$$\int f(\sin\theta, \cos\theta)\, d\theta = \int f\left(\boxed{\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}}\right)\left(\frac{2\,dt}{1+t^2}\right).$$

$$\int \boxed{\sec\theta}\, d\theta = \int \frac{1}{\cos\theta}\, d\theta$$

$$= \int \frac{2}{1+t^2}\left(\frac{1+t^2}{1-t^2}\right) dt$$

$$= \int \frac{2}{1-t^2}\, dt$$

$$= \int \left(\frac{1}{1-t} + \frac{1}{1+t}\right) dt$$

$$= \boxed{\ln\left|\frac{1+t}{1-t}\right| + c}$$

$$= \ln\left|\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}\right| + c.$$

$$\ln|\sec\theta + \tan\theta| + c'$$

$$= \ln\left|\frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}\right| + c' = \ln\left|\frac{(t+1)^2}{1-t^2}\right| + c'$$

$$= \ln\left|\frac{1+t}{1-t}\right| + c'$$

$$\boxed{\int \sin^3\theta \cos\theta\, d\theta} = \boxed{\frac{1}{4}\sin^4\theta} + c$$

$$\frac{1}{4}\left(\frac{2t}{1+t^2}\right)^4 = \frac{4t^4}{(1+t^2)^4} = \boxed{\frac{4u^2}{(1+u)^4}}$$

$$= \int \left(\frac{2t}{1+t^2}\right)^3 \left(\frac{1-t^2}{1+t^2}\right)\left(\frac{2}{1+t^2}\right) dt$$

$$= \int \frac{16t^3(1-t^2)}{(1+t^2)^3}\, dt$$

$$= \int \frac{8u(1-u)}{(1+u)^3}\, du \qquad u = t^2,\quad du = 2t\,dt$$

$$\frac{8u(1+u)^4 - 4u^2(4)(1+u)^3}{(1+u)^6}.$$

$$= \frac{8u(1+u) - 16u^2}{(1+u)^3}$$

$$= \frac{8u(1-u)}{(1+u)^3}$$