UNISYS INNOVATION PROGRAM

PROJECT TITLE :- STRUCTURAL ANALYSIS USING QUANTUM COMPUTING

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STRUCTURAL ANALYSIS USING QUANTUM COMPUTING FOR KATHIPARA BRIDGE

INTRODUCTION:

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This project leverages the Variational Quantum Linear Solver (VQLS) to optimize Finite Element Analysis (FEA) for the Kathipara Flyover, enhancing structural assessment through quantum parallelism.
By tackling computational challenges in large-scale stress analysis, the hybrid quantum-classical approach enables efficient real-time evaluation and long-term performance optimization.

Approach Summary:

This project employs a hybrid quantum-classical method for solving FEA problems efficiently. Key steps include:
Problem Formulation: Discretizing structural elements via FEA, leading to a large sparse linear system.

Preprocessing: Applying classical preconditioning to improve quantum solver convergence.

VQLS Implementation: Mapping FEA equations to quantum circuits using parameterized quantum models.

Hybrid Optimization: Using classical optimizers (e.g., gradient descent, Nelder-Mead) to refine quantum circuit parameters

Validation: Comparing quantum results with classical solvers for accuracy and performance assessment.

FEA (Finite Element Analysis)

WHAT IS FEA?

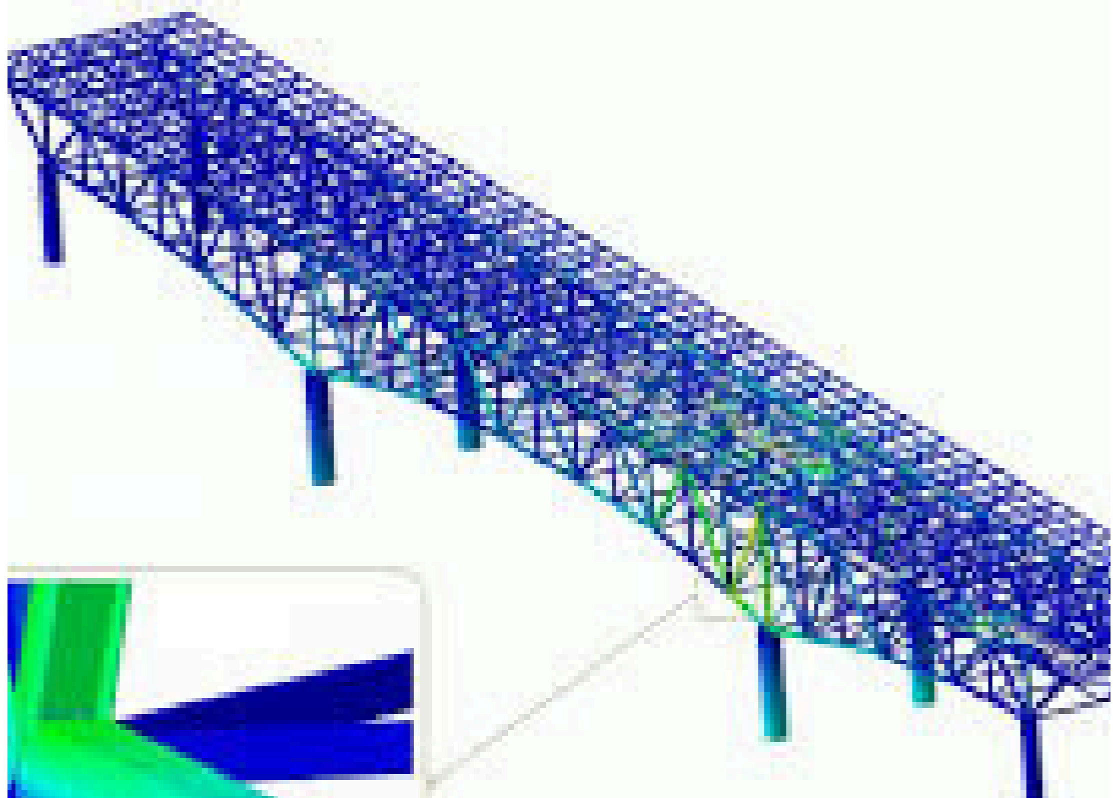
FEA predicts structural responses by discretizing complex structures into smaller elements to analyze stress, deformation, and failure points under various forces.

WHY IS FEA USED IN CIVIL ENGINEERING?

- **Structural Safety:** Ensures infrastructures withstand expected loads.
- **Material Optimization:** Aids in selecting materials by analyzing stress-strain behavior.
- **Predicting Failures:** Identifies weak points to prevent structural issues.
- **Cost Efficiency:** Reduces physical testing through simulations.

FEA PROCESS FOR KATHIPARA BRIDGE

- **Model Creation:** Develop a 3D structural model for accurate simulation.
- **Meshing:** Discretize the structure into finite elements for precise analysis.
- **Load Application:** Apply traffic, wind, seismic, and dead loads for evaluation.
- **Boundary Conditions:** Define support constraints to simulate real-world behavior.
- **Analysis & Results:** Identify stress concentration, displacement, and failure zones for optimization.



