Dsp assignment

Q.1) A discrete time signal $x[n]$ is defined as $x(n)=\begin{cases} 1+\frac{n}{3} & -3\le n\le 1 \\ 1 & 0\le n\le 3 \\ 0 & \text{elsewhere}\end{cases}$

a) Determine the values and sketch the signal $x(n)$?

A) $1+\frac{n}{3}$; $-3\le n\le 1$ $\Rightarrow$ for $-3 \Rightarrow 1+\frac{(-3)}{3}=0$

$\Rightarrow$ for $-2 \Rightarrow 1+\left(-\frac{2}{3}\right)=\frac{1}{3}$

$\Rightarrow$ for $-1 \Rightarrow 1+\left(-\frac{1}{3}\right)=\frac{2}{3}$

$\Rightarrow$ for $0 \Rightarrow 1=1$

1 ; for $1 \Rightarrow x(n)=1$

for $2 \Rightarrow x(n)=1$

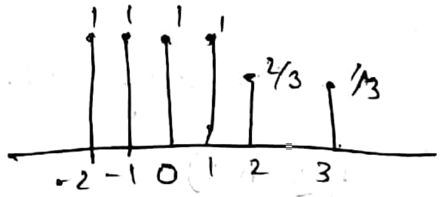
for $3 \Rightarrow x(n)=1$

b) sketch the signals that result if we:-

i) First fold $x(n)$ and then delay the resulting signal by four samples.

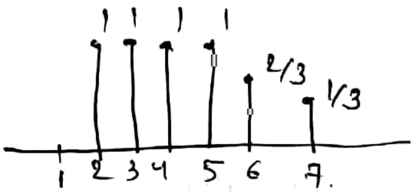
A) After folding $x(n)$, we get

$x(-n)=\{\ldots 0, 1, 1, 1, 1, 2/3, 1/3, 0 \ldots\}$



After delaying the folded signal by 4 samples.

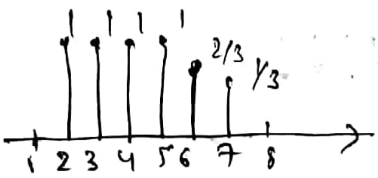
$x[-n+4]=\{\ldots 0, 0, 1, 1, 1, 1, 2/3, 1/3, 0 \ldots\}$



2) First delay $x(n)$ by four samples and then fold the resulting signal:-

$\Rightarrow$ Delaying $x[n]$ by four samples:-

$\Rightarrow$ Folding the above signal:-



c) sketch the signal $x(-n+4)$

A) $x(-n+4) = \{0, 1, 1, 1, 1, 2/3, 1/3, 0 \cdots\}$

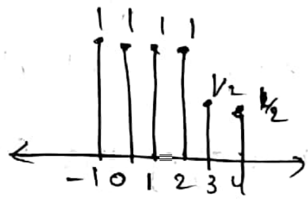
d) compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

A) By comparing results in parts (b) & (c) we can say that to get $x(-n+k)$ from $x(n)$ first we need to fold $x(n)$ which result in $x(-n)$ and then we need shift by k sample to right if $k>0$ (or) to left if $k<0$, results in $x(-n+k)$

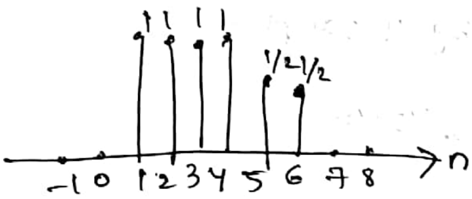
e) can you express the signal $x(n)$ in terms of signals $\delta(n)$ & $u(n)$?

A) $x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n+1) + u(n) - u(n-4)$

2.2) A discrete time signal $x(n)$ is shown in fig. sketch and label carefully each of the following signals. $x(n)$

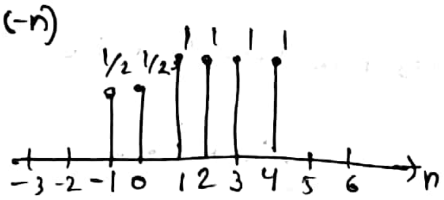


A) a) $x(n-2)$

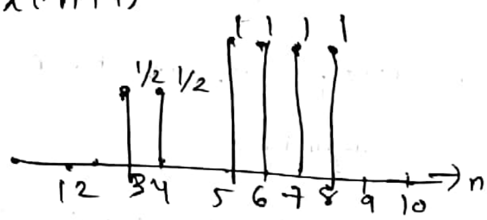


b) $x(4-n)$ (or) $x(-n+4)$

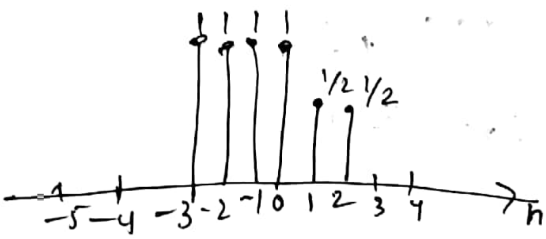
$x(-n)$



$x(-n+4)$



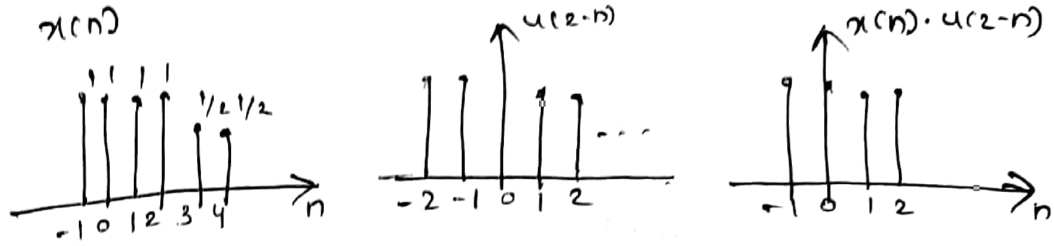
c) $x(n+2)$



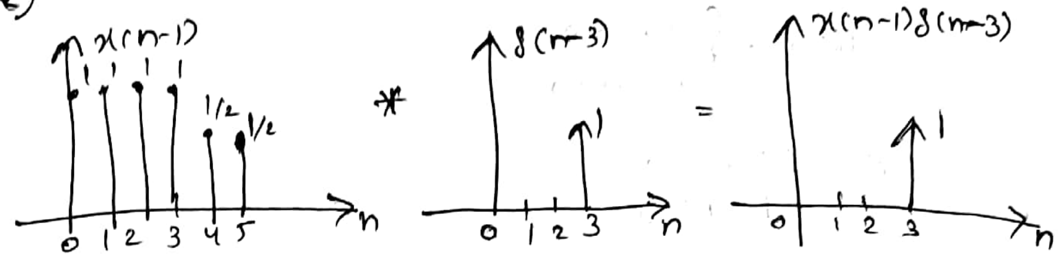
Scanned by CamScanner

d) $x(n) \cdot u(2-n)$

u(2-n)



e) $x(n-1) \cdot \delta(n-3)$



f) $x(n^2)$

⇒ for 0 ; x(0)

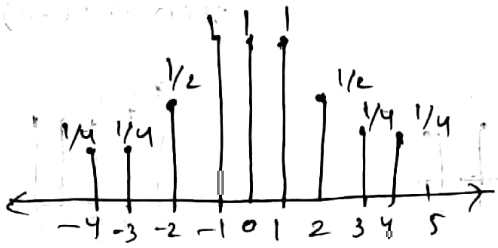
1 ; x(1)

for 2 & -2 ; x(4)

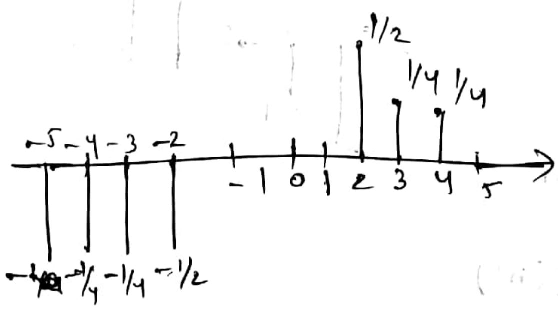
$x(n^2) = \{x(4), x(1), x(0), x(1), x(4), x(-1)\}$

$= \{-\frac{1}{2}, 1, 1, 1, 1/2, 0, 0\}$

g) even part ; $x_e(n) = \frac{x(n) + x(-n)}{2}$



h) odd part ; $x_o(n) = \frac{x(n) - x(-n)}{2}$



2.3) show that

a) $\delta(n) = u(n) - u(n-1)$

$$\Rightarrow u(n) - u(n-1) = \delta(n) = \begin{cases} 0 & n<0 \\ 1 & n=0 \\ 0 & n>0 \end{cases}$$

b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

A) $\Rightarrow \sum_{k=-\infty}^{\infty} \delta(k) = u(n) = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$

$$\sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$$

2.4) show that any signal can be decomposed into an even and an odd component. Is the decomposition unique? Illustrate your arguments using the signal, $x[n] = [2,3,4,5,6]$

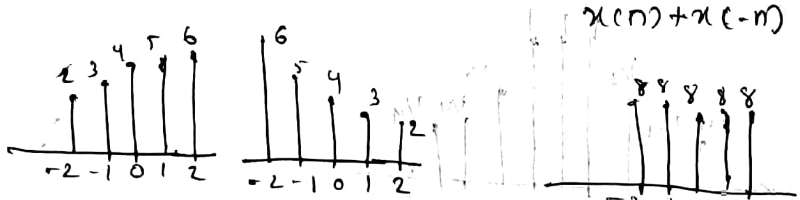
A) $x_e(n) = \dfrac{x(n) + x(-n)}{2}$ $\quad x_e(n) = x_e(-n)$

$x_o(n) = \dfrac{x(n) - x(-n)}{2}$ $\quad x_o(n) = -x_o(-n)$

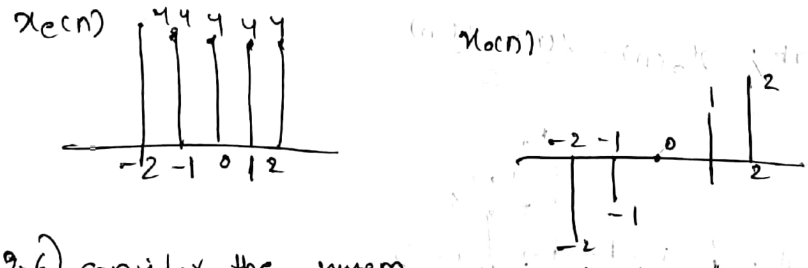
$\Rightarrow x(n) = x_e(n) + x_o(n)$

$x(n) = \{2,3,4,5,6\}$

$x(n) + x(-n)$



$x_e(n)$ $\quad x_o(n)$



2.6) consider the system

$y(n) = T[x(n)] = x(n^2)$

a) Determine if the system is time invariant.

The system is time variant.

A) $x(n) \rightarrow y(n) = x(n^2)$

$x(n-k) \rightarrow y_1(n) = x[(n-k)^2]$

$= x(n^2 + k^2 - 2nk)$

$\neq y(n-k)$

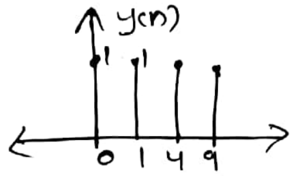
b) To clarify the result in part a) assume that the signal.

$x(n) = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$ is applied into the system.

1) sketch the signal $x(n)$

2) Determine and sketch the signal, $y(n) = T[x(n)]$

A) $y(n) = x(n^2)$

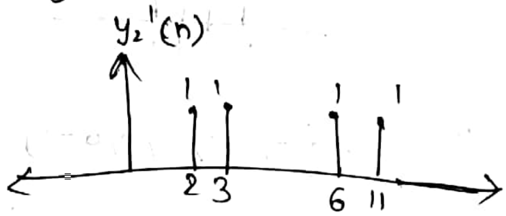


$y(n) = T[x(n)] = x[n^2] \Rightarrow \{x(0), x(1), x(2^2), x(3^2) \ldots\}$

$y(n) = x(n^2) = \{\underset{0}{1}, \underset{1}{1}, \underset{2}{0}, \underset{3}{0}, \underset{4}{0} \ldots\}$

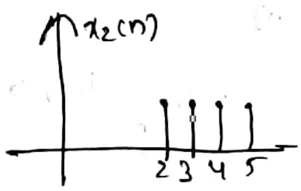
3) sketch the signal $y_2'(n) = y(n-2)$

A) $y(n-2) = \{0, 0, 1, 1, 0, 0 \ldots\}$
↑



4) Determine & sketch the signal $x_2(n) = x(n-2)$

A) $x(n-2) = \{\underset{0}{0}, \underset{1}{0}, \underset{2}{1}, \underset{3}{1}, \underset{4}{1}, \underset{5}{1}, 0 \ldots\}$



5) Determine & sketch the signal $y_2(n) = T[x_2(n)]$

A) $y_2(n) = x_2(n^2)$ $\quad n = 0 \Rightarrow x(0)$

$n = 1$ or $1 \Rightarrow x(1)$

$n = -2, 2 \Rightarrow x(4)$

$T[x_2(n-2)] = \{\ldots 0, 1, 0, \underset{\uparrow}{0}, 0, 1, 0\}$

Scanned by CamScanner

6) compare the signals $y_2(n)$ and $y(n-2)$. what is your conclusion.

