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Optimizing Doctor-Patient Allocation in an OPD Using Q-Learning

by

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1.1 Introduction

Efficient doctor-patient allocation in an outpatient department (OPD) is crucial for reducing patient wait times and ensuring optimal doctor utilization. Traditional scheduling methods often face challenges in balancing patient load among doctors. Assigning patients manually may result in long queues and overburdening specific doctors while others remain underutilized. To address these inefficiencies, artificial intelligence (AI) techniques such as reinforcement learning provide a promising solution.

In this report, we explore how **Q-learning**, a reinforcement learning algorithm, can be applied to optimize patient allocation in an OPD setting. The goal is to minimize the overall consultation time by learning an optimal doctor-patient assignment strategy. This report provides a detailed breakdown of Q-learning, formulates the problem as a reinforcement learning task, and implements a solution using Python.

1.2 Understanding the OPD Scheduling Problem

1.2.1 Challenges in OPD Scheduling

- Hospital outpatient departments typically face several challenges in managing patient appointments and consultations. Some of the key challenges include:
- **Unpredictable Patient Inflow:** The number of patients visiting an OPD varies daily, making it difficult to pre-allocate doctors efficiently.
- **Varying Consultation Time:** Different doctors take different amounts of time for consultation depending on experience, specialization, and patient complexity.
- **Doctor Availability:** Some doctors may have fixed consultation hours or be required in emergency cases, affecting the scheduling process.
- **Long Waiting Times:** Poor patient assignment leads to congestion, increasing waiting times and patient dissatisfaction.
- **Uneven Workload Distribution:** Without an optimized approach, some doctors may be overburdened while others remain idle.

1.3 Need for Optimization

Given these operational challenges, there is a critical need to design an intelligent, adaptive system that can:

- Dynamically assign patients to doctors based on current availability and efficiency.
- Minimize the overall waiting time and maximize patient throughput.
- Balance the workload across all doctors fairly to avoid exhaustion and ensure better service quality.
- Improve patient satisfaction by reducing idle waiting and providing faster consultations.

Traditional rule-based or fixed slot scheduling is insufficient to handle the dynamic environment of an OPD. Reinforcement Learning, especially Q-learning, is an appropriate choice because it learns by interacting with the environment and can continuously improve its policy based on real-time feedback.

1.4 Introduction to Q-Learning

1.4.1 Basics of Reinforcement Learning

Reinforcement Learning (RL) is a machine learning paradigm where an agent learns optimal behavior by interacting with an environment. The agent makes decisions, receives feedback (rewards or penalties), and updates its knowledge accordingly.

1.4.2 The key components of RL are

- **Agent:**The decision-making entity (in our case, the scheduling system).
- **Environment:** The system in which the agent operates (hospital OPD with doctors and patients).
- **Actions (A):**The possible moves the agent can make (assigning a patient to a doctor).
- **States (S):**The current status of the environment (which patients are assigned to which doctors).
- **Rewards (R):** A numerical score representing the effectiveness of an action (negative consultation time to minimize time spent).

1.5 Why Use Q-Learning for OPD Scheduling?

Q-learning is particularly suited for OPD scheduling because it allows the system to learn optimal doctor-patient assignments through trial-and-error interaction with the environment, without needing a pre-built model of the OPD dynamics. OPD environments are dynamic and uncertain — the number of patients, consultation times, and doctor availabilities can change daily. Traditional rule-based or static optimization techniques struggle to adapt to such variability.

Q-learning, being a model-free reinforcement learning algorithm, helps in handling this uncertainty by learning directly from the environment. It enables:

- **Adaptability:** The model can adjust its scheduling strategies over time based on real patient and doctor behavior.
- **Scalability:** It can easily scale to larger OPDs with more doctors and patients.
- **Efficiency:** By learning to minimize consultation time, it leads to faster service and better utilization of doctor time.
- **No Need for Prior Knowledge:** Q-learning does not require knowledge of patient inflow models or consultation distributions beforehand.

Thus, Q-learning offers an intelligent, flexible, and efficient approach to optimize real-world OPD operations compared to traditional hard-coded scheduling systems.

1.6 What is Q-Learning?

Q-learning is a type of reinforcement learning that helps an agent learn an optimal action policy using a **Q-table**. The Q-table stores the expected rewards for state-action pairs and is updated using the Bellman Equation:

$$Q(s,a) = Q(s,a) + \alpha[R + \gamma \max Q(s',a') - Q(s,a)]$$

Where:
- $Q(s,a)$ is the Q-value of taking action a in state s.
- α (alpha) is the learning rate.
- R is the reward received.
- γ (gamma) is the discount factor.
- $\max Q(s',a')$ is the maximum future reward for the next state.

