Localization of (super)gravity

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Based on

- Discovery of the model
  - Dvali, Gabadadze, Porrati, hep-th/0005106
- Cosmological implication: late-time self-expanding universe
  - Deffayet, Dvali and Gabadadze, astro-ph/0105068
  - Lue and Starkman, astro-ph/0212083
- Field theory analysis: ghost, strong coupling scales
  - Dubovsky and Rubakov, hep-th/0212222
  - Rubakov, hep-th/0303125
  - Kolanovic, Porrati and Rombouts, hep-th/0304148
  - T. Tanaka, hep-th/0305031
  - Luty, Porrati and Rattazzi, hep-th/0303116
  - Gabadadze and Shifman, hep-th/0312289
  - Porrati and Rombourts, hep-th/0401211
- String theory realisation
  - Antoniadis, Minasian, Vanhove hep-th/0209030
  - Kohlprath, hep-th/0311251

Infra-Red modification of Gravitation law

what?

Find a consitent general covariant scheme for large cosmological distances (Infra-Red) modification of Gravitationnal law.

$$M_{pl}^2 \left(1 + \mathcal{F}(L_{IR}^2 \Box)\right) \left(\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\, \mathcal{R}_{(4)}\right) = T_{\mu\nu}$$

$$1\ \mathrm{mm} \quad \underbrace{\ll r \ll}_{\text{4d Einstein Gravity}} \quad R_c \simeq H_o^{-1} \sim \mathrm{Gpc}$$

why?

The cosmological constant/Dark Energy problem:

- ▷ explaining the observed acceleration of the universe without introducing extra (dark) matter.
- ▷ An interesting laboratory for Quantum Gravity Effects

how?

Effective action: non-compact bulk and localized 4-dim EH term

[Dvali, Gabadadze, Porrati'00]

$$M_*^{2+N} \int d^{4+N}y\, \sqrt{G}\, \mathcal{R}_{(4+N)} + M_{pl}^2 \int_{\mathcal{M}_4} d^4x\, \sqrt{g} \mathcal{R}_{(4)}$$

dominate in IR

dominate in UV

- ▷ quasi-localisation of the graviton. Radiation into the continuum of KK-states weakens the Newton force
- ▷ Large distance modification of 4d Newton law:

$$V(r) \propto \int_0^\infty dm \left| \frac{1}{1 + m^2 R_c^2} \right|^2 \frac{e^{-mr}}{r}$$

Non-compact extra dimensions

$$M_*^{2+N}\int d^4x d^N y\,\sqrt{G}\,\mathcal{R}_{(4+N)} + M_{pl}^2\int_{\mathcal{M}_4} d^4x d^N y\, f_w(y)\sqrt{g}(\alpha\,\mathcal{R}_{(4+N)} + \beta\,\mathcal{R}_{(4)})$$

function with width w

[Dvali, Gabadadze, Porrati'00] model: $\alpha = 0$, $\beta \neq 0$ $N = 1$ and $f^N(y) = \delta^{(N)}(y)$.

- Planck Masses: bulk $M_* \gg 1$TeV and localized $M_{pl}^2 \sim 10^{19}$GeV
- localized matter $\mathcal{S}_{\rm matter}$ for matter interaction, and a cosmological constant (Remark: this model does not solve the CC problem)

$$T_{AB} = \begin{pmatrix} T_{\mu\nu}\,\delta^{(N)}(y) & 0 \\ 0 & 0 \end{pmatrix}$$

- 4d induced metric g:

$$g_{\mu\nu} = \partial_\mu X^M \partial_\nu X^N\, G_{MN}(x\,;y=0) \qquad \mu,\nu = 0,\cdots,3$$

- Gauss-Codazzi equations: $\mathcal{R}_{(4+N)} = \mathcal{R}_{(4)} + \Omega^2$ with $\Omega^2 = -(\partial \ln h)^2$, with $h = \mathrm{Tr}(G)$ the transverse "volume".
- Choice of gauge: $G_{MN} = \eta_{MN} + \kappa H_{MN}$

