# Integration by Parts

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{d}{dx}\left(u v^{-1}\right).$$

$c_1$ ~ "Product rule". $c_2$

functions of $x$. $u = u(x)$, $v = v(x)$

$$\boxed{\int u \frac{dv}{dx}\,dx} = uv - \boxed{\int v \frac{du}{dx}\,dx}$$

$$\iff \int \left( u\frac{dv}{dx} + v\frac{du}{dx} \right) dx = \boxed{uv} + c.$$

Define $\int \left( u\frac{dv}{dx} + v\frac{du}{dx} \right) dx = F(x). = uv \boxed{+C}$

Then, $F'(x) = u\frac{dv}{dx} + v\frac{du}{dx}$

$= \frac{d}{dx}(uv).$

$$\int \underline{u}\,\underline{v'}\,dx = uv - \int u'v\,dx$$

Any choice of $u, v$ works.

$$\boxed{\int \frac{f'(x)}{f(x)}\,dx} = 1 - \int \frac{-f'(x)}{f(x)}\,dx = 1 + \boxed{\int \frac{f'(x)}{f(x)}\,dx}$$

$\ln|f(x)| + c_1$ $\quad$ $\ln|f(x)| + c_2$

$u = \frac{1}{f(x)}$, $v' = f'(x)$

$u' = -\frac{f'(x)}{(f(x))^2}$ $\quad$ $v = f(x) + c$ $\quad$ Choose $\underline{v = f(x)}$

$\Rightarrow c_1 = 1 + c_2$ ✓.

$$\int f(x) g'(x)\,dx = f(x)g(x) + cf(x) - \int \left( f'(x)g(x) + cf'(x) \right) dx$$

$$= f(x)g(x) - \int f'(x)g(x)\,dx + \boxed{cf(x) - \int cf'(x)\,dx}.$$

$$\underbrace{cf(x) - \int cf'(x)\,dx}_{=0}$$

$u = f(x)$ $\quad$ $v' = g'(x)$

$u' = f'(x)$ $\quad$ $\boxed{v = g(x) + c.}$

(*) Any $v$ works (i.e. pick a convenient $c \in \mathbb{R}$).