Closed Strings from SO(8) Yang-Mills Instantons

based on `hep-th/0106096`

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We want to strength the link between open string and closed string.

Closed Strings are known to be related to open string by

- Unitarity: S-matrix has closed string poles in open string loop diagrams.
- Anomaly: Green-Schwarz mechanism requires propagating B-field.
- Non-perturbatively: S-duality between SO(32) heterotic string and type I.
- AdS/CFT: in $AdS_5 \times S^5$ background type IIb supergravity $\simeq \mathcal{N}_4 = 4$ super Yang-Mills.
- Singularities: Double scaling limits near singularities of (supersymmetric) gauge theory.

We consider supersymmetric effective Yang–Mills theory and exhibit gravity (F-string) from fluctuations around a well chose background.

An infinite set of non-renormalisable interactions is expected, but will be decoupled with considerations about (super)symmetries.

The vacuum will be characterized by supersymmetries more than by an action. The action can have pathological behaviour (vanishing or being infinite), the supersymmetries will be well behaved.

Effective supersymmetric Yang–Mills Action

$$
\begin{aligned}
\mathcal{L} = &- \frac{(\alpha')^2}{4} F^A{}_{\mu\nu} F^A{}_{\mu\nu} - (\alpha')^2\, 8\bar{\chi}^A \not{D} \chi^A \\
&+ (\alpha')^4 M_{ABCD}\, t_{(8)}^{\mu_1\cdots\mu_8} F^A_{\mu_1\mu_2} F^B_{\mu_3\mu_4} F^C_{\mu_5\mu_6} F^D_{\mu_7\mu_8} \\
&+ (\alpha')^4 M_{ABCD} \left[4F^A_{\mu\lambda} F^B_{\lambda\nu} (\bar{\chi}^C \Gamma_\mu (D_\nu \chi)^D) + 2F^A_{\mu\nu} F^B_{\rho\lambda} (\bar{\chi}^C \Gamma_{\mu\nu\rho} (D_\lambda \chi)^D) \right] \\
&+ \text{quartic fermions}\,.
\end{aligned}
$$

linear supersymmetry fixes M_{ABCD} to be a completely symmetric tensor

[Cederwall, Nilsson, Tsimpis]

$$
\begin{aligned}
M_{(ABCD)} &= m_1\, \mathrm{tr}\left(T_{(A} T_B T_C T_{D)}\right) + m_2\, \delta_{(AB}\delta_{CD)} \\
&= m_1\, \mathrm{Str}(\cdots) + m_2\, \mathrm{tr}(\cdots)\mathrm{tr}(\cdots)
\end{aligned}
$$

independent coeff

In open string vacuum: $m_1/g_s = 1/g_s + 1 + o(g_s)$ and $m_2/g_s = 1 + o(g_s)$.

$$
\mathcal{S} = -\frac{1}{2^9 \pi^5 (\alpha')^5 g_s} \int d^{10}x\, \mathcal{L}
$$

D9-brane tension

Introducing the Instanton

We consider an 8-dimensional SO(8) instanton (co-dimension 2 in $\mathbf{R}^{1,9}$).

$$F^{ab}_{mn} = (\Gamma_{mn}\mathcal{P}_+)^{ab} \times f(\rho, x), \quad f(\rho, x) = \frac{\rho^2}{(\rho^2 + x^2)^2}$$

$SO(8)_{\text{lorentz}}$ $\qquad$ $SO(8)_{\text{gauge}}$

[Grossman, Kephart, Stasheff]

SO(8) generalisations of 't Hooft η-symbols

$$(\Gamma_{mn}\mathcal{P}_+)^{ab} = (\gamma_{mn})^{ab} = \delta^{ab}_{mn} + c^{ab}{}_{mn}$$

Identifies the gauge and Lorentz SO(8). In particular the spacetime chirality will be correlated to the chirality of the instanton.

The instanton interpolate between Yang–Mills (open string) vacuum $f = 0$ and the closed string vacuum $\lim_{\rho\to 0} f(\rho, 0) = \infty$

Between the two vacuum we say nothing. Only then end points are studied.

The instanton solves the eom from the "F^4" part of the action but not the "F^2". It is not an extremum of the action.

Supersymmetry is a better friend to this instanton than the action

Conformal properties of SO(8) Instantons

The solution is not invariant under dilatation and translations but is invariant under SO(8) rotation and conformal transformation (modulo gauge transformations).

