

Integration by Substitution.

①  $u = g(x)$ ,  $g$  differentiable

then  $\int \boxed{f(g(x))} \boxed{g'(x)} dx = \int \underline{f(u)} du$ .

Define  $\int f(u) du = F(u) + C$

$$\begin{aligned} \boxed{\frac{d}{dx} F(u)} &= \boxed{\frac{du}{dx}} \cdot \boxed{\frac{d}{du} F(u)} \\ &= g'(x) \cdot f(u) \\ &= \boxed{g'(x) f(g(x))}. \end{aligned}$$

$$\int g'(x) f(g(x)) dx = F(u) + C = \int f(u) du$$

•  $\int \frac{x}{1+x^2} dx$   $u = g(x) = x^2$

$$\begin{aligned} &\frac{1}{2} \int \underbrace{\left( \frac{1}{1+x^2} \right)}_{f(x^2)} \underbrace{(2x)}_{\frac{du}{dx}} dx = \frac{1}{2} \int \frac{1}{1+u} du \\ &= f(x^2) \Rightarrow f(y) = \frac{1}{1+y} \quad = \frac{1}{2} \ln|u+1| + C \\ &\quad \quad \quad = \frac{1}{2} \ln(x^2+1) + C \end{aligned}$$

$\hookrightarrow v = 1+x^2, \frac{dv}{dx} = 2x$

$$\frac{1}{2} \int \frac{2x}{1+x^2} dx = \frac{1}{2} \int \frac{\frac{dv}{dx}}{v} dx$$

Made with Goodnotes

$$= \frac{1}{2} \int \frac{1}{v} dv = \ln(x^2+1) + C$$