Quantum Corrections to Superspace Constraints

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what?

Find superspace manifestations of higher-derivative corrections to low-energy field theories arising from superstring theory, e.g. $((\alpha'_M)^3 = 4\pi(l_P)^6)$

$$\int \mathrm{d}^{11}x\, e\, \left[R + (\alpha'_M)^3 R^4 + \cdots\right]$$

or

$$\int \mathrm{d}^{p+1}\xi \sqrt{-\det(g+\mathcal{F})}\left[1 + (\alpha')^2 R^2 + \cdots\right]$$

why?

Understand symmetry constraints and

quantum consistency:
$\mathcal{M}$-theory effective action,
Supersymmetric vacua,
$\mathcal{M}$/D-brane quantum physics

how?

classical supersymmetry $\rightarrow$ classical superspace
stringy supersymmetry $\rightarrow$ stringy superspace

Quantum corrections to $\mathcal{M}$-theory

The existence for higher-derivative terms has been motivated by:

- ▷ Anomaly considerations (heterotic, IIA)

  [Vafa, Witten], [Duff, Liu, Minasian]

$$(\alpha'_M)^3 \times \int C_{(3)} \wedge t_8 R^4$$

- ▷ Universal from four-point S-matrix analysis

  [Deser, Seminara], [Bern et al.], [Green et al.]

$$(\alpha'_M)^3 \times \int t_8 t_8 R^4$$

- ▷ Corrections to $d = 4$ hypermultiplet geometry

  [Antoniadis et al.], [Strominger]

- ▷ Supersymmetry considerations

  [Berkovits], [Green, Sethi], [de Roo et al.], [Bergshoeff, de Roo]

$$\int d^{10}x \; \underbrace{R^4 + B \wedge R^4 + \psi^2 (R^3 + R^2 DR)}_{I_X} + \ldots$$

- ▷ Consistency of extended object puts constraints on the background fields: σ-model β-function approach, κ-symmetry, superembedding.

  [Grisaru et al.], [Witten], [Sorokin et al.] ,[Bergshoeff et al.]

Supersymmetric Branes are Supersymmetric

▷ D-brane actions receive corrections:

$$-\int d^{p+1}\xi\sqrt{-\det(g+\mathcal{F})}\left[1+(\alpha')^2R^2+\cdots\right]+\int C\wedge e^{\mathcal{F}}\wedge\sqrt{\frac{\hat{A}(\alpha' R_{\mathrm{T}})}{\hat{A}(\alpha' R_{\mathrm{N}})}}$$

require quantum kappa symmetry

[Green et al.], [Bachas et al.], [Cheung, Yin]

▷ Supersymmetry for $\mathcal{M}$/D-brane force the specific backgrounds $E_M{}^A$, B_{MN}, C_{MN}... to satisfy their supergravity equation of motion.

Supersymmetric objects $\Longleftrightarrow$ supergravity effective action

kappa symmetry

$$\delta_\kappa Z^M = \kappa^A E_A{}^M, \quad \delta_\kappa A = i_\kappa B^{(NS)}, \quad \kappa^a = 0$$

$$Z^M(\xi) := (X^\mu(\xi), \theta^\alpha(\xi)), \quad \kappa^\alpha(\xi) = P_+\zeta$$

$$\Updownarrow$$

$$T^r{}_{ab} = 2\,(C\Gamma^r)_{ab}\,, \quad T^r{}_{as} = 0, \ldots$$

[Bergshoeff et al.], [Witten], [Grisaru et al.]

Supersymmetric Vacua of $\mathcal{M}$-theory

Supersymmetry transformations get modified

$$\left(\delta_\epsilon^{(0)} + (\alpha'_M)^3 \delta_\epsilon^{(3)}\right)\left(S^{(0)} + (\alpha'_M)^3 S^{(3)}\right) = 0$$

▷ Existence of Killing spinors $\Longleftrightarrow$ supersymmetric vacuum

$$\delta_\epsilon \psi_\mu = \left[D_\mu(\omega) + \mathcal{T}_\mu \cdot G_{(4)} + (\alpha'_M)^3\,(\Gamma_{[n]} D R^3)_\mu + \cdots\right]\epsilon = 0$$

The higher-derivative terms are affected by field redefinitions on ψ_μ and ϵ.

