⑥ $\int \tan x \, dx$ " $k\int \frac{f'(x)}{f(x)} dx$ "

$= \int \frac{\sin x}{\cos x} dx$

$= -\int \frac{-\sin x}{\cos x} dx$

$= -\ln|\cos x| + C$

⑦ $\int \frac{\sin 2x}{\sin^2 x} dx \longrightarrow \int \frac{2\sin x \cos x}{\sin^2 x} dx$

$\cos 2x = \cos^2 x - \sin^2 x$
$\quad = 1 - 2\sin^2 x$
$\Rightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$

$= 2\int \frac{2\sin 2x}{1-\cos 2x} dx$

$= \ln|1-\cos 2x| + c$

$= \ln|\sin x| + c'$

$= \int \frac{2\cos x}{\sin x} dx$

$= 2\ln|\sin x| + c''$

⑧ $\int \frac{\tan^4 x - 1}{\tan x - 1} dx \quad \overset{u=\tan x}{\longrightarrow} \quad \int \frac{u^4 - 1}{u-1} \quad \cdots$

$= \int \frac{(\tan x - 1)(\tan^3 x + \tan^2 x + \tan x + 1)}{\tan x - 1} dx$

$= \int (\tan x + 1)(\tan^2 x + 1) dx$

$\qquad \underbrace{}_{f(\tan x)} \qquad \int x \sin x \, dx$

$= \int (\tan x + 1) \sec^2 x \, dx$

Try $\boxed{u = \tan x}$
$\quad du = \sec^2 x \, dx$

$= \int (u+1) du$

$= \frac{u^2}{2} + u + c$

$= \tan x + \frac{\tan^2 x}{2} + c$