1.7 Formulating the Problem as an RL Task

1.7.1 Defining States, Actions, and Rewards

To apply Q-learning to OPD scheduling, we define:

1.8 State Representation

The state S_t at time t in the OPD environment is defined as a tuple comprising three key components:

$$S_t = \big(\text{Patient\_Count}, \text{Doctor\_Status}, \text{Time\_Matrix}\big) \in \mathcal{S} \tag{1.1}$$

where:

- $\text{Patient\_Count} \in \mathbb{Z}^+$ represents the current number of patients waiting in the queue
- $\text{Doctor\_Status} \in \{0,1\}^D$ is a binary vector indicating availability of D doctors:

$$\text{Doctor\_Status} = \begin{cases} 0 & \text{if doctor is idle} \\ 1 & \text{if doctor is busy} \end{cases} \tag{1.2}$$

- $\text{Time\_Matrix} \in \mathbb{R}^{P \times D}$ is the consultation time matrix where:

$$T_{p,d} = \text{Estimated time for patient } p \text{ with doctor } d \tag{1.3}$$

The state space $\mathcal{S}$ is therefore the Cartesian product:

$$\mathcal{S} = \mathbb{Z}^+ \times \{0,1\}^D \times \mathbb{R}^{P \times D} \tag{1.4}$$

1.9 Action Space Definition

The action A_t at time t represents the assignment of an available patient to a suitable doctor, subject to constraints:

$$A_t \in \mathcal{A}(S_t) = \{a \in \{0, 1, \ldots, D-1\} \,|\, \text{ValidAssignment}(a, S_t)\} \tag{1.5}$$

where:

- D is the total number of doctors ($D = 3$ in our case)
- $\mathcal{A}(S_t)$ denotes the state-dependent action space
- $\text{ValidAssignment}(a, S_t)$ requires:
    1. **Availability**: Doctor a must be idle (from S_t's Doctor_Status)

2. **Specialization**: Doctor a must have matching expertise for the patient's condition

For the concrete case of 3 doctors:

$$\mathcal{A} \subseteq \{\text{Doctor}_0, \text{Doctor}_1, \text{Doctor}_2\} \tag{1.6}$$

with the constraint that:

$$a = \text{Doctor}_i \text{ is valid only if Doctor\_Status}[i] = 0 \text{ and SpecializationMatch}(i, p) \tag{1.7}$$

1.10 Reward Function and Q-Learning Update

1.10.1 Reward Function

The reward R_t at time t combines immediate consultation time with a penalty for doctor overload:

$$R_t = -\text{Consultation\_Time}(p, d) - \lambda \cdot \text{Overload\_Penalty}(d) \tag{1.8}$$

where:

- $\text{Consultation\_Time}(p, d) \in \mathbb{R}^+$ is the time taken by doctor d for patient p (from Time Matrix)
- $\lambda \in \mathbb{R}^+$ is the penalty weighting factor
- $\text{Overload\_Penalty}(d) = \max(0, \text{Workload}(d) - \text{Threshold})$ quantifies excessive workload

1.10.2 Q-Table Structure

The Q-table is a matrix mapping state-action pairs to expected cumulative rewards:

$$Q : \mathcal{S} \times \mathcal{A} \to \mathbb{R} \tag{1.9}$$

with dimensions $|\mathcal{S}| \times |\mathcal{A}|$.

1.10.3 Bellman Update Rule

The Q-values are updated iteratively:

$$Q(s_t, a_t) \leftarrow (1-\alpha)Q(s_t, a_t) + \alpha \left[R_t + \gamma \max_{a' \in \mathcal{A}(s_{t+1})} Q(s_{t+1}, a') \right] \tag{1.10}$$

where:

- $\alpha \in (0, 1]$ is the learning rate
- $\gamma \in [0, 1)$ is the discount factor
- s_{t+1} is the resulting state after taking action a_t

1.10.4 By Example:

$$\begin{aligned} &\text{If Consultation\_Time} = 12\text{min}, \lambda = 0.5, \text{Overload} = 2 \\ &\quad R = -12 - 0.5 \times 2 = -13 \end{aligned}$$

$$Q = \begin{bmatrix} -12.3 & -8.7 & -15.1 \\ -10.5 & -18.2 & -9.4 \\ \vdots & \vdots & \vdots \end{bmatrix} \tag{1.11}$$