▷ Maximally supersymmetric classical solutions $AdS_{4|7} \times S^{7|4}$ are not corrected by higher-order corrections.

[Kallosh, Rajaraman]

▷ Compactifications of $\mathcal{M}$-theory to $d = 3$ and $d = 4$ have been shown to be compatible with classical supersymmetry and the corrected Bianchi identity for the $\star G_{(4)}$

$$d \star G_{(4)} = G_{(4)} \wedge G_{(4)} + (\alpha'_M)^3 t_8 R^4$$

Quantization requires no zero $G_{(4)}$ fluxes

$$-\int G_{(4)} \wedge G_{(4)} = \int t_8 R^4 = \frac{1}{24}\chi$$

[Becker, Becker]

Supersymmetry Algebra

▷ What about the on-shell super-algebra?

$$\left(\delta_\epsilon^{(0)} + (\alpha'_M)^3 \delta_\epsilon^{(3)}\right)\left(S^{(0)} + (\alpha'_M)^3 S^{(3)}\right) = 0$$

$$[\bar\epsilon_1 Q, \bar\epsilon_2 Q] e_\mu{}^r = (\bar\epsilon_1 \Gamma^\nu \epsilon_2)\partial_\nu e_\mu{}^r + \cdots$$

field dependent parameters

In eleven dimensions:

$$\begin{aligned}[\delta_{1,\mathrm{sg}}, \delta_{2,\mathrm{sg}}] = {} & \delta^{\text{translation}}(\xi^\nu) \\ & + \delta^{\text{susy}}(-\xi^\nu \psi_\nu) \\ & + \delta^{\text{gauge}}(-\xi^\sigma C_{\sigma\nu\rho} - 2\bar\epsilon_2 \Gamma_{\nu\rho}\epsilon_1) \\ & + \delta^{\text{Lorentz}}(\xi^\nu \hat\omega_\nu{}^r{}_t + 4\,\bar\epsilon_2 S^{r\ \nu\rho\sigma\kappa}{}_t \epsilon_1 \hat F_{\nu\rho\sigma\kappa})\,,\end{aligned}$$

ξ^μ receives higher-order derivative corrections

$$\xi^\mu = \bar\epsilon_1 \Gamma^\mu \epsilon_2 + (\alpha'_M)^3 X(W, \psi, G)$$

Superspace Geometry

- Superspace is the group manifold of the super-Poincaré group divided by the Lorentz group.
  Supersymmetry reflected in algebra generated by covariant derivatives:

$$\{D_a, D_b\} = T^r{}_{ab} D_r$$

fermionic translation

bosonic translation

- Supertorsions and supercurvatures are restricted (just as bosonic ones) by Bianchi identities

$$D_{[A} T_{BC)}{}^D - T_{[AB|}{}^E T_{E|C)}{}^D - \tfrac{1}{2} R_{[ABC)}{}^D = 0$$

$$\tfrac{1}{24} D_{[A_1} G_{A_2 \cdots A_5)} - \tfrac{1}{12} T_{[A_1 A_2|}{}^B G_{B|A_3 A_4 A_5)} = 0$$

- Relaxed constraints : consider SO(10,1) representations:

$$\underbrace{T^r{}_{ab}}_{\mathbf{5808}} = \underbrace{2(C\Gamma^r)_{ab}}_{\mathbf{1}} + \underbrace{(C\Gamma^{r_1 r_2})_{ab} X^r{}_{r_1 r_2}}_{\mathbf{429}+165+11} + \underbrace{(C\Gamma^{r_1 \cdots r_5})_{ab} X^r{}_{r_1 \cdots r_5}}_{\mathbf{4290}+462+330}$$

[Cederwall,Gran,Nielsen,Nilsson]

- In $d = 11$ Howe showed that because of field redefinition freedom, the standard dimension zero constraint

$$T^r{}_{ab} = 2(C\Gamma^r)_{ab}$$

  inevitably leads to classical supergravity, without higher derivative terms.

