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School of Information Technology and Engineering

Digital Assignment, FEBRUARY 2020

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COURSE NAME	TRANSFORMATION TECHNIQUES
SLOT	F1+TF1
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Course Code : ITE1017

Course Name : Transformation Techniques

Faculty : Prof. Pradeepa M.

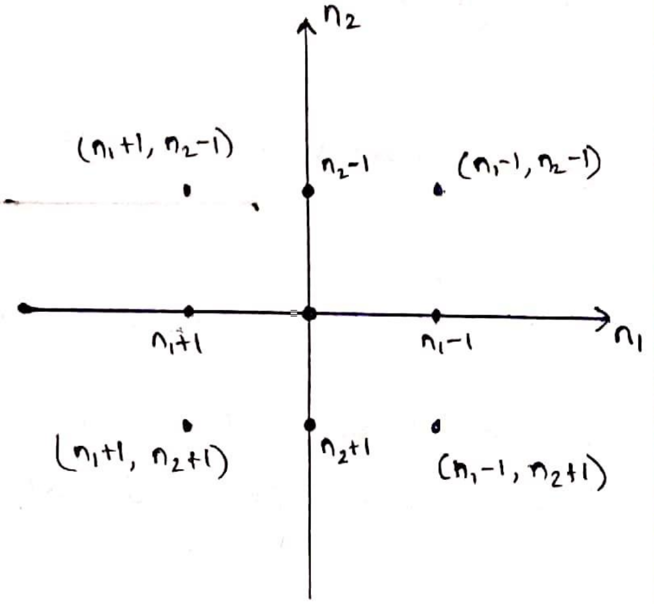
Name : Priyal Bhardwaj

Reg. No. : 18BIT0272

Slot : F1 + TF1

Due Date : 10/02/2020

Q.1 Calculate & plot frequency response of the 2D digital filter, whose impulse response sequence is shown in figure. Use appropriate software for implementation.



A.1 The expression for 2D-sequence is :

$$x(n_1,n_2) = \delta(n_1,n_2) + \delta(n_1-1) + \delta(n_1+1) + \delta(n_2-1) + \delta(n_2+1) + \delta(n_1-1,n_2-1) + \delta(n_1-1,n_2+1) + \delta(n_1+1,n_2-1) + \delta(n_1+1,n_2+1)$$

Taking Z-transform on both sides :-

$$X[z_1,z_2] = 1 + z_1^{-1} + z_1^{+1} + z_2^{-1} + z_2^{+1} + z_1^{-1}z_2^{-1} + z_1^{-1}z_2^{+1} + z_1^{+1}z_2^{-1} + z_1^{+1}z_2^{+1}$$

Frequency response :-

Replace $z_1 = e^{j\omega_1}$ & $z_2 = e^{j\omega_2}$ in above equation

$$X[\omega_1,\omega_2] = 1 + \left(\frac{e^{-j\omega_1}+e^{j\omega_1}}{2}\right)\times 2 + \left(\frac{e^{-j\omega_2}+e^{j\omega_2}}{2}\right)\times 2 + e^{-j\omega_1}\left(\frac{e^{-j\omega_2}+e^{j\omega_2}}{2}\right)\times 2 + e^{j\omega_1}\left(\frac{e^{-j\omega_2}+e^{j\omega_2}}{2}\right)\times 2$$

$$X[\omega_1,\omega_2] = 1 + 2\cos\omega_1 + 2\cos\omega_2 + 2\cos\omega_2\cdot e^{-j\omega_1} + 2\cos\omega_2 e^{j\omega_1}$$

$$= 1 + 2\cos\omega_1 + 2\cos\omega_2 + 4\cos\omega_2\left(\frac{e^{-j\omega_1}+e^{j\omega_1}}{2}\right)$$

$$X[\omega_1,\omega_2] = 1 + 2\cos\omega_1 + 2\cos\omega_2 + 4\cos\omega_1\cos\omega_2$$

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[X,Y] = meshgrid(-pi: .09: pi);
Z = 1 + 2*cos(X) + 2*cos(Y) + 4*cos(X)*cos(Y);
surf(X,Y,Z);
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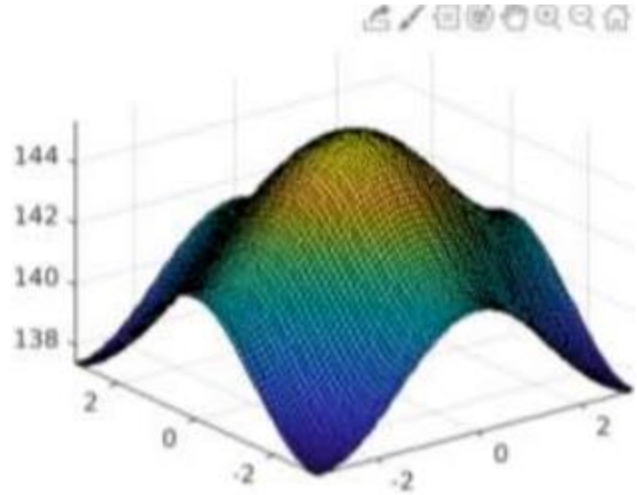

