

Ohm's Law

The ratio of potential difference (V) between any two points on a conductor to the current (I) flowing between them, is constant, provided the temperature of the conductor does not change.

$$\frac{V}{I} = \text{constant or } \frac{V}{I} = R$$

Assi: 1. Derive the vector form of Ohm's law.

OR

Relation b/w current Density and electric field

v_d = drift velocity.

$$\vec{J} = ne\vec{v}_d$$

$$\vec{J} = ne \left(\frac{e \vec{E} I}{m} \right)$$

$$I = Tao.$$

$$\Rightarrow \vec{J} = \left(\frac{n e^2 I}{m} \right) \vec{E}$$

$$\text{Also, } J = \frac{m}{n e^2 \tau}$$

$$\Rightarrow \vec{J} = \frac{\vec{E}}{\rho} \quad [\sigma = \frac{1}{\rho}]$$

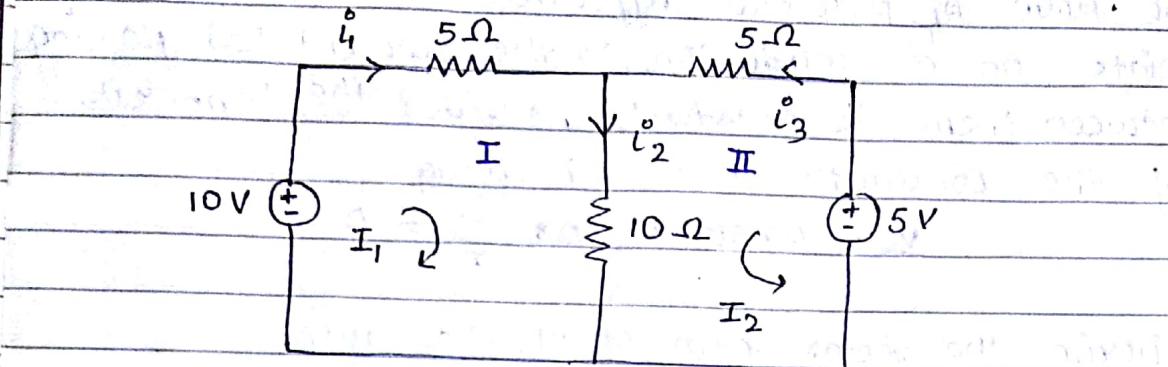
$$\Rightarrow \boxed{\vec{J} = \sigma \vec{E}}$$

KVL The sum of voltage rises = sum of voltage drop in closed loop.

KCL The sum of current entering through a junction is equal to the sum of current leaving through junction.

KVL

1. Find the branch currents and loop currents in the circuit



Branch currents are: i_1 , i_2 and i_3

Loop currents are: I_1 and I_2

From loop I,

$$5i_1 + 10i_2 = 10 \quad \text{--- (1)}$$

From loop II,

$$5i_3 + 10i_2 = 5 \quad \text{--- (II)}$$

$$i_1 = I_1; i_2 = I_1 + I_2; i_3 = I_2$$

Replace it in (1) and (II),

$$5I_1 + 10I_1 + 10I_2 = 10$$

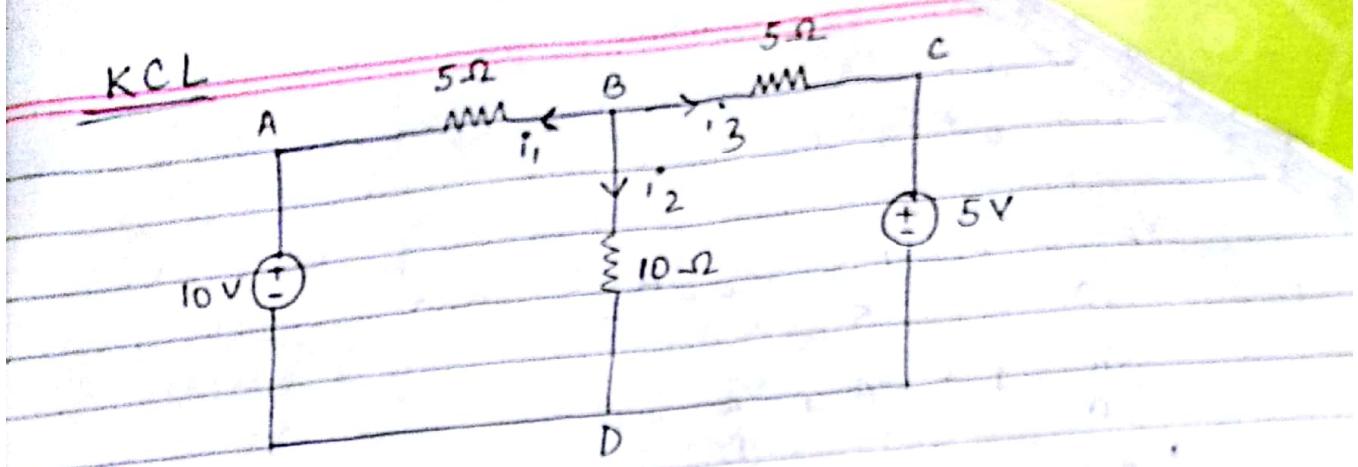
$$5I_2 + 10I_1 + 10I_2 = 5$$

$$3I_1 + 2I_2 = 2$$

$$2I_1 + 3I_2 = 1$$

$$-5I_2 = 1$$

$$I_2 = -\frac{1}{5} \text{ A} \quad I_1 = \frac{4}{5} \text{ A}$$



From KCL,

$$i_1 + i_2 + i_3 = 0$$

$$\frac{v_B - 10}{5} + \frac{v_B - 0}{10} + \frac{v_B - 5}{5} = 0$$

$$\Rightarrow \frac{v_B}{5} - 2 + \frac{v_B}{10} + \frac{v_B}{5} - 1 = 0$$

$$\Rightarrow \frac{2v_B}{5} + \frac{v_B}{10} = 3$$

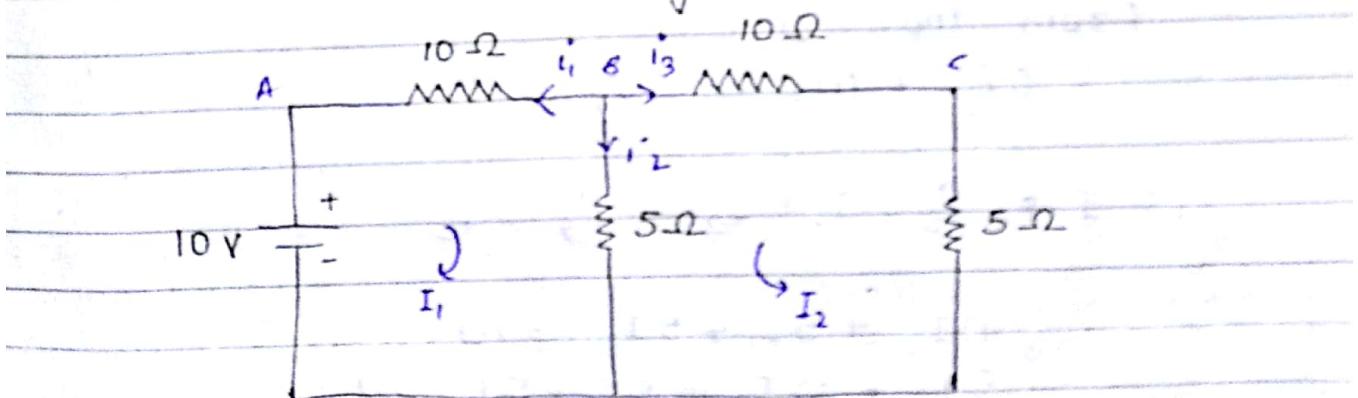
$$\Rightarrow \frac{4v_B + v_B}{10} = 3 \Rightarrow \frac{5v_B}{10} = 3$$

$$\Rightarrow v_B = 6$$

Now,

$$i_1 = \frac{v_B - 10}{5} = -\frac{4}{5} \quad i_2 = \frac{3}{5} \quad i_3 = \frac{1}{5}$$

2. Find the branch currents using both KCL and KVL in the following circuit:



I) KCL

From KCL,

$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_B - 10}{10} + \frac{V_B - 0}{5} + \frac{V_B - 0}{15} = 0$$

$$\frac{V_B}{10} - 1 + \frac{V_B}{5} + \frac{V_B}{15} = 0$$

$$\frac{2V_B}{10} + \frac{V_B}{5} = 1 \Rightarrow \frac{2V_B}{5} = 1$$

$$V_B = \frac{5}{2}$$

$$\frac{V_B}{5} \cancel{\frac{V_B}{2}} + V_B$$

$$\frac{V_B}{5} \left(\frac{1}{2} + 1 + \frac{1}{3} \right) = 1 \Rightarrow \frac{V_B}{5} \times \frac{11}{6} = 1$$

$$\Rightarrow V_B = \frac{30}{11}$$

$$i_1 = \frac{30}{11} - 10 \\ = 0.7272$$

$$\frac{30 - 10}{11} = \frac{-80 \times 10}{11} = -0.7272$$

$$\text{or } \frac{30}{11} = 0.7272$$

II) KVL

From loop I,

$$10i_1 + 5i_2 = 10$$

From loop II,

$$5i_3 + 10i_2 + 5i_1 = 0$$

$$i_1 = I_1; i_2 = I_1 + I_2; i_3 = I_2$$

$$10I_1 + 5I_1 + 5I_2 = 10$$

$$5I_2 + 10I_2 + 5I_1 + 5I_2 = 0$$

$$\Rightarrow 15I_1 + 5I_2 = 10$$

$$5I_1 + 20I_2 = 0$$

$$\begin{array}{l} 3I_1 + I_2 = 0 \\ I_1 + 4I_2 = 0 \end{array}$$

$$\begin{array}{l} 3I_1 + I_2 = 0 \\ 3I_1 + 12I_2 = 0 \end{array}$$

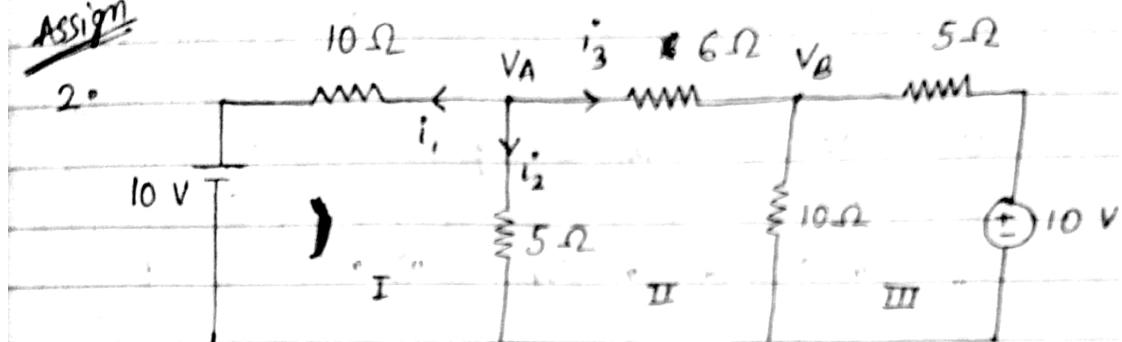
$$-11I_2 = 2$$

$$\begin{aligned} I_1 &= -4I_2 \\ &= \frac{4 \times 2}{11} = \frac{8}{11} \end{aligned}$$

$$I_2 = \frac{-2}{11}$$

$$I_1 = \frac{8}{11} = 0.727272$$

ASSIGN



At node A,

~~$i_1 + i_2 + i_3 = 0$~~

$$\frac{V_A - 10}{10} + \frac{V_A - 0}{5} + \frac{V_A - V_B}{6} = 0 \quad \textcircled{1}$$

~~At node B,~~ At node B,

$$\frac{V_B - V_A}{6} + \frac{V_B - 0}{10} + \frac{V_B - 10}{5} = 0 \quad \textcircled{2}$$

From $\textcircled{1}$,

$$\frac{V_A}{10} - 1 + \frac{V_A}{5} + \frac{V_A}{6} - \frac{V_B}{6} = 0$$

$$\Rightarrow \frac{V_A}{5} \times \frac{3}{2} + \frac{V_A}{6} - \frac{V_B}{6} = 1$$

$$\Rightarrow \frac{3V_A}{10} + \frac{V_A}{6} - \frac{V_B}{6} = \frac{7V_A}{15} - \frac{V_B}{6} = 1$$

$$\Rightarrow \frac{7V_A}{15} - \frac{V_B}{6} = 1 \quad \textcircled{3}$$

$$\frac{V_B}{6} - \frac{V_A}{6} + \frac{V_A}{10} + \frac{V_B}{5} - 2 = 0$$

$$\frac{7}{15} V_B - \frac{V_A}{6} = 2 \quad \text{--- (iv)}$$

$$\frac{7}{15} V_A - \frac{V_B}{6} = 1 \quad \times 1$$

~~$$\frac{7}{15} V_B - \frac{V_A}{6} = 2$$~~
$$\times \frac{42}{15}$$

$$\frac{7 \times 6}{15} \cdot \frac{6}{6}$$

~~$$42 \cdot \frac{6}{15}$$~~

~~$$\bullet \quad \frac{7V_A}{15} - \frac{V_B}{6} = 1$$~~

$$\frac{7 \times 6}{15} \cdot \frac{6}{15}$$

~~$$\bullet \quad -\frac{7V_A}{15} + \frac{7 \times 42}{15 \times 15} V_B = \frac{84}{15}$$~~

$$\frac{984}{225}$$

$$1.14 V_B = 6 \cdot 6$$

$$1.306 \\ 0.166$$

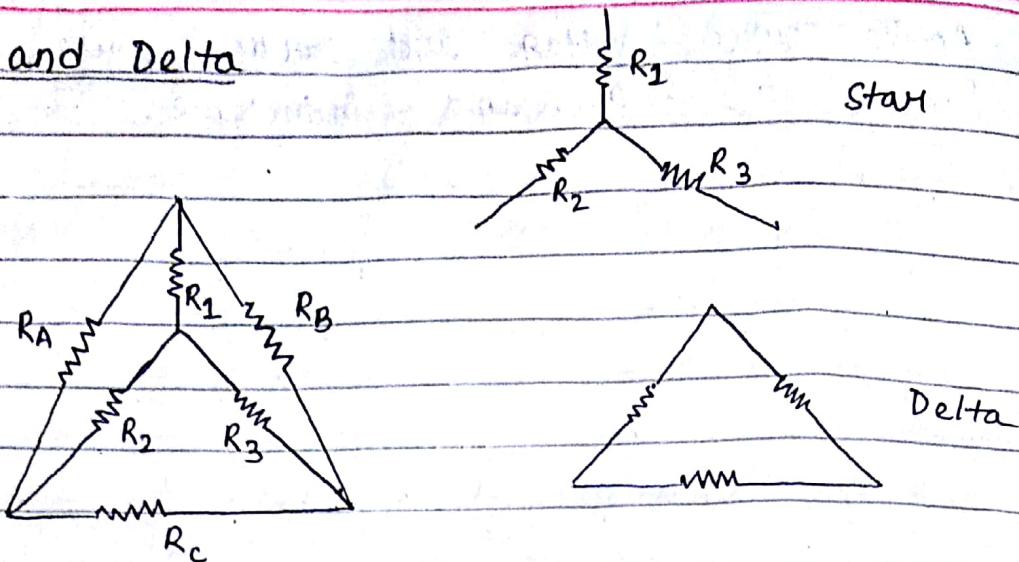
$$V_B = 5.78$$

$$\frac{7}{15} \times 5.78 - 2 = \frac{V_A}{6}$$

$$V_A = 4.184$$

$$i_2 = \frac{V_A}{5} = 0.83$$

Star and Delta



1. Delta \rightarrow Star

R_A , R_B and R_C are known

$$R_1 = \frac{R_A \cdot R_B}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_B \cdot R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A \cdot R_C}{R_A + R_B + R_C}$$

