

Assignment-12: TensorFlow and Keras: Build various MLP architectures for MNIST dataset [M]

Objective:

- Model with three different architecture:
 - 1) 2-Hidden layer architecture (784-472-168-10 architecture)
 - 2) 3-Hidden layer architecture (784-352-164-124-10 architecture)
 - 3) 5-Hidden layer architecture (784-216-170-136-80-38-10 architecture)
- Train-Test error plot
- Activation='relu'+ Adam Optimizer+Batch_Normalization +Drop_out

```
In [1]: # if you keras is not using tensorflow as backend set "KERAS_BACKEND=tensorflow" use this command
from keras.utils import np_utils
from keras.datasets import mnist
import seaborn as sns
from keras.initializers import RandomNormal
```

Using TensorFlow backend.

```
In [2]: %matplotlib inline
import matplotlib.pyplot as plt
```

```

import numpy as np
import time
# https://gist.github.com/greydanus/f6eee59eaf1d90fcb3b534a25362cea4
# https://stackoverflow.com/a/14434334
# this function is used to update the plots for each epoch and error
def plt_dynamic(x, vy, ty, ax, colors=['b']):
    ax.plot(x, vy, 'b', label="Validation Loss")
    ax.plot(x, ty, 'r', label="Train Loss")
    plt.legend()
    plt.grid()
    fig.canvas.draw()

```

In [3]: *# the data, shuffled and split between train and test sets*
 (x_train, y_train), (x_test, y_test) = mnist.load_data()

Downloading data from <https://s3.amazonaws.com/img-datasets/mnist.npz>
 11493376/11490434 [=====] - 1s 0us/step

In [4]: `print("Number of training examples :", x_train.shape[0], "and each image is of shape (%d, %d)"%(x_train.shape[1], x_train.shape[2]))`
`print("Number of training examples :", x_test.shape[0], "and each image is of shape (%d, %d)"%(x_test.shape[1], x_test.shape[2]))`

Number of training examples : 60000 and each image is of shape (28, 28)
 Number of training examples : 10000 and each image is of shape (28, 28)

In [5]: *# if you observe the input shape its 3 dimensional vector*
*# for each image we have a (28*28) vector*
*# we will convert the (28*28) vector into single dimensional vector of 1 * 784*

```

x_train = x_train.reshape(x_train.shape[0], x_train.shape[1]*x_train.shape[2])
x_test = x_test.reshape(x_test.shape[0], x_test.shape[1]*x_test.shape[2])

```

In [6]: *# after converting the input images from 3d to 2d vectors*

```
print("Number of training examples :", x_train.shape[0], "and each image is of shape (%d)"%(x_train.shape[1]))
print("Number of training examples :", x_test.shape[0], "and each image is of shape (%d)"%(x_test.shape[1]))
```

Number of training examples : 60000 and each image is of shape (784)
 Number of training examples : 10000 and each image is of shape (784)

In [7]: *# An example data point*

```
print(x_train[0])
```

```
[ 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 0  0  0  0  0  0  0  0  3  18  18  18 126 136 175  26 166 25
5 247 127  0  0  0  0  0  0  0  0  0  0  0  0  30  36  94 15
4 170 253 253 253 253 253 225 172 253 242 195  64  0  0  0  0  0
0  0  0  0  0  49 238 253 253 253 253 253 253 253 251  93  8
2 82  56  39  0  0  0  0  0  0  0  0  0  0  0  0  18 219 25
3 253 253 253 253 198 182 247 241  0  0  0  0  0  0  0  0  0
0  0  0  0  0  0  0  0  80 156 107 253 253 205  11  0  43 15
```

4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	14	1	154	253	90	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	139	253	190	2
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	11	190	253	70	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	35 24
1	225	160	108	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	81	240	253	253	119	25	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	45	186	253	253	150	27	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	93	252	253 18
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	249	253	249	64	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	46	130	183	25
3	253	207	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	39	148	229	253	253	253	250	182	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	24	114	221	253	253	25
3	253	201	78	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	23	66	213	253	253	253	253	198	81	2	0	0	0	0	0
0																	

5	0	0	0	0	0	0	0	0	0	0	18	171	219	253	253	253	253	19
0	80	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	55	172	226	253	253	253	253	244	133	11	0	0	0	0	0	0	0	
6	0	0	0	0	0	0	0	0	0	0	136	253	253	253	212	135	132	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0]								

```
In [8]: # if we observe the above matrix each cell is having a value between 0-255
# before we move to apply machine learning algorithms lets try to normalize the data
#  $X \Rightarrow (X - X_{min}) / (X_{max} - X_{min}) = X / 255$ 

x_train = x_train/255
x_test = x_test/255
```

```
In [9]: # example data point after normalizing
print(x_train[0])
```

[illegible]

0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.05490196	0.00392157	0.60392157	0.99215686	0.35294118
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.54509804	0.99215686	0.74509804	0.00784314	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.04313725
0.74509804	0.99215686	0.2745098	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.1372549	0.94509804
0.88235294	0.62745098	0.42352941	0.00392157	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.31764706	0.94117647	0.99215686
0.99215686	0.46666667	0.09803922	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.17647059	0.72941176	0.99215686	0.99215686
0.58823529	0.10588235	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.0627451	0.36470588	0.98823529	0.99215686	0.73333333
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.97647059	0.99215686	0.97647059	0.25098039	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.

0.	0.	0.	0.	0.	0.
0.	0.	0.18039216	0.50980392	0.71764706	0.99215686
0.99215686	0.81176471	0.00784314	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.15294118	0.58039216
0.89803922	0.99215686	0.99215686	0.99215686	0.98039216	0.71372549
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.09411765	0.44705882	0.86666667	0.99215686	0.99215686	0.99215686
0.99215686	0.78823529	0.30588235	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.09019608	0.25882353	0.83529412	0.99215686
0.99215686	0.99215686	0.99215686	0.77647059	0.31764706	0.00784314
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.07058824	0.67058824
0.85882353	0.99215686	0.99215686	0.99215686	0.99215686	0.76470588
0.31372549	0.03529412	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.21568627	0.6745098	0.88627451	0.99215686	0.99215686	0.99215686
0.99215686	0.95686275	0.52156863	0.04313725	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.53333333	0.99215686
0.99215686	0.99215686	0.83137255	0.52941176	0.51764706	0.0627451
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.

[illegible]

```
In [10]: # here we are having a class number for each image
print("Class label of first image :", y_train[0])

# lets convert this into a 10 dimensional vector
# ex: consider an image is 5 convert it into 5 => [0, 0, 0, 0, 0, 1, 0, 0, 0, 0]
# this conversion needed for MLPs

y_train = np_utils.to_categorical(y_train, 10)
y_test = np_utils.to_categorical(y_test, 10)

print("After converting the output into a vector : ", y_train[0])

Class label of first image : 5
After converting the output into a vector : [0. 0. 0. 0. 0. 1. 0. 0. 0. 0.]
```

Softmax classifier

```
In [11]: # https://keras.io/getting-started/sequential-model-guide/

# The Sequential model is a linear stack of layers.
# you can create a Sequential model by passing a list of layer instances
# to the constructor:

# model = Sequential([
#     Dense(32, input_shape=(784,)),
#     Activation('relu'),
#     Dense(10),
```

```

#     Activation('softmax'),
# ])

# You can also simply add layers via the .add() method:

# model = Sequential()
# model.add(Dense(32, input_dim=784))
# model.add(Activation('relu'))

###

# https://keras.io/layers/core/

# keras.layers.Dense(units, activation=None, use_bias=True, kernel_initializer='glorot_uniform',
# bias_initializer='zeros', kernel_regularizer=None, bias_regularizer=None, activity_regularizer=None,
# kernel_constraint=None, bias_constraint=None)

# Dense implements the operation: output = activation(dot(input, kernel) + bias) where
# activation is the element-wise activation function passed as the activation argument,
# kernel is a weights matrix created by the layer, and
# bias is a bias vector created by the layer (only applicable if use_bias is True).

# output = activation(dot(input, kernel) + bias) => y = activation(WT.X + b)

####

# https://keras.io/activations/

# Activations can either be used through an Activation layer, or through the activation argument supported by all forward layers:

# from keras.layers import Activation, Dense

```

```
# model.add(Dense(64))
# model.add(Activation('tanh'))

# This is equivalent to:
# model.add(Dense(64, activation='tanh'))

# there are many activation functions ar available ex: tanh, relu, soft
max

from keras.models import Sequential
from keras.layers import Dense, Activation
from keras.initializers import he_normal
```

```
In [12]: # some model parameters

output_dim = 10
input_dim = x_train.shape[1]

batch_size = 128
nb_epoch = 20
```

