# Homework 7

**Problem 1:**

1. are the eigenvalues of and are the corresponding p eigenvectors
2. The matrix can be written as: . From this equation and from part c., we can see that has p eigenvalues and corresponding eigenvectors. The rest of the eigenvalues are 0.

Thus and have the same p eigenvalues

1. If M has rank k, then

**Problem 2:**

1. The best rank-1 approximation to M is:
2. The decomposition is not unique. We can find orthonormal vectors *a* and *b* such that

. This is when

1. Best k-rank approximation of M can be written as:

**Problem 3:**

The Gram matrix for the given data points is as follows:

The other set of points with exactly the same Gram matrix are:

(-1, 0, 0), (-1, 0, -1), (-1, -1, 0), (-1, -1, -1). This is because, the Gram matrix B can be written as and

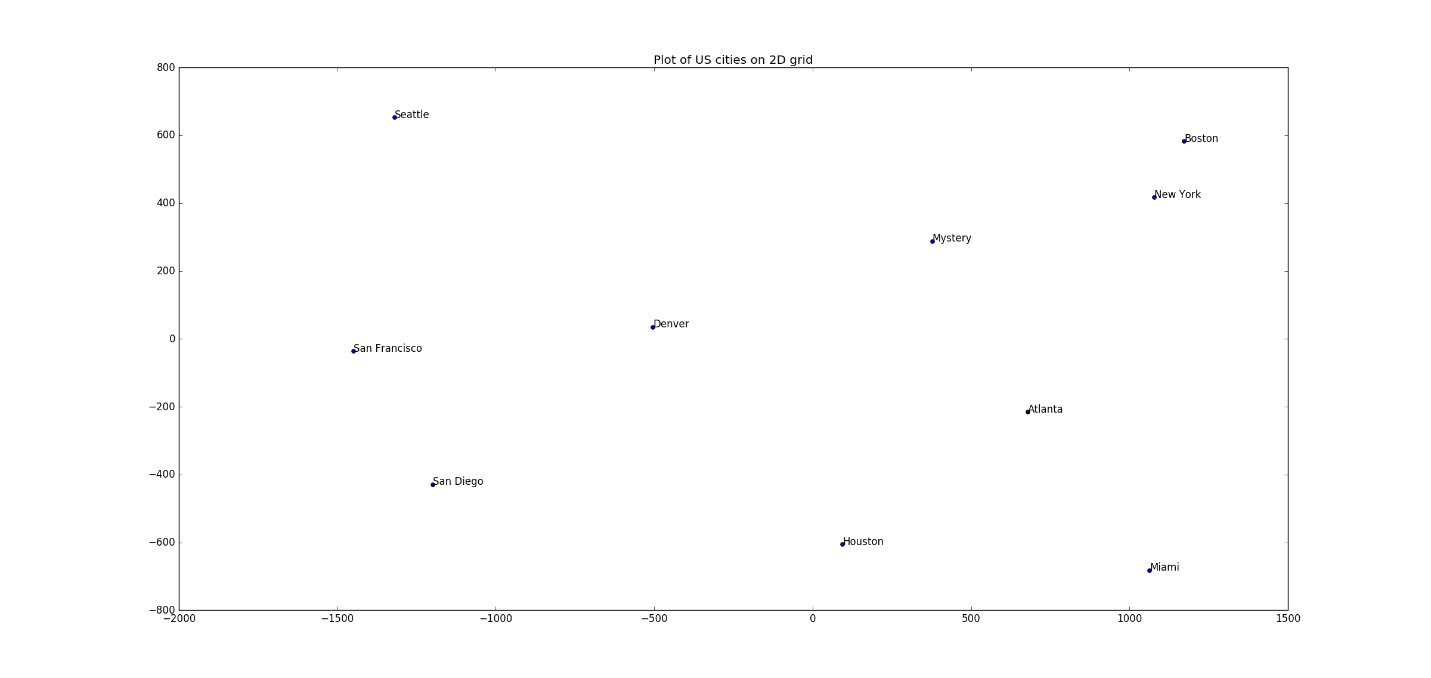
Thus the second solution is negative of the first solution.

**Problem 4:**

The plot of the cities was inverted with respect to the actual map of U.S cities. This was fixed by multiplying the 2D points by (-1). Thus we plot (-x, -y) instead of (x, y).

The two set of points have the same Gram matrix, since we can decompose the matrix B to get two solutions for calculating the square root of the Eigen value matrix (:

The following is the corrected plot of U.S. cities:



The mystery city appears to be **Chicago**.