④ (less) common integrals

$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$

$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C'$.

⑤ $\int f(ax+b)\,dx$?   Define $\int f(x)\,dx = F(x) + C$.

$a=1$? $\int f(x+b)\,dx = F(x+b) + C'$

$\frac{d}{dx}F(x+b)$? ①.

$= f(x+b) \cdot \cancel{\frac{d}{dx}(x+b)}$

General $a \in \mathbb{R}$? $\frac{d}{dx}F(ax+b)$

$= (f(ax+b))\,a.$

$\int a\boxed{f(ax+b)}\,dx$

$= F(ax+b) + C''$

(*) want to integrate $\int f(ax+b)\,dx$. $= \frac{1}{a}F(ax+b) + C'''$.

$\int (\boxed{3x-2})^2\,dx = \int (9x^2 - 12x + 4)\,dx$

$= 9\int x^2\,dx - 12\int x\,dx + 4\int dx = \cdots$

$\int \boxed{x}^2\,dx = \frac{1}{3}x^3 + C$

$F(x) = \frac{1}{3}x^3$.

$f(x) := x^2$, then $(3x-2)^2 = f(3x-2)$.

$\int (3x-2)^2\,dx = \int f(3x-2)\,dx$

$= \frac{1}{3}F(3x-2) + C''$

$= \frac{1}{3}(3x-2)^3 + C''$.

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