Show that for all integers $n>1$,

$$\int \sin^n x \, dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x \, dx$$

(*) Define $I_n = \int \sin^n x \, dx$, $n \in \mathbb{Z}$, $n \geq 0$

Integration by Parts: $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Try:

$u = \sin^n x$	$\frac{dv}{dx} = 1$
$\frac{du}{dx} = n\sin^{n-1}x\cos x$	$v = x$

$$\Rightarrow \int \sin^n x \, dx = x\sin^n x - \int n x \sin^{n-1}x \cos x \, dx$$

$$= x\sin^n x - n\int \boxed{x}\,\boxed{\sin^{n-1}x\cos x}\, dx$$

hard to evaluate

Try:

$u = \sin^{n-1}x$	$\frac{dv}{dx} = \sin x$
$\frac{du}{dx} = (n-1)\sin^{n-2}x\cos x$	$v = -\cos x$

$$\Rightarrow \int \sin^n x \, dx = -\cos x \sin^{n-1}x - \int (-\cos x)(n-1)\sin^{n-2}x\cos x \, dx$$

$$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x \cos^2 x \, dx$$

$$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x (1-\sin^2 x)\, dx$$

$$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x \, dx - (n-1)\int \sin^n x \, dx$$

Show that for all integers $n>1$,

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$$

(*) Define $I_n = \int \sin^n x\,dx$

$\Rightarrow I_n = -\sin^{n-1}x\cos x + (n-1)I_{n-2} - (n-1)I_n$

$\Rightarrow I_n = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}I_{n-2}$

$\Rightarrow \int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

$\int \sin^3 x\,dx \rightarrow \sin 3x = P(\underbrace{\sin x})$

$= \int \left(\boxed{}\sin 3x + \boxed{}\sin x\right)dx$

Higher powers: $\int \sin^5 x\,dx$

$= \frac{1}{5}\sin^4 x\cos x + \frac{4}{5}\int \sin^3 x\,dx$

$\int \sin^{13}x\,dx \rightsquigarrow \int \sin^{11}x\,dx \rightsquigarrow \int \sin^9 x\,dx$

$\int \sin x\,dx$