

BACHELOR OF COMPUTER SCIENCE & ENGG. EXAMINATION, 2010
(2nd Year, 2nd Semester)

NUMERICAL METHODS

Time : 3 hours.

Full Marks : 100

Answer question no.1 and any 4 from the rest.

1. a) What are the different types of errors introduced in a computational process? 5

- b) Let $x = -\log_e x$
Can iterative method using repeated substitution be applied to find a solution to this equation? Why? 2

- c) Is it possible to get a solution of the following system of equations by LU decomposition method? 3

$$\begin{aligned} -2y + 3z &= 15 \\ 3x + 2y + 9z &= 18 \\ x + y + 3z &= 10 \end{aligned}$$

- d) Can Gauss-Seidel method be used for solving the following system of equations? Why? 2

$$\begin{aligned} 3x + 9y - 2z &= 2 \\ 4x + 2y + 13z &= 5 \\ 4x - 2y + z &= 10 \end{aligned}$$

- e) The velocity of a moving particle is measured at various time instants and shown in the following table:

t (sec.)	0	0.1	0.2	0.3	0.4
v (cm. / sec.)	10.5	11.5	12.75	14.5	17.0

Compute the total distance traversed in 0.4 sec.

Also compute the acceleration of the particle at $t = 3.75$ sec.

8

2. a) Discuss Gaussian elimination method for solution of a system of linear simultaneous equations. 8

- b) Calculate the number of multiplications / divisions required in the elimination process. 6

- c) Solve the following system of equations by the above method. 6

$$\begin{aligned} 2x - 3y + 4z &= 13 \\ x + y + z &= 9 \\ 3x + 4y + 5z &= 40 \end{aligned}$$

3. a) Define the terms eigenvalue and eigenvector. 2
 b) Prove that a $n \times n$ matrix has n different eigenvalues and corresponding n different eigenvectors. 4
 c) Discuss transform technique for computing all the eigenvalues and corresponding eigenvectors of a matrix. 6
 d) Find all the eigenvalues and eigenvectors of the following matrix by Jacobi's method. 8

$$\begin{bmatrix} 1 & -2 & 4 \\ -2 & 5 & -2 \\ 4 & -2 & 1 \end{bmatrix}$$

4. a) Discuss curve fitting by the method of least squares. 10
 b) Given the following table of values:

p	0.5	1.0	1.5	2.0	2.5	3.0
v	1.62	1.00	0.75	0.62	0.52	0.46

Obtain a least squares fit of the following form to the tabular values.

$$pv^{\lambda} = k \quad 10$$

5. a) Derive multi-point iteration formula for solution of non-linear equations. 5
 b) Derive the order of convergence of this method. 10
 c) Derive the recursive formula for evaluating the n -th root of a number A ($A^{1/n}$) using Newton-Raphson method. 5

6. a) Consider the following non-linear simultaneous equations:

$$x^2 - 2x - y + 0.5 = 0$$

$$x^2 + 4y^2 - 4 = 0$$

Find the solution of the above set of equations corrected upto 3 decimal places using Newton - Raphson method. Take initial approximations as (2.00, 0.25). 10

- b) Derive Lagrange interpolation formula. 6
 c) Fit a polynomial to the following table of values using Lagrange interpolation formula.

x	0	1	3	4
y	-12	0	6	12

Find the value of y when $x = 2$. 4

7. a) Derive numerical integration formula using Trapezoidal rule. 6
 b) Derive the expression for total truncation error associated with this method. 8

- c) evaluate the integral $\int_0^{1/2} (x / \sin x) dx$ using Trapezoidal method with $h = 0.25$ and $h = 0.125$. Hence compute the Romberg term at first level. 6

8. a) Discuss Modified Euler's method for solution of ordinary first order differential equations. 6
 b) Derive the expression for truncation error associated with this method. 4
 c) Solve the following initial value problem using Modified Euler's method.

$$\frac{dy}{dx} = x + y \quad \text{with } y(0) = 1$$

Solution is required to be corrected up to 2 decimal places over the interval $(0, 0.5)$ with $h = 0.1$. 10