1) 2-Hidden layer architecture (784-472-168-10 architecture)

1.1 MLP + ReLU + ADAM

```
In [13]: model_relu = Sequential()
model_relu.add(Dense(472, activation='relu', input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))
model_relu.add(Dense(168, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(output_dim, activation='softmax'))

print(model_relu.summary())
```

```

model_relu.compile(optimizer='adam',
                   loss='categorical_crossentropy',
                   metrics=['accuracy'])

history11 = model_relu.fit(x_train, y_train,
                           batch_size=batch_size,
                           epochs=nb_epoch, verbose=1,
                           validation_data=(x_test, y_test))

```

Layer (type)	Output Shape	Param #
dense_1 (Dense)	(None, 472)	370520
dense_2 (Dense)	(None, 168)	79464
dense_3 (Dense)	(None, 10)	1690

```

Total params: 451,674
Trainable params: 451,674
Non-trainable params: 0

```

```

None
Train on 60000 samples, validate on 10000 samples
Epoch 1/20
60000/60000 [=====] - 4s 65us/step - loss: 0.2342 - acc: 0.9321 - val_loss: 0.1046 - val_acc: 0.9690
Epoch 2/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0859 - acc: 0.9736 - val_loss: 0.0847 - val_acc: 0.9727
Epoch 3/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0549 - acc: 0.9831 - val_loss: 0.0754 - val_acc: 0.9770
Epoch 4/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0379 - acc: 0.9877 - val_loss: 0.0749 - val_acc: 0.9757
Epoch 5/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0269 - acc: 0.9916 - val_loss: 0.0797 - val_acc: 0.9754

```

```
Epoch 6/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
213 - acc: 0.9936 - val_loss: 0.0743 - val_acc: 0.9776
Epoch 7/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
187 - acc: 0.9939 - val_loss: 0.0746 - val_acc: 0.9782
Epoch 8/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
161 - acc: 0.9946 - val_loss: 0.0806 - val_acc: 0.9780
Epoch 9/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
131 - acc: 0.9957 - val_loss: 0.0746 - val_acc: 0.9798
Epoch 10/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
143 - acc: 0.9949 - val_loss: 0.0998 - val_acc: 0.9754
Epoch 11/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
099 - acc: 0.9970 - val_loss: 0.0709 - val_acc: 0.9822
Epoch 12/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
099 - acc: 0.9967 - val_loss: 0.0737 - val_acc: 0.9814
Epoch 13/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
098 - acc: 0.9966 - val_loss: 0.0865 - val_acc: 0.9797
Epoch 14/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
111 - acc: 0.9963 - val_loss: 0.1118 - val_acc: 0.9743
Epoch 15/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
074 - acc: 0.9977 - val_loss: 0.0829 - val_acc: 0.9815
Epoch 16/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
072 - acc: 0.9978 - val_loss: 0.0882 - val_acc: 0.9812
Epoch 17/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
087 - acc: 0.9971 - val_loss: 0.0959 - val_acc: 0.9797
Epoch 18/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
067 - acc: 0.9977 - val_loss: 0.0795 - val_acc: 0.9831
```

```
Epoch 19/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
064 - acc: 0.9980 - val_loss: 0.0994 - val_acc: 0.9797
Epoch 20/20
60000/60000 [=====] - 2s 34us/step - loss: 0.0
080 - acc: 0.9974 - val_loss: 0.0804 - val_acc: 0.9824
```

```
In [14]: score = model_relu.evaluate(x_test, y_test, verbose=0)
score1=score[0]
score2=score[1]
train_acc1=history11.history['acc']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax11 = plt.subplots(1,1)
ax11.set_xlabel('epoch') ; ax11.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1,nb_epoch+1))

# print(history.history.keys())
# dict_keys(['val_loss', 'val_acc', 'loss', 'acc'])
# history = model_drop.fit(x_train, y_train, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(x_test, y_test))

# we will get val_loss and val_acc only when you pass the parameter validation_data
# val_loss : validation loss
# val_acc : validation accuracy

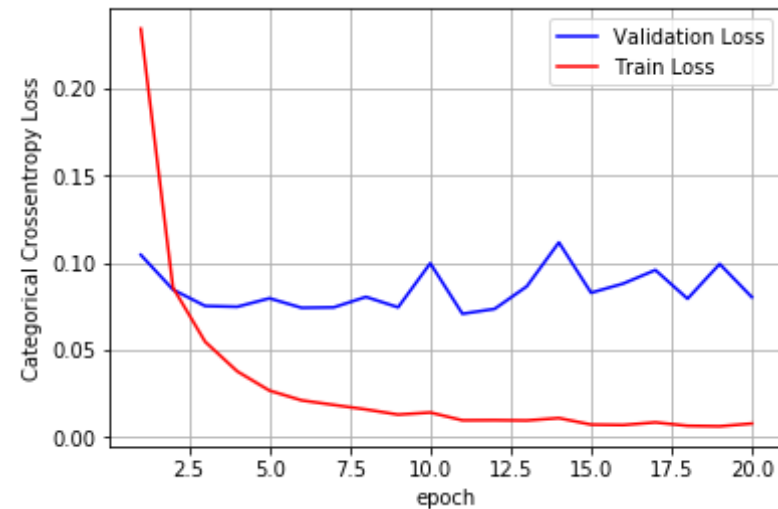
# loss : training loss
# acc : train accuracy
# for each key in history.history we will have a list of length equal to number of epochs

vy11 = history11.history['val_loss']
```

```
tyl1 = history11.history['loss']  
plt_dynamic(x, vy11, tyl1, ax11)
```

Test score: 0.0804028440125881

Test accuracy: 0.9824



1.2 MLP + Batch-Norm on hidden Layers + AdamOptimizer

```
In [15]: from keras.layers.normalization import BatchNormalization  
  
model_batch = Sequential()  
  
model_batch.add(Dense(472, activation='relu',  
                      input_shape=(input_dim,),  
                      kernel_initializer=he_normal(seed=None)))  
model_batch.add(BatchNormalization())  
  
model_batch.add(Dense(168, activation='relu',  
                      kernel_initializer=he_normal(seed=None)) )  
model_batch.add(BatchNormalization())
```

```
model_batch.add(Dense(output_dim, activation='softmax'))

model_batch.summary()
```

