0-1 Knapsack

$W = 8$ $P = \{1, 2, 5, 6\}$

$n = 4$ $\omega = \{2, 3, 4, 5\}$

$\sum \omega_i x_i \leq W$ $x_i = \{1/0, 1/0, 1/0, 1/0\}$

↳ constraint ↳ 2^n

Maximise Profit

$\max \sum P_i x_i$

Memoization → Recursion

↳ Storing the recursive values

Net Pending Profit = 8 − 6 = 2

Tabulation → No Recursion

W = 8 ⟷ 2 − 2 = 0

W ⟶

	P_i	ω_i		0	1	2	3	4	5	6	7	8
			0	0	0	0	0	0	0	0	0	0
obj1	1	2	1	0	0	1	1	1	1	1	1	1
obj2	2	3	2	0	0	1	2	2	3	3	3	3
obj3	5	4	3	0	0	1	2	5	5	6	7	7
obj4	6	5	4	0	0	1	2	5	6	6	7	8

	2		6
x_1	x_2	x_3	x_4
0	1	0	1

final Result ← Max Profit

Generalized formula

not including current object including current object

↳ $V[i, W] = \max \{ V[i-1, W],\ V[i-1, W - \omega t[i]] + P[i] \}$

$$V[4,1] = \max\{\underline{V[3,1]},\ V[3, \overset{W - wt(i)}{1-5}] + 6\}$$

$$\max\{0,\ \underline{V[3,-4]} + 6\}$$

$$V(4,5) = \max\{\underline{V(3,5)},\ V(3, \overset{\overset{5 \quad 5}{W - wt(i)}}{0}) + 6\}$$

$$\{5,\ 0+6\}$$

$$= \underline{6}$$

$wt(i-1) > W$

↳ skip that object

$i = 0$ ↳ filled values with 0

Problem 2

$N = 3,\ W = 4$

$P = \{\overset{0}{1}, \overset{1}{2}, \overset{2}{3}\}$

$wt = \{\overset{0}{4}, \overset{1}{5}, \overset{2}{1}\}$

6

max Profit =

$i = 1$

↳ Picking values of P & wt.

$V(3,4) =$ ____

x_1	x_2	x_3
0	0	1

$W \rightarrow$

P	wt		0	1	2	3	4
		0	0	0	0	0	0
1	4	1	0	0	0	0	1
2	5	2	0	0	0	0	1
3	1	3	0	3	3	3	3

Net Pending Profit =

$3 - 3 = 0$

$$\overset{i\ W}{V[1,3]} = \max\{\underline{\overset{0}{V(0,3)}},\ \underline{V(0,-1)} + 1\}$$

$$V(1,4) = \max\{V(0,4),\ V(0,0)+1\}$$
$$= \max\{0,1\}$$

$$V(2,4) = \max\{\underbrace{V(1,4)}_{1},\ \underline{V(1,-1)}+2\}$$
$$= 1$$

$$V(3,4) = \max\{V(2,4),\ V(2,3)+3\}$$
$$= \max\{1,\ 0+3\}$$
$$= 3$$

↳ Display the object names which contribute to maximum profit.

Time complexity → $\Theta(n * W)$

Space complexity → $\Theta(n * W)$

Brute force Approach

↳ $\Theta(2^n)$ → Exponential Time Complexity

↳ $\Theta(n * W)$

↳ Polynomial Time Complexity

Matrix Chain Multiplication

$$M_1 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{\frac{2\times 3}{a \times b}} \qquad M_2 = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}_{\substack{b \times c \\ 3 \times 2}}$$

$2 \times 3 \quad 3 \times 2$ Output Matrix   Total $a * b * c$

$M_1 * M_2 =$ 2×2 $2 * 3 * 2 = 12$

Num of multiplication = **12** **(3*4)**

$$\begin{array}{c|cc} & 0 & 1 \\ \hline 0 & a_{11} * b_{11} + a_{12} * b_{21} + a_{13} * b_{31} & a_{11} * b_{21} + a_{12} * b_{22} + a_{13} * b_{32} \\ 1 & a_{21} * b_{11} + a_{22} * b_{21} + a_{23} * b_{31} & a_{21} * b_{21} + a_{22} * b_{22} + a_{23} * b_{32} \end{array}$$

$$M_1 = \overset{P_0}{2} \times \overset{P_1}{3}$$

$$M_2 = \underset{P_1}{3} \times \underset{P_2}{4}$$

$$M_3 = \underset{P_2}{4} \times \underset{P_3}{2}$$

→ Matrix multiplication (Associative Property)

$$M_1 * M_2 * M_3$$

Min number of multip.

I

$$(M_1 * M_2) * \underline{M_3}$$

II

$$\boxed{M_1 * (M_2 * M_3)}$$

$$\left.\begin{array}{l} M_1 = 2*3 \\ M_2 = 3*4 \end{array}\right\} = 2 \times 3 \times 4 = 24 \left(\begin{array}{l} O_1 = M_1 * M_2 \\ \to 2*4 \end{array} \right)$$

$$\left.\begin{array}{l} O_1 = 2*4 \\ M_3 = 4*2 \end{array}\right\} = \underset{P_0}{\underline{2}} \times \underset{P_2}{\underline{4}} \times \underset{P_3}{\underline{2}} = 16$$

Total num of multiplications $= 24 + 16 = 40$

$$\left.\begin{array}{l} M_2 = 3 \times 4 \\ M_3 = 4 \times 2 \end{array}\right\} = 3 \times 4 \times 2 = 24$$

$$O_1 = M_2 * M_3$$

$$\underline{(3 * 2)}$$

$$M_1 = 2 \times 3$$

$$O_1 = 3 \times 2$$

$$\underset{P_0}{2} \times \underset{P_1}{3} \times \underset{P_3}{2} = 12$$

$$24 + 12 = 36$$

Commutative Property

$$\boxed{M_1 * O_1 \neq O_1 * M_1}$$

$M_1 \quad M_2 \quad M_3 \quad - - - - - \quad M_{10}$

↳ Generalized formula

$$C(1,3) = \min_{i \le k < J} \begin{cases} \overset{i\ k}{C[1,2)} + \overset{k+1\ J}{C(3,3]} + \overset{P_{i-1}\ P_K\ P_J}{P_0 P_2 P_3} \\ \\ \underset{i\ k}{C(1,1)} + \underset{k+1\ J}{C(2,3)} + P_0 P_1 P_3 \end{cases}$$

(with $C(1,3)$ labelled $i\ J$)

$k=1, \ k=2, \ k=3$

$$\overset{i\ J}{C(1,4)} = \min \begin{cases} \\ \end{cases}$$

Task

$A = 1 \times 3$, $B = 3 \times 1$, $C = 1 \times 2$, $D = 2 \times 3$

$E = ABCD$

1) How many min number of multiplications required to get an output matrix E ??

2) In what sequence ??

3) Try to do same thing using above generalization & verify that in both ways answers are same or not ??