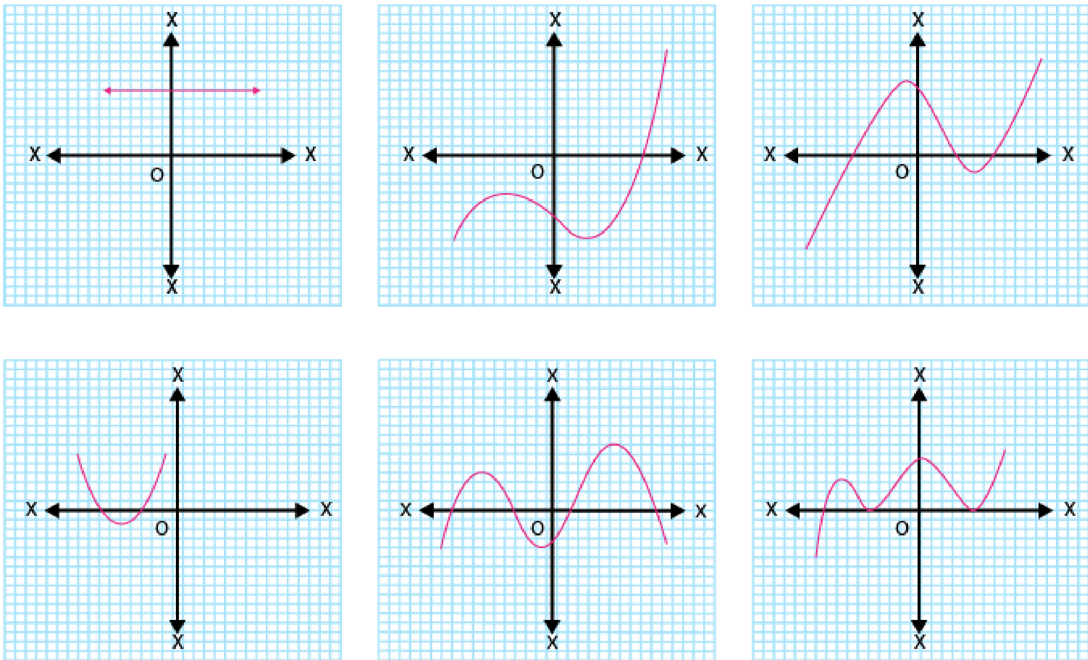
POLYNOMIALS

EXERCISE 2.1

1. The graphs of y = p(x) are given in Fig. 2.10 below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.



Solutions:

FORMULA:

Total number of zeroes in any polynomial equation = total number of times the curve intersects x-axis.

In the given graphs,

(i) The number of zeroes of p(x) is 0 because the graph is parallel to x-axis does not cut it at any point.

(ii)The number of zeroes of p(x) is 1, because the graph intersects the x-axis at only one point.

(iii)The number of zeroes of p(x) is 3, because the graph intersects the x-axis at any three points.

(iv)The number of zeroes of p(x) is 2, because the graph intersects the x-axis at two points.

(v)The number of zeroes of p(x) is 4, because the graph intersects the x-axis at four points.

(vi)The number of zeroes of p(x) is 3, because the graph intersects the x-axis at three points.

EXERCISE 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

Solutions:

(i) $x^2–2x–8$

$\Rightarrow$**p(x)=** $x^2– 4x+2x–8$

$\Rightarrow x(x–4)+2(x–4)$

$\Rightarrow (x-4)(x+2)$

Therefore, zeroes of polynomial equation p(x) are (4, -2)

Sum of zeroes = -(Coefficient of x)/(Coefficient of x^2)

- ⇨ $4–2 = -(-2) /1$
- ⇨ $2 =-(-2)$
- ⇨ LHS = RHS

Product of zeroes == (Constant term)/(Coefficient of x^2)

- ⇨ $4×(-2) = -(8)/1$
- ⇨ $-8 =-(8)/1$
- ⇨ LHS = RHS

(ii) $4s^2–4s+1$

=> $4s^2–2s–2s+1$

=> $2s(2s–1)–1(2s-1)$

=>$(2s–1)(2s–1)$

Therefore, zeroes of polynomial equation P(X) are (1/2, 1/2)

Sum of zeroes = = -(Coefficient of s)/(Coefficient of s^2)