Finite Element Analysis (FEA) Mathematical Model for Kathipara Flyover

The Kathipara Flyover is modeled using FEA by discretizing it into finite elements and applying boundary conditions to simulate real-world constraints. Material properties and computational methods help analyze stress, deformation, and potential failure points for structural optimization.

1.Structural Modeling Using FEM

The Kathipara Flyover is discretized into finite elements like beams, trusses, and plates, with beam elements used for primary analysis. The governing equation for a beam element follows the Euler-Bernoulli beam theory:

$$EI\frac{d^4w}{dx^4} = q(x)$$

2. Element Stiffness Matrix Formulation

For a beam element, the stiffness matrix Ke is given by:

$$K^e = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

3. Load and Boundary Conditions

Traffic Load Modeling For dynamic traffic load, we use the moving load function:

$$F(x,t) = P\delta(x - vt)$$

4. Dynamic Analysis (Modal Analysis)

To analyze natural frequencies and vibrations, we solve the eigenvalue problem:

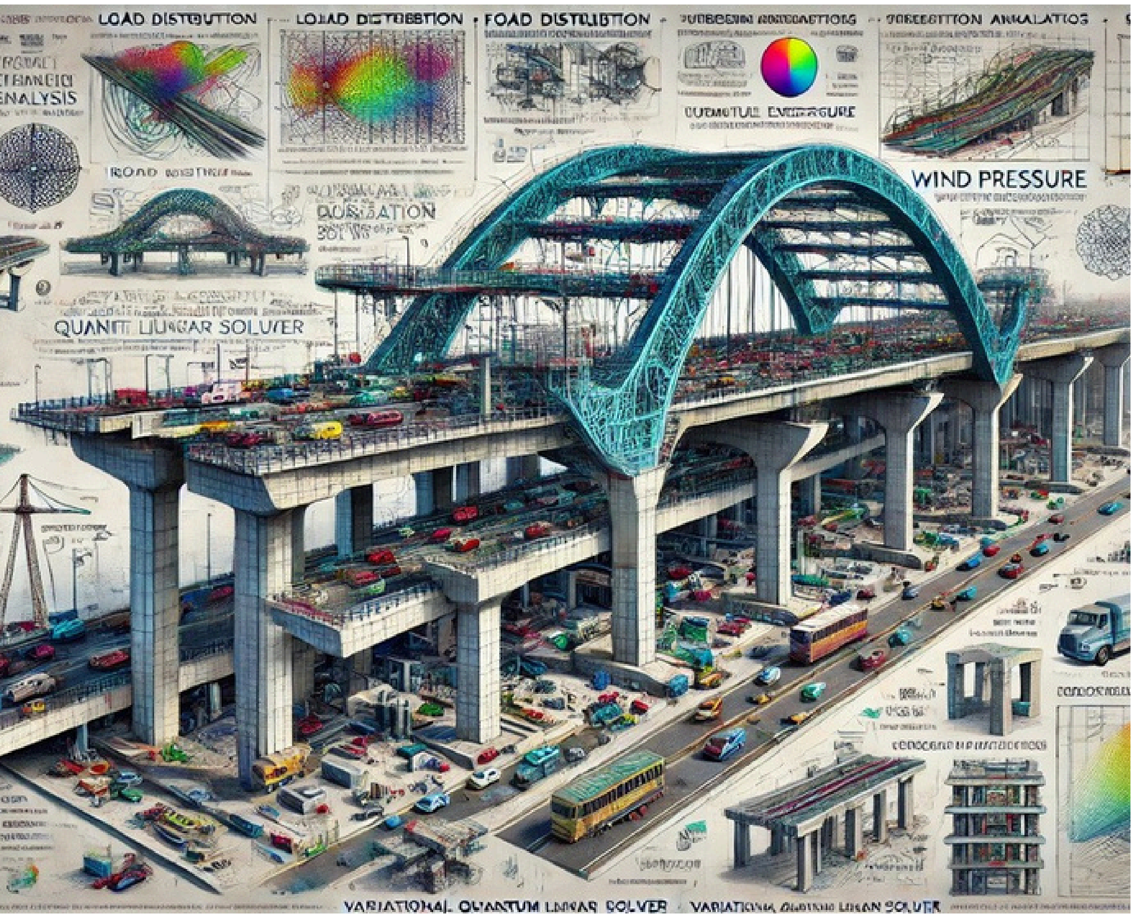
$$(K - \lambda M)\Phi = 0$$

5. Seismic & Wind Load Analysis

For seismic loading, we model the ground motion using:

