① $\int \frac{1}{(x^2-4)^{3/2}}\,dx$

$= \int \frac{1}{(x^2-4)\boxed{\sqrt{x^2-4}}}\,dx$

$x = 2\sec\theta \rightarrow \theta \in [0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$

$dx = 2\sec\theta\tan\theta\, d\theta$

$\frac{x}{2}$, $\sqrt{\frac{x^2}{4}-1}$, θ, 1

$= \int \frac{2\sec\theta\tan\theta}{4\tan^2\theta(2\tan\theta)}\,d\theta$

$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$

$= \frac{1}{4}\int \frac{\cos\theta}{\boxed{\sin^2\theta}}\,d\theta$

$u = \sin\theta$

$= \frac{1}{4}\int \frac{1}{u^2}\,du = -\frac{4}{u} + c$

$= -\frac{4}{\sin\theta} + c.$

$= -\frac{4}{(\frac{x^2}{4}-1)\div(\frac{x}{2})} + c$

$= -\frac{4}{\left(\frac{x^2-4}{4}\right)\left(\frac{2}{x}\right)} + c$

$= -\frac{8x}{x^2+4} + c.$

② $\int \sqrt{\boxed{4x}-x^2}\,dx$

$\int \sqrt{1-x^2}\,dx$

$\int \frac{1}{\sqrt{a^2-u^2}}\,du \rightarrow$ $\sin^{-1}$ identity, $u = a\sin\theta$

$u = bx+c$

$\int \frac{b}{\sqrt{a^2-\boxed{(bx+c)}^2}}\,dx$

complete sq.!

$= \int \sqrt{2^2-\boxed{(x-2)}^2}\,dx$

$u = x-2$

$= \int \sqrt{2^2-u^2}\,du$

$u = 2\sin\theta$

$= \int (2\cos\theta)(2\cos\theta)\,d\theta$

$= 4\int \boxed{\cos^2\theta}\,d\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\int (1+\cos 2\theta)\,d\theta$

$= 2\theta + \boxed{\sin 2\theta} + c$ $\rightarrow 2\sin\theta\cos\theta$

$= 2\sin\theta\sqrt{1-\sin^2\theta}$

$= u\sqrt{1-\frac{u^2}{4}} = \frac{u}{2}\sqrt{4-u^2}$

$= 2\sin^{-1}\left(\frac{u}{2}\right)$

$+ \frac{u}{2}\sqrt{4-u^2} + c$

$= 2\sin^{-1}\left(\frac{x-2}{2}\right) + \left(\frac{x-2}{2}\right)\sqrt{4-\left(\frac{x-2}{2}\right)^2} + c.$