

Q.1 A computer employs RAM chips of 512×8 & ROM chips of 2048×4 . The computer system needs $2M \times 16$ of RAM & $4K \times 16$ of ROM & 2 interface units with 256 registers each. A memory mapped I/O configuration is used.

a.) How many RAM & ROM chips are needed?

$$\Rightarrow (i) \text{ p-factor} = \frac{N'}{N} = \frac{2 \times 2^{10} \times 2^{10}}{2^9} = 2^{12}$$

$$q\text{-factor} = \frac{W'}{W} = \frac{2^4}{2^3} = 2$$

$$\therefore \text{Number of RAM chips needed} = p \times q = 2^{13} = \boxed{8192 \text{ chips}}$$

$$(ii) \text{ p-factor} = \frac{N'}{N} = \frac{2^2 \times 2^{10}}{2^{11}} = 2$$

$$q\text{-factor} = \frac{W'}{W} = \frac{2^4}{2^2} = 2^2 = 4$$

$$\therefore \text{Number of ROM chips needed} = p \times q = 2 \times 4 = \boxed{8 \text{ chips}}$$

S.No.	Type	$N \times W$	$N' \times W'$	p	q	x	y	z	Total (x+y+z)
1	RAM	512×8	$2M \times 16$	2^{12}	2	9	12	2	23
2	ROM	2048×4	$4K \times 16$	2	4	11	1	2	14
3	Interface	256		1	1	8	1	2	11

(b.) Tell how many lines of the address bus must be used to access total memory?

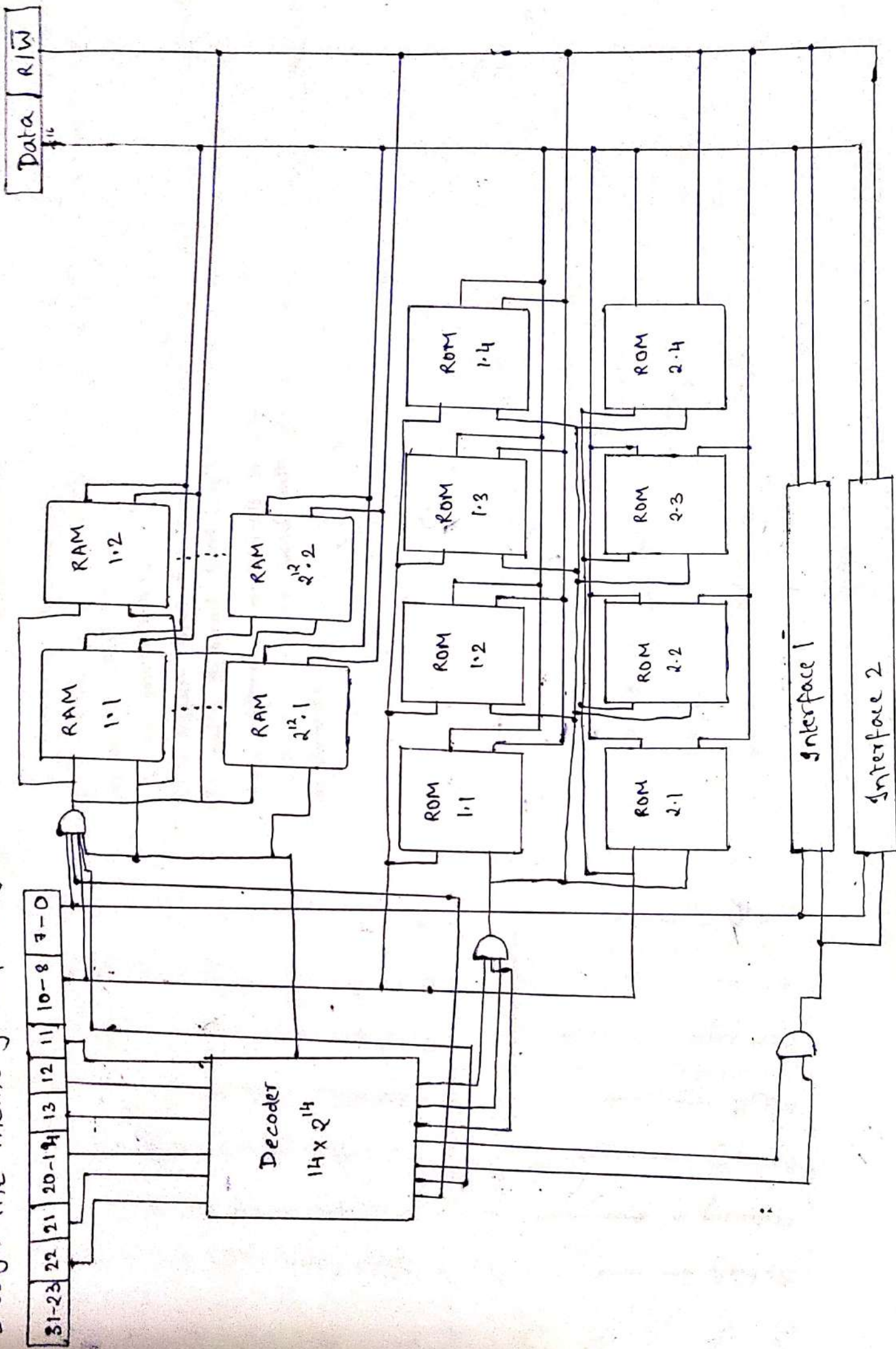
\Rightarrow RAM:- 9 lines of address bus must be used to access total memory.