A) $y_2(n) \neq y(n-2)$

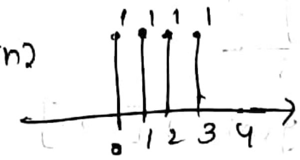
Because the system is time variant.

c) 1)

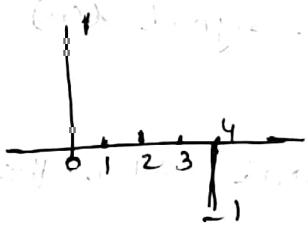
$= \{1,1,1,1\}$

2) $\Rightarrow y(n) = x(n) - x(n-1)$

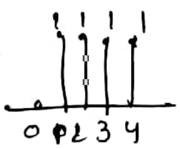
$x(n)$



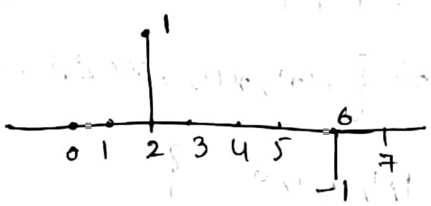
$y(n) =$



$x(n-1)$



3) $y(n-2) =$



$\Rightarrow \{0,0,1,0,0,0-1\}$

4) $x(n-2) \Rightarrow$

$\Rightarrow \{0,0,1,1,1,1\}$

5) $y_2(n) = \{0,0,1,0,0,0-1\}$

6) $y_2(n) = y(n-2) \Rightarrow$ system is time invariant.

d) 1) $y(n) = n \cdot x(n)$

$x(n) = \{ \ldots 0,1,1,1,1,0 \ldots \}$ n ≠ integer value from 0

2) $y(n) = \{ \ldots 0,1,2,3,4 \ldots \}$

3) $y(n-2) = \{ \ldots 0,0,0,1,2,3,4 \ldots \}$

4) $x(n-2) = \{ \ldots 0,0,0,1,1,1,1, \ldots \}$

5) $y_2(n) = T[x(n-2)] = \{\ldots 0, 0, 2, 3, 4, 5 \ldots\}$

6) $y_2(n) \neq y(n-2)$ ⇒ system is time variant

2.7) A discrete system can be

1) static & dynamic

2) Linear & Nonlinear

3) Time variant & time varying

4) causal & non causal

5) stable & unstable

Examine the following systems with respect to the properties above.

a) $y[n] = \cos[x(n)]$

A) 1) It is a static system. Because if we give any value as i/p it shows only present values only. For example, if we take $[x(n)]$ as we get o/p as 1, like that for any values of x, the o/p is in the limit of -1 to 1.

2) It is non-linear.

For example :- $x=0, y=0$; $\cos(x+y) = \cos(0+0) = \cos(0) = 1$

For, $\cos(x) + \cos(y) = \cos(0) + \cos(0) = 1+1 = 2$

So, $\cos(x+y) \neq \cos(x) + \cos(y)$

3) It is time-invariant.

$y[n] = \cos[x[n]]$

$y(n-n_0) = \cos[x[n-n_0]]$

$y_2[n] = \cos[x](n-n_0)]$

$y[n-n_0] = y_2[n]$

4) It is a casual system. Because it depends on present value.

5) It is a stable system. Because if we give only bounded i/p then we get bounded o/p.

b) $y[n] = \sum_{k=-\infty}^{n+1} x[n]$

1) It is a dynamic system, because it depends on present & past value.

2) Linear,

For Example:- $x[k_0]+x[k_1]+x[k_2]=x_1[n]$

$x[k_0+k_1+k_2]=x_1[n]$

$x[n]=x_1[n]$

3) Time invariant system.

4) Non-causal system.

5) unstable system. Because we don't get bounded o/p for bounded i/p.

c) $y[n]=x[n]\cdot \cos[w_0 n]$

A) 1) It is a static system. 2) Linear system.

Ex:- $x[n_0+n_1]\cdot \cos w_0[n]$

$y[n_0+n_1] = x[n_0]\cdot \cos[w_0 n] + x[n_1] \cos[w_0 n]$

$x[n_0]\cdot \cos[w_0 n]+x[n_1] \cos[w_0 n] = y_2[n]$

$y[n+n_0]=y_2[n]$

3) time variant system.

4) causal system 5) stable system.

d) $y[n]= x[-n+2]$

A) 1) Dynamic system. Because it depends on both past & present values

2) Linear system

3) Time Invariant system.

4) Non-causal system.

5) stable system.

e) $y[n] = Trun[x[n]]$, where $Trun[x[n]]$ denotes the integer part of $x[n]$, obtained by Truncation.

A) static, non linear, time invariant, causal, stable.

f) $y[n] = Round[x[n]]$, where $Round[x[n]]$ denotes the integer part of $x[n]$ obtained by Rounding.

A) 1) static

2) Nonlinear

3) time invariant.

4) causal

5) stable.

g) $y[n] = |x[n]|$

1) static; 2) Non-linear 3) Time invariant 4) causal 5) stable.

h) $y[n] = x[n] \cdot u[n]$

1) static 2) Linear 3) Time invariant 4) causal 5) stable.

i) $y[n] = x[n] + n \cdot x[n+1]$

1) Dynamic 2) Linear 3) Time variant 4) noncausal 5) ~~un~~stable.

j) $y[n] = x[2n]$

1) Dynamic 2) Linear 3) Time variant 4) noncausal 5) stable.

k) $y[n] = \begin{cases} x[n], & \text{if } x[n] \geq 0 \\ 0, & \text{if } x[n] < 0 \end{cases}$

1) Static 2) non linear 3) time invariant 4) causal 5) stable.

l) $y[n] = x[-n]$

1) Dynamic 2) linear 3) time invariant 4) non causal 5) stable.

m) $y[n] = x[n]$

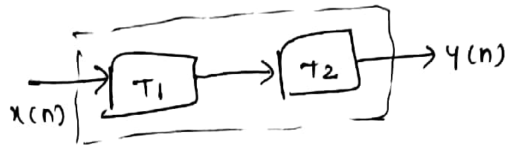
1) static 2) non linear 3) time invariant 4) Causal 5) stable.

7) The ideal sampling system with input $x_0(t)$ and output

$x[n] = x_0(nT)$; $-\infty < n < \infty$

1) static 2) linear 3) timeinvariant 4) causal 5) stable

2.8) Two discrete-time systems T_1 and T_2 are connected in cascade to form a new system T as shown in fig. prove or disprove the following statements.



$T = T_1 T_2$

a) if T_1 & T_2 are linear. then T is linear (i.e, the cascade connection of two linear systems is linear)

A) for T_1

if $u_1(n) = T_1[x_1[n]]$ & $u_2[n] = T_1[x_2[n]]$

if for T_2, $\alpha_1 x_1(n) + \alpha_2 x_2(n) = \alpha_1 u_1(n) + \alpha_1 u_2(n)$

$y_1(n) = T_2[u_1[n]]$ & $y_2[n] = T_2[u_2[n]]$

$\beta_1 u_1(n) + \beta_2 u_2(n) \Rightarrow y(n) = \beta_1 y_1(n) + \beta_2 y_2(n)$

since, $u_1(n) = T_1[x_1(n)]$ & $u_2[n] = T_2[x_2(n)]$

$A_1 x_1(n) + A_2 x_2(n) = A_1 T[x_1(n)] + A_2 T[x_2(n)]$

Here, $T = T_1 + T_1$; Hence T is linear.

b) If T_1 & T_2 are time invariant, then T is time invariant.

A) True, if for T_1

$x(n) \rightarrow v(n)$ &

$x(n-k) \rightarrow v(n-k)$

if for T_2

$v(n) \rightarrow y(n)$

$v(n-k) \rightarrow y(n-k)$

Scanned by CamScanner

for T_1, T_2, if $x(n) \rightarrow y(n)$

$$x(n-k) \rightarrow y(n-k)$$

$T = T_1 T_2$ is time invariant.

c) if T_1 & T_2 are causal, then T is causal.

A) True.

T_1 is causal $\rightarrow v(n)$ depends only on $x(k)$ for $k \leq n$

T_2 is causal $\rightarrow y(n)$ depends only on $v(k)$ for $k \leq n$.

$\therefore$ $y(n)$ depends only on $x(k)$ for $k \leq n$. Hence, T is causal.

d) If T_1 and T_2 are linear and time invariant, the same holds for T.

A) True.

if $v_1(n) = T_1[x(n)]$ & $v_2(n) = T_1[x_2(n)]$

$$\alpha_1 x_1(n) + \alpha_2 v_2(n) = \alpha_1 v_1(n) + \alpha_2 v_2(n)$$

similarly,

$$y_1(n) = T_2[v_1(n)] \text{ \& } y_2(n) = T_2[v_2(n)]$$

$$\beta_1 v_1(n) + \beta_2 v_2(n) \Rightarrow y(n) = \beta_1 y_1(n) + \beta_2 y_2(n)$$

$$v_1(n) = T_1[x_1(n)] \text{ \& } v_2(n) = T_2[x_2(n)]$$

$$A_1 x_1(n) + A_2 x_2[n] = A_1 T[x_1(n)] + A_2 T[x_2(n)]$$

for $T = T_1 T_2$

& if for T_1; $x(n) \rightarrow v(n)$ &

$$x(n-k) \rightarrow v(n-k)$$

if for T_2; $v(n) \rightarrow y(n)$

$$v(n-k) \rightarrow y(n-k)$$

For $T_1 T_2$, if $x(n) \rightarrow y(n)$

$$x(n-k) \rightarrow y(n-k)$$

$T = T_1 T_2$ is time invariant.

e) If T_1 & T_2 are linear and time invariant, then interchanging their order does not change the system T.

A) True. This follows from $h_1(n) * h_2(n) = h_2(n) * h_1(n)$

f) As in part (e) except that $T_1 T_2$ are now time varying

(Hint: use an example)

A) False. $T_1: y(n) = n \cdot x(n)$ &

$T_2: y(n) = n \cdot x(n+1)$

Then, $T_2[T_1[\delta(n)]] = T_2[0] = 0$

$T_1[T_2[\delta(n)]] = T_1[\delta(n+1)]$

$= -\delta(n+1)$

$\neq 0$

g) If T_1 & T_2 are nonlinear, then T is nonlinear.

A) False. $T_1: y(n) = x(n) + b$ &

$T_2: y(n) = x(n) - b \qquad b \neq 0$

$T[x(n)] = T_2[T_1[x(n)]] = T_2[x(n)+b] = x[n]$

Hence T is linear.

h) If T_1 & T_2 are stable, the T is stable.

A) True. T_1 is stable $\Rightarrow$ $v(n)$ is bounded if $x(n)$ is bounded.

T_2 is stable $\Rightarrow$ $y(n)$ is bounded if $x(n)$ is bounded.

Hence, $y(n)$ is bounded if $x(n)$ is bounded.

$\Rightarrow T = T_1 T_2$ is stable.

i) Show by an example that the inverse of parts (c) and (h) do not hold in general.

A) Inverse of (c). T_1 & T_2 are noncausal $\rightarrow$ T is non causal.

$T_1: y(n) = x(n+1)$ &

$T_2: y(n) = x(n-2)$

$\Rightarrow$ T: $y(n) = y(n-1)$

which is causal. Hence the inverse of (c) is false.

inverse of (b): T_1 & T_2 is unstable implies T is unstable.

Ex:- $T_1 : y(n) = e^{x(n)}$, stable & $T_2 : y(n) = \ln[x(n)]$ which is unstable.

But T: $y(n) = x(n)$, which is stable. Hence the inverse of (h) is false.

2.9) Let T be an LTI, relaxed and BIBO stable system with input $x(n)$ and output $y(n)$. Show that

a) if $x(n)$ is periodic with period N [ie, $x(n) = x(n+N)$ for all $n \geq 0$], the output $y(n)$ tends to a periodic signal with the same period.