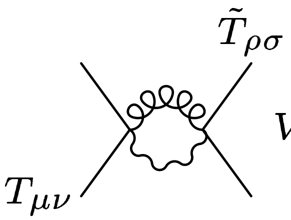
$$\begin{aligned} \partial^M H_{MN} &= \tfrac{1}{2}\,\partial_N H^M{}_M && \text{Harmonic Gauge} \\ H_{\mu\perp} &= 0 && \text{Rigid brane hypothesis} \end{aligned}$$

In the harmonic gauge the Einstein tensor simplies to

$$G^D_{MN} = \partial_D^2 H_{MN} - \frac{1}{2}\eta_{MN}\partial_D^2 H^M{}_M$$

Behaviour of the effective potential

$$\delta^{(N)}(y) M_{pl}^2 \left(\alpha\, G^{(4)}_{\mu\nu} + \beta G^{(4+N)}_{\mu\nu}\right) \delta^{\mu\nu}_{MN} + M_*^{2+N} G^{(4+N)}_{MN} = -T_{\mu\nu}\, \delta^{\mu\nu}_{MN}\, \delta^{(N)}(y)$$



$$V(p,y) = f_1(p,y) \left\{ \overbrace{\left[T_{\mu\nu}\tilde{T}^{\mu\nu} - \tfrac{1}{2}(T^\alpha{}_\alpha)^2\right]}^{\text{spin 2}} - \frac{1}{2} f_2(p,y)\, \overbrace{(T^\alpha{}_\alpha)^2}^{\text{spin 0}} \right\}$$

Propagation of bulk modes:

UV cut-off

$$(-p^2 - \Delta_y - i\epsilon) D_\omega(p,y) = \delta^{(N)}(y) \qquad D_\omega(p,q) = \int d^N q\, \frac{f_w(q)}{p^2 + q^2}$$

$$f_1(p,y) = \frac{1}{M_{pl}^2} \frac{D_\omega(p,y)}{p^2\, D_\omega(p,0) - r_c^{-N}}, \qquad f_2(p,y) = 1 - \frac{p^2 D_\omega(p,0) - K_1\, r_c^{-N}}{p^2 D_\omega(p,0) - K_2\, r_c^{-N}}$$

$f_w(y) = \delta^{(N)}(y)$ then $D(p)$ UV divergent and the potential is always always 4d.

The width is a UV cutoff

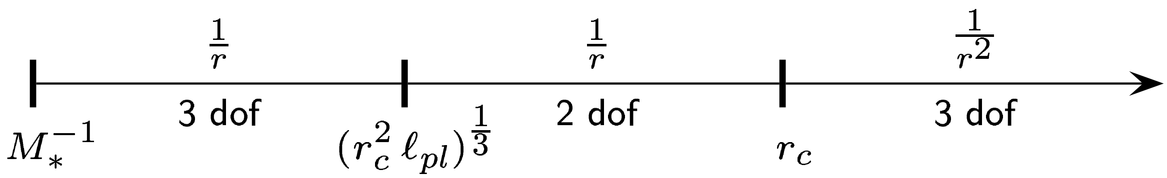
$$R_c = w \left(\frac{r_c}{w}\right)^{\frac{N}{2}}, \qquad r_c^N = \frac{M_{pl}^2}{M_*^{2+N}}$$

- ▷ $r \ll R_c$ $V(r) \sim 1/r$ 4d Einstein Gravity
- ▷ $r \gg R_c$ $V(r) \sim 1/r^{1+N}$ modified gravity: the bulk graviton dominates.

Is gravity Einstein at intermediate scales?

4d Einstein gravity is tricky to recover: extra (physical or spurios) states can propate.

