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Q.3) Using the bilinear Transformation design a HPF monotonic in passband with Cutoff freq. of 1000Hz & down 10dB at 350Hz. The sampling freq is 5000Hz. Implement using basic building blocks. Show the derivation for this filter. Demonstrate the filter's output for 5 different frequencies ranging from 100 Hz to 10000 Hz. Choose the freq. smartly.

Ans Given :-

Pass band attenuation $= \alpha_p = 3dB$

Stopband attenuation $= \alpha_s = 10dB$

Sampling freq $= f_{samp} = 5000 Hz$

$$\therefore T_s = \frac{1}{5000} = 2\times10^{-4} sec.$$

f_p = Pass band freq = 1000 Hz

f_s = Stopband freq = 350 Hz.

$\therefore$ The prewarping digital frequencies

$$\therefore \Omega_p = \frac{2}{T} \tan \frac{\omega_p T_s}{2}$$

$$\therefore \Omega_p = \frac{2}{2\times10^{-4}} \tan\left[\frac{2\pi f_p T_s}{2}\right]$$

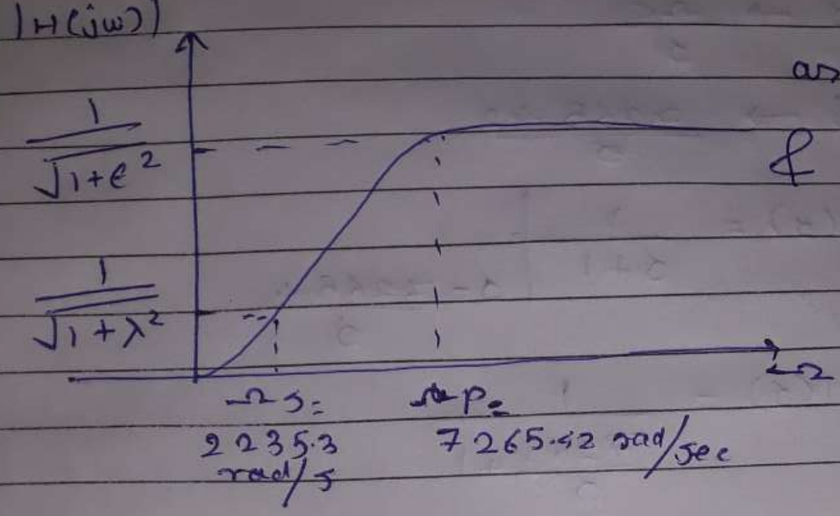
$$= 10^4 \tan\left[\frac{\pi \times 1000 \times 2}{5000}\right]$$

$$= 10^4 \tan[0.2\pi] = 7265.42 \text{ rad/sec}$$

$$\therefore \Omega_s = \frac{2}{T} \tan\left[\frac{\omega_s T_{sam}}{2}\right]$$

$$\therefore \Omega_s = \frac{2}{T} \tan\left[\frac{2\pi f_s T_{samp}}{2}\right]$$

$\therefore \Omega_s = 10^4 \tan(0.07\pi) = 2235.3 \text{ rad/s}$



as $\epsilon = \sqrt{10^{0.1\alpha_p} - 1}$

& $\lambda = \sqrt{10^{0.1\alpha_s} - 1}$

$\therefore$ The order of filter

$$\cancel{N \geq} \; \therefore \epsilon = \sqrt{10^{0.1\alpha_p} - 1} = 1$$

$$\lambda = \sqrt{10^{0.1\alpha_s} - 1} = 3$$

$$\therefore N \geq \frac{\log(\lambda/\epsilon)}{\log(\Omega_p/\Omega_s)}$$

$$N \geq \frac{\log(3)}{\log\left(\frac{726542}{2235.3}\right)} = 0.932$$

$$\therefore N \approx 1$$

$\therefore$ The 1st order butterworth filter for $\Omega_c = 1$ rad/sec

$$\therefore H(s) = \frac{1}{s+1}$$

$\therefore$ Highpass filter for $\Omega c = \Omega p = 7265.42$ rad/sec
Can be obtained by transformation

$$s \to \frac{\Omega c}{s}$$

$$\therefore s \to \frac{7265.42}{s}$$

$$\therefore \quad H(s) = \frac{1}{s+1}\Bigg|_{s=\frac{7265.4}{s}}$$

$$\therefore \quad H(s) = \frac{1}{\frac{7265}{s}+1}$$

$$= \frac{s}{s+7265}$$

Analog to digital transformation using bilinear transformation

$$s \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

$$\therefore s \to 10^4\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

$$\therefore \quad H(z) = \frac{s}{s+7265}\Bigg|_{10^4\left[\frac{1-z^{-1}}{1+z^{-1}}\right]=s}$$

$$= \frac{10000\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}{10000\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+7265}$$

$$= \frac{0.5792(1-z^{-1})}{1-0.1584z^{-1}}$$

$$H(z) = \frac{0.5792 - 0.5792 z^{-1}}{1 - 0.1584 z^{-1}}$$

$$\therefore \frac{Y(z)}{X(z)} = \frac{0.5792 - 0.5792 z^{-1}}{1 - 0.1584 z^{-1}}$$

$$\therefore Y(z)\left[1 - 0.1584 z^{-1}\right] = X(z)\left[0.5792 - 0.5792 z^{-1}\right]$$

$$\therefore Y(z) = X(z)\,0.5792 - X(z)\,0.5792\, z^{-1} + 0.1584\, z^{-1} Y(z)$$

Taking IZT

$$\therefore y(n) = x(n)(0.5792) - x(n-1)\,0.5792 + y(n-1)\,0.1584$$

$$\therefore y(n) = 0.5792\, x(n) - 0.5792\, x(n-1) + 0.1584\, y(n-1)$$

$$v(n) = 0.5792\, x(n) - 0.5792\, x(n-1)$$

$$\therefore y(n) = v(n) + 0.1584\, y(n-1)$$

Direct realization