Field Content

The eight-derivative invariant in $d = 11$

$$I_X = \underbrace{R^4}_{\text{4-point}} - \underbrace{C_{(3)} \wedge R^4 + \varepsilon_{11}\varepsilon_{11}R^4}_{\text{5-point}} + (G_{(4)})^2 R^3 + \cdots$$

We restrict ourselves to an invariant independent of $G_{(4)}$ so

- ${X^r}_{r_1 r_2} = 0$ because requires $G_{(4)} \neq 0$
- ${X^r}_{r_1 \cdots r_5} = (\mathsf{Weyl})^3 + (G_{(4)})^2 W^2 + G_{(4)} W \psi^2 + \psi^4 + \cdots$

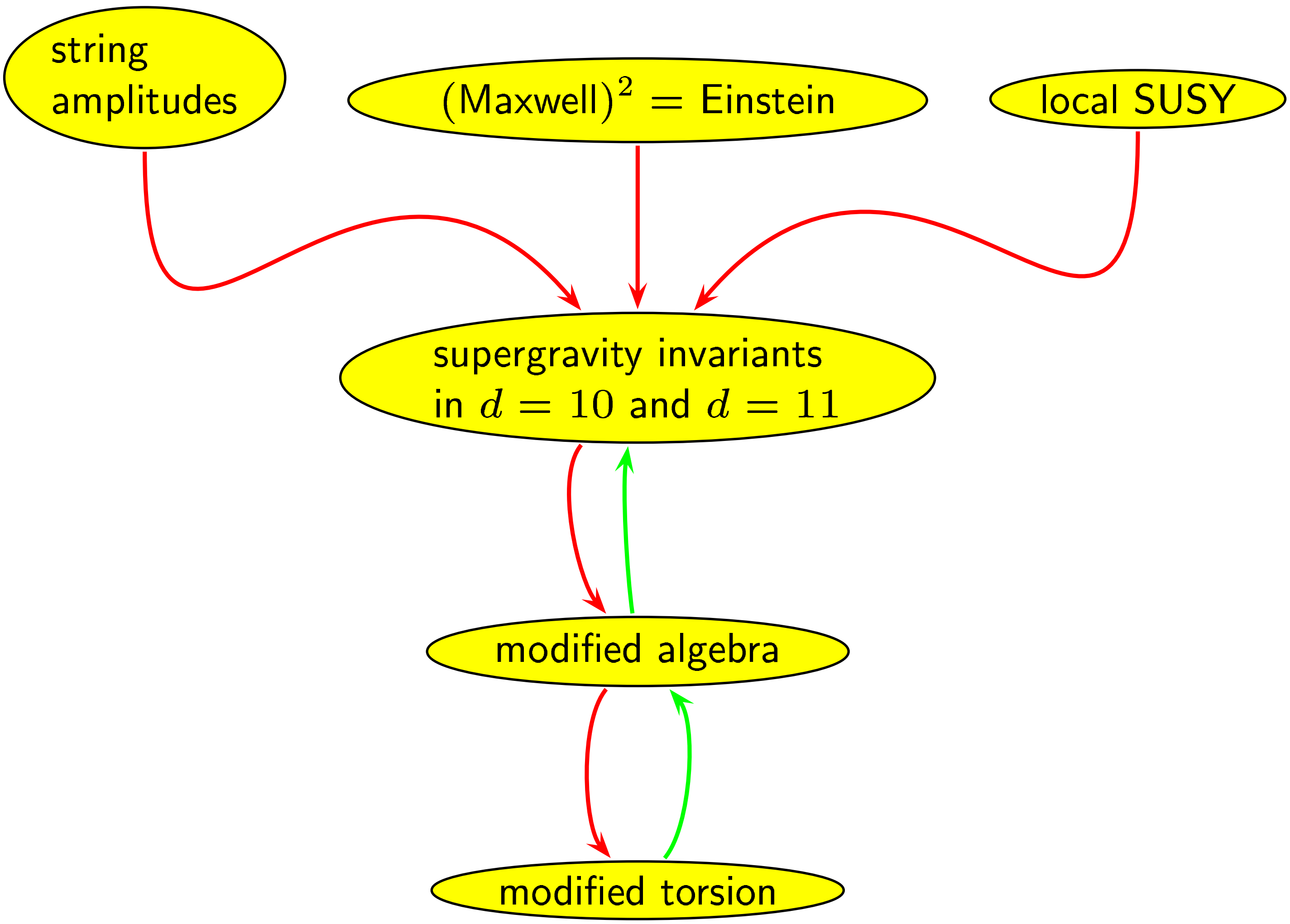
Group theory analysis gives a candidate

$${X^r}_{r_1 \cdots r_5} = a \times t_8^{rts_1 \cdots s_6} {W_{[r_1 r_2}}^{s_1 s_2} {W_{r_3 r_4}}^{s_3 s_4} {W_{r_5]t}}^{s_5 s_6}$$

contains $2 \times \mathbf{4290}$.

[Cederwall et al.]

The Procedure



The Super-Maxwell Invariant

Computing string amplitude to get the tensorial structures, fixing the relative normalisations with supersymmetry

[Metsaev, Rakhmanov], [Bergshoeff et al.]

$$
\begin{aligned}
S_{F^4} = \frac{(\alpha')^2}{32} \int \mathrm{d}^{10}x \Big[& \tfrac{1}{6} e\, t_8^{(r)} F_{r_1 r_2} \cdots F_{r_7 r_8} \\
& + \tfrac{1}{3\sqrt{2}} \varepsilon_{10}^{(r)} B_{r_1 r_2} F_{r_3 r_4} \cdots F_{r_9 r_{10}} \\
