Monte Carlo Method for Numerical Integration:

The Monte Carlo Method (or the method of statistical trials) consists ~~of~~ in solving various problems of computational mathematics by constructing some random process for each problem with the parameters of the process equal to the required quantities of the problem. These quantities are then determined approximately by observation of the random process and the computation of its statistical characteristics. The results are often approximately equal to the required parameters.

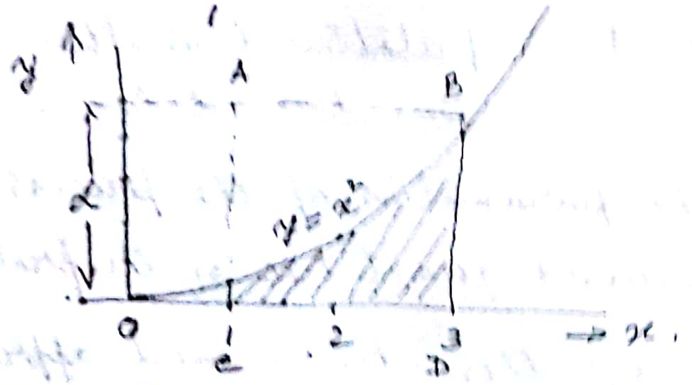
Limitations of modelling:

In reality, we often replace the process being investigated by a simple artificial process which can be modeled on a computer. The necessity of such a simplification arise from the incompleteness of information about the actual process and also from tailoring the problem to fit itself for solution by a digital computer.

Numerical integration.

Let us consider the area under the curve $y = x^2$ over the interval [1, 3]

$$I = \int_1^3 x^2\,dx$$



To solve the problem of evaluation of the integral by Monte Carlo method, darts are thrown at random onto the rectangle ABCD (whose area is $d(3-1)$) and which covers the whole of the shaded area). The probability that a dart will hit the shaded area is

$$P = \frac{\text{no. of darts hitting the shaded area}}{\text{total no. of darts thrown}}.$$

The probability P evidently depends on the shaded area since a low probability of hitting the ~~shad~~ shaded area implies that this area is relatively small. Hence the area of the shaded region is given by

$$I = P \times \text{area ABCD}.$$

However the no. of darts thrown should be infinitely large.

Algorithm :

$$I = \int_{a=0}^{b=1} f(x)\,dx$$

step 1) : Set N = a large +ve integer

2) $i = 1$, $Hit = 0$, d = a value greater than or equal to $\max f(x)$ in $[a, b]$.

3) generate a random no. x_i in $[a, b]$.
Let $y_i = d.x_i$. Here $d \geq y_i \geq 0$

4) generate another random no. x_i in $[a, b]$
Let $z_i = f(x_i)$.

5) If $z_i > y_i$, $Hit = Hit + 1$

6) If $i = N$ then ~~compute~~ $P = Hit / N$
$I = P \times d \times (b - a)$.
else $i = i+1$, go to step 3.

Generation of pseudorandom nos :

1) mid square method.
r_{i+1} = mid part of 32 bits of $~~x~~_i^2$

2). Power residue method.
$r_{i+1} = a r_i \pmod{m}$.
For computer word length of k bits,
a = an integer of the form $8x \pm 3$ and close to $2^{k/2}$.
$m = 2^k$.
r_i = an odd integer close to $2^{k/2}$

3) mid square bit method.

let r_i be an 8-bit no.

a) compute r_i^2 which is a 16 bit no.

b) Pick the 8th (or 9th) bit of r_i^2 and place it in the 8th bit position of r_{i+1}

c) pick the middle 8 bits of r_i^2 & square it. place the 8th (or 9th) bit of it in the 7th bit position of r_{i+1}

d) repeat step c) until all bit position or r_{i+1} is filled up.

~~comparative results~~

Results of the computation of the integral

$\int_0^1 x^2 dx$ by Monte-carlo Method.

i) $a=0$, $b=1$, $d=1$ (max^m value of x^2 in $[a,b]$)

after 5,000 darts thrown, $I = 0.33680$

ii) $a=0$, $b=1$, $d=2$. (larger than max x^2)

after 5,000 darts thrown, $I = 0.333599$.

iii) $a=0$, $b=1$, $d=4$

after 5000 darts thrown, $I = 0.3263998$.

I actual $= 1/3 = 0.33333$.