A) $x(n) = x(n+N) \;\forall\; n \geq 0$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$y(n+N) = \sum_{k=-\infty}^{n+N} h(k) \cdot x(n+N-k)$$

for BIBO system $\lim_{n\to\infty} |h(n)| = 0$

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k) \cdot x(n-k) = 0$$

$$\lim_{n\to\infty} y(n+N) = y(N)$$

$\therefore y(N) = y(N+N)$

b) If $x(n)$ is bounded and tends to a constant, the output will also tend to a constant.

A) $x(n) = x_0(n) + a \cdot u(n)$; $x_0(n) \rightarrow$ bounded with $\lim_{n\to\infty} x_0(n)$

$\Rightarrow y(n) = a \sum_{k=0}^{\infty} h(k) \cdot u(n-k) + \sum_{k=0}^{\infty} h(k) \cdot x_0(n-k) = a \sum_{k=0}^{n} h(k) \cdot u(n-k)$

$\Rightarrow \sum_n x_0^2(n) < \infty \Rightarrow \sum_n y_0^2(n) < \infty$. Hence $\lim_{n\to\infty} |y_0(n)|$

10) The following input-output pairs have been observed during the operation of a time invariant system:

$x_1(n) = \{1, 0, 2\} \xleftrightarrow{T} y_1(n) = \{0, \underset{\uparrow}{1}, 2\}$

$x_2(n) = \{0, 0, 3\} \xleftrightarrow{T} y_2(n) = \{0, \underset{\uparrow}{1}, 0, 2\}$

$x_3(n) = \{0, 0, 0, 1\} \xleftrightarrow{T} y_3(n) = \{1, \underset{\uparrow}{2}, 1\}$

Can you draw any conclusions regarding the linearity of the system.

What is the impulse response of the system?

A) As this is a time-invariant system.

$y_2(n)$ should have only 3 elements and

$y_3(n)$ should have 4 elements.

So, it is non-linear.

11) The following input-output pairs have been observed.

$x_3(n) = \{0, 1\} \xrightarrow{T} y_3(n) = \{1, 2, 1\}$

Can you draw any conclusions about the time invariance of this system?

$x_1(n) + x_2(n) = \delta(n)$

If system is linear, the impulse response of the system is,

$y_1(n) + y_2(n) = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$

if system was time invariant, the response of $x_3(n)$ would be $\{\underset{\uparrow}{3}, 2, 1, 3, 1\}$

12) The only available information about the system consists of N input-output pairs of signals $y_i(n) = T[x_i(n)]$; $i = 1, 2 \dots N$

a) What is the class of i/p signals for which we can determine the output, using the information above, if the system is known to be linear?

A) Linear combination of signall in the form

of $x_i(n)$; $i = 1, 2 \ldots N$

because if we take $i = 1, 3$

$y_1(n) = x_1(n) \Rightarrow y(n) = y_2(n) + y_3(n)$

$y_3(n) = x_3(n) \qquad = x_1(n) + x_3(n)$

$y(n) = x_1(n) + x_3(n)$ ∴ linear

b) same repeat, for the system is invariant.

Any $x_i(n-k)$ where k is any integer; $i = 1, 2 \ldots n$

i replace, $n = n - n_0 \Rightarrow x_i(n - n_0 - k)$

$x(n)$ by $x(n-n_0) \Rightarrow x_i(n-k-n_0)$
(Time invariant)

2.13) show that the necessary and sufficient condition for a relaxed LTI system to be BIBO stable is,

$$\sum_{n=-\infty}^{\infty} |h(n)| \leq M_b < \infty \text{ for some constant } M_0.$$

A) A system to be BIBO stable, only when bounded output should produce bounded input.

$$y(n) = \sum_{k}^{\infty} |h(k)| \cdot |x(n-k)|$$

$$= \sum_{k} |x(n-k)| \leq M_n \text{ (some constant)}$$

So, $\sum_{n=-\infty}^{\infty} |y(n)|$

$\Rightarrow$ A system to be BIBO stable only when bounded input produce bounded output.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k) \; ; \; n \leq n-k, \; k \geq 0$$

$$|y(n)| = \sum_{x=-\infty}^{\infty} |h(k)| \, |x(n-k)|$$

as, $\sum_{k=-\infty}^{\infty} |x(n-k)| \le m_n$ for some constant.

$|y(n)|$, if $< \infty$ If & only if $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$.

So, $\sum_{k=-\infty}^{\infty} |h(n)| < \infty$

2.14) show that

a) A relaxed linear system is causal if & only if for any input $x(n)$ such that

$x(n)=0$ for $n<n_0 \Rightarrow y(n)=0$ for $n<n_0$

A) If a system is causal output depends only on the present and past inputs as $x(n)=0$ for $n<0$, then $y(n)$ also becomes for $n<n_0$.

b) A relaxed LTI system is caused if and only if $h(n)=0$ for $n<0$.

A) $y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$

$h(n)=0$; $n<0$ and $n \ge m$

So, $y(n)$ reduces to $y(n) = \sum_{k=m}^{n-1} h(k) \cdot x(n-k)$

If it is infinite impulse response.

Then $y(n) = \sum_{k=0}^{\infty} h(k) \cdot x(n-k)$

2.15) a) show that for any real or complex constant a, and any finite integer numbers M and N, we have

$$\sum_{n=M}^{N} a^n = \begin{cases} \dfrac{a^M - a^{N+1}}{1-a} ; & \text{if } a \ne 1 \\ N-M+1 ; & \text{if } a=1 \end{cases}$$

for $a=1$; $\sum_{n=M}^{N} a^n = N-M+1$

for $a \ne 1$; $\sum_{n=M}^{N} a^n = \dfrac{a^M - a^{N+1}}{1-a}$

Scanned by CamScanner

$$(1-a)\sum_{n=m}^{N} a^n = a^m + a^{m+1} - a^{m+1} + \cdots a^N - a^N - a^{N+1}$$

$$= a^m - a^{N+1}$$

2.15) b) show that if $|a| < 1$, then

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

for $M=0$, $|a|<1$ and $N \to \infty$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \; ; \; |a|<1$$

2.16) a) if $y(n) = x(n) * h(n)$. show that $\Sigma_y = \Sigma_x \Sigma_h$, where

$$\Sigma_x = \sum_{n=-\infty}^{\infty} x(n)$$

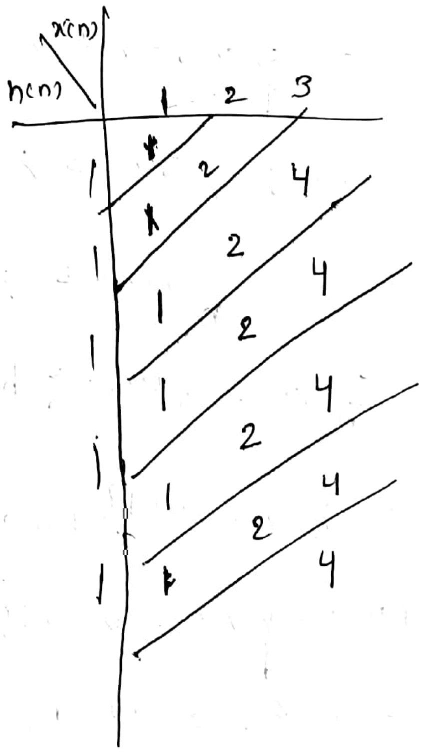
A) $y(n) = \sum_k h(k) \cdot x(n-k)$

$$\sum_n y(n) = \sum_n \sum_k h(k) \cdot x(n-k)$$

$$\sum_n y(n) = \sum_k h(k) \cdot \sum_{n=-\infty}^{\infty} x(n-k)$$

$$\sum_n y(n) = \left(\sum_k h(k) \cdot \left(\sum_n x(n) \right) \right]$$

b) compute the convolution $y(n) = x(n) * h(n)$ of the following signals and check the corrections of the results by using the test in (a).



1) $x(n) = \{1, 2, 4\}$, $h(n) = (1, 1, 1, 1)$

A) $y(n) = \{1, 3, 7, 7, 7, 6, 4\}$

$$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$$

$35 = 7 \times 5$

$35 = 35$

2) $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

A) $x(n) = \{1, 2, -1\}$; $h(n) = \{1, 2, -1\}$

$y(n) = x(n) * h(n)$

$y(n) = \{1, 4, 2, -4, 1\}$

Scanned by CamScanner

$\sum_n y(n) = 4$; $\sum_n x(n) = 2$; $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n (x) \cdot \sum h(n)$

$4 = 4$

h(n) \ x(n)	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

3) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{1/2, 1/2, 1, 1/2\}$

A) $y(n) = \{0, 1/2, -1/2, 3/2, -2, 0, -5/2, 2\}$

$\sum_n y(n) = -5$; $\sum_n x(n) = -2$

$\sum_n h(n) = \frac{5}{2}$

$\sum_n y(n) = \sum_n x(n) \cdot h(n)$

$-5 = -5$

h(n) \ x(n)	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1/2	0	1/2	-1	3/2	-2
1/2	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

4) $x(n) = \{1, 2, 3, 4, 5\}$, $h(n) = \{1\}$

A) $y(n) = \{1, 2, 3, 4, 5\}$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$

$15 = 15(1)$

5) $x(n) = \{1, 2, 3\}$, $h(n) = \{0, 0, 1, 1, 1, 1\}$

A) $y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$
$\uparrow$

$\sum_n y(n) = 8$; $\sum_n x(n) = 2$; $\sum_n h(n) = 4$

$\sum_n y(n) = \sum_n x(n) = 2$; $\sum_n h(n) = 4$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n) \Rightarrow 8 = 8$

h(n) \ x(n)	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

6) $x(n) = \{0, 0, 1, 1, 1, 1\}$; $h(n) = \{1, -2, 3\}$

A) $y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$
$\uparrow$

$\sum y(n) = 8$; $\sum_n x(n) = 4$; $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$

$8 = 4 \cdot 2$

$8 = 8$

h(n) \ x(n)	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

7) $x(n) = \{0, 1, 4, -3\}$; $h(n) = \{1, 0, -1, -1\}$

A) $y(n) = \{\underset{\uparrow}{0}, 1, 4, -4, -5, -1, 3\}$

h(n) \ x(n)	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

$\sum_n y(n) = -2$; $\sum_n x(n) = -2$

$\sum_n h(n) = 1$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$

$-2 = -2 \cdot 1$

$-2 = -2.$

8) $x(n) = \{1, 1, 2\}$, $h(n) = u(n)$

A) $y(n) = \{\underset{\uparrow}{1}, 2, 4, 3, 2\}$

h(n) \ x(n)	1	1	2
1	1	1	2
1	1	1	2
1	1	1	2

$\sum_n y(n) = 12$; $\sum_n x(n) = 4$; $\sum_n h(n) = 3$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$

$12 = 4 \cdot 3$

$12 = 12$

9) $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$; $h(n) = \{1, -2, -3, 4\}$

A) $y(n) = \{1, -1, -5, 2, 3, -5, 1, 4\}$

h(n) \ x(n)	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
4	-4	-4	0	-4	-4

$\sum_n y(n) = 0$; $\sum_n x(n) = 4$; $\sum_n h(n) = 0$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$

$0 = 0$

10) $x(n) = \{1, 2, \underset{\uparrow}{0}, 2, 1\}$; $h(n) = x(n)$

A) $y(n) = \{1, 4, 4, 4, \underset{\uparrow}{10}, 4, 4, 4\}$

h(n) \ x(n)	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

$\sum_n y(n) = 36$; $\sum_n x(n) = 6$

$\sum_n h(n) = 6$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$

$36 = 36$

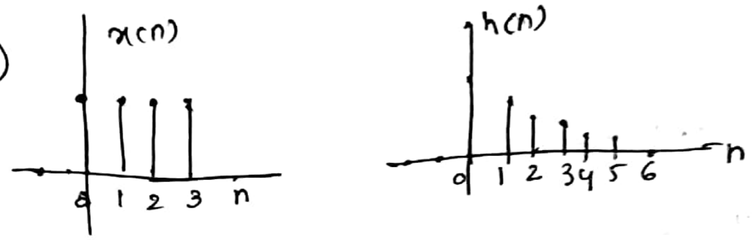
1) $x(n) = \left(\frac{1}{2}\right)^n \cdot u(n)$; $h(n) = \left(\frac{1}{4}\right)^n \cdot u(n)$

