

CS246: Mining Massive Data Sets

Assignment number: 3

Fill in and include this cover sheet with each of your assignments. It is an honor code violation to write down the wrong time. Assignments and code are due at 5:00 PM on Scoryst and SNAP respectively. Failure to include the coversheet with your assignment will be penalized by 2 points. Each student will have a total of *two* free late periods. *One late period expires at the start of each class.* (Assignments are due on Thursdays, which means the first late period expires on the following Tuesday at 5:00 PM.) Once these late periods are exhausted, any assignments turned in late will be penalized 50% per late period. However, no assignment will be accepted more than one late period after its due date. (If an assignment is due to Thursday then we will not accept it after the following Thursday.)

Your name: Priyank Mathur

Email: priyankm@stanford.edu **SUNet ID:** priyankm

Collaborators: Emrah Budur, Shundan Xiao, Dibyajyoti Ghosh, Arkajyoti Misra

I acknowledge and accept the Honor Code.

(Signed) Priyank Mathur

Answer to Question 1.a

$$\epsilon_{iu} = 2 \times (R_{iu} - q_i \cdot p_u^T)$$

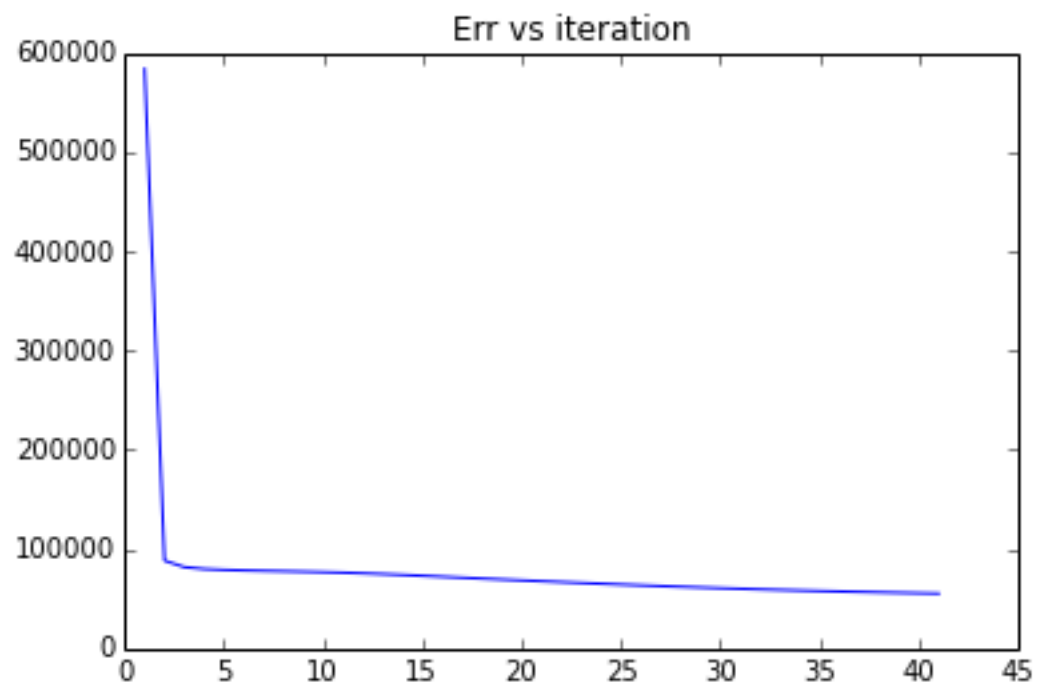
$$q_i = q_i + \eta(\epsilon_{iu} p_u - \lambda q_i)$$

$$p_u = p_u + \eta(\epsilon_{iu} q_i - \lambda p_u)$$

Note that the factor 2 obtained during differentiation w.r.t p_u and q_i has been consumed within the learning rate η .