2. Star \rightarrow Delta

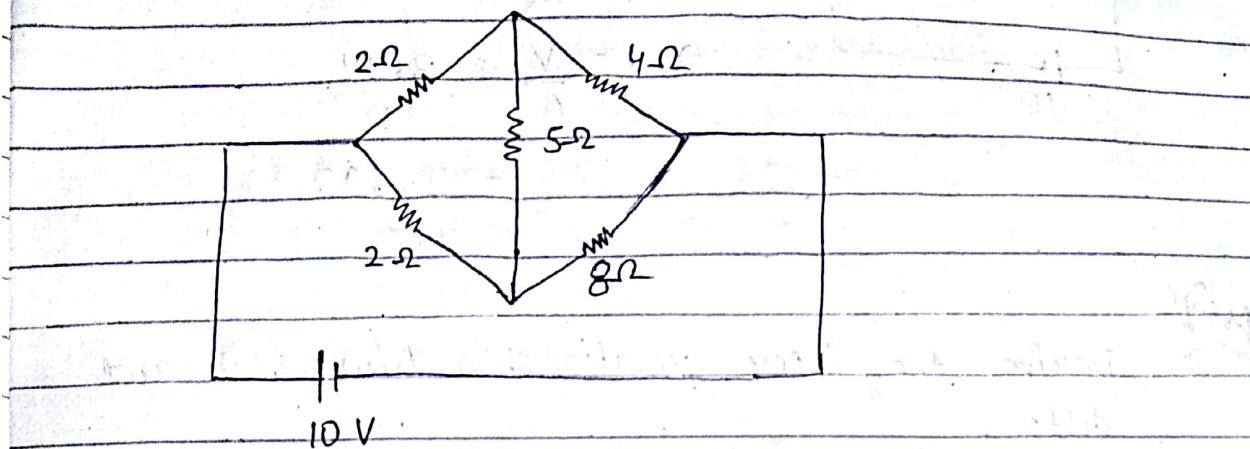
R_1 , R_2 and R_3 are known

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

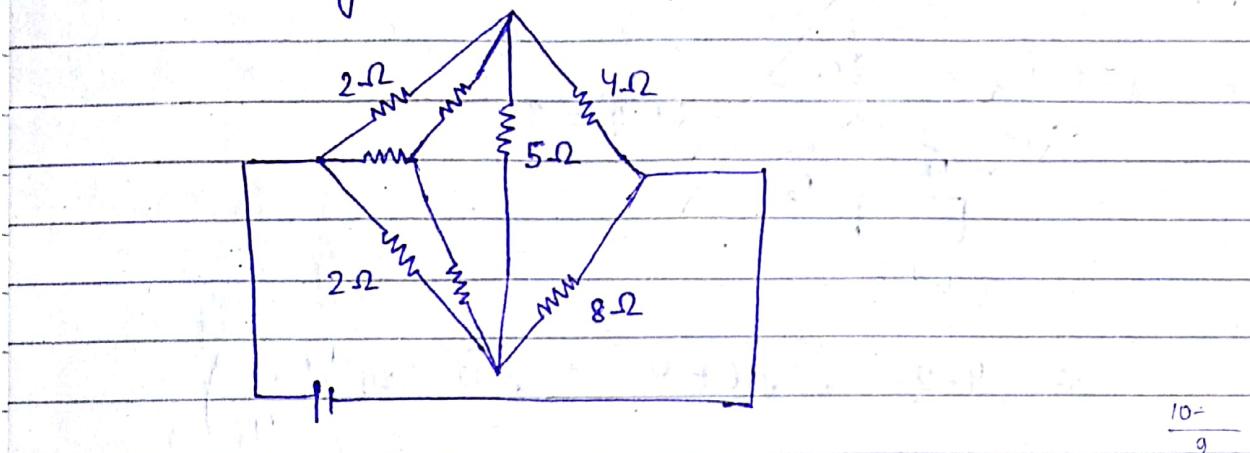
$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

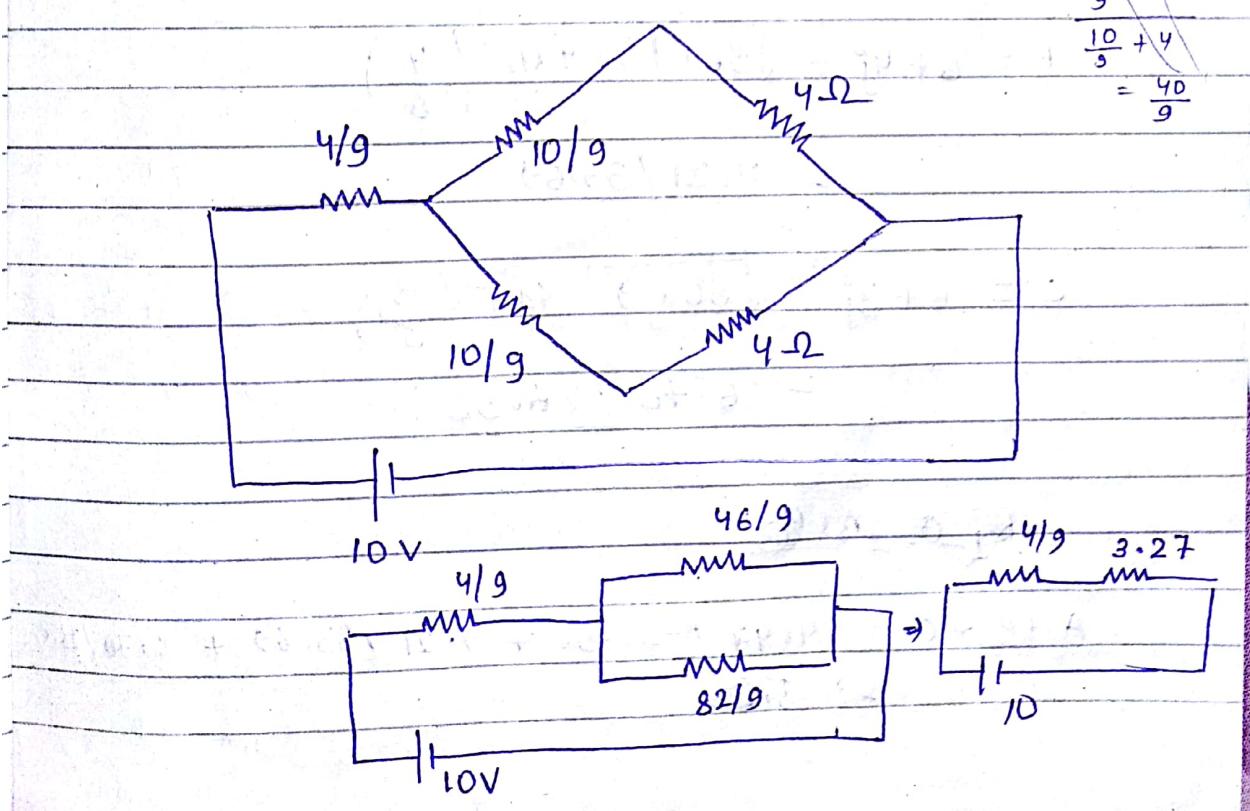
3. Find out the total current in the circuit



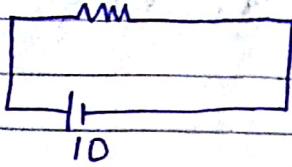
Assuming star,



After rearranging it,



3.71



$$V = IR,$$

$$\Rightarrow I = \frac{V}{R} = \frac{3.71}{10}$$

$$= 2.695$$

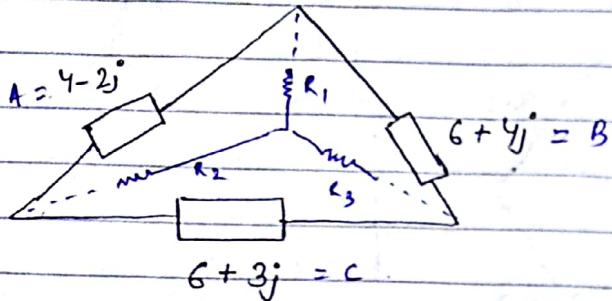
$$\approx 2.7$$

Assign

3. Derive the inter relationship b/w star and delta.

Assign

4.



$$A = 4 - 2j = \sqrt{16 + 4} = \sqrt{20} \tan^{-1}\left(\frac{-2}{4}\right)$$

$$= 4.47 \angle -26.56^\circ$$

$$B = 6 + 4j = \sqrt{36 + 16} \tan^{-1}\left(\frac{4}{6}\right)$$

$$= 7.21 \angle 33.69^\circ$$

$$C = 6 + 3j = \sqrt{36 + 9} \tan^{-1}\left(\frac{3}{6}\right)$$

$$= 6.70 \angle 26.56^\circ$$

R₁ + R₂ + R₃

$$A + B + C = 4.47 \angle -26.56^\circ + 7.21 \angle 33.69^\circ + 6.70 \angle 26.56^\circ$$

~~DO NOT DO~~

$$253.1281 \\ 24.9001$$

$$= 4.47(0.89 + j0.44) + 7.21(0.83 + j0.55) \\ + 6.70(0.89 + j0.447)$$

$$= 3.97 - 1.96j + 5.98 + 3.96j + 5.96 + 2.99j$$

$$= 15.91 + 4.99j \approx 16 + 5j = 16.76 \angle 17.35$$

$$= \sqrt{278.0282} \tan^{-1}\left(\frac{4.99}{15.91}\right) = 16.67 \angle 17.41$$

~~R₁ = 16.67 ∠ 17.41~~

$$R_1 = \frac{(4-2j)(6+4j)}{16.67 \angle 17.41} = \frac{24 + 16j - 12j^2 + 8}{16.67 \angle 17.41}$$

$$= \frac{32 + 4j}{16.67 \angle 17.35}$$

$$= \frac{32.24 \angle 17.12}{16.76 \angle 17.35}$$

$$= 1.92 \angle -10.23$$

$$= 1.88 + j(-0.33)$$

$$\begin{aligned} &= \frac{32 + 8.4j}{16.67 \angle 17.41} \\ &= \frac{8.06 + j(8+j)}{16.67 \angle 17.41} \\ &= \frac{4 \times 8.06 \angle 7.12}{16.67 \angle 17.41} \\ &= 1.93 \angle -10.29 \\ &= 1.89 + j(-0.34) \end{aligned}$$

$$R_2 = \frac{A \cdot C}{A + B + C} = \frac{(4-2j)(6+3j)}{16.76}$$

$$= \frac{24 + 12j - 12j^2 + 6}{16.76}$$

$$= \frac{30.00}{16.76 \angle 17.35}$$

$$= 1.78 \angle -17.35$$

$$= 1.7 + j(-0.53)$$

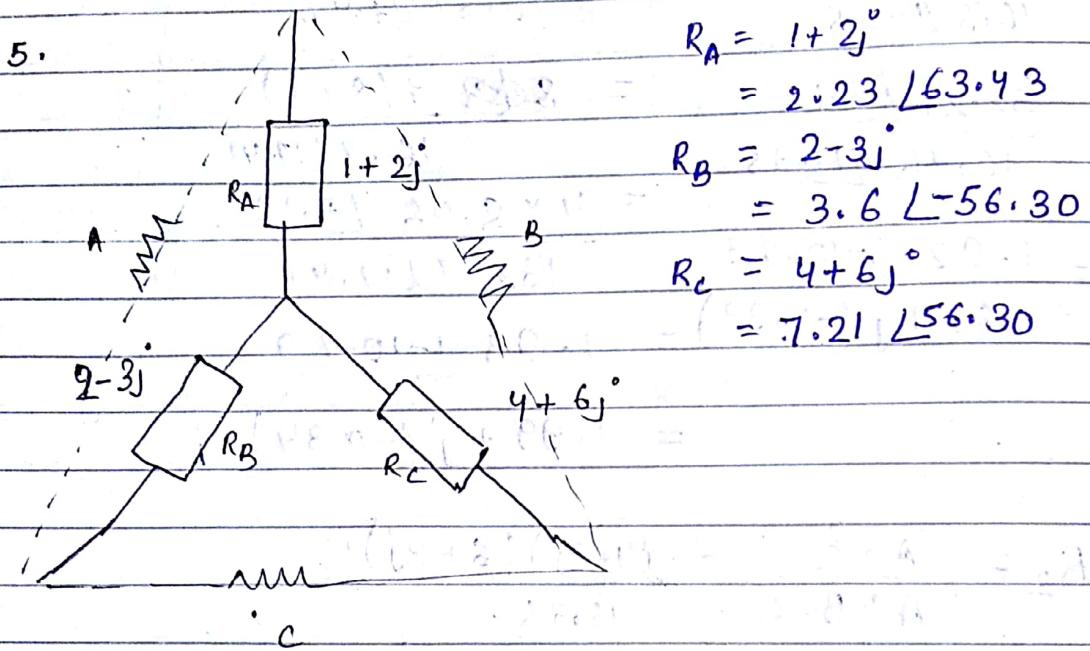
$$\begin{aligned}
 R_g &= \frac{(6+4j)(6+3j)}{16 \cdot 76 \angle 17.35} \\
 &= \frac{36 + 18j + 24j - 12}{16 \cdot 76 \angle 17.35} \\
 &\approx \frac{24 + 42j}{16 \cdot 76 \angle 17.35} = \frac{48.37 \angle 60.25}{16 \cdot 76 \angle 17.35} \\
 &= 2.88 \angle 42.09 \\
 &= 2.01 + j(1.96)
 \end{aligned}$$

$$R_1 = 1.88 + j(-0.33)$$

$$R_2 = 1.7 + j(-0.53)$$

$$R_g = 2.01 + j(1.96)$$

5.



$$\begin{aligned}
 R_A &= 1 + 2j \\
 &= 2.23 \angle 63.43
 \end{aligned}$$

$$\begin{aligned}
 R_B &= 2 - 3j \\
 &= 3.6 \angle -56.30
 \end{aligned}$$

$$\begin{aligned}
 R_C &= 4 + 6j \\
 &= 7.21 \angle 56.30
 \end{aligned}$$

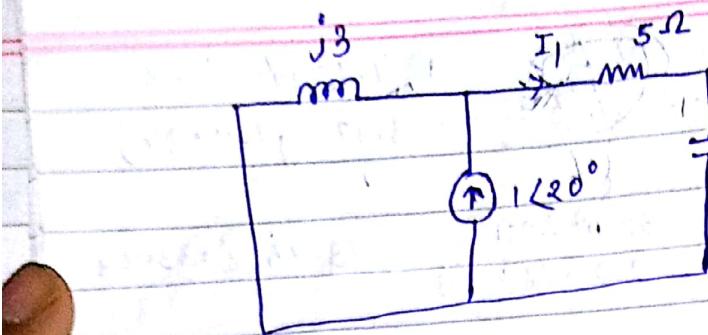
$$\begin{aligned}
 D &= R_A R_B + R_B R_C + R_C R_A \\
 &= (1+2j)(2-3j) + (2-3j)(4+6j) + (4+6j)(1+2j) \\
 &= 2-3j + 4j + 6 + 8 + 12j - 12j + 18 + 4 + 8j + 6j - 12 \\
 &= 26 + 15j \\
 &= 30.01 \angle 29.98 \approx 30.01 \angle 30
 \end{aligned}$$

$$\begin{aligned}
 A &= \frac{D}{2-3j} = \frac{30.01 \angle 30}{3.6 \angle -56.30} = 8.33 \angle 86.3 \\
 &= 0.53 + j(8.31)
 \end{aligned}$$

$$B = \frac{D}{4+6j} = \frac{30.01 \angle 30^\circ}{7.21 \angle 56.30^\circ} = 4.16 \angle -26.3^\circ$$
$$= 3.72 + j(-1.84)$$