Layer (type)	Output Shape	Param #
dense_4 (Dense)	(None, 472)	370520
batch_normalization_1 (Batch Normalization)	(None, 472)	1888
dense_5 (Dense)	(None, 168)	79464
batch_normalization_2 (Batch Normalization)	(None, 168)	672
dense_6 (Dense)	(None, 10)	1690
Total params: 454,234		
Trainable params: 452,954		
Non-trainable params: 1,280		

```
In [16]: model_batch.compile(optimizer='adam', loss='categorical_crossentropy',
                             metrics=['accuracy'])

history12 = model_batch.fit(x_train, y_train,
                             batch_size=batch_size,
                             epochs=nb_epoch, verbose=1,
                             validation_data=(x_test, y_test))
```

```
Train on 60000 samples, validate on 10000 samples
Epoch 1/20
60000/60000 [=====] - 4s 65us/step - loss: 0.1
869 - acc: 0.9439 - val_loss: 0.1017 - val_acc: 0.9669
Epoch 2/20
60000/60000 [=====] - 3s 54us/step - loss: 0.0
706 - acc: 0.9788 - val_loss: 0.0791 - val_acc: 0.9757
Epoch 3/20
```



```
60000/60000 [=====] - 3s 55us/step - loss: 0.0
439 - acc: 0.9867 - val_loss: 0.0820 - val_acc: 0.9743
Epoch 4/20
60000/60000 [=====] - 3s 54us/step - loss: 0.0
325 - acc: 0.9893 - val_loss: 0.0782 - val_acc: 0.9767
Epoch 5/20
60000/60000 [=====] - 3s 54us/step - loss: 0.0
249 - acc: 0.9919 - val_loss: 0.0910 - val_acc: 0.9729
Epoch 6/20
60000/60000 [=====] - 3s 55us/step - loss: 0.0
203 - acc: 0.9936 - val_loss: 0.0749 - val_acc: 0.9752
Epoch 7/20
60000/60000 [=====] - 3s 54us/step - loss: 0.0
195 - acc: 0.9933 - val_loss: 0.0861 - val_acc: 0.9743
Epoch 8/20
60000/60000 [=====] - 3s 55us/step - loss: 0.0
159 - acc: 0.9945 - val_loss: 0.0724 - val_acc: 0.9794
Epoch 9/20
60000/60000 [=====] - 3s 55us/step - loss: 0.0
170 - acc: 0.9943 - val_loss: 0.0812 - val_acc: 0.9778
Epoch 10/20
60000/60000 [=====] - 3s 54us/step - loss: 0.0
129 - acc: 0.9959 - val_loss: 0.0657 - val_acc: 0.9815
Epoch 11/20
60000/60000 [=====] - 3s 54us/step - loss: 0.0
135 - acc: 0.9956 - val_loss: 0.0743 - val_acc: 0.9795
Epoch 12/20
60000/60000 [=====] - 3s 55us/step - loss: 0.0
114 - acc: 0.9964 - val_loss: 0.0838 - val_acc: 0.9762
Epoch 13/20
60000/60000 [=====] - 3s 54us/step - loss: 0.0
099 - acc: 0.9967 - val_loss: 0.0798 - val_acc: 0.9788
Epoch 14/20
60000/60000 [=====] - 3s 54us/step - loss: 0.0
083 - acc: 0.9973 - val_loss: 0.0695 - val_acc: 0.9806
Epoch 15/20
60000/60000 [=====] - 3s 54us/step - loss: 0.0
081 - acc: 0.9974 - val_loss: 0.0745 - val_acc: 0.9814
Epoch 16/20
```

```

60000/60000 [=====] - 3s 54us/step - loss: 0.0
083 - acc: 0.9970 - val_loss: 0.0783 - val_acc: 0.9799
Epoch 17/20
60000/60000 [=====] - 3s 55us/step - loss: 0.0
105 - acc: 0.9965 - val_loss: 0.0791 - val_acc: 0.9803
Epoch 18/20
60000/60000 [=====] - 3s 54us/step - loss: 0.0
094 - acc: 0.9968 - val_loss: 0.0753 - val_acc: 0.9815
Epoch 19/20
60000/60000 [=====] - 3s 54us/step - loss: 0.0
055 - acc: 0.9982 - val_loss: 0.0764 - val_acc: 0.9805
Epoch 20/20
60000/60000 [=====] - 3s 55us/step - loss: 0.0
057 - acc: 0.9981 - val_loss: 0.0790 - val_acc: 0.9797

```

```

In [17]: score = model_batch.evaluate(x_test, y_test, verbose=0)
print('Test score:', score[0])
print('Test accuracy:', score[1])
score3=score[0]
score4=score[1]
train_acc2=history11.history['acc']

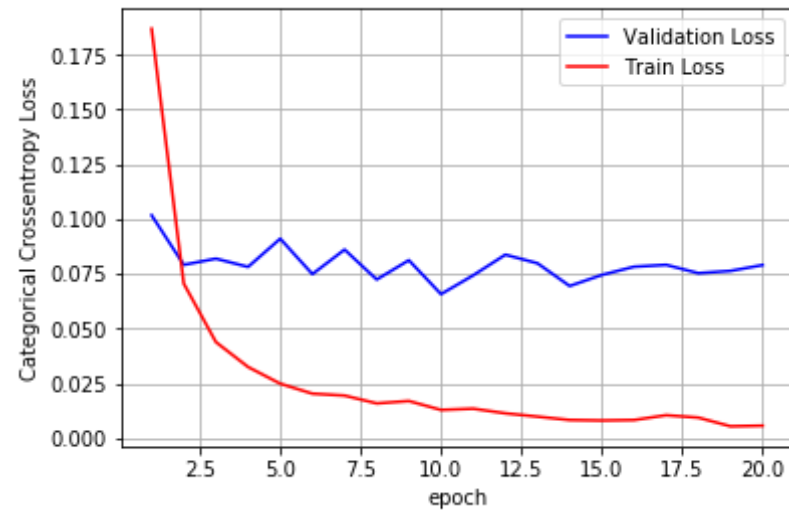
fig,ax12 = plt.subplots(1,1)
ax12.set_xlabel('epoch') ; ax12.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1,nb_epoch+1))

vy12 = history12.history['val_loss']
ty12 = history12.history['loss']
plt_dynamic(x, vy12, ty12, ax12)

Test score: 0.07902511259189378
Test accuracy: 0.9797

```



1.3 MLP + Dropout + AdamOptimizer

```
In [18]: # https://stackoverflow.com/questions/34716454/where-do-i-call-the-batch-normalization-function-in-keras

from keras.layers import Dropout

model_drop = Sequential()

model_drop.add(Dense(472, activation='relu',
                    input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_drop.add(Dense(168, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_drop.add(Dense(output_dim, activation='softmax'))
```

```
model_drop.summary()
```

Layer (type)	Output Shape	Param #
dense_7 (Dense)	(None, 472)	370520
batch_normalization_3 (Batch Normalization)	(None, 472)	1888
dropout_1 (Dropout)	(None, 472)	0
dense_8 (Dense)	(None, 168)	79464
batch_normalization_4 (Batch Normalization)	(None, 168)	672
dropout_2 (Dropout)	(None, 168)	0
dense_9 (Dense)	(None, 10)	1690
Total params: 454,234		
Trainable params: 452,954		
Non-trainable params: 1,280		

```
In [19]: model_drop.compile(optimizer='adam',
                             loss='categorical_crossentropy',
                             metrics=['accuracy'])

history13 = model_drop.fit(x_train, y_train,
                           batch_size=batch_size,

                           epochs=nb_epoch, verbose=1,
                           validation_data=(x_test, y_test))

Train on 60000 samples, validate on 10000 samples
Epoch 1/20
60000/60000 [=====] - 4s 70us/step - loss: 0.4
343 - acc: 0.8696 - val_loss: 0.1400 - val_acc: 0.9560
```

```
Epoch 2/20
60000/60000 [=====] - 3s 56us/step - loss: 0.2
069 - acc: 0.9376 - val_loss: 0.1039 - val_acc: 0.9668
Epoch 3/20
60000/60000 [=====] - 3s 57us/step - loss: 0.1
648 - acc: 0.9501 - val_loss: 0.0917 - val_acc: 0.9695
Epoch 4/20
60000/60000 [=====] - 3s 56us/step - loss: 0.1
389 - acc: 0.9583 - val_loss: 0.0773 - val_acc: 0.9762
Epoch 5/20
60000/60000 [=====] - 3s 56us/step - loss: 0.1
204 - acc: 0.9626 - val_loss: 0.0778 - val_acc: 0.9744
Epoch 6/20
60000/60000 [=====] - 3s 57us/step - loss: 0.1
042 - acc: 0.9674 - val_loss: 0.0693 - val_acc: 0.9764
Epoch 7/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0
984 - acc: 0.9691 - val_loss: 0.0708 - val_acc: 0.9786
Epoch 8/20
60000/60000 [=====] - 3s 57us/step - loss: 0.0
922 - acc: 0.9708 - val_loss: 0.0714 - val_acc: 0.9787
Epoch 9/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0
895 - acc: 0.9716 - val_loss: 0.0615 - val_acc: 0.9817
Epoch 10/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0
834 - acc: 0.9735 - val_loss: 0.0655 - val_acc: 0.9797
Epoch 11/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0
762 - acc: 0.9759 - val_loss: 0.0589 - val_acc: 0.9829
Epoch 12/20
60000/60000 [=====] - 3s 57us/step - loss: 0.0
748 - acc: 0.9766 - val_loss: 0.0620 - val_acc: 0.9814
Epoch 13/20
60000/60000 [=====] - 3s 57us/step - loss: 0.0
691 - acc: 0.9783 - val_loss: 0.0595 - val_acc: 0.9835
Epoch 14/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0
690 - acc: 0.9780 - val_loss: 0.0562 - val_acc: 0.9828
```

```

Epoch 15/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0
652 - acc: 0.9790 - val_loss: 0.0579 - val_acc: 0.9820
Epoch 16/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0
597 - acc: 0.9808 - val_loss: 0.0549 - val_acc: 0.9830
Epoch 17/20
60000/60000 [=====] - 3s 57us/step - loss: 0.0
582 - acc: 0.9815 - val_loss: 0.0607 - val_acc: 0.9819
Epoch 18/20
60000/60000 [=====] - 3s 57us/step - loss: 0.0
581 - acc: 0.9811 - val_loss: 0.0549 - val_acc: 0.9833
Epoch 19/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0
540 - acc: 0.9826 - val_loss: 0.0567 - val_acc: 0.9826
Epoch 20/20
60000/60000 [=====] - 3s 56us/step - loss: 0.0
533 - acc: 0.9832 - val_loss: 0.0636 - val_acc: 0.9820