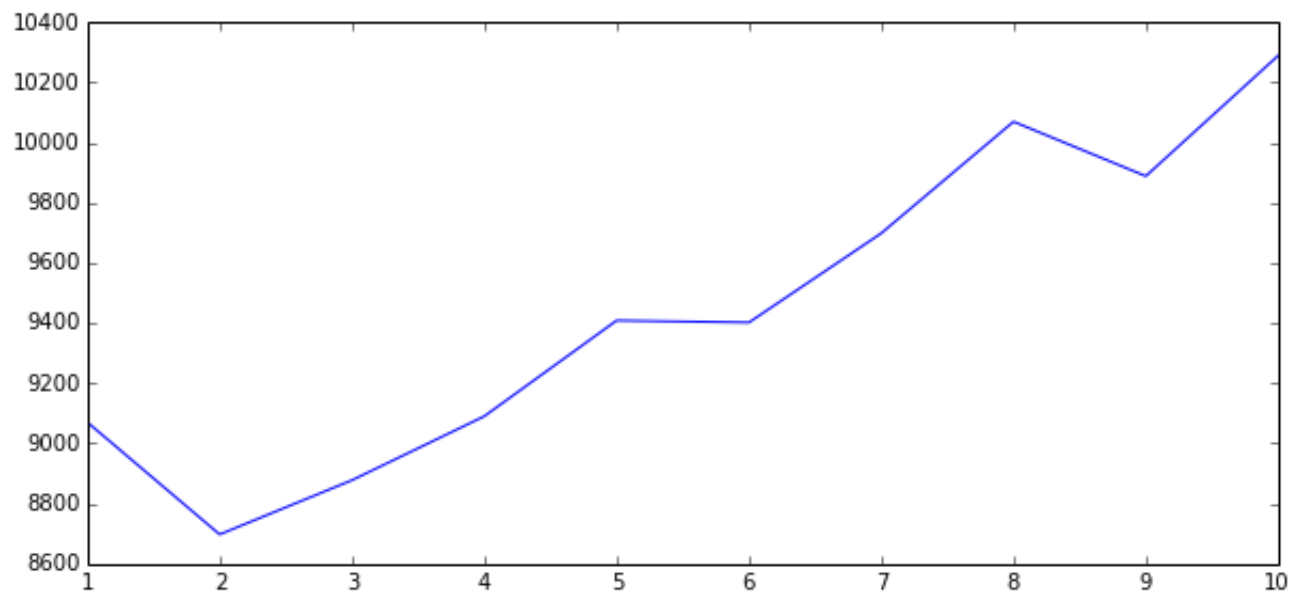
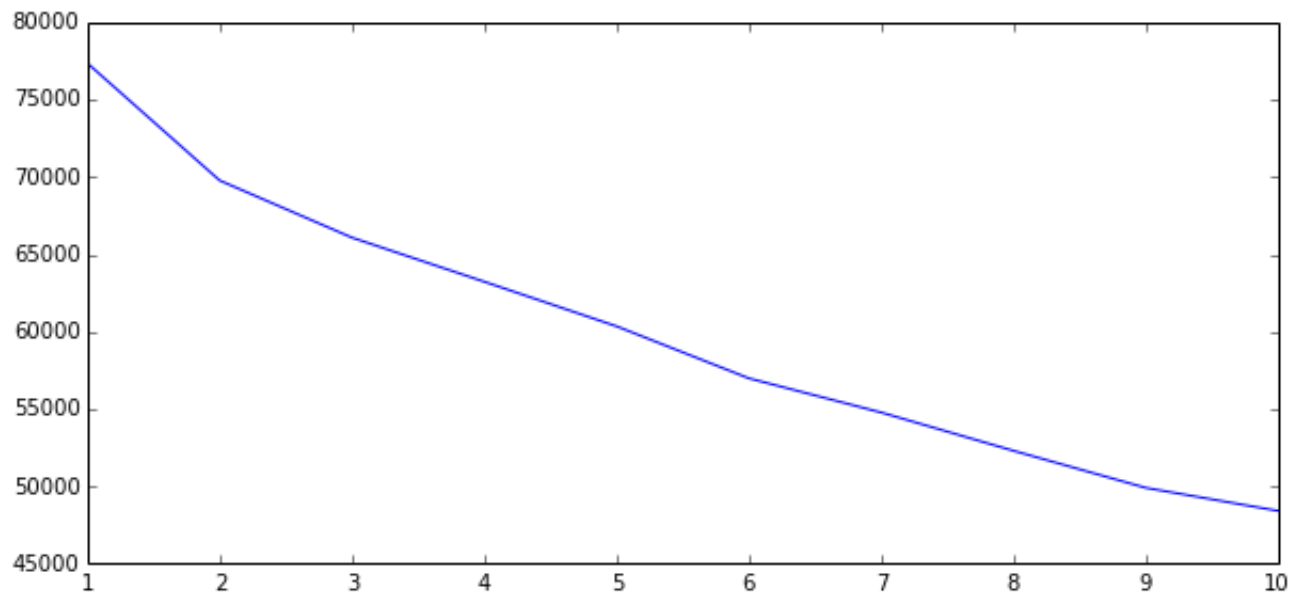
Answer to Question 1.b

$$\eta = 0.005$$



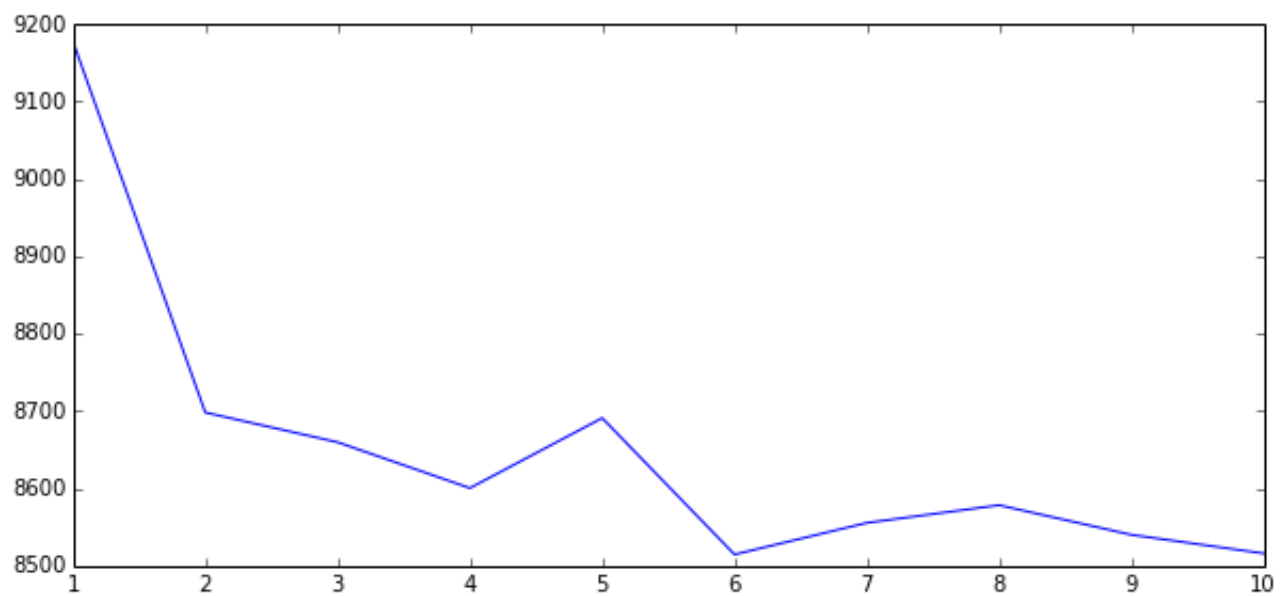
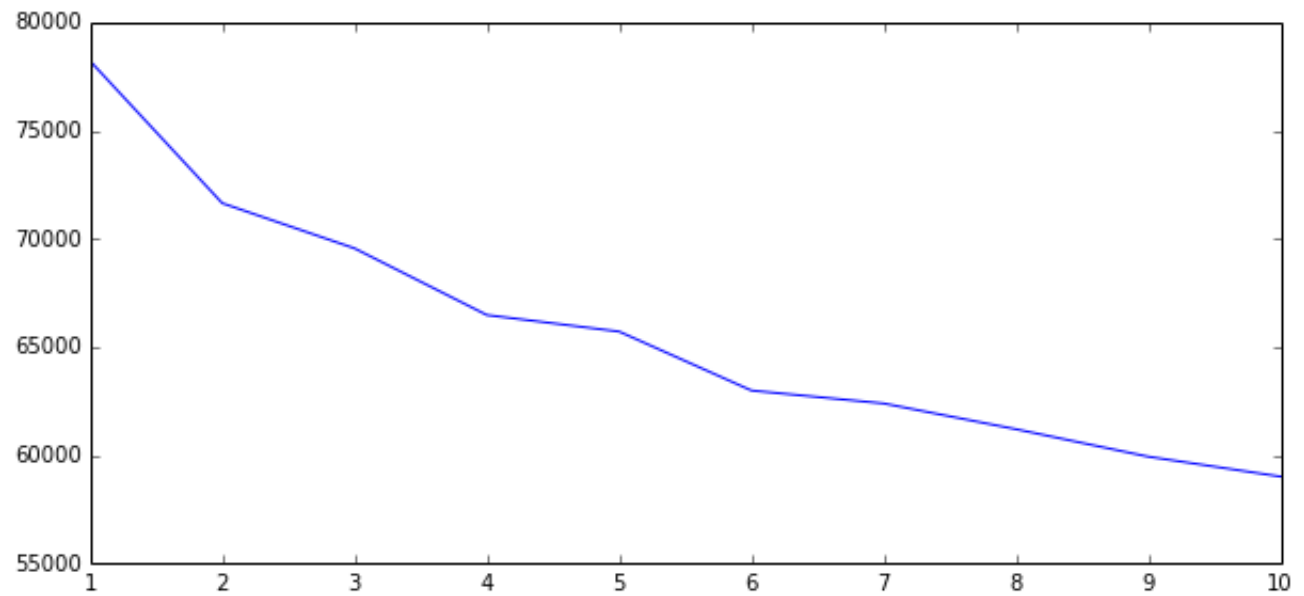
Answer to Question 1.c