- ⇨ $(½)+(1/2) = -(4/4)$
- ⇨ $1 = 1$
- ⇨ LHS = RHS

Product of zeros = (Constant term)/(Coefficient of s^2)

- ⇨ $(1/2)×(1/2) = ¼$
- ⇨ LHS = RHS

(iii) $6x^2–3–7x$

$\Rightarrow 6x^2–7x–3 = 6x^2 – 9x + 2x – 3$

=> $3x(2x – 3) +1(2x – 3)$

=> $(3x+1)(2x-3)$

Therefore, zeroes of polynomial equation P(x) are (-1/3, 3/2)

Sum of zeroes = -(Coefficient of x)/(Coefficient of x^2)

- ⇨ -(1/3)+(3/2) = -(7/6)
- ⇨ -7/6 = -7/6
- ⇨ 7/6=7/6
- ⇨ LHS = RHS

Product of zeroes = (Constant term) /(Coefficient of x^2)

- ⇨ -(1/3)×(3/2) =-(3/2)
- ⇨ -(3/6)=-(3/2)
- ⇨ LHS = RHS

(iv) $4u^2$+8u

⇒ 4u(u+2)=0

- ⇨ 4u=0 => u+2=0
- ⇨ u=0 => u=-2

Therefore, zeroes of polynomial equation P(X) are (0, -2).

Sum of zeroes = -(Coefficient of u)/(Coefficient of u^2)

- ⇨ 0+(-2) = -(8/4)
- ⇨ -2 = -2
- ⇨ LHS = RHS

Product of zeroes = (Constant term)/(Coefficient of u^2).

- ⇨ 0×-2 = 0/4
- ⇨ 0=0
- ⇨ LHS = RHS

(v) t^2–15

⇒ t^2 = 15 or

- ⇨ t = ±√15

Therefore, zeroes of polynomial equation P(X) are (√15, -√15)

Sum of zeroes =-(Coefficient of t) / (Coefficient of t^2)

- ⇨ √15+(-√15) = -(0/1)
- ⇨ 0 = 0
- ⇨ LHS = RHS

Product of zeroes = (Constant term) / (Coefficient of t^2)

- ⇨ = √15×(-√15) = -15/1
- ⇨ LHS = RHS

(vi) $3x^2$–x–4

⇒ $3x^2$–4x+3x–4

- ⇨ x(3x-4)+1(3x-4)

⇨ $(3x – 4)(x + 1)$

Therefore, zeroes of polynomial equation $3x^2 – x – 4$ are (4/3, -1)

Sum of zeroes = -(Coefficient of x) / (Coefficient of x^2)

⇨ (4/3)+(-1) = -(-1/3)
⇨ (1/3) = (1/3)
⇨ LHS = RHS

Product of zeroes= (Constant term) /(Coefficient of x^2)

⇨ (4/3)×(-1) = (-4/3)
⇨ -(4/3)=-(4/3)
⇨ LHS = RHS

---

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) 1/4 , -1

Solution:

From the formulas of sum and product of zeroes, we know,

Sum of zeroes = $\alpha+\beta$

Product of zeroes = $\alpha\beta$

Sum of zeroes = $\alpha+\beta$ = 1/4

Product of zeroes = $\alpha\beta$ = -1

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2–(\alpha+\beta)x +\alpha\beta = 0$

$x^2–(1/4)x +(-1) = 0$

$4x^2–x-4 = 0$

Thus,$4x^2–x–4$ is the quadratic polynomial.

(ii)√2, 1/3

Solution:

Sum of zeroes = $\alpha + \beta = \sqrt{2}$

Product of zeroes = $\alpha\beta$ = 1/3

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$x^2–(\alpha+\beta)x +\alpha\beta = 0$

$\mathbf{x^2 - (}\sqrt{2}\mathbf{)x + (1/3) = 0}$

$\mathbf{3x^2 - 3}\sqrt{2}x+1 = 0$

Thus, $3x^2-3$ $\sqrt{2}x+1$ **is the** quadratic polynomial.

(iii) 0, √5

Solution:

Given,

Sum of zeroes = $\alpha+\beta = 0$

Product of zeroes = $\alpha\,\beta = \sqrt{5}$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly

as:-

$\mathbf{x^2-(\alpha+\beta)x + \alpha\beta = 0}$

$\mathbf{x^2-(0)x} + \sqrt{5} \mathbf{= 0}$

Thus, x^2+$\sqrt{5}$ **is the** quadratic polynomial.