ROM:- 11 lines of address bus must be used to access total memory.

Interface:- 8 lines of address bus must be used to access total memory.

[consider 'x' for address bus lines accessing total memory].

e.) Design the memory chip layout for that address map.



Q. 2 Consider a computer with the following characteristics:
total of 1M byte of main memory; word size of 1 byte; block size of 16 bytes; & cache size of 64K bytes.

\Rightarrow Main memory = 1M = 2^{20} bytes

1 Block = 16 bytes = 2^4 bytes/words (as 1 word = 1 byte)

Number of blocks in main memory = $\frac{2^{20}}{2^4} = 2^{16}$ bytes

Cache = 64K = 2^{16} bytes

Number of cache lines = $\frac{2^{16}}{2^4} = 2^{12}$ bytes

(A) For the main memory address of F0010, 01234, & CABBE, give the corresponding tag, cache line address, & word offsets for a direct-mapped cache.

\Rightarrow Bits 0-3 indicate the word offset (4 bits)

Bits 4-15 indicate the cache line address slot (12 bits)

Bits 16-19 indicate the tag (remaining bits)

(i) F0010 = 1111 0000 0000 0001 0000

Word offset = 0000 = 0

Cache line address = 0000 0000 0001 = 001

Tag = 1111 = F

F	001	0
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(ii) 01234 = 0000 0001 0010 0011 0100

Word offset = 0100 = 4

Cache line address = 0001 0010 0011 = 123

Tag = 0000 = 0

0	123	4
---	-----	---

(iii) CABBE = 1100 1010 1011 1011 1110

Word offset = 1110 = E

Cache line address = 1010 1011 1011 = ABB

Tag = 1100 = C

C	ABB	E
---	-----	---

(B) Give any 2 main memory addresses with different tags that map to the same cache slot for a direct mapped cache

⇒ (i) 3FFF0 →

3	FFF	0
---	-----	---

(ii) 2FFF0 →

2	FFF	0
---	-----	---

(C) For the main memory addresses of F0010 & CABBE, give the corresponding tag & offset values for a fully-associative cache

⇒ Tag bits = 16 bits (for a fully-associative cache)

(i) F0010

word offset = 0h

Tag = F001h

(ii) CABBE

word offset = Eh

Tag = CABBh

(D) For the main memory addresses of F0010 & CABBE, give the corresponding tag, cache set, & offset values for a 2-way set-associative cache.