- For $N = 1$ beyong the linearized approximation arises a strong coupling scale: The longitudinal modes $H_{\mu\perp} = \partial_\mu \pi_\perp$ are getting massive because of brane bending effects (breaking of the y-reparametrisation). The model is strongly coupled at non-linear level at a scale fixed by the inverse mass of π:



[Rubakov'03] [Luty,Porrati,Rattazzi'03] [Deffayet,Dvali,Gabadadze,Vainshtein'01]

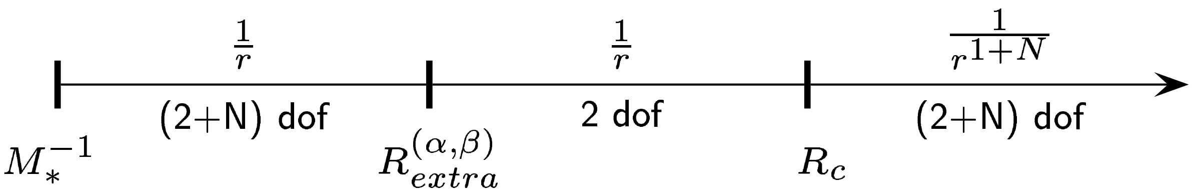
- For $N > 1$ the strong coupling problem depends on the high energy behaviour of the propagator as can be seen from the dependence of the IR scale on the UV cutoff.

[Dubovsky, Rubakov'02] [Kolanovic,Porrati,Rombouts'03]

The resolution is to remark that since the theory non-local in 4d one should construct a unitary but not analytic Green function. Their $i\epsilon$-prescription gives the Green function [Gabadadze, Shifman'03]

$$\mathcal{G}_N = \frac{D_\omega(p,y)}{p^2\,D_\omega(p,0) - K(\alpha,\beta,N)\,r_c^{-N}} + i\pi D_\omega(p,y)\delta(p^2 D_\omega(p,0) - K(\alpha,\beta,N) r_c^{-N})$$

All the tachyonic ghosts states of [Dubovsky, Rubakov'02] , [Rubakov'03] are not asymptotic states of the theory (not on the physical sheet) [Porrati, Rombouts'03] [Gabadadze, Shifman'03]



Depending on the values of α and β extra physical polarisations can propagate:

$$R_{extra}^{(0,\beta)} \ll M_*^{-1}, \qquad R_{extra}^{(\alpha,0)} = R_c$$

Everything depends on the UV cut- off ω: need of a consistent UV completion: String theory

Planck Mass Renormalisation

Localized term = Renormalisation of Newton's constant from loops of matter fields coupled to the external classical metric

- ▷ Field theory mechanism [Adler'83] , [Pauli'73] , [Sakharov'67]
- ▷ String theory mechanism: localization by the twisted fields [Antoniadis,Minasian,Vanhove'02]

$$M_*^8 \int d^{10}x \sqrt{G}\, \mathcal{R}_{(10)} + M_{pl}^2 \int_{\mathcal{M}_4} d^4x \int d^6y f_w(y) \sqrt{g} (\alpha\, \mathcal{R}_{(10)} + \beta\, \mathcal{R}_{(4)})$$

We derive all the parameters by considering string theory on non-compact background

Localization arises from Planck mass renormalisation for models with $\mathcal{N}_{4d} \leq 2$

$$\int d^4x \left(e^{-2\phi^4} + \delta\right) \mathcal{R}_{(4)}$$

- $\delta_h = 0$ for heterotic string on $K3 \times T^2$

  [Antoniadis, Gava, Narain'92]
- δ_I depends on the moduli of T^2 for type I on $K3 \times T^2$.

  [Antoniadis, Bachas, Fabre, Partouche, Taylor'97]

  Decompactification to 6d $\delta_I = 0$ for type I on $K3$ [local tadpoles cancellation]

  [Antoniadis,Minasian,Vanhove'02]
- $\delta_{II} = \chi$ moduli independent type IIA/B on CY$_3$

  [Antoniadis, Ferrara, Minasian,Narain'97]
- $\delta_M = \chi$ M-theory on CY$_3$ but no interesting phenomenology

  [Antoniadis,Minasian,Vanhove'02]

Localization occurs only for type II on CY$_3$

Working setup: Compact CY$_3^{(n_v,n_h)} \equiv \mathbb{C}^3/\mathbb{Z}_N$ with N-large

[Kohlprath'03] , [Antoniadis,Minasian,Vanhove'02]

Type II on compact CY_3

$\mathcal{N}=2$ supergravity imposes factorisation of the hyper and vector multiplet manifold in the supergravity variables $\mathcal{M}_H \otimes \mathcal{M}_V$. $\mathcal{N}=2$ 4d local supersymmetry imposes the sigma-model for N hypermulitplet is a Quaternionic-Kähler manifold with holonomy $Sp(n)\cdot Sp(1)$.