(iv) 1, 1

Solution:

Given,

Sum of zeroes = $\alpha+\beta = 1$

Product of zeroes = $\alpha\,\beta = 1$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$\mathbf{x^2-(\alpha+\beta)x + \alpha\beta = 0}$

$\mathbf{x^2-x+1 = 0}$

Thus , $x^2-x+1$is the quadratic polynomial.

(v) -1/4, 1/4

Solution:

Given,

Sum of zeroes = $\alpha+\beta = -1/4$

Product of zeroes = $\alpha\,\beta = 1/4$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2-(\alpha+\beta)x+\alpha\beta = 0$$

$$x^2-(-1/4)x+(1/4) = 0$$

$$4x^2+x+1 = 0$$

Thus, $4x^2+x+1$ **is the** quadratic polynomial.

(vi) 4, 1

Solution:

Given,

Sum of zeroes = $\alpha+\beta$ =

Product of zeroes = $\alpha\beta = 1$

∴ If α and β are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2-(\alpha+\beta)x+\alpha\beta = 0$$

$$x^2-4x+1 = 0$$

Thus, x^2-4x+1 **is the** quadratic polynomial.

EXERCISE 2.3

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

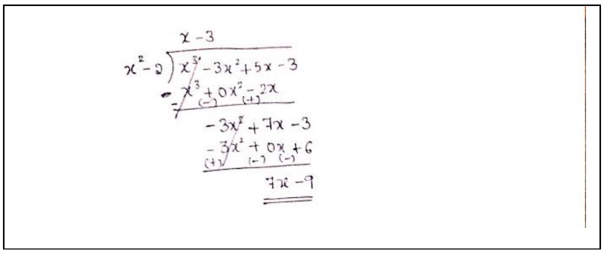
(i) $p(x) = x^3-3x^2+5x-3$ **,** $g(x) = x^2-2$

Solution:

Given,

Dividend = $p(x) = x^3-3x^2+5x-3$

Divisor = $g(x) = x^2-2$



Therefore, upon division we get,

Quotient = $x-3$

Remainder = $7x-9$

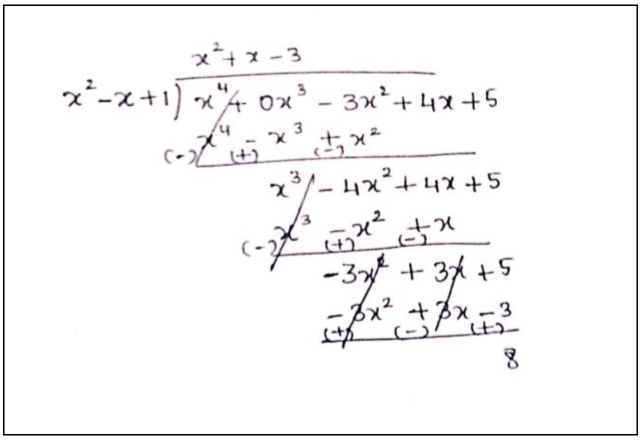
(ii) $p(x) = x^4-3x^2+4x+5$ **,** $g(x) = x^2+1-x$

Solution:

Given,

Dividend = p(x) = $x^4 – 3x^2 + 4x +5$

Divisor = g(x) = $x^2 +1-x$



Therefore, upon division we get,

Quotient = $x^2 + x–3$

Remainder = 8

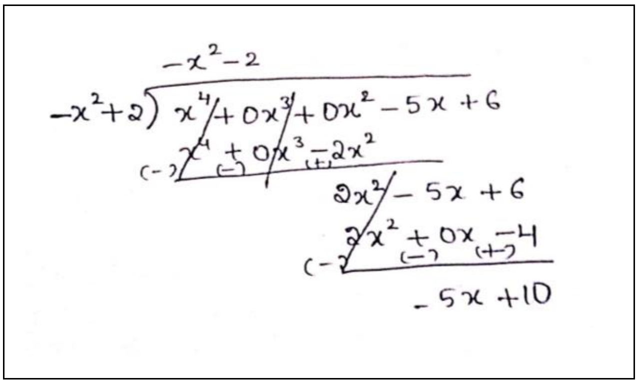
(iii) p(x) =x^4–5x+6, g(x) = 2–x^2

Solution:

Given,

Dividend = p(x) =$x^4 – 5x + 6 = x^4 +0x^2–5x+6$

Divisor = g(x) = $2–x^2 = –x^2+2$



Therefore, upon division we get,

Quotient = $-x^2-2$

Remainder = $-5x + 10$

2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

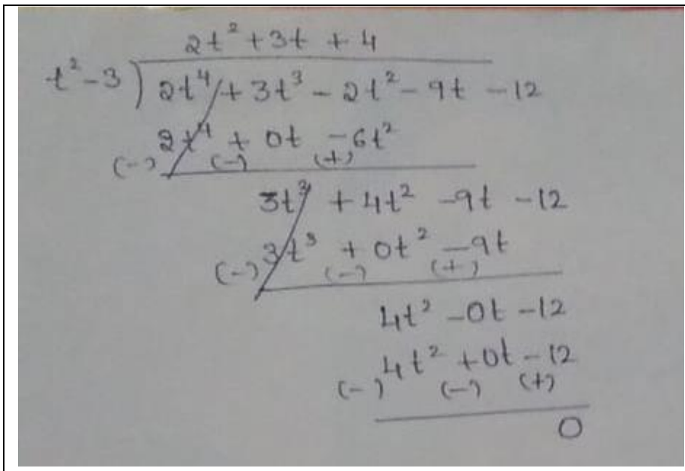
(i) t^2-3, 2t^4 +3t^3-2t^2-9t-12

Solutions:

Given,

First polynomial = t^2-3

Second polynomial = 2t^4 +3t^3-2t^2 -9t-12



As we can see, the remainder is left as 0. Therefore, we say that, t^2-3 is a factor of 2t^2+3t+4.

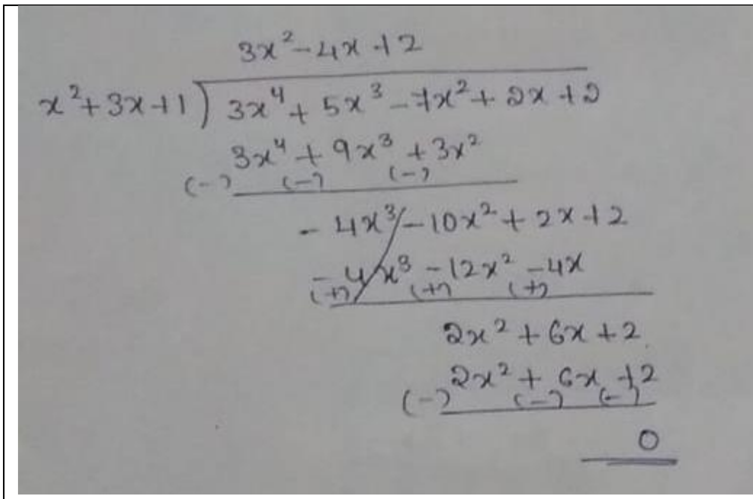
(ii)x^2+3x+1 , 3x^4+5x^3-7x^2+2x+2

Solutions:

Given,

First polynomial = x^2+3x+1

Second polynomial = 3x^4+5x^3-7x^2+2x+2



As we can see, the remainder is left as 0. Therefore, we say that, x^2 + 3x + 1 is a factor of 3x^4+5x^3-7x^2+2x+2.

