Integration by Substitution.

① $u = g(x)$, g differentiable,

then $\int \boxed{f(g(x))}\,\boxed{g'(x)}\,dx = \int f(u)\,du$.

Define $\int f(u)\,du = F(u) + C$

$$\boxed{\frac{d}{dx}F(u)} = \boxed{\frac{du}{dx}} \cdot \boxed{\frac{d}{du}F(u)}$$

$$= g'(x) \cdot f(u)$$

$$= \boxed{g'(x)\,f(g(x))}.$$

$$\int g'(x)\,f(g(x))\,dx = F(u) + C' = \int f(u)\,du$$

- $\int \frac{x}{1+x^2}\,dx$ $\boxed{u = g(x) = x^2}$

$\frac{1}{2}g'(x)$.

$$\frac{1}{2}\int \left(\frac{1}{1+x^2}\right)(2x)\,dx = \frac{1}{2}\int \frac{1}{1+u}\,du$$

$$= f(x^2) \Rightarrow f(y) := \frac{1}{1+y}$$

$$= \frac{1}{2}\ln|u+1| + C$$

$$= \frac{1}{2}\ln(x^2+1) + C$$

$v = 1+x^2$, $\frac{dv}{dx} = 2x$.

$$\frac{1}{2}\int \frac{2x}{1+x^2}\,dx = \frac{1}{2}\int \frac{\frac{dv}{dx}}{v}\,dx$$

$$= \frac{1}{2}\int \frac{1}{v}\,dv = \ln(x^2+1) + C.$$