

Page Replacement Algorithms

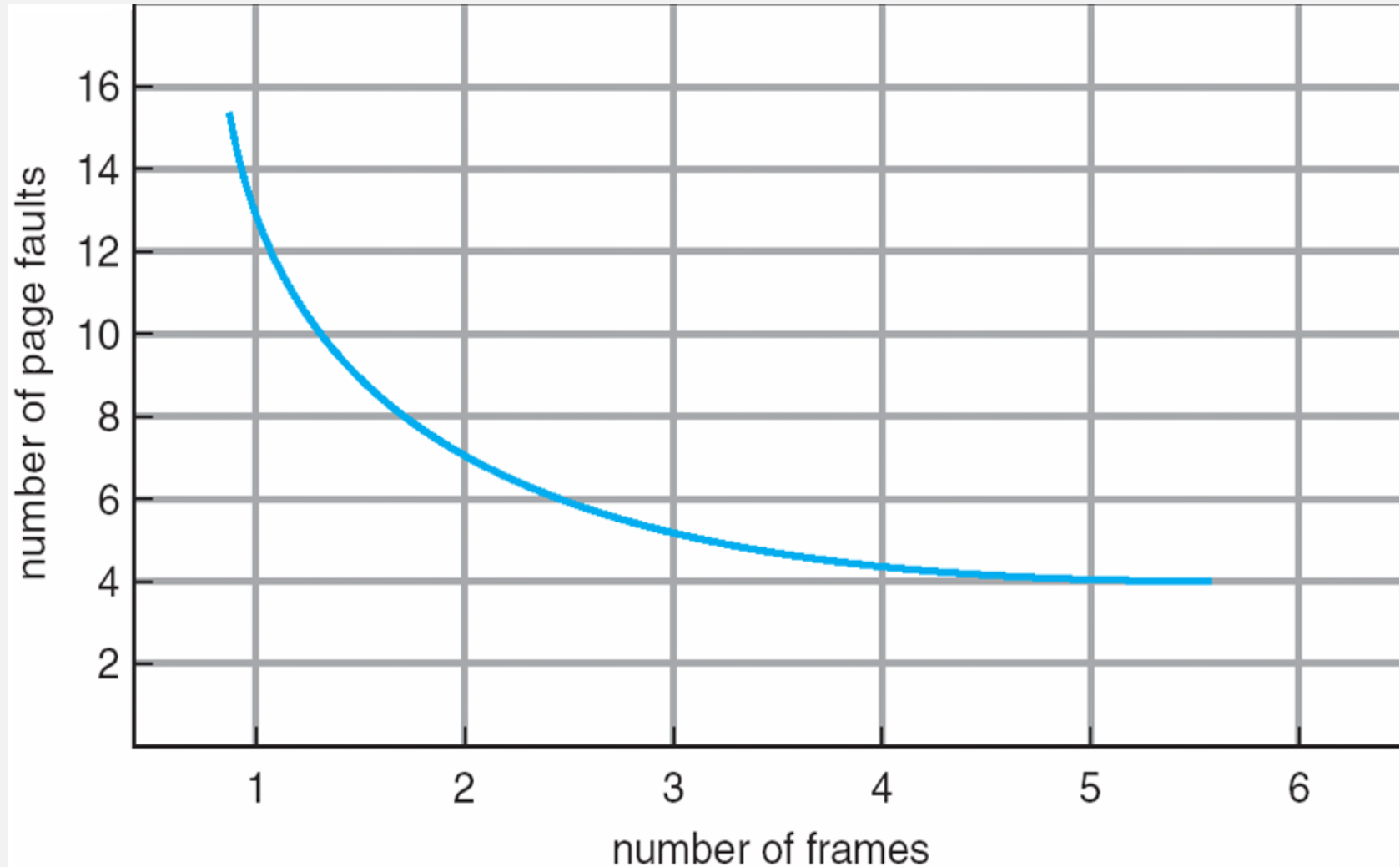
Virtual Memory Management

- Background
- Demand Paging
- Demand Segmentation
- Paging Considerations
- Page Replacement Algorithms
- Virtual Memory Policies

Page Replacement Algorithms

- Want lowest page-fault rate.
- Evaluate algorithm by running it on a particular string of memory references (reference string) and computing the number of page faults and page replacements on that string.
- In all our examples, we use a few recurring reference strings.