$$M\ddot{U} + C\dot{U} + KU = F_{\text{seismic}}$$

STRUCTURAL ANALYSIS



Introduction to Structural Analysis

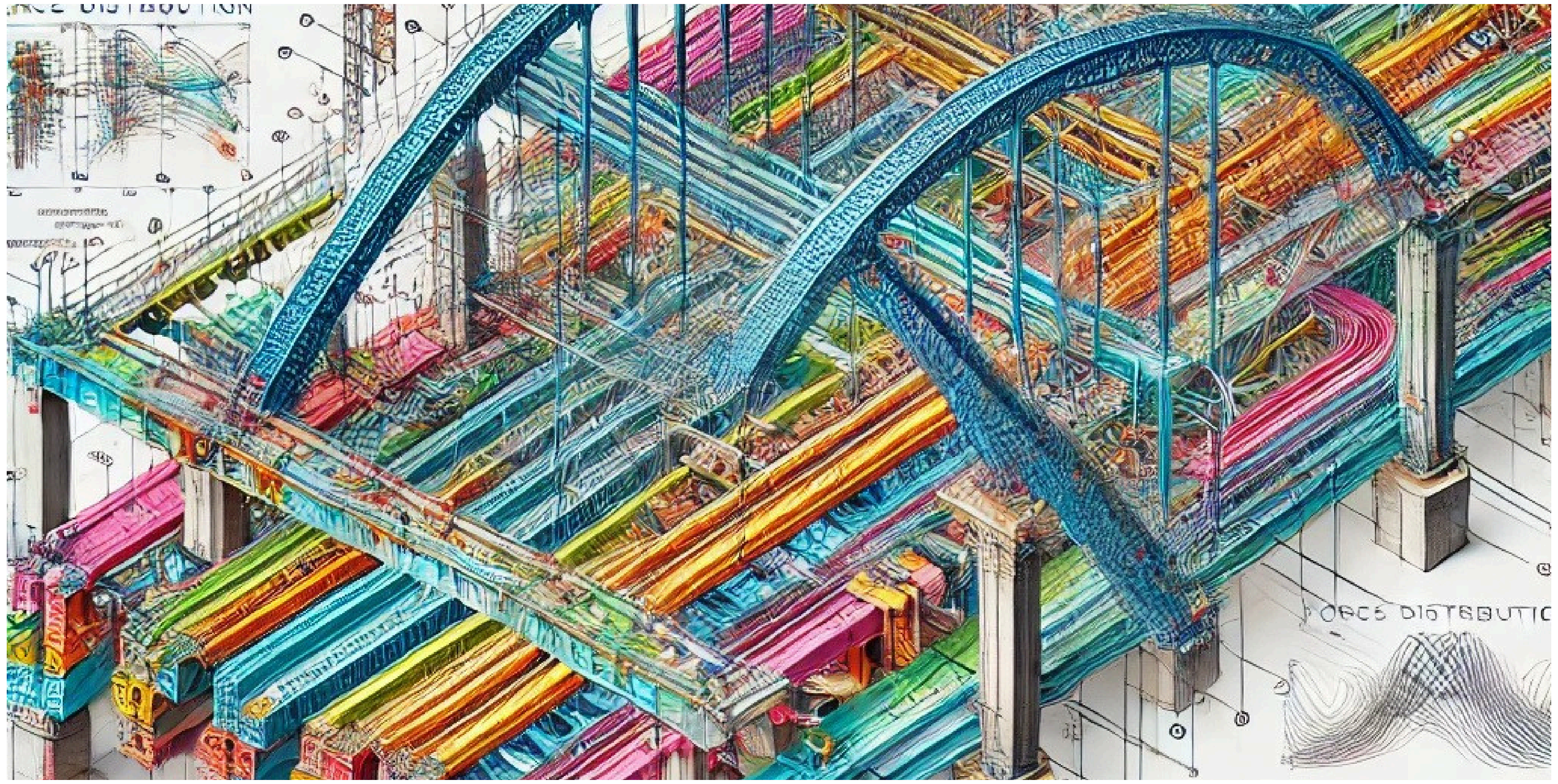
In civil engineering, a structure is a system of interconnected members designed to support external loads. Structural analysis predicts how a structure responds to these loads by estimating forces, stresses, and displacements. During preliminary design, expected loads determine member sizes, ensuring safety and compliance with building codes.

What is Structure Analysis in Civil Engineering ?

Structural analysis helps civil engineers assess the strength and stability of structures like buildings, bridges, and towers. It involves calculating internal forces, moments, and deformations in structural members under applied loads.

STRUCTURAL ANALYSIS

The primary objectives of structural analysis include:



Safety	Ensures structural integrity under static and dynamic loads.
Serviceability	Maintains permissible deflection, vibration, and deformation limits.
Cost Efficiency	Cost Efficiency: Optimizes material usage while minimizing construction and maintenance costs.

Applications of Structure Analysis in Civil Engineering

1. Bridge Design
2. Dams and Hydraulic Structures
3. Building Design

Mathematical Framework for Structural Analysis

Mathematical Model of Structural Analysis for Kathipara Bridge

The Kathipara Flyover's structural analysis uses continuum mechanics, beam theory, and FEM to assess load-bearing capacity. Dynamic analysis evaluates its response to traffic, wind, and seismic forces, ensuring stability and durability.