A) $y(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right] u(n)$

$\sum_n y(n) = \frac{8}{3}$; $\sum_n h(n) = \frac{4}{3}$; $\sum_n x(n) = 2$

2.14) compute and plot the convolutions $x(n) * h(n)$ and $h(n) * x(n)$ for the pairs of signals shown in fig.

a)



A) $x(n) = \{1, 1, 1, 1\}$; $h(n) = \{6, 5, 4, 3, 2, 1, 0\}$

$y(n) = x(n) * h(n)$

$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$

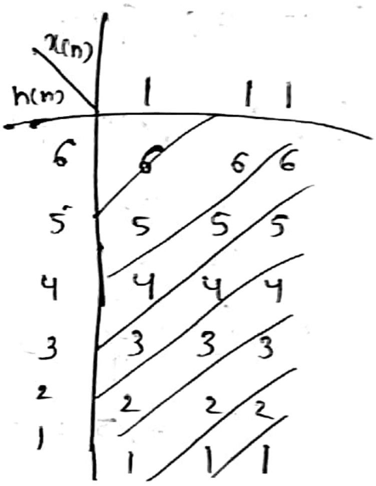


fig a

b)

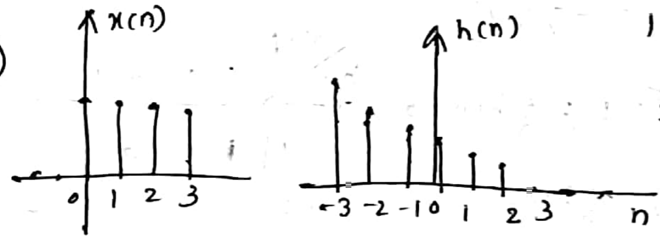


fig a

$x(n) = \{1, 1, 1, 1\}$; $h(n) = \{6, 5, 4, 3, 2, 1, 0\}$

$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$

c)

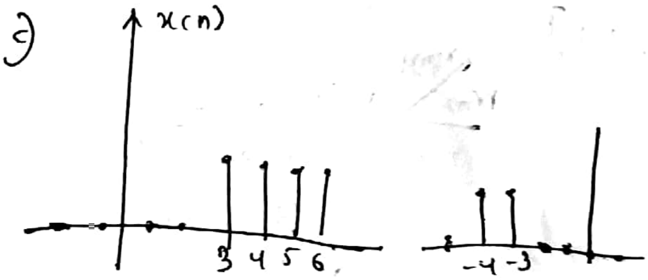


fig a

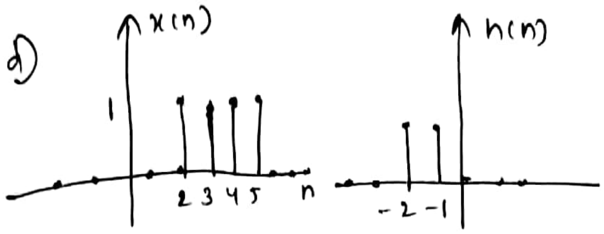
A) $x(n) = \{0, 0, 0, 1, 1, 1, 1\}$

$h(n) = \{1, 1, 0, 0, 0\}$

$y(n) = \{0, 0, 0, 1, 2, 2, 2, 1, 0, 0, 0\}$

h(n) \ x(m)	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

d)



A) $x(n) = \{0, 0, \underset{\uparrow}{1}, 1, 1, 1\}$

$h(n) = \{1, 1, \underset{\uparrow}{0}, 0, 0\}$

$y(n) = \{0, 0, \underset{\uparrow}{1}, 2, 2, 2, 1, 0, 0, 0\}$

h(n) \ x(n)	0	0	1	1	1	1
1	0	0	1	1	1	1
1	0	0	1	1	1	1
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

2.18 Determine & sketch the convolution $y(n)$ of the signals

$$x(n) = \begin{cases} \frac{1}{3}n, & 0 \le n \le 6 \\ 0, & \text{elsewhere} \end{cases}, \quad h(n) = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

a) Graphically
b) Analytically.

A) a) $x(n) = \{0, \frac{1}{3}, \frac{2}{3}, 1, 4/3, 5/3, 2\}$

$h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = \{0, \frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$

h(n) \ x(m)	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2

b) $x(n) = \frac{1}{3}n\,(u(n) - u(n-7))$

$h(n) = u(n+2) - u(n-3)$

$y(n) = \frac{1}{3}n\,(u(n) - u(n-7)) * (u(n+2) - u(n-3))$

$= \frac{1}{3}n\, u(n) * u(n+2) - \frac{1}{3}n\,(u(n) * u(n-3)) - \frac{1}{3}n\,(u(n-7) * u(n+2)) + \frac{1}{3}n\,(u(n-7) * u(n-3))$

Scanned by CamScanner

2.19) Compute the convolution $y(n)$ of the signal

$$x(n) = \begin{cases} \alpha^n ; & -3 \le n \le 5 \\ 0 ; & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1 ; & 0 \le n \le 4 \\ 0 ; & \text{elsewhere} \end{cases}$$

A) $y(n) = \sum_{k=0}^{4} h(k) \cdot x(n-k)$

$x(n) = \{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, 1, \alpha \dots \alpha^5\}$

$h(n) = \{1, 1, 1, 1, 1\}$

$$y(n) = \begin{cases} \sum_{k=0}^{4} x(n-k) ; & -3 \le n \le 9 \\ 0 & \text{elsewhere} \end{cases}$$

$\therefore y(-3) = \alpha^3$

$y(-2) = x(-3) + x(-2) = \alpha^{-3} + \alpha^{-2}$; $y(-1) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1}$

$y(0) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1$

$y(1) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha$; $y(2) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^2$

$y(3) = \alpha^{-1} + 1 + \alpha + \alpha^2 + \alpha^3$; $y(4) = \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1$

$y(5) = \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$; $y(6) = \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$

$y(7) = \alpha^3 + \alpha^4 + \alpha^5$; $y(8) = \alpha^4 + \alpha^5$; $y(9) = \alpha^5$

2.20) Consider the following 3 operations.

a) Multiply the integer numbers: 131 and 122

A) $131 \times 122 = 15982$

b) Compute the convolution of signals.

$\{1, 3, 1\} * \{1, 2, 2\}$

x(n) / y(n)	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

A) $y(n) = \{15, 9, 8, 2\}$

c) Multiply the polynomials:-

$1 + 3z + z^2$ and $1 + 2z + 2z^2$

$(z^2+3z+1) * (2z^2+2z+1)$

$= 2z^4 + 8z^3 + 9z^2 + 5z + 1$

d) Repeats (a) for the numbers 1.31 and 12.2

A) $1.31 * 12.2 = 15.98$

e) comment on your results.

There are different ways to perform convolution.

2.21) compute the convolution $y(n) = x(n) * h(n)$ of the following pairs of signals.

a) $x(n) = a^n . u(n)$; $h(n) = b^n . u(n)$ when $a \neq b$ and when $a = b$

A) $y(n) = x(n) * h(n)$

$= a^n . u(n) * b^n . u(n)$

$= [a^n * b^n] . u(n)$

$y(n) = \sum_{k=0}^{n} a^k u(k) . b^{n-k} u(n-k)$

$= b^n \sum_{k=0}^{n} a^k . u(k) . b^{-k}$

$= b^n \sum_{k=0}^{n} (ab)^{-k}$

if $a \neq b$, then $y(n) = \dfrac{b^{n+1} - a^{n+1}}{b-a} . u(n)$

if $a = b \Rightarrow b^n (n+1) . u(n)$

b) $x(n) = \begin{cases} 1 ; & n = -2, 0, 1 \\ 2 ; & n = 1 \\ 0 ; & \text{elsewhere} \end{cases}$ $\quad h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

A) $x(n) = \{1, 2, 1, 1\}$; $h(n) = \{1, -1, 0, 0, 1, 1\}$

$y(n) = \{1, 1, -1, 0, 0, 3, 3, 2, 1\}$

h(n) \ x(n)	1	2	1	1
1	1	2	1	1
-1	-1	-2	-1	-1
0	0	0	0	0
0	0	0	0	0
1	1	2	1	1
1	1	2	1	1

c) $x(n) = u(n+1) - u(n-4) - \delta(n+3)$

$h(n) = [u(n+2) - u(n-3)] (3 - |n|)$

A) $x(n) = \{1, 1, 1, 1, 1, 0, -1\}$

$h(n) = \{1, 2, 3, 2, 1\}$; $y(n) = \{1, 3, 6, 8, 9, 5, 1, -2, -2, -1\}$

d) $x(n) = u(n) - u(n-5)$

$h(n) = u(n-2) - u(n-8) + u(n-11) - u(n-17)$

A) $x(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1, 0, -1\}$

$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$y(n) = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, -1\}$

h(n) \ x(n)	1	1	1	1	1	0	-1
1	1	1	1	1	1	0	-1
2	2	2	2	2	2	0	-2
3	3	3	3	3	3	0	-3
2	2	2	2	2	2	0	-2
1	1	1	1	1	1	0	-1

⇒ d) $x(n) = \{1, 1, 1, 1, 1\}$

$h'(n) = \{0, 0, 1, 1, 1, 1, 1, 1\}$

$h(n) = h'(n) + h'(n-9)$

$y(n) = y'(n) + y'(n-9)$, when

$y'(n) = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

2.22) Let $x(n)$ be the input signal to a discrete time filter with impulse response $h_1(n)$ and let $y_1(n)$ be the corresponding output.

a) compute and sketch $x(n)$ and $y_1(n)$ in the following cases, using the same scale in all figures.

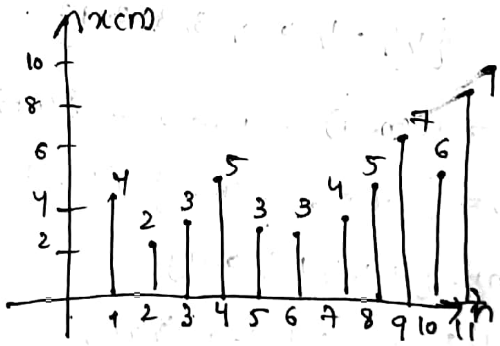
$x(n) = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

$h_1(n) = \{1, 1\}$; $h_2(n) = \{1, 2, 1\}$; $h_3(n) = \{1/2, 1/2\}$

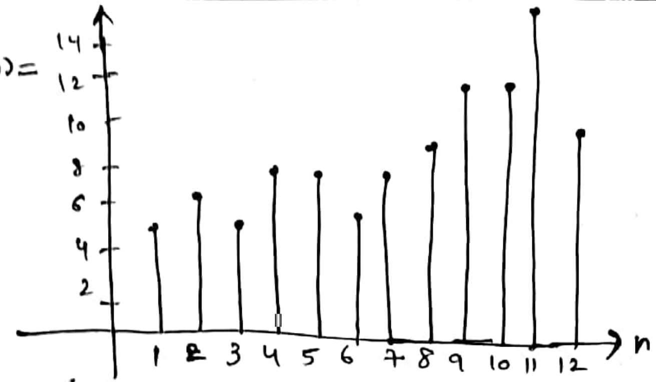
$h_4(n) = \{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$; $h_5(n) = \{\frac{1}{4}, -1/2, 1/4\}$

Sketch $x(n)$, $y_1(n)$, $y_2(n)$ on one graph and $x(n)$, $y_3(n)$, $y_5(n)$ on another graph.

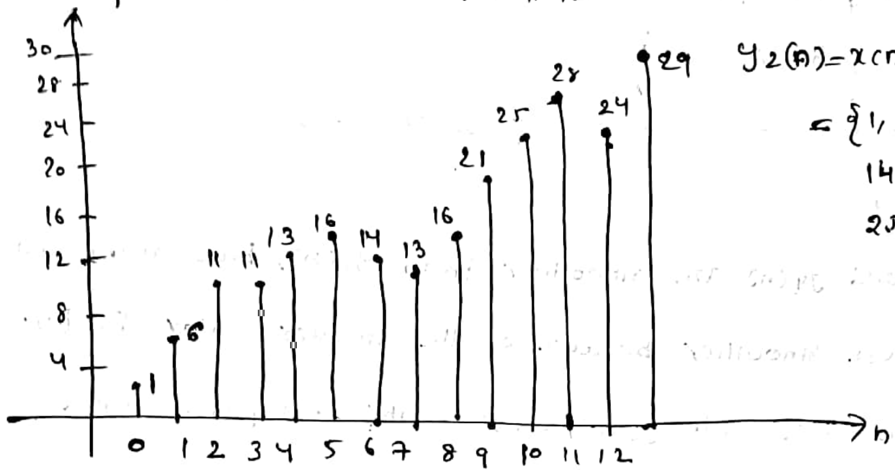
A) $y_1(n) = x(n) * h_1(n)$.