Graph of Page Faults vs. the Number of Frames



The FIFO Policy

- Treats page frames allocated to a process as a circular buffer:
 - When the buffer is full, the oldest page is replaced. Hence first-in, first-out:
 - A frequently used page is often the oldest, so it will be repeatedly paged out by FIFO.
 - Simple to implement:
 - requires only a pointer that circles through the page frames of the process.

FIFO Page Replacement

reference string

7 0 1 2 0 3 0 4 2 3 0 3 2 1 2 0 1 7 0 1

7	7	7	2
	0	0	0
		1	1

2	2	4	4	4	0
3	3	3	2	2	2
1	0	0	0	3	3

0	0
1	1
3	2

7	7	7
1	0	0
2	2	1

page frames

First-In-First-Out (FIFO) Algorithm

- Reference string: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

- 3 frames (3 pages can be in memory at a time per process):

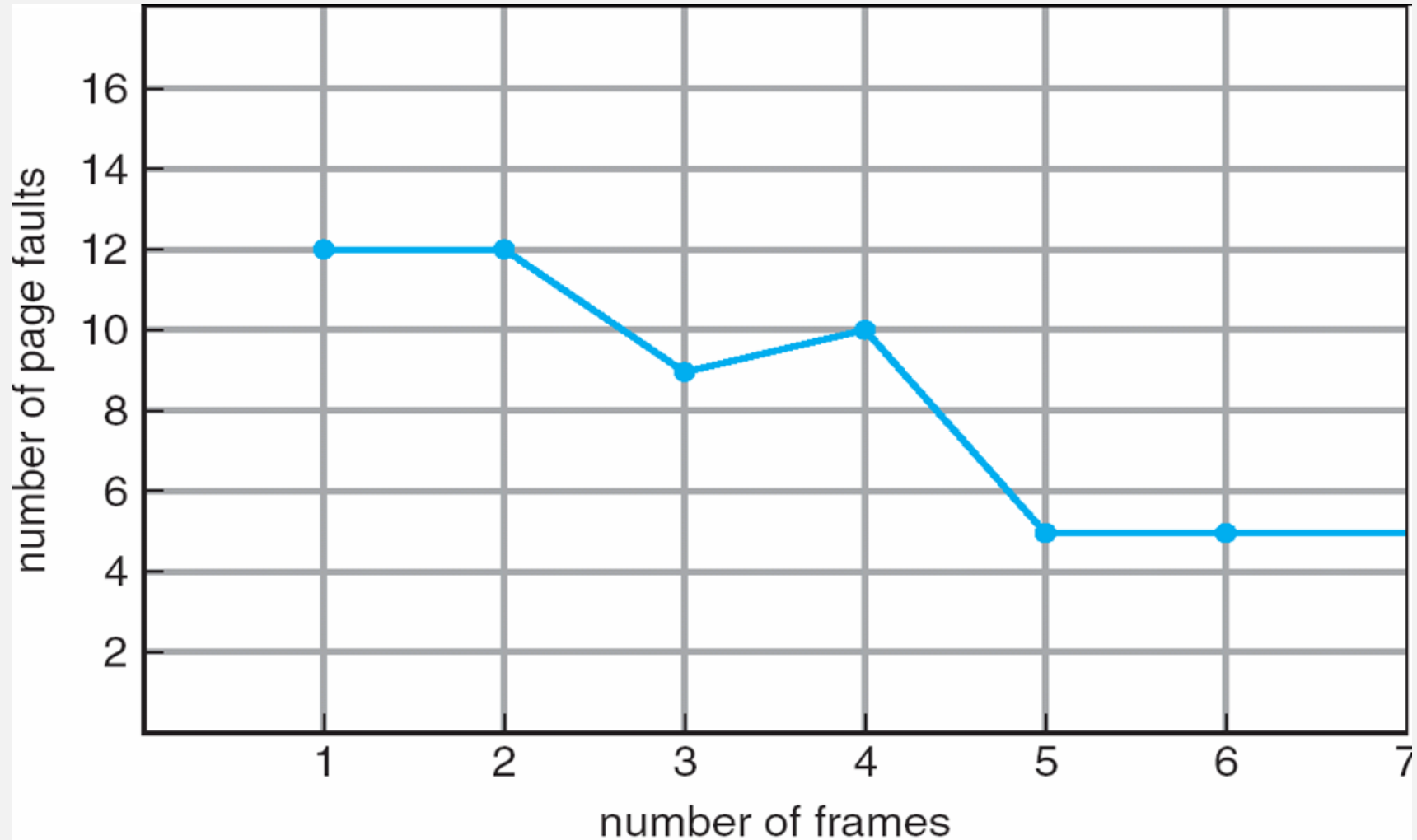
1	1	4	5	9 page faults
2	2	1	3	
3	3	2	4	

- 4 frames:

1	1	5	4	10 page faults
2	2	1	5	
3	3	2		
4	4	3		

- FIFO Replacement manifests Belady's Anomaly:
 - more frames \Rightarrow more page faults

FIFO Illustrating Belady's Anomaly



Optimal Page Replacement

- The Optimal policy selects for replacement the page that will not be used for longest period of time.
- Impossible to implement (need to know the future) but serves as a standard to compare with the other algorithms we shall study.

Optimal Page Replacement

reference string

7 0 1 2 0 3 0 4 2 3 0 3 2 1 2 0 1 7 0 1

7	7	7	2		2		2		2								7		
	0	0	0		0		0		0								0		
		1	1		3		3		3								1		

page frames

Optimal Algorithm