```

```

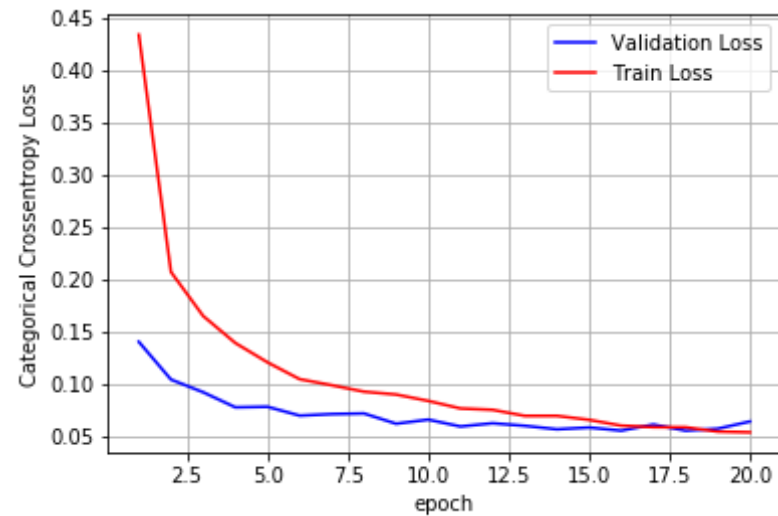
In [20]: score = model_drop.evaluate(x_test, y_test, verbose=0)
score5=score[0]
score6=score[1]
train_acc3=history11.history['acc']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax13 = plt.subplots(1,1)
ax13.set_xlabel('epoch') ; ax13.set_ylabel('Categorical Crossentropy Loss')

vy13 = history13.history['val_loss']
ty13 = history13.history['loss']
plt_dynamic(x, vy13, ty13, ax13)

Test score: 0.06363873664251878
Test accuracy: 0.982

```



2) 3-Hidden layer architecture (784-352-164-124 architecture)

2.1 MLP + ReLU + ADAM

```
In [21]: model_relu = Sequential()
model_relu.add(Dense(352, activation='relu', input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))
model_relu.add(Dense(164, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )

model_relu.add(Dense(124, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(output_dim, activation='softmax'))

print(model_relu.summary())

model_relu.compile(optimizer='adam',
```

```

        loss='categorical_crossentropy',
        metrics=['accuracy'])

history21 = model_relu.fit(x_train, y_train,
                           batch_size=batch_size,
                           epochs=nb_epoch, verbose=1,
                           validation_data=(x_test, y_test))

```

Layer (type)	Output Shape	Param #
dense_10 (Dense)	(None, 352)	276320
dense_11 (Dense)	(None, 164)	57892
dense_12 (Dense)	(None, 124)	20460
dense_13 (Dense)	(None, 10)	1250

```

Total params: 355,922
Trainable params: 355,922
Non-trainable params: 0

```

```

None
Train on 60000 samples, validate on 10000 samples
Epoch 1/20
60000/60000 [=====] - 3s 43us/step - loss: 0.2384 - acc: 0.9297 - val_loss: 0.1191 - val_acc: 0.9635
Epoch 2/20
60000/60000 [=====] - 2s 35us/step - loss: 0.0917 - acc: 0.9718 - val_loss: 0.0898 - val_acc: 0.9727
Epoch 3/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0595 - acc: 0.9813 - val_loss: 0.0870 - val_acc: 0.9731
Epoch 4/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0431 - acc: 0.9861 - val_loss: 0.0912 - val_acc: 0.9733
Epoch 5/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0326 - acc: 0.9894 - val_loss: 0.0768 - val_acc: 0.9779

```



```
Epoch 6/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
278 - acc: 0.9912 - val_loss: 0.0760 - val_acc: 0.9781
Epoch 7/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
239 - acc: 0.9917 - val_loss: 0.0852 - val_acc: 0.9739
Epoch 8/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
174 - acc: 0.9942 - val_loss: 0.0831 - val_acc: 0.9772
Epoch 9/20
60000/60000 [=====] - 2s 35us/step - loss: 0.0
139 - acc: 0.9951 - val_loss: 0.1033 - val_acc: 0.9744
Epoch 10/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
169 - acc: 0.9942 - val_loss: 0.0819 - val_acc: 0.9794
Epoch 11/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
133 - acc: 0.9955 - val_loss: 0.0802 - val_acc: 0.9820
Epoch 12/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
154 - acc: 0.9949 - val_loss: 0.1081 - val_acc: 0.9754
Epoch 13/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
126 - acc: 0.9959 - val_loss: 0.0866 - val_acc: 0.9809
Epoch 14/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
095 - acc: 0.9968 - val_loss: 0.0990 - val_acc: 0.9788
Epoch 15/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
121 - acc: 0.9963 - val_loss: 0.0841 - val_acc: 0.9803
Epoch 16/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
126 - acc: 0.9961 - val_loss: 0.0825 - val_acc: 0.9822
Epoch 17/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
088 - acc: 0.9971 - val_loss: 0.0915 - val_acc: 0.9803
Epoch 18/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
081 - acc: 0.9974 - val_loss: 0.1088 - val_acc: 0.9784
```

```
Epoch 19/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
123 - acc: 0.9960 - val_loss: 0.0885 - val_acc: 0.9816
Epoch 20/20
60000/60000 [=====] - 2s 36us/step - loss: 0.0
077 - acc: 0.9976 - val_loss: 0.0931 - val_acc: 0.9801
```

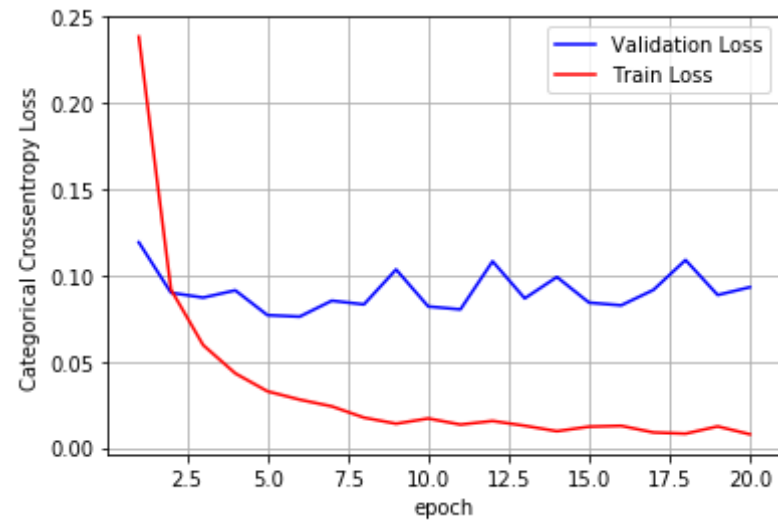
```
In [22]: score = model_relu.evaluate(x_test, y_test, verbose=0)
score7=score[0]
score8=score[1]
train_acc4=history11.history['acc']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax21 = plt.subplots(1,1)
ax21.set_xlabel('epoch') ; ax21.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1,nb_epoch+1))

vy21 = history21.history['val_loss']
ty21 = history21.history['loss']
plt_dynamic(x, vy21, ty21, ax21)

Test score: 0.0930518318862476
Test accuracy: 0.9801
```



