B.C.S.E. 2nd year 2nd Semester Examination 2011

NUMERICAL METHODS

Time : 3 hours.   Full Marks : 100

Answer question no.1 and any 4 from the rest.

1. a) What is the maximum absolute error when an approximate number p
        is divided by another approximate number q.   3
    b) Draw a comparison between regula falsi method and secant method.   3
    c) Derive the modifier formula for Adams-Bashforth method.   5
    d) Write down the expressions for truncation errors for Trapezoidal method,
        Simpson's ⅓ rule and Simpson's ⅜ rule.   3
    e) Define Δ, ∇ and E. Hence prove that

$$\Delta - \nabla = \Delta\nabla$$ 3

    f) State the condition of convergence for Gauss- Seidel iterative method
        for solution of linear simultaneous equations.   1
    g) Derive the iteration formula for finding $(1/N)^{1/3}$ using Newton –
        Raphson method.   2

2. a) Describe Newton- Raphson method for solution of non-linear equations.   4
    b) Derive the condition of convergence and the order of convergence for
        the above method.   5+5
    c) Solve the following equation using secant method.   6

$$3x + \sin x - e^{x} = 0$$

        Take $r_0 = 1$ and $r_1 = 0$.

3. a) Describe the iterative method for finding the inverse of a matrix. What is
        its order of convergence.   8
    b) Given the following set of linear simultaneous equations:

$$x - y + 2z = -8$$
$$x + y + z = -2$$
$$2x - 2y + 3z = -20$$

        Find the inverse of the above co-efficient matrix by Gauss – Jordan
        elimination method. Hence find the solution of this set of equations.   8

c) Derive the Chebyshev iteration formula for solution of non-linear
    equations.   4

4. a) Derive divided difference formula for interpolation. How is it different
    from Newton's interpolation formula.   10
   b) Given the following table of values:

x	- 5	-3	-1	1	3	5
y	5.5	9.1	14.9	22.8	33.3	46

   Obtain a least squares fit of the following form to the tabular values. Show
   each step clearly.
   $$y = a + bx + cx^2$$
       10

5. a) Find all the eigenvalues and corresponding eigenvectors of a 2 x 2
    symmetric matrix as given below using Jacobi's method.   8

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

   b) Describe Lin's method for finding the complex roots of a polynomial
    equation.   12

6. a) Prove by the method induction
    $$\Delta^m y_r = \nabla^m y_{r+m}$$
        5
   b) If $y = U(x).V(x)$ then
    $$y[x_0, x_1] = U(x_0).V[x_0, x_1] + V(x_1).U[x_0, x_1]$$
        3

   c) Use Newton's formula to find the area of a circle of diameter 98 cm
    from the following area (A) vs. diameter (D) table:

D (cm)	80	85	90	95	100
A (cm^2)	5026	5674	6362	7088	7854

   Derive the necessary formula.   12

7. a) Derive numerical integration formula using Simpson's ⅓ rule.   6
   b) Derive the expression for total truncation error associated with this method.   8

   c) Evaluate the integral $\int_0^2 (1/(x^2 + 4)) \, dx$ using Trapezoidal method.
    Take $h = 0.25$. Hence obtain the value of π.   6

8. a) Discuss Modified Euler's method for solution of ordinary first order differential equations. 6

   b) Derive the expression for truncation error associated with this method. 4

   c) Solve the following initial value problem using Euler's method.

   $$\frac{dy}{dx} = x^2 + y \quad \text{with} \quad y(0) = 1$$

   Compute the first 5 steps of the solution with $h = 0.1$. Compare the results (% relative error) with those obtained from the exact solution

   $$y = 3e^x - x^2 - 2x - 2$$

   10