Numerical pricing of financial derivatives using Jain's high-order compact scheme

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1 Introduction

We consider high-order compact(HOC) schemes for quasilinear parabolic partial differential equations and use it for pricing of European options and then cover some of its more novel applications for pricing interest rate derivatives.
Since it might not always possible to obtain an analytical solution, for ex. options with early exercise features. The Crank-Nicolson scheme is the most commonly used technique for pricing in these cases, and hence will be the scheme that is used as a benchmark for such derivatives.

We consider the generalised Chan-Karolyi-Longstaff-Sanders (CKLS) family of term structure models for pricing interest rate derivatives. For the CIR and Vasicek models belonging to this family analytical solutions exists but for other cases pricing through numerical schemes is required. We compute numerical solutions to bonds and European options on bonds for both CIR and Vasicek models as well as for some more general CKLS models using the given HOC scheme with 4^{th} order of convergence.

2 PDE Frameworks

2.1 Black-Scholes Model

The dynamics of the Black-Scholes model is given as -

$$\frac{dS_t}{S_t} = (r-\delta)dt + \sigma dW_t$$

where,

$$\{S_t\}_{t\geqslant 0} := \text{Price process for the underlying stock}$$

$$\sigma > 0 := \text{Volatility constant}$$

$$r > 0 := \text{Fixed rate of return in a risk neutral economy.}$$

$$\delta := \text{Continuous dividend yield.}$$

$$W_t := \text{Standard Brownian motion.}$$

A European call option with strike price K and maturity date T gives the holder the right to buy the underlying risky asset or not at the maturity date. The payoff for this call option is thus:

$$g(S_T) = max(S_T - K, 0) = (S_T - K)^+$$

Let $V(S,t)$ denote the price of such a financial instrument at time t, with $V(S,T) = g(S_T)$. Using the Feyman-Kac theorem it can be shown that $V(S,t)$ is the solution of the Black-Scholes equation:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial_2 V}{\partial S^2} + (r-\delta)S\frac{\delta V}{\delta S} - rV = 0, \quad S > 0, 0 \leqslant t \leqslant T \tag{1}$$

The above PDE can be explicitly solved for $V(S,t)$ giving the analytical solution:

$$V(S,t) = Se^{-\delta(T-t)}\phi(d_1) - Ke^{-r(T-t)}\phi(d_2)$$

where ϕ is the distribution function of the standard normal distribution $N(0,1)$ and,

$$d_2 = \frac{log(S/K) + ((r-\delta) - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_1 = d_2 + \sigma\sqrt{T-t}$$

Similarly, for a European put option the price $P(S,T)$ can be given as:

$$V(S,t) = Ke^{-r(T-t)}\phi(-d_2) - Se^{-\delta(T-t)}\phi(-d_1)$$

2.2 The CKLS Stochastic interest rate model

Assumption: Spot rate r_t is stochastic at time t, and is governed by the stochastic differential equation:

$$dr(t) = \kappa(\theta - r(t))dt + \sigma r(t)^{\gamma} dW_t$$

. where,

$$\theta = \text{The long run mean}$$

$$\sigma = \text{Volatility}$$

$$\kappa = \text{The rate of reversion about } \theta$$

$$\gamma = \text{Parameter used for nesting various models}$$

Using no arbitrage arguments one can show that the price $V(r,t)$ of some financial instrument with payoff at maturity as g(r) is solution of the following PDE:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 r^{2\gamma}\frac{\partial^2 V}{\partial r^2} + \kappa(\theta - r)\frac{\partial V}{\partial r} - rV = 0 \tag{2}$$

The payoff function $g(r)$ can take various forms depending on the financial contract being considered. For instance for a zero-coupon bond with maturity T and face value = 1, $g(r) = 1$, and for a European call option on the zero-coupon bond with maturity $T_0 < T$ and strike price K, $g(r) = (P(r, T_0, T) - K)^+$, where $P(r, t, T)$ denotes the zero-coupon bond price at time t.

3 Numerical Methods

We described the problems that we will be considering in the section above and now we move on to solving these problems using the HOC Jain's scheme. We will first try to solve the transformed Black-Scholes equation followed by some schemes for pricing interest rate derivatives.