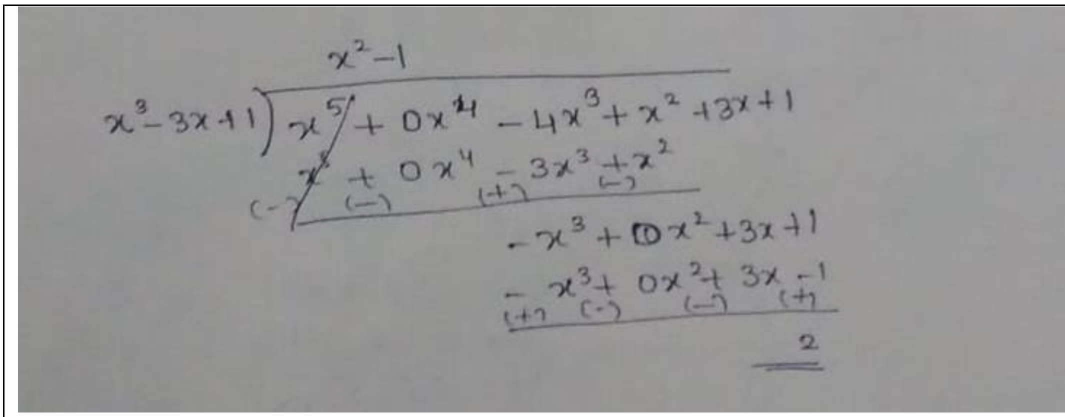
(iii) x^3-3x+1, x^5-4x^3+x^2+3x+1

Solutions:

Given,

First polynomial = x^3-3x+1

Second polynomial = x^5-4x^3+x^2+3x+1



As we can see, the remainder is not equal to 0. Therefore, we say that, x^3-3x+1 is not a factor of

x^5-4x^3+x^2+3x+1 .

3. Obtain all other zeroes of 3x^4+6x^3-2x^2-10x-5, if two of its zeroes are √(5/3) and – √(5/3).

Solutions:

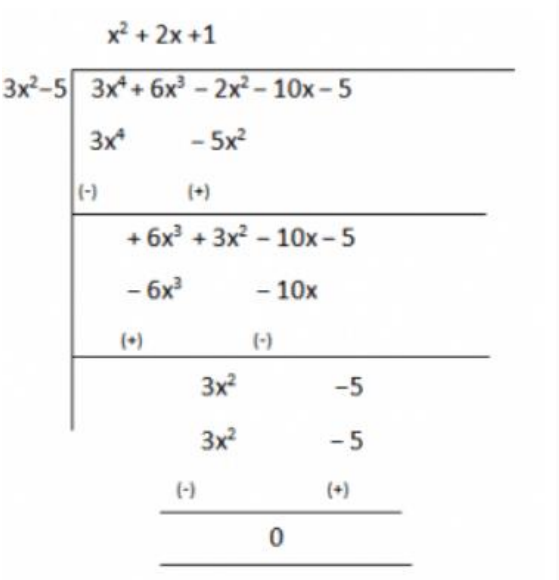
Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

√(5/3) and – √(5/3) are zeroes of polynomial f(x).

∴ (x –**√(5/3)**) (x+**√(5/3)** = x^2-(5/3) = 0

(3x^2−5)=0, is a factor of given polynomial f(x).

Now, when we will divide f(x) by (3x^2−5) the quotient obtained will also be a factor of f(x) and the remainder will be 0.



Therefore, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)$**$(x^2+2x+1)$**

Now, on further factorizing (x^2+2x+1) we get,

x^2+2x+1 $= x^2+x+x+1 = 0$

$x(x+1)+1(x+1) = 0$

$(x+1)(x+1) = 0$

So, its zeroes are given by: **$x = -1$** and **$x = -1$.**

Therefore, all four zeroes of given polynomial equation are:

$\sqrt{(5/3)}$,- $\sqrt{(5/3)}$, -1 and -1.

Hence, is the answer.

4. On dividing x^3-3x^2+x+2 by a polynomial g(x), the quotient and remainder were $x-2$ and $-2x+4$, respectively. Find g(x).

Solution:

Given,

Dividend, $p(x) = x^3-3x^2+x+2$

Quotient = $x-2$

Remainder = $-2x+4$

We have to find the value of Divisor, g(x) =?

As we know,

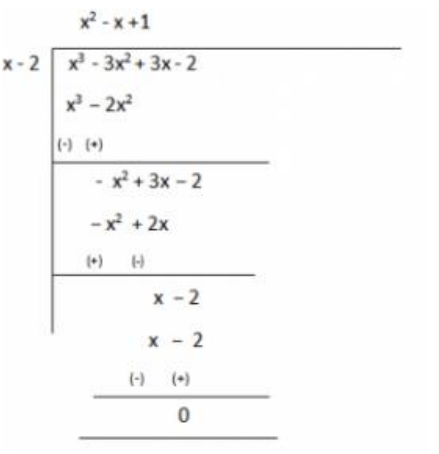
Dividend = Divisor × Quotient + Remainder