2.2 MLP + Batch-Norm on hidden Layers + AdamOptimizer

```
In [24]: from keras.layers.normalization import BatchNormalization

model_batch = Sequential()

model_batch.add(Dense(352, activation='relu', input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))
model_batch.add(BatchNormalization())
model_batch.add(Dense(164, activation='relu',
                    kernel_initializer=he_normal(seed=None)))
model_batch.add(BatchNormalization())

model_batch.add(Dense(124, activation='relu',
                    kernel_initializer=he_normal(seed=None)))
model_batch.add(BatchNormalization())

model_batch.add(Dense(output_dim, activation='softmax'))
```

```
In [25]: model_batch.compile(optimizer='adam', loss='categorical_crossentropy',
                             metrics=['accuracy'])

history22 = model_batch.fit(x_train, y_train,
                             batch_size=batch_size,
                             epochs=nb_epoch, verbose=1,
                             validation_data=(x_test, y_test))

Train on 60000 samples, validate on 10000 samples
Epoch 1/20
60000/60000 [=====] - 4s 67us/step - loss: 0.4
890 - acc: 0.8559 - val_loss: 0.8223 - val_acc: 0.7690
Epoch 2/20
60000/60000 [=====] - 3s 52us/step - loss: 0.3
046 - acc: 0.9136 - val_loss: 0.6800 - val_acc: 0.8069
Epoch 3/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
873 - acc: 0.9194 - val_loss: 1.3266 - val_acc: 0.6626
Epoch 4/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
784 - acc: 0.9214 - val_loss: 1.2652 - val_acc: 0.6949
Epoch 5/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
724 - acc: 0.9234 - val_loss: 0.7312 - val_acc: 0.8170
Epoch 6/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
677 - acc: 0.9246 - val_loss: 2.0437 - val_acc: 0.5762
Epoch 7/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
664 - acc: 0.9250 - val_loss: 1.6328 - val_acc: 0.6136
Epoch 8/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
636 - acc: 0.9261 - val_loss: 8.1621 - val_acc: 0.3043
Epoch 9/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
629 - acc: 0.9260 - val_loss: 1.4145 - val_acc: 0.6829
Epoch 10/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
606 - acc: 0.9267 - val_loss: 3.2574 - val_acc: 0.5500
```

```

Epoch 11/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
575 - acc: 0.9282 - val_loss: 1.2263 - val_acc: 0.7405
Epoch 12/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
570 - acc: 0.9291 - val_loss: 0.6801 - val_acc: 0.8525
Epoch 13/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
564 - acc: 0.9275 - val_loss: 1.3066 - val_acc: 0.6563
Epoch 14/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
559 - acc: 0.9280 - val_loss: 1.2406 - val_acc: 0.6985
Epoch 15/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
552 - acc: 0.9277 - val_loss: 0.6215 - val_acc: 0.8638
Epoch 16/20
60000/60000 [=====] - 3s 53us/step - loss: 0.2
541 - acc: 0.9282 - val_loss: 2.2431 - val_acc: 0.5557
Epoch 17/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
526 - acc: 0.9285 - val_loss: 4.0150 - val_acc: 0.5063
Epoch 18/20
60000/60000 [=====] - 3s 52us/step - loss: 0.2
539 - acc: 0.9302 - val_loss: 1.2628 - val_acc: 0.6957
Epoch 19/20
60000/60000 [=====] - 3s 53us/step - loss: 0.2
513 - acc: 0.9294 - val_loss: 7.2882 - val_acc: 0.2021
Epoch 20/20
60000/60000 [=====] - 3s 53us/step - loss: 0.2
532 - acc: 0.9285 - val_loss: 0.6960 - val_acc: 0.8508

```

In [26]: `model_batch.summary()`

Layer (type)	Output Shape	Param #
=====		
batch_normalization_8 (Batch Normalization)	(None, 784)	3136
batch_normalization_9 (Batch Normalization)	(None, 784)	3136

batch_normalization_10 (Batch Normalization)	(None, 784)	3136
dense_21 (Dense)	(None, 10)	7850

Total params: 17,258
 Trainable params: 12,554
 Non-trainable params: 4,704

```

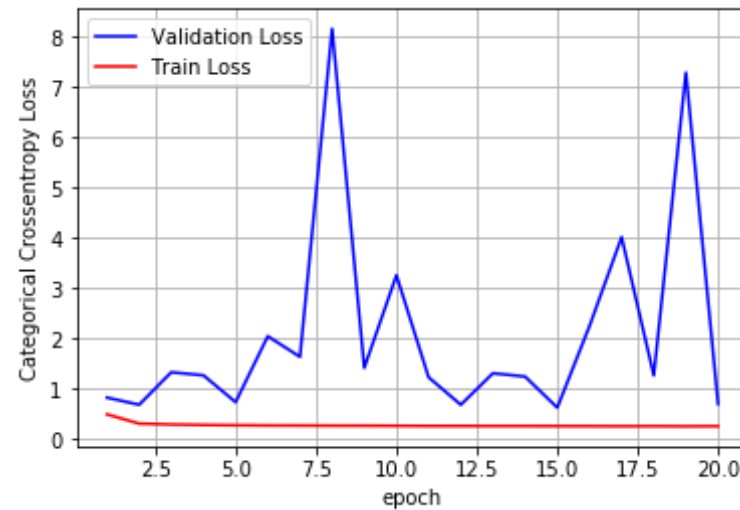
In [27]: score = model_batch.evaluate(x_test, y_test, verbose=0)
score9=score[0]
score10=score[1]
train_acc5=history11.history['acc']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax22 = plt.subplots(1,1)
ax22.set_xlabel('epoch') ; ax22.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1,nb_epoch+1))

vy22 = history22.history['val_loss']
ty22 = history22.history['loss']
plt_dynamic(x, vy22, ty22, ax22)

Test score: 0.6960192083239556
Test accuracy: 0.8508
  
```



2.3 MLP + Dropout + AdamOptimizer

In [28]: *# <https://stackoverflow.com/questions/34716454/where-do-i-call-the-batch-normalization-function-in-keras>*

```
from keras.layers import Dropout

model_drop = Sequential()
model_relu.add(Dense(352, activation='relu', input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))

model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))
model_relu.add(Dense(164, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_relu.add(Dense(124, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
```

```
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_drop.add(Dense(output_dim, activation='softmax'))
```

```
In [29]: model_drop.compile(optimizer='adam',
                           loss='categorical_crossentropy',
                           metrics=['accuracy'])

history23 = model_drop.fit(x_train, y_train,
                           batch_size=batch_size,

                           epochs=nb_epoch, verbose=1,
                           validation_data=(x_test, y_test))
```

Train on 60000 samples, validate on 10000 samples

Epoch 1/20

60000/60000 [=====] - 4s 73us/step - loss: 1.3624 - acc: 0.5675 - val_loss: 0.5117 - val_acc: 0.8760

Epoch 2/20

60000/60000 [=====] - 3s 56us/step - loss: 0.8656 - acc: 0.7185 - val_loss: 0.4472 - val_acc: 0.8913

Epoch 3/20

60000/60000 [=====] - 3s 56us/step - loss: 0.8441 - acc: 0.7262 - val_loss: 0.4391 - val_acc: 0.8962

Epoch 4/20

60000/60000 [=====] - 3s 56us/step - loss: 0.8296 - acc: 0.7280 - val_loss: 0.4273 - val_acc: 0.8928

Epoch 5/20

60000/60000 [=====] - 3s 56us/step - loss: 0.8278 - acc: 0.7296 - val_loss: 0.4224 - val_acc: 0.8943

Epoch 6/20

60000/60000 [=====] - 3s 56us/step - loss: 0.8288 - acc: 0.7291 - val_loss: 0.4213 - val_acc: 0.8978

Epoch 7/20

60000/60000 [=====] - 3s 56us/step - loss: 0.8293 - acc: 0.7306 - val_loss: 0.4187 - val_acc: 0.8945

Epoch 8/20

60000/60000 [=====] - 3s 55us/step - loss: 0.8


```
172 - acc: 0.7332 - val_loss: 0.4207 - val_acc: 0.8940
Epoch 9/20
60000/60000 [=====] - 3s 56us/step - loss: 0.8
207 - acc: 0.7344 - val_loss: 0.4140 - val_acc: 0.8981
Epoch 10/20
60000/60000 [=====] - 3s 56us/step - loss: 0.8
187 - acc: 0.7332 - val_loss: 0.4115 - val_acc: 0.8972
Epoch 11/20
60000/60000 [=====] - 3s 56us/step - loss: 0.8
115 - acc: 0.7365 - val_loss: 0.4126 - val_acc: 0.8920
Epoch 12/20
60000/60000 [=====] - 3s 56us/step - loss: 0.8
167 - acc: 0.7337 - val_loss: 0.4119 - val_acc: 0.8953
Epoch 13/20
60000/60000 [=====] - 3s 56us/step - loss: 0.8
166 - acc: 0.7349 - val_loss: 0.4081 - val_acc: 0.8979
Epoch 14/20
60000/60000 [=====] - 3s 56us/step - loss: 0.8
216 - acc: 0.7356 - val_loss: 0.4093 - val_acc: 0.8997
Epoch 15/20
60000/60000 [=====] - 3s 56us/step - loss: 0.8
193 - acc: 0.7368 - val_loss: 0.4048 - val_acc: 0.8976
Epoch 16/20
60000/60000 [=====] - 3s 56us/step - loss: 0.8
108 - acc: 0.7388 - val_loss: 0.4082 - val_acc: 0.8980
Epoch 17/20
60000/60000 [=====] - 3s 55us/step - loss: 0.8
070 - acc: 0.7393 - val_loss: 0.4074 - val_acc: 0.8965
Epoch 18/20
60000/60000 [=====] - 3s 56us/step - loss: 0.8
123 - acc: 0.7373 - val_loss: 0.4039 - val_acc: 0.8977
Epoch 19/20
60000/60000 [=====] - 3s 56us/step - loss: 0.8
093 - acc: 0.7388 - val_loss: 0.4032 - val_acc: 0.8983
Epoch 20/20
60000/60000 [=====] - 3s 56us/step - loss: 0.8
050 - acc: 0.7399 - val_loss: 0.4032 - val_acc: 0.8992
```

```
In [30]: model_drop.summary()
```