The universal sectors $\mathcal{N}=2$ type IIa on CY_3 is composed by the graviton multiplet, and two universal multiplets: hypermultiplet (containing the dilation) and vector multiplet (the volume of the CY).

Under SO(1,9) $\to$ SO(1,3) $\times$ SO(6) the vertex operators are

$$
\begin{aligned}
V_{NS}^{(-1,-1)} &= \zeta_{\mu\nu} : \psi^{\mu}\tilde{\psi}^{\nu} : e^{-(\varphi+\bar{\varphi})}\, \mathbb{I}\, e^{ik\cdot X} \\
V_{F}^{(-1/2,-1/2)} &= F_{\mu} : S^{\alpha}(\sigma_{\mu})_{\alpha\dot{\beta}}\tilde{S}^{\dot{\beta}} : e^{-(\varphi+\bar{\varphi})/2}\, \Sigma\bar{\Sigma}\, e^{ik\cdot X}
\end{aligned}
$$

The loop corrections to the hypermultiplet metric are counted by the *dilaton* and the loop corrections to the vector multiplet metric by the *volume*. [Berkovits,Siegel'95]

$$
\int d^4x\sqrt{g^E}\,\mathcal{R}_{(4)} + \underbrace{f(\tilde{\phi}_4)\,G_{hh}(\partial h)^2}_{\text{hypers}} + \underbrace{g(\tilde{v}_6)\,G_{vv}(\partial v)^2}_{\text{vectors}}
$$

Non universal direction where analyzed by [Antoniadis, Ferrara, Minasian, Narain'97]

Factorization imposes a mixing between the physical dilaton ϕ_4 and volume v_6

$$
\begin{aligned}
e^{-2\tilde{\phi}_4} &= e^{-2\phi_4}\left(1+\mu_T\,\chi\,\frac{2\zeta(3)}{v_6}+\cdots\right) \\
\tilde{v}_6 &= v_6\left(1+\mu_1\,\chi\,4\zeta(2)\,e^{2\phi_4}+\cdots\right)
\end{aligned}
$$

For type IIa:

- Tree-level corrections (sigma-model β-function) affects the vectormultiplet metric
- 1-loop corrections **affects** the hypermultiplet metric: the 1-loop corrected metric is not Kähler anymore [Antoniadis, Minasian, Theisen, Vanhove'03]

Type II on compact CY$_3$ (amplitude)

The width is derived from the 1-loop amplitude between 2 graviton and 1 Kaluza-Klein of the graviton for $T^6/\mathbb{Z}_N$ model

$$\left\langle (V_g)^2 V_{KK} \right\rangle = \mathcal{R} \frac{1}{N^2} \underbrace{\sum_{f,k} e^{i\gamma^k\, q\cdot x_f}}_{\boxed{\text{localized}}} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \int \prod_{1\leq i\leq 3} \frac{d^2 z_i}{\tau_2} \underbrace{\sum_{(h,g)}{}' e^{\alpha' q^2 F_{(h,g)}(\tau, z_i)}}_{\boxed{\text{width}}}$$

- ▷ The amplitude is **localized** at the position of the fixed point. We can decompactify and focus on the origin (always a fixed point).
- ▷ A saddle-point analysis gives the width to be the 4d Planck length (for $N \to \infty$)

$$w \simeq \ell_{pl} = \ell_s \left[\chi \left(\frac{2\zeta(3)}{g_s^2} + 4\zeta(2) \right) \right]^{-1/2} .$$

- ▷ General structure of the localized action

$$\begin{aligned}
&\int d^4x \sqrt{g^\sigma} \Big\{ (e^{-2\phi_{10}}(v_6 + a_T\chi) + a_1\chi)\, \mathcal{R}_{(4)} \\
&+ \int d^4x \sqrt{g^\sigma} [e^{-2\phi_{10}}(v_6 - a_T\chi) + a_1\chi]\, (d\ln v_6)^2 \\
&+ \int d^4x \sqrt{g^\sigma} [e^{-2\phi_{10}}(v_6 + a_T\chi) - a_1\chi]\, (d\phi_{10})^2 \\
&- \int d^4x \sqrt{g^\sigma} [e^{-2\phi_{10}}(v_6 + b_T\chi) + b_1\chi]\, d\ln v_6\, d\phi_{10} \Big\}
\end{aligned}$$

