{15}$$

Of course, this leads to the upper bound on the scalar mass

$$m\lesssim\bar{m}_\nu=(m_1^4+m_2^4+m_3^4)^{\frac{1}{4}}\simeq 0.1\ \mathrm{eV}\,. \tag{16}$$

In terms of the inverse mass m^{-1} of the scalar interaction, we get the noticeably short-range constraint on the interaction mediated by the scalaron

$$2\ \mu\mathrm{m}\lesssim m^{-1}\lesssim 68\ \mu\mathrm{m}\,. \tag{17}$$

What is the implication of this result? First of all, if the scalaron exists as a low energy degree of freedom, it would manifest itself in a modification of the Newtonian potential (11). It turns out that this range is within reach of the new runs[21] of the Eöt-Wash experiment which have been recently presented. If no signal in such laboratory experiments were found in this small range, this would signify that the scalaron has a much larger mass with a much smaller coefficient c_0. In a sense, the scalaron would disappear from the low energy realm and at sufficiently low energy the Einstein–Hilbert term of General Relativity would be all that remains together with a tuned (possibly fine-tuned) vacuum energy. Moreover this would also imply that dark energy would not be explained by a low energy manifestation of gravity alone but, if differing from a nondynamical dark energy, by new light degrees of freedom.[22,23]

The existence a short-ranged scalaron could be tested by different types of experiments. Preliminary results of new runs of the Eöt-Wash experiment[21] indicate a possible new upper bound of 40 μm, therefore reducing the range of allowed mass almost by half. Another extremely promising possibility would be a torsion pendulum experiment of the Eöt-Wash type aboard a satellite. Such a project has already been considered[24] with a target of force ranges around 10 μm which would be sensitive to coupling of order one or below such as $\beta\simeq 1/\sqrt{6}$.[25] Of course, such a future experiment would have the power to vindicate or exclude the scalaron that we have considered in this paper.

4. Where Does the Scalaron Come From?

We have just seen that a scalaron with a short-range, of the sub-millimeter type, could appear as a low energy amendment to General Relativity. Such a scenario requires a very large coupling c_0. This seems very contrived and unnatural. We have also seen that renormalization effects and the running of c_0 at low energy cannot lead to large values as the dependence of c_0 on the renormalization scale is very weak. Could it be that such large couplings be generated at high energy? The most likely candidate would be that c_0 is generated and is very large when compactifying extra-dimensions as in string theory.[26] Let us focus on such compactifications of the effective action of string theory and evaluate the order of magnitude of the c_0 coefficient at very high energy. We can estimate the effects of the higher derivative corrections to the string theory effective action, here at tree level in the string coupling constant, using

$$S_{10d}^{\text{string}} = \frac{1}{(2\pi)^7}\int d^{10}x\sqrt{-g}\frac{1}{g_s^2\ell_s^8}(\mathcal{R} + \alpha_3\ell_s^6 R^4 + \alpha_5\ell_s^{10}R^6 + \cdots)\,, \tag{18}$$

where the α_n are numerical coefficients.[26] After compactification on a nonwarped product of four-dimensional space–time augmented with the compactification manifold, the effective action in four-dimensions takes the typical form

$$S_{4d}^{\text{string}} = \int d^4x\sqrt{-g}\left\{V_{\text{vac}} + M_{\text{Pl}}^2\left[\frac{\mathcal{R}}{2} + R^2\ell_s^2\sum_{p=0}^{\infty}d_p(\ell_s^2 R)^p\right]\right\}. \tag{19}$$

Typically, we consider supersymmetric constructions where the vacuum energy vanishes $V_{\text{vac}} \equiv 0$. The structure of the other terms in the action follows by direct integration. The four-dimensional Planck mass is given by

$$M_{\text{Pl}}^2 \simeq \frac{l_6^6}{g_s^2\ell_s^8}\left(1 + \sum_{n\geq 3}\alpha_n\frac{\ell_s^{2n}}{\tilde{l}_6^{2n}}\right) \quad \text{with} \quad \int\frac{d^6x}{V_6}\sqrt{-g}R^n \sim \tilde{l}_6^{-2n}\,. \tag{20}$$