- Reference string : 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5
- 4 frames example

1	4	6 page faults
2		
3		
4	5	

- How do you know future use? You don't!
- Used for measuring how well your algorithm performs.

The LRU Policy

- Replaces the page that has not been referenced for the longest time:
 - By the principle of locality, this should be the page least likely to be referenced in the near future.
 - performs nearly as well as the optimal policy.

LRU Page Replacement

reference string

7 0 1 2 0 3 0 4 2 3 0 3 2 1 2 0 1 7 0 1

7	7	7	2	2	4	4	4	0	1	1	1
	0	0	0	0	0	0	3	3	3	0	0
		1	1	3	3	2	2	2	2	2	7

page frames

Least Recently Used (LRU) Algorithm

- Reference string: 1, 2, 3, 4, 1, 2, **5**, 1, 2, **3**, **4**, **5**

1	1	1	1	5
2	2	2	2	2
3	5	5	4	4
4	4	3	3	3

8 page faults

Comparison of OPT with LRU

- Example: A process of 5 pages with an OS that fixes the resident set size to 3.

Page address
stream

2 3 2 1 5 2 4 5 3 2 5 2

OPT

2	2	2	2	2	2	4	4	4	2	2	2
	3	3	3	3	3	3	3	3	3	3	3
			1	5	5	5	5	5	5	5	5
				F		F			F		

LRU

2	2	2	2	2	2	2	2	3	3	3	3
	3	3	3	5	5	5	5	5	5	5	5
			1	1	1	4	4	4	2	2	2
				F		F		F	F		

Comparison of FIFO with LRU

Page address
stream

2 3 2 1 5 2 4 5 3 2 5 2

LRU

2	2	2	2	2	2	2	2	3	3	3	3
	3	3	3	5	5	5	5	5	5	5	5
			1	1	1	4	4	4	2	2	2
				F		F		F	F		

FIFO

2	2	2	2	5	5	5	5	3	3	3	3
	3	3	3	3	2	2	2	2	2	5	5
			1	1	1	4	4	4	4	4	2
				F	F	F		F		F	F

- LRU recognizes that pages 2 and 5 are referenced more frequently than others but FIFO does not.