3.1 Numerical scheme for Black-Scholes

The following transformations are made to the Black-Scholes pde(1) to transform it to a constant coefficient problem:

$$S = Ke^x$$

$$\tau = \sigma^2(T-t)/2$$

$$p_\delta = 2(r-\delta)/\sigma^2$$

$$T' = \sigma^2 T/2$$

$$p = 2r/\sigma^2$$

This gives us the constant coefficient problem:

$$\frac{\partial V}{\partial \tau} = \frac{\partial^2 V}{\delta x^2} + (p_\delta - 1)\frac{\partial V}{\partial x} - pV, \quad -\infty < x < +\infty, 0 < \tau \leqslant T' \tag{3}$$

with boundary conditions:

$$V(x,0) = K(e^x - 1)^+, \quad -\infty < x < +\infty,$$

$$V(x,\tau) = 0, x \to -\infty$$

$$V(x,\tau) = K(e^{x-2\delta\tau/\sigma^2} - e^{-p\tau}), x \to +\infty$$

A final substitution $u(x,\tau) = e^{p\tau}V(x,\tau)$ is used to reduce (3) to:

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2} + (p_\delta - 1)\frac{\partial u}{\partial x}, \quad -\infty < x < +\infty, 0 < \tau \leqslant T' \tag{4}$$

with boundary conditions:

$$u(x,0) = K(e^x - 1)^+, \quad -\infty < x < +\infty,$$

$$u(x,\tau) = 0, x \to -\infty$$

$$u(x,\tau) = K(e^{x+(p-2\delta/\sigma^2)\tau} - 1), x \to +\infty$$

Since, the domain is unbounded along $x-axis$ it is required that we truncate it to some finite domain $\Omega = (x_{min}, x_{max}) \times [0, T']$.

Let M and N be the number of divisions along the x and time directions respectively. Then the mesh spacings are $h = (x_{max} - x_{min}/M)$ along the x-direction and $k = T'/N$ along the time direction. Thus we get a uniform mesh of grid points

$$\Omega_\Delta = \{(x_m, \tau_n) \in \Omega, x_m = x_{min} + mh, 0 \leqslant m \leqslant M, \tau_n = nk, 0 \leqslant k \leqslant N\}$$

Taking $b = (1 - p_\delta)$ we have from (4):

$$\frac{\partial^2 u}{\partial x^2} = b\frac{\partial u}{\partial x} + \frac{\partial u}{\partial \tau} = f(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial \tau})$$

The Jain's scheme derived from Numerov discretisation of the form:

$$-\frac{1}{h^2}\delta_x^2 u_m^{n+1/2} + \frac{1}{12}(f_{m+1}^{n+1/2} + 10f_m^{n+1/2} + f_{m-1}^{n+1/2}) = 0 \tag{5}$$

Using the standard approximations we get the scheme:

$$(\beta_{-1}-\gamma_{-1})u_{m-1}^{n+1}+(1-2\beta_{-1})u_m^{n+1}+(\beta_{-1}+\gamma_{-1})u_{m+1}^{n+1} = (\beta_1+\gamma_1)u_{m-1}^n+(1-2\beta_1)u_m^n+(\beta_1-\gamma_1)u_{m+1}^n \tag{6}$$

where,

$$\beta_{-1} = \frac{1}{12} - \frac{k}{2h^2} - \frac{b^2 k}{24}$$
$$\beta_1 = \frac{1}{12} + \frac{k}{2h^2} + \frac{b^2 k}{24}$$
$$\gamma_{-1} = b(\frac{k}{4h} - \frac{h}{24})$$
$$\gamma_1 = b(\frac{k}{4h} + \frac{h}{24})$$

We let $U^n = [u_0^n, u_1^n \cdots u_M^n]$ denote the vector of our numerical solutions at time level n.

Then we can write (6) in matrix form as:

$$AU^{n+1} = BU^n, \quad n \geqslant 0$$

where,

$$A = tridiag[\beta_{-1} - \gamma_{-1}, 1 - 2\beta_{-1}, \beta_{-1} + \gamma_{-1}]$$

$$B = tridiag[\beta_{-1} + \gamma_{-1}, 1 - 2\beta_{-1}, \beta_{-1} - \gamma_{-1}]$$

3.2 Numerical scheme for CKLS

We see a high order fully discretised scheme for pricing bonds and bond options. We start with the numerical scheme for pricing a unit discount bond, that is a zero coupon bond with face value of one dollar.