& - \tfrac{32}{5} e\, t_8^{(r)} \eta_{r_2 r_3} (\bar{\chi} \Gamma_{r_1} \chi_{r_4}) F_{r_5 r_6} F_{r_7 r_8} \\
& + \tfrac{12 \cdot 32}{5} e (\bar{\chi} \Gamma_{r_1} \chi_{r_2}) F^{r_1 m} F_m{}^{r_2} \\
& - \tfrac{16}{5!} \varepsilon_{10}^{(r)} (\bar{\chi} \Gamma_{r_1 \cdots r_5} \chi_{r_6}) F_{r_7 r_8} F_{r_9 r_{10}}
\end{aligned}
$$

four-point functions

five-point functions

$$
\begin{aligned}
& + \tfrac{16}{3} e\, t_8^{(r)} (\bar{\psi}_{r_1} \Gamma_{r_2} \chi) F_{r_3 r_4} F_{r_5 r_6} F_{r_7 r_8} \\
& + \tfrac{8}{3} e\, (\bar{\psi}_m \Gamma^{m r_1 \cdots r_6} \chi) F_{r_1 r_2} \cdots F_{r_5 r_6} \Big]
\end{aligned}
$$

$(\textbf{Maxwell})^2 = \textbf{Einstein}$

Super-Maxwell and supergravity shares similarities at the level of the supersymmetry algebra

[Bergshoeff, de Roo], [Gates et al.]

$$\delta\chi^a = \tfrac{1}{8}\Gamma^{\mu\nu}\epsilon\, F^a{}_{\mu\nu}\,,$$

$$\delta F^a{}_{\mu\nu} = -\,8D_{[\mu}(\epsilon\Gamma_{\nu]}\chi^a)\,.$$

compare

$$\delta\psi^{rs} = \tfrac{1}{8}\Gamma^{\mu\nu}\epsilon\, R_{\mu\nu}{}^{rs} + \cdots\,,$$

$$\delta R_{\mu\nu}{}^{rs} = -8D_{[\mu}\left(\bar\epsilon\Gamma_{\nu]}\psi^{rs}\right) + 4D_{[\mu}\left(\bar\epsilon\Gamma_{\nu]}\psi^{rs} + 2\bar\epsilon\Gamma^{[r}\psi^{s]}{}_{\nu]}\right) + \cdots\,.$$