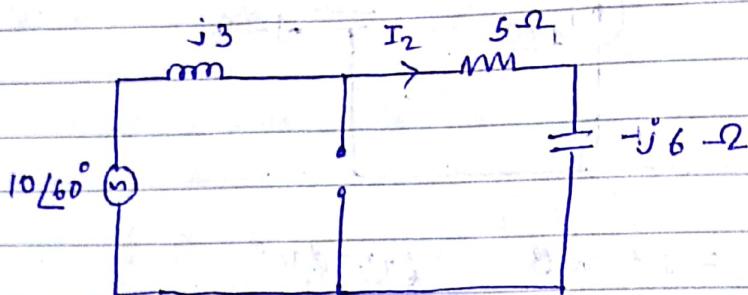
$$C = \frac{30.01 \angle 30^\circ}{1+2j} = \frac{30.01 \angle 30^\circ}{2.23 \angle 63.43^\circ} = 13.45 \angle -33.43^\circ$$

$$I_1 = \frac{j3}{j3 + 5 - j6} \times 1 \angle 20^\circ$$



$$I_2 = \frac{5 - j6}{j3 + 5 - j6} \times 1 \angle 20^\circ$$

$$\begin{aligned} I_1 &= \frac{j3}{5 - j6 + j3} \times 1 \angle 20^\circ \\ &= 0.515 \angle 120^\circ \text{ A} \\ &= -0.264 + j0.442 \text{ A} \end{aligned}$$



$$I_2 = \frac{10 \angle 60^\circ}{5 - j3} = 1.71 \angle 90.9^\circ = -0.026 + j1.715 \text{ A}$$

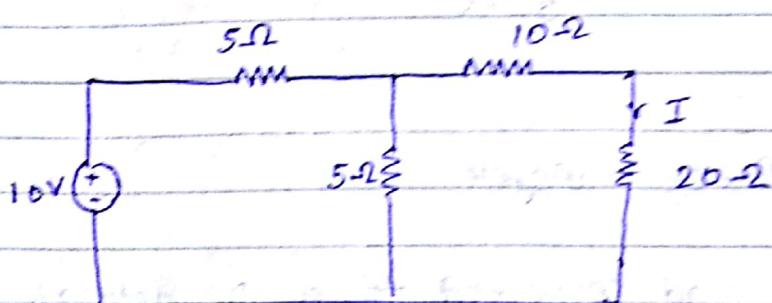
$$\begin{aligned} I &= I_1 + I_2 \\ &= -0.264 + j0.442 + -0.026 + j1.715 \\ &= -0.29 + j2.157 \text{ A} \end{aligned}$$

Thevenin's th

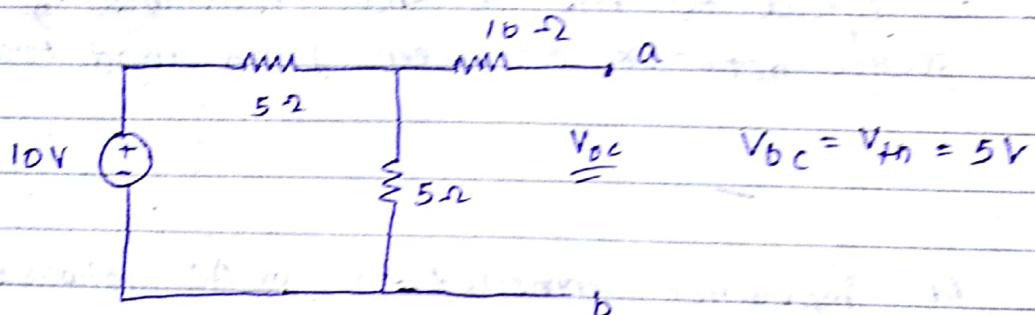
Any linear active network consisting of dependent/independent sources and linear bilateral network elements (resistances, conductance, inductance) can be replaced by an equivalent circuit consisting of a voltage source in a series with a resistance. The voltage source being a open circuited ^{voltage} across the open-circuited load resistance.

and the resistance being the internal resistance of load ~~between~~ source network looking ~~through~~ through open circuit load terminal.

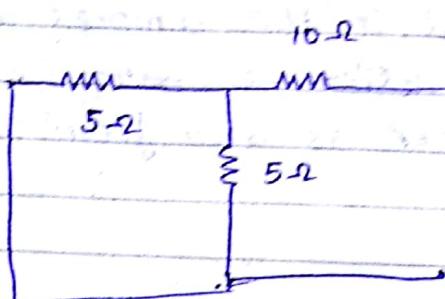
With all the active sources either replaced by their internal resistances (if any) or short-circuited (in case of voltage source) or open circuited (in case of current source).



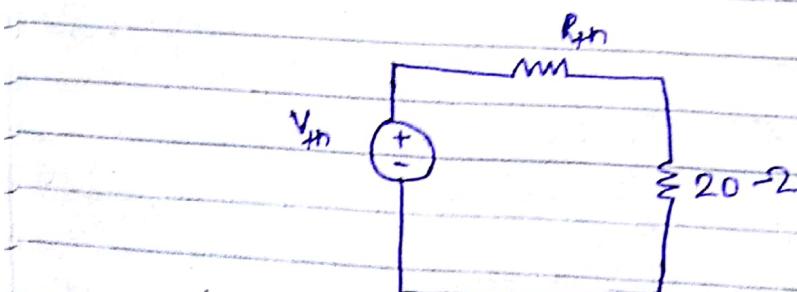
$$R_L = 20\Omega$$



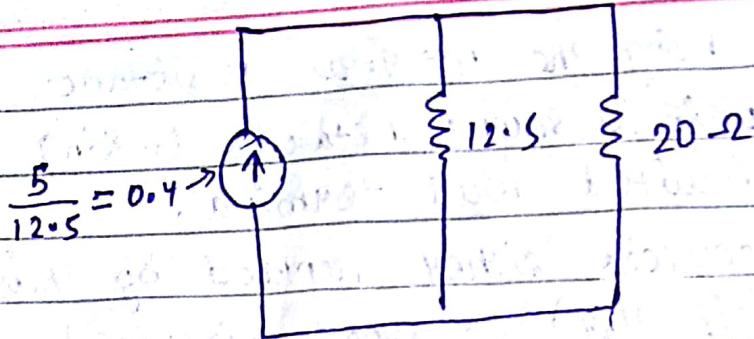
$$R_{th} = 5 \parallel 10 = 12.5$$



$$I_{th} = \frac{5}{12.5 + 20} = 0.153 A$$



Noonon.

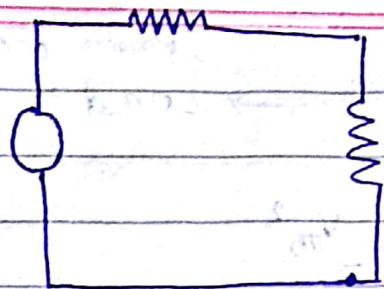


$$I_N = \frac{12.5 \times 0.4}{32.5}$$

Maximum Power transfer

A resistive load connected to a DC network receives maximum power when the load resistance is equal to the internal resistance of source network as seen from load terminal.

An impedance connected to an AC network receives maximum power when the load impedance is equal to the complex conjugate of the internal resistance same as above of source network as seen from load terminal.



$$P_L = I_L^2 R_L$$

$$= \left(\frac{V_m}{R_m + R_L} \right)^2 \cdot R_L$$

$\frac{dP_L}{dR_L} = 0$, then power will be maximum.

$$\frac{2V_m}{(R_m + R_L)} \cdot \frac{R_L}{(R_m + R_L)}$$

$$\frac{V_m^2}{(R_m + R_L)^2} \cdot R_L$$

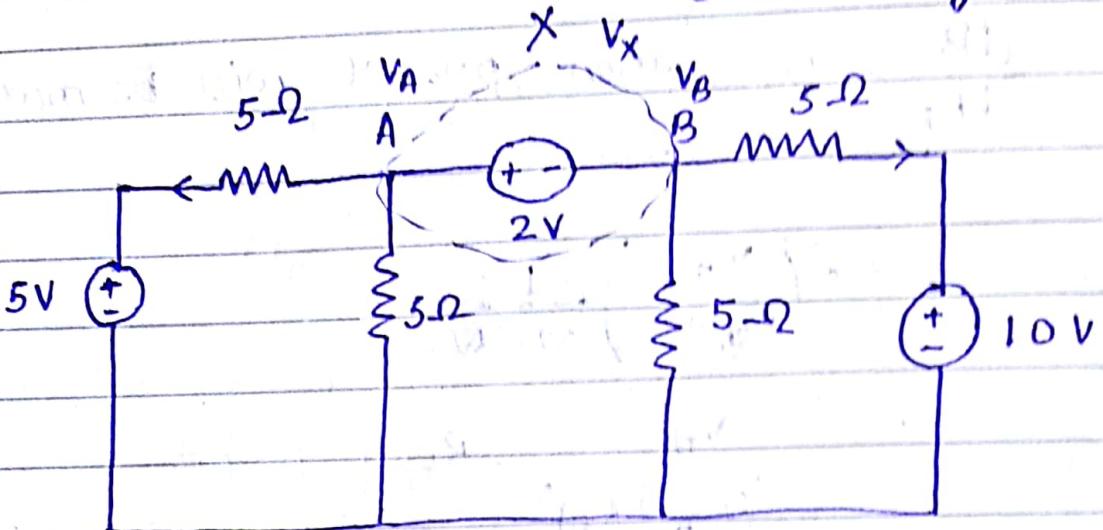
$$\Rightarrow V_m^2 [2(R_m + R_L) \cdot R_L - (R_m + R_L)^2 \cdot 2V_m] = 0$$

$$= (R_m + R_L) \cdot 2V_m$$

$$\Rightarrow R_m = R_L$$

Proof the Maximum power transfer theorem
for AC circuit.

Super Node Analysis (In b/w two nodes
is a voltage source)



At Node X,

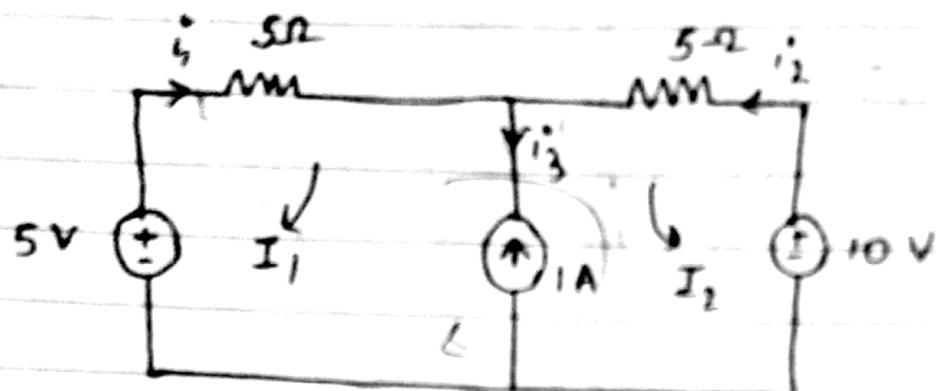
$$\frac{V_A - 5}{5} + \frac{V_A}{5} + \frac{V_B - 10}{5} + \frac{V_B}{5} = 0 \quad \text{--- (1)}$$

$$V_A - V_B = 2 \quad \text{--- (II)}$$

$$V_A =$$

$$V_B =$$

Super Mesh. (b/w two mesh a current source)



Considered as 1 mesh

$$5I_2 + 5 = 5I_1 + 10 \quad \text{--- (1)}$$

Voltage rises = Voltage down drops

$$I_1 + I_2 = -1 \quad \text{--- (2)}$$

$$I_1 = -1 - I_2$$

$$5I_2 + 5 = 5(-1 - I_2) + 10$$

$$\Rightarrow 5I_2 + 5 = -5 - 5I_2 + 10$$

$$\Rightarrow 10I_2 = 0$$

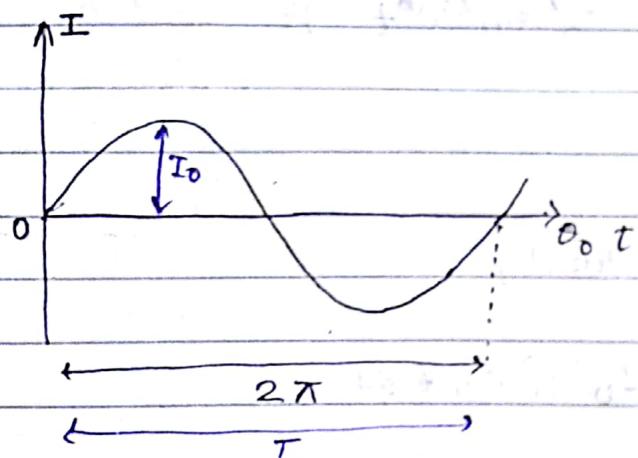
$$\Rightarrow I_2 = 0$$

$$I_1 = -1$$

RMS : value of AC = value of DC ~~not~~ flowing in a circuit producing same amount of ~~use~~ heat as of AC circuit

Avg value.

value of DC charge flowing in a circuit same amount of charge as of AC circuit.



Symmetrical electrical signal ;

In a symmetrical electrical signal : average value half of over a half period.