Here, we have introduced $\tilde{l}_6^{-2}$ as the typical averaged value of the scalar curvature R over the six-dimensional compactification manifold. Typically, it is expected that $\ell_s \lesssim \tilde{l}_6 \lesssim l_6$. The infinite tower of contributions coming from the higher-order terms in the 10d action are suppressed when $\tilde{l}_6 \gg \ell_s$, which is thus a necessary condition for the expansion in R of the four-dimensional action to make any sense at all. In this construction, the R^2 model only applies up to the curvature scale $R \leq \tilde{l}_6^{-2}$, beyond which the terms R^3 and R^4 become greater in the action (19), and higher-order terms successively appear at higher curvature. These estimates give for the coefficients c_0[13]

$$c_0 \sim \frac{1}{g_s^2}\left(\frac{l_6}{\ell_s}\right)^6\left(\frac{\tilde{l}_6}{\ell_s}\right)^{-4}. \tag{21}$$

We want to reach $c_0 \sim \lambda^{-2} \simeq M_{\text{Pl}}/H_0$, and upon using $\tilde{l}_6 \gg \ell_s$ this would give $\ell_s^2 \gg 1/(M_{\text{Pl}}H_0)$, hence, $M_s \ll 10^{-3}$ eV. This value for the string scale is much below the scales probed by colliders. As a result, the c_0 term must have a different physical origin for the dominance of the $\mathcal{R}^2$ term.

One way of obtaining a $\mathcal{R}^2$ model with a large c_0 coupling is to consider the inverse process. Is it possible to generate an effective field theory for a very light scalar field coupled to matter with a fixed strength $\beta = 1/\sqrt{6}$ at high energy in the context of fundamental models coming from string theory or more phenomenological approaches like extra-dimensional models? This requires to generate a hierarchy of scales between the energy scale of the model and the mass of the scalar. If this scenario can be designed, then by reverse engineering, it becomes equivalent to an $\mathcal{R}^2$ model which is characterized by its scalaron mass and the coupling β.

In the following section, we will describe such a scenario using $N = 1$ supergravity which may describe the low energy features of string compactifications with antibranes and warping. This is a highly debated subject connected to the existence of de Sitter vacua in string theory.[27–29] We will not treat our model as a string model per se and avoid these thorny issues. We will simply use it as a four-dimensional model described by $N = 1$ supergravity at second-order in the derivatives whose behavior is equivalent to the $\mathcal{R}^2$ model. Using this analogy, the scalaron originates from the breathing mode of the compactification. Its small mass is due to the warping while the higher-order corrections in the curvature invariants are not affected by the dynamics of the scalaron and are small thanks to $\tilde{l}_6 \gg \ell_s$.

5. $N = 1$ Supergravity Embedding

The $\mathcal{R}^2$ model with $c_0 \gg 1$ can be described by a massive scalar field of small mass compared to the Planck scale and the uniquely fixed coupling to matter $\beta = 1/\sqrt{6}$. In this section, we will construct a supergravity model inspired by string theory where a Kähler modulus T will be associated to ϕ and the coupling to matter is also uniquely defined to be $\beta = 1/\sqrt{6}$. In what follows, we use a string vocabulary for illustration purposes only. The model is strictly defined at the level of a $N = 1$ supergravity action with two derivatives only. We also include a nilpotent field S such that $S^2 = 0$ which corresponds to the Goldstino fermion field associated to the breaking of supersymmetry by an antibrane. We do not consider the issue of the full supergravity action when the F-term of the S field may vanish.[29] We restrict ourselves to field values where this is not the case. The Kähler potential is given by

$$K = -3m_{\rm Pl}^2 \ln\left(\frac{T+\bar{T}}{m_{\rm Pl}} - \frac{S\bar{S}}{3m_{\rm Pl}^2}\right) + m_{\rm Pl}\frac{C\bar{C}}{T+\bar{T}} + \cdots\,, \tag{22}$$

where C is a matter superfield containing the fermion ψ_C. The volume of compactification would be given by

$$\mathcal{V}_6 \sim l_s^6 \left(\frac{T+\bar{T}}{m_{\rm Pl}}\right)^{3/2}. \tag{23}$$