$Error_{tr}$ vs. k for $\lambda = 0$



$Error_{te}$ vs. k for $\lambda = 0$

$Error_{tr}$ vs. k for $\lambda = 0.2$



$Error_{te}$ vs. k for $\lambda = 0.2$

Based on these graphs, we find the following to be true -

- B: Regularization decreases the test error for $k \geq 5$
- D: Regularization increases the training error for all (or almost all) k
- H: Regularization decreases overfitting

Answer to Question 1.d

$$\epsilon_{iu} = 2 \times (R_{iu} - (\mu + b_u + b_i + q_i \cdot p_u^T))$$

$$q_i = q_i + \eta_{LF}(\epsilon_{iu}p_u - \lambda q_i)$$

$$p_u = p_u + \eta_{LF}(\epsilon_{iu}q_i - \lambda p_u)$$

$$b_{i_i} = b_{i_i} + \eta_{b_i}(\epsilon_{iu} - \lambda b_{i_i})$$

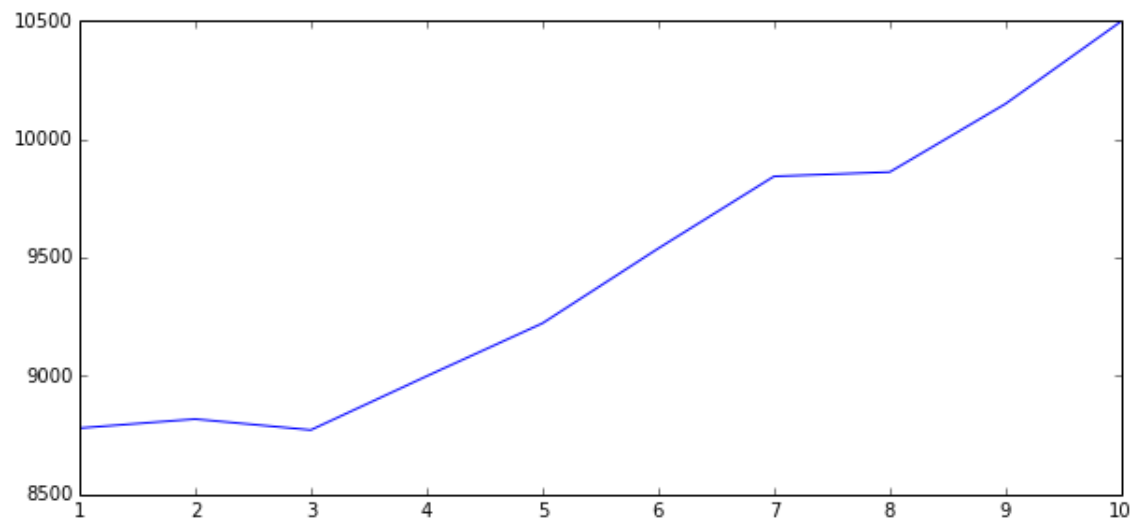
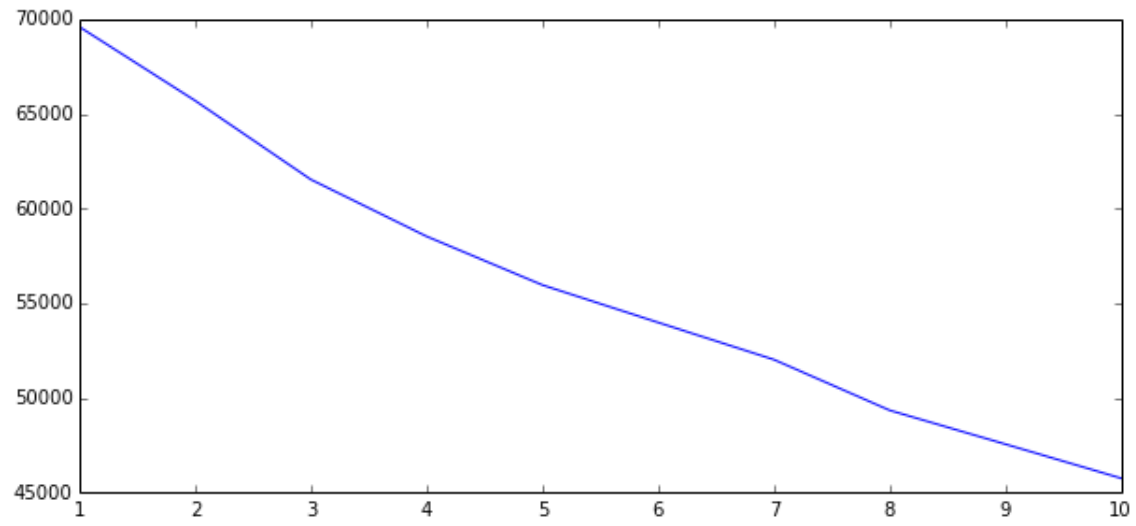
$$b_{u_u} = b_{u_u} + \eta_{b_u}(\epsilon_{iu} - \lambda b_{u_u})$$