In a unsymmetrical electrical signal : average value over a complete period

$$I_{av} = \frac{1}{T/2} \int_0^{T/2} i dt = \frac{1}{\pi} \int_0^{\pi} i d\theta$$

$$i = I_0 \sin \omega t$$

$$I_{\text{avg}} = \frac{1}{\pi} \int_0^{\pi} I_0 \sin \omega t \, dt$$

$$= \frac{I_0}{\pi} \left[-\cos \omega t \right]_0^{\pi}$$

$$= \frac{I_0}{\pi} - [-1 - 1]$$

$$I_{\text{avg}} = \left(-\frac{2I_0}{\pi} \right) = \frac{2I_0}{\pi}$$

$$I_{\text{rms}}^2 = \frac{1}{2\pi} \int_0^{2\pi} i^2 d\theta \quad \text{mean} = 2\pi$$

$$= \frac{1}{2\pi} \int_0^{2\pi} I_0^2 \sin^2 \omega t \, dt$$

~~cancel~~

For half period,

$$= \frac{1}{\pi} \int_0^{\pi} I_0^2 \sin^2 \omega t \, dt$$

$$= \frac{I_0^2}{\pi} \left[\frac{\sin^2 \omega t}{\omega} \right]_0^{\pi}$$

$$= \frac{I_0^2}{2\pi} \left[\frac{1 - \cos 2\omega t}{2} \right]_0^{\pi}$$

$$= \frac{I_0^2}{4\pi} \left[0 - \frac{\sin 2\omega \pi}{2} \right]$$

$$= \frac{I_0^2}{\sqrt{2}}$$

Form factor : Ratio of rms value by Avg

$$= \frac{I_{rms}}{I_{avg}}$$

$$= \frac{I_0}{\sqrt{2}} \times \frac{\pi}{2I_0}$$

$$= \frac{\pi}{2\sqrt{2}} = 1.11$$

Peak factor: Ratio of peak value by rms value

$$= \frac{\text{Ipeak value}}{\text{Irms}}$$

$$= I_0 \times \frac{\sqrt{2}}{I_0}$$

$$= \sqrt{2} = 1.414$$



FOH full-value period.

$$I_{avg} = \frac{1}{T} \int_0^T i dt$$

$$= \frac{1}{T} \left[\int_0^{T/2} i dt + \int_{T/2}^T i dt \right]$$

$$= \frac{1}{T} \int_0^{T/2} i dt$$

$$= \frac{1}{2\pi} \int_0^\pi i dt$$

$$= \frac{I_0}{\pi} A$$

$$T_{avg} = \pi$$

$$T = 2\pi$$

$$I_{rms}^2 = \frac{1}{2\pi} \int_0^\pi i^2 dt = \frac{1}{2\pi} \int_0^\pi i'^2 dt + \frac{1}{2\pi} \int_\pi^{2\pi} i''^2 dt$$

$$= \frac{1}{2\pi} \int_0^\pi i'^2 dt = \frac{I_0}{2} A$$

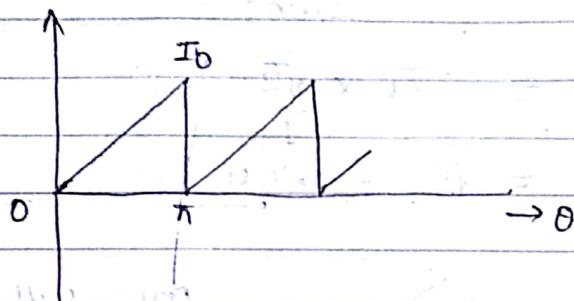
$$\text{Form factor} = \frac{I_0}{2} \times \frac{\pi}{I_0}$$

$$= \frac{\pi}{2} = 1.57$$

$$\text{Peak factor} = I_0 \times \frac{\pi}{I_0}$$

$$= 2.$$

1. Find form factor and peak factor of :



$$y = mx + c$$

$$y = \begin{cases} A_1 - B_2 \frac{x}{\pi}, & x_1 < x \\ A_2 - B_1 \frac{x}{\pi}, & x > x_2 \end{cases}$$

$$I_{av} = \frac{1}{\pi/2} \int_0^{\pi/2} i dt$$

$$= \frac{1}{\pi} \int_0^{\pi} i d\theta$$

$$i = \frac{I_0}{\pi} \cdot \theta$$

$$= \frac{1}{\pi} \int_0^{\pi} \frac{I_0}{\pi} \theta dt$$

$$= \frac{I_0}{\pi^2} \left[\frac{t^2}{2} \right]_0^\pi$$

$$= \frac{I_0}{2\pi^2} [\pi^2]$$

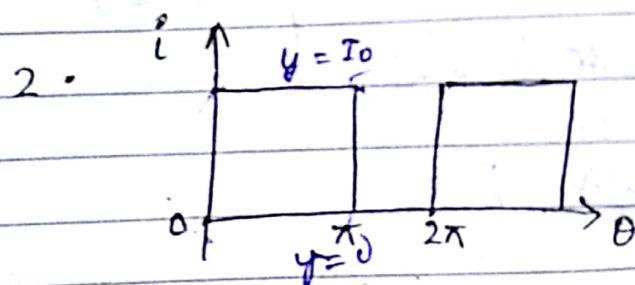
$$= \frac{I_0}{2}$$

$$\begin{aligned}
 I_{rms}^2 &= \frac{1}{T} \int_0^{2\pi} i^2 d\theta \\
 &= \frac{1}{2\pi} \int_0^{2\pi} \frac{I_0^2}{\pi^2} t^2 dt = \text{[redacted]} \\
 &= \frac{I_0^2}{\pi^3} \left[\frac{t^3}{3} \right]_0^{2\pi} \\
 &= \frac{I_0^2}{3\pi^3} \left[t^3 \right]_0^{2\pi} \\
 &= \frac{I_0^2}{3\pi^3} [8\pi^3] = \frac{8I_0^2}{3} = \frac{8}{3} I_0 \\
 &= \frac{I_0^2}{3} \text{ rms} = \frac{I_0}{\sqrt{3}} = 0.63 I_0
 \end{aligned}$$

Peak factor

$$\begin{aligned}
 \text{Form factor} &= \frac{I_0 \times 2}{\sqrt{3} I_0} \\
 &= 1.15
 \end{aligned}$$

$$\begin{aligned}
 \text{Peak factor} &= I_0 \times \frac{\sqrt{3}}{I_0} = \sqrt{3} \\
 &= 1.73
 \end{aligned}$$



$$i = \frac{I_0 - I_0}{\pi - 0} \theta$$

$$i = \frac{I_0}{2\pi} t = 0$$

$$I_{av} = \frac{1}{T} \int_0^T i dt$$

$$= \frac{1}{T} \int_0^{T/2} i dt + \frac{1}{T} \int_{T/2}^T 0 dt$$

$$= \frac{1}{T} \int_0^{T/2} i dt + 0$$

$$I_{av} = \frac{1}{2\pi} \int_0^{\pi} I_0 dt = \frac{I_0 \times \pi}{2\pi} = \frac{I_0}{2}$$

$$I_{rms}^2 = \frac{1}{\pi} \int_0^{2\pi} i^2 d\theta$$

$$i = \frac{I_0}{2} \cos \theta$$

$$= \frac{1}{\pi} \int_0^{2\pi} \frac{I_0^2}{4} \cos^2 \theta d\theta$$

~~$\frac{I_0^2}{4}$~~ ~~$\frac{1}{4}$~~ ~~$\frac{1}{2}$~~ ~~$\frac{1}{2}$~~

$$= \frac{I_0^2}{4} \left[\frac{\theta}{2} + \frac{1}{2} \sin 2\theta \right]_0^{2\pi}$$

~~$\frac{1}{2}\pi^3$~~ ~~$\frac{1}{3}$~~ ~~$\frac{1}{2}$~~ ~~$\frac{1}{2}$~~

$$= \frac{I_0^2}{4} \times \pi$$

$$I_{rms} = I_0$$

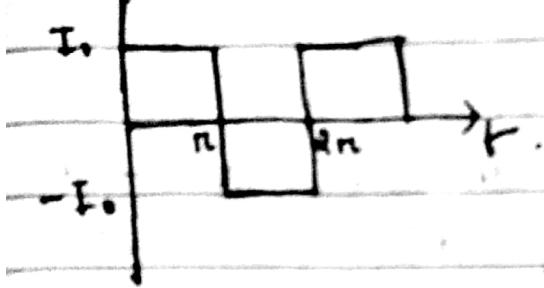
Form factor

$$\text{Power factor} = \frac{I_0 \times 2}{I_0}$$

$$= 2$$

$$\text{Peak factor} = \frac{I_0 \times 2}{I_0}$$

$$= 2$$



$$i = I_0, -I_0$$

$$I_{\text{avg}}^2 = \frac{1}{T} \int_0^{T/2} i^2 dt$$

$$i = \begin{cases} I_0 \text{ for } 0 < t < \pi \\ -I_0 \text{ for } \pi < t < 2\pi \end{cases}$$

$$\begin{aligned} &= \frac{1}{2\pi} \left[\int_0^{\pi} I_0^2 dt + \int_{\pi}^{2\pi} (-I_0)^2 dt \right] \\ &= \frac{1}{2\pi} \left[I_0^2 (t) \Big|_0^\pi + I_0^2 (t) \Big|_\pi^{2\pi} \right] \end{aligned}$$

$$= \frac{I_0^2}{2\pi} (\pi + \pi) = \frac{I_0^2}{2\pi} \times 2\pi = I_0^2$$

$$I_{\text{avg}} = I_0$$

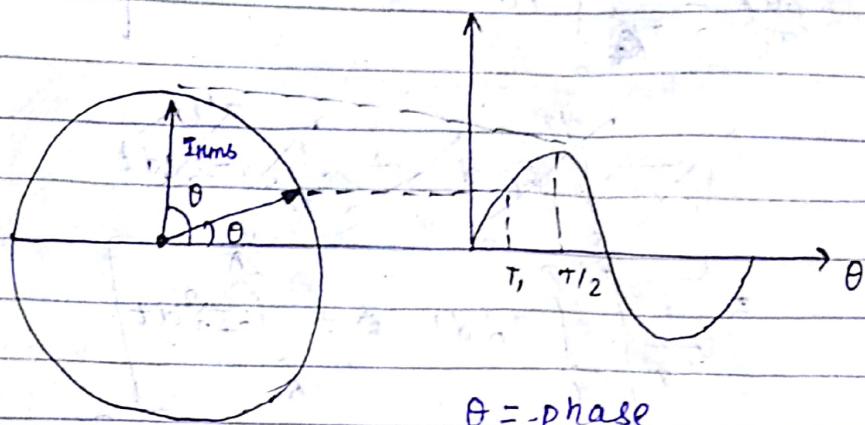
$$\begin{aligned} I_{\text{avg}} &= \frac{1}{T/2} \int_0^{T/2} i dt = \frac{1}{\pi} \int_0^{\pi} I_0 dt \\ &= \frac{I_0 \times \pi}{\pi} = I_0 \end{aligned}$$

Form factor = 1

$$\text{Peak factor} = \frac{I_0}{I_0} = 1$$

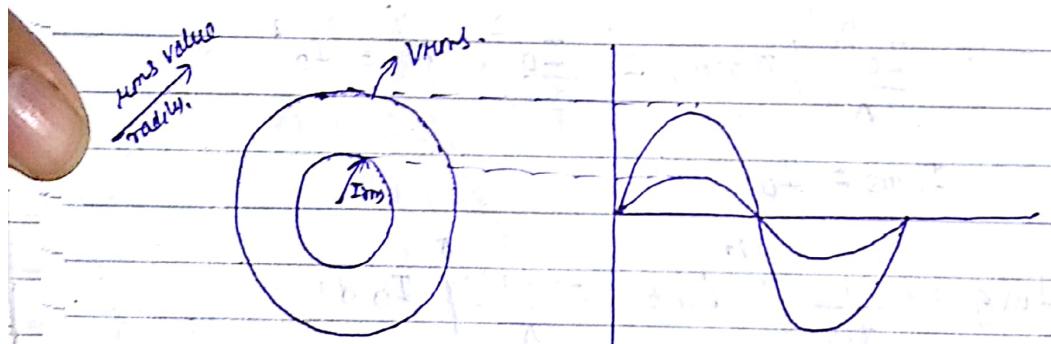
Phasor quantity

Only sinusoidal quantity can be represented as a phasor diagram and the magnitude of that phasor is rms. value of that quantity.



$\theta = \text{-phase}$

$$V = IR$$

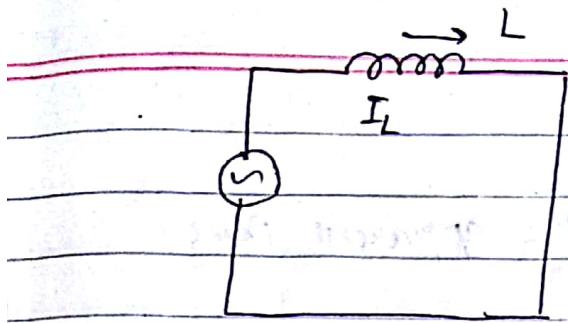


In a resistive ckt, current and voltage are in same phase.

$$E = E_0 \sin \omega t$$

$$I = \frac{E_0}{R} \sin \omega t$$

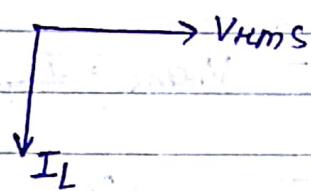
$$= I_0 \sin \omega t$$



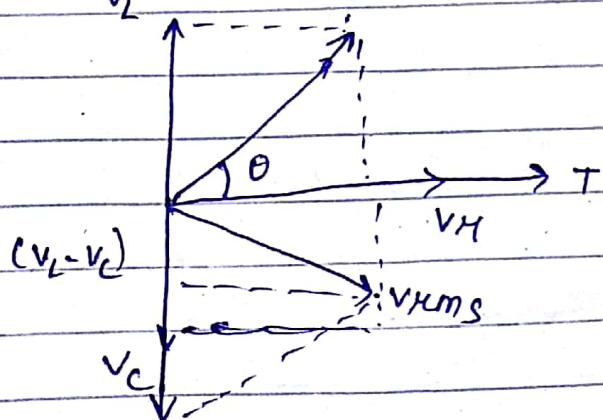
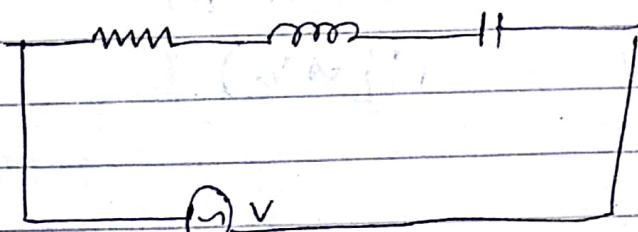
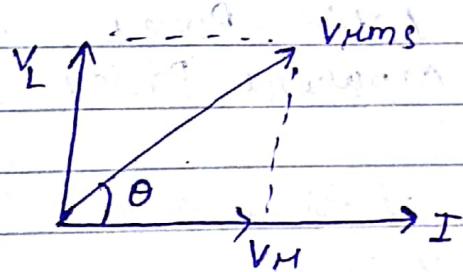
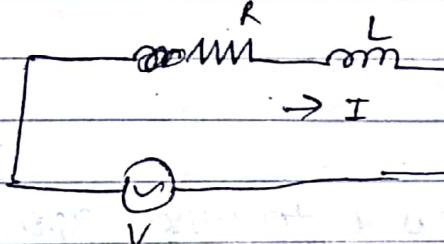
$$V = V_0 \sin \omega t$$

$$I_L = \frac{V}{jX_L} = \frac{V_0 \sin \omega t}{jX_L}$$

$$= \frac{V_0}{X_L} \sin(\omega t - 90^\circ)$$



V is reference.