Computational Methods in Structural Analysis

- **Structural Stability:** Analyzes load distribution under traffic, wind, and seismic forces.
- **Linear Solvers:** Enable quick computation of deformation patterns and structural responses.
- **Classical Mathematical Models:** Assess stress conditions, temperature changes, and seismic effects.
- **Quantum Mathematical Models:** Accelerate analysis for real-time monitoring and safety enhancements

Mathematical Models for Structural Analysis of Kathipara Bridge

Linear Algebra

Purpose:

Linear algebra is fundamental to formulating equilibrium equations and matrix operations in Finite Element Analysis (FEA) of the Kathipara Bridge.

Applications:

- Stiffness Matrix Representation: The global stiffness matrix [K][K][K] is constructed to model the force-displacement relationship:

where:

- [K] [K] [K] i s t h e g l o b a l s t i f f n e s s m a t r i x .
- {U}\{U\}{U} is the nodal displacement vector.
- {F}\{F\}{F} is the external force vector (traffic loads, wind, seismic forces).

Mass and Damping Matrix in Vibration Analysis

$$[M]\{\ddot{U}\}+[C]\{\dot{U}\}+[K]\{U\}=\{F\}$$

where:

- [M][M][M] is the mass matrix (representing structural inertia).
- [C][C][C] is the dampingmatrix(representing energy dissipation).
- {U¨}are acceleration, velocity, and displacement vectors.

Quantum Mathematical Models

Purpose:

Quantum algorithms, such as Variational Quantum Linear Solver (VQLS), enhance computational speed in solving large structural equations for real-time analysis of the Kathipara Bridge.

Applications:

- **VQLS for FEA:**
  - Used for solving large-scale stiffness matrices faster than classical solvers.
  - Minimizes the residual error using quantum optimization: **θmin⟨ψ(θ) ∣ H ∣ ψ(θ)⟩**
- **Quantum Phase Estimation for Vibration Analysis:**
  - Computes eigenvalues for dynamic response analysis :**e iHt ∣ ψ⟩=eiλt ∣ ψ⟩**
- **Hybrid Quantum-Classical Approach:**
  - Integrates quantum algorithms with classical FEA solvers for improved speed and efficiency in analyzing the Kathipara Bridge.

Linear Solver

1.**Purpose:**–

To solve large systems of equations derived from FEA efficiently, ensuring accurate stress distribution and deformation analysis for the bridge.

Applications:

- **Classical Solvers:**

1.Direct Solvers (LU & Cholesky Decomposition): Factorizes the stiffness matrix for small-scale substructures

$$[K]=[L][U]$$

2.terative Solvers (CGM & GMRES): Used for sparse matrices in large-scale bridge modeling

xk+1=xk+αkpk

Quantum Solvers (VQLS):

- Reduces computational complexity in solving high-dimensional structural models.
- Uses quantum circuits to iteratively approximate solutions.
- Enhances the efficiency of solving stiffness and mass matrix equations for structural analysis.

Classical Mathematical Models

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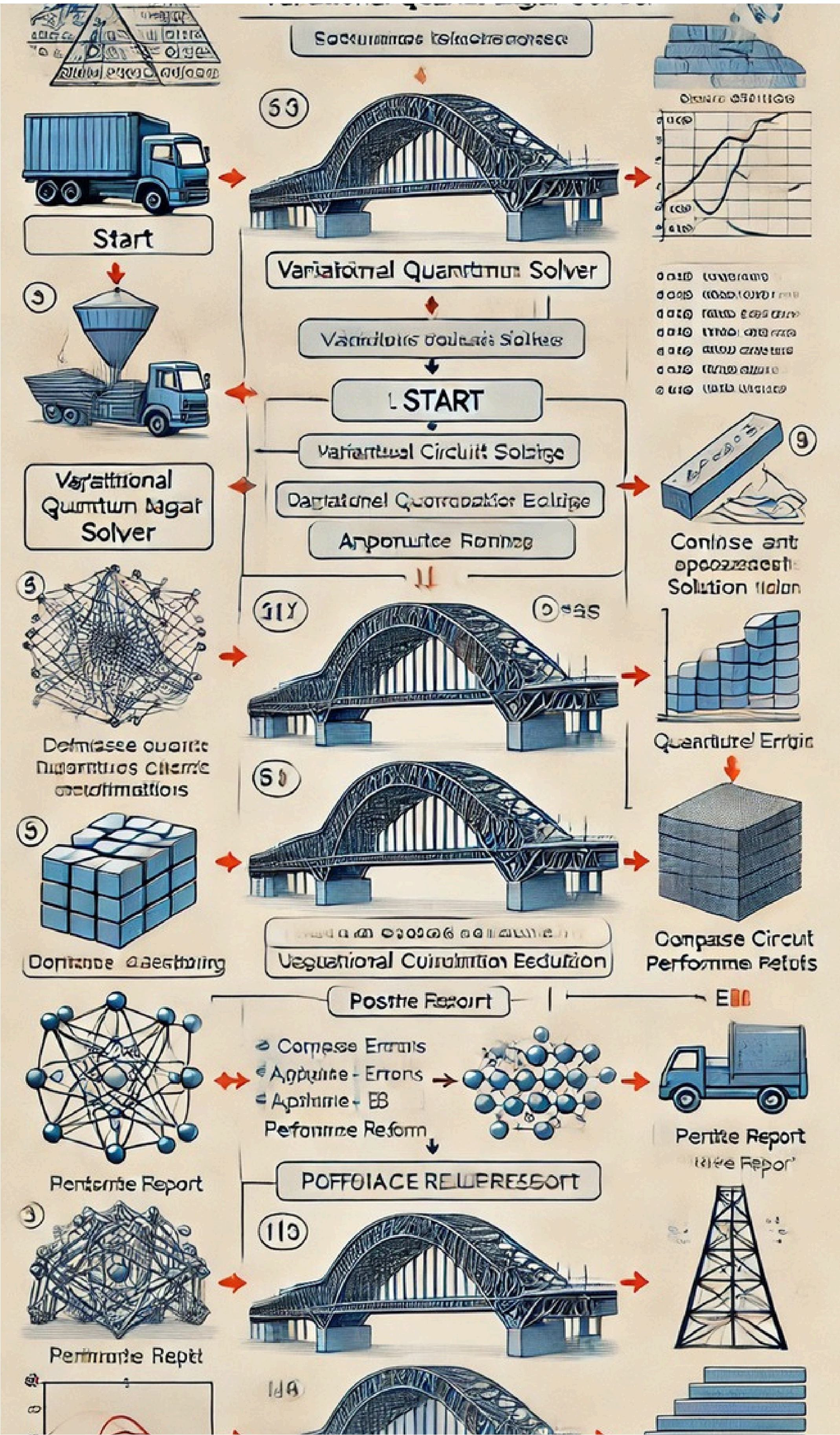
- **Purpose:**
- Classical mathematical models provide numerical techniques for stress-strain analysis, load distribution, and stability checks in the Kathipara Bridge.
- **Applications:**
- Finite Element Method (FEM):
- Discretizes the bridge structure into elements to compute stress and strain.
- **Element stiffness matrix:**