$y_1(n) =$

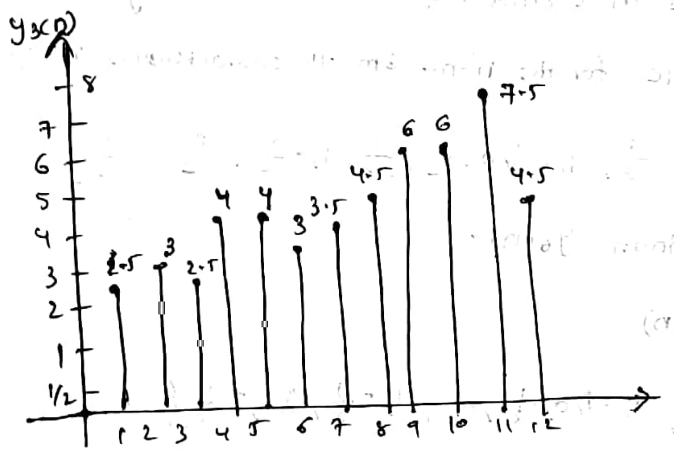


⇒

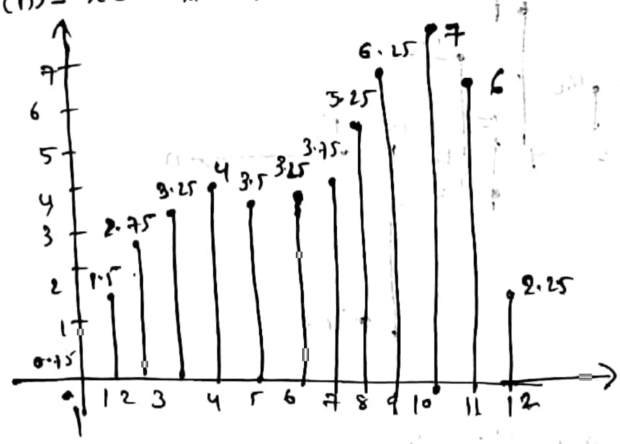


$y_2(n) = x(n) * h_2(n)$

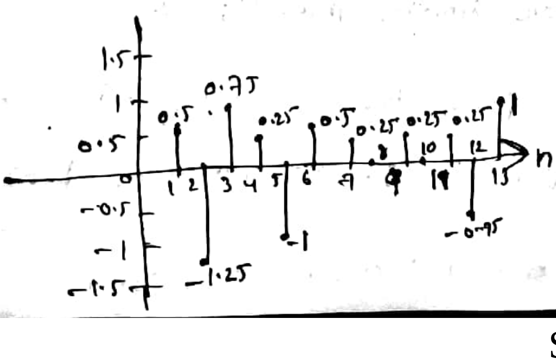
$= \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28, 24, 29\}$



⇒ $y_4(n) = x(n) * h_4(n)$



⇒ $y_5(n) = x(n) * h_5(n)$



b) what is the difference between $y_1(n)$ & $y_2(n)$ and between $y_3(n)$ and $y_4(n)$?

A) $y_3(n) = \frac{1}{2} y_1(n)$, because

$h_3(n) = \frac{1}{2} h_1(n)$

$y_4(n) = \frac{1}{4} y_2(n)$, because

$h_4(n) = \frac{1}{4} h_2(n)$

c) comment on the smoothness of $y_2(n)$ and $y_1(n)$ and between $y_3(n)$ and $y_4(n)$?

A) $y_2(n)$ and $y_4(n)$ are smoother than $y_1(n)$, but $y_4(n)$ will appear even smoother because of the smaller scalar factor.

d) Compare $y_4(n)$ with $y_5(n)$. what is the output, can you explain it?

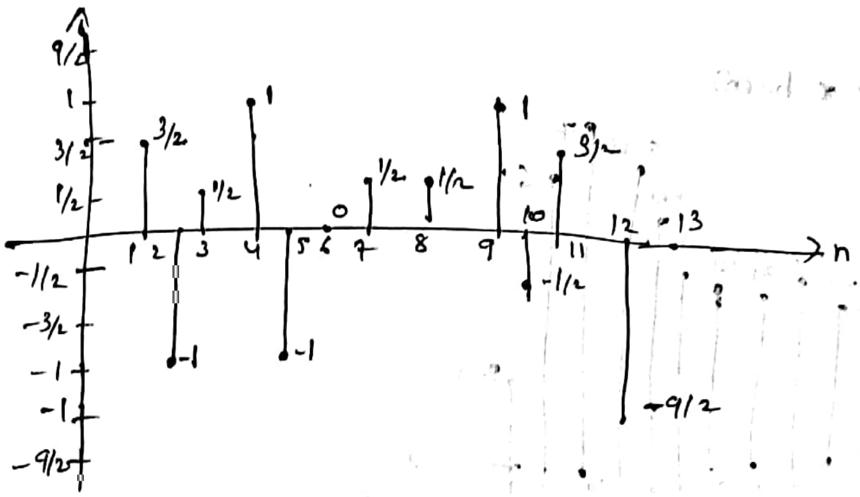
A) $y_4(n)$. system result in a smoother output. The negative value of $h_5(0)$ is responsible for the non-smooth characteristics of $y_5(n)$.

e) $y_6(n) = \{\frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{3}{2}, -\frac{9}{2}\}$

$y_2(n)$ is smoother than $y_6(n)$.

A) $y_6(n) = x(n) * h_6(n)$

$y_6(n) = \{\frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{3}{2}, -\frac{9}{2}\}$



$y_2(n)$ is more smother than $y_6(n)$.

Q.23) we can express the unit sample in terms of the unit step function as $\delta(n) = u(n) - u(n-1)$. then, $h(n) = h(n) * \delta(n)$

$= h(n) * [u(n) - u(n-1)]$

$\dots(n) = h(n) * u(n-1)$; $= \delta(n) - \delta(n-1)$

using this defination of $h(n)$

$$y(n) = h(n) * x(n)$$

$$= [\delta(n) - \delta(n-1)] * x(n)$$

$$= \delta(n) * x(n) - \delta(n-1) * x(n)$$

2.24) Consider the signal $\gamma(n) = a^n u(n)$, $0 < a < 1$.

a) Show that any sequence $x(n)$ can be decomposed as,

$x(n) = \sum_{n=-\infty}^{\infty} c_k \gamma(n-k)$ and express c_k in terms of $x(n)$.

A) $\delta(n) = \gamma(n) - a\gamma(n-1)$

$$\delta(n-k) = \gamma(n-k) - a\gamma(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot \delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k) \cdot [\gamma(n-k) - a\gamma(n-k-1)]$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot \gamma(n-k) - a \sum_{k=-\infty}^{\infty} x(k) \cdot \gamma(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot \gamma(n-k) - a \sum_{k=-\infty}^{\infty} x(k-1)\gamma(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - a \cdot x(k-1)] \cdot \gamma(n-k)$$

Thus, $c_k = x(k) - a x(k-1)$

2.24) b) use the properties of linearity and time invariance to express the output $y(n) = T[x(n)]$ in terms of the input $x(n)$ and the signal.

$y(n) = T[\gamma(n)]$, where $T[\cdot]$ is an LTI system.

A) $y(n) = T[x(n)]$

$$= T\left[\sum_{k=-\infty}^{\infty} c_k \cdot \gamma(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} c_k \cdot T[\gamma(n-k)]$$

$$= \sum_{k=-\infty}^{\infty} c_k \cdot \gamma(n-k)$$

c) Express the impulse response $h(n) = T[\delta(n)]$ in terms of $g(n)$

A) $h[n] = T[\delta(n)]$

$h[n] = T[\delta(n) - a.\delta(n-1)]$

$= g(n) - a.g(n-1)$

2.25) Determine the zero-input response of the system described by the second order difference equation.

$x(n) - 3y(n-1) - 4.y(n-2) = 0.$

A) with $x(n)=0$

$-3y(n-1) - 4y(n-2) = 0$ (divide by -3)

$y(n-1) + \frac{4}{3}y(n-3) = 0$

at $n=0$

$y(-1) = \frac{-4}{3}y(-3)$

at $n=1$

$y(0) = \frac{-4}{3}y(-1) = \left(\frac{-4}{3}\right)^2 . y(-2)$

$y(1) = \left(-\frac{4}{3}\right)^3 . y(-2)$

$y(k) = \left(-\frac{4}{3}\right)^{k+2} . y(-2)$

zero input response.

2.26) Determine the particular solution of the difference equation.

$y(n) = \frac{5}{6}.y(n-1) - \frac{1}{6}y(n-2) + x(n)$

when the forcing function is $x(n) = 2^n u(n)$.

A) at $y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$

$x(n) = y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2)$

characteristic equation is,

$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0$; $\lambda = 1/2, 1/3$

So, $y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{3}\right)^n$

$x(n) = 2^n \cdot u(n)$

$y_i(n) = k(2^n)u(n)$

So, $k(2^n) \cdot u(n) - k\left(\frac{5}{6}\right)(2^{n-1}) \cdot u(n-1) + k\left(\frac{1}{6}\right)(2^{n-2}) \cdot$

$u(n-2) = 2^n \cdot u(n)$

for $n=2$

$$4k - \frac{5k}{3} + \frac{k}{6} = 4$$

$$k = \frac{8}{6}$$

Total solution is, $y(n) + y_h(n) = y(n)$

$y(n) = \frac{8}{5}(2^n) \cdot u(n) + c_1\left(\frac{1}{2}\right)^n \cdot u(n) + c_2\left(\frac{1}{3}\right)^n \cdot u(n)$.

Assume, $y(-2) = y(-1) = 0$; so $y(0) = 1$.

then, $y(1) = \frac{5}{6}y(0) + 2 = \frac{17}{6}$

So, $\frac{8}{5} + c_1 + c_2 = 1$; $c_1 + c_2 = \frac{3}{5} \rightarrow ①$

$\frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 = \frac{17}{6}$

$3c_1 + 2c_2 = -\frac{11}{5} \rightarrow ②$

By solving ① & ②

$c_1 = -1$ & $c_2 = \frac{2}{5}$

So the total solution is,

$$y[n] = \left[\frac{8}{5}(2)^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right]u(n)$$

2.27) Determine the response $y(n)$, $n \geq 0$ of the system described by the second order difference equation.

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ to input, $x(n) = 4^n u(n)$

A) $y(n) - 3 \cdot y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

characteristic equation is, $\lambda^2 - 3\lambda - 4 = 0$

$\lambda = 4, -1$

So $y_h(n) = c_1 4^n + c_2(-1)^2$

$x(n) = 4^n \cdot u(n)$; $y_p(n) = k_n 4^n \cdot u(n)$

$$k n 4^n u(n) - 3k(n-1)4^{n-1} \cdot u(n-1) - 4k(n-2)4^{n-2} u(n-2)$$

$$= 4^n u(n) - 1 \cdot 2(4)^{n-1} u(n-1)$$

for $n=2$; $\quad k(32 \cdots = 4^2+8$

$$= 24$$

$$k = \frac{6}{5}$$

The total solution is;

$$y(n) = y_p(n) + y_h(n)$$

$$= \left\{ \frac{6}{5} n \cdot 4^{n-1} + c_1 4^n + c_2 (-1)^n \right\} u(n)$$

to find c_1 and c_2, Let $y(-2)$ & $y(-1)=0$ then,

$$y(1) = 3 \cdot y(0) + 4 + 2 = 9$$

$$c_2 + c_1 = 1 \rightarrow ①$$

$$\frac{24}{5} + 4c_1 - c_2 = 9 \Rightarrow 4c_1 - c_2 = \frac{21}{5} \rightarrow ②$$

from ① & ②

$$c_1 = \frac{26}{25} ; \quad c_2 = \frac{-1}{25}$$

So, $y[n] = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25} (-1)^n \right] u[n]$

to find, c_1 and c_2, Let $y(-2)$ & $y(-1)=0$ then,

$$y(1) = 3 \cdot y(0) + 4 + 2 = 9$$

$$c_2 + c_1 = 1 \rightarrow ①$$

$$\frac{24}{5} + 4c_1 - c_2 = 9 \Rightarrow 4c_1 - c_2 = \frac{21}{5} \rightarrow ②$$

from ① & ②

$$c_1 = \frac{26}{25} ; \quad c_2 = \frac{-1}{25}$$

So, $y[n] = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25} (-1)^n \right] u(n)$

2.28) Determine the impulse response of the following causal system:-

Scanned by CamScanner

$y[n] - 3y[n-1] - 4y(n-2) = x(n) + 2x(n-1)$

a) characteristic equation,

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda = -4, 1$$

$$y_h(n) = c_1 4^n + c_2 (-1)^n$$

$$x(n) = \delta(n)$$

$y(0) = 1$ and $y(1) - 3 \cdot y(0) = 2$

$$y(1) = 5$$

So, $c_1 + c_2 = 1 \rightarrow (1)$

$$4c_1 - c_2 = 5 \rightarrow (2)$$

from, (1) & (2) ; $c_1 = \frac{6}{5}$ & $c_2 = \frac{-1}{5}$

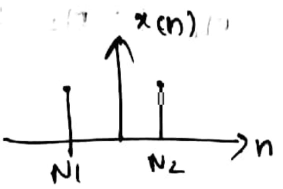
$$\therefore h[n] = \left[\frac{6}{5} 4^n - \frac{1}{5}(-1)^n\right] u(n)$$

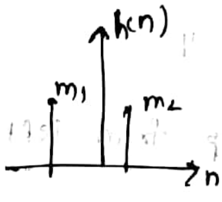
2.29) Let $x(n)$, $N_1 \le n \le N_2$ and $h(n)$, $M_1 \le n \le M_2$ be two finite-duration signals.

a) Determine the range $L_1 \le n \le L_2$ of their convolution, in terms of N_1, N_2, M_1 and M_2.