Power factor angle

$$P = V_{\text{rms}} I_{\text{rms}}$$

$P = \text{apparent Power}$

$$\text{Unit of } P = \text{VA} = \text{Volt-Ampere.}$$

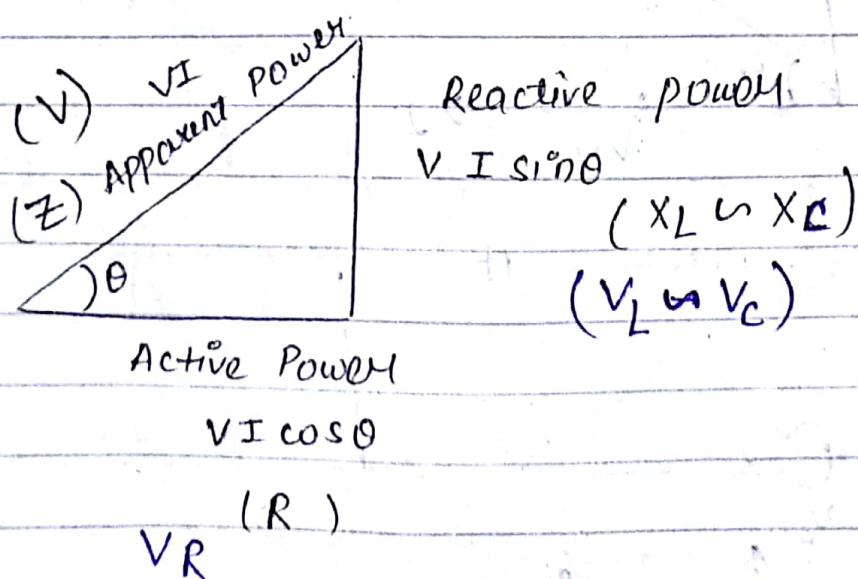
$$\text{Active / Available Power} = V_{\text{rms}} \cdot I_{\text{rms}} \cos \theta$$

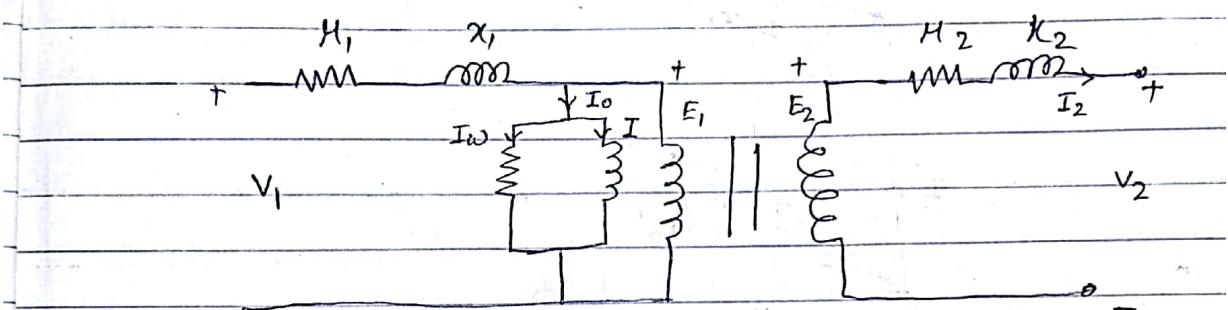
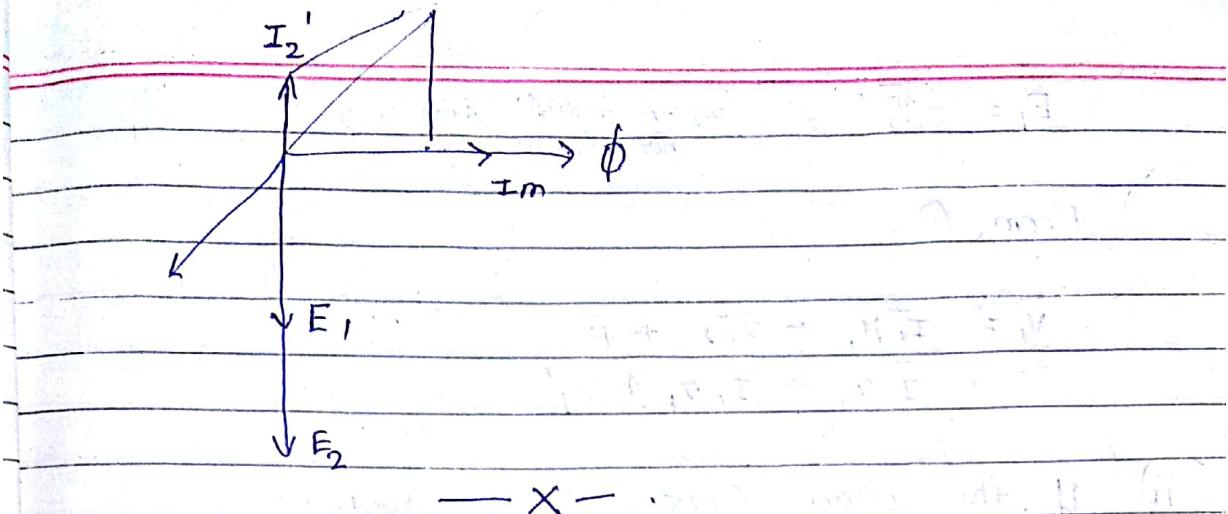
$$\text{Reactive Power} = V_{\text{rms}} \cdot I_{\text{rms}} \sin \theta.$$

Power factor

The ratio of active power to the apparent power.

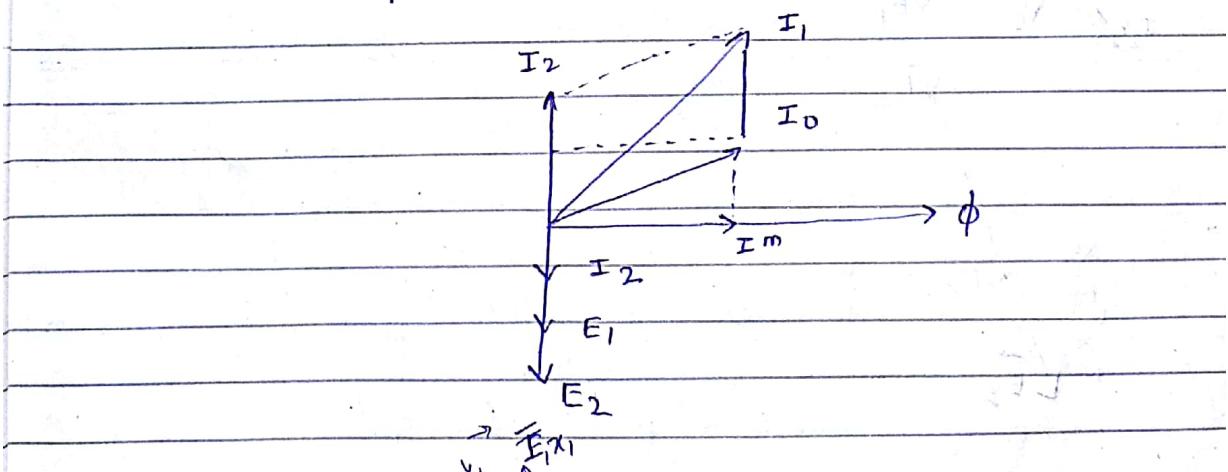
$$\text{PF} = \frac{\text{Active Power}}{\text{apparent Power}}$$



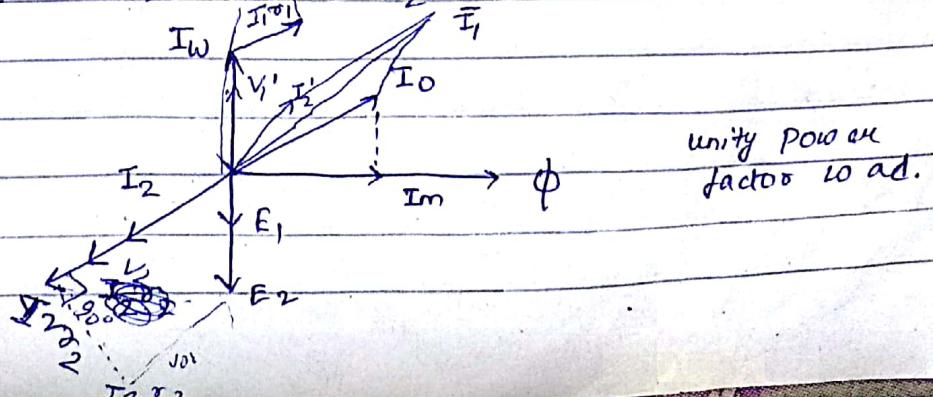


Applying KVL at Primary, phasor eqⁿ

$$V_1 = \bar{I}_1 H_1 + \bar{I}_1 X_1 + \bar{E}_1 \quad \text{--- (1)}$$



$$E_2 = \bar{I}_2 X_2 + \bar{I}_2 x_2 + \bar{V}_2$$

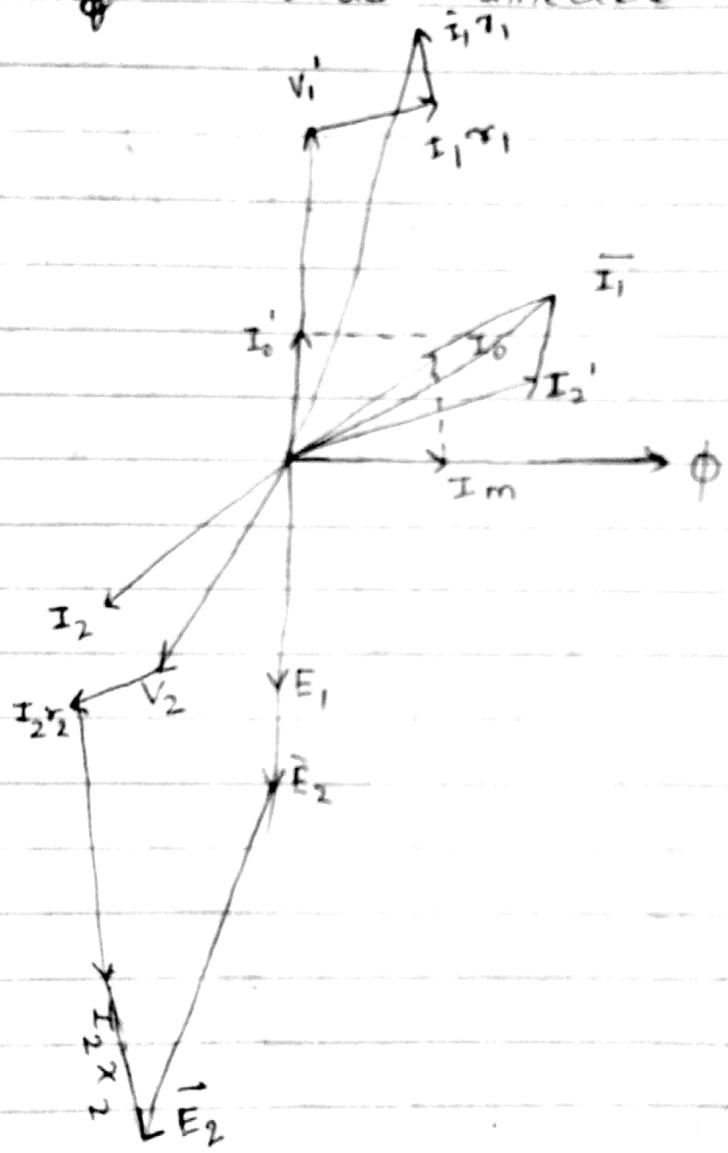


$$E_1 = -\bar{V}_1' \quad [(-) \text{ to indicate direction, not magnitude.}]$$

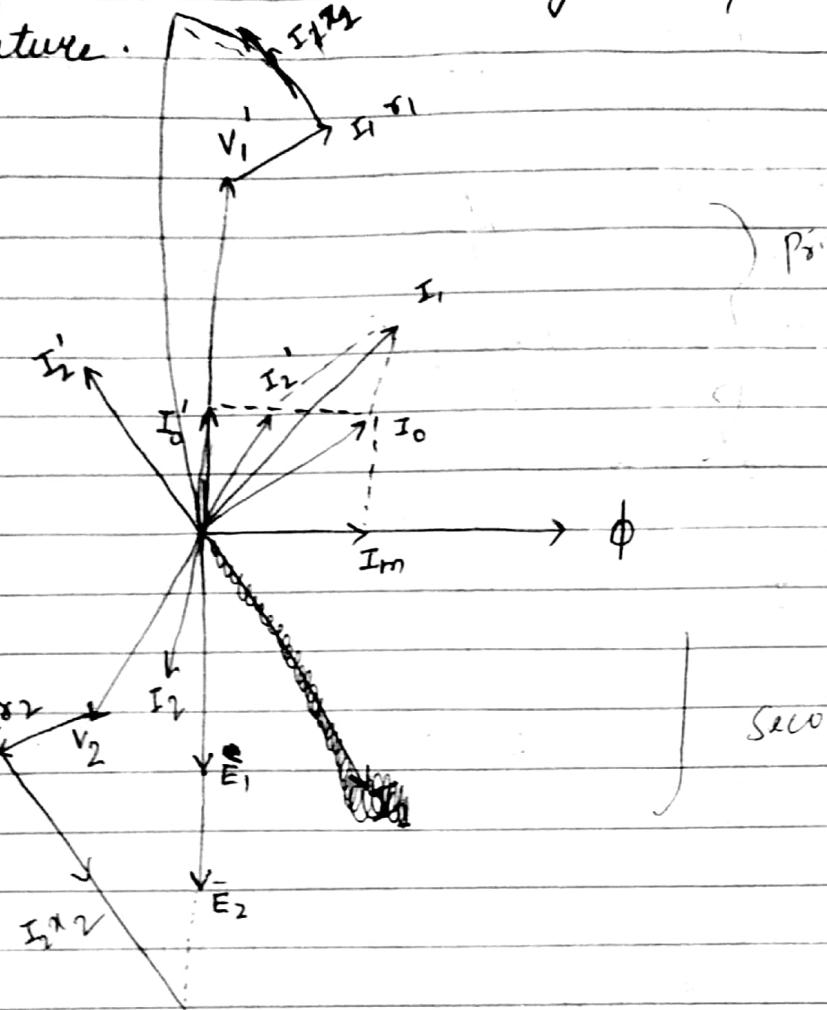
From ①,

$$\begin{aligned}\bar{V}_1 &= \bar{I}_1 R_1 + \bar{I}_1 x_1 + \bar{E}_1 \\ &= \bar{I}_1 r_1 + \bar{I}_1 x_1 + v_1'\end{aligned}$$

ii) If the Load connected is lagging

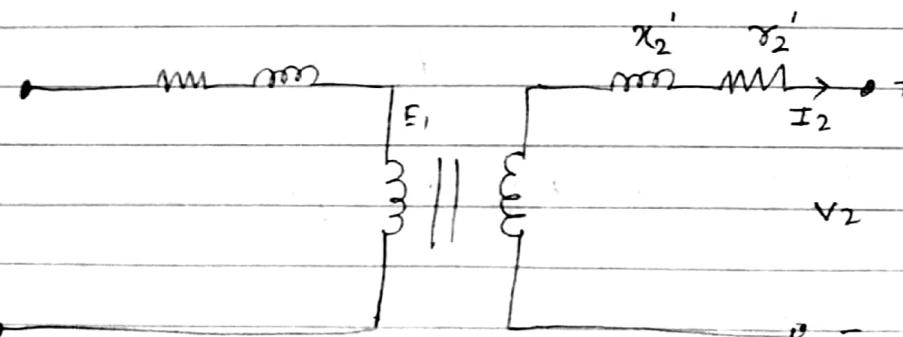


iii) If the Load is leading or capacitive in nature.



i) Referred to primary

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} \Rightarrow E_2 = \frac{N_2}{N_1} E_1$$



Value of E_2 is referred to primary = E_2'

$$\frac{E_2'}{E_2} = \frac{N_1}{N_2}$$

$$N_2 \rightarrow \sigma_2 \quad 2 \\ N_1 \rightarrow \frac{\sigma_2 / N_1}{\sigma_2 / N_2}$$

$$E_2' = \frac{N_1}{N_2} E_2$$

$$N_2 \rightarrow \gamma_2 \\ \text{For 1 turn} \rightarrow \frac{\gamma_2}{N_2^2}$$