- **[Ke]=∫VBTDBdV**

- **BBB is the strain-displacement matrix.**
- **DDD is the material stiffness matrix.**
- **VVV is the element volume.**

VARIATIONAL QUANTUM LINEAR SOLVER (VQLS) IN STRUCTURAL ANALYSIS



what is vqls

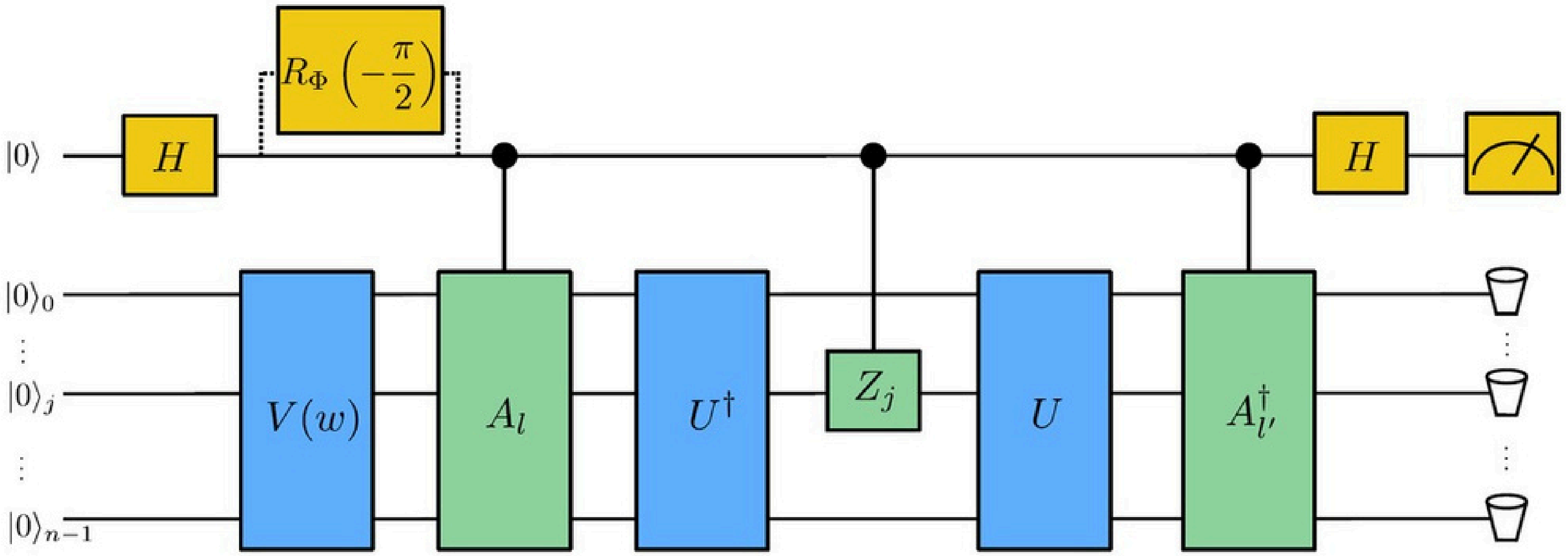
VQLS efficiently solves large linear systems in structural analysis, aiding in stiffness matrix computations for the Kathipara Bridge. By leveraging quantum computing, it accelerates FEA, enabling real-time structural simulations.

