DIGITAL ASSIGNMENT – 1

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Q.

Explain any three scenarios where Heisenberg Uncertainty is applied successfully.

A.

Uncertainty principle, also called **Heisenberg uncertainty principle** or indeterminacy **principle**, statement, articulated (1927) by the German physicist Werner **Heisenberg**, that the position and the velocity of an object cannot both be measured exactly, at the same time, even in theory.

1. NON-EXISTENCE OF ELECTRON IN NUCLEUS.

→ This can be proved by using Heisenberg's Uncertainty Principle by method of contradiction.

→ The principle states that :- $\Delta p \cdot \Delta x \geq \frac{h}{2\pi}$ — ①

where $h = 6.626 \times 10^{-34}$ $kg\,m^2/s$

Let us assume that an electron "CAN" exist in nucleus.

Now we know that $p = mv$ and therefore $\Delta p = m\Delta v$ — ②

Also mass of electron is 9.1×10^{-31} kg.

putting ② in ①

$$\Rightarrow \quad m\Delta v \cdot \Delta x \geq \frac{h}{2\pi}$$

$$\Delta v \geq \frac{h}{2\pi m \Delta x} \quad — ③$$

It is also known that the uncertainty in position of electron i.e $\Delta x = 2 \times 10^{-14}$ m (experimental value).

So now we know the values of h, π, m and Δx.

Hence we can find Δv, by putting these values in equation ③,

$$\Rightarrow \quad \Delta v \geq \frac{6.626 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 2 \times 10^{-14}}$$

$$\Delta v \geq 5.797 \times 10^{9} \text{ m/s}$$

But we know that the speed of light, $c = 3 \times 10^{8}$ m/s.

So $\Delta v > c$ which is not possible. So our assumption that an electron can exist in nucleus is wrong.

2) RADIUS OF BOHR'S FIRST ORBIT.

→ Using Heisenberg's Uncertainty Principle, we can find the radius of Bohr's first orbit.

→ The principle states that $\Delta p \cdot \Delta x \geq \frac{h}{2\pi}$ or $\Delta p \cdot \Delta x \geq \hbar$

$$\Rightarrow \quad \Delta p \geq \frac{\hbar}{\Delta x} \quad \text{—} \ (1)$$

Now, the total energy (E) of the orbit will be sum of the kinetic and potential energies. i.e

$$E = K.E + P.E \qquad \begin{pmatrix} K.E = T \\ P.E = V \end{pmatrix}$$

$$E = T + V$$

$$\Delta E = \Delta T + \Delta V$$

We need to find ΔT & ΔV

We know $T = \frac{1}{2}mv^2 \Rightarrow \Delta T = \frac{1}{2}m(\Delta v)^2$

Also $T = \frac{p^2}{2m} \Rightarrow \Delta T = \frac{(\Delta p)^2}{2m}$

We know $V = \frac{-Ze^2}{x}$ (Z = Atomic number)

$$\therefore \Delta V = \frac{-Ze^2}{\Delta x}$$

So $$\Delta E = \frac{(\Delta p)^2}{2m} - \frac{Ze^2}{\Delta x} = \frac{\hbar^2}{2m(\Delta x)^2} - \frac{Ze^2}{\Delta x}$$

↳ from the principle

To find minimum ΔE; $\frac{d^2(\Delta E)}{d(\Delta x)^2} > 0$ (positive)

To find Δx; $\frac{d(\Delta E)}{d(\Delta x)} = 0$

i.e $$\frac{-\hbar^2}{m(\Delta x)^3} + \frac{Ze^2}{(\Delta x)^2} = 0$$

$$\therefore \left(\frac{-h^2}{m\Delta x} + Ze^2\right)\frac{1}{(\Delta x)^2} = 0$$

$$\therefore \quad Ze^2 = \frac{h^2}{m\Delta x} \Rightarrow \Delta x = \frac{h^2}{mZe^2} \quad \text{—— (2)}$$

Now $$\frac{d^2(\Delta E)}{d(\Delta x)^2} = \frac{3h^2}{m(\Delta x)^4} - \frac{2Ze^2}{(\Delta x)^3} \quad \text{—— (3)}$$

put (2) in (3)

$$\frac{d^2(\Delta E)}{d(\Delta x)^2} = \frac{m^3Z^4e^8}{h^6}\left[3 - \frac{2mZe^2}{h^2}\right]$$

Clearly $\frac{d^2(\Delta E)}{d(\Delta x)^2} > 0$ i.e positive

$$\therefore \quad \Delta x = \frac{h^2}{mZe^2} \longrightarrow \text{minimum}$$

Since uncertainty in position is minimum, we can say that Δx = radius (r)

$$\therefore \quad r = \frac{h^2}{mZe^2}$$

$$\boxed{r = \frac{h^2}{4\pi^2 mZe^2}} \qquad \left[\because \Delta p \cdot \Delta x \geq \frac{h}{2\pi}\right]$$

→ Radius of Bohr's 1st orbit.

Hence radius of Bohr's 1st orbit is found using Heisenberg's Uncertainty Principle.

3) QUANTUM HARMONIC OSCILLATOR

→ The ground state energy for the quantum harmonic oscillator can be shown to be the minimum energy allowed by the uncertainty principle.

→ The energy of the quantum harmonic oscillator must be at least

$$E = \frac{(\Delta p)^2}{2m} + \frac{1}{2} m\omega^2 (\Delta x)^2$$

Δx = uncertainty in position
Δp = uncertainty in moment

→ Taking the lower limit from the uncertainty principle

$$\Delta p \cdot \Delta x = \frac{h}{2}$$

→ Then the energy expressed in terms of the position uncertainty can be written

$$E = \frac{h^2}{8m(\Delta x)^2} + \frac{m\omega^2(\Delta x)^2}{2}$$

→ Minimizing the energy by taking the derivative with respect to the position uncertainty and setting it equal to zero gives

$$\frac{-h^2}{4m(\Delta x)^3} + m\omega^2(\Delta x) = 0$$

$$\Rightarrow \quad \Delta x = \sqrt{\frac{h}{2m\omega}}$$

→ Substituting gives the minimum value of energy allowed

$$E_0 = \frac{h^2}{8m(\Delta x)^2} + \frac{1}{2} m\omega^2 (\Delta x)^2 = \frac{h\omega}{4} + \frac{h\omega}{4}$$

$$\boxed{E_0 = \frac{h\omega}{2}}$$

- This is a very significant physical result because it tells us that the energy of a system described by a harmonic oscillator potential cannot have zero energy.

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