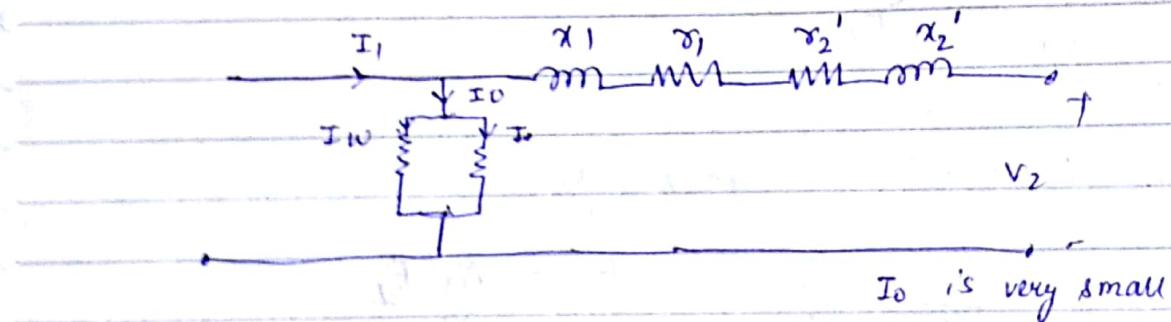
$$N_1' \rightarrow \gamma_2 \times \left(\frac{N_1}{N_2} \right)^2$$

$$\gamma_2' = \gamma_2 \left(\frac{N_1}{N_2} \right)^2$$

$$x_2' = x_2 \left(\frac{N_1}{N_2} \right)^2$$

$$I_2' = I_2 \left(\frac{N_2}{N_1} \right)$$

After Neffing, it is known as approximate equivalent circuit



$$\gamma_1 + \gamma_2' = R_{01}$$

$$\gamma_1 + \gamma_2' = x_{01}$$

R and γ referred to primary.

i) Referred to secondary

value of E_1' is referred to secondary = E_1'

$$\frac{E_1'}{E_1} = \frac{N_2}{N_1}$$

$$E'_1 = \left(\frac{N_2}{N_1}\right) E_1 \quad x'_1 = \left(\frac{N_2}{N_1}\right)^2 x_1$$

$$\gamma'_1 = \left(\frac{N_2}{N_1}\right)^2 \gamma_1 \quad I'_1 = \frac{N_1}{N_2} I_1$$

$$I_N' = \frac{N_1}{N_2} I_N \quad I_m' = \frac{N_1}{N_2} I_m$$

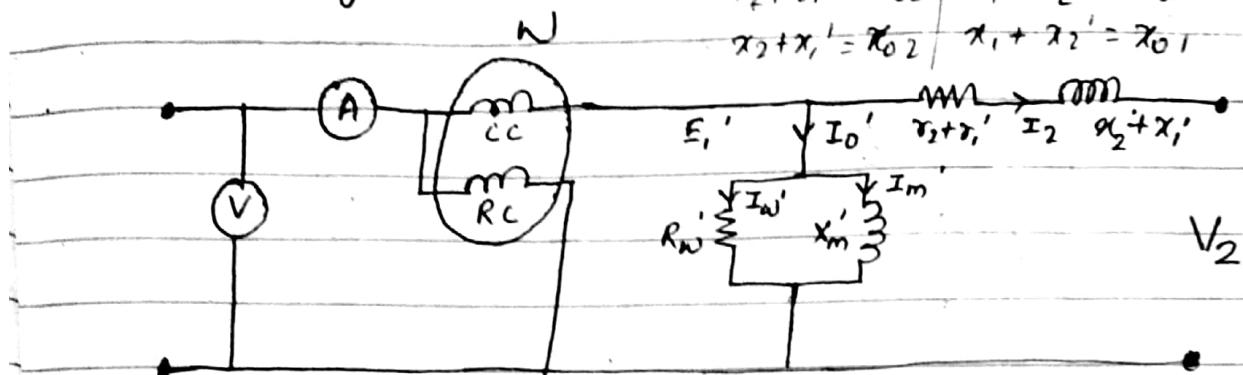
$$R_N' = \left(\frac{N_2}{N_1}\right)^2 R_N \quad X_m' = \left(\frac{N_2}{N_1}\right)^2 X_m$$

$$\gamma'_1 + \gamma_2 = R_{02}$$

$$x'_1 + x_2 = X_{02}$$

R_{02} and X_{02} are referred to secondary

As the name suggest, we will open the circuit
Apply rated voltage. Open-circuit test perform on
low voltage side



$$1 \text{ KVA}, 100 \text{ V} / 400 \text{ V} \quad I_0 \quad V_{\text{NO}} \cancel{\text{LOAD}}$$

OC test : 100 V, 1A, 50 W (Readings)

$$I_m = I_0 \sin \phi$$

$$I_w = I_0 \cos \phi$$

$$V I_0 \cos \phi = 50$$

$$\cos \phi = \frac{50}{1 \times 100}$$

$$= \frac{1}{2}$$

$$\cos \phi = 0.5$$

$$\sin \phi = 0.867$$

$$I_m = 0.867 \text{ A}$$

$$I_w = 0.5$$

$$X_m = \frac{100}{0.867}$$

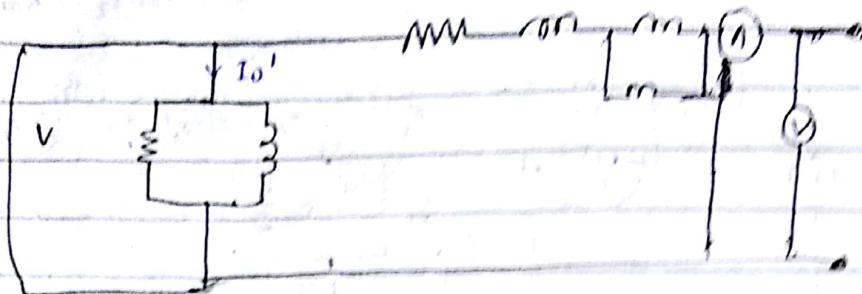
$$R_w = \frac{100}{0.5} = 200 \Omega$$

$$= 115.34 \Omega$$

why low voltage NOT high voltage.

- 1) Very high voltage needed i.e 400 V.
- 2) Dangerous

SC perform on high voltage side. LV side is short circuited.



1 kVA, 1000 V / 400 V

DC test : 100V, 1A, 50W

SC test : 40V, 2.3A, 80W \rightarrow Ohmic or Variable loss
or
D.P. loss

$$I^2 R_{02} = 80$$

$$R_{02} = \frac{80}{(2.3)^2} \therefore R_{02} = 12.8 \Omega$$

$$Z_{02} = \frac{40}{2.3} = 17.4 \Omega$$

$$X_{02} = \sqrt{Z_{02}^2 - R_{02}^2}$$

$$= \sqrt{17.4^2 - (12.8)^2} = \sqrt{296 - 163.84} \\ = 9.6 \Omega$$

$$R_{01} = 12.8 \times \left(\frac{100}{400}\right)^2$$

$$= \frac{12.8}{16}$$

$$X_{01} = 9.6 \times \left(\frac{100}{400}\right)^2 \Omega$$

$$X_m' = X_m \times \left(\frac{100}{400}\right)^2$$

$$R_w' = R_w \times \left(\frac{100}{400}\right)^2$$

Primary

OC test \rightarrow Pow voltage side

Convert high voltage side param to low voltage side.

Sec

SC test \rightarrow high voltage side

convert low voltage side param to high voltage side.

Find parameters for primary and secondary re

5 kVA, 500 V / 200 V $\frac{I_0}{V I_0 \cos \phi}$
OC test : $200 V, 1.5 A, 80 W \xrightarrow{\text{no load losses}}$

SC test : 50 V, 10 A, 200 W.

Primary :

$$I_m = I_0 \sin \phi$$

$$I_w = I_0 \cos \phi$$

$$\phi = 60^\circ$$

$$V I_0 \cos \phi = 80$$

$$V = 200$$

$$\cos \phi = \frac{80}{200 \times 25}$$

$$I_0 = \frac{5000}{200} = 25$$

$$= \frac{8}{25 \times 20} = \frac{8}{500} = 0.016$$

$$co = \sqrt{-}$$

$$\cos \phi = 0.016$$

$$\cos \phi = \frac{80}{200 \times 1.5} = 0.26$$

$$\sin \phi = 0.965$$

$$I_m = 1.5 \times 0.965 = 1.44$$

$$I_w = 1.5 \times 0.26 = 0.49$$

$$X_m = \frac{200}{1.44} = 138.88 \quad R_w = \frac{200}{0.49} = 408.16 \approx 500$$

Secondary :

$$I^2 R_{02} = 200$$

$$R_{02} = \frac{200}{10 \times 10} = 2$$

$$Z_{02} = \frac{50}{10} = 5$$

$$X_{02} = \sqrt{25 - 4} = \sqrt{21} = 4.58$$

$$R_{02}' = R_{02} \times \left(\frac{200}{500}\right)^2$$

$$= 2 \times \frac{2 \times 2}{5 \times 5} = \frac{8}{25} = 0.32$$

$$X_{02}' = X_{02} \times \left(\frac{200}{500}\right)^2$$

$$X_m' = X_m \times \left(\frac{500}{200}\right)^2$$

$$R_w' = R_w \times \left(\frac{500}{200}\right)^2$$

2. 10 kVA, 1000 V / 500 V

OC test \rightarrow 500V, 2A, 100W

SC test \rightarrow 50V, 10A, 300W

parameters

Find the eq. ckt i) OC

ii) SC.

Low voltage :

$$I_0 = 2 \text{ A}$$

$$V I_0 \cos \phi = 100$$

$$\cos \phi = \frac{100}{500 \times 2}$$

$$= \frac{1}{10} = 0.1$$

$$\sin \phi = \sqrt{1 - (0.1)^2} = 0.99$$

$$I_m = I_0 \sin \phi$$

$$= 2 \times 0.99 = 1.98$$

$$I_w = I_0 \cos \phi$$

$$= 2 \times 0.1 = 0.2$$

$$X_m = \frac{500}{1.98} = 252.52$$

$$R_w = \frac{500}{0.2} = 2500$$

High Voltage :

~~$$R_{01} I^2 R_{01} = 300$$~~

$$R_{01} = \frac{300}{10 \times 10} = 3 \Omega$$

$$Z_{01} = \frac{50}{10} = 5 \Omega$$

$$X_{01} = \sqrt{25 - 9} = \sqrt{16} = 4 \Omega$$

$$X_{02} = X_{01} \times \frac{(500)^2}{(1000)^2}$$

$$= 4 \times \frac{1}{4} = 1$$

~~$$R_{02} = R_{01} \times \frac{(500)^2}{(1000)^2} = 3 \times \frac{1}{4} = 0.75$$~~

$$X_m' = 252.52 \times \left(\frac{25}{4}\right) = 9864.0 \quad R_w' = 2500 \times \left(\frac{25}{4}\right)$$

Voltage Regulation.

Fractional change of E_2 from No Load to Full Load.

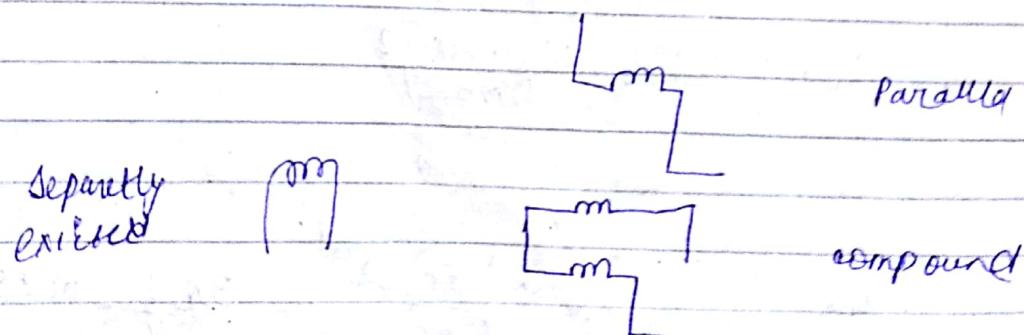
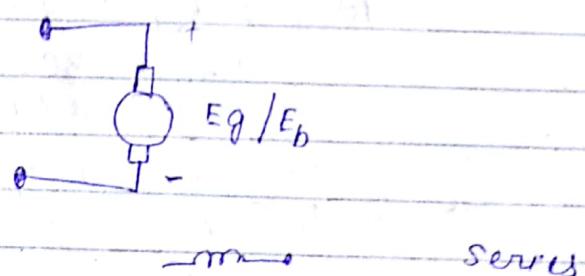
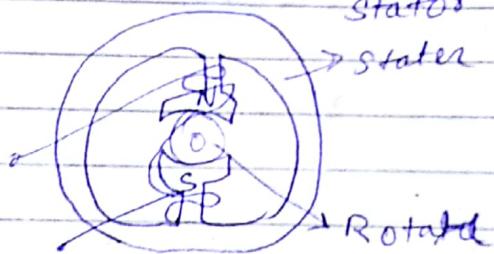
$$V_{NL} = \text{No load Voltage} = E_2$$

$$\text{Full load voltage} = V_2$$

$$\text{Voltage regulation} = \frac{E_2 - V_2}{E_2}$$

DC Machine

- Rotating Machine
- Stator : stationary part
- Rotor : Rotating part
- Electrical part : Field and Armature.
- Field placed on Armature and Rotor in Stator



DC Motor

- Work both as generators and motor.
- Brush and commutator assembly : Rectification
convert AC to DC.
- Field windings ~~are~~ are placed in the stator
and armature windings are placed in the rotor.
- Flux due to each pole is ϕ

Amount of flux cut / revolution ~~rotates~~
~~rotates~~ is $P\phi$ where P no. of poles.

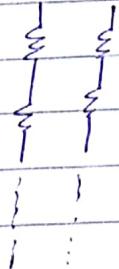
speed of machine N rpm

No. of conductors / parallel path $\frac{Z}{A}$

$$\text{emf. induced} = \frac{d\phi}{dt} = \text{In one rotation flux change}$$

$$= P\phi$$

$$\begin{aligned} N &\rightarrow 60 \text{ s} \\ 1 &\rightarrow \frac{60}{N} \\ &= \frac{P\phi N}{60} \end{aligned}$$



$$\text{emf. induced in one conductor} = \frac{P\phi N}{60}$$

No. of conductors per parallel path

$$= \frac{P\phi N}{60} \times \frac{Z}{A}$$

In case of generator \rightarrow Emf induced = $\frac{P\phi N}{60} \times \frac{Z}{A}$
(Generator emf)

(Back emf) \rightarrow In case of motor.