Implementation of the LRU Policy

- Each page could be tagged (in the page table entry) with the time at each memory reference.
- The LRU page is the one with the smallest time value (needs to be searched at each page fault).
- This would require expensive hardware and a great deal of overhead.
- Consequently very few computer systems provide sufficient hardware support for true LRU replacement policy.
- Other algorithms are used instead.

LRU Implementations

- Counter implementation:
 - Every page entry has a counter; every time a page is referenced through this entry, copy the clock into the counter.
 - When a page needs to be changed, look at the counters to determine which are to change.
- Stack implementation – keep a stack of page numbers in a double link form:
 - Page referenced:
 - move it to the top
 - requires 6 pointers to be changed
 - No search for replacement.

Use of a stack to implement LRU

- Stack implementation – keep a stack of page numbers in a double link form:
 - Page referenced:
 - move it to the top
 - requires 6 pointers to be changed
 - No search for replacement – always take the bottom one.

reference string

4 7 0 7 1 0 1 2 1 2 7 1 2



stack
before
a



stack
after
b



Hardware Matrix LRU Implementation

Pages are referenced in the order 0, 1, 2, 3, 2, 1, 0, 3, 2, 3

	Page			
	0	1	2	3
0	0	1	1	1
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0

(a)

	Page			
	0	1	2	3
0	0	0	1	1
1	1	0	1	1
2	0	0	0	0
3	0	0	0	0

(b)

	Page			
	0	1	2	3
0	0	0	0	1
1	1	0	0	1
2	1	1	0	1
3	0	0	0	0

(c)

	Page			
	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	1	1	0	0
3	1	1	1	0

(d)

	Page			
	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	1	1	0	1
3	1	1	0	0

(e)

	0	1	2	3
0	0	0	0	0
1	1	0	1	1
2	1	0	0	1
3	1	0	0	0

(f)

	0	1	2	3
0	0	1	1	1
1	0	0	1	1
2	0	0	0	1
3	0	0	0	0

(g)

	0	1	2	3
0	0	1	1	0
1	0	0	1	0
2	0	0	0	0
3	1	1	1	0

(h)

	0	1	2	3
0	0	1	0	0
1	0	0	0	0
2	1	1	0	1
3	1	1	0	0

(i)

	0	1	2	3
0	0	1	0	0
1	0	0	0	0
2	1	1	0	0
3	1	1	1	0

(j)

LRU Approximation Algorithms (1)

- Reference Bit:
 - With each page associate a bit, initially = 0
 - When page is referenced, bit is set to 1.
 - Replace the one which is 0 (if one exists) – we do not know the real order of use, however.

LRU Approximation Algorithms (2)

- Reference Byte:
 - Idea is to record reference bits at regular intervals; Keep a byte of reference bits for each page.
 - At regular intervals (say, every 20 ms), left shift the reference bit of each page into the high-order bit of the byte.
 - Each reference byte keeps the history of the page use (aging) for the last eight time intervals.
 - If we interpret the reference byte as an unsigned integer, the page with the lowest number is the LRU page.