[Grossman, Kephart, Stasheff], [Jackiw, Rebbi]

- Rotations $R : x^m \to x^m - \omega^m{}_n x^n$ with $(\omega_{mn} = -\omega_{nm})$

$$\delta_{\mathcal{R}} F_{mn} = 2f(\rho, x) \times \omega_{[m}{}^l \Gamma_{n]l} = \delta_{\text{gauge}} F_{mn}$$

for the gauge transformation generated by $U = \exp(i\Theta)$ with $\Theta = \frac{1}{2}\omega_{mn}\Gamma_{mn}$.

The instanton is invariant under $\mathcal{R}_{mn} = M_{mn} + \Gamma_{mn}$.

- Conformal transformation: $C_m = \frac{1}{2}(K_m + P_m)$: generated by $\Omega_{mn} = 2a_{[m}x_{n]}/\rho^2$

$$\delta_{\mathcal{C}} F_{mn} = 2f(\rho, x) \times \Omega_{[m}{}^l \Gamma_{n]l} = \delta_{\text{gauge}} F_{mn}$$

for the gauge transformation generated by $U = \exp(i\Theta)$ with $\Theta = \frac{1}{2}\Omega_{mn}\Gamma_{mn}$.

The instanton is invariant under $\mathcal{C}_m = C_m + \Gamma_{mn}x^n$.

$\{\mathcal{R}_{mn}, \mathcal{C}_m\}$ close on spacetime SO(9) group of invariance.

SO(9) invariant formulation

Reformulate in terms of the last Hopf map $S^{15} \to S^7 \to S^8$

[Grossman, Kephart, Stasheff]

On $\mathbf{S}^8$ of radius ρ: $r^m r_m + r^9 r_9 = \rho^2$:

- ▷ 9-dimensional connection constrained by $\hat{A} \cdot r = 0$
- ▷ "generalized" field strength $\hat{F}_{abc} = L_{ab}\hat{A}_c + r_a[\hat{A}_b, \hat{A}_c]$ with $L_{ab} = 2r_{[a}\partial_{b]}$ the SO(9) angular momentum

$$\hat{A}_a = 2i\Gamma_{ab}r^b, \qquad F_{abc} = i\, r_{(a}\Gamma_{bc)}$$

Projecting on $\mathbf{R}^8$ with coordinates x^m

$$r^m = 2ix^m \frac{\rho^2}{\rho^2 + x^2}, \qquad r^9 = \rho\frac{\rho^2 - x^2}{\rho^2 + x^2}, \qquad \frac{\rho^2 + x^2}{2}A_m = \rho\hat{A}_m - \rho\, x_m\hat{A}_9$$

gives

$$A_m = f(\rho, x) \times \Gamma_{mn}x^n, \qquad F_{mn} = f(\rho, x) \times \Gamma_{mn}$$

$\hat{A}_9$ is re-absorbed by gauge transformations: $U = \exp(\frac{i}{2}f(r_9)\hat{A}_9)$.

The position of the 8-plane can be shifted by $r_9 \to r_9 + C$ with $C \in [-\rho, \rho]$ and rotational invariance makes C periodic.

Localization

$$\int d^{10}x \lim_{\rho \to 0} f(\rho, x)^4 = \int d^{10}x\, \delta^{(8)}(x^m).$$

preserves the periodicity along r_9

Supersymmetric and the Instanton

The open string vacuum has sixteen real supercharges linear supersymmetries compatible with non Abelian gauge group G

$$\delta_\epsilon \chi^A = \frac{(\alpha')^2}{8}\Gamma^{\mu\nu} F^A_{\mu\nu}\,\epsilon$$
$$+\frac{(\alpha')^4}{2^5\cdot 4!} M^A{}_{BCD}\left[\left(t_{(8)}^{\nu_1\cdots\nu_8}\Gamma_{\nu_7\nu_8} - \Gamma^{\nu_1\cdots\nu_6}\right) F^B_{\nu_1\nu_2} F^C_{\nu_3\nu_4} F^D_{\nu_5\nu_6}\right]\epsilon$$

$$\delta_\epsilon^{(2)}\mathcal{L}^{(4)} + \delta_\epsilon^{(4)}\mathcal{L}^{(2)} = 0 \iff \delta^{(2)}\mathcal{L}^{(4)} = \mathcal{E}\left(A^\mu\right)\delta_\epsilon^{(4)}(A_\mu) + \mathcal{E}\left(\bar\chi\right)\delta_\epsilon^{(4)}(\chi)$$