NOTE: Q2 done after Q5

Q.3 The 2D-signal represented by the following mathematical representation is periodic or not.

$$x(n_1, n_2) = \cos(n_1 + \pi n_2)$$

A.3 The signal $x(n_1, n_2)$ is of the form $x(n_1, n_2) = \cos(\omega_1 n_1 + \omega_2 n_2)$

For the signal to be periodic, $\frac{\omega_1}{2\pi} = N_1^{-1}$, $\frac{\omega_2}{2\pi} = N_2^{-1}$ & $N_1, N_2 \in Z$

Here $\omega_1 = 1$; $\omega_2 = \pi$

$N_1 = 2\pi$; $N_2 = 2$ $\quad$ N_1 is not an integer

Hence the given sequence is not periodic.

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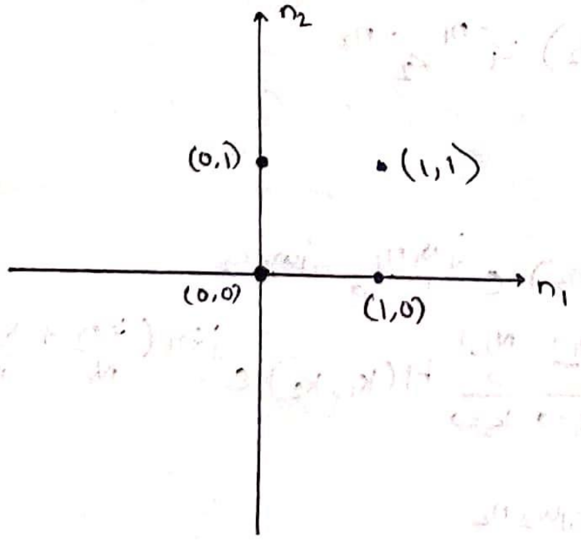
Q.4 Determine the frequency response of the system

$$h(n_1,n_2) = \begin{cases} 1/4 & (n_1,n_2) = (0,0),(0,1),(1,0),(1,1) \\ 0 & \text{otherwise} \end{cases}$$

A.4



$$h(n_1,n_2) = \frac{1}{4}\left[\delta(0,0)+\delta(1,0)+\delta(0,1)+\delta(1,1)\right] \quad \text{--- (1)}$$

Taking z- transform on both sides of equation (1)

$$H[z_1,z_2] = \frac{1}{4}\left[1+z_1^{-1}+z_2^{-1}+z_1^{-1}z_2^{-1}\right]$$

$$= \frac{1}{4}\left[(1+z_1^{-1}) + z_2^{-1}(1+z_1^{-1})\right]$$

$$= \frac{1}{4}(1+z_1^{-1})(1+z_2^{-1})$$

For frequency response, replace z_1^{-1} with $e^{-j\omega_1}$ & z_2^{-1} with $e^{-j\omega_2}$

$$H[\omega_1,\omega_2] = \frac{1}{4}(1+e^{-j\omega_1})(1+e^{-j\omega_2})$$

$$e^{-j\omega_1} = \cos\omega_1 - j\sin\omega_1 \;;\; e^{-j\omega_2} = \cos\omega_2 - j\sin\omega_2$$

$$H[\omega_1,\omega_2] = \frac{1}{4}(1+\cos\omega_1 - j\sin\omega_1)(1+\cos\omega_2 - j\sin\omega_1)$$

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Q.5 Compare & contrast the different approaches for designing a 2D digital filter. Discuss the advantages & disadvantages of each approach.

A.5 The 2D- digital filter can be broadly classified into 2 types :-

(i) Finite impulse response (FIR) filter

(ii) Infinite impulse response (IIR) filter

FIR digital filters of the non- recursive type can be realised by means of simple hardware or software. The choice between FIR & IIR filters depends upon the application.