Layer (type)	Output Shape	Param #
batch_normalization_11 (Batch Normalization)	(None, 784)	3136
dropout_3 (Dropout)	(None, 784)	0
batch_normalization_12 (Batch Normalization)	(None, 784)	3136
dropout_4 (Dropout)	(None, 784)	0
batch_normalization_13 (Batch Normalization)	(None, 784)	3136
dropout_5 (Dropout)	(None, 784)	0
dense_25 (Dense)	(None, 10)	7850
Total params: 17,258		
Trainable params: 12,554		
Non-trainable params: 4,704		

```
In [31]: score = model_drop.evaluate(x_test, y_test, verbose=0)
score1=score[0]
score12=score[1]
train_acc6=history11.history['acc']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax23 = plt.subplots(1,1)
ax23.set_xlabel('epoch') ; ax23.set_ylabel('Categorical Crossentropy Loss')

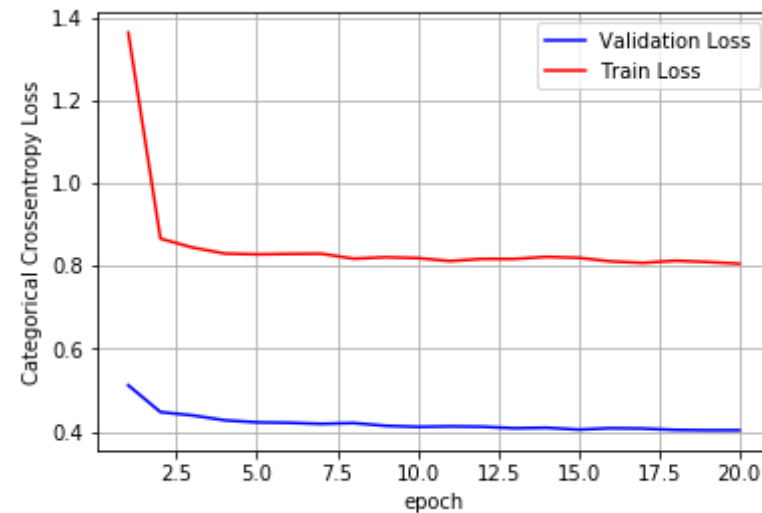
# list of epoch numbers
x = list(range(1,nb_epoch+1))

vy23 = history23.history['val_loss']
```

```
ty23 = history23.history['loss']  
plt_dynamic(x, vy23, ty23, ax23)
```

Test score: 0.40321056950092315

Test accuracy: 0.8992



3) 5-Hidden layer architecture (784-216-170-136-80-38-10 architecture)

3.1 MLP + ReLU + ADAM

```
In [33]: model_relu = Sequential()  
model_relu.add(Dense(216, activation='relu', input_shape=(input_dim,),  
                    kernel_initializer=he_normal(seed=None)))  
model_relu.add(Dense(170, activation='relu',  
                    kernel_initializer=he_normal(seed=None)) )  
  
model_relu.add(Dense(136, activation='relu',  
                    kernel_initializer=he_normal(seed=None)) )
```

```

model_relu.add(Dense(80, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )

model_relu.add(Dense(38, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(output_dim, activation='softmax'))

print(model_relu.summary())

model_relu.compile(optimizer='adam',
                  loss='categorical_crossentropy',
                  metrics=['accuracy'])

history31 = model_relu.fit(x_train, y_train,
                          batch_size=batch_size,
                          epochs=nb_epoch, verbose=1,
                          validation_data=(x_test, y_test))

```

Layer (type)	Output Shape	Param #
=====	=====	=====
dense_32 (Dense)	(None, 216)	169560
dense_33 (Dense)	(None, 170)	36890
dense_34 (Dense)	(None, 136)	23256
dense_35 (Dense)	(None, 80)	10960
dense_36 (Dense)	(None, 38)	3078
dense_37 (Dense)	(None, 10)	390
=====	=====	=====
Total params: 244,134		
Trainable params: 244,134		
Non-trainable params: 0		
None		
Train on 60000 samples, validate on 10000 samples		
Epoch 1/20		

```
60000/60000 [=====] - 3s 53us/step - loss: 0.2
755 - acc: 0.9179 - val_loss: 0.1245 - val_acc: 0.9616
Epoch 2/20
60000/60000 [=====] - 2s 41us/step - loss: 0.1
043 - acc: 0.9681 - val_loss: 0.1133 - val_acc: 0.9654
Epoch 3/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
731 - acc: 0.9773 - val_loss: 0.0926 - val_acc: 0.9729
Epoch 4/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
552 - acc: 0.9824 - val_loss: 0.0833 - val_acc: 0.9762
Epoch 5/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
432 - acc: 0.9857 - val_loss: 0.0780 - val_acc: 0.9764
Epoch 6/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
349 - acc: 0.9882 - val_loss: 0.0838 - val_acc: 0.9736
Epoch 7/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
308 - acc: 0.9897 - val_loss: 0.1178 - val_acc: 0.9668
Epoch 8/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
267 - acc: 0.9912 - val_loss: 0.0904 - val_acc: 0.9755
Epoch 9/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
236 - acc: 0.9924 - val_loss: 0.0814 - val_acc: 0.9793
Epoch 10/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
224 - acc: 0.9930 - val_loss: 0.0928 - val_acc: 0.9777
Epoch 11/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
202 - acc: 0.9935 - val_loss: 0.0894 - val_acc: 0.9774
Epoch 12/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
193 - acc: 0.9936 - val_loss: 0.0859 - val_acc: 0.9809
Epoch 13/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
143 - acc: 0.9954 - val_loss: 0.0868 - val_acc: 0.9795
Epoch 14/20
```

```

60000/60000 [=====] - 2s 41us/step - loss: 0.0
180 - acc: 0.9942 - val_loss: 0.0873 - val_acc: 0.9806
Epoch 15/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
160 - acc: 0.9952 - val_loss: 0.1039 - val_acc: 0.9770
Epoch 16/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
145 - acc: 0.9951 - val_loss: 0.0803 - val_acc: 0.9793
Epoch 17/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
132 - acc: 0.9956 - val_loss: 0.0820 - val_acc: 0.9812
Epoch 18/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
129 - acc: 0.9959 - val_loss: 0.0982 - val_acc: 0.9794
Epoch 19/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
115 - acc: 0.9962 - val_loss: 0.1187 - val_acc: 0.9754
Epoch 20/20
60000/60000 [=====] - 2s 41us/step - loss: 0.0
120 - acc: 0.9961 - val_loss: 0.0872 - val_acc: 0.9816

```

```

In [34]: score = model_relu.evaluate(x_test, y_test, verbose=0)
score13=score[0]
score14=score[1]
train_acc7=history11.history['acc']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax31 = plt.subplots(1,1)
ax31.set_xlabel('epoch') ; ax31.set_ylabel('Categorical Crossentropy Loss')

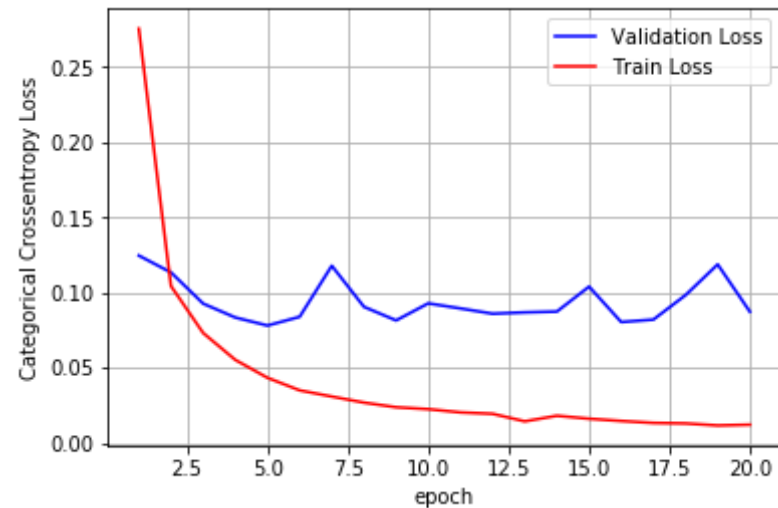
# list of epoch numbers
x = list(range(1,nb_epoch+1))

vy31 = history31.history['val_loss']
ty31 = history31.history['loss']
plt_dynamic(x, vy31, ty31, ax31)

