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Chapter 4: General Vector Spaces (Multiple Choice Questions)

1. Let $V = R^2$ and define addition and scalar multiplication as follow

$u+v = (u_1, u_2) + (v_1, v_2) = (u_1 + v_1, 0)$.

$ku = k(u_1, u_2) = (ku_1, 0)$

Which of the following vector space axioms does not hold?

(A) $k(u+v) = ku+kv$ (B) $1u = u$ (C) Closure under scalar multiplication

(D) None of the above

Ans: (B)

$u = (u_1, u_2)$

$1u = 1(u_1, u_2) = (1u_1, 0)$ $\because ku = k(u_1, u_2) = (ku_1, 0)$

$\therefore$ $1u \neq u$

$\therefore$ $1u = u$ axiom does not hold.

2. Let V be a vector space, let k be a scalar, and let u, v, w be vectors in V. Which of the following statements does not hold?

(A) $(u+v) - w = u - (w - v)$ (B) If $ku = 0$, then $k = 0$ or $u = 0$

(C) $-k(u+v-w) = -(ku - k(v-w))$ (D) $0 - k(1u - v) + 0w = k(1v - u) + 0$

Ans: (C)

$-k(u+v-w) = -ku - kv + kw$ — (1)

$-(ku - k(v-w)) = -ku + k(v-w) = -ku + kv - kw$ — (2)

$\therefore$ (1) $\neq$ (2), So (C) does not hold

3. Which of the following is a subspace of R^3?

(A) All vectors of the form $(0, a, a^2)$.

(B) All vectors of the form $(a+2, a, 0)$

(C) All vectors of the form $(a, b, 2)$

(D) All vectors of the form $(a, b, a-2b)$.

Ans: (D)

$W = (a, b, a-2b)$; $u, v \in W$

$u = (u_1, u_2, u_1 - 2u_2)$; $v = (v_1, v_2, v_1 - 2v_2)$

$u+v = (u_1, u_2, u_1-2u_2) + (v_1, v_2, v_1-2v_2)$.

$= (u_1+v_1, u_2+v_2, u_1-2u_2+v_1-2v_2)$

$= ((u_1+v_1), (u_2+v_2), (u_1+v_1)-2(u_2+v_2))$

So $u+v \in W$

$ku = k(u_1, u_2, u_1-2u_2) = (ku_1, ku_2, ku_1 - 2(ku_2))$

$\therefore$ $ku \in W$

So (D) belongs to subspace of R^3

4. Which of the following is not a linear combination of A and B?.

$A = \begin{bmatrix} 1 & 4 \\ 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

(A) $\begin{bmatrix} 1 & 12 \\ 8 & -2 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 4 \\ 4 & 2 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & -4 \\ -1 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 8 \\ 6 & -1 \end{bmatrix}$

Ans (D)

$\because$ (A) $= 3A-B$; (B) $= A+B$; (C) $= 2B-A$

$\therefore$ By elimination, (D) is not a linear combination of A & B. Also (D) cannot be obtained by any operations on A & B.

5. Which of the following sets of vectors does not span R^3?

(A) $\{(2,3,4), (6,0,7), (0,9,5)\}$ (B) $\{(1,3,2), (1,0,1), (0,2,2)\}$

(C) $\{(1,4,6), (2,3,1), (1,1,0)\}$ (D) $\{(6,7,1), (2,2,3), (1,1,2)\}$

Ans (A)

$\rightarrow \begin{bmatrix} 2 & 3 & 4 \\ 6 & 0 & 7 \\ 0 & 9 & 5 \end{bmatrix} \xrightarrow{\times -3} \sim \begin{bmatrix} 2 & 3 & 4 \\ 0 & -9 & -5 \\ 0 & 9 & 5 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & 4 \\ 0 & -9 & -5 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore$ $3(2,3,4) - (6,0,7) = (0,9,5)$

So the set of vectors in (A) are not linearly independent and thus do not span R^3.

In (B), (C), (D) ~~the matrices formed~~ all the vectors are linearly independent & therefore span R^3.

6. Which of the following vectors in R^3 does not ~~sp~~ lie on the same line as the others?

(A) $(1,-7,4)$ (B) $(-4,-28,16)$ (C) $(-3,21,-12)$ (D) $(2,-14,8)$

Ans (B)

$\because$ (C) $= -3$(A) ; (D) $= 2$(A) & $\frac{-1}{4}$(B) $= (1,7,-4)$

$\therefore$ (B) does not lie on the same line as (A), (C), (D)

7. Which of the following sets of matrices are linearly independent?

(A) $\left\{\begin{bmatrix}2&3\\1&0\end{bmatrix}, \begin{bmatrix}4&0\\1&0\end{bmatrix}, \begin{bmatrix}3&1\\-1&1\end{bmatrix}\right\}$ (B) $\left\{\begin{bmatrix}1&0\\2&1\end{bmatrix}, \begin{bmatrix}0&1\\2&2\end{bmatrix}, \begin{bmatrix}1&-1\\0&-1\end{bmatrix}\right\}$