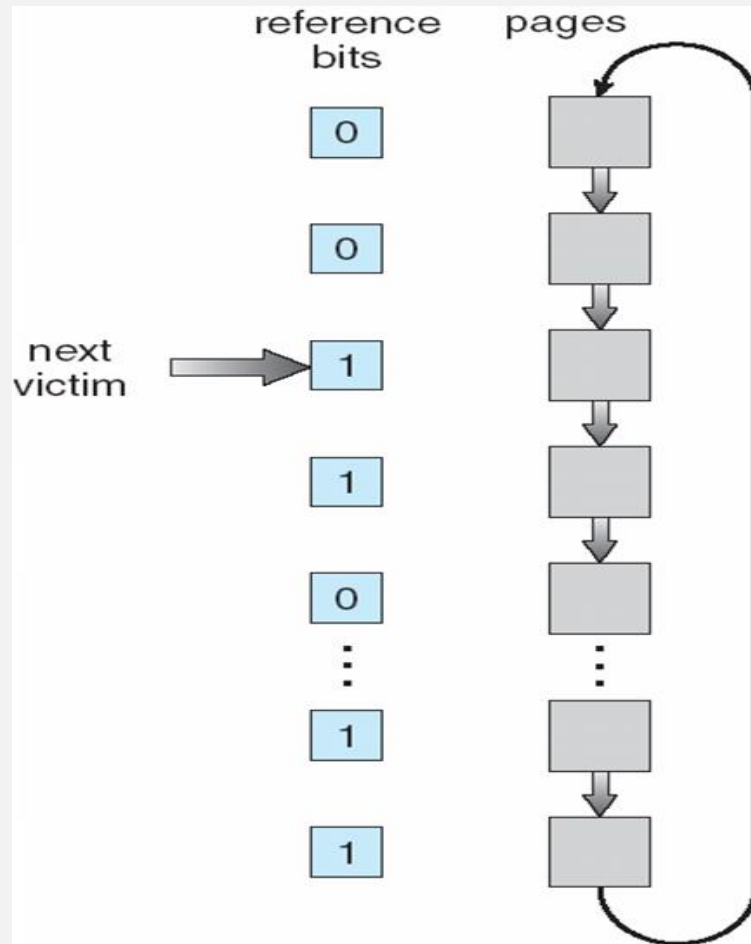
Reference Byte Example

	R bits for pages 0-5, clock tick 0	R bits for pages 0-5, clock tick 1	R bits for pages 0-5, clock tick 2	R bits for pages 0-5, clock tick 3	R bits for pages 0-5, clock tick 4
	1 0 1 0 1 1	1 1 0 0 1 0	1 1 0 1 0 1	1 0 0 0 1 0	0 1 1 0 0 0
Page					
0	10000000	11000000	11100000	11110000	01111000
1	00000000	10000000	11000000	01100000	10110000
2	10000000	01000000	00100000	00100000	10010000
3	00000000	00000000	10000000	01000000	00100000
4	10000000	11000000	01100000	10110000	01011000
5	10000000	01000000	10100000	01010000	00101000
	(a)	(b)	(c)	(d)	(e)

The Clock (Second Chance) Policy

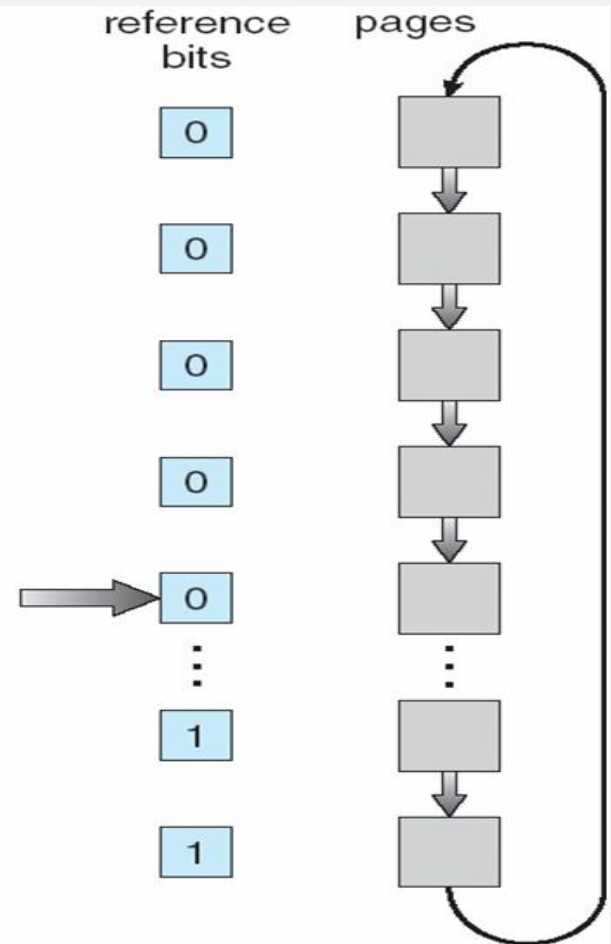
- The set of frames candidate for replacement is considered as a circular buffer.
- When a page is replaced, a pointer is set to point to the next frame in buffer.
- A reference bit for each frame is set to 1 whenever:
 - a page is first loaded into the frame.
 - the corresponding page is referenced.
- When it is time to replace a page, the first frame encountered with the reference bit set to 0 is replaced:
 - During the search for replacement, each reference bit set to 1 is changed to 0.