```

Test score: 0.08723179607167986

Test accuracy: 0.9816



3.2 MLP + Batch-Norm on hidden Layers + AdamOptimizer

```
In [35]: from keras.layers.normalization import BatchNormalization

model_batch = Sequential()

model_batch.add(Dense(216, activation='relu', input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))
model_batch.add(BatchNormalization())
model_batch.add(Dense(170, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_batch.add(Dense(136, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())
```

```

model_relu.add(Dense(80, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_relu.add(Dense(38, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_batch.add(Dense(output_dim, activation='softmax'))

```

```

In [36]: model_batch.compile(optimizer='adam', loss='categorical_crossentropy',
                           metrics=['accuracy'])

history32 = model_batch.fit(x_train, y_train,
                           batch_size=batch_size,
                           epochs=nb_epoch, verbose=1,
                           validation_data=(x_test, y_test))

```

```

Train on 60000 samples, validate on 10000 samples
Epoch 1/20
60000/60000 [=====] - 6s 96us/step - loss: 0.4
860 - acc: 0.8559 - val_loss: 14.5498 - val_acc: 0.0973
Epoch 2/20
60000/60000 [=====] - 4s 71us/step - loss: 0.3
060 - acc: 0.9119 - val_loss: 14.5482 - val_acc: 0.0974
Epoch 3/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
868 - acc: 0.9190 - val_loss: 14.2799 - val_acc: 0.1140
Epoch 4/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
793 - acc: 0.9210 - val_loss: 14.2871 - val_acc: 0.1136
Epoch 5/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
719 - acc: 0.9233 - val_loss: 14.5417 - val_acc: 0.0978
Epoch 6/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
689 - acc: 0.9239 - val_loss: 14.4837 - val_acc: 0.1014
Epoch 7/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2

```



```
641 - acc: 0.9253 - val_loss: 14.5530 - val_acc: 0.0971
Epoch 8/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
616 - acc: 0.9276 - val_loss: 14.2758 - val_acc: 0.1143
Epoch 9/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
613 - acc: 0.9268 - val_loss: 14.2887 - val_acc: 0.1135
Epoch 10/20
60000/60000 [=====] - 4s 72us/step - loss: 0.2
600 - acc: 0.9268 - val_loss: 14.4515 - val_acc: 0.1034
Epoch 11/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
574 - acc: 0.9276 - val_loss: 14.2855 - val_acc: 0.1137
Epoch 12/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
575 - acc: 0.9277 - val_loss: 14.2822 - val_acc: 0.1139
Epoch 13/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
564 - acc: 0.9284 - val_loss: 14.2871 - val_acc: 0.1136
Epoch 14/20
60000/60000 [=====] - 4s 72us/step - loss: 0.2
552 - acc: 0.9279 - val_loss: 14.5514 - val_acc: 0.0972
Epoch 15/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
552 - acc: 0.9284 - val_loss: 14.5434 - val_acc: 0.0977
Epoch 16/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
551 - acc: 0.9282 - val_loss: 14.5401 - val_acc: 0.0979
Epoch 17/20
60000/60000 [=====] - 4s 72us/step - loss: 0.2
542 - acc: 0.9289 - val_loss: 14.5691 - val_acc: 0.0961
Epoch 18/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
521 - acc: 0.9290 - val_loss: 14.5514 - val_acc: 0.0972
Epoch 19/20
60000/60000 [=====] - 4s 71us/step - loss: 0.2
533 - acc: 0.9290 - val_loss: 14.2774 - val_acc: 0.1142
Epoch 20/20
```

```
60000/60000 [=====] - 4s 71us/step - loss: 0.2514 - acc: 0.9294 - val_loss: 14.5482 - val_acc: 0.0974
```

```
In [37]: model_batch.summary()
```

Layer (type)	Output Shape	Param #
batch_normalization_14 (Batch Normalization)	(None, 784)	3136
batch_normalization_15 (Batch Normalization)	(None, 784)	3136
batch_normalization_16 (Batch Normalization)	(None, 784)	3136
batch_normalization_17 (Batch Normalization)	(None, 784)	3136
batch_normalization_18 (Batch Normalization)	(None, 784)	3136
dense_43 (Dense)	(None, 10)	7850
Total params: 23,530		
Trainable params: 15,690		
Non-trainable params: 7,840		

```
In [39]: score = model_batch.evaluate(x_test, y_test, verbose=0)
score15=score[0]
score16=score[1]
train_acc8=history32.history['acc']
print('Test score:', score[0])
print('Test accuracy:', score[1])

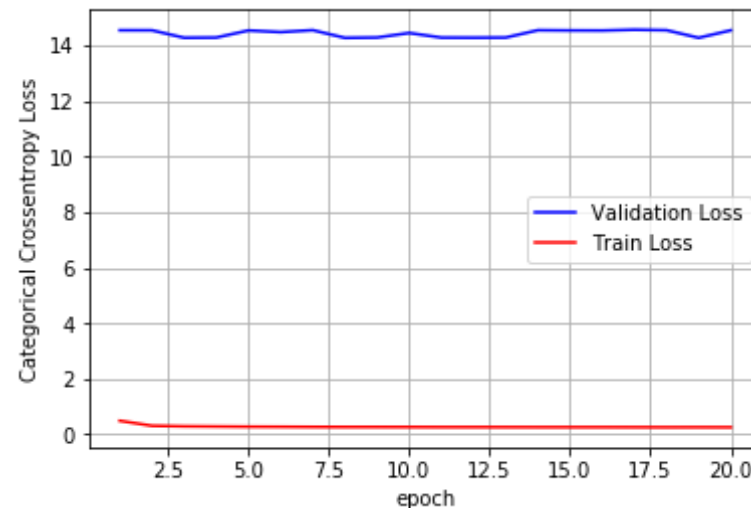
fig,ax32 = plt.subplots(1,1)
ax32.set_xlabel('epoch') ; ax32.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1,nb_epoch+1))
```

```
vy32 = history32.history['val_loss']  
ty32 = history32.history['loss']  
plt_dynamic(x, vy32, ty32, ax32)
```

Test score: 14.548192663574218

Test accuracy: 0.0974



3.3 MLP + Dropout + AdamOptimizer

In [40]: *# <https://stackoverflow.com/questions/34716454/where-do-i-call-the-batch-normalization-function-in-keras>*

```
from keras.layers import Dropout  
  
model_drop = Sequential()  
model_drop.add(Dense(216, activation='relu', input_shape=(input_dim,),  
                    kernel_initializer=he_normal(seed=None)))  
model_drop.add(BatchNormalization())  
model_drop.add(Dropout(0.5))  
model_drop.add(Dense(170, activation='relu',  
                    kernel_initializer=he_normal(seed=None)))
```

```

model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_relu.add(Dense(136, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))
model_relu.add(Dense(80, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_relu.add(Dense(38, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_drop.add(Dense(output_dim, activation='softmax'))

```

```

In [41]: model_drop.compile(optimizer='adam',
                        loss='categorical_crossentropy',
                        metrics=['accuracy'])

history33 = model_drop.fit(x_train, y_train,
                        batch_size=batch_size,

                        epochs=nb_epoch, verbose=1,
                        validation_data=(x_test, y_test))

```

Train on 60000 samples, validate on 10000 samples

Epoch 1/20

60000/60000 [=====] - 6s 106us/step - loss: 2.1378 - acc: 0.3107 - val_loss: 1.0547 - val_acc: 0.8325

Epoch 2/20

60000/60000 [=====] - 5s 76us/step - loss: 1.6071 - acc: 0.4430 - val_loss: 0.9704 - val_acc: 0.8447