2D - FIR filter through Frequency sampling :-

Let $h(n_1, n_2)$ represent the impulse response of 2D digital filters

$$H(z_1, z_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} h(n_1, n_2)\, z_1^{-n_1} z_2^{-n_2}$$

Frequency response is given by :-

$$H(e^{jw_1}, e^{jw_2}) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} h(n_1, n_2)\, e^{-jw_1 n_1} e^{-jw_2 n_2}$$

$$= \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \frac{1}{N_1 N_2} \left[\sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} H(k_1, k_2)\, e^{j2\pi\left(\frac{k_1 n_1}{N_1} + \frac{k_2 n_2}{N_2}\right)} \right] \times$$

$$\times\, e^{-jw_1 n_1} e^{-jw_2 n_2}$$

This equation represents the main basis of 2D- ~~signal~~ digital filter with sampling method.

2D IIR filter through recursive filter:-

A 2D -digital filter (recursive) is characterised by :-

$$H(z_1, z_2) = \frac{\sum_{m=0}^{P} \sum_{n=0}^{P} a_{mn} z_1^m z_2^n}{\sum_{m=0}^{P} \sum_{n=0}^{P} b_{mn} z_1^m z_2^n} \qquad a_{mn}, b_{mn} \rightarrow \text{constants}$$

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Advantages

1) FIR filter : (i) Can be done by simple hardware or software

(ii) Mostly stable as impulse response is summable

(iii) Exhibit linear phase characteristics

2) IIR filter : (i) Has the potential of saving computation time

(ii) Higher efficiency than FIR filters

Disadvantages

1) FIR filter: ~~(x)~~

→ The frequency response assumes the values $H(k_1, k_2)$ in the fixed points, while it presents fluctuations in the intervals.

→ Thus sampled impulse response need to be modified in order to reduce fluctuations

2) IIR filter :

→ The approximation problem in recursive filter may lead to instability propagation through the output.

→ Thus stability of 2D-digital filter is not always guaranteed.

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Q.2 Sketch the following sequences

(i) $x(n_1, n_2) = \delta(n_1-1, n_2-1) + \delta(n_1-2, n_2-1)$

(ii) $x(n_1, n_2) = u(n_1, n_2) - u(n_1-1, n_2-1)$

A.2 (i) $x(n_1, n_2) = \delta(n_1-1, n_2-1) + \delta(n_1-2, n_2-1)$

Take z-transform on both sides

$$X[z_1, z_2] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} x(n_1, n_2)\, z_1^{-n_1} z_2^{-n_2}$$

$\cancel{\delta(n_1-1, n_2-1)}$

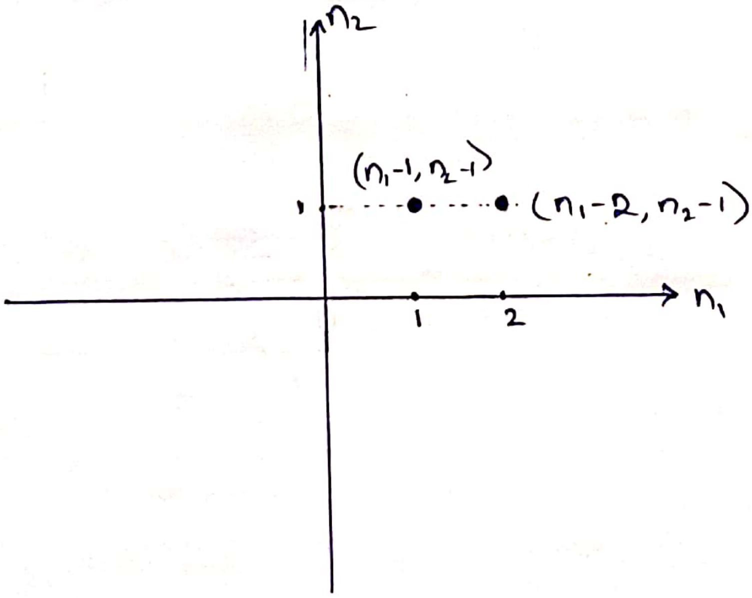
$$\boxed{X[z_1, z_2] = z_1^{-1} z_2^{-1} + z_1^{-2} z_2^{-1}}$$

Because $\delta(n_1, n_2) = \begin{cases} 1 & n_1 = n_2 = 0 \\ 0 & \text{otherwise} \end{cases}$

So for : $\delta(n_1-1, n_2-1) = 1$ at $n_1 = n_2 = 1$

$\delta(n_1-2, n_2-1) = 1$ at $n_1 = 2$ & $n_2 = 1$

The region of convergence :- Entire z_1 & z_2 plane except where $z_1 = 1, 2$ & $z_2 = 1$



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