Clock Page-Replacement Algorithm



circular queue of pages

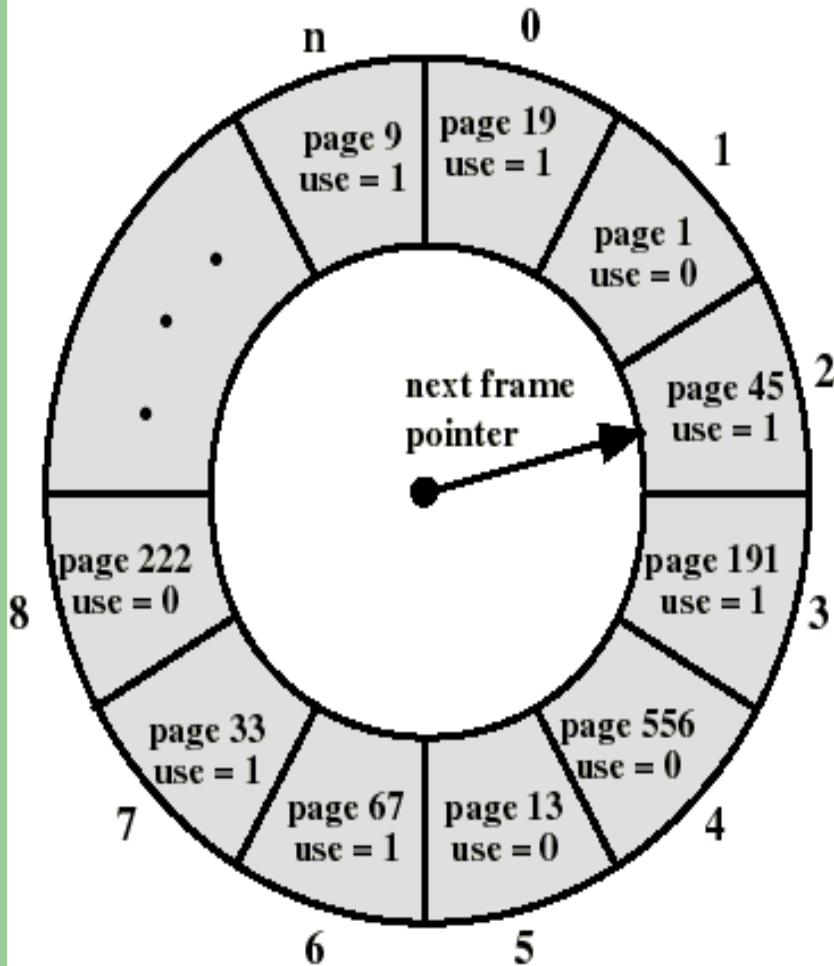
(a)