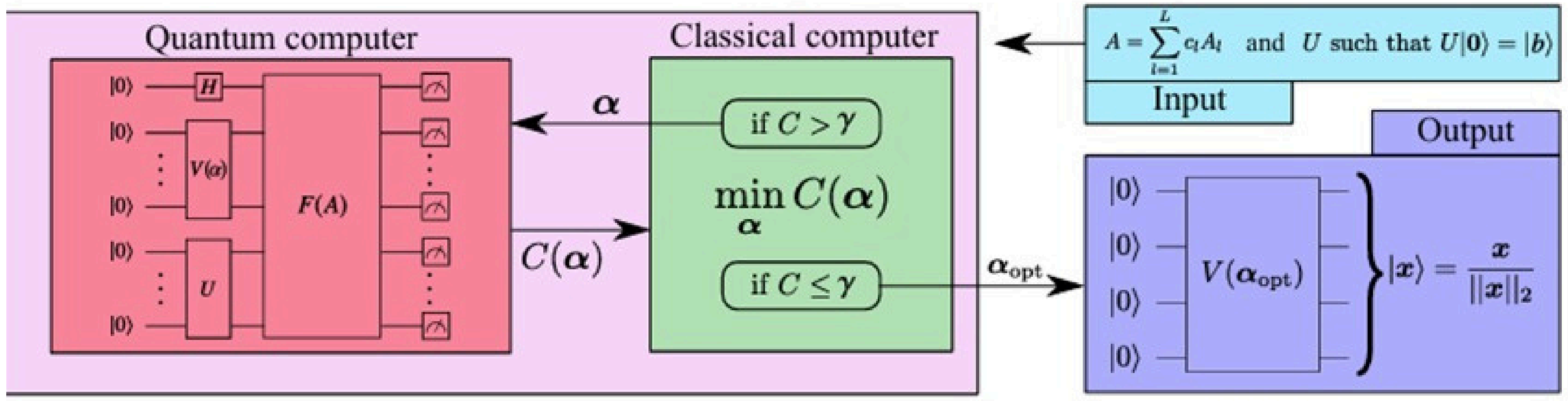
Why Variational Quantum Linear Solver (VQLS) ?

In FEA, structural problems are formulated as large linear systems, making traditional solvers like Direct Stiffness Method and Conjugate Gradient computationally expensive for large structures. VQLS, a hybrid quantum-classical algorithm, efficiently solves these equations by encoding them into a quantum state and using a variational approach for iterative approximation.

