B.C.S.E. 2nd year 1st Semester Examination 2016

NUMERICAL METHODS

Time : 3 hours. Full Marks : 100

Answer question no.1 and any 4 from the rest.
All parts of same question should be answered together.

1. a) Define round-off and truncation errors. 2
   b) Draw a comparison between regula falsi method and secant method. 3
   c) Derive Aitken's acceleration formula 3
   d) Modify Newton- Raphson iteration formula for solution of a nonlinear equation with multiple roots at a point. 2
   e) Define Δ and E. Prove that $(E^{\frac{1}{2}} + E^{-\frac{1}{2}})(1+\Delta)^{\frac{1}{2}} = 2 + \Delta$. 3
   f) Define the terms eigenvalue and eigenvector. 2
   g) Given 4 points: (x_0, y_0), (x_1, y_1), (x_2, y_2), (x_3, y_3) such that $x_i = x_0 + ih$. Fit a polynomial of degree 3 through these points using Newton's forward difference interpolation method. Find the area under this curve over the interval $[x_0, x_3]$. 5

2. a) Given the following table of values:

p	0.5	1.0	1.5	2.0	2.5	3.0
v	1.62	1.00	0.75	0.62	0.52	0.46

   Obtain a least squares fit of the following form to the tabular values.

$$\mathbf{pv^{\lambda} = k}$$ 10

   b) Derive Lagrange interpolation formula. 6
   c) Fit a polynomial to the following table of values using Lagrange interpolation formula.

x	0	1	3	4
y	-12	0	6	12

   Find the value of **y** when **x** = 2. 4

3. a) Discuss Jacobi's iterative method for finding the roots of linear simultaneous equations. 6
   b) Write down the method in matrix notation. 4
   c) Hence find the convergence of the method. 4

d) Solve the following system of equations by Gauss- Seidel iterative method. Solution is required corrected upto 4 decimal places. 6

$$x + 3y + 10z = 24$$
$$28x + 4y - z = 32$$
$$2x + 17y + 4z = 35$$

4. a) Discuss Modified Euler's formula for solution of differential equations. 6
b) Derive the expression for truncation error of the above method. 4
c) Solve the following differential equation by **Euler's** method.

$$\frac{dy}{dx} = x^2 + y \qquad \text{with } y(0) = 1.0$$

Compute the first **5** steps of the solution with step size $h = 0.1$
Compare the results with those obtained from the exact solution 10

$$y = 3e^x - x^2 - 2x - 2$$

5. a) Prove that Power method finds the largest eigenvalue of a square matrix. 8
b) Discuss predictor-corrector method for solution of ordinary differential equations. Derive the predictor, modifier and corrector formulae for Adams-Bashforth method. 12

6. a) Given the following tabular values:

x	50	60	70	80	90
y	19.96	36.65	58.81	77.21	94.61

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 51$. 12
Derive the requisite formulae.
c) Find the inverse of the following matrix using Gauss-Jordan method. 8

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

7. a) Discuss Romberg's method for evaluating the integral of the following form.

$$\int_b^a f(x)\,dx$$ 12

b) Evaluate the following integral by Gaussian quadrature formula. Take n = 2.

$$\int_0^1 \frac{1}{x^2 + 1}\,dx$$ 8

8. a) Describe Bairstow's method for finding the complex roots of a polynomial equation. 12

b) Solve the following equation by iterative method using repeated substitution corrected upto 4 decimal places. Take $\mathbf{x_0} = 0$. 8

$$\mathbf{3x - 1 - \cos(x) = 0}$$

Take an appropriate form for iteration.