We use the substitution $\tilde{\tau} = T - t$ in (2) to get a forward problem where the discount bond price at time $\tilde{\tau}$, denoted by $P(r, \tilde{\tau}, T)$ is the solution of:

$$\frac{\partial P}{\partial \tilde{\tau}} = \frac{1}{2}\sigma^2 r^{2\gamma}\frac{\partial^2 P}{\partial r^2} + \kappa(\theta - r)\frac{\partial V}{\partial r} - rP \tag{7}$$

with initial condition $P(r, 0, T) \equiv 1$.

We again need to truncate the r domain in a fashion similar to the Black-Scholes case to be able to apply the numerical scheme. So, $\Omega_r = (r_m in, r_{max})$

Again let M and N be the number of divisions along the r and time directions respectively. Then the mesh spacings are $h = (r_{max} - r_{min}/M)$ along the $r - direction$ and $k = T'/N$ along the time direction. Thus we get a uniform mesh of grid points

$$\Omega_\Delta = \{(r_m, \tau_n) \in \Omega, r_m = r_{min} + mh, 0 \leqslant m \leqslant M, \tau_n = nk, 0 \leqslant k \leqslant N\}$$

We again express (7) in the form:

$$\frac{\partial^2 P}{\partial r^2} = \frac{2}{\sigma^2 r^{2\gamma}}(\frac{\partial P}{\partial \tilde{\tau}} - \kappa(\theta - r)\frac{\partial P}{\partial r} + rP) = f(r, p, \frac{\partial P}{\partial \tilde{\tau}}, \frac{\partial P}{\partial r}) \tag{8}$$

Let P_m^n denote the bond price at the grid point (r_m, t_n), then we obtain the Jain's scheme for (8) from (5) after some standard approximations of the derivative terms as:

$$b_{m-1}P_{m-1}^{n+1} + b_m P_m^{n+1} + b_{m+1}P_{m+1}^{n+1} = c_{m-1}P_{m-1}^n + c_m P_m^n + c_{m+1}P_{m+1}^n \tag{9}$$

where,

$$b_{m\pm1} = r_{m-1}^{-2\gamma} r_m^{-2\gamma} r_{m+1}^{-2\gamma}\left((\sigma)^2 r_m^{2\gamma}\left[\pm hk(\xi)_{m\pm1} r_{m\pm1}^{2\gamma} + r_{m\pm1}^{2\gamma}\left(4h^2 \pm 3hk(\xi)_{m\pm1} + 2h^2 kr_{m\pm1} - 12k(\sigma)^2 r_{m\pm1}^{2\gamma}\right)\right]\right)$$
$$\left(\pm h(\xi)_m\left(\pm hk(\xi)_{m\pm1} r_{m\pm1}^{2\gamma} + r_{m\pm1}^{2\gamma}\left[4h^2 \pm 3hk(\xi)_{m\pm1} + 2h^2 kr_{m\pm1} - 10k(\sigma)^2 r_{m\pm1}^{2\gamma}\right]\right)\right),$$

$$c_{m\pm1} = r_{m-1}^{-2\gamma} r_m^{-2\gamma} r_{m+1}^{-2\gamma}\left((\sigma)^2 r_m^{2\gamma}\left[\pm hk(\xi)_{m\pm1} r_{m\pm1}^{2\gamma} + r_{m\pm1}^{2\gamma}\left(4h^2 \pm 3hk(\xi)_{m\pm1} - 2h^2 kr_{m\pm1} + 12k(\sigma)^2 r_{m\pm1}^{2\gamma}\right)\right]\right)$$
$$\left(+h(\xi)_m\left(hk(\xi)_{m\pm1} r_{m\pm1}^{2\gamma} + r_{m\pm1}^{2\gamma}\left[\pm 4h^2 + 3hk(\xi)_{m\pm1} \pm 2h^2 kr_{m\pm1} \pm 10k(\sigma)^2 r_{m\pm1}^{2\gamma}\right]\right)\right),$$