(C) $\left\{\begin{bmatrix}0&2\\2&1\end{bmatrix}, \begin{bmatrix}1&1\\1&1\end{bmatrix}, \begin{bmatrix}1&3\\3&2\end{bmatrix}\right\}$ (D) $\left\{\begin{bmatrix}-1&1\\1&0\end{bmatrix}, \begin{bmatrix}1&0\\2&1\end{bmatrix}, \begin{bmatrix}1&1\\5&2\end{bmatrix}\right\}$

Ans (A)

$\because$ In (B) : $\begin{bmatrix}1&0\\2&1\end{bmatrix} = \begin{bmatrix}0&1\\2&2\end{bmatrix} + \begin{bmatrix}1&-1\\0&-1\end{bmatrix}$

(C) : $\begin{bmatrix}1&3\\3&2\end{bmatrix} = \begin{bmatrix}0&2\\2&1\end{bmatrix} + \begin{bmatrix}1&1\\1&1\end{bmatrix}$

(D) : $\begin{bmatrix}1&1\\5&2\end{bmatrix} = \begin{bmatrix}-1&1\\1&0\end{bmatrix} + 2\begin{bmatrix}1&0\\2&1\end{bmatrix}$

$\therefore$ only (A) is linearly independent

8. The following set forms a basis for M_{22}. Which of the following is a possible value for the matrix A?

$\left\{\begin{bmatrix}1&1\\1&0\end{bmatrix}, \begin{bmatrix}1&1\\0&1\end{bmatrix}, \begin{bmatrix}1&0\\1&1\end{bmatrix}, A\right\}$

(A) $\begin{bmatrix}2&1\\1&2\end{bmatrix}$ (B) $\begin{bmatrix}-1&-1\\0&-1\end{bmatrix}$ (C) $\begin{bmatrix}0&1/2\\1/2&1/2\end{bmatrix}$ (D) $\begin{bmatrix}0&0\\2&-2\end{bmatrix}$

Ans (C)

In basis, all elements should be linearly independent.

In (A) : $\begin{bmatrix}2&1\\1&2\end{bmatrix} = \begin{bmatrix}1&1\\1&0\end{bmatrix} + \begin{bmatrix}1&0\\1&1\end{bmatrix}$

(B) : $\begin{bmatrix}-1&-1\\0&-1\end{bmatrix} = -1\begin{bmatrix}1&1\\0&1\end{bmatrix}$

(D) $\begin{bmatrix} 0 & 0 \\ 2 & -2 \end{bmatrix}$ is not valid because of all 0's in R_1.

$\therefore$ (C) is a possible value for matrix A

9. Given that the set $\{1, 1+x+x^2, p(x)\}$ is a basis for P_2, which of the following is a possible value for $p(x)$? (A) 0 (B) $1+x$ (C) -1 (D) $2+x+x^2$

Ans (B)

In (A) since it is a basis for polynomial P_2, 0 cannot be in the basis

In (C), $-1 = (-1) \times (1)$; i.e it can be obtained from 1 which is already in the basis

In (D) $2+x+x^2 = 1 + (1+x+x^2)$ i.e it is a ~~linearly combin~~ linearly dependent on the 2 vectors.

$\therefore$ (B) $1+x$ is ~~the~~ a possible for $p(x)$

10. Which of the following sets in R^3 is linearly dependent?

(A) $\{(1,4,6), (1,-4,0), (4,5,2), (1,3,-5)\}$ (D) $\{(1,4,-2), (3,0,0)\}$

(B) $\{(3,2,4), (2,4,3), (0,1,3)\}$ (C) $\{(0,2,0), (2,3,3), (4,2,4)\}$

Ans (A)

$\because$ $M = \begin{bmatrix} 1 & 4 & 6 \\ 1 & -4 & 0 \\ 4 & 5 & 2 \\ 1 & 3 & -5 \end{bmatrix}$ $R_2 \to (R_2 - R_1) \times -1/2$, $R_3 \to (R_3 - 4R_1) \times -1/11$, $R_4 \to (R_4 - R_1) \times -1$

$M = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 4 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 11 \end{bmatrix}$ $R_2 \to (R_2 - 4R_3) \times -1/5$, $R_4 \to (R_4 - R_2) \times 1/9$

$M = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ $R_4 \to R_4 - R_2 \Rightarrow M = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

Since after row reduction, $R_4 \to (0,0,0)$, we can conclude that the set of vectors in (A) are linearly dependent

Q.11 How many vectors are in the standard basis for the vector space P_5?

(A) 4 (B) 5 (C) 6 (D) 0

Ans (C)

Standard basis for $P_5 = \{\underset{1}{1}, \underset{2}{x}, \underset{3}{x^2}, \underset{4}{x^3}, \underset{5}{x^4}, \underset{6}{x^5}\}$

Q.12 What is the transition matrix from S_1 to S_2 given $S_1 = \{u_1 = (1,-2), u_2 = (3,-4)\}$ & $S_2 = \{v_1 = (1,3), v_2 = (3,8)\}$?