A) $L_1 = N_1 + M_1$

$L_2 = N_2 + M_2$

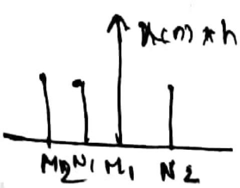


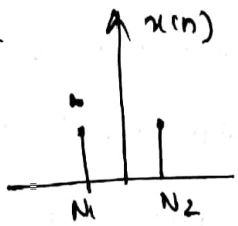


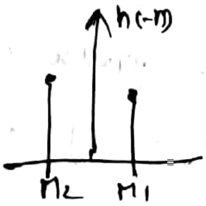
b) Determine the limits of the cases of partial overlap from the left, full overlap, and partial overlap from the right for convenience, assume that $h(n)$ has shorter duration than $x(n)$.

A) Partial overlap from left

$\rightarrow x(n) * h(n) \Rightarrow$



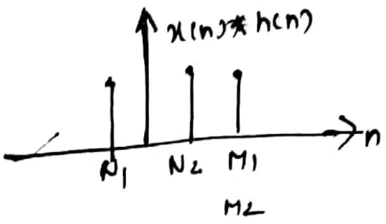




Low N_1+M_1 & high M_2+N_1-1

If fully overlap then N_1+m_2 (low) & high N_2+M_1

partial overlap from the right



$low \Rightarrow N_2+M_1+1$

$high \Rightarrow N_2+M_2$.

If fully overlapped high, N_2+M_1 ; $Low = N_1+M_2$

c) Illustrate the validity of your results by computing the convolution of the signals.

$$x(n) = \begin{cases} 1; & -2 \le n \le 4 \\ 0; & \text{elsewhere} \end{cases} \quad ; \quad h(n) = \begin{cases} 2; & -1 \le n \le 2 \\ 0; & \text{elsewhere} \end{cases}$$

A) $x(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$ $h(n) = \{2, \underset{\uparrow}{2}, 2, 2\}$

$N_1 = -2$; $N_2 = 4$ $M_1 = -1$; $M_2 = 2$

partial overlap from left,

Low $N_1+M_1 = -3$

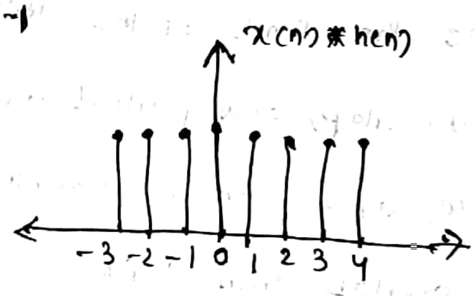
high $M_2+N_1-1 = 2-2 = -1$

$= -1$

full overlap ; $n=0, n=3$

Partial right ; $n=4, n=6$

$L_2 = 6$



2.30) Determine the impulse response and the unit step response of the systems describe by the difference equation.

a) $y(n) = 0.6\, y(n-1) - 0.08\, y(n-2) + x(n)$

$x(n) = y(n) - 0.6y(n-1) - 0.08y(n-2)$

Characteristic equation

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$\lambda = \frac{1}{2} / 2/5$$

$$y.\ h(n) = c_1\left(\frac{1}{5}\right)^n + c_2\left(\frac{4}{5}\right)^n$$

Impulse response $x(n) = \delta(n)$; with $y(0) = 1$

$y(1) - 0.6y(0) = 0 \Rightarrow y(1) = 0.6$

So, $c_1 + c_2 = 1 \rightarrow ①$

$\frac{1}{5}c_1 + \frac{2}{5}c_2 = 0.6 \rightarrow ②$

from ① & ② $c_1 = -1$; $c_2 = 3$

$\therefore h[n] = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right] . u(n)$

Step response $x(n) . u(n)$

$$\delta(n) = \sum_{k=0}^{n} h(n-k) \quad , n \geq 0$$

$$= \sum_{k=0}^{n}\left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= 2 . \left(\frac{2}{5}\right)^n \sum_{k=0}^{n}\left(\frac{5}{2}\right)^k - \left(\frac{1}{5}\right)^n \sum_{k=0}^{n}(5)^k .$$

b) $y(n) = 0.7y(n-1) - 0.1y(n-2) + 2-x(n) - x(n-2)$

$\Rightarrow$ $2x(n) - x(n-2) = y(n) - 0.7y(n-1) + 0.1y(n-2)$

Characteristic equation,

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda = \frac{1}{2}, \frac{1}{5}$$

$$y.\ h(n) = c_1\left(\frac{1}{2}\right)^n + c_n\left(\frac{1}{5}\right)^n$$

Impulse response, $x(n) = \delta(n)$; $y(0) = 2$

$y(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4$

$c_1 + c_2 = 2$

$\frac{1}{2}c_1 + \frac{1}{5}c_2 = \frac{1}{5} \rightarrow ①$

$$c_1 + \frac{2}{5}c_2 = \frac{14}{5} \rightarrow (2)$$

solving (1) & (2)

$$c_1 = \frac{10}{3}; \quad c_2 = -\frac{4}{3}$$

$$\text{so, } h[n] = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] \cdot u(n)$$

step response, $s[n] = \sum_{k=0}^{n} h(n-k)$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \cdot \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left[\frac{1}{2}^n\left(2^{n+1}-1\right) \cdot u(n)\right] - \frac{4}{3}\left[\left(\frac{1}{5}\right)^n\left(5^{n+1}-1\right) u(t)\right.$$

2.31) Consider a system with impulse response.

$$h(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

Determine the output input $x(n)$ for $0 \le n \le \infty$ that will generate the output sequence.

A) $h(n) = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \dots\}$

$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0 \dots\}$

$y(0) = x(0) \cdot h(0)$

$y(0) = x(0) \cdot 1 \Rightarrow x(0) = 1$

$y(1) = x(1) + h(1) \cdot x(0)$

$2 = x(1) + \frac{1}{2}(1) \Rightarrow x(1) = \frac{3}{2}$

$y(2) = x(2) + h(2) \cdot x(1) + h_1 x(0)$

$2.5 = x(2) + \frac{1}{4}\left(\frac{3}{2}\right) + \frac{1}{2}(1)$

$x(2) = \frac{7}{4}$

So, $x(n) = \{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2} \dots\}$

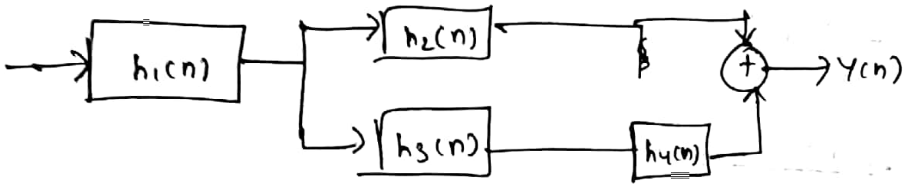
2.32) Consider the Interconnection of LTI systems as shown in fig.

a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$ and $h_4(n)$.

b) Determine $h(n)$, when

$$h_1(n) = \left\{\tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{2}\right\} \; ; \; h_2(n) = h_3(n) = (n+1)\cdot u(n)$$

$$h_4(n) = \delta(n-2)$$



c) Determine the response of the system in part (b) if,

$$x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$$

A) a) $h(n) = h_1(n) * [h_2(n) - \{h_3(n) * h_4(n)\}]$

b) $h_3(n) * h_4(n) = (n+1)\, u(n) * \delta(n-2)$

$= (n+1)\, u(n-2)$

$= (n+1)\cdot u(n-2)$

$h_2(n) - [h_3(n) * h_4(n)] = (n+1)\, u(n) - (n+1)\, u(n-2)$

$= 2\, u(n) - \delta(n)$

$h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$

$h(n) = \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)\right] * [2u(n) - \delta(n)]$

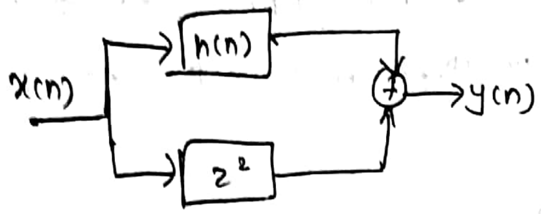
$= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$

c) $x(n) = \{1, 0, 0, 0, 3, 0, -4\}$

$y(n) = \left\{\frac{1}{2}, \frac{5}{4}, \underset{\uparrow}{2}, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, 0 \ldots\right\}$

2.33) Consider the system fig, with $h(n) = a^n u(n)$ $-1 < a < 1$. Determine the response $y(n)$ of the system to the excitation.

$$x(n) = u(n+5) - u(n-10)$$



$S(n) = u(n) * h(n)$

$$s(n) = \sum_{k=0}^{\infty} u(k) \cdot h(n-k)$$

$$= \sum_{k=0}^{\infty} h(n-k)$$

$$= \sum_{k=0}^{\infty} a^{n-k}$$

$$= \frac{a^{n+1}-1}{a-1} \; ; \; n \geq 0$$

for, $x(n) = u(n+5) - u(n-10)$, then,

$$s(n+5) - s(n-10) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10).$$

from given fig. $y(n) = x(n) * h(n) - x(n) * h(n-2)$.

$$y(n) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10) - \frac{a^{n+4}-1}{a-1} u(n+3)$$

$$+ \frac{a^{n-11}-1}{a-1} u(n-12).$$

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$$

$$h[n] = \frac{[u(n) - u(n-m)]}{M}$$

$$S(n) = \sum_{k=-\infty}^{\infty} u(k) \cdot h(n-k)$$

2.25) The discrete time system ; $y(n) = n \cdot y(n-1) + x(n)$; $n \geq 0$ is at rest [i.e, $y(-1) = 0$]. check if the system is linear time invariant and BIBO stable.

A) $y(n) = n \cdot y(n-1) + x(n)$; $n \geq 0$

$$\left.\begin{aligned} y_1(n) &= n y_1(n-1) + x_1(n) \\ y_2(n) &= n y_2(n-1) + x_2(n) \end{aligned}\right\} \text{①} \Rightarrow y(n) = n y_1(n-1) + x_1(n) + n y_2(n+1) x_2(n)$$

$y(n) = n\,y(n-1) + x(n)$

$x(n) = a \cdot x_1(n) + b \cdot x_2(n)$

$y(n) = a \cdot y_1(n) + b \cdot y_2(n)$

Hence the system is linear.

$\Rightarrow y(n-1) = (n-1) \cdot y(n-2) + x(n-1)$

delayed $\Rightarrow y(n-1) = n \cdot y(n-2) + x(n-1)$

So, the system is time variant.