$\therefore x^3-3x^2+x+2 = g(x)\times(x-2) + (-2x+4)$

$x^3-3x^2+x+2-(-2x+4) = g(x)\times(x-2)$

Therefore, $g(x) \times (x-2) = x^3-3x^2+x+2$

Now, for finding g(x) we will divide x^3-3x^2+x+2 with (x-2)



Therefore, **$g(x) = (x^2–x+1)$**

5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x)

(ii) deg q(x) = deg r(x)

(iii) deg r(x) = 0

Solutions:

According to the division algorithm, dividend p(x) and divisor g(x) are two polynomials, where $g(x)\neq0$. Then we can find the value of quotient q(x) and remainder r(x), with the help of below given formula;

Dividend = Divisor × Quotient + Remainder

$\therefore p(x) = g(x)\times q(x)+r(x)$

Where r(x) = 0 or degree of r(x)< degree of g(x).

Now let us proof the three given cases as per division algorithm by taking examples for each.

(i) deg p(x) = deg q(x)

Degree of dividend is equal to degree of quotient, only when the divisor is a constant term.

Let us take an example, $3x^2+3x+3$ is a polynomial to be divided by 3.

So, $(3x^2+3x+3)/3 = x^2+x+1 = q(x)$

Thus, you can see, the degree of quotient is equal to the degree of dividend.

Hence, division algorithm is satisfied here.

(ii) deg q(x) = deg r(x)

Let us take an example , $p(x)=x^2+x$ is a polynomial to be divided by $g(x)=x$.

So, $(x^2+x)/x = x+1 = q(x)$

Also, remainder, $r(x) = 0$

Thus, you can see, the degree of quotient is equal to the degree of remainder.

Hence, division algorithm is satisfied here.

(iii) deg r(x) = 0

The degree of remainder is 0 only when the remainder left after division algorithm is constant.

Let us take an example, $p(x) = x^2+1$ is a polynomial to be divided by $g(x)=x$.

So,$(x^2+1)/x= x=q(x)$

And $r(x)=1$

Clearly, the degree of remainder here is 0.

Hence, division algorithm is satisfied here.

EXERCISE 2.4

1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2$; -1/2, 1, -2

Solution:

Given, $p(x) = 2x^3+x^2-5x+2$

And zeroes for p(x) are = 1/2, 1, -2

$\therefore p(1/2) = 2(1/2)^3+(1/2)^2-5(1/2)+2 = (1/4)+(1/4)-(5/2)+2 = 0$

$p(1) = 2(1)^3+(1)^2-5(1)+2 = 0$

$p(-2) = 2(-2)^3+(-2)^2-5(-2)+2 = 0$

Hence, proved 1/2, 1, -2 are the zeroes of $2x^3+x^2-5x+2$.

Now, comparing the given polynomial with general expression, we get;

$\therefore ax^3+bx^2+cx+d = 2x^3+x^2-5x+2$

a=2, b=1, c= -5 and d = 2

As we know, if α, β, γ are the zeroes of the cubic polynomial ax^3+bx^2+cx+d , then;

$\alpha +\beta+\gamma = -b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha = c/a$

$\alpha \beta\gamma = – d/a$.

Therefore, putting the values of zeroes of the polynomial,

$\alpha+\beta+\gamma = ½+1+(-2) = -1/2 = –b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha = (1/2\times1)+(1 \times-2)+(-2\times1/2) = -5/2 = c/a$

$\alpha \beta \gamma = ½\times1\times(-2) = -2/2 = -d/a$

Hence, the relationship between the zeroes and the coefficients are satisfied.

(ii) x^3-4x^2+5x-2 ;2, 1, 1

Solution:

Given, $p(x) = x^3-4x^2+5x-2$

And zeroes for p(x) are 2,1,1.

$\therefore$ $p(2)= 2^3-4(2)^2+5(2)-2 = 0$

$p(1) = 1^3-(4\times1^2)+(5\times1)-2 = 0$

Hence proved, 2, 1, 1 are the zeroes of x^3-4x^2+5x-2

Now, comparing the given polynomial with general expression, we get;

$\therefore$ $ax^3+bx^2+cx+d = x^3-4x^2+5x-2$

a = 1, b = -4, c = 5 and d = -2

As we know, if α, β, γ are the zeroes of the cubic polynomial ax^3+bx^2+cx+d , then;

$\alpha + \beta + \gamma = –b/a$

$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$

$\alpha \beta \gamma = – d/a$.

