Vellore Institute of Technology
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School of Information Technology and Engineering
Digital Assignment, June 2020
B.Tech, Winter-2019-2020

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COURSE CODE	MAT3005
COURSE NAME	APPLIED NUMERICAL METHODS
SLOT	A2+TA2+TAA2+V3
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Q.1 Find the unique polynomial $P(x)$ of degree 2 or less such that $P(1) = 1$, $P(3) = 27$, $P(4) = 64$ using each of the following methods

(i) Lagrange's interpolation formula

(ii) Newton's divided difference formula

A-1

x	$f(x)$	Ist div diff	II div diff
1	1		
		13 $[x_0, x_1]$	
3	27		8 $[x_0, x_1, x_2]$
		37 $[x_1, x_2]$	
4	64		

(i) $\because$ By Lagrange interpolation formula,

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$= \frac{x^2-7x+12}{6} + \frac{64}{3}(x^2-4x+3) + \frac{(-27)}{2}(x^2-5x+4)$$

$$= \frac{1}{6}(48x^2 - 114x + 172)$$

$$y = 8x^2 - 19x + 12$$

(ii) By Newton's divided difference formula (Refer table above)

$$\therefore \; y = y_0 + (x-x_0)[x_0, x_1] + (x-x_0)(x-x_1)[x_0, x_1, x_2]$$

$$= 1 + (x-1)(13) + (x-1)(x-3)(8)$$

$$y = 8x^2 - 19x + 12$$

Ans $P(x) = y = 8x^2 - 19x + 12$

Q.2 Obtain the cubic spline fit for the data under the conditions ~~$f(0)=0$~~ $f'(0) = 0 = f'(3)$ & valid in the interval $[1, 2]$. Hence obtain the estimate of $f(1.5)$.

x	0	1	2	3
y	1	4	12	8

A-2 $M_0 = M_3 = 0$

$$h_i M_{i-1} + (2h_i + h_{i+1}) M_i + h_{i+1} M_{i+1} = 6\left(\frac{y_{i+1} - y_i}{h_{i+1}} - \frac{y_i - y_{i-1}}{h_i}\right)$$

Here $h_{i-1} = h_i = h_{i+1} = 1$

$$\therefore \quad M_0 + 4M_1 + M_2 = 6[y_2 - 2y_1 + y_0]$$

$$4M_1 + M_2 = 30 \quad \text{——— (1)}$$

$$M_1 + 4M_2 + M_3 = 6[y_3 - 2y_2 + y_1]$$

$$M_1 + 4M_2 = -72 \quad \text{——— (2)}$$

From (1) & (2) $M_1 = 12.8$ & $M_2 = -21.2$

For cubic spline :-

$$S_i(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i\right] + \left[y_{i-1} - \frac{M_{i-1}}{6}\right](x_i - x) + \left[y_i - \frac{M_i}{6}\right](x - x_{i-1})$$

$i = 2 \rightarrow$ interval $[1, 2]$ since $f(1.5) = ?$ & 1.5 lies in $[1, 2]$

$$S_2(x) = \frac{1}{6}\left[(2-x)^3\, 12.8 + (x-1)^3(-21.2)\right] + \left[4 - \frac{12.8}{6}\right](2-x) + \left[12 + \frac{21.2}{6}\right](x-1)$$

$$S_2(x) = \frac{1}{15}\left[-85x^3 + 351x^2 - 388x + 132\right]$$

Ans $S_2(1.5) = f(1.5) = \frac{341}{40} = 8.525$

Q.3 Calculate $\int_0^{1/2} \frac{x}{\sin x}\, dx$ using Romberg integration with step size $h = 1/16$

A-3

x	0	0.0625	0.125	0.1875	0.25
y	1	1.000651	1.002609	1.005883	1.010493
x	0.3125	0.375	0.4375	0.5	
y	1.016463	1.023828	1.032628	1.042915	