$\Rightarrow$ If $x(n) = u(n)$, then $|x(n)| \le 1$, for this bounded input output is $y(0) = 0$, $y(1) = 2$, $y(2) = 5$, unbounded, so system is unstable.

$$\Rightarrow \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{M}; & n < M \\ 1; & n \ge M. \end{cases}$$

3) Determine the range of values of the parameter a for which the linear time-invariant system with impulse response:

$$h(n) = \begin{cases} a^n, & n \ge 0, \quad n \text{ even is stable.} \\ 0, & \text{otherwise} \end{cases}$$

A) $\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0}^{\infty} |a|^n \quad n = \text{even}$

$$= \sum_{n=0}^{\infty} |a|^{2n} = \frac{1}{1-|a|^2} \quad \text{stable if } |a| < 1$$

39) Determine the response of the system with impulse response, $h(n) = a^n u(n)$ to i/p signal $x(n) = u(n) - u(n-10)$

A) $h(n) = a^n \cdot u(n)$

$$y_1(n) = \sum_{k=0}^{\infty} u(k) \cdot h(n-k)$$

$$= \sum_{k=0}^{n} a^{n-k}$$

$$= \sum_{k=0}^{\infty} a^{n-k} \Rightarrow a^n \sum_{k=0}^{n} a^{-k}$$

$$= \frac{1-a^{n+1}}{1-a} \cdot u(n)$$

$y(n) = y_1(n) + y_1(n-10) \Rightarrow \frac{1}{1-a}\left[(1-a^{n+1})\, u(n) - (1-a^{n-9})\right] u(n-10)$

40) Determine the response of the relaxed system characterized by the signal response, $h(n) = (\frac{1}{2})^n u(n)$ is the input signal,

$$x(n) = \begin{cases} 1, & 0 \le n \le 10 \\ 0, & \text{elsewhere} \end{cases}$$

A) $a = \frac{1}{2}$ from above problems.

$$y(n) = 2\left\{1 - (\tfrac{1}{2})^{n+1}\right. 2(n) - 2\left\{1 - (\tfrac{1}{2})^{n-9}\right\} u(n-10)$$

$$y(n) = 2\left[1 - (\tfrac{1}{2})^{n+1} - 2\left(1 - (\tfrac{1}{2})^{n-9}\right) u(n) \left[u(n-10)\right]\right.$$

$$y(n) = 2\left[1 - (\tfrac{1}{2})^{n+1} - \left[1 - (\tfrac{1}{2})^{n-9}\right] u(n)\, u(n-10)\right.$$

41) Determine the response of system characteristic by the impulse response $h(n) = (\frac{1}{2})^n u(n)$ to the input signals.

a) $x(n) = 2^n u(n)$

A) $$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$= \sum_{k=0}^{n} (\tfrac{1}{2})^k 2^{n-k}$$

$$= 2^n \sum_{k=0}^{n} (\tfrac{1}{4})^k = \frac{2}{3}\left[2^{n+1} - (\tfrac{1}{2})^{n+1}\right] u(n)$$

b) $x(n) = u(-n)$

A) $$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$= \sum_{k=0}^{\infty} h(k) = \sum_{k=0}^{\infty} (\tfrac{1}{2})^k$$

$$= \sum_{k=0}^{\infty} (\tfrac{1}{2})^k - \sum_{k=0}^{n-1} (\tfrac{1}{2})^k$$

$$= 2 - \left(\frac{1 - (\frac{1}{2})^n}{\frac{1}{2}}\right)$$

$$= 2(\tfrac{1}{2})^n ;\ n \ge 0$$

42) Three systems with impulse responses $h(n) = \delta(n) - \delta(n$

$h_2(n) = h(n)$ and $h_3(n) = u(n)$ are connected in cascade.

a) what is the impulse response, $h_T(n)$ of the ...

A) $h_T(n) = h_1(n) * h_2(n) * h_3(n)$

$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$

$= [u(n) - u(n-1)] * h(n)$

$= \delta(n) * h(n)$

$= h(n)$.

b) Does the order of interconnection affect the overall system? A) No

B) c) prove and explain graphically the difference between the relations $x(n) \cdot \delta(n-n_0) = x(n_0)\,\delta(n-n_0)$ and $x(n) * \delta(n-n_0) = x(n-n_0)$.

$x(n)\,\delta(n-n_0) = x(n_0)$. They only the value of $x(n)$ at $n = n_0$ is interest.

A) $x(n) * \delta(n-n_0) = x(n-n_0)$. Thus, we obtained shifted version of

b) show that a discrete time system, where convolution summation, is [T] and relaxed.

A) $y[n] = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$

$= h(n) * x(n)$

Linearity $= x_1(n) \rightarrow y_1(n) = h(n) * x_1(n)$

$= x_2(n) \rightarrow y_2(n) = h(n) * x_2(n)$

$= \alpha\, h(n) * x_1(n) + \beta\, h(n) * x_2(n)$

$= \alpha\, y_1(n) + \beta\, y_2(n)$

Time invariance :-

$x(n) \rightarrow y_1(n) = h(n) * x(n)$

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$

$= \sum_k h(k) \cdot x(n-n_0-k)$

$= y(n-n_0)$.

c) what is the impulse response of the system described by $y(n) = x(n-n_0)$. A) $h(n) = \delta(n-n_0)$.

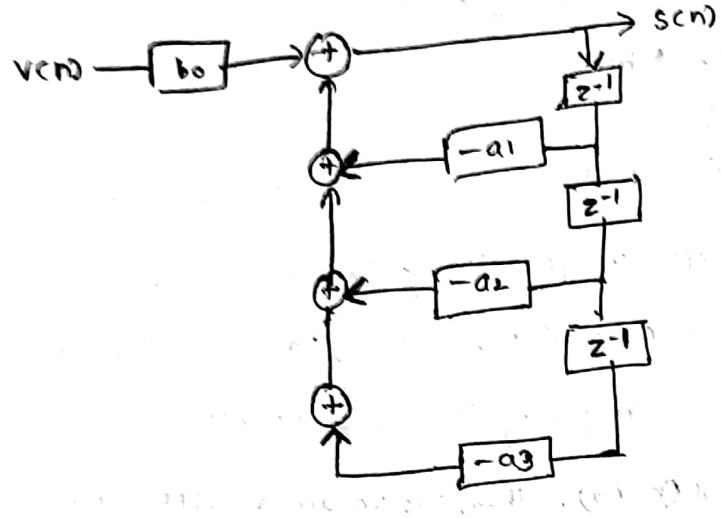
f) compute the zero state response of system described by

44) $\quad$ 46)

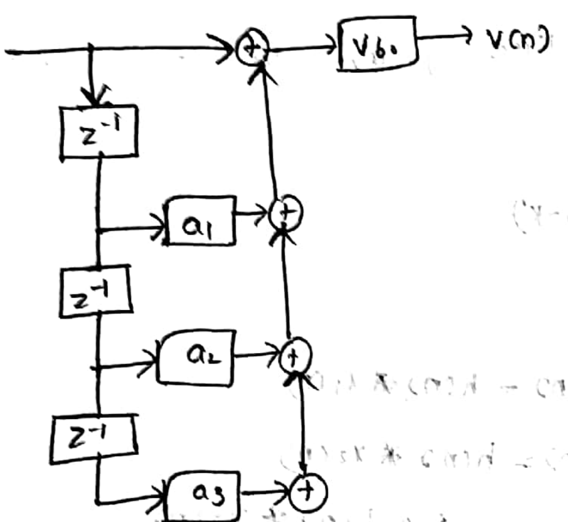
a) $s(n) = -a_1 \delta(n-1) - a_2 \delta(n-2) \ldots a_2 \delta(n-2) + b_0 (v(n))$

b) $v(n) = \frac{1}{b_0} [\delta(n) + a_1 \delta(n-1)\ a_2\ \delta(n-2) + \ldots + a_N \delta(n-N)]$

c)



d)



What is the impulse response of the system described by

$y(n) = x(n-n_0)$

A) $h(n) = \delta(n-n_0)$

45) Compute the zero state response of the system described by,

$y(n) = x(n-n_0)$

A) $y(n) = -\frac{1}{2} y(n-1) + x(n) + 2x(n-2)$

at $y(-2) = -\frac{1}{2} y(-3) + x(-2) + 2x(-4) = 1$

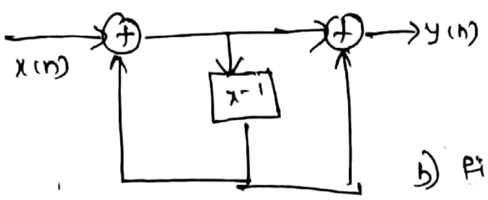
$y(-1) = -\frac{1}{2}(y(-2) + x(-1) + 2x(-3)) = \frac{3}{2}$

$y(0) = -\frac{1}{2} y(-1) + 2x(-2) + x(0) = \frac{17}{4}$

$y(1) = -\frac{1}{2} y(0) + x(1) + 2x(-1)$

$\Rightarrow \frac{47}{8}$

46) Consider the discrete time system shown in fig.



a) compute the 10 first samples of its impulse response.

b) Find the input - output relation.

c) Apply the input $x(n) = \{1, 1, 1 \ldots\}$ and. d) .. e) ..

a) $x(n) = \{\frac{1}{4}, 0, 0 \ldots\}$

$y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

$y(0) = x(0) = 1$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) = \frac{3}{2}$

$y(2) = \frac{1}{2} y(1) + x(2) + x(1) = \frac{3}{4}$. thus, we obtain.

$y(n) = \{1, 3/2, 3/4, 3/8, 3/16, 3/32 \ldots\}$

b) $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

c) as in part (a) we obtain

$y(n) = \{1, 5/2, 13/4, \frac{29}{8}, \frac{61}{16}, \ldots\}$

d) $y(n) = u(n) * h(n)$

$= \sum_k u(k) \cdot h(n-k)$

$= \sum_{k=0}^{n} h(n-k)$

$y(0) = h(0) = 1$; $y(1) = h(0) + h(1) = 5/2$

$y(2) = h(0) + h(1) + h(2) = \frac{13}{4}$ etc.

e) from part a), $h(n) = 0$ for the system is causal

$$\sum_{n=0}^{\infty} |h(n)| = 1 + \frac{3}{2}\left(1 + \frac{1}{2} + \frac{1}{4} + \ldots\right) = 4$$

system is stable.

48) a) $y(n) = a \cdot y(n-1) + b x(n)$

$h(n) = b a^n \cdot u(n) \Rightarrow \sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$

$b = 1-a$

b) $s(n) = \sum_{k=0}^{n} h(n-k)$

$= b\left[\frac{1-a^{n+1}}{1-a}\right]u(n)$

$s(\infty) = \frac{b}{1-a} = 1$

$b = 1-a$

c) $b = 1-a$ in both the cases.

4Q) a) $y(n) = 0.8y(n-1) + 2x(n) + 3x(n-1)$

$y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$

$\lambda - 0.8 = 0$

$\lambda = 0.8$

$y_h(n) = c(0.8)^n$

Let us first consider the response of the system.

$y(n) - 0.8y(n-1) = x(n)$

to $x(n) = \delta(n)$. since $y(0) = 1$, it follows that $c = 1$

then the impulse response of the original s/m is,

$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$

$= 2\delta(n) + 4.6(0.8)^{n-1} \cdot u(n-1)$

b) The inverse system is characterized by the equation,

$x(n) = -1.5x(n-1) + \frac{1}{2}y(n) - 0.4y(n-1)$

5Q) $y(n) = 0.9y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

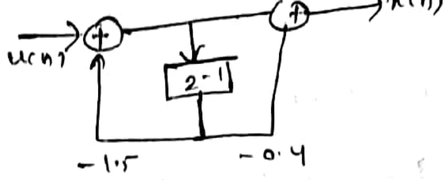
$y(n) - 0.9y(n-1) = x(n) + 2x(n-1) + 3x(n-2)$.

a) For $x(n) = \delta(n)$, we have

$y(0) = 1$, $y(1) = 2.9$, $y(2) = 5.61$, $y(3) = 5.049$

$y(4) = 4.544$, $y(5) = 4.090$

Scanned by CamScanner



b) $s(0) = y(0) = 1$; $s(1) = y(0) + y(1) = 3.91$; $s(2) = y(0) + y(1) + y(2)$
$= 9.51$

$s(3) = y(0) + y(1) + y(2) + y(3)$, $s(4) = \sum_0^4 y(n)$, $s(5) = \sum_0^5 y(n)$
$= 14.56$ $= 19.10$ $= 23.19$

c) $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$
$= \delta(n) + 2.9\delta(n-1) + 5.61 (0.9)^{n-2} u(n-2$

5) a) $y(n) = \frac{1}{3} x(n) + \frac{1}{3} x(n-3) + y(n-1)$

for, $x(n) = \delta(n)$, we have

$h(n) = \{1/3, 1/3, 1/3, 1/3, 1/3, 2/3, \dots\}$

b) $y(n) = \frac{1}{2} y(n-1) + \frac{1}{2} y(n-2) + \frac{1}{2} x(n-2)$

with $x(n) = \delta(n)$ and $y(-1) = y(-2) = 0$, we obtain

$h(n) = \{0, 0, 1/2, 1/4, 3/16, 1/3, 11/16, 81, \frac{15}{256} \dots\}$

c) $y(n) = 1.4 y(n-1) - 0.48 y(n-2) + x(n)$

with $x(n) = \delta(n)$ and
$y(-1) = y(-2) = 0$ we obtain

$h(n) = \{1, 1.4, 1.48, 14, 1.2496, 1.0774 \dots\}$

d) All three systems are IIR

$y(n) = 1.4 y(n-1) - 0.48 y(n-2) + x(n)$.

