① $\int \frac{1}{x\ln x}\,dx$

$f(g(x)) \cdot g'(x)$

$u=\ln x$; $g(x)=\ln x$, $g'(x)=\frac{1}{x}$

$\int \frac{1}{u}\,du = \ln|u|+C$

$= \ln|\ln x|+C$

② $\int \frac{2x}{\sqrt{x^2+1}}\,dx$ $\quad u=x^2+1,\ \frac{du}{dx}=2x$

$= \int \frac{du}{\sqrt{u}}$

$= 2\sqrt{u}+C = 2\sqrt{x^2+1}+C$

③ $\int \frac{3x^2}{(x^3+3)^{3/2}}\,dx$ $\quad u=x^3+3$

$= \int \frac{du}{u^{3/2}}$

$= -2u^{-\frac{1}{2}}+C$

$= -\frac{2}{\sqrt{x^3+3}}+C$

$\int f'(x)(f(x))^n\,dx$ ($n<0$ permitted)

Sub $u=f(x)$

$\int u^n\,du = \begin{cases} \frac{u^{n+1}}{n+1}+C, & n\neq -1 \\ \ln|u|+C, & n=-1 \end{cases}$

④ $\int \frac{(x+3)^2}{x^2+4}\,dx$

$= \int \frac{x^2+6x+9}{x^2+4}\,dx$

$= \int \left(1 + \frac{6x+5}{x^2+4}\right) dx$

$= \int 1\,dx + \int \frac{6x+5}{x^2+4}\,dx$

$= \int 1\,dx + \int \frac{6x}{x^2+4}\,dx + \int \frac{5}{x^2+4}\,dx$

$= x + 3\int \frac{2x}{x^2+4}\,dx + 5\int \frac{1}{x^2+4}\,dx$

$= x + 3\ln(x^2+4) + \frac{5}{2}\tan^{-1}\frac{x}{2} + c$

$\int \frac{1}{x^2+9x+13}$

$\int \frac{P(x)}{Q(x)}\,dx$ → Partial fractions

or $\frac{1}{x^2+(bx+c)x}$ ...

⑤ $\int x\sqrt{x-2}\,dx$

$= \int x\sqrt{u}\,du$ $\quad u = x-2,\ du = dx$

$= \int (u+2)\sqrt{u}\,du$ (bad notation)

$= \int u^{3/2}\,du + 2\int u^{1/2}\,du$

$= \frac{2}{5}u^{5/2} + \frac{4}{3}u^{3/2} + c$

$= \frac{2}{5}(x-2)^{5/2} + \frac{4}{3}(x-2)^{3/2} + c$

$\int x\sqrt{3x-4}\,dx$, $u = 3x-4$