$$\eta_{LF} = 0.005$$

$$\eta_{b_i} = 0.01$$

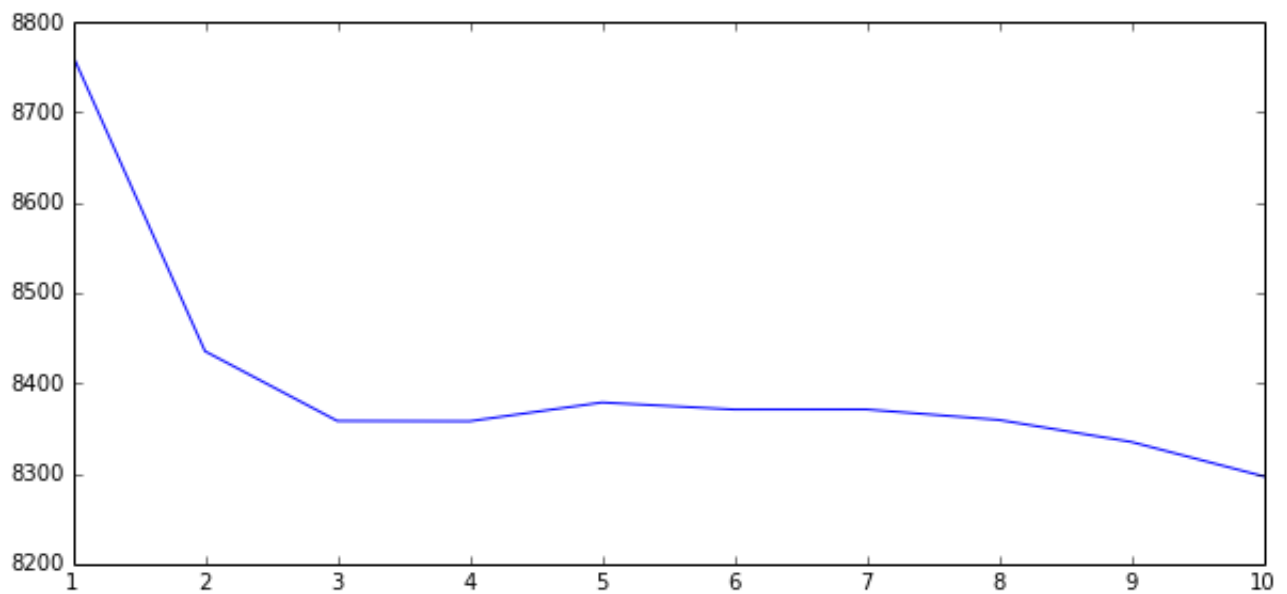
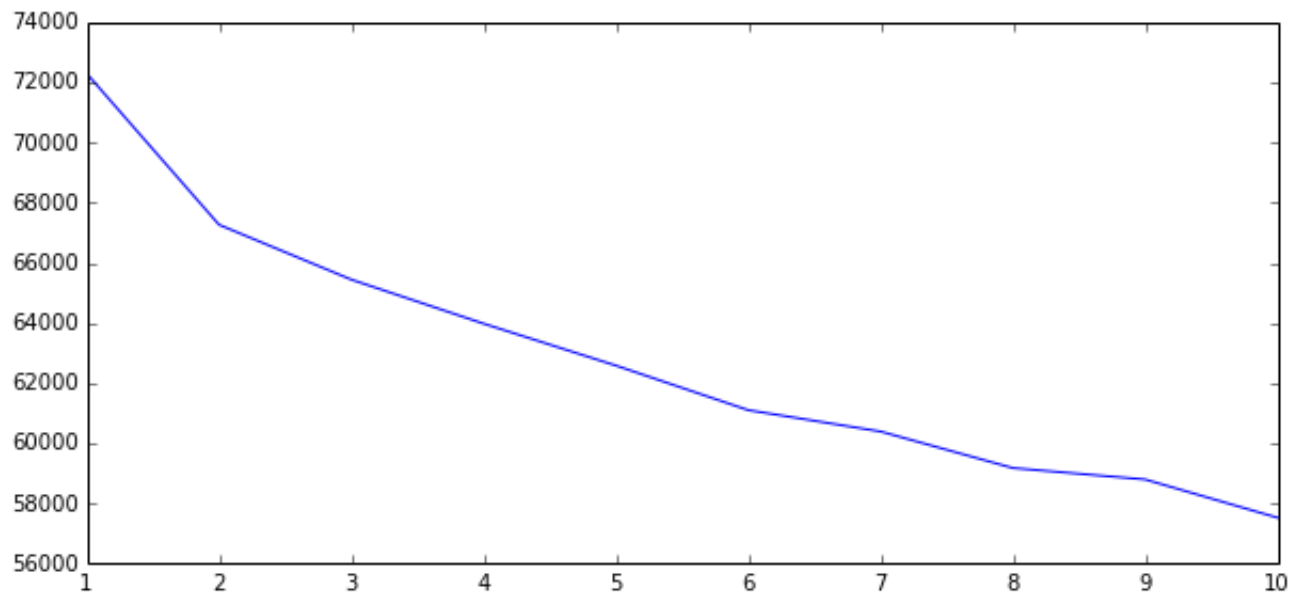
$$\eta_{b_u} = 0.01$$

