Emmy's $\mathcal{M}$-Theory Recipes

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What ?

From non-perturbative string emerged $\mathcal{M}$-Theory.

What are its complete degree of freedom?

How ?

Quantum deformations by $\alpha'_M \propto (\ell_P)^2$ allowed by:

▷ the reparametrisation invariance (GR)

▷ supersymmetry algebra (SUGRA)

▷ unitarity of the theory (QGR)

▷ $\mathcal{U}$-duality symmetries ($\mathcal{M} \cdots$)

Why ?

$\mathcal{M}$-Theory is equivalent to a

local quantum field theory of gravitation.

Quantum Gravity

Classical General Relativity exists for every $D \geq 4$.
Topologically trivial in $D = 2$ and non-dynamical in $D = 3$.

- ▷ Defined perturbatively by expanding around a vacuum

$$g_{\mu\nu} := \eta_{\mu\nu} + \kappa_{(d)} h_{\mu\nu}$$

dimensionfull

$$S_{\text{Einstein}} := \frac{1}{\kappa^2_{(d)}} \int \mathrm{d}^D x \sqrt{-g} R$$

$$= -\tfrac{1}{2} \int \mathrm{d}^D x (\partial_\alpha h_{\mu\nu})^2 + \sum_{n=1}^{\infty} \int \mathrm{d}^D x (\partial h)^2 (\kappa_{(d)} h)^n$$

spin 2

vertices

Quantum General Relativity receives loop corrections:

- ▷ $\kappa^2_{(d)} = (\ell_P)^{D-2} = (\alpha'_M)^{(D-2)/2}$ $(\hbar = c = 1)$
- ▷ Infinite set of counter-terms (non-renormalizability) ['t Hooft, Veltman]

$$\Delta S_{D=4} = \infty \times \sum_{n \geq 0} \int \mathrm{d}^4 x \left(\kappa^2_{(d)} R\right)^n R^2$$

- ▷ Corrections do *not* violate unitarity nor introduce tachyons or ghosts
- ▷ Field redefinitions or symmetries can forbid *some* terms [Goroff, Sagnotti], [van de Ven]
- ▷ Ultra-violet behaviour is background independent

Eight Spoonfuls of SUGRA

Supergravity = GR + fermions with local grassmanian invariance.

▷ d.o.f. bosons = d.o.f. fermions

spin 3/2

$$S \sim \frac{1}{\kappa_{(d)}^2} \int \mathrm{d}^4x\, \sqrt{-g}\, \Big(R - \tfrac{i}{2} \bar{\psi}^{\alpha}{}_{\mu}\, (\Gamma^{\mu\nu\rho})_{\alpha\beta}\, D_\nu(\omega) \psi^{\beta}{}_{\rho} \Big)$$

$$\delta^{(0)} e_\mu{}^r = \bar{\epsilon} \Gamma^r \psi_\mu, \quad \delta^{(0)} \psi_\mu = D_\mu(\omega)\epsilon$$

[Deser, Zumino], [Ferrara et al]

▷ $\alpha = 1, \cdots, 4N$ with $N \leq 8$ otherwise exist massless spin higher than 2 and more than one particle with spin 2. [Nahm]

▷ A "better" Ultra-violet behaviour [Grisaru], [Deser et al.], [Gates et al.], [Kallosh]

$$\Delta S_{D=4} = 0 \times \int d^4x\, R^2 + 0 \times \int d^4x\, \kappa_{(d)}^2 R^3 + \int d^4x\, \kappa_{(d)}^4 R^4 + \cdots$$

Field Redefinition Supersymmetry Quantum correction

Boil Everything to D=11

Classical $N = 1$ $D = 11$ is [Cremmer, Julia, Scherk]

$$S = \frac{1}{\kappa^2_{(d)}} \int \mathrm{d}^{11}x e \left(R + \bar{\psi}_\mu \Gamma^{\mu\nu\rho} D_\nu \psi_\rho + G^2_{(4)} \right) + \frac{\lambda}{6\kappa^2_{(d)}} \int C_{(3)} \wedge G_{(4)} \wedge G_{(4)}$$

▷ Maximal: no supergravity theories in $D > 11$ [Nahm]

▷ Minimal: in its field content: $(e_\mu{}^r,\ \psi_\mu{}^\alpha,\ C_{(3)})$. [Nahm]

▷ Unique: no cosmological constant allowed [Bautier et al]

▷ Non-trivial: has exact maximally supersymmetric vacua: $AdS_{4|7} \times S^{7|4}$

[Kallosh, Rajaraman]