$y = \frac{x}{\sin x}$

$h = \frac{1}{16}$

By trapezoidal Rule

$$I_{.} = \frac{0.0625}{2}\left[1 + 1.042915 + 2(y_1+y_2+y_3+\cdots y_{n-1})\right]$$

$$I = 0.507125$$

For $\frac{h}{2} = \frac{1}{32}$ $\quad I = 0.507074$

For $h = \frac{1}{64}$ $\frac{h}{4}$

x	0	0.15625	0.3125	0.046875	0.0625	0.78125
y	1	1.000041	1.00163	1.000366	1.000651	1.001018
x	0.0625	0.78125	0.09375	0.109375	0.125	0.140625
y	1.000651	1.001018	1.001466	1.001997	1.002609	1.003304
x	0.15625	0.171875	0.1875	0.203125	0.28125	0.296875
y	1.004081	1.004941	1.005883	10.0691	1.013306	1.014842
x	0.25	0.265625	0.28125	0.296875	0.3125	0.390625
y	1.010443	1.011357	1.013306	1.015762	1.023828	1.025891
x	0.40625	0.421875	0.375	0.390625	0.40625	0.421875
y	1.028046	1.030291	1.016463	1.018172	1.019969	1.021854
x	0.4375	0.453125	0.46875	0.484375	0.5	
y	1.032628	1.035058	1.037582	1.040201	1.042915	

$$\therefore I = \frac{0.015625}{2}\left[1 + 1.042915 + 2(y_1+y_2+\cdots y_{n-1})\right]$$

$$I = 0.507071$$

By Romberg's Method

I	Val. of I	Val. of I	Val. of I
$h=\frac{1}{16}$	0.507125		
		0.507057	
$h=\frac{1}{32}$	0.507074		0.507074
		0.50707	
$h=\frac{1}{64}$	0.507071		

Ans $\quad I = \int_0^{1/2} \frac{x}{\sin x}\,dx = 0.507074$

Q·4 Use the classical Runge-Kutta 4^{th} order formula to find the numerical solution at $x = 0.8$ for $\frac{dy}{dx} = \sqrt{x+y}$, $y(0.4) = 0.41$. Assume step length $h = 0.2$

A-4 $f(x, y) = \frac{dy}{dx} = \sqrt{x+y}$

By Runge - Kutta Method

$k_1 = hf(x_0, y_0) = 0.2f(0.4, 0.41) = 0.18$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.2$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = 0.200997$

$k_4 = hf(x_0 + h, y_0 + k_3) = 0.22009$

$k = \frac{1}{6}\left[k_1 + 2k_2 + 2k_3 + k_4\right]$

$k = 0.200347$

$y_1 = y_0 + k = 0.41 + 0.200347 = 0.610347$

$x_1 = 0.4 + 0.2 = 0.6$

Now

$k_1 = 0.220031$

$k_2 = 0.238357$

$k_3 = 0.256863$

$k_4 = \cancel{0.238643}\ 0.247347$

$k = \cancel{0.238643}\ 0.242969$

$y_2 = y_1 + k = 0.853316$

$x_2 = 0.6 + 0.2 = 0.8$

~~Now~~

$\cancel{k_1 = \cancel{0.264825}\ 0.2571626}$

$\cancel{k_2 = 0.274364}$

$\cancel{k_3 = 0.274990}$

$\cancel{k_4 = 0.271774}$

$\cancel{k = 0.274607}$

$\cancel{y_3 = y_2 + k = 1.12792}$

Ans $\cancel{y(0.8) =}\ y(0.8) \simeq 0.853316$

Q.6 Find difference approximations of the solution $y(x)$ of the boundary value problem $y'' + 8\sin^2(\pi x)y = 0$, $0 \le x \le 1$, $y(0) = y(1) = 1$ taking step-lengths $h = 1/4$ & $h = 1/6$. Also find an approximate value for $y'(0)$.