Epoch 3/20

60000/60000 [=====] - 5s 77us/step - loss: 1.5920 - acc: 0.4488 - val_loss: 0.9579 - val_acc: 0.8452

```
Epoch 4/20
60000/60000 [=====] - 5s 77us/step - loss: 1.5
923 - acc: 0.4474 - val_loss: 0.9549 - val_acc: 0.8456
Epoch 5/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
842 - acc: 0.4502 - val_loss: 0.9419 - val_acc: 0.8458
Epoch 6/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
851 - acc: 0.4478 - val_loss: 0.9492 - val_acc: 0.8455
Epoch 7/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
809 - acc: 0.4495 - val_loss: 0.9455 - val_acc: 0.8507
Epoch 8/20
60000/60000 [=====] - 5s 77us/step - loss: 1.5
809 - acc: 0.4507 - val_loss: 0.9446 - val_acc: 0.8530
Epoch 9/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
800 - acc: 0.4545 - val_loss: 0.9391 - val_acc: 0.8507
Epoch 10/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
730 - acc: 0.4524 - val_loss: 0.9367 - val_acc: 0.8446
Epoch 11/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
697 - acc: 0.4563 - val_loss: 0.9396 - val_acc: 0.8498
Epoch 12/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
773 - acc: 0.4530 - val_loss: 0.9417 - val_acc: 0.8515
Epoch 13/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
672 - acc: 0.4578 - val_loss: 0.9393 - val_acc: 0.8496
Epoch 14/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
659 - acc: 0.4565 - val_loss: 0.9358 - val_acc: 0.8436
Epoch 15/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
699 - acc: 0.4541 - val_loss: 0.9395 - val_acc: 0.8475
Epoch 16/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
641 - acc: 0.4596 - val_loss: 0.9332 - val_acc: 0.8458
```

```

Epoch 17/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
722 - acc: 0.4563 - val_loss: 0.9375 - val_acc: 0.8522
Epoch 18/20
60000/60000 [=====] - 5s 76us/step - loss: 1.5
593 - acc: 0.4602 - val_loss: 0.9359 - val_acc: 0.8533
Epoch 19/20
60000/60000 [=====] - 5s 77us/step - loss: 1.5
578 - acc: 0.4607 - val_loss: 0.9285 - val_acc: 0.8512
Epoch 20/20
60000/60000 [=====] - 5s 77us/step - loss: 1.5
583 - acc: 0.4602 - val_loss: 0.9346 - val_acc: 0.8529

```

In [42]: `model_drop.summary()`

Layer (type)	Output Shape	Param #
=====		
batch_normalization_19 (Batch Normalization)	(None, 784)	3136
dropout_6 (Dropout)	(None, 784)	0
batch_normalization_20 (Batch Normalization)	(None, 784)	3136
dropout_7 (Dropout)	(None, 784)	0
batch_normalization_21 (Batch Normalization)	(None, 784)	3136
dropout_8 (Dropout)	(None, 784)	0
batch_normalization_22 (Batch Normalization)	(None, 784)	3136
dropout_9 (Dropout)	(None, 784)	0
batch_normalization_23 (Batch Normalization)	(None, 784)	3136
dropout_10 (Dropout)	(None, 784)	0
dense_49 (Dense)	(None, 10)	7850

```
=====
Total params: 23,530
Trainable params: 15,690
Non-trainable params: 7,840
=====
```

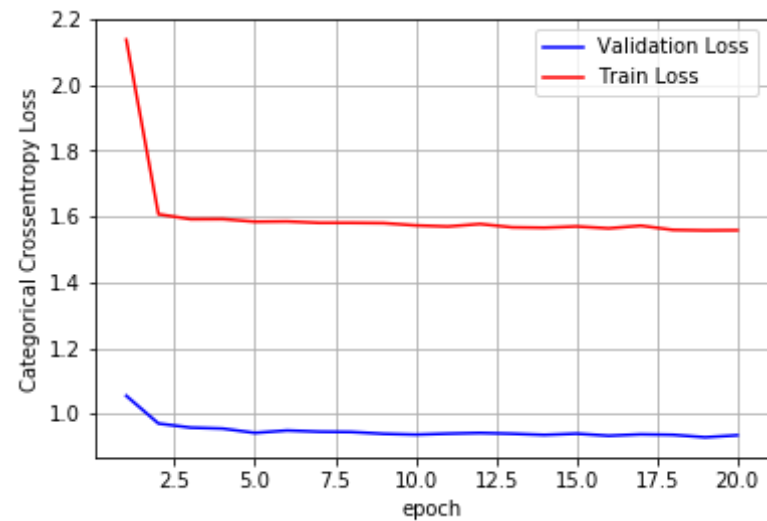
```
In [43]: score = model_drop.evaluate(x_test, y_test, verbose=0)
score17=score[0]
score18=score[1]
train_acc9=history33.history['acc']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax33 = plt.subplots(1,1)
ax33.set_xlabel('epoch') ; ax33.set_ylabel('Categorical Crossentropy Loss')

# list of epoch numbers
x = list(range(1,nb_epoch+1))

vy33 = history33.history['val_loss']
ty33 = history33.history['loss']
plt_dynamic(x, vy33, ty33, ax33)

Test score: 0.9346290021896362
Test accuracy: 0.8529
```



Final observation:

```
In [44]: from prettytable import PrettyTable
```

```
In [45]: models=['2_hidden_layer MLP+ReLu+Adam',  
                '2_hidden_layer MLP+Relu+adam+BN',  
                '2_hidden_layer MLP+reLu+Adam+BN+Drop-out',  
                '3_hidden_layer MLP+ReLu+Adam',  
                '3_hidden_layer MLP+Relu+adam+BN',  
                '3_hidden_layer MLP+reLu+Adam+BN+Drop-out',  
                '5_hidden_layer MLP+ReLu+Adam',  
                '5_hidden_layer MLP+Relu+adam+BN',  
                '5_hidden_layer MLP+reLu+Adam+BN+Drop-out']
```

```
In [46]: training_accuracy=[train_acc1,train_acc2,train_acc3,train_acc4,  
                             train_acc5,train_acc6,train_acc7,train_acc8,  
                             train_acc9  
                             ]
```



```
In [47]: test_score=[score1,score3,score5,score7,score9,score11,score13,score15,
                score17]
```

```
In [48]: test_accuracy=[score2,score4,score6,score8,score10,score12,score14,
                score16,
                score18]
INDEX = [1,2,3,4,5,6,7,8,9]
```

```
In [ ]: # Initializing prettytable
Model_Performance = PrettyTable()
# Adding columns
Model_Performance.add_column("INDEX.",INDEX)
Model_Performance.add_column("MODEL_NAME",models)
Model_Performance.add_column("TRAINING ACCURACY",training_accuracy)
Model_Performance.add_column("TESTING ACCURACY",test_accuracy)
Model_Performance.add_column("TEST SCORE",test_score)

# Printing the Model_Performance
print(Model_Performance)
```

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+-----+-----+-----+-----+-----+-----+-----+-----+
|
|
|
|-----+-----+-----+-----+ | INDEX.
| MODEL_NAME | TRAINING ACCURACY | TESTING ACCURACY | TEST SCORE | +-----+-----+
|
|-----+-----+-----+-----+
|-----+-----+-----+-----+ | 1 | 2_hidden_layer
MLP+ReLu+Adam | 0.9974 | 0.9824 | 0.0804028440125881 | | 2 | 2_hidden_layer
MLP+Relu+adam+BN | 0.9974 | 0.9797 | 0.07902511259189378 | | 3 | 2_hidden_layer
MLP+reLu+Adam+BN+Drop-out | 0.9974 | 0.982 | 0.06363873664251878 | | 4 | 3_hidden_layer
MLP+ReLu+Adam | 0.9974 | 0.9801 | 0.0930518318862476 | | 5 | 3_hidden_layer
MLP+Relu+adam+BN | 0.9974 | 0.8508 | 0.6960192083239556 | | 6 | 3_hidden_layer
MLP+reLu+Adam+BN+Drop-out | 0.9974 | 0.8992 | 0.40321056950092315 | | 7 | 5_hidden_layer
```

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MLP+ReLu+Adam | 0.9974 | 0.9816 | 0.08723179607167986 | | 8 | 5_hidden_layer
MLP+Relu+adam+BN | 0.9293 | 0.974 | 14.548192663574218 | | 9 | 5_hidden_layer
MLP+reLu+Adam+BN+Drop-out | 0.9345 | 0.8529 | 0.9346290021896362 | +-----+-----+
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```