(A) $\begin{bmatrix} -14 & -36 \\ 5 & 13 \end{bmatrix}$ (B) $\begin{bmatrix} -13/2 & -18 \\ 5/2 & 7 \end{bmatrix}$ (C) $\begin{bmatrix} -7 & -18 \\ 5/2 & 13/2 \end{bmatrix}$ (D) $\begin{bmatrix} -13 & -36 \\ 5 & 14 \end{bmatrix}$

Ans (A)

S_1 is old basis & S_2 is new basis & Let T be the transition matrix.

$\therefore \quad [S_2 \mid S_1] \xrightarrow[\text{operations}]{\text{row}} [I_2 \mid T]$

$$\cancel{\left[\begin{array}{cc|cc} 1 & 3 & 1 & -2 \\ 3 & 8 & 3 & -4 \end{array}\right]} \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ \cancel{R_2 \to -1 \times R_2} \\ R_1 \to R_1 - 3R_2 \end{array} \sim \cancel{\left[\begin{array}{cc|cc} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & -2 \end{array}\right]}$$

$$\cancel{T = \begin{bmatrix} 1 & 4 \\ 0 & -2 \end{bmatrix}}$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 3 \\ 3 & 8 & -2 & -4 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_2 \to -1 \times R_2 \\ R_1 \to R_1 - 3R_2 \end{array} \sim \left[\begin{array}{cc|cc} 1 & 0 & -14 & -36 \\ 0 & 1 & 5 & 13 \end{array}\right]$$

$T = \begin{bmatrix} -14 & -36 \\ 5 & 13 \end{bmatrix}$ $\therefore$ (A) is the transition matrix from S_1 to S_2

Q.13 Given the basis $B = \{(4,0), (0,1)\}$ and the vector $w = (2,3)$, what is the coordinate vector $[w]_B$?

(A) $(1/2, 3)$ (B) $(8,3)$ (C) $(2,6)$ (D) $(2,3)$

Ans (A)

Let $[w]_B = (x, y)$

$\therefore \quad (2,3) = x(4,0) + y(0,1) = (4x, 0) + (0, y) = (4x, y)$

$\therefore \; 4x = 2 \Rightarrow x = 1/2$

$y = 3 \qquad \therefore [w]_B = (1/2, 3) \Rightarrow$ (A)

Q.14. Which is ~~the~~ equivalent to the following matrix multiplication?

$$\begin{bmatrix} 2 & 2 & 0 \\ 0 & 4 & 2 \\ 5 & 4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

(A) $1\begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} + 3\begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix} + 5\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ (B) $1\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix} + 5\begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$

(C) $2\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} + 2\begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix} + 0\begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix}$ (D) $1\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + 3\begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} + 5\begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}$

Ans (A)

$$\begin{bmatrix} 2 & 2 & 0 \\ 0 & 4 & 2 \\ 5 & 4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 22 \\ 27 \end{bmatrix} ; \quad —(1)$$

$$1\begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} + 3\begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix} + 5\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} + \begin{bmatrix} 6 \\ 12 \\ 12 \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 22 \\ 27 \end{bmatrix} \quad —(2)$$

$(1) = (2)$ $\therefore$ Ans is (A)

Q.15 Which of the following is not in the column space of the following matrix?

$$\begin{bmatrix} 9 & 4 & 14 \\ 4 & 1 & 7 \\ 6 & 6 & 6 \end{bmatrix}$$

(A) $\begin{bmatrix} 8 \\ 2 \\ 12 \end{bmatrix}$ (B) $\begin{bmatrix} 13 \\ 5 \\ 12 \end{bmatrix}$ (C) $\begin{bmatrix} 12 \\ 7 \\ 6 \end{bmatrix}$ (D) $\begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix}$

Ans (C)

Let c_1, c_2, c_3 be the column vectors in the given matrix

$$c_1 = \begin{bmatrix} 9 \\ 4 \\ 6 \end{bmatrix}, \quad c_2 = \begin{bmatrix} 4 \\ 1 \\ 6 \end{bmatrix}, \quad c_3 = \begin{bmatrix} 14 \\ 7 \\ 6 \end{bmatrix}$$

$(A) = 2c_2$; $(B) = c_1 + c_2$; $(D) = c_1 - c_2$

$\therefore$ (A), (B), (D) ~~cannot~~ can be in column space of given matrix as they are linearly dependent on the column vectors

(C) is linearly independent from c_1, c_2, c_3 vectors $\therefore$ it is not in the column space.

Q.16 If rank(A) = 4 and nullity(A^T) = 2, which of the following is a possible dimension of A? (A) 6×5 (B) 6×3 (C) 3×6 (D) 5×6

Ans (A)

$\text{rank}(A) = \text{rank}(A^T) = 4$ $\because$ row rank = column rank. $\rightarrow (A_{m\times n} ; A^T_{n\times m})$

$\text{rank}(A^T) + \text{nullity}(A^T) = m \Rightarrow m = 4+2 = 6 \rightarrow (A_{6\times n} ; A^T_{n\times 6})$

Now rank $\leq$ no. of rows $\therefore$ $4 \leq n$. So (A) fits (6×5)