Notice that the coupling of matter to T in $C\bar{C}/(T+\bar{T})$ is reminiscent of the coupling of matter fields on $D3$ branes to T. The superpotential is given by

$$W = W_0 + (P + CS)e^{-aT/m_{\rm Pl}} + BS\,, \tag{24}$$

where in the string context, the constant W_0 comes from integrating out the complex moduli of the compactification manifold, the exponential factor would come from gaugino condensation on $D7$ branes with $P \sim \Lambda_C^3$ and would be related to the condensation scale Λ_C. The B term would be related to the warped down brane tension along a throat of the compactification manifold

$$B \sim e^{-2A_0}\sqrt{T}_3\,, \tag{25}$$

where T_3 is the antibrane tension and $A_0 \gg 1$ would be associated to the warping along the throat, where the antibrane sits. The factor C is only phenomenological here and will be taken to be of order of $\sqrt{T}_3$, i.e. not warped down. We will also assume that the gaugino condensation scale is much smaller than the brane tension $P/m_{\mathrm{Pl}} \ll \sqrt{T}_3$.

In a first step, let us assume that $P = 0$. We will take into account the corrections introduced by P in the scalar potential below. Using the nilpotent constraint, the scalar potential reads

$$V_0 = m_{\mathrm{Pl}}^2 \frac{|B + Ce^{-aT/m_{\mathrm{Pl}}}|^2}{(T+\bar{T})^2}. \tag{26}$$

This potential asymptotically reaches zero for large T. There is always another Minkowski minimum t_0 with

$$e^{-at_0/m_{\mathrm{Pl}}} = \left|\frac{B}{C}\right| \tag{27}$$

and putting $T_0 = t_0 + i\theta$ we have $\theta = 0$ if $BC < 0$ while $\theta = \pi/a$ for $BC > 0$. We have also

$$\frac{t_0}{m_{\mathrm{Pl}}} \sim \frac{2A_0}{a} + \frac{1}{a} \ln \left| \frac{C}{\sqrt{T}_3} \right| , \tag{28}$$

which is large and of order $2A_0/a$ corresponding to a relatively large volume of compactification compared to the string scale as can be seen in (23). On the other hand, this volume is not large enough to imply the existence of low-lying Kaluza–Klein excitations at low energy. Notice that this minimum corresponds to a vanishing $F_S = 0$ from which the full embedding of this model in supergravity may be problematic[29] and may require to modify the model slightly to be fully consistent. Expanding around the minimum $T = T_0 + \delta T$, and introducing the normalized scalar

$$\frac{T}{t_0} = e^{-2\beta\phi/m_{\mathrm{Pl}}} , \tag{29}$$

where $\beta = 1/\sqrt{6}$, we find that the mass of the scalar at the minimum becomes

$$m^2 = \beta^2 a^2 \frac{|B|^2}{m_{\mathrm{Pl}}^2} \tag{30}$$

implying that the scalar received a warped down mass term

$$m \sim e^{-2A_0} \frac{\sqrt{T}_3}{m_{\mathrm{Pl}}}. \tag{31}$$

The coupling of the scalar ϕ can be inferred from the mass term of the fermions ψ_C after normalization $\psi = \psi_C \sqrt{\frac{m_{\mathrm{Pl}}}{T+\bar{T}}}$ which implies that

$$\mathcal{L}_\psi \supset e^{K/2m_{\mathrm{Pl}}^2} m_0 \bar{\psi}_C \psi_C \sim e^{\beta\phi/m_{\mathrm{Pl}}} m_0 \bar{\psi}\psi \tag{32}$$

and matter couples to ϕ with a strength $\beta = 1/\sqrt{6}$. As a result, the theory behaves at low energy after modulus stabilization as an $\mathcal{R}^2$ theory with a coefficient

$$c_0 = \frac{m_{\rm Pl}^4}{2a^2|B|^2} \sim \frac{m_{\rm Pl}^4}{T_3} e^{4A_0}, \tag{33}$$

which is large due to the hierarchies $T_3 \ll m_{\rm Pl}^4$ and $A_0 \gg 1$.

