Trigonometric Substitutions

$x \overset{\text{bijects}}{\longleftrightarrow} \theta$

Expression	Substitute	Domain
$\sqrt{a^2-x^2}$	$x=a\sin\theta$	$\lvert\theta\rvert \le \frac{\pi}{2}$
$\sqrt{a^2+x^2}$	$x=a\tan\theta$	$\lvert\theta\rvert < \frac{\pi}{2}$
$\sqrt{x^2-a^2}$	$x=a\sec\theta$	$\theta \in [0,\frac{\pi}{2}) \cup [\pi,\frac{3\pi}{2})$

fixed $|x|<a$

$x \in \mathbb{R}$

$\tan\theta > 0$

"$\sec\theta$ bijective"

① Domain allows θ to take on (hopefully) all "permitted" values of x

$\sqrt{\cdots}$ defined.

② In the domain, $\nexists\, a, b$ of. when $\theta=a$ & $\theta=b$, we get the same x.

③ $\sqrt{\cdots} > 0 \ldots$

Ex. $\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = |\cos\theta|$

$x=\sin\theta$

should have same sign on interval. (preferably +ve)

$x=a\sinh\theta$:

Range of x: $[-a, a]$

$\cos\theta > 0$.

$[-\frac{\pi}{2}, \frac{\pi}{2}]$