DIGITAL ASSIGNMENT-1

Course: Calculus for Engineers

(MAT1011) Slot: ~~B1~~D2

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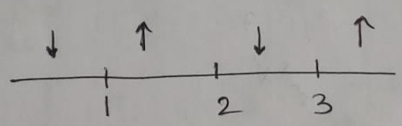
1) $f(x) = \frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x$

$f'(x) = x^3 - 6x^2 + 11x - 6$

$= (x-1)(x-2)(x-3)$

Now, for increasing; $f'(x) \geq 0$

i.e $(x-1)(x-2)(x-3) \geq 0$



$\therefore$ $f'(x) \geq 0$ in $x \in [1, 2] \cup [3, \infty)$

$\therefore$ $f(x)$ is increasing in $x \in [1, 2] \cup [3, \infty)$

Similarly for decreasing; $f'(x) \leq 0$

i.e $(x-1)(x-2)(x-3) \leq 0$

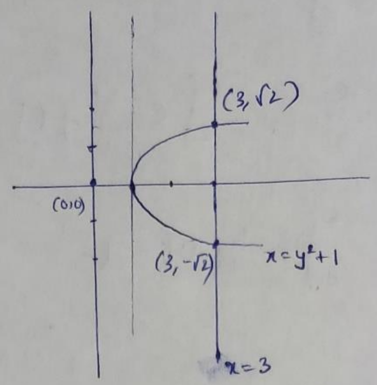
$\therefore$ $f'(x) \leq 0$ in $x \in (-\infty, 1] \cup [2, 3]$

$\therefore$ $f(x)$ is decreasing in $x \in (-\infty, 1] \cup [2, 3]$

2. $x = y^2+1$ & $x = 3$

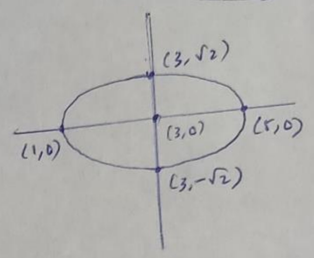
When volume is generated by revolving region around $x=0$, then

V ~~area~~ $= \pi \int_a^b x^2 \cdot dy$



$a=\sqrt{2}$; $b=-\sqrt{2}$

on revolving



∴ volume generated by revolving region around $X = (x-3) = 0$,

$$V = \pi \int_a^b X^2 \cdot dy$$

$$= \pi \int_a^b (x-3)^2 \cdot dy$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} (y^2+1-3)^2 \cdot dy$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} (y^4 - 4y^2 + 4) \, dy = 2\pi \int_0^{\sqrt{2}} (y^4 - 4y^2 + 4) \, dy$$

$$= 2\pi \left[\frac{y^5}{5} - \frac{4}{3} y^3 + 4y \right]_0^{\sqrt{2}}$$

Ans → $V = \dfrac{64\sqrt{2}\,\pi}{15}$

$$\left[\text{Volume} = \int_a^b (\text{Area}) \cdot dy = \int_a^b A(y) \cdot dy \right]$$

(Since on cutting the ellipse generated, the cross-section is a solid disk, the area is πr^2.)

3) $L\left\{\frac{\sin 2t \cos t}{t}\right\} = ?$

Let $f(t) = \sin 2t \cos t = 2\sin t \cos^2 t = 2\sin t(1-\sin^2 t)$

$= 2\sin t - 2\sin^3 t = 2\sin t - 2\left(\frac{3\sin t - \sin 3t}{4}\right)$

$= 2\sin t - \frac{3}{2}\sin t + \frac{1}{2}\sin 3t$

$f(t) = \frac{1}{2}(\sin t + \sin 3t)$

$[\sin 2t = 2\sin t \cos t]$; $[\cos^2 t = 1-\sin^2 t]$; $[\sin 3t = 3\sin t - 4\sin^3 t]$

$\bar{f}(s) = L[f(t)] = L\left[\frac{1}{2}(\sin t + \sin 3t)\right] = \frac{1}{2}\left\{L[\sin t] + L[\sin 3t]\right\}$

$= \frac{1}{2}\left\{\frac{1}{s^2+1} + \frac{3}{s^2+9}\right\} = \frac{1}{2(s^2+1)} + \frac{3}{2(s^2+9)}$

Now, $L\left[\frac{f(t)}{t}\right] = \int_s^\infty \bar{f}(s)\cdot ds$

$\therefore L\left\{\frac{\sin 2t \cos t}{t}\right\} = \frac{1}{2}\int_s^\infty \frac{1}{s^2+1} + \frac{3}{s^2+9}\cdot ds = \frac{1}{2}\left\{\int_s^\infty \frac{1}{s^2+1}\cdot ds + 3\int_s^\infty \frac{1}{s^2+9}\cdot ds\right\}$

$= \frac{1}{2}\left\{\left[\tan^{-1}s + \frac{3}{3}\tan^{-1}(s/3)\right]_s^\infty\right\}$

$= \frac{1}{2}\left\{\left[\tan^{-1}s + \tan^{-1}(s/3)\right]_s^\infty\right\}$

$= \frac{\pi}{2} - \frac{1}{2}\left\{\tan^{-1}(s) + \tan^{-1}(s/3)\right\}$

4. $\int_0^\infty e^{-t} t\sin t = ?$

W.K.T $\int_0^\infty e^{-st} f(t) \cdot dt = L[f(t)]$

$\therefore \int_0^\infty e^{-t} t\sin t \cdot dt = L[t\sin t]$, (where $s=1$ & $f(t) = t\sin t$)

$= (-1)^1 \dfrac{d^1}{ds^1} (L[\sin t])$

$= (-1)^1 \dfrac{d}{ds} \left(\dfrac{1}{s^2+1}\right)$

$= \dfrac{2s}{(s^2+1)^2}$

putting $s = 1$

Ans $\int_0^\infty e^{-t} t\sin t \cdot dt = \dfrac{1}{2}$