Reintroducing P, the scalar potential reads now[13]

$$V = V_0 + \frac{a^2 P^2}{3m_{\rm Pl}^2} \frac{m_{\rm Pl}}{T+\bar{T}} e^{-2aT/m_{\rm Pl}} + \frac{2a}{3(T+\bar{T})^2} \Re\mathrm{e}(Pe^{-aT/m_{\rm Pl}}(W_0 + Pe^{-aT/m_{\rm Pl}}))\,. \tag{34}$$

Working to linear order in P, i.e. with $W_0 \gg Pe^{-at_0/m_{\rm Pl}}$, we find that the correction terms behave like

$$\delta V \sim \frac{aP}{6t_0^2} W_0 e^{-at_0/m_{\rm Pl}} - \frac{\delta T}{6t_0^3} e^{-at_0/m_{\rm Pl}} W_0 \left(2 + a\frac{t_0}{m_{\rm Pl}}\right), \tag{35}$$

while

$$V_0 \sim \frac{a^2|B|^2}{4} \left(\frac{\delta T}{t_0}\right)^2. \tag{36}$$

The potential admits a new minimum for

$$\frac{\delta T}{T_0} = \frac{P}{3t_0^2|B|^2} \left(2 + a\frac{t_0}{m_{\rm Pl}}\right) e^{-at_0/m_{\rm Pl}} W_0\,. \tag{37}$$

The resulting vacuum energy becomes

$$V_{\rm vac} = \frac{aP}{6t_0^2} W_0 e^{-at_0/m_{\rm Pl}} \tag{38}$$

at leading order, i.e. a positive contribution if $P > 0$. The mass at the new minimum is hardly modified. Notice too that $F_S \neq 0$ now at the minimum of the potential. Of course, this result could be taken as a proof of the nonembeddability of this $N = 1$ supergravity model in string theory if de Sitter vacua cannot be accommodated at the string level.

In conclusion, we have found a model where $c_0 \gg 1$ coming from the dynamics of the modulus T. This should provide the boundary at $\mu = \tilde{l}_6$ for both the renormalization evolution of c_0 and the vacuum energy. It is quite likely that decoupling states of masses $m_i \ll M_{\rm Pl}$ will give rise to contributions to c_0 of order unity. Hence, the boundary value of c_0 at $\mu = \tilde{l}_6$ is hardly modified if the warping is such that $c_0 \sim \lambda^2 \gg 1$. On the other hand, the boundary value of the vacuum energy is very close to its Minkowski value for $P = 0$ and is therefore, largely modified by the decoupling of large mass states and phase transitions. This is the main difference between the c_0 coefficient which is essentially determined at high energy, whereas the vacuum energy at low energy is affected by all the physics between the high energy physics described here and the low energy realm described earlier in this paper.

6. Conclusion

In this contribution, we have described the consequences of the $\mathcal{R}^2$ dark sector at low energy where the scalaron could be detectable by the next generation, on earth or in satellites, of experiments testing the validity of Newton's law to short distances. We have also described how the $\mathcal{R}^2$ model could emerge in the context of $N = 1$ supergravity and maybe extra-dimensional models with warped throats. In this scenario, the scalaron would be the only detectable consequence of dark energy in the laboratory with a sub-millimeter deviation from Newton's law with a range never lower than a few microns. This last bound is highly dependent on a bound on the scalar mass deduced from the modelization of the quantum vacuum inside galaxy clusters. Validating the hypothesis that the scalaron and neutrino decouple from the vacuum fluctuations in these clusters is left for more detailed work. On the other hand, the next generation of Eöt-Wash experiments and possible satellite-based experiments will vindicate the existence of a short-ranged scalaron, or disprove this scenario and therefore imply that all the higher-order corrections to the gravity actions do not play a role in the tests of gravity at low energy.

Acknowledgments

This work is supported in part by the EU Horizon 2020 research and innovation programme under the Marie–Sklodowska Grant No. 690575. This paper is based upon work related to the COST Action CA15117 (CANTATA) supported by COST (European Cooperation in Science and Technology).