A.6 $h = 1/4$

x	0	0.25	0.5	0.75	1
y	1	y_1	y_2	y_3	1

$$\frac{1}{h^2}\left[y_{i+1} - 2y_i + y_{i-1}\right] + 8\sin^2(\pi x)y = 0$$

$$\left(y_i'' = \frac{1}{h^2}\left[y_{i+1} - 2y_i + y_{i-1}\right]\right)$$

$$16\left[y_{i+1} - 2y_i + y_{i-1}\right] + 8\sin^2(\pi x_i)y_i = 0$$

$i = 1$

$$2\cancel{16}\left[y_2 - 2y_1 + 1\right] + \cancel{8}\sin^2(0.25\pi)y_1 = 0$$

$$2y_2 - 4y_1 + 2 + \frac{y_1}{2} = 0 \quad \rightarrow \quad 7y_1 - 4y_2 = 4$$

$$\rightarrow y_2 = \frac{7y_1 - 4}{4}$$

$i = 2$

$$2\left[y_3 - 2y_2 + y_1\right] + \sin^2(0.5\pi)y_2 = 0$$

$$2y_3 - 4y_2 + 2y_1 + y_2 = 0$$

$$2y_1 - 3y_2 + 2y_3 = 0 \quad \rightarrow \quad y_3 = \frac{13y_1 - 12}{8}$$

$i = 3$

$$2\left[1 - 2y_3 + y_2\right] + \sin^2(0.75\pi)y_3 = 0$$

$$2 - 4y_3 + 2y_2 + \frac{y_3}{2} = 0$$

$$4y_2 - 7y_3 + 4 = 0$$

$$y_1 = 2.4, \quad y_2 = 3.2, \quad y_3 = 2.4$$

$$y(0.25) = y(0.75) = 2.4, \quad y(0.5) = 3.2$$

$h = 1/6$

x	0	1/6	1/3	1/2	2/3	5/6	1
y	1	y_1	y_2	y_3	y_4	y_5	1

$$36\left[y_{i+1} - 2y_i + y_{i-1}\right] + 8\sin^2(\pi x_i)y_i = 0$$

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$i=1$ $\quad 9[y_2 - 2y_1 + 1] + 2\sin^2(\pi/6)\, y_1 = 0$

$$9y_2 - 18y_1 + 9 + \frac{y_1}{2} = 0 \Rightarrow 35y_1 - 18y_2 = 18$$

$i=2$ $\quad 9y_3 - 18y_2 + 9y_1 + \frac{3y_2}{2} = 0 \Rightarrow 18y_1 - 33y_2 + 18y_3 = 0$

$i=3$ $\quad 9y_4 - 18y_3 + 9y_2 + 2y_3 = 0 \Rightarrow 9y_2 - 16y_3 + 9y_4 = 0$

$i=4$ $\quad 9y_5 - 18y_4 + 9y_3 + \frac{3y_4}{2} = 0 \Rightarrow 18y_3 - 33y_4 + 18y_5 = 0$

$i=5$ $\quad 9 - 18y_5 + 9y_4 + \frac{y_5}{2} = 0 \Rightarrow 18y_4 - 35y_5 + 9 = 0$

$y_1 = \cancel{1.6433744}\ 2.358481 = y(1/6)$

$y_2 = \cancel{2.1954503}\ 3.585935 = y(1/3)$

$y_3 = 7.703542 = y(1/2)$

$y_4 = 6.096904 = y(2/3)$

$y_5 = 3.360947 = y(5/6)$

Q.7 Consider the initial value problem

$$\frac{dy}{dx} = x^2y + y^2, \qquad y(0) = 1$$

Find the solution of this equation at $x = 0.4, 0.5, 0.6$, using Adams-Bashforth - Moulton predictor-corrector method.

N.7

x	0	0.1	0.2	0.3	0.4	0.5	0.6
y	1	1.11149	1.227613	1.408836	1.66271	2.038901	2.649021
f	1	1.246524	1.556138	2.111614	3.030641	4.660842	7.970959

$$f = \frac{dy}{dx} = x^2y + y^2$$

For $y_1, y_2, y_3 \rightarrow$ R-K method

For $y_4, y_5, y_6 \rightarrow$ ABM method

$h = 0.1$ $\quad k_1 = 0.1 \qquad k_3 = 0.140688 \qquad k = 0.116123$

$k_2 = 0.\cancel{126143}\ 1105125 \qquad k_4 = 0.161783 \qquad y_2 = 1.227613$

Again $k_1 = 0.155613$

$k_2 = 0.17857$

$k_3 = 0.181652$

$k_4 = 0.211286$

$k = \frac{1}{6}(1.087343) = 0.181223$

$y_3 = 1.227613 + 0.181223 = 1.408836$

Predictor formula ; $y_4(p) = y_3 + \frac{h}{24}[55f_3 - 59f_2 - 37f_1 - 9f_0]$

corrector formula ; $y_4(c) = y_3 + \frac{h}{24}[9f_4 + 19f_3 - 5f_2 + f_1]$

$\rightarrow$ $y_4(p) = 1.408836 + \frac{0.1}{24}[55(2.111614) - 59(1.566138) + 37(1.11149) - 9(1)]$