$$
\begin{aligned}
b_m &= 4r_m^{-2\gamma}\left(hk\left(\xi\right)_{m-1} r_{m-1}^{-2\gamma}\left(h\left(\xi\right)_m + \left(\sigma\right)^2 r_m^{2\gamma}\right)\right)\\
&\quad + r_{m+1}^{-2\gamma}\left(\left[h^2k\left(\xi\right)_m\left(\xi\right)_{m+1} + \left(\sigma\right)^2\left(-hk\left(\xi\right)_{m+1} r_m^{2\gamma} + r_{m+1}^{2\gamma}\left[10h^2 + 5h^2kr_m + 6k\left(\sigma\right)^2 r_m^{2\gamma}\right]\right)\right]\right)\\
c_m &= 4r_m^{-2\gamma}\left(hk\left(\xi\right)_{m+1} r_{m+1}^{-2\gamma}\left(-h\left(\xi\right)_m + \left(\sigma\right)^2 r_m^{2\gamma}\right)\right) + r_{m-1}^{-2\gamma}\left(\left[-h^2k\left(\xi\right)_m\left(\xi\right)_{m-1} - hk\left(\sigma\right)^2\left(\xi\right)_{m-1} r_m^{2\gamma}\right]\right.\\
&\quad \left. + \left(\sigma\right)^2\left[10h^2 - 5h^2kr_m - 6k\left(\sigma\right)^2 r_m^{2\gamma}\right]\right.
\end{aligned}
$$

We let $P^n = [P_0^n, P_1^n \cdots P_M^n]$ denote the vector of our numerical solutions at time level n.

Then we can write (6) in matrix form as:

$$
AP^{n+1} = BP^n, \quad n \geqslant 0 \tag{10}
$$

where,

$$
\begin{aligned}
&A = tridiag[b_{m-1}, b_m, b_{m+1}]\\
&B = tridiag[c_{m-1}, c_m, c_{m+1}]\\
&P^0 = \mathbf{1}, \text{ is the initial condition}
\end{aligned}
$$

3.3 Pricing bond options

Both bond prices and bond option prices satisfy (7) given appropriate boundary conditions. The boundary conditions for European Call option with maturity T_0 and strike price K on a discount bond with maturity T. We solve the numerical scheme (9) using (10) for $V(r, \tau^*, T_0)$ which is the option price at time τ^* and satisfies (7) with the initial condition $V(r, 0, T) = (P(r, T_0, T) - K)^+$. Here $\tau^* = (T_0 - t)$.

3.4 Pricing coupon bonds

Now let us consider pricing a bond with face value $\breve{f}$, which makes annual coupon payments of amount $\breve{a}$ at regular intervals. The price for such a bond is obtained by solving (10) with the initial condition $P(r, 0, T) = \breve{f} + \breve{a}$. At each time level P_{n+1} is obtained by computing $P(r, \tau^*, T)$, and if the time level coincides with a coupon payment date, we simply add the coupon, *e.i.* $P(r, \tau^*, T) = P(r, \tau^*, T) + \breve{a}$.

3.5 European Call option on a coupon bond

If the underlying security is a coupon paying bond with maturity T, and the option has a maturity T_0, we solve (10) using the initial condition $V(r, 0, T_0) = (P(r, T_0, T) - K)^+$, and at each time level we compute $V(r, \tau_{n+1}^*, T_0)$ using $V(r, \tau_n^*, T_0)$ as the initial condition. The coupon amount $\breve{a}$ is then added if the time level corresponds to a coupon payment date similar to previous case.

3.6 Bermudan call option on a coupon bond

A Bermudan call option allows us to exercise the option on only some specified dates. We again solve (10) using the initial condition $V(r, 0, T_0) = (P(r, T_0, T) - K)^+$ similar to the previous case, and add the coupon amount $\breve{a}$ if the time level corresponds to a coupon payment date. Now if the time level corresponds to one of the exercise dates then, the option price becomes $V(r, \tau_{n+1}^*, T_0) = max(V(r, \tau_{n+1}^*, T_0), (P(r, \tau_{n+1}^*, T) - K)^+)$.

4 Experiments & Results

Finally we perform various experiments with the above mentioned numerical schemes for pricing European options and various other interest rate derivatives.