(A)

Wave Connection = No. of parallel path = 2

Lap Connection = No. of parallel path = $P = \frac{\text{No. of poles}}{2}$

1. A wave connected 200 V DC machine has 400 conductors it has 4 poles and the useful flux/pole is 0.003 wb. It runs at the speed of 1000 rpm. Find the emf induced.

$$e = \frac{P\phi N}{60} \times \frac{Z}{A}$$

$$N = 1000 \text{ rpm}$$

$$\therefore P = 4 \quad Z = 400$$

$$A = 2$$

$$\phi = 0.03$$

$$e = \frac{2}{4 \times 0.03 \times \frac{100}{60} \times \frac{400}{2}}$$

$$= 400 \text{ V}$$

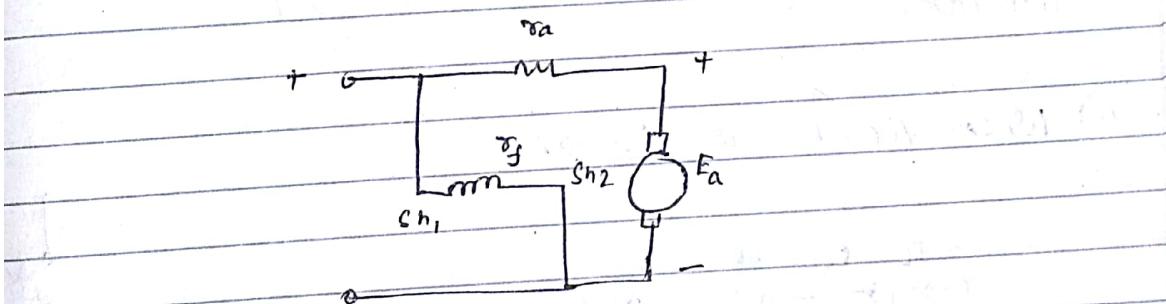
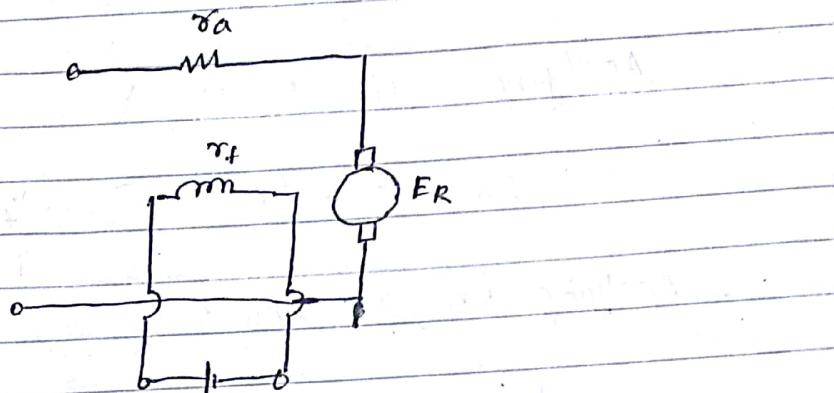
\therefore It is a Generator.

emf induced > Terminal voltage = Generator

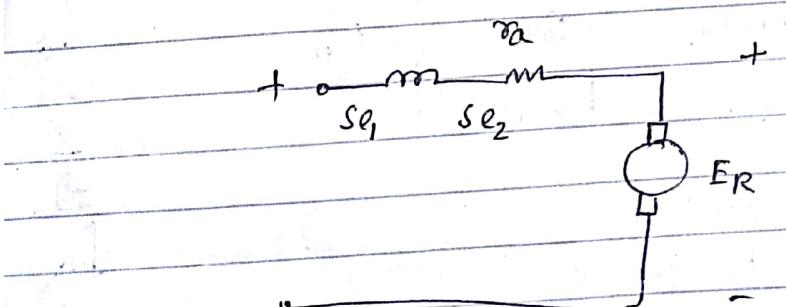
emf induced < Terminal voltage = Motor

Based on excitation DC machine is classified into 4 types:

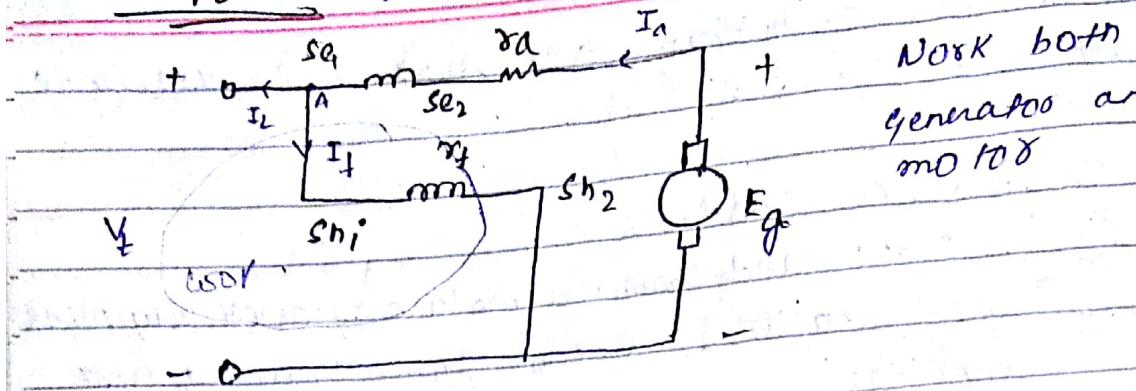
1) Separately excited
Field and armature power supplies are externally, excited from two different supplies.



Series excited: If armature field same supply and series



Compound : Both series and shunt same.



i) When works as Generator.

Applying KCL at pt. A.

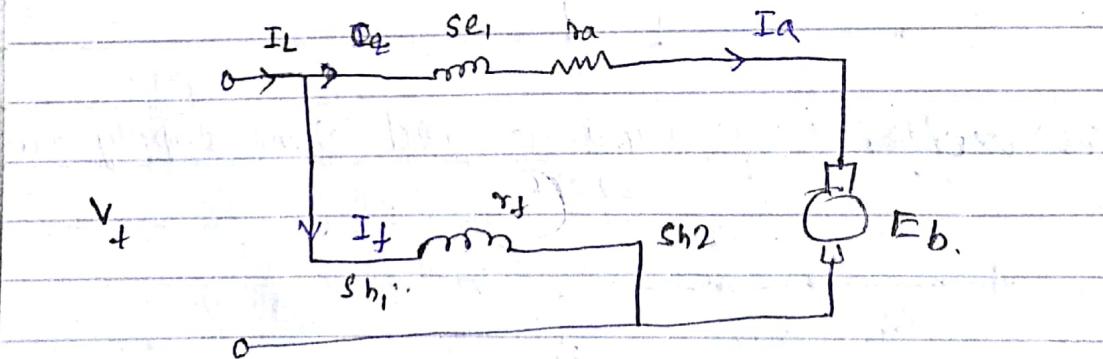
$$I_a = I_L + I_f \quad I_f = \frac{V_f}{r_f}$$

Applying KVL at loop 1

$$E_g = V_f + I_a r_a$$

As it is generator $E_g > V_f$ (terminal voltage)

ii) When works as Motor.



Only given for any DC machine

$$I_L = I_a + I_f \quad V_f = E_b + I_a r_a$$

~~Long time~~

$$E_g = \frac{P\phi N g}{60A}$$

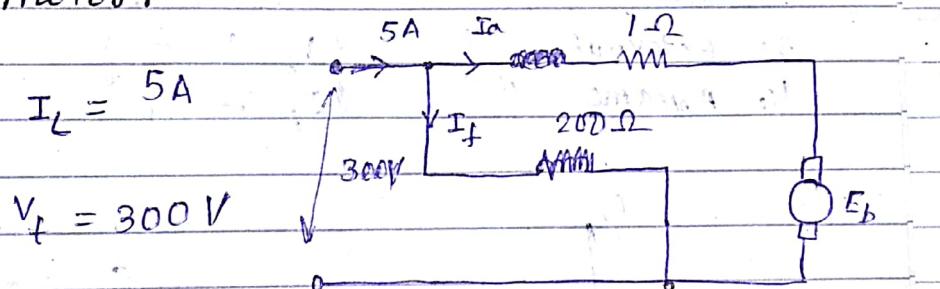
$$E_b = \frac{P\phi_m N_m Z}{60A}$$

$$\left| \frac{E_g}{E_b} = \frac{\phi g N_g}{\phi_m N_m} \right|$$

* For DC shunt machine ϕ is constant.

Shunt

2. A 300V DC machine takes a load current of 5A the field resistance is 200Ω and armature resistance is 1Ω . Find out the ratio of the speeds of the machine as the generator and a motor.



$$V_f = E_b + I_a R_a$$

$$300 = E_b + I_a$$

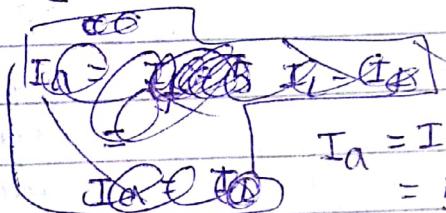
$$E_b = I_a R_a$$

$$300 = I_f \times 200$$

$$I_f = \frac{300}{200}$$

$$= \frac{3}{2}$$

$$I_L = I_a + I_f$$



$$I_a = I_L - I_f$$

$$= 5 - 1.5 = 3.5 V$$

$$E_b = 300 - 3.5$$

$$= 296.5$$

Armature Reaction : Reaction to the armature current

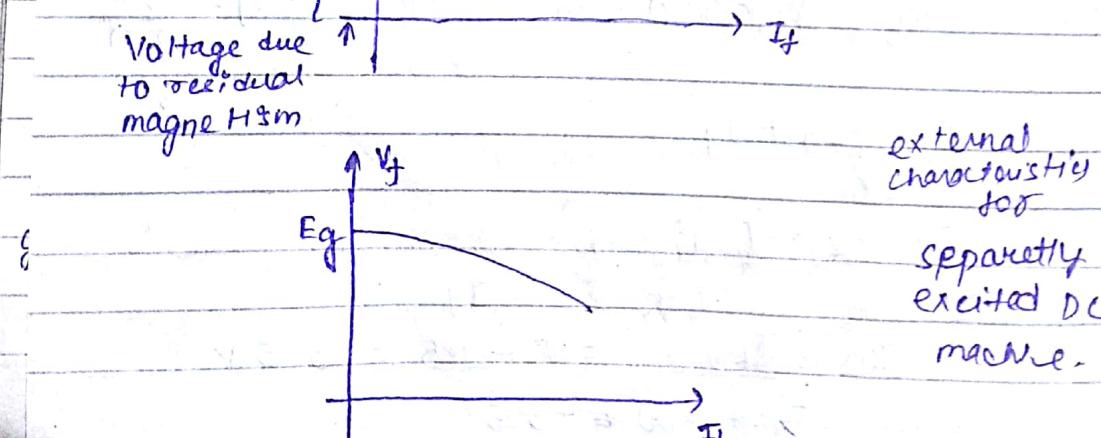
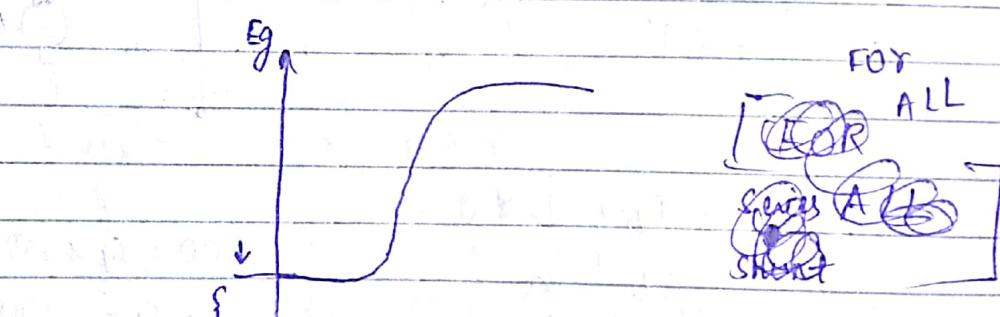
$$\begin{aligned} E_g &= 300 + 3.5 \times 1 \\ &= 300 + 3.5 \\ &= 303.5 \end{aligned}$$

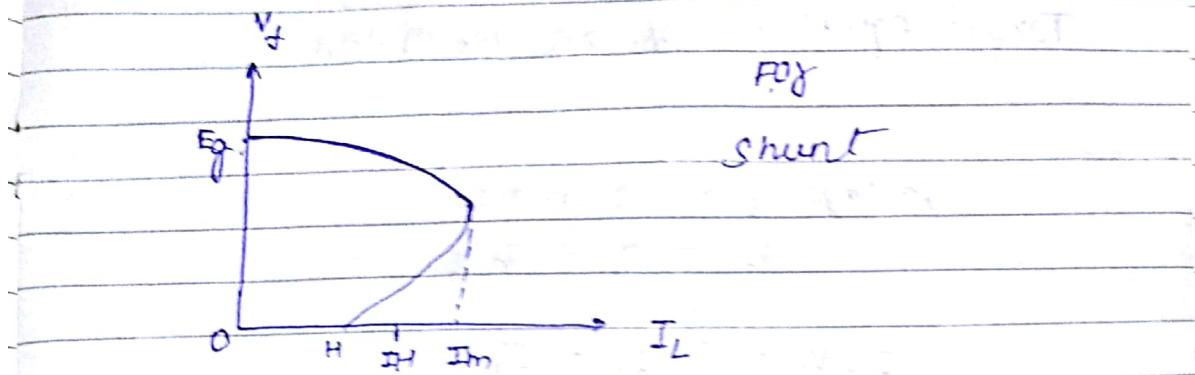
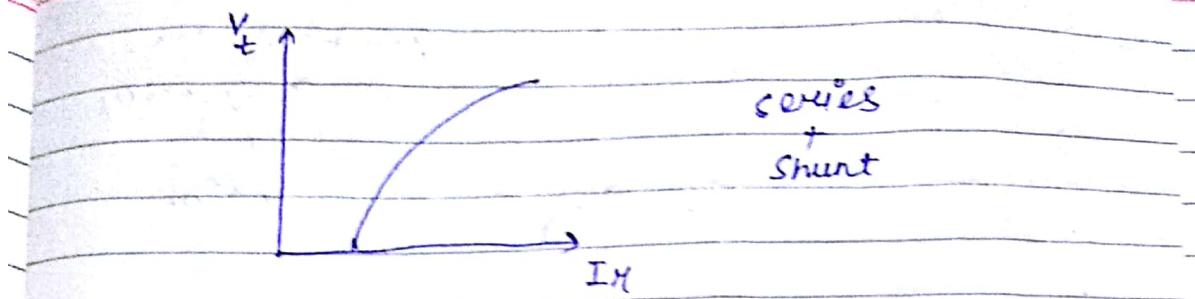
$$\begin{aligned} V_f &= I_f \gamma_f \\ &= \frac{3 \times 200}{2} \\ &= 300 \end{aligned}$$

$$\frac{E_g}{E_b} = \frac{N_g}{N_m} \Rightarrow \frac{N_g}{N_m} = \frac{303.5}{296.5} = 1.024.$$

Characteristics of DC generators.