So just replace

what to do with

$$F^a{}_{r_1 r_2} \to R_{r_1 r_2}{}^{s_1 s_2}\,,$$

$$\chi^a \to \psi^{s_1 s_2}\,,$$

$$D_r\chi^a \to D_r\psi^{s_1 s_2}\,,$$

String theory input

Compare type II to heterotic amplitudes

$$\text{gauge field}: \quad V_A^{(0)}(k) \sim \int \mathrm{d}^2 z\, A_\mu : (i\partial X^\mu + k\cdot\Psi\,\Psi^\mu)\, e^{ik\cdot X} :$$

$$\text{graviton}: \quad V_g^{(0,0)}(k) \sim \int \mathrm{d}^2 z\, g_{\mu\nu} : (i\partial X^\mu + k\cdot\Psi\,\Psi^\mu)$$

$$\times\, (i\bar\partial X^\nu + k\cdot\tilde\Psi\,\tilde\Psi^\nu)\, e^{ik\cdot X} :$$

The gravitino is seen as a Lorentz gaugino and the Riemann tensor as a Lorentz curvature.

The trace over the gauge group is converted into a trace over the $\mathbf{8}_\mathrm{s}$ for SO(8) giving

$$t_8^{(r)}\mathrm{Tr}(\bar\psi_r\Gamma_r\chi^a)F_{rr}{}^aF_{rr}{}^aF_{rr}^a \quad\rightarrow\quad t_8^{(r)}t_8^{(s)}(\bar\psi_r\Gamma_r\psi^{ss})W_{rr}{}^{ss}W_{rr}{}^{ss}W_{rr}{}^{ss}$$

origin of the extra $t_8^{(s)}$ in gravity

The Ten-dimensional $N = 1$ invariant

$$
\begin{aligned}
S_X \sim \int \mathrm{d}^{10}x \Big[& \tfrac{1}{6} e\, t_8^{(r)} t_8^{(s)} W_{r_1 r_2 s_1 s_2} \cdots W_{r_7 r_8 s_7 s_8} \\
& + \tfrac{1}{3\sqrt{2}} \varepsilon_{10}^{(r)} t_8^{(s)} B_{r_9 r_{10}} W_{r_1 r_2 s_1 s_2} \cdots W_{r_7 r_8 s_7 s_8} \\
& + \tfrac{4}{3} e\, t_8^{(r)} t_8^{(s)} \eta_{r_8 s_8} (\bar{\psi}_m \Gamma_{r_7} \psi_{s_7 m}) W_{r_1 r_2 s_1 s_2} \cdots W_{r_5 r_6 s_5 s_6}
\end{aligned}
$$

left/right mixing

$$
- \tfrac{32}{5} e\, t_8^{(r)} t_8^{(s)} \eta_{r_2 r_3} (\bar{\psi}_{s_1 s_2} \Gamma_{r_1} D_{r_4} \psi_{s_3 s_4}) W_{r_5 r_6 s_5 s_6} W_{r_7 r_8 s_7 s_8}
$$

dimension dep.

$$
\begin{aligned}
& + \tfrac{12\cdot 32}{5} e\, t_8^{(s)} (\bar{\psi}_{s_1 s_2} \Gamma_{r_1} D_{r_2} \psi_{s_3 s_4}) W_{r_1 m s_5 s_6} W_{m r_2 s_7 s_8} \\
& - \tfrac{16}{5!} \varepsilon_{10}^{(r)} t_8^{(s)} (\bar{\psi}_{s_1 s_2} \Gamma_{r_1 \cdots r_5} D_{r_6} \psi_{s_3 s_4}) W_{r_7 r_8 s_5 s_6} W_{r_9 r_{10} s_7 s_8} \\
& + \tfrac{8}{3} e\, t_8^{(r)} t_8^{(s)} (\bar{\psi}_{r_1} \Gamma_{r_2} \psi_{s_1 s_2}) W_{r_3 r_4 s_3 s_4} \cdots W_{r_7 r_8 s_7 s_8} \\
& + \tfrac{16}{3} e\, t_8^{(r)} t_8^{(s)} (\bar{\psi}_{r_1} \Gamma_{s_1} \psi_{r_2 s_2}) W_{r_3 r_4 s_3 s_4} \cdots W_{r_7 r_8 s_7 s_8} \\
& + \tfrac{8}{3} e\, t_8^{(s)} (\bar{\psi}_m \Gamma_{m r_1 \cdots r_6} \psi_{s_7 s_8}) W_{r_1 r_2 s_1 s_2} \cdots W_{r_5 r_6 s_5 s_6} \Big] ,
\end{aligned}
$$

Superspace geometry

From the closure of the algebra on the elfbein

$$[\delta_1^{(0)} + (\alpha'_M)^3 \delta_1^{(3)}, \delta_2^{(0)} + (\alpha'_M)^3 \delta_2^{(3)}] e_m{}^r$$

we find only $\Gamma_{[1]}$ corrections to the translation parameter ξ^μ, so

$$X_5 = 0 \times (\mathrm{Weyl})^3$$

▷ Like for the super-Maxwell case the superspace geometry is not modified at this order in the field (with $G_{(4)}$).