$Error_{tr}$ vs. k for $\lambda = 0$



$Error_{te}$ vs. k for $\lambda = 0$

$Error_{tr}$ vs. k for $\lambda = 0.2$



$Error_{te}$ vs. k for $\lambda = 0.2$

Based on these graphs, we find the following to be true -

- B: Regularization decreases the test error for $k \geq 5$
- D: Regularization increases the training error for all (or almost all) k
- H: Regularization decreases overfitting

Answer to Question 2a

Answer to Question 2b

Answer to Question 2c

Answer to Question 2d

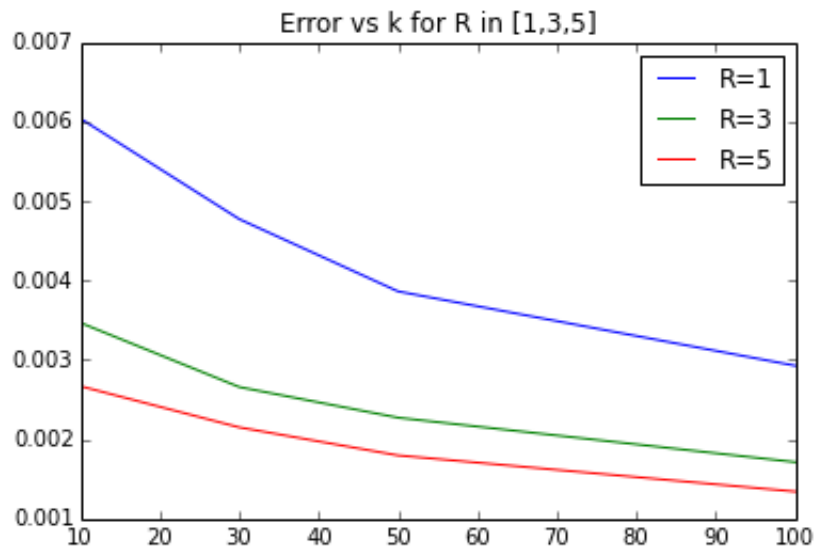
Runtime for 40 iterations of power iteration - $791 \mu s$.

Avg runtime for Monte Carlo with $R = 1$ - $7.83 ms$.

Avg runtime for Monte Carlo with $R = 3$ - $20.6 ms$.

Avg runtime for Monte Carlo with $R = 5$ - $35.57 ms$.

R	K	Error
1	10	0.0060365444268
1	30	0.0047717255253
1	50	0.0038620385493
1	100	0.0029277461094
3	10	0.0034659740412
3	30	0.0026577309646
3	50	0.0022720065398
3	100	0.0017132042790
5	10	0.0026678865383
5	30	0.0021502352960
5	50	0.0017972828358
5	100	0.0013425298758



Answer to Question 3.a

$$s_A(\text{cameras}, \text{phones}) = \frac{C1}{3 \times 2} [s_B(\text{nokia.com}, \text{nokia.com}) + s_B(\text{nokia.com}, \text{apple.com}) + s_B(\text{kodak.com}, \text{nokia.com}) + s_B(\text{kodak.com}, \text{apple.com}) + s_B(\text{cannon.com}, \text{nokia.com}) + s_B(\text{cannon.com}, \text{apple.com})]$$
$$s_A(\text{cameras}, \text{phones}) = \frac{0.8}{6} [1 + 0 + 0 + 0 + 0 + 0] = 0.13333$$

Iteration 1 -

$s_A(\text{'cameras'}, \text{'cameras'}) : 1,$
 $s_A(\text{'cameras'}, \text{'phones'}) : \mathbf{0.13333333333333333},$
 $s_A(\text{'printers'}, \text{'printers'}) : 1,$
 $s_A(\text{'phones'}, \text{'phones'}) : 1,$
 $s_A(\text{'cameras'}, \text{'printers'}) : \mathbf{0.0},$
 $s_A(\text{'phones'}, \text{'printers'}) : 0.0$

$s_B(\text{'cannon.com'}, \text{'nokia.com'}) : 0.4,$
 $s_B(\text{'hp.com'}, \text{'nokia.com'}) : 0.0,$
 $s_B(\text{'kodak.com'}, \text{'kodak.com'}) : 1,$
 $s_B(\text{'apple.com'}, \text{'kodak.com'}) : 0.0,$
 $s_B(\text{'cannon.com'}, \text{'kodak.com'}) : 0.8,$
 $s_B(\text{'cannon.com'}, \text{'cannon.com'}) : 1,$
 $s_B(\text{'hp.com'}, \text{'kodak.com'}) : 0.0,$
 $s_B(\text{'apple.com'}, \text{'hp.com'}) : 0.0,$
 $s_B(\text{'kodak.com'}, \text{'nokia.com'}) : 0.4,$
 $s_B(\text{'apple.com'}, \text{'apple.com'}) : 1,$
 $s_B(\text{'apple.com'}, \text{'nokia.com'}) : 0.4,$
 $s_B(\text{'nokia.com'}, \text{'nokia.com'}) : 1,$
 $s_B(\text{'cannon.com'}, \text{'hp.com'}) : 0.0,$
 $s_B(\text{'hp.com'}, \text{'hp.com'}) : 1,$
 $s_B(\text{'apple.com'}, \text{'cannon.com'}) : 0.0$