References

1. E. J. Copeland, M. Sami and S. Tsujikawa, *Int. J. Mod. Phys. D* **15**, 1753 (2006), arXiv:hep-th/0603057.
2. T. Clifton, P. G. Ferreira, A. Padilla and C. Skordis, *Phys. Rep.* **513**, 1 (2012), arXiv:1106.2476 [astro-ph.CO].
3. S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
4. LIGO Scientific and Virgo Collabs. (B. P. Abbott *et al.*), *Phys. Rev. Lett.* **116**, 221101 (2016), arXiv:1602.03841.
5. LIGO Scientific and VIRGO Collab. (B. P. Abbott *et al.*), *Phys. Rev. Lett.* **118**, 221101 (2017), arXiv:1706.01812.
6. G. 't Hooft and M. J. G. Veltman, *Annu. Inst. Henri Poincaré Phys. Theor. A* **20**, 69 (1974).
7. M. H. Goroff and A. Sagnotti, *Nucl. Phys. B* **266**, 709 (1986).
8. C. D. Hoyle, D. J. Kapner, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, U. Schmidt and H. E. Swanson, *Phys. Rev. D* **70**, 042004 (2004), arXiv:hep-ph/0405262.
9. D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle and H. E. Swanson, *Phys. Rev. Lett.* **98**, 021101 (2007), arXiv:hep-ph/0611184.
10. P. Brax, L. A. Rizzo and P. Valageas, *Phys. Rev. D* **94**, 044027 (2016), arXiv:1605.02938 [astro-ph.CO].
11. E. S. Fradkin and A. A. Tseytlin, *Nucl. Phys. B* **201**, 469 (1982).
12. A. Salvio and A. Strumia, *J. High Energy Phys.* **1406**, 080 (2014), arXiv:1403.4226 [hep-ph].

13. P. Brax, P. Valageas and P. Vanhove, *Phys. Rev. D* **97**, 103508 (2018), arXiv:1711.03356 [astro-ph.CO].
14. T. P. Sotiriou and V. Faraoni, *Rev. Mod. Phys.* **82**, 451 (2010), arXiv:0805.1726.
15. C. P. Burgess, *Annu. Rev. Nucl. Part. Sci.* **57**, 329 (2007), arXiv:hep-th/0701053.
16. I. L. Shapiro and J. Sola, *Phys. Lett. B* **475**, 236 (2000), arXiv:hep-ph/9910462.
17. H. Georgi, *Annu. Rev. Nucl. Part. Sci.* **43**, 209 (1993).
18. Planck Collab. (P. A. R. Ade *et al.*), *Astron. Astrophys.* **594**, A13 (2016), arXiv:1502.01589.
19. K. S. Stelle, *Gen. Relativ. Gravit.* **9**, 353 (1978).
20. P. Brax, P. Valageas and P. Vanhove, in preparation.
21. E. Adelberger, Probing quantum-gravity notions via classical tests of the Equivalence Principle and the Inverse-square Law, talk at the *13th Central European Seminar on Particle Physics and Quantum Field Theory*, November 30–December 1 2017, University of Vienna.
22. T. Banks, *Nucl. Phys. B* **309**, 493 (1988).
23. S. R. Beane, *Gen. Relativ. Gravit.* **29**, 945 (1997), arXiv:hep-ph/9702419.
24. J. Bergé, The inverse square law and Newtonian dynamics space explorer (ISLAND), in *Proc. 52nd Rencontres de Moriond*, 2017.
25. J. Bergé, The inverse square law and Newtonian dynamics space explorer (ISLAND), http://moriond.in2p3.fr/grav/2017/program.php.
26. M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory, Vol.* 1: *Introduction*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, UK, 1987), p. 469.
27. G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, De Sitter space and the swampland, arXiv:1806.08362 [hep-th].
28. P. Agrawal, G. Obied, P. J. Steinhardt and C. Vafa, *Phys. Lett. B* **784**, 271 (2018), doi:10.1016/j.physletb.2018.07.040, arXiv:1806.09718 [hep-th].
29. R. Kallosh, A. Linde, E. McDonough and M. Scalisi, 4d models of dS uplift in KKLT, arXiv:1809.09018 [hep-th].