Decompactification limit $v_6 \to \infty$ is safe and we get the general action

$$M_*^8 \int d^{10}x \sqrt{G}\, \mathcal{R}_{(10)} + M_{pl}^2 \int_{\mathcal{M}_4} d^4x\, f_w(y) \sqrt{g} (\alpha\, \mathcal{R}_{(10)} + \beta\, \mathcal{R}_{(4)})$$

[Antoniadis, Gabadadze, Vanhove work in progress]

Localization from R^4 terms: $\mathcal{R}_{(4)}$ from R^4

In the string frame the corrections are

$$\frac{1}{l_s^8} \qquad \int_{M_{10}} \frac{1}{g_s^2}\mathcal{R}_{(10)} + \frac{1}{l_s^2}\int_{M_{10}} \left(\frac{2\zeta(3)}{g_s^2} + 4\zeta(2)\right) t_8 t_8 R^4$$
$$- \quad \frac{1}{l_s^2}\int_{M_{10}} \left(\frac{2\zeta(3)}{g_s^2} \mp 4\zeta(2)\right) \underbrace{R\wedge R\wedge R}_{\text{Euler characteristic}} \wedge R\wedge e^2 + \cdots$$

$$\frac{1}{l_s^8}\int_{M_4\times M_6} \frac{1}{g_s^2}\mathcal{R}_{(10)} + \frac{\chi}{l_s^2}\int_{M_4} \left(-\frac{2\zeta(3)}{g_s^2} \pm 4\zeta(2)\right) \mathcal{R}_{(4)}\,,$$

these corrections can be seen as descending from the $\alpha'^3\, R^4$ correction in 10d.

Field redefinition ambiguities

$$R\wedge R\wedge R\wedge R\wedge e^2 = (\mathrm{Weyl})^4 + (R^3)^{MN} R_{MN} + (\mathcal{R}_{(10)} + \mathcal{R}_{(4)})\,(R^3) + \mathrm{Ricci}^2$$

All the Ricci terms are off-shell informations in 10d. The supersymmetry argument and string analysis in 4d shows that the above off-shell action is the correct one.

Parameters from String Theory

- ▷ UV completion of co-dimension ≥ 1 DGP model $\equiv$ string on singular background
- ▷ Parameters expressed in terms of string variables.
  - Bulk Planck mass fixed by the string scale
  - 4d Planck mass

$$M_{pl}^2 = \frac{1}{\ell_{pl}^2} = \frac{\chi}{\ell_s^2}\,[e^{-2\phi_4} + 1] \sim 10^{19}\,\mathrm{GeV}$$

  - cut-off

$$\omega = \frac{1}{\ell_{pl}}$$

  - Critical radius

$$R_c = g_s\,\frac{\ell_s^4}{\ell_P^3} = g_s\,10^{32}\,\mathrm{cm}$$

confer Cosmological Friedman's equation

$$H^2 - H/R_c = \frac{8\pi}{3}\,G_N\,\rho$$

late-time cosmology $\rho = 0$ ask for $R_c \sim H_o^{-1} \sim 10^{28}\,\mathrm{cm}$ [Deffayet,Dvali,Gabadadze'01] and lost of causality [Arkani-Hamed et al.'02]

- ▷ $g_s \sim 10^{-4}$ weak coupling regime
- ▷ $\chi \sim 10^{24}$ large number twisted fields (hidden matter)

Open problems

- Large number of twisted fields: strong coupling in loops [Kohlprath, Vanhove work in progress]
- What is the meaning of the width? Does it carry any physical informations (localisation of the zero modes) [Bachas, Minasian, Vanhove work in progress]
- How can we supersymmetrize this system? (idea: bulk+ brane = supersymmetric, but each are not)