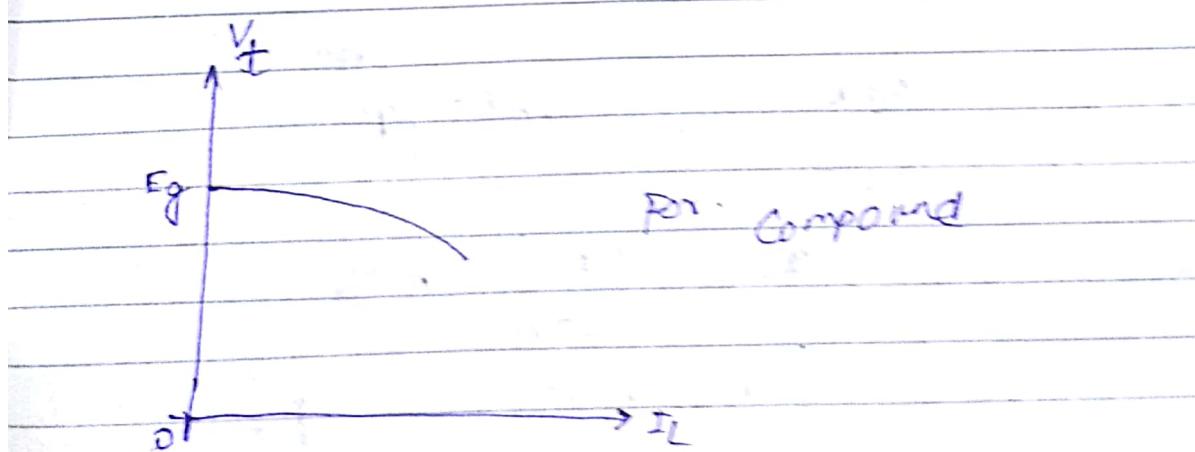
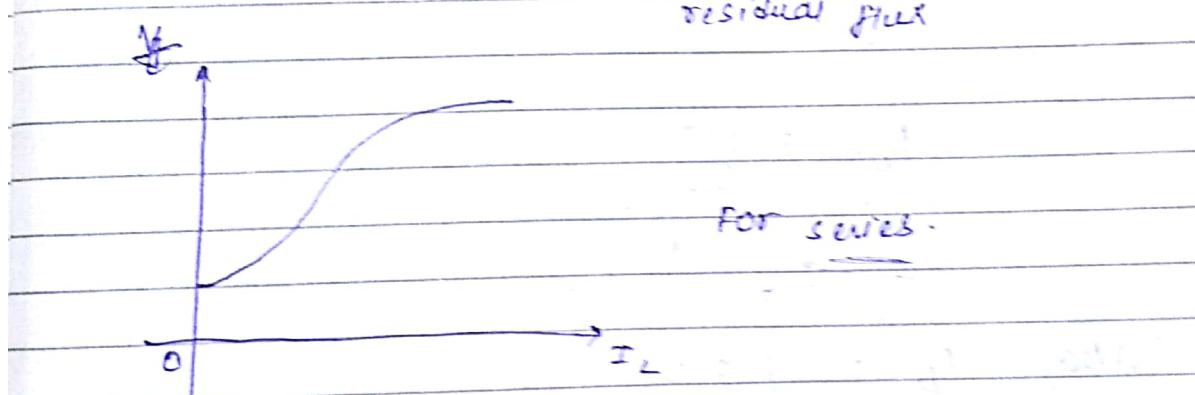
1. No Load characteristics (E_g vs I_f)
2. Load " (V_f vs I_f)
3. External " (V_f vs I_L)
4. Armature " (I_f vs I_a)



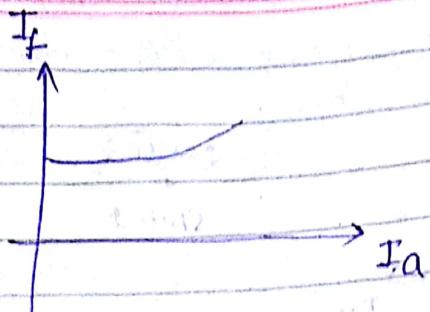


I_m = Max. current can flow through the DC source.

OH = current flow through the resistive part



$$E_g = \frac{N}{R} \Phi$$



Armature char-
for no separately
excited DC
generators.

Torque equation of DC motor

Voltage for DC motor.

$$V_f = E_b + I_a \delta_s$$

$$I_a V_f = E_b I_a + I_a^2 \delta_a$$

~~$E_b I_a$~~ $E_b I_a = \text{Output mechanical Power. (P_m)}$

$$P_m = \omega T$$

$$\begin{aligned} E_b I_a &= \omega T \\ &= 2\pi n T \\ &\downarrow \\ &\text{rps} \end{aligned}$$

$$\text{Also, } E_b = \frac{P \phi N Z}{60 A}$$

$$\frac{P \phi N Z}{60 A} I_a = \frac{2\pi N}{60} \times T$$

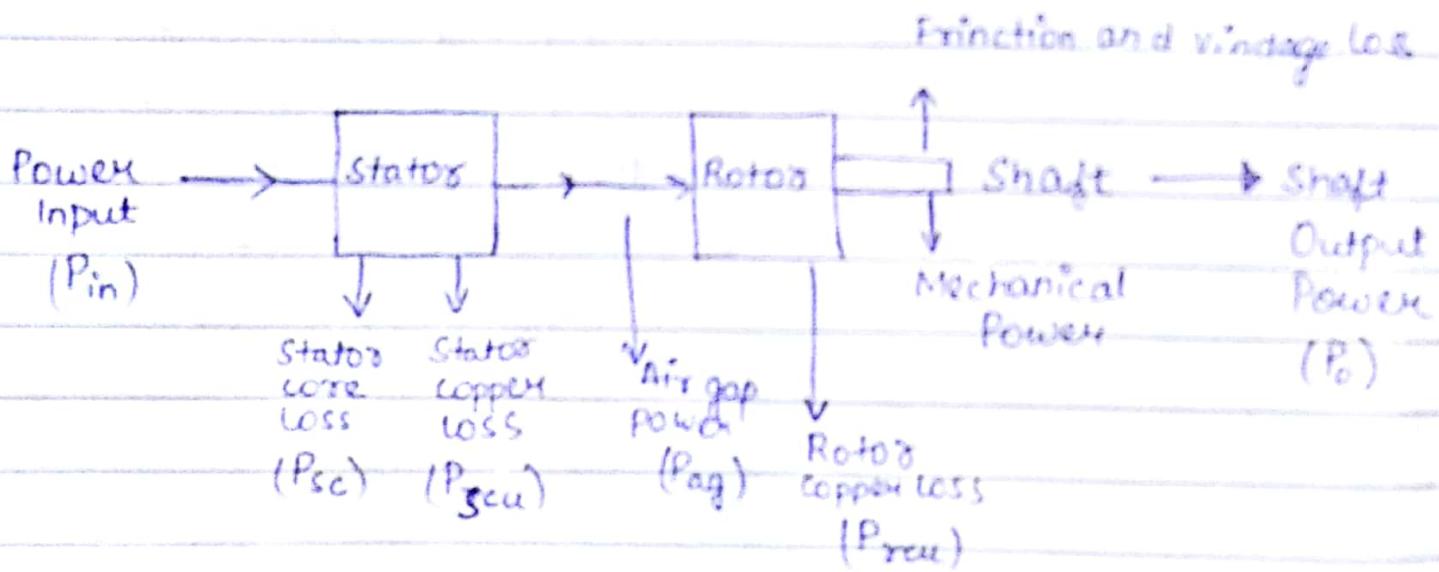
$$\therefore T = \left(\frac{P Z}{2\pi A} \right) \phi I_a$$

$T = R \phi I_a$

$R = \text{constant}$

$$T \propto \phi I_a$$

Power and losses



$$\text{Rotor Copper loss (Prcu)} = \text{Slip} \times \text{Pag}$$

$$\text{Mechanical Power (Pm)} = (1 - \text{Slip}) \times \text{Pag}$$

$$\begin{aligned}
 &= \text{Pag} - \text{Prcu} \\
 &= \text{Pag} - S \cdot \text{Pag} \\
 &= (1 - S) \cdot \text{Pag}
 \end{aligned}$$

S = Slip

2. A 3 phase 4 pole 3-phase induction motor is running at 1440 rpm. Rotor input power is 2 kW. Find the rotor copper loss and rotor output power.

$$N_S = \frac{120 \times F}{P}$$

$$= \frac{120 \times 50}{4}$$

$$= 1500 \text{ rpm}$$

$$N_D = 1440 \text{ rpm.}$$

$$\frac{1500 - 1440}{1500} = S.$$

$$S = \frac{60}{1500} = 4\%$$

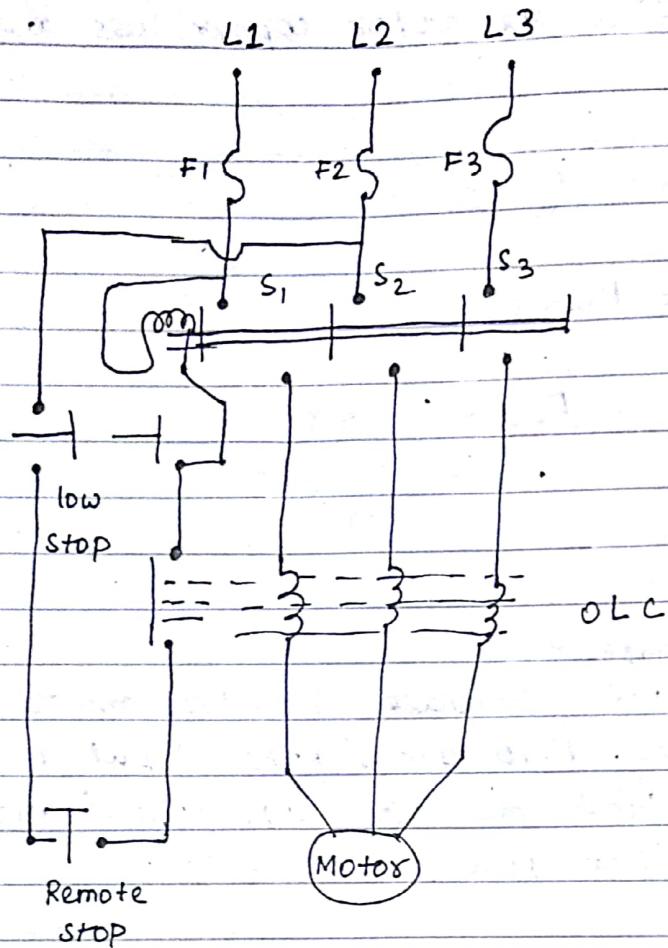
$$P_{ag} = 2 \text{ kW}$$

$$P_{cu} = 0.04 \times 2 \times 10^3 = 0.08 \times 10^3$$

$$= 80$$

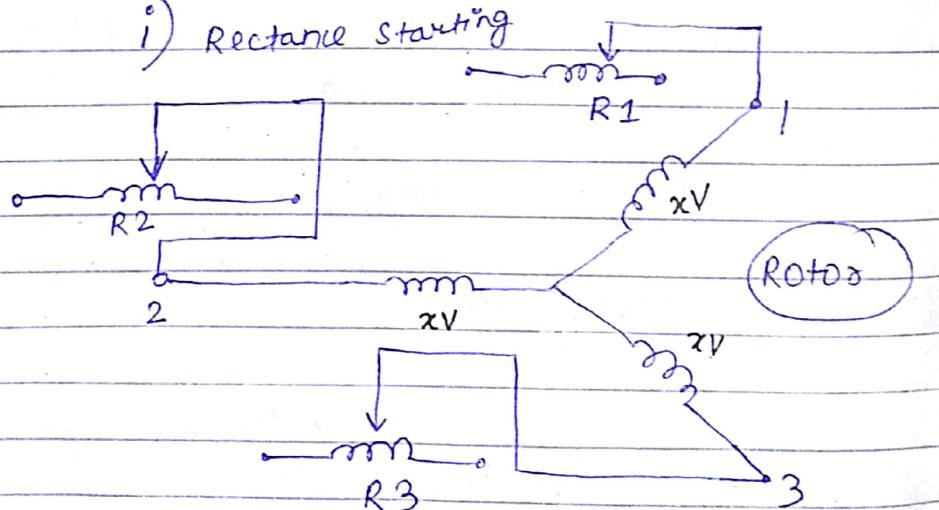
$$\text{Mechanical Power} = (1 - 0.04) \times 2 \times 10^3 = 1920.$$

Direct ON Line



The starting current is very high of DOL starter

i) Reactance starting



$$I_{st} = \chi I_{sc}$$

I_{sc} = initial current

I_{st} = starting current.

$$\gamma \propto B V^2$$

$$\gamma \propto (2V)^2$$

Torque is decreased x^2 times

$$T_{st} = T_{2st}^2 \frac{\gamma_2}{S} \quad S \uparrow p = 1$$

$$T_{st} = I_{2st}^2 \gamma_2$$

$$T_{fl} = I_{2fl}^2 \cdot \frac{\gamma_2}{S_{fl}}$$

$$\frac{T_{st}}{T_{fl}} = \left(\frac{I_{st}}{I_{fl}} \right)^2 S_{fl}$$

[Proof of $T = I_2^2 \frac{\gamma_2}{S}$]

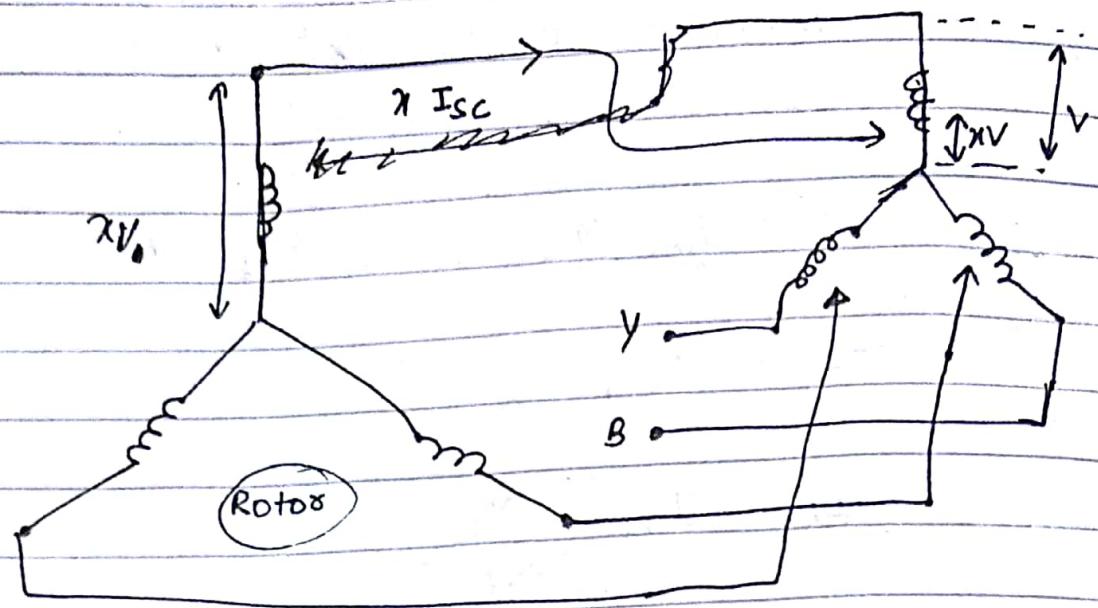
$$T = \frac{K \cdot E_1^2 \cdot \gamma_2 \cdot S}{\gamma_2^2 + (Sx_2)^2}$$

$$= K \frac{E_1^2 \cdot S \cdot \gamma_2}{S^2 \left\{ \left(\frac{\gamma_2}{S} \right)^2 + x_2^2 \right\}}$$

$$= K \frac{E_1^2 \cdot S \cdot \gamma_2}{S^2 \sqrt{\left(\frac{\gamma_2}{S} \right)^2 + x_2^2}} = K \cdot \frac{I_2^2 \gamma_2}{S}$$

$$\sqrt{\left(\frac{\gamma_2}{S} \right)^2 + x_2^2} = \text{Impedance}_{(2)}$$

ii) Auto - transformer starting



3- ϕ auto transformer.

current decreases x^2 times.

$$I_{st} = x^2 \cdot I_{sc}$$

~~Top 22 Base 22 22 V
SC~~

$$xV \times x I_{sc} = V_o I_{st}$$

$$I_{st} = x^2 I_{sc}$$