Iteration 2 -

$s_A(\text{'cameras'}, \text{'cameras'}) : 1,$
 $s_A(\text{'cameras'}, \text{'phones'}) : \mathbf{0.29333333333333333},$
 $s_A(\text{'printers'}, \text{'printers'}) : 1,$
 $s_A(\text{'phones'}, \text{'phones'}) : 1,$
 $s_A(\text{'cameras'}, \text{'printers'}) : \mathbf{0.0},$
 $s_A(\text{'phones'}, \text{'printers'}) : 0.0$

$s_B(\text{'cannon.com'}, \text{'nokia.com'}) : 0.45333333333333337,$
 $s_B(\text{'hp.com'}, \text{'nokia.com'}) : 0.0,$
 $s_B(\text{'kodak.com'}, \text{'kodak.com'}) : 1,$
 $s_B(\text{'apple.com'}, \text{'kodak.com'}) : 0.10666666666666667,$
 $s_B(\text{'cannon.com'}, \text{'kodak.com'}) : 0.8,$
 $s_B(\text{'cannon.com'}, \text{'cannon.com'}) : 1,$
 $s_B(\text{'hp.com'}, \text{'kodak.com'}) : 0.0,$
 $s_B(\text{'apple.com'}, \text{'hp.com'}) : 0.0,$

$s_B('kodak.com', 'nokia.com') : 0.4533333333333337,$
 $s_B('apple.com', 'apple.com') : 1,$
 $s_B('apple.com', 'nokia.com') : 0.4533333333333337,$
 $s_B('nokia.com', 'nokia.com') : 1,$
 $s_B('cannon.com', 'hp.com') : 0.0,$
 $s_B('hp.com', 'hp.com') : 1,$
 $s_B('apple.com', 'cannon.com') : 0.10666666666666667$

Iteration 3 -

$s_A('cameras', 'cameras') : 1,$
 $s_A('cameras', 'phones') : 0.34311111111111114,$
 $s_A('printers', 'printers') : 1,$
 $s_A('phones', 'phones') : 1,$
 $s_A('cameras', 'printers') : 0.0,$
 $s_A('phones', 'printers') : 0.0$

$s_B('cannon.com', 'nokia.com') : 0.5173333333333333,$
 $s_B('hp.com', 'nokia.com') : 0.0,$
 $s_B('kodak.com', 'kodak.com') : 1,$
 $s_B('apple.com', 'kodak.com') : 0.23466666666666663,$
 $s_B('cannon.com', 'kodak.com') : 0.8,$
 $s_B('cannon.com', 'cannon.com') : 1,$
 $s_B('hp.com', 'kodak.com') : 0.0,$
 $s_B('apple.com', 'hp.com') : 0.0,$
 $s_B('kodak.com', 'nokia.com') : 0.5173333333333333,$
 $s_B('apple.com', 'apple.com') : 1,$
 $s_B('apple.com', 'nokia.com') : 0.5173333333333333,$
 $s_B('nokia.com', 'nokia.com') : 1,$
 $s_B('cannon.com', 'hp.com') : 0.0,$
 $s_B('hp.com', 'hp.com') : 1,$
 $s_B('apple.com', 'cannon.com') : 0.23466666666666663$

Final result after 3 iterations -

$s_A('cameras', 'phones') : 0.34311111111111114,$
 $s_A('cameras', 'printers') : 0.0$

Answer to Question 3.b

$$s_A(X, Y) = \frac{C1}{\sum_{i=1}^{|O(X)|} \sum_{j=1}^{|O(Y)|} W_{X, O_i(X)} \cdot W_{Y, O_j(Y)}} \times \sum_{i=1}^{|O(X)|} \sum_{j=1}^{|O(Y)|} W_{X, O_i(X)} \cdot W_{Y, O_j(Y)} \cdot s_B(O_i(X), O_j(Y)) \quad (3)$$

$$s_B(x, y) = \frac{C2}{\sum_{i=1}^{|I(x)|} \sum_{j=1}^{|I(y)|} W_{I_i(x), x} W_{I_j(y), y}} \times \sum_{i=1}^{|I(x)|} \sum_{j=1}^{|I(y)|} W_{I_i(x), x} \cdot W_{I_j(y), y} \cdot s_A(I_i(x), I_j(y)) \quad (4)$$

Answer to Question 3.c

Let the graph $K_{2,1}$ have nodes $A1, A2$ in A and $B1$ in B . Similarly, let the graph $K_{2,2}$ have nodes $A1, A2$ in A and $B1, B2$ in B .

Therefore, for iteration 1 -

$$s_A(A1, A2)_{K_{2,1}} = \frac{C1}{1 \times 1} \times s_B(B1, B1) = 0.8$$

$$s_A(A1, A2)_{K_{2,2}} = \frac{C1}{2 \times 2} \times [s_B(B1, B1) + s_B(B1, B2) + s_B(B2, B1) + s_B(B2, B2)] = \frac{0.8}{2 \times 2} \times 2 = 0.4$$