▷ Finite ? $\kappa^2_{(11)} = (\ell_P)^9$ does not pair with powers of Riemann tensor R^n

[Naive Argument]

Incorporate Higher-order Derivative

Finite higher-derivative corrections are necessary:

▷ Anomaly considerations [Duff, Liu, Minasian], [Vafa, Witten], [Green, Schwarz]

$$S = (\alpha'_M)^3 \int C_{(3)} \wedge t_8 R^4 \,.$$

versus

▷ Strong coupling limit of string theory amplitudes gives finite R^4, $\Box^2 R^4$, $\ldots$

[Green, Kwon, Vanhove], [Green, Gutperle, Vanhove], [Green, Vanhove]

This is an effective theory of a microscopic quantum theory of gravitation.

Add some Brane

Branes are solitonic extended solutions of supergravity theories

$$S = -\int \mathrm{d}^{p+1}\xi\,\sqrt{-\det(g + \mathrm{d}A - B)} + \int C \wedge e^{\mathrm{d}A - B}$$

background fields

Consistency of the world-sheet theory constraints the background fields

- ▷ higher-loop σ-model β-functions [Grisaru et al]

$$S = \frac{1}{\alpha'} \int \mathrm{d}^2\sigma\, \partial_i X^\mu \partial^i X^\nu G_{\mu\nu}(X)$$

- ▷ κ-symmetry invariance

$$\delta_\kappa(X^\mu(\xi), \theta^\alpha(\xi)) = \kappa^a E_a{}^M\,, \quad \delta_\kappa A = \mathrm{i}_\kappa B\,, \quad \kappa^\alpha(\xi) = P_+\,\zeta$$

  only if the backgrounds $E_M{}^A$, B_{MN} and $C_{M_1 \ldots M_k}$ satisfy the on-shell constraints that imply their equation-of-motions

Quantum corrections to the effective brane world-volume theory $\Longleftrightarrow$ corrections to $\mathcal{M}$-Theory.

Serve with Strings

Noether's deformations of the $(D=11)$ supersymmetry algebra

$$\left(\delta_\epsilon^{(0)} + \sum_{n\geq 3} a_n \,(\alpha'_M)^n \delta_\epsilon^{(n)}\right)\left(S^{(0)} + \sum_{n\geq 3} a_n \,(\alpha'_M)^n S^{(n)}\right) = 0$$

field dependent parameters

The super-algebra closes on-shell:

$$\begin{aligned}[\delta_{1,\text{sg}}, \delta_{2,\text{sg}}] = \delta^{\text{translation}}(\xi^\nu) \\ + \delta^{\text{susy}}(-\xi^\nu \psi_\nu) \\ + \delta^{\text{gauge}}(-\xi^\sigma C_{\sigma\nu\rho} - 2\bar\epsilon_2 \Gamma_{\nu\rho}\epsilon_1) \\ + \delta^{\text{Lorentz}}(\xi^\nu \hat\omega_\nu{}^r{}_t + 4\,\bar\epsilon_2 S^{r\;\nu\rho\sigma\kappa}{}_t \epsilon_1 \hat F_{\nu\rho\sigma\kappa})\,,\end{aligned}$$

ξ^μ receives quantum corrections

$$\xi^\mu = \bar\epsilon_1 \Gamma^\mu \epsilon_2 + a_3 \times 0 \times (\alpha'_M)^3 \, t_8^{\mu t s_1 \cdots s_6} R_{[r_1 r_2}{}^{s_1 s_2} R_{r_3 r_4}{}^{s_3 s_4} R_{r_5]t}{}^{s_5 s_6}$$

[Peeters, Vanhove, Westerberg]

Improve with $\mathcal{U}-$duality toppings

Is the procedure unique ? Field theory reasonning are not enough!

▷ a_n not fixed by supersymmetry alone

All extended supergravity theories in $D \leq 10$ presents non-linear symmetries

$\mathcal{U}-$duality groups

	Classical	Quantum
D=10 (chiral)	$U(1)\backslash Sl(2,\mathbb{R})$	$Sl(2,\mathbb{Z})$
D=8	$K\backslash Sl(3,\mathbb{R}) \times Sl(2,\mathbb{R})$	$Sl(3,\mathbb{Z}) \times Sl(2,\mathbb{Z})$

▷ Invariance under the quantum symmetry fixes the a_n by constraining the

- ▷ Supersymmetry algebra [Green, Sethi]
- ▷ S-matrix elements [Green, Kwon, Vanhove], [Green, Gutperle, Vanhove], [Green, Vanhove]

More recipes:

how to accomodate the Membrane with these higher-derivative corrections?