No background fermions so $\delta_\epsilon^{(4)}(A_\mu) = 0$ and thanks to

$$M_{(ABCD)}\left(t_{(8)}^{r_1\cdots r_8} \mp \frac{1}{2}\varepsilon_{(8)}^{r_1\cdots r_8}\right) F^A_{r_1r_2} F^B_{r_3r_4} F^C_{r_5r_6} = 0,$$

and

$$\left(\Gamma^{mn}\right)_{\alpha\beta} F^{ab}{}_{mn} = \left(\mathcal{P}_+\right)^{ab}_{\alpha\beta} : \qquad SO(8)_{\text{Gauge}} \equiv SO(8)_{\text{Lorentz}}$$

$$\delta_\epsilon\chi^{ab}_\alpha = -\,7\,f(\rho,x)\,(\mathcal{P}_+)^{ab}_{\alpha\beta}\left((\alpha')^2 + \frac{4}{3}(\alpha')^4 m_1 (f(\rho,x))^2\mathcal{P}_-\right)_{\beta\gamma}\epsilon_\gamma$$
$$= -\,7(\alpha')^2\,f(\rho,x)\,(\mathcal{P}_+)^{ab}_{\alpha\beta}\epsilon_\beta + 0\times(\alpha')^4$$

for the classical background

$$\delta_\epsilon^{(2)}\mathcal{L}^{(4)} = 0\,.$$

By the same token supersymmetry decouple the F^4-action from F^6-action and higher.

The String Solitons

In the $\rho \to 0$ limit we obtain a D-string (m_1) and radiate F-string (m_2).

Recall that m_1 and m_2 are independent coefficient

The energy of the instanton $\mathcal{E} \sim m_1/g_s$ gives a D-string ($\Pi_7(G) = \mathbf{Z}$). [Witten]

From now we set $m_1 = 0$.

F-strings come from $m_2/g_s = 1 + o(g_s)$

Bosonic zero modes

- ▷ translation invariance along "time" direction: $\sigma^0 = x^0$
- ▷ shift $r_9 \to r_9 + C$ and SO(9) symmetry: periodic $\sigma^1 \in [-1, 1]$

$$A_\mu = (0, A_m(\sigma^0, \sigma^1))$$

$$\hat{F}_{\alpha m} = \partial_\alpha A_m = -4i\gamma_{mn}\left(\partial_\alpha z^n f(\rho, x_0) + x^n(\partial_\alpha z \cdot x) f'(\rho, x_0) + \cdots\right) .$$

$$\lim_{\rho \to 0} \mathcal{S}^{(4)} = \frac{1}{2\pi\alpha'} 1 \int d^2\sigma \ \partial_\alpha z_m \partial^\alpha z^m$$

F-string tension

$$\Delta\mathcal{L}^{(4)} \sim 1\left[\mathrm{tr}_{\mathrm{SO}(8)}(\bar{\chi}\Gamma_\mu D_\nu \chi)\, \mathrm{tr}_{\mathrm{SO}(8)}(\hat{F}_{\mu\lambda}\hat{F}_{\lambda\nu}) + \Gamma_{[3]}\right]$$

The fluctuations are no longer supersymmetric invariant under $\delta_\epsilon^{(2)}$

Extra non-linear supersymmetry

An extra supersymmetry is needed for having supersymmetric σ-model with introducing a new supermultiplet

The original open string vacuum had non-linear supersymmetry δ_η on Abelian field

[Cederwall, Nilsson, Tsimpis], [Hara, Yoneya], [Bagger, Galperin]

The SO(8) fermionic fluctuation can be interpreted as abelian Majorana-Weyl fermions

$$\chi = \xi(x) \otimes S(\sigma), \text{ with } \chi^{ab}_{\alpha\beta} = S_\beta(\mathcal{P}_+)^{ab}_{\alpha\beta}$$

$SO(8)_L = SO(8)_G$

$$\delta^{(2)}_\epsilon(\Delta\mathcal{L}^{(4)}) \sim (f(\rho,x))^3 \times \mathrm{tr}_{\mathrm{SO}(8)}(\bar\epsilon\Gamma\mathcal{P}_+ D\chi) \sim W(T) \times (\bar\eta\Gamma DS)$$
$$\sim \delta^{(2)}_\eta(\Delta\mathcal{L}^{(4)}) \,,$$