Crank Nicolson				
M	Price	Error	Order	cpu(s)
10	0.494119	4.299682e-04	-	0.126701
20	0.493887	9.006585e-04	-1.066750	0.001917
40	0.494358	5.215585e-05	4.110079	0.001609
80	0.494334	3.025054e-06	4.107796	0.009457
160	0.494332	9.626092e-07	1.651939	0.193439
320	0.494332	2.520807e-07	1.933065	1.891023
Exact Price = 0.494332				

Table 1: Bond prices under the CIR model for T = 5

Jains Scheme				
M	Price	Error	Order	cpu(s)
10	0.077237	8.437550e-01	-21.674507	0.134101
20	0.496278	3.936669e-03	7.743705	0.002160
40	0.494340	1.558863e-05	7.980338	0.001976
80	0.494332	9.016281e-07	4.111818	0.010164
160	0.494332	6.187253e-08	3.865162	0.147673
320	0.494332	3.841779e-09	4.009452	1.758576
Exact Price = 0.494332				

Table 2: Bond prices under the CIR model for T = 5

CIR Model				
M	Price	Error	Order	cpu(s)
10	-0.070230	2.036439e+00	-	0.111856
20	0.068492	1.078395e-02	7.561019	0.004561
40	0.067783	3.310128e-04	5.025855	0.007088
80	0.067761	2.553277e-06	7.018393	0.055595
160	0.067761	5.493039e-08	5.538602	0.865904
320	0.067761	1.704477e-09	5.010203	11.254604
Exact Price = 0.067761				

Table 3: CIR bond prices for T = 30

Vasicek Model				
M	Price	Error	Order	cpu(s)
10	0.225421	1.746344e-02	-23.288505	0.001316
20	0.220957	2.686124e-03	2.700740	0.000553
40	0.220960	2.674648e-03	0.006177	0.001816
80	0.221128	1.912379e-03	0.483980	0.016452
160	0.221373	8.102519e-04	1.238926	0.263427
320	0.221544	3.594945e-05	4.494329	2.778937
CN Price = 0.221552				

Table 4: Vasicek bond prices for T = 30

M	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$	$\gamma = 1$
10	0.263470	-0.045416	0.877085	-7.068155
20	0.495876	0.486904	0.500708	0.566150
40	0.495851	0.493174	0.491772	0.492524
80	0.495872	0.493243	0.491933	0.491273
160	0.495876	0.493244	0.491934	0.491281
320	0.495877	0.493244	0.491934	0.491281
CN	0.495877	0.493244	0.491935	0.491281

Table 5: Bond prices under CKLS model for different values of the parameter γ

Bond options - Jain's Scheme $T_0 = 5$				
M	Price	Error	Order	cpu(s)
10	-0.630244	6.959978e+00	-	0.114196
20	0.093379	1.169493e-01	5.895127	0.004168
40	0.107834	1.974350e-02	2.566434	0.003878
80	0.105735	1.048494e-04	7.556915	0.027074
160	0.105745	1.184094e-05	3.146464	0.484453
320	0.105746	5.039296e-07	4.554417	5.361848
CN Price = 0.105746				

Table 6: European bond option prices under the CIR model for option maturities $T_0 = 5$

Bond options - Jain's Scheme $T_0 = 2$				
M	Price	Error	Order	cpu(s)
10	-0.903869	6.502737e+00	-	0.001319
20	0.146344	1.090612e-01	5.897837	0.000740
40	0.170098	3.555198e-02	1.617136	0.002763
80	0.164403	8.793394e-04	5.337366	0.025284
160	0.164417	9.673465e-04	-0.137613	0.412210
320	0.164258	4.934918e-07	10.936791	5.251805
CN Price = 0.164258				

Table 7: European bond option prices under the CIR model for option maturities $T_0 = 2$

M	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$	$\gamma = 1$
10	-0.248573	-0.882009	3.999268	202.308964
20	0.079077	0.096393	0.350941	-0.117450
40	0.096527	0.102405	0.067157	-0.028959
80	0.104636	0.104537	0.103076	0.077795
160	0.106719	0.104502	0.103184	0.102636
CN	0.107358	0.104502	0.103184	0.102636

Table 8: European call option prices under CKLS for different values of the parameter γ

Crank Nicolson				
M	Price	Error	Order	cpu(s)
10	0.669083	2.291907e-01	-	0.002466
20	0.668785	2.286436e-01	0.003448	0.000886
40	0.543442	1.628485e-03	7.133426	0.001454
80	0.544090	4.372947e-04	1.896853	0.014356
160	0.544271	1.045224e-04	2.064793	0.141440
320	0.544317	2.085409e-05	2.325410	1.816633
Exact Price = 0.544328				