Results -

Iteration 1

$K_{2,1}$

$s_A('A1', 'A2') : 0.8,$

$s_A('A1', 'A1') : 1,$

$s_A('A2', 'A2') : 1$

$s_B('B1', 'B1') : 1$

$K_{2,2}$

$s_A('A1', 'A2') : 0.4,$

$s_A('A1', 'A1') : 1,$

$s_A('A2', 'A2') : 1$

$s_B('B1', 'B2') : 0.4,$

$s_B('B1', 'B1') : 1,$

$s_B('B2', 'B2') : 1$

Iteration 2

$K_{2,1}$

$s_A('A1', 'A2') : 0.8,$

$s_A('A1', 'A1') : 1,$

$s_A('A2', 'A2') : 1$

$s_B('B1', 'B1') : 1$

$K_{2,2}$

$s_A('A1', 'A2') : 0.5599999999999999,$

$s_A('A1', 'A1') : 1,$

$s_A('A2', 'A2') : 1$

$s_B('B1', 'B2') : 0.5599999999999999,$

$s_B('B1', 'B1') : 1,$

$s_B('B2', 'B2') : 1$

Iteration 3

$K_{2,1}$

$s_A('A1', 'A2') : 0.8,$

$s_A('A1', 'A1') : 1,$

$s_A('A2', 'A2') : 1$

$s_B('B1', 'B1') : 1$

$K_{2,2}$

$s_A('A1', 'A2') : 0.6240000000000001,$

$s_A('A1', 'A1') : 1,$

