

$$\begin{aligned}
 ⑨ \int \cos^5 x dx &= \int (\cos^2 x)(\cos^3 x) dx \\
 &= \int (\cos x) (\cos^4 x) dx = \int (1 - \sin^2 x) \cos^3 x dx \\
 &= \int \cos x (\cos^2 x)^2 dx = \int \cos^3 x dx - \int \frac{\sin^2 x}{\cos^3 x} dx \\
 &= \int \cos x (1 - \sin^2 x) dx
 \end{aligned}$$

$\textcircled{8}$ $u = \sin x \quad \cos x \quad P(\sin x)$

$$= \int (1 - u^2) du$$

$$= u - \frac{u^3}{3} + C$$

$$= \sin x - \frac{\sin^3 x}{3} + C$$

$$\begin{aligned}
 &\int \cos^7 x dx \quad P(\sin x) \\
 &= \int \cos x (1 - \sin^2 x)^3 dx \quad \int \sin^7 x \cos^2 x dx \quad P(\cos x) \\
 &= \int \sin x (1 - \cos^2 x)^3 \cos^2 x dx
 \end{aligned}$$

$$\int \sin^a x \cos^b x dx,$$

$\left\{ \begin{array}{l} a \text{ odd} \\ b \text{ even} \end{array} \right. \text{ or vice versa}$

$$\begin{aligned}
 &= \int \sin x (\sin^{a-1} x \cos^b x) dx \\
 &\quad (\sin^a x)^{\frac{a-1}{2}} = (1 - \cos^2 x)^{\frac{a-1}{2}}
 \end{aligned}$$

$$(10) \int \sec x \, dx = \int \frac{d}{dx} \left(\frac{\sec x + \tan x}{\sec x + \tan x} \right) \, dx = \ln |\sec x + \tan x| + C$$

$\frac{f'(x)}{f(x)}$

$$\begin{aligned} & \frac{d}{dx} (\sec x + \tan x) \\ &= \sec^2 x + \sec x \tan x \\ &= \sec x (\sec x + \tan x) \\ \Rightarrow \sec x &= \frac{d}{dx} \frac{(\sec x + \tan x)}{\sec x + \tan x} \end{aligned}$$

$$\begin{aligned} & \int \sec x \, dx \\ &= \int \frac{1}{\cos x} \, dx \quad \left\{ \begin{array}{l} \cos x \rightarrow P(\sin x) \\ \cos x \\ \hline P(\sin x) \end{array} \right. \\ &= \int \frac{\cos x}{\cos^2 x} \, dx \\ &= \int \frac{\cos x}{1 - \sin^2 x} \, dx \\ &= \int \frac{1}{1 - u^2} \, du \quad (u = \sin x) \\ &= \int \left(\frac{1}{1+u} + \frac{1}{1-u} \right) \, du \\ &= \frac{1}{2} \ln \left| \frac{1+u}{1-u} \right| + C' \\ &= \frac{1}{2} \ln \left| \frac{1+\sin x}{1-\sin x} \right| + C' \\ &= \frac{1}{2} \ln \left| \frac{(1+\sin x)^2}{1-\sin^2 x} \right| + C' \\ &= \frac{1}{2} \ln \left| \frac{(1+\sin x)^2}{\cos^2 x} \right| + C' \\ &= \ln \left| \frac{(1+\sin x)^2}{\cos^2 x} \right| + C' \\ &= \ln |\sec x + \tan x| + C \end{aligned}$$