Trigonometric Substitutions

Expression	Substitute	Domain
$\sqrt{a^2-x^2}$	$x=a\sin\theta$	$\lvert\theta\rvert\le\frac{\pi}{2}$
$\sqrt{a^2+x^2}$	$x=a\tan\theta$	$\lvert\theta\rvert<\frac{\pi}{2}$
$\sqrt{x^2-a^2}$	$x=a\sec\theta$	$\theta\in[0,\frac{\pi}{2})\cup[\pi,\frac{3\pi}{2})$

$$\int_0^{\infty}\frac{1}{x^2\sqrt{1+x^2}}\,dx$$

$x=\tan\theta$, $dx=\sec^2\theta\,d\theta$

$$=\int_0^{\pi/2}\frac{\sec^2\theta\,d\theta}{\tan^2\theta\sqrt{1+\tan^2\theta}}$$

$$=\sqrt{\sec^2\theta}=\sec\theta,\ \text{since } \lvert\theta\rvert<\tfrac{\pi}{2}$$

$$=\int\frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$=\int\frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta$$

$$=\int\frac{\cos\theta}{\sin^2\theta}\,d\theta$$

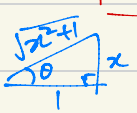
$u=\sin\theta$

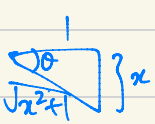
$$=\int\frac{1}{u^2}\,du=-\frac{1}{u}+C$$

$$=-\frac{1}{\sin\theta}+C$$

$u=\sin\theta$

$$=\sin(\tan^{-1}x)$$





$$=-\frac{\sqrt{x^2+1}}{x}+C$$