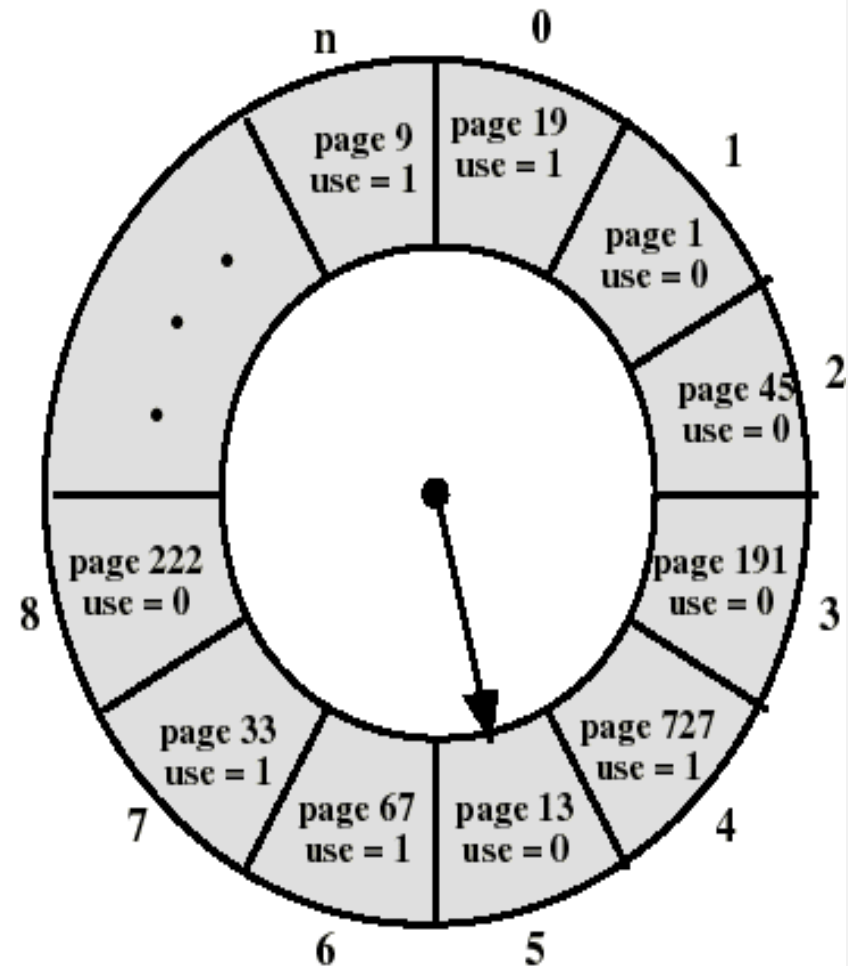
circular queue of pages

(b)

The Clock Policy: Another Example



(a) State of buffer just prior to a page replacement



(b) State of buffer just after the next page replacement

Comparison of Clock with FIFO and LRU (1)

