

Equation-Writing in Latex

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1 Linear Differential Equation

$$\frac{d^2w}{dx^2} - \frac{4u^2w}{l^2} = \frac{-qlx}{2D} + \frac{qx^2}{2D}$$

Here u, l, q, D are constants.

Auxiliary Equation is

$$D^2 - \frac{4u^2}{l^2}w = 0$$

$$(D + \frac{2u}{l})(D - \frac{2u}{l}) = 0$$

$$D = \frac{-2u}{l}, \frac{2u}{l}$$

So C.F is

$$C_1e^{\frac{-2ux}{l}} + C_2e^{\frac{2ux}{l}}$$

or

$$C_1 \cosh(\frac{2u}{l}x)$$

Because

$$\cosh(x) = \frac{e^{-x} + e^x}{2}$$

Now Particular Integral (P. I) is

$$\begin{aligned} PI &= \frac{1}{(D^2 - \frac{4u^2}{l^2})} \left(\frac{-qlx}{2D} + \frac{qx^2}{2D} \right) \\ &= \frac{-q}{2D} \frac{1}{(D^2 - \frac{4u^2}{l^2})} (lx - x^2) \\ &= \frac{-ql}{2D} \frac{1}{(D^2 - (\frac{2u}{l})^2)} x + \frac{q}{2D} \frac{1}{(D^2 - (\frac{2u}{l})^2)} x^2 \end{aligned}$$

Taking $4u^2/l^2$ common from denominator

$$= \frac{-ql}{2D} \frac{l^2}{-4u^2} \left(1 + \frac{-l^2 D^2}{4u^2} \right)^{-1} + \frac{q}{2D} \frac{l^2}{-4u^2} \left(1 + \frac{-l^2 D^2}{4u^2} \right)^{-1}$$

Solving binomial as it is of the form

$$\begin{aligned}
 (1-x)^{-1} &= 1 + x^2 + x^3 + \dots \\
 &= \frac{ql^3}{8Du^2} \left(1 + \frac{l^4 D^4}{16u^4} + \dots\right)x + \frac{-ql^2}{8Du^2} \left(1 + \frac{l^4 D^4}{16u^4} + \dots\right)x^2 \\
 &= \frac{ql^3 x}{8Du^2} - \frac{ql^2 x^2}{8Du^2}
 \end{aligned}$$

Solution of Differential Equation is C. S = A. E + P. I

$$w = C_1 \cosh\left(\frac{2u}{l}\right)x + \frac{ql^3 x}{8Du^2} - \frac{ql^2 x^2}{8Du^2}$$