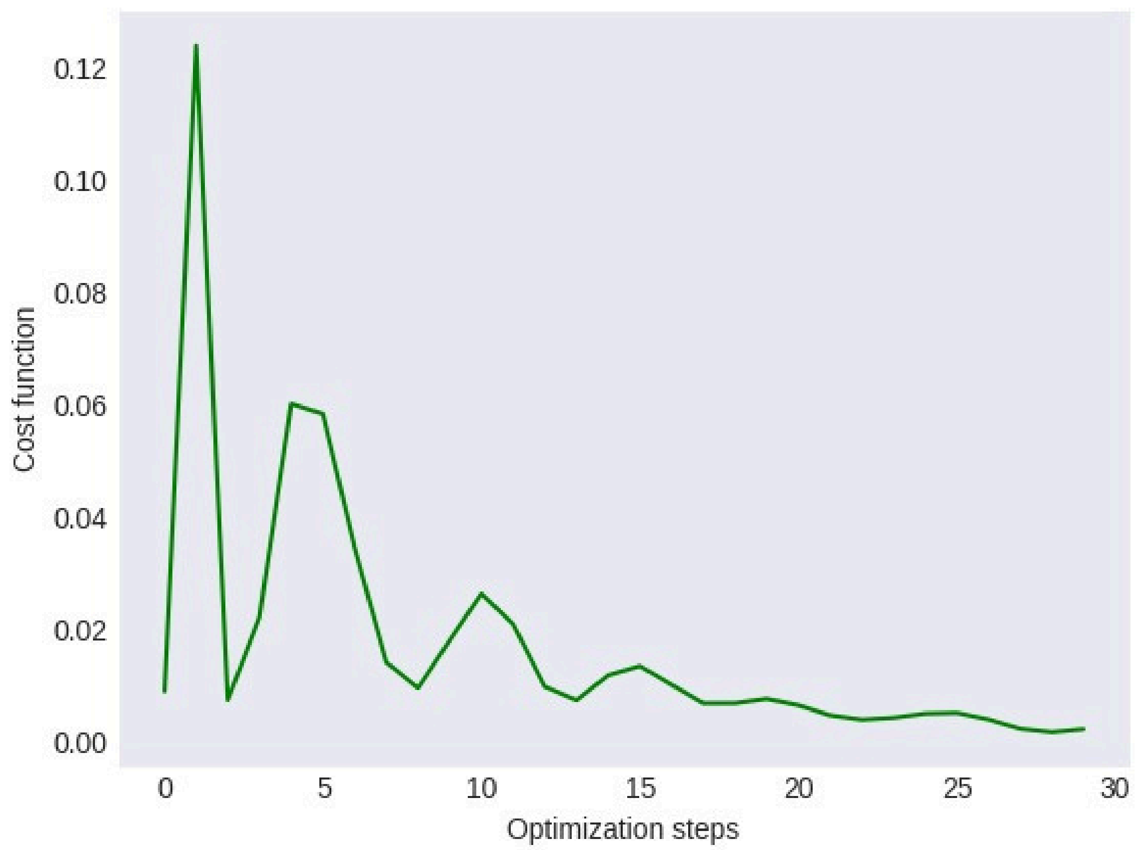
Schematic Diagram of the VQLS Algorithm

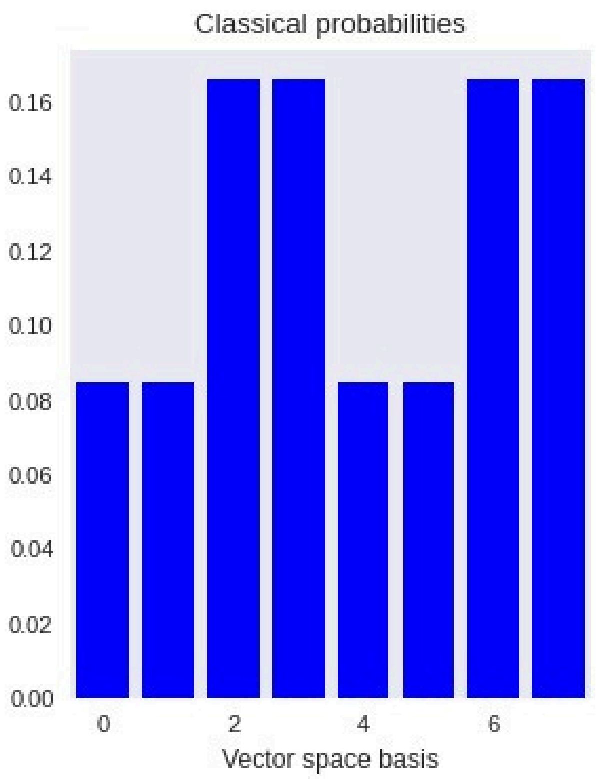
VQLS solves Ax = b by encoding A as unitaries and preparing |b〉 with a quantum circuit. A hybrid optimization loop adjusts V(α) until the cost C(α) meets a threshold, yielding |x〉, an approximate solution for computing observables with bounded error.

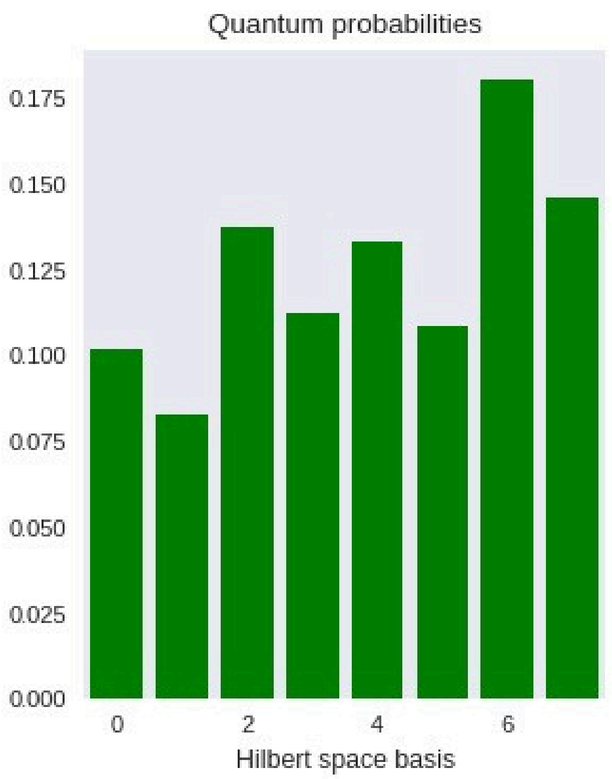




VARIATIONAL QUANTUM LINEAR SOLVER (VQLS) IN STRUCTURAL ANALYSIS







PennyLane for VQLS in Structural Analysis

PennyLane enables hybrid quantum-classical computing for VQLS in Kathipara Bridge analysis, optimizing FEA with quantum speedup.

