INDIAN INSTITUTE OF TECHNOLOGY MADRAS

Department of Chemical Engineering

CH5230 System Identification

Project

Due: Saturday, May 06, 2017

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1. Multiscale systems are characterized by wide separation in time scales and their identification is in general more challenging than the single-scale systems. Suppose for an over-damped continuous-time system of N^{th} order, with time constants $\tau_1 > \tau_2 > ...\tau_i > \tau_{i+1}.. > \tau_N$, if there exists at least one pair of time constants, $\tau_i >> \tau_{i+1}$, then the system is said to be multiscale. Essentially, there exists subsystems that evolve at different speeds or rates.

   In a second-order multiscale system, for example, wherein $\tau_1 \gg \tau_2$, the subsystem associated with τ_2 (call it G_2) evolves at a much faster rate than that with τ_1 (call that G_1). From a frequency-domain viewpoint, G_2 has a much broader bandwidth than G_1. As a result, when an identification experiment is performed on a multiscale system, depending on the nature of the input, either one or both of the subsystem(s) gets predominantly excited.

   There exists many challenges in identification of multiscale systems. One of these challenges is that direct application of conventional discrete-time identification techniques to identify multiscale systems can yield poor results. The question under consideration asks the student to first experience this challenge and then propose a solution.

   Three independent experiments are conducted and the input-output data for each of the experiments are recorded. It is known that the process is of **second order** and the noise model has white noise characteristics. The input-output data for each experiment are given in the datasets `data1`, `data2` and `data3`.

   Using the given information, answer the following questions:

   (a) Identify the output-error models by directly applying the conventional identification techniques for each of the three data sets. In each case, use the full length data for training purposes while using the remaining two data sets as test data. Make sure you perform the usual residual analysis in the training phase. Do you visually observe good predictions in all the three cases? If yes/no explain.

   (b) If you are not satisfied with the predictions in part (1a), propose a method to identify the multiscale system. For this purpose, please note that only among the given data sets only one of them has maximum information on both fast and slow subsystems (**best dataset** for identifying the overall model) while others have information predominantly on only one of the subsystems.

      i. Propose a method to identify the **discrete-time model** accurately using the ascertained **best dataset**.

      ii. Report the values of poles and zeros of the identified discrete-time model.

      iii. Validate the identified model using the test data.

2. This question is concerned with the estimation of Input-output delays, which are coventionally estimated using cross-correlation, more specifically, using impulse response functions. In this problem, you are required to investigate the use of a frequency-domain method (based on the phase of the FRF) as outlined in Chapter 22 of the text. In this method, the delay D is estimated by solving the optimization problem

$$J(D) = \sum_{n=0}^{N-1} W(\omega_n) \cos \varepsilon(\omega_n) \tag{1}$$

$$\text{where } \varepsilon(\omega) = \hat{\phi}_{yu}(\omega) - \arg\ \bar{G}(\omega) + D\omega, \quad W(\omega) = 1/\sigma^2_{\hat{\phi}(\omega)} = \frac{|\kappa(\omega)|^2}{1 - |\kappa(\omega)|^2} \tag{2}$$

where $\bar{G}(\omega)$ is the delay-free part $\bar{G}(\omega)$ of the LTI system $G(\omega) = \bar{G}(\omega)e^{-D\omega}$. The key idea is that $\bar{\phi}_{yu}(\omega)$ can be estimated without the knowledge of the delay, purely from the magnitude of the FRF as:

$$\arg\ \bar{G}(\omega_l) = -\frac{1}{2M} \sum_{n=1, n\neq l}^{M} \log|G(\omega_n)| \left(\cot\left(\frac{\omega_l - \omega_n}{2}\right) + \cot\left(\frac{(\omega_l + \omega_n)}{2}\right)\right) \tag{3}$$

The quantity $\hat{\phi}_{yu}(\omega)$ is the estimate of phase obtained from the smoothed estimate of FRF (using the `spa` routine in MATLAB, for example) and $|\kappa(\omega)|^2$ is the squared coherence (use `mscohere` in MATLAB).

(a) Develop a MATLAB function to estimate the delay using the above algorithm (also given as Algorithm 22.1 in MATLAB).

(b) Test your algorithm on the transfer function given in Equation (22.60) of the text. Simulate by assuming white-noise error with SNR set to 10.

(c) For the data given in `delest_data.mat`, estimate the time-delay. Compare the estimate with that obtained from the impulse response estimates.