⇒ $K = 2$ ∴ Number of sets in a cache = $\frac{2^{12}}{2} = 2^{11}$

Bits 0-3 indicate the word offset (4)

Bits 4-14 indicate the cache set (11)

Bits 15-~~20~~¹⁹ indicate the tag (remaining)

$$(i) F0010 = 11110\ 000\ 0000\ 0001\ 0000$$

$$\text{Word offset} = 0000 = 0$$

$$\text{Cache set} = 000\ 0000\ 0001 = 001$$

$$\text{Tag} = 11110 = 1\ 1110 = 1E$$

$$(ii) CABBE = 11001\ 010\ 1011\ 1011\ 1110$$

$$\text{Word offset} = 1110 = E$$

$$\text{Cache set} = 010\ 1011\ 1011 = 2BB$$

$$\text{Tag} = 11001 = 19$$

Q.3 For a direct-mapped cache design with a 32-bit address, the following bits of the addresses are used to access to the cache

Tag (31-10)	Index (9-5)	offset (4-0)
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(a.) what is the cache block size (in words)?

Block size = 2^N

N = number of offset bits - 2

'-2' because we are using 32 bit words & byte-addressed memory

\therefore Cache line / Block size = $2^{5-2} = 2^3 = 8$ words

(b) How many entries does the cache have?

\Rightarrow Entries = $2^{\text{index bits}} = 2^5 = 32$ entries

(c) What is the ratio between total bits required for such a cache implementation over the data storage bits?

\Rightarrow Total bits = Tag bits + valid bit + data bit = $22 + 1 + 32 \times 8 = 279$

Data bits = $32 \times 8 = 256$

Ratio = $\frac{279}{256} = 1.089$

(d) Starting from power on, the following byte-addressed cache references are recorded:

Addresses											
0	4	16	132	232	160	1024	30	140	3100	180	2180

Address	Index	Offset	Hit/Miss	Replace?	Final value?
0	00000	00000	M	N	N
14	00000	00100	H	N	N
16	00000	10000	H	N	N
132	000100	00100	M	N	N
232	00111	01000	M	N	Y
160	00101	00000	M	N	Y
1024	00000	00000	M	Y	N
30	00000	1101100	M	Y	N
140	00100	01100	H	N	N
3100	00000	11100	M	Y	Y
180	00101	10100	H	N	Y
2180	00100	0100	M	Y	Y

(e.) How many blocks are replaced?

⇒ From the table : 4 blocks are replaced

(f.) What is the hit ratio?

$$\Rightarrow \text{Hit ratio} = \frac{\text{No. of hits}}{\text{No. of hits} + \text{No. of misses}} = \frac{4}{4+8} = \frac{4}{12} = \frac{1}{3} = 0.33 \text{ i.e. } 33\%$$

(g.) List the final state of the cache with each valid entry represented as a record <index, tag, data>.

⇒ From the last column in the above table.

<00100, 0010, mem[2176]>

<00101, 0000, mem[160]>

<00000, 0011, mem[3072]>

<00111, 0000, mem[224]>

Q.4 A process references 5 pages A, B, C, D, and E, in the following order: A; B; C; D; A; B; E; A; B; C; D; E. Assume that the replacement algorithm is (i) First-in-first-out & (ii) LRU.

Find the number of page transfers during this sequence of references starting with an empty main memory with 3 page frames. Repeat for 4 page frames. Calculate hit ratio.