The characteristic equation: $\lambda^2 - 1.4\lambda + 0.48 = 0$ hence
$\lambda = 0.8, 0.6$ and

$y_h(n) = c_1 (0.8)^n + c_2 (0.6)^n$ for $x(n) = \delta(n)$ we have
$c_1 + c_2 = 1$ and $0.8c_1 + 0.6c_2 = 1.4$
$c_1 = 4$; $c_2 = -3$; $h[n] = [4(0.8)^n$

52) a) $h_1(n) = c_0\delta(n) + c_1\delta(n-1) + c_2\delta(n-2)$

$h_2(n) = b_2\delta(n) + b_1\delta(n-1) + b_0\delta(n-2)$

$h_3(n) = a_0\delta(n) + (a_1 + a_0 a_2)\delta(n-1) + a_1 a_2\delta(n-2)$

b) The only question is whether,

$h_3(n) \neq h_2(n) = h_1(n)$

let $a_0 = c_0$

$a_1 + a_2 c_0 = c_1 \Rightarrow a_1 + a_2 c_0 - c_1 = 0$

$\Rightarrow c_0 a_2^2 - c_1 a_2 + c_2 = 0$

For $c_0 \neq 0$, the quadratic has a real solution if and only if $c_1^2 - 4c_0 c_2 \geq 0$

53) $y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$

a) for $y(n) - \frac{1}{2}y(n-1) = x(n) + x(n-1)$, $x(n) = \delta(n)$

$h(n) = (\frac{1}{2})^n u(n) + (\frac{1}{2})^{n-1} u(n-1)$

b) $h_1(n) [\delta(n) + \delta(n-1)] = (\frac{1}{2})^n u(n) + (\frac{1}{2})^{n-1} u(n-1)$

54) a) Convolution : $y_1(n) = \{1, 3, 7, 7, 7, 6, 4\}$
$\uparrow$ (under 3)

Correlation $\gamma_1(n) = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

b) Convolution : $y_2(n) = \{1/2, \underset{\uparrow}{0}, 3/2, -2, 1/2, -6, -5/2, -2\}$

Correlation = $\gamma_2(n) = \{1/2, 0, 3/2, -2, 1/2, -6, -5/2, -2\}$

Note, $y_2(n) = \gamma_2(n) = h_2(-n) = h_2(n) \cdot (c)$

Convolution $\gamma_1(n) = \{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\}$

55) $x(n) * y(n) = h(n)$

length of $h(n) = 2$ $\quad h(n) = \{h_0, h_1\}$

$h_0 = 1$

$3h_0 + h_1 = 4$

$\Rightarrow h_0 = 1$ & $h_1 = 1$

56) $y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k \cdot x(n-k) \quad (2.5.6)$

$\omega(n) = -\sum_{k=1}^{N} a_k\, \omega(n-k) + x(n) \quad (2.5.9)$

$y(n) = \sum_{k=0}^{M} b_k \cdot \omega(n-k) \quad (2.5.10)$

from 2.5.9 we obtain $x(n) = \omega(n) + \sum_{k=1}^{N} a_k\, \omega(n-k)$ by substituting

(2.5.10) ~~by sub~~ for $y(n)$ into (2.5.6) we obtain L.H.S = R.H.S

57) $y(n) = -4y(n-1) + 4 \cdot y(n-2) = x(n) - x(n-1)$

The characteristic equation is,

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$. Hence, $y_h(n) = c_1 2^n + c_2 n 2^n$

The particular solution is,

$y_p(n) = k(-1)^n u(n)$

substituting this solution into the difference equation. we obtain

$k(-1)^n \cdot u(n) - 4k(-1)^{n-1} \cdot u(n-1) + 4k(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$

for $n=2$, $k(1+4+4) = 2 \Rightarrow k = \frac{2}{9}$

The total solution is $y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$

from the initial conditions. we obtain $y(0) = 1$, $y(1) = 2$, thus.

$c_1 + \frac{2}{9} = 1 \Rightarrow c_1 = \frac{7}{9}$

$2c_1 + 2c_2 - \frac{2}{9} = 2 \Rightarrow c_1 = \frac{1}{3}$

58) from Q.57

$h(n) = [c_1 2^n + c_1 n 2^n] \cdot u(n)$ with $y(0) = 1$, $y(1) = 3$:

We have $c_1 = 1$; $2c_1 + 2c_2 = 2$

$c_2 = \frac{1}{2}$

Thus $h(n) = [2^n + \frac{1}{2} n 2^n] \cdot u(n)$

5) $x(n) = x(n) * \delta(n)$

$= x(n) * [u(n) - u(n-1)]$

$= [x(n) - x(n-1)] * u(n)$

$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] \cdot u(n-k)$ (2)

6) Let $h(n)$ be the impulse response of the system.

$\delta(k) = \sum_{m=-\infty}^{k} h(m)$

$h(k) = \delta(k) - \delta(k-1)$

$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$

$= \sum_{k=-\infty}^{\infty} [\delta(k) - \delta(k-1)] \cdot x(n-k)]$

7) $x(n) = \begin{cases} 1 ; & n_0 - N \leq n \leq n_0 + N \\ 0 ; & \text{otherwise} \end{cases}$

$y(n) = \begin{cases} 1 ; & -N \leq n \leq N \\ 0 ; & \text{otherwise} \end{cases}$

$\delta_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$

The range of non-zero values of $\gamma_{xx}(l)$ is determined by,

$n_0 - N \leq n \leq n_0 + N$

$n_0 - N \leq n - l \leq n_0 + N$

which implies

$-2N \leq l \leq 2N$

for a given shift l, the number of terms in the summation for

which the both $x(n)$ and non zero is $2N+1 - |x|$ and the

Value of each terms is 1, then

$$r_{xx}(l) = \begin{cases} 2N+1-|l| & ; -2N \le l \le 2N \\ 0 & ; \text{otherwise.} \end{cases}$$

for $r_{xy}(l)$, we have

$$r_{xy}(l) = \begin{cases} 2N+1-|l-N_0| & ; N_0-2N \le l \le N_0+2N \\ 0 & ; \text{otherwise.} \end{cases}$$

(2) a) $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$

$r_{xx}(-3) = x(0) \cdot x(3) = 1$

$r_{xx}(-2) = x(0) \cdot x(2) + x(1) \cdot x(3) = 3$

$r_{xx}(-1) = x(0) \cdot x(1) + x(1) \cdot x(2) + x(2) \cdot x(3) = 5$

$r_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$

also, $r_{xx}(-l) = r_{xx}(l)$

$\therefore r_{xx}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

b) $r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n) \cdot y(n-l)$

we obtain $r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

We obtain $y(n) = x(-n+3)$ which is equivalent to reversing the sequence $x(n)$. This has not changed the autocorrelation sequence.

63) $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$

$$= \begin{cases} 2N+1-|l| & ; -2N \le l \le 2N \\ 0 & ; \text{otherwise} \end{cases}$$

$r_{xx}(0) = 2N+1$

$\therefore$ The normalized autocorrelations.

$$\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|) & ; -2N \le l \le 2N \\ 0 & ; \text{otherwise} \end{cases}$$

64)

a) $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$

$= \sum_{n=-\infty}^{\infty} [\delta(n) + r_1 \delta(n-k) + r_2 \delta(n-k_2)]$ *

$[\delta(n-l) + r_1 \delta(n-l-k_1) + r_2 \delta(n-l-k_2)]$

$= (1 + r_1^2 + r_2^2) \cdot r_{ss}(l) + r_1 [r_{ss}(l+k) + r_{ss}(l-k)] +$

$r_2 [r_{ss}(l+k_2) + r_{ss}(l-k_2)] + r_1 r_2 [r_{ss}(l+k_1-k_2) +$

$r_{ss}(l+k_2-k_1)$

b) $r_{xx}(l)$ has peaks at $x=0$; $\pm k_1$; $\pm k_2$ and $\pm(k_1+k_2)$.

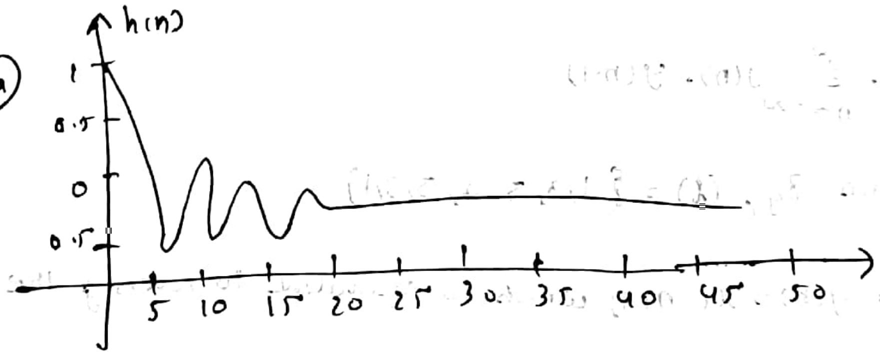
So that $k_1 < k_2$. Then, we can determine r_1 and k_1. the problem is

to determine r_2 and k_2 from the other peak.

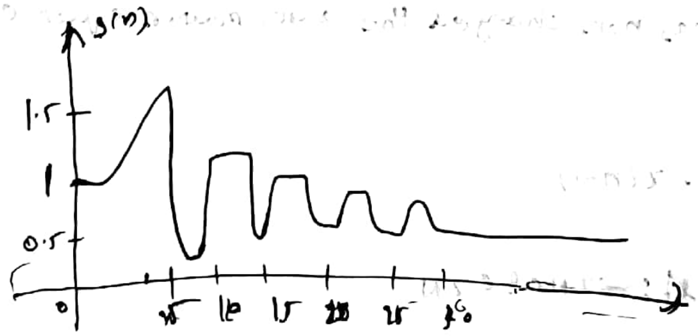
c) If $r_2 = 0$; the peaks occur at $l=0$ and $l=1$. Then it is

easy to obtain r_1 and k_1

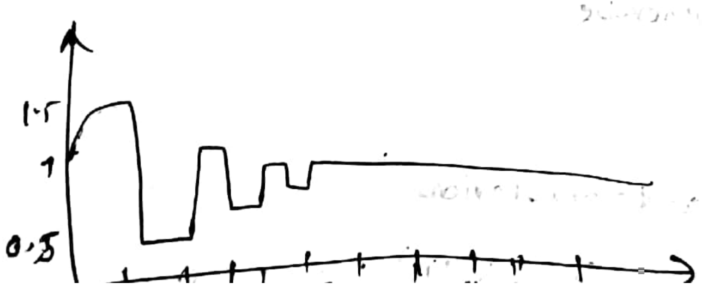
66) a)



b)



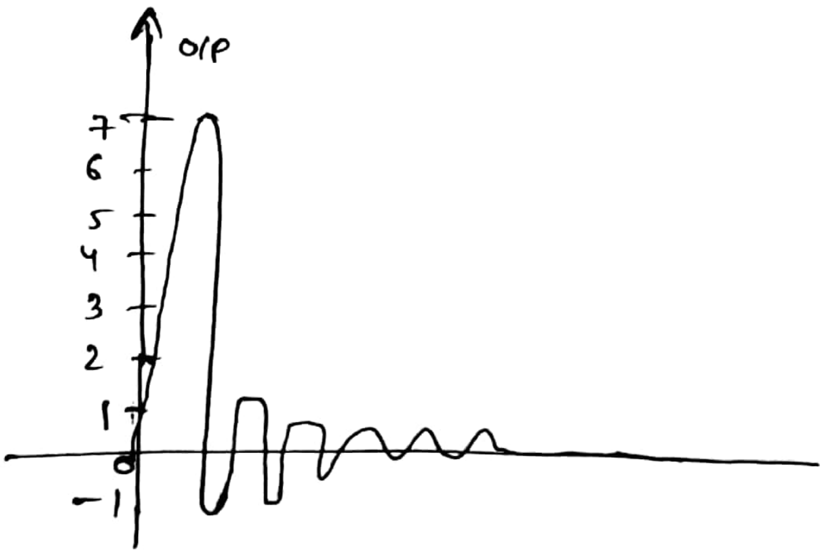
c)



d) c, b are similar except c have steady state after n=20

where b have nearly at n=30

67)



=x=