$s_A('A2', 'A2') : 1$

$s_B('B1', 'B2') : 0.6240000000000001,$

$s_B('B1', 'B1') : 1,$

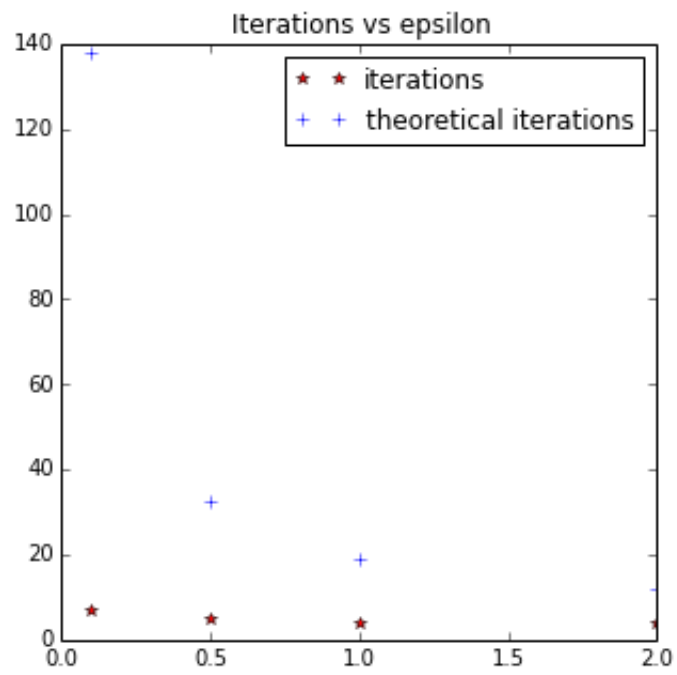
$s_B('B2', 'B2') : 1$

Answer to Question 4a

Answer to Question 4b

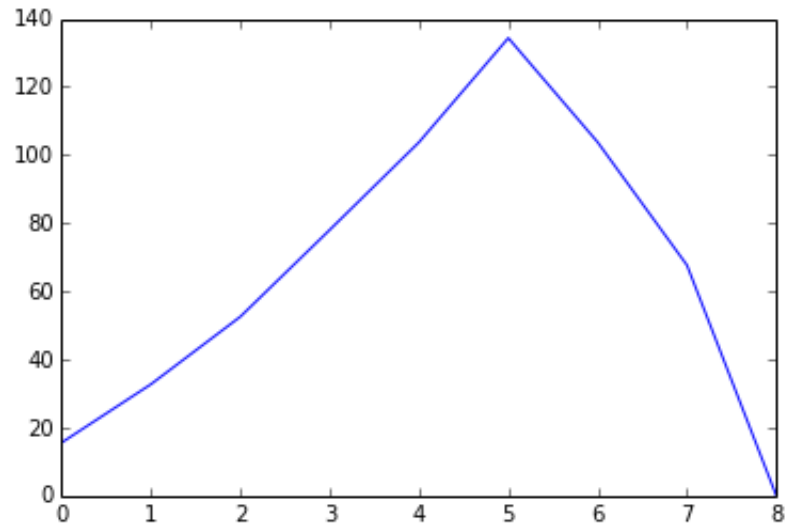
Answer to Question 4c

1

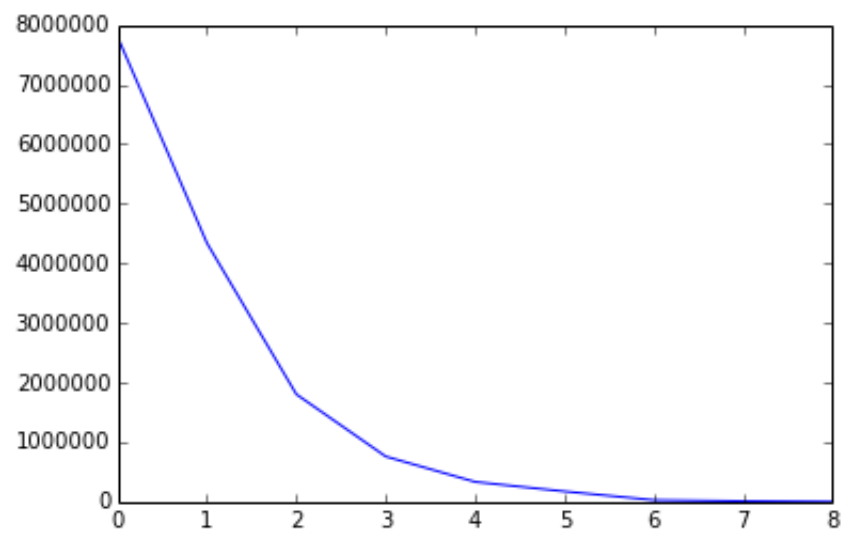


ϵ	Iterations	Theoretical iterations
0.1	7	137.67
0.5	5	32.36
1	4	18.93
2	3	11.94

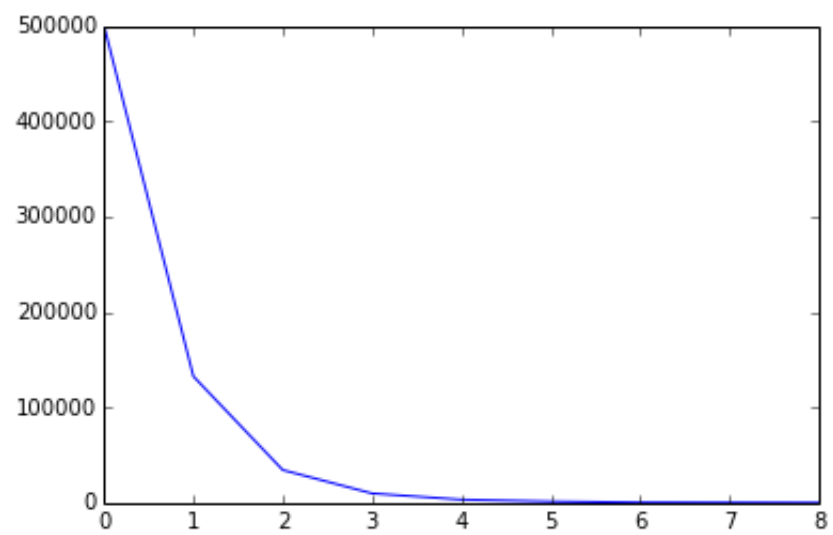
Density ($\rho(S_i)$) vs iteration



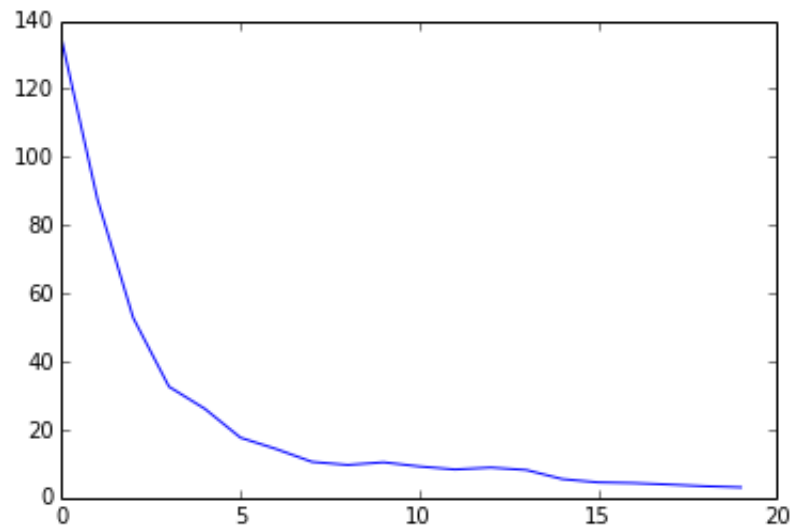
$|E(S_i)|$ vs iteration



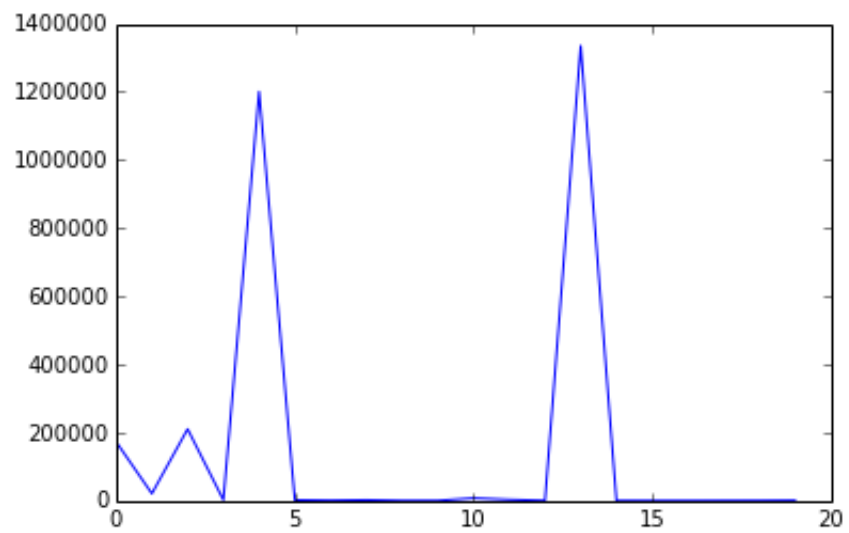
$|S_i|$ vs iteration



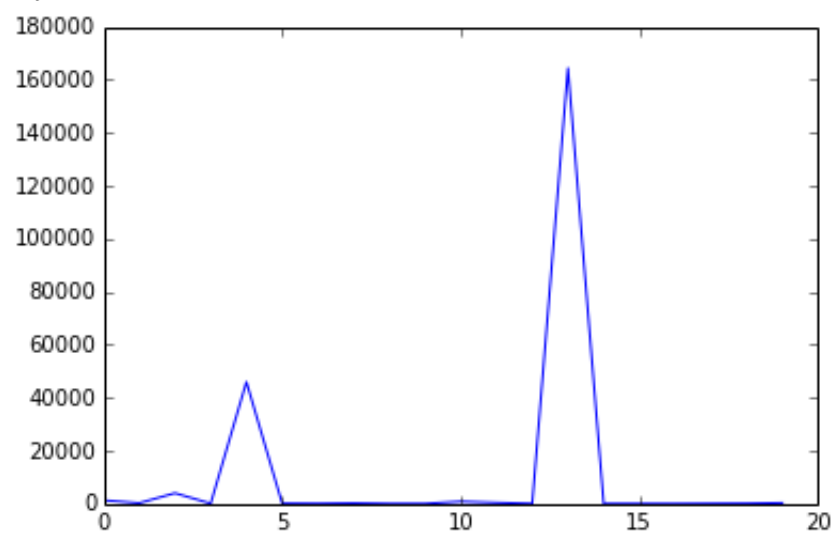
Density ($\rho(\bar{S}_j)$) vs iteration



$|E(\bar{S}_j)|$ vs iteration



$|\bar{S}_j|$ vs iteration



Code for Q1

Code for Q2

Code for Q4