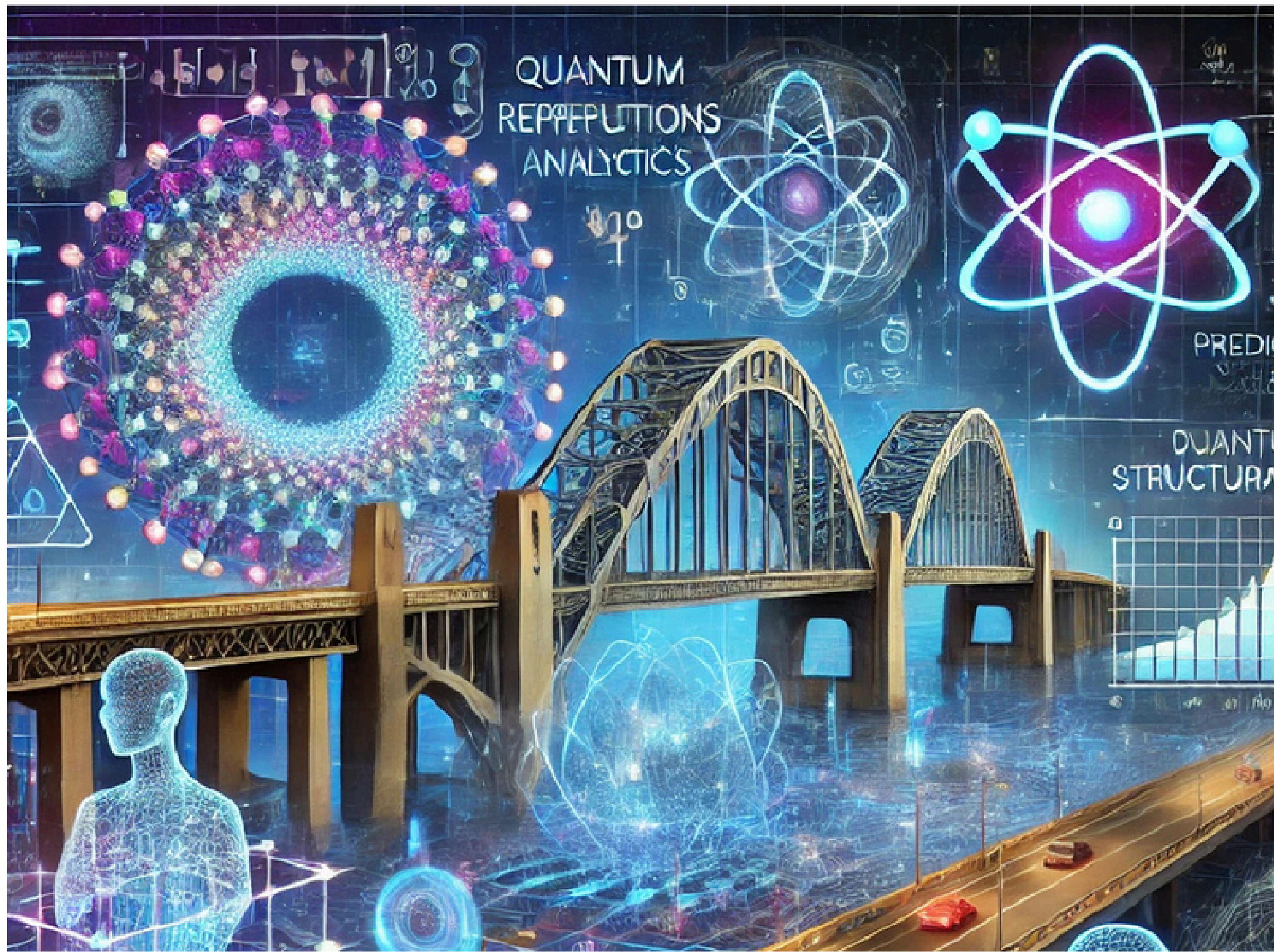
Key Benefits:

- **Hybrid Approach:** Combines quantum circuits with classical optimization.
- **Variational Circuits:** Optimizes stiffness and mass matrices.
- **Multi-Device Support:** Works with IBM Q, Google Cirq, Amazon Braket.
- **Cost Optimization:** Minimizes Ax = b errors.
- **Quantum Simulation:** Models load, stress, and vibration analysis.
- **Scalability:** Supports real-time structural monitoring.

Python implementation for vqls

https://colab.research.google.com/drive/1WnZ9P2w6e6FkYt_xRGh5K4wbSFLQOOmP

Quantum Advantage in Structural Analysis



Why Quantum Computing Outperforms Classical Methods

- Exponential Speedup: Classical solvers have $O(N^3)$ complexity, while VQLS achieves $O(\log N)$, optimizing large-scale FEA.
- Efficient Sparse Matrix Handling: Qubits enable compact representation, reducing storage and computation time.
- Superior Structural Simulations: Quantum algorithms analyze traffic loads, wind forces, and seismic activity efficiently.
- Hybrid Optimization in FEA: VQLS refines stress distribution, load capacity, and failure predictions with improved accuracy.
- Real-Time Monitoring: Faster eigenvalue computations enhance vibration and stability analysis for predictive maintenance.

Scalability for Future Engineering Applications: Quantum-enhanced structural analysis ensures scalability, adaptability, and optimized real-time decision-making for smart bridge systems.

Thankyou