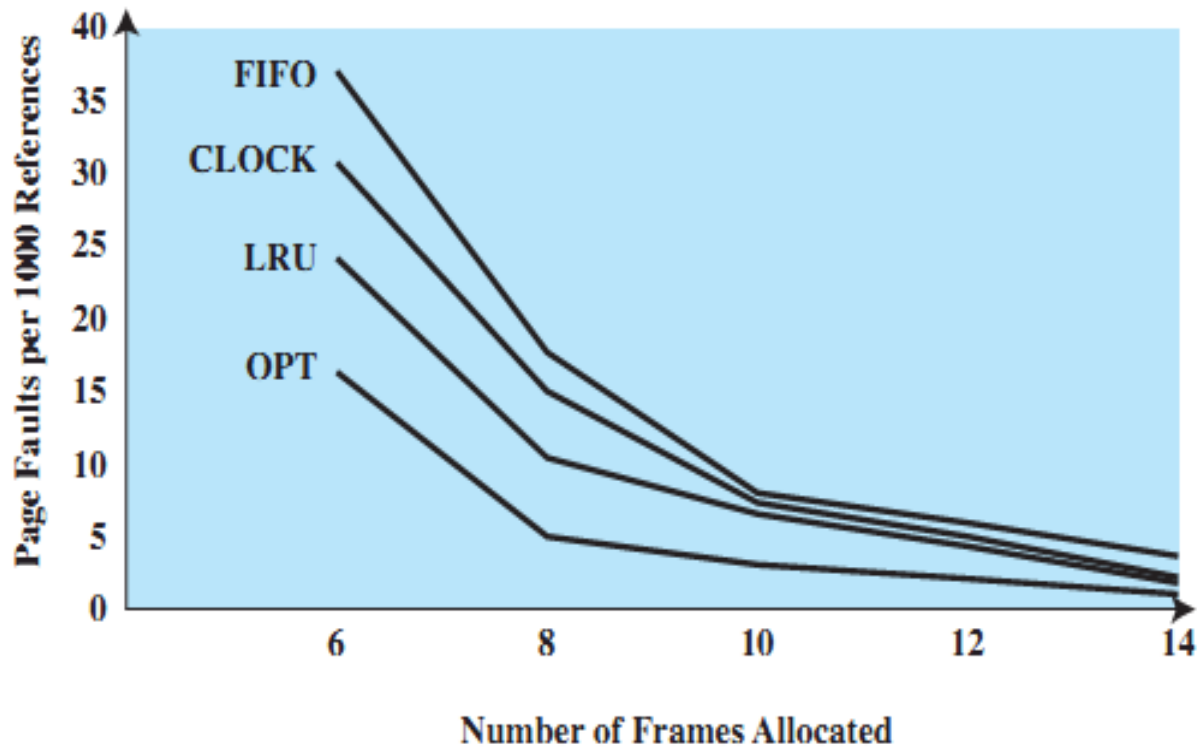
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- Asterisk indicates that the corresponding use bit is set to 1.
- The arrow indicates the current position of the pointer.
- Note that the clock policy is adept at protecting frames 2 and 5 from replacement.

Comparison of Clock with FIFO and LRU (2)

- Numerical experiments tend to show that performance of Clock is close to that of LRU.
- Experiments have been performed when the number of frames allocated to each process is fixed and when pages local to the page-fault process are considered for replacement:
 - When few (6 to 8) frames are allocated per process, there is almost a factor of 2 of page faults between LRU and FIFO.
 - This factor reduces close to 1 when several (more than 12) frames are allocated. (But then more main memory is needed to support the same level of multiprogramming).

Fixed-Allocation, Local Page Replacement



Counting-based Algorithms

- Keep a counter of the number of references that have been made to each page.
- Two possibilities: Least/Most Frequently Used (LFU/MFU).
- LFU Algorithm: replaces page with smallest count; others were and will be used more.
- MFU Algorithm: based on the argument that the page with the smallest count was probably just brought in and has yet to be used.

Page Buffering (1)

- Pages to be replaced are kept in main memory for a while to guard against poorly performing replacement algorithms such as FIFO.
- Two lists of pointers are maintained: each entry points to a frame selected for replacement:
 - a free page list for frames that have not been modified since brought in (no need to swap out).
 - a modified page list for frames that have been modified (need to write them out).
- A frame to be replaced has a pointer added to the tail of one of the lists and the present bit is cleared in corresponding page table entry; but the page remains in the same memory frame.

Page Buffering (2)

- At each page fault the two lists are first examined to see if the needed page is still in main memory:
 - If it is, we just need to set the present bit in the corresponding page table entry (and remove the matching entry in the relevant page list).
 - If it is not, then the needed page is brought in, it is placed in the frame pointed by the head of the free frame list (overwriting the page that was there); the head of the free frame list is moved to the next entry.
 - (the frame number in the page table entry could be used to scan the two lists, or each list entry could contain the process id and page number of the occupied frame).
- The modified list also serves to write out modified pages in cluster (rather than individually).