Table 9: Coupon bond prices under the CIR model with an annual coupon of 5% and face value 100

Jains Scheme				
M	Price	Error	Order	cpu(s)
10	0.225386	5.859382e-01	-14.778130	0.149883
20	0.672187	2.348919e-01	1.318752	0.002340
40	0.550025	1.046621e-02	4.488186	0.001879
80	0.547292	5.445476e-03	0.942609	0.009910
160	0.545112	1.440180e-03	1.918809	0.156448
320	0.544455	2.323919e-04	2.631618	1.815153
Exact Price = 0.544328				

Table 10: Coupon bond prices under the CIR model with an annual coupon of 5% and face value 100

Crank Nicolson				
M	Price	Error	Order	cpu(s)
10	0.780053	3.746525e+00	-	0.003316
20	0.777185	3.729079e+00	0.006734	0.001655
40	0.161900	1.486120e-02	7.971125	0.002517
80	0.163737	3.682374e-03	2.012843	0.023734
160	0.164220	7.394183e-04	2.316173	0.270184
320	0.164342	6.084912e-12	26.856579	3.570016
Exact Price = 0.164342				

Table 11: European option for CIR model on bond with face value 100 with an annual coupon of 10% compounded semiannually

Jains Scheme				
M	Price	Error	Order	cpu(s)
10	0.438384	1.667512e+00	-37.995597	0.198804
20	0.745727	3.537657e+00	-1.085097	0.003461
40	0.171579	4.403502e-02	6.327999	0.004141
80	0.170064	3.482024e-02	0.338725	0.026837
160	0.166132	1.089452e-02	1.676324	0.460876
320	0.164657	1.915871e-03	2.507530	5.796097
Exact Price = 0.164342				

Table 12: European option for CIR model on bond with face value 100 with an annual coupon of 10% compounded semiannually

Strike Price(K)					
M	95.0	97.5	100	102.5	105.0
0.5	0.753720(0.753650)	0.741362(0.741292)	0.729005(0.728936)	0.716725(0.716656)	0.705808(0.705738)
1	0.780780(0.780667)	0.768421(0.768308)	0.756064(0.755951)	0.743716(0.743602)	0.731438(0.731325)
1.5	0.807165(0.806974)	0.794806(0.794616)	0.782448(0.782258)	0.770092(0.769902)	0.757747(0.757557)
2	0.832819(0.832514)	0.820461(0.820155)	0.808103(0.807797)	0.795745(0.795439)	0.783389(0.783084)
3	0.881831(0.881168)	0.869473(0.868809)	0.857114(0.856451)	0.844756(0.844093)	0.832398(0.831735)
4	0.927745(0.926542)	0.915387(0.914184)	0.903029(0.901826)	0.890671(0.889468)	0.878312(0.877109)

Table 13: European option for CIR model on bond with face value 100 and an annual coupon of 10% compounded semiannually at different strike prices

Crank Nicolson				
M	Price	Error	Order	cpu(s)
16	0.476050	2.215275e-01	-	0.002723
32	0.475983	2.213535e-01	0.001134	0.000992
64	0.389114	1.548907e-03	7.158958	0.002705
128	0.389575	3.647856e-04	2.086129	0.062997
256	0.483218	2.399198e-01	-9.361288	0.689437
Exact Price = 0.389717				

Table 14: Bermudan put option under CIR model on a bond with face value 1000 with coupon of 4% compounded annually

Jains Scheme				
M	Price	Error	Order	cpu(s)
16	0.475490	2.200898e-01	0.124460	0.151726
32	0.475993	2.213802e-01	-0.008434	0.003406
64	0.389160	1.429594e-03	7.274776	0.004711
128	0.389665	1.348059e-04	3.406650	0.109706
256	0.483437	2.404822e-01	-10.800831	0.610421
Exact Price = 0.389717				

Table 15: Bermudan put option under CIR model on a bond with face value 1000 with coupon of 4% compounded annually

5 Limitations and future work

The results for HOC Jain scheme fail to beat those obtained from Crank Nicolson. The only reason we could come up with was missing mesh refinement techniques in our implementation at boundaries. Although the paper itself is somewhat ambiguous on its implementation of mesh refinement techniques for the presented results, we came across a few other papers that document the necessity of using mesh refinement for such a HOC scheme at the boundaries.