$y_4(p) = 1.64869$

$\therefore f_4(p) = 3.038167$

$\rightarrow$ $y_4(c) = 1.408836 + \frac{0.1}{24}[9(3.038167) + 19(2.111614) + 5(1.556138) + 1.11149]$

$y_4 = 1.662711$

$f_4 = 3.036641$

$\rightarrow$ $y_5(p) = 1.6627 + \frac{0.1}{24}[55(3.30641) - 59(2.111614) + 37(1.556138) - 9(1.11149)]$

$y_5(p) = 2.031287$ $\quad \therefore f_5(p) = 4.633948$

$\rightarrow$ $y_5(c) = 1.662711 + \frac{0.1}{24}[9(4.633948) + 19(3.030641) - 5(2.111614) + 1.55613)]$

$y_5(c) = 2.038901$, $f_5 = 4.660842$

$\rightarrow$ $y_6(p) = 2.038901 + \frac{0.1}{24}[55(4.666842) - 59(3.30641) + 37(2.111614) - 9(1.556138)]$

$y_6(p) = 2.630538$ $\quad \therefore f_6(p) = 7.866723$

$\rightarrow$ $y_6(c) = 2.038901 + \frac{0.1}{24}[9(7.866723) + 19(4.666842) - 5(3.03641) + 2.11161$

$y_6 = 2.649021$, $f_6 = 7.970959$

Q.5 The solution of the system of eqns $y' = u$, $y(0) = 1$, $u' = -4y - 2u$, $u(0) = 1$ is to be obtained by the Runge-Kutta 4th order method. Can a step length of $h = 0.1$ be used for integration. If so find the approximate values of $y(0.2)$ & $u(0.2)$.

A.5 $y' = u$ $\therefore$ $u' = y'' = -4y - 2u$

$y' = f_1(x, y, u) = u$

$y'' = u' = f_2(x, y, u) = -4y - 2u$

$k_1 = hf_1(x_0, y_0, u_0)$

$k_2 = hf_1(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, u_0 + \frac{l_1}{2})$

$k_3 = hf_1(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, u_0 + \frac{l_2}{2})$

$k_4 = hf_1(x_0 + h, y_0 + k_3, u_0 + l_3)$

$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$l_1 = hf_2(x_0, y_0, u_0)$

$l_2 = hf_2(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, u_0 + \frac{l_1}{2})$

$l_3 = hf_2(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, u_0 + \frac{l_2}{2})$

$l_4 = hf_2(x_0 + h, y_0 + k_3, u_0 + l_3)$

$l = \frac{1}{6}[l_1 + 2l_2 + 2l_3 + l_4]$

$x_0 = 0$, $y_0 = u_0 = 1$

$k_1 = 0.1$ $\quad l_1 = -0.6$

$k_2 = 0.07$ $\quad l_2 = -0.56$

$k_3 = 0.072$ $\quad l_3 = -0.558$

$k_4 = 0.044$ $\quad l_4 = -0.5178$

$k = 0.071$ $\quad l = -0.559$

$y_1 = 1.071$ $\quad u_1 = 0.441$

$k_1 = 0.0441$ $\quad l_1 = -0.576$

$k_2 = 0.0183$ $\quad l_2 = -0.472$

$k_3 = 0.02$ $\quad l_3 = -0.473$

$k_4 = -0.0032$ $\quad l_4 = -0.43$

$k = \cancel{0.0834}\ 0.0195$ $\quad l = -0.472$

$y_2 = 1.0905$ $\quad u_2 = -0.0316$

$y(0.2) = 1.0905$, $u(0.2) = -0.0316$