Warnings: the map $(\mathrm{Maxwell})^2 = \mathrm{Einstein}$ does not commute with supersymmetry.

▷ Subtraction of lowest-order equation of motions

$$R_{\mu\nu}{}^{rs} \to W_{\mu\nu}{}^{rs} - \frac{16}{d-2} \delta_{[m}{}^{[p]} \left(\bar{\psi}_{|r|} \Gamma^{|r|} \psi_{n]}{}^{q]} - \bar{\psi}^{|r|} \Gamma^{q]} \psi_{n]r} \right)$$

give extra $\Gamma_{[1]}$ contributions to ξ^μ eliminated by field redefinitions.

▷ This map has no reasons to be valid when $G_{(4)} \neq 0$.

▷ Some R^4 structure have no super-Maxwell equivalent and are not obtained by the map:

$$\varepsilon_{10}^{\mu\nu\, r_1 \cdots r_8} \varepsilon_{10}^{\mu\nu\, s_1 \cdots s_8} R^4$$

Typically $N = 2$ contribution in $d = 10$ since this term does not appear in the one-loop heterotic $N = 1$ invariant.

Aiming to the Supermembrane

The supermembrane action is supersymmetric if the background field satisfies the equation of motion derived from the Cremmer-Julia-Scherk action

$$S = T_3 \int d^3\zeta \left[-\sqrt{-\det\left(\Pi_i{}^r \Pi_j{}^s \eta_{rs}\right)} + \frac{1}{3!}\epsilon^{ijk}\Pi_i{}^A \Pi_j{}^B \Pi_k{}^C C_{CBA} \right]$$

$$\Pi_i{}^A := (\partial_i Z^M) E_M{}^A \ .$$

$$\delta_\kappa \mathcal{L} = T_3 (\delta_\kappa E^a) \left(\sqrt{-g} g^{ij} \Pi_i{}^B T_{Ba}{}^r \Pi_{jr} + \frac{1}{3}\epsilon^{ijk}\Pi_i{}^B \Pi_j{}^C \Pi_k{}^D G_{DCBa} \right)$$

Modifying the torsion constraint $T_{Ba}{}^r$ spoils the κ-symmetry by

$$\delta_\kappa \mathcal{L} = T_3^{-1} (\delta_\kappa E^a) \left(\sqrt{-g} g^{ij} \Pi_i{}^b (\Gamma_5 \cdot X)_{ba}{}^r \Pi_{jr} \right)$$

These contributions can be cured by two-loop world-volume κ anomalies.

Constructing non-minimal supergravity in eleven-dimension will obviously leads to some new insights for the supermembrane world-volume actions.

It is urging to have a better control of its world-volume theory.

[Sugino et al. work in progress], [Nicolai et al.], [Dasgupta et al.], [Green et al.]

Summary and open questions

We got a compact expression for the $N = 1$ R^4 invariant in $d = 10$ and $d = 11$ at independent of $G_{(4)}$, in a compact form.

We showed that at this order in the fields the corrections to $d = 11$ supersymmetry transformations rules are trivial because eliminated by field redefinitions.

Next steps:

▷ Getting the $N = 2$ structure: inclusion of $\varepsilon\varepsilon R^4$

▷ Inclusion of the gauge field dependence $G_{(4)}$ and the modifications of the super-Bianchi identities for G_{ABCD}

▷ Membrane loop computation?

[Nicolai et al.], [Dasgupta et al.], [Green et al.]

▷ $\mathcal{M}$/D-brane world-volume Field Theory: relation between our background and higher-order terms on the brane world-volume theory.