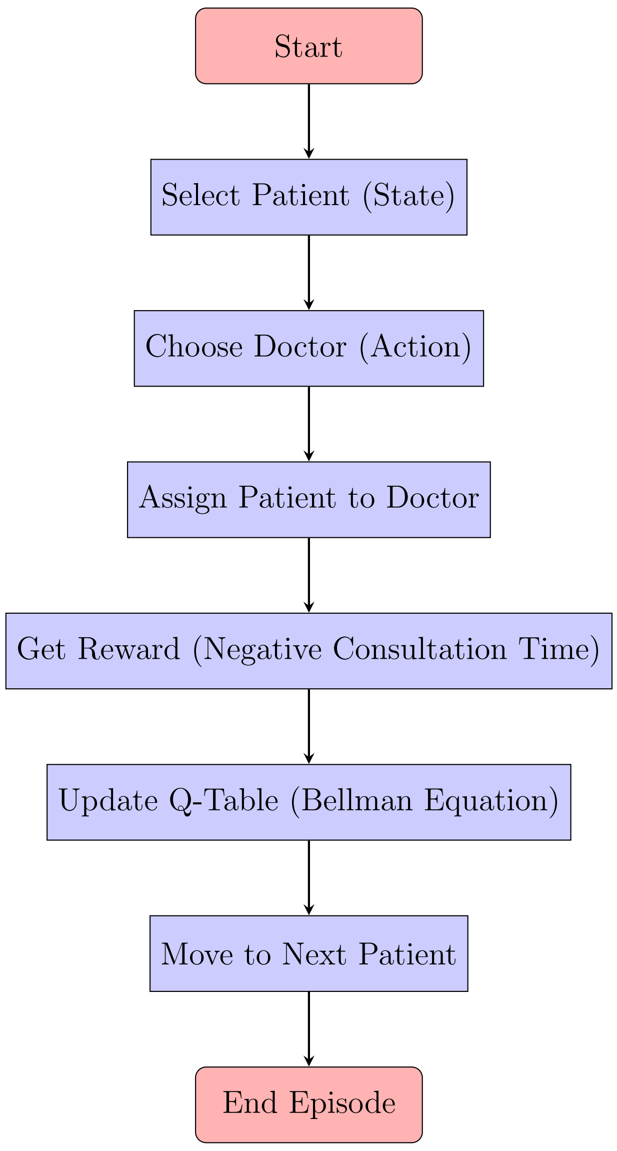
1.11 Q-Learning Algorithm

Algorithm 1 Q-Learning Algorithm

```
    Initialize $Q(s, a)$ arbitrarily each episode Choose $a$ from $s$ using policy
  derived from $Q$ Take action $a$, observe reward $r$ and next state $s'$ Update
  $Q(s, a)$ using Bellman equation:
```

$$Q(s, a) \leftarrow Q(s, a) + \alpha \left[r + \gamma \max_{a'} Q(s', a') - Q(s, a) \right]$$

1.12 Flowchart of Q-Learning Process



1.13 Real-Life Example: Traditional Scheduling vs Q-Learning Scheduling

1.13.1 Traditional Scheduling (Manual Assignment)

Consider an OPD with 3 doctors and 5 patients. In traditional manual scheduling:

Patient	Assigned Doctor	Time Taken (minutes)
P0	D0	12
P1	D0	15
P2	D1	9
P3	D1	10
P4	D2	14

Problems:

- Doctor 0 is overloaded with two long consultations (total 27 minutes).
- Doctor 2 is underutilized (only one patient).
- Longer waiting time for patients.

1.13.2 Q-Learning Based Scheduling (Optimized)

After applying Q-Learning:

Patient	Assigned Doctor	Time Taken (minutes)
P0	D2	13
P1	D1	9
P2	D0	11
P3	D2	14
P4	D0	7

Advantages:

- Balanced workload among doctors.
- Shorter total consultation time.
- Reduced patient waiting time.

1.13.3 Conclusion

- **Traditional Scheduling:** Leads to workload imbalance and inefficiencies.
- **Q-Learning Scheduling:** Provides an adaptive, efficient, and balanced patient allocation automatically.

1.14 Explanation of the Time Matrix

In our doctor-patient assignment problem, each doctor may take a different amount of time to treat each patient. Some doctors may be faster for certain patients based on their experience, specialization, or familiarity with specific diseases. To model this real-world behavior mathematically, we introduce the concept of a **Time Matrix**.

The Time Matrix is a two-dimensional array where:

- Each **row** represents a patient.
- Each **column** represents a doctor.
- Each **entry** in the matrix represents the consultation time (in minutes) that a doctor would take to treat a specific patient.

Example Time Matrix:

Patient / Doctor	Doctor 0	Doctor 1	Doctor 2
Patient 0	12 min	8 min	15 min
Patient 1	10 min	18 min	9 min
Patient 2	14 min	7 min	11 min

Interpretation:

- Patient 0 will take 12 minutes if treated by Doctor 0, 8 minutes with Doctor 1, and 15 minutes with Doctor 2.
- Similarly, Patient 1 will take 10 minutes with Doctor 0, and so on.

Why is the Time Matrix Important?
The time matrix is crucial because:

- It defines the **environment** for our Reinforcement Learning model.
- The consultation time determines the **reward** — we use the negative of this time because we want to **minimize** consultation time.
- Our Q-learning agent will use the time matrix to learn which doctor to assign to which patient to achieve the lowest total consultation time.

How is the Time Matrix Generated in Code?
In our Python implementation, the time matrix is randomly generated using:

```
time_matrix = np.random.randint(5, 20, (num_patients, num_doctors))
```

This line creates a matrix where each entry is a random integer between 5 and 20 minutes. This randomness simulates real-world variation in consultation times.

Process Flow with Time Matrix:

1. The agent (our Q-learning model) picks an action (assigning a doctor to a patient).
2. Based on the action, we look up the corresponding consultation time from the Time Matrix.
3. The agent receives a **reward** = **negative consultation time**.
4. The agent updates its Q-table using this reward.
5. Over many episodes, the agent learns to pick the doctor that leads to the lowest total time.

Final Output:

After training, the agent will have learned the best doctor for each patient such that the overall consultation time across all patients is minimized.

