B.C.S.E. 2nd year 2nd Semester Examination 2012

NUMERICAL METHODS

Time : 3 hours. Full Marks : 100

Answer question no.1 and any 4 from the rest.

1. a) Define round-off and truncation errors. 2
   b) Derive the condition of convergence for Newton -Raphson method. 3
   c) Derive Aitken's acceleration formula. 4
   d) Can Gauss- Seidel method be used for solving the following system of equations? Why? 2

$$9x + 2y + 3z = 2$$
$$3x + y + z = 5$$
$$x - 6y + 2z = 10$$

   e) Define divided difference of order k and derive divided difference interpolation polynomial. 5
   f) Derive the condition of stability for Euler's method. 2
   g) Derive the recursive formula to find 1/N using Newton- Raphson method. 2

2. a) Describe secant method for solution of non-linear equations. 5
   b) Derive the order of convergence for the above method. 10
   c) Solve the following equation using secant method.

$$x^3 - 3x + 1 = 0$$

   Take $r_0 = 1$ and $r_1 = 2$. 5

3. a) Discuss curve fitting by the method of least squares. 10
   b) Given the following table of values:

x	0.2	0.3	0.5	1.0	2.0
f(x)	16	14	11	6	3

Obtain a least squares fit of the following form to the tabular values.

$f(x) = c_0 x + c_1/\sqrt{x}$ 10

4. a) Derive Newton Forward Difference interpolation polynomial for a given set of tabular values. 6

b) From the above polynomial, derive the expressions for evaluating first order and second order derivatives. 8

c) A particle is moving in a circular path. The following table gives the angle of rotation θ (radian) at time instant t (second).

t	0.0	0.2	0.4	0.6	0.8	1.0
θ	0.0	0.12	0.49	1.12	2.02	3.20

Calculate the angular velocity and angular acceleration of the particle at $t = 0.1$ second. 6

5. a) Prove that Power method finds the largest eigenvalue of a square matrix. 10

b) Find all the eigenvalues and eigenvectors of the following matrix by Jacobi's method. 10

$$\begin{bmatrix} 1 & \sqrt{3} & 4 \\ \sqrt{3} & 5 & \sqrt{3} \\ 4 & \sqrt{3} & 1 \end{bmatrix}$$

6. a) Discuss Gauss- Jordan elimination method for matrix inversion. 6

b) Calculate the total number of multiplications / divisions required by this method. 6

c) Find the inverse of the following matrix. 8

$$\mathbf{A} = \begin{bmatrix} \mathbf{8} & \mathbf{-4} & \mathbf{0} \\ \mathbf{-4} & \mathbf{8} & \mathbf{-4} \\ \mathbf{0} & \mathbf{-4} & \mathbf{8} \end{bmatrix}$$

Also find $\mathbf{A}\,\mathbf{A}^{-1}$.

7. a) Write down the algorithm for implementing Trapezoidal rule for evaluating the integral of the following form corrected upto a finite number of decimal places.

$$\int_b^a f(x)\,dx$$ 6

b) Calculate the amount of truncation error involved in this method. 6

c) Evaluate the following integral corrected upto 4 decimal places by the above method. Hence obtain the value of π.

$$\int_0^1 \frac{1}{x^2 + 1}\,dx$$ 8

8. a) Discuss Runge-Kutta 4th order formula for solution of ordinary first order differential equations. 6

b) Extend this method to find the solution of second order differential equations. 4

c) Describe Lin's method for finding the complex roots of a polynomial equation. 10