Therefore, putting the values of zeroes of the polynomial,

$\alpha +\beta+\gamma = 2+1+1 = 4 = -(-4)/1 = –b/a$

$\alpha\beta+\beta\gamma+\gamma\alpha = 2\times1+1\times1+1\times2 = 5 = 5/1= c/a$

$\alpha\beta\gamma = 2\times1\times1 = 2 = -(-2)/1 = -d/a$

Hence, the relationship between the zeroes and the coefficients are satisfied.

2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Solution:

Let us consider the cubic polynomial is ax^3+bx^2+cx+d and the values of the zeroes of the polynomials be α, β, γ.

As per the given question,

$\alpha+\beta+\gamma = -b/a = 2/1$

$\alpha\beta + \beta\gamma + \gamma\alpha = c/a = -7/1$

$\alpha\beta\gamma = -d/a = -14/1$

Thus, from above three expressions we get the values of coefficient of polynomial.

a = 1, b = -2, c = -7, d = 14

Hence, the cubic polynomial is $x^3-2x^2-7x+14$

3. If the zeroes of the polynomial x^3-3x^2+x+1 are a – b, a, a + b, find a and b.

Solution:

We are given with the polynomial here,

$p(x) = x^3-3x^2+x+1$

And zeroes are given as a – b, a, a + b

Now, comparing the given polynomial with general expression, we get;

$\therefore px^3+qx^2+rx+s = x^3-3x^2+x+1$

p = 1, q = -3, r = 1 and s = 1

Sum of zeroes = a – b + a + a + b

-q/p = 3a

Putting the values q and p.

-(-3)/1 = 3a

a=1

Thus, the zeroes are 1-b, 1, 1+b.

Now, product of zeroes = 1(1-b)(1+b)

$-s/p = 1-b^2$

$-1/1 = 1-b^2$

$b^2 = 1+1 = 2$

$b = \sqrt{2}$

Hence, $1-\sqrt{2}$, 1, $1+\sqrt{2}$ are the zeroes of x^3-3x^2+x+1.

4. If two zeroes of the polynomial $x^4-6x^3-26x^2+138x-35$ are $2 \pm\sqrt{3}$, find other zeroes.

Solution:

Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

Let $f(x) = x^4-6x^3-26x^2+138x-35$

Since $2+\sqrt{3}$ and $2-\sqrt{3}$ are zeroes of given polynomial f(x).

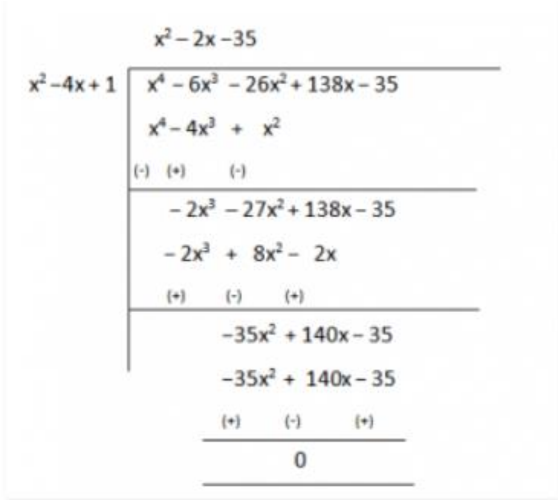
$\therefore [x-(2+\sqrt{3})]\,[x-(2-\sqrt{3})] = 0$

$(x-2-\sqrt{3})(x-2+\sqrt{3}) = 0$

On multiplying the above equation we get,

x^2-4x+1, this is a factor of a given polynomial f(x).

Now, if we will divide f(x) by g(x), the quotient will also be a factor of f(x) and the remainder will be 0.



So, $x^4-6x^3-26x^2+138x-35 = (x^2-4x+1)(x^2-2x-35)$

Now, on further factorizing $(x^2-2x-35)$ we get,

x^2**$-(7-5)x-35$** $= x^2-7x+5x+35 = 0$

$x(x-7)+5(x-7) = 0$

$(x+5)(x-7) = 0$

So, its zeroes are given by:

$x = -5$ and $x = 7$.

Therefore, all four zeroes of given polynomial equation are: $2+\sqrt{3}$, $2-\sqrt{3}$, **-5 and 7.**