Tangent half-angle Substitution

$$\circledast \int f(\sin\theta, \cos\theta)\, d\theta = \int f\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right)\left(\frac{2}{1+t^2}\right) dt$$

$$\left(t = \tan\frac{\theta}{2}\right).$$

✓ $t = \boxed{\tan\frac{\theta}{2}}$ (defn). $\Rightarrow \frac{dt}{d\theta} = \frac{1}{2}\sec^2\frac{\theta}{2} = \frac{1}{2}(1+t^2) \Rightarrow d\theta = \frac{2}{1+t^2}dt$

$\sin\theta$, $\cos\theta$? (as rational func. in terms of t).

$$\sin\theta = \boxed{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = 2\boxed{\tan\frac{\theta}{2}}\boxed{\cos^2\frac{\theta}{2}}$$

$$= \frac{2\tan\frac{\theta}{2}}{\sec^2\frac{\theta}{2}} = \frac{2t}{1+t^2}.$$

$$\cos\theta = \cos^2\frac{\theta}{2} - \boxed{\sin^2\frac{\theta}{2}} = \left(1 - \tan^2\frac{\theta}{2}\right)\cos^2\frac{\theta}{2}$$

$$= \frac{1-\tan^2\frac{\theta}{2}}{\sec^2\frac{\theta}{2}} = \frac{1-t^2}{1+t^2}.$$

$$\int f(\sin\theta, \cos\theta)\, d\theta = \int f\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right) d\theta$$

$$= \int \frac{2}{1+t^2} f\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right) dt$$

$$\int \boxed{\sin^6\theta \cos\theta}\, d\theta = \int \boxed{\frac{2}{1+t^2}\left(\frac{2t}{1+t^2}\right)^6\left(\frac{1-t^2}{1+t^2}\right)} dt.$$

$$\int \frac{1}{\sin\boxed{2\theta} + 4\cos\boxed{2\theta}}\, d\theta$$

"$f(ax+b)$"

sub $u = \tan\theta$.

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