(i) FIFO:-

A → B → C → D → A → B → E → A → B → C → D → E												Reference Frame
A	A	A	B	C	D	A	A	A	B	E	E	Frame 1
	B	B	C	D	A	B	B	B	E	C	C	Frame 2
		C	D	A	B	E	E	E	C	D	D	Frame 3
1	2	3	4	5	6	7	-	-	8	9	-	Count
M	M	M	M	M	M	M	H	H	M	M	H	Hit/Miss

Page transfers during given sequence of frames references with 3 page frames = 9

A → B → C → D → A → B → E → A → B → C → D → E												Reference
A	A	A	A	A	A	B	C	D	E	A	B	Frame 1
	B	B	B	B	B	C	D	E	A	B	C	Frame 2
		C	C	C	C	D	E	A	B	C	D	Frame 3
			D	D	D	E	A	B	C	D	E	Frame 4
1	2	3	4	-	-	5	6	7	8	9	10	Count
M	M	M	M	H	H	M	M	M	M	M	M	Hit/Miss

Page transfer count for 4 page frames = 10

$$\text{Hit ratio} = \frac{3}{3+9} = \frac{3}{12} = 0.25 \quad (\text{for 3 page frames})$$

$$\text{Hit ratio} = \frac{2}{2+10} = \frac{2}{12} = 0.166 \quad (\text{for 4 page frames})$$

(ii) LRU:- (Least Recently Used)

→ 3 page frames

Reference	A	B	C	D	A	B	E	A	B	C	D	E
Frame 1	A	A	A	D	D	D	E	E	E	C	C	C
Frame 2		B	B	B	A	A	A	A	A	A	D	D
Frame 3			C	C	C	B	B	B	B	B	B	E
Count	1	2	3	4	5	6	7	-	-	8	9	10
Hit/Miss	M	M	M	M	M	M	M	H	H	M	M	M

Page transfer count for 3 page frames = 10

$$\text{Hit ratio} = \frac{2}{2+10} = 0.166$$

→ 4 page frames

Reference	A	B	C	D	A	B	E	A	B	C	D	E
Frame 1	A	A	A	A	A	A	A	A	A	A	A	E
Frame 2		B	B	B	B	B	B	B	B	B	B	B
Frame 3			C	C	C	C	E	E	E	E	D	D
Frame 4				D	D	D	D	D	D	C	C	C
Count	1	2	3	4	-	-	5	-	-	6	7	8
Hit/Miss	M	M	M	M	H	H	M	H	H	M	M	M

Page transfer count for 4 page frames = 8

$$\text{Hit ratio} = \frac{4}{4+8} = \frac{4}{12} = 0.33$$

Q.5 A bit stream 10011101 is transmitted using the standard CRC method. The generator polynomial is $x^3 + 1$. What is the actual bit transmitted? Suppose the 3rd bit from the left is inverted during transmission. How will the receiver detect this error?

$$\Rightarrow x^3 + 1 \equiv 1001$$

Since generator polynomial is of 3rd degree, append 3 zeros to the lower end of frame to be transmitted.

\therefore Bit stream = 10011101000

$$\begin{array}{r}
 1001 \overline{) 10011101000} \\
 \underline{1001} \\
 0001 \\
 \underline{0000} \\
 0011 \\
 \underline{0000} \\
 0110 \\
 \underline{0000} \\
 1101 \\
 \underline{1001} \\
 1000 \\
 \underline{1001} \\
 0010 \\
 \underline{0000} \\
 0100 \\
 \underline{0000} \\
 100 \text{ (remainder)}
 \end{array}$$

CRC = 100

Actual frame transmitted :-
$$\begin{array}{r}
 10011101000 \\
 + 100 \\
 \hline
 10011101100
 \end{array}$$

\therefore Actual bit transmitted = 10011101100

Now, 3rd bit from left gets inverted during transmission.

So, the bit stream received by the receiver = 1011101100

Receiver performs binary division with same generator polynomial

$$\begin{array}{r} 1001 \overline{) 101010000} \\ \underline{10111101100} \\ 0101 \\ \underline{0000} \\ 1011 \\ \underline{1001} \\ 0100 \\ \underline{0000} \\ 1001 \\ \underline{1001} \\ 0001 \\ \underline{0000} \\ 0010 \\ \underline{0000} \\ 0100 \\ \underline{0000} \\ 100 \text{ (remainder)} \end{array}$$

Remainder is non-zero which indicates that an error occurred in the data during the transmission.

Therefore, receiver rejects the data & asks the sender for transmission