Normalization

The action is supersymmetric under $\delta^{(2)}_\epsilon + \delta_\eta$ iff

$$\eta = \frac{(f(\rho,x))^3}{W(T)}\mathcal{P}_+\epsilon, \text{ and } \lim_{\rho\to 0}\frac{f(\rho,x)^3}{W(T)} = \text{finite number} = 1$$

Gives $(8,0)$ σ-model with $\mathcal{P}_-\eta = 0$, in the light-cone gauge with the GS fermions S_a

The $(8,0)$ sigma model

Fluctuations along SO(8)

▷ Kinetic term for the GS fermions: $m_2 \int d^2\sigma\, \bar{S}\gamma_\alpha \partial_\beta S\, h_{\alpha\beta}$,

$$m_2 \lim_{\rho \to 0} \int d^{10}x \,\mathrm{tr}_{\mathrm{SO}(8)} \left(\bar{\chi}\Gamma_\alpha \partial_\beta \chi\right) \mathrm{tr}_{\mathrm{SO}(8)}(F_{\alpha n} F_{n\beta})$$

▷ The induce metric $h_{\alpha\beta} := \partial_\alpha z^n \partial_\beta z^n \times \lim_{\rho\to 0} \int d^8x \, \left(f(\rho, x)\right)^2 \, \bar{\xi}\xi(x)$.

Fluctuations orthogonal to SO(8)

We consider that the gauge d.o.f. are in SO(24) (for $G = SO(32)$)

▷ Kinetic term for gauge fermions: $m_2 \int d^2\sigma \,\mathrm{tr}_{\mathrm{SO}(24)}(\bar{\lambda}\gamma_\alpha \partial_\beta \lambda)\, h_{\alpha\beta}$,

$$m_2 \lim_{\rho \to 0} \int d^{10}x \,\mathrm{tr}_{\mathrm{SO}(24)} \left(\bar{\chi}\Gamma_\alpha \partial_\beta \chi\right) \mathrm{tr}_{\mathrm{SO}(8)}(F_{\alpha n} F_{n\beta})$$

▷ coupling to the background field: $m_2 \int d^2\sigma \,\mathrm{tr}_{\mathrm{SO}(24)}(\bar{\lambda}\gamma_\alpha [a_m, \lambda])\, e_\alpha{}^m$,

$$m_2 \lim_{\rho \to 0} \int d^{10}x \,\mathrm{tr}_{\mathrm{SO}(24)} \left(\bar{\chi}\Gamma_\alpha [a_m, \chi]\right) \mathrm{tr}_{\mathrm{SO}(8)}(F_{\alpha n} F_{nm})$$

with the induced vielbein $e_\alpha{}^m := \partial_\alpha z^m \times \lim_{\rho\to 0} \int d^8x \, \left(f(\rho, x)\right)^2 \, \bar{\xi}\xi(x)$.

This is an heterotic string in the light-cone gauge in the gravitational background with the spin connection embedded in the $SO(8) \in G$.

Open Questions

Achievement

- We found a mechanism for generating gravity from Yang–Mills degrees of freedom.
- Vacua are characterized by their (super)symmetries not by the action.

Remarks

- Linear and non-linear supersymmetry necessary for having supersymmetric fluctuations. [Cederwall, Nilsson, Tsimpis]
- The heterotic string is found in a gravity + Yang-Mills background: $\omega_\mu \in SO(8)$.
- The string is closed by the SO(9) (conformal) symmetry of the solution. Different from flux confinement mechanism of [Kleban, Lawrence, Shenker], [Bergman, Hori, Yi], [Yi]

Questions:

- Is it possible to control the field redefinition between linear and non-linear supersymmetry? constraining the form of Tachyon potential?
- Using $SO(8)_G$ as a "center" and the extra field T,

$$T \to T + \mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$$

  completed by linear supersymmetry into $t_8(\mathrm{tr}F^2)^2$. Yang–Mills kinetic term is generated from $G = SO(8) \oplus G'$

$$t_8 \mathrm{tr}_{SO(8)}(F^2)\mathrm{tr}_{G'}(F^2) \to \mathrm{tr}_{G'}(F^2)$$

- Different background? Is it possible to look at some non commutative version of this instanton and get a closed string in a non trivial B-field background? Is it difficult to give a non-commutative version of the SO(8) instanton (one modulus)