FTC: $\int f(x)\,dx = F(x) + C$

$\iff \frac{d}{dx}F(x) = f(x).$

constant multiple

① $\frac{d}{dx}(\boxed{a}f(x)) = a\frac{d}{dx}f(x)$ $(\in \mathbb{R})$

How does $\int af(x)\,dx$ relate to $\int f(x)\,dx$?

Let $\int f(x)\,dx = F(x) + C$

$\int af(x)\,dx = G(x) + C'$

$C \neq C'$ is allowed!

FTC

$\frac{d}{dx}F(x) = f(x)$

$\frac{d}{dx}G(x) = \boxed{a}f(x) = \frac{d}{dx}(aF(x))$

$$\int af(x)\,dx = aF(x) + C''$$

✓ $\int af(x)\,dx = a\int f(x)\,dx$

fixed real

arbitrary real

$= aF(x) + aA$ $\quad a(F(x) + A)$

Can biject every $B \in \mathbb{R}$ to some $A \in \mathbb{R}$ st. $B = aA$.

② $\frac{d}{dx}\left(f(x) \pm g(x)\right) = \left(\frac{d}{dx} f(x)\right) \pm \left(\frac{d}{dx} g(x)\right)$.

✓ First deal with addition.

$\int \left(f(x)+g(x)\right) dx$ relates to $\int f(x)\,dx$ & $\int g(x)\,dx$?

set $\int f(x)\,dx = F(x) + \boxed{C} \in \mathbb{R}$.

$\int g(x)\,dx = G(x) + \boxed{D}$

$$\frac{d}{dx}\left(\boxed{F(x)+G(x)}\right) = \frac{d}{dx}F(x) + \frac{d}{dx}G(x)$$

$$= \boxed{f(x)+g(x)}.$$

$\hookrightarrow \int \left(f(x)+g(x)\right) dx = F(x) + G(x) + E.$

⊛ $\boxed{\int \left(f(x)-g(x)\right) dx \,?}$ $= F(x) - G(x) + \boxed{X_1 - X_2}$

arb. constant.

$$= \int \left[f(x) + (\underbrace{\boxed{-g(x)}}_{h(x)})\right] dx$$

$$= \int f(x)\,dx + \int \left(\boxed{-}g(x)\right) dx$$

$$= \boxed{\int f(x)\,dx} - \boxed{\int g(x)\,dx}$$

$= F(x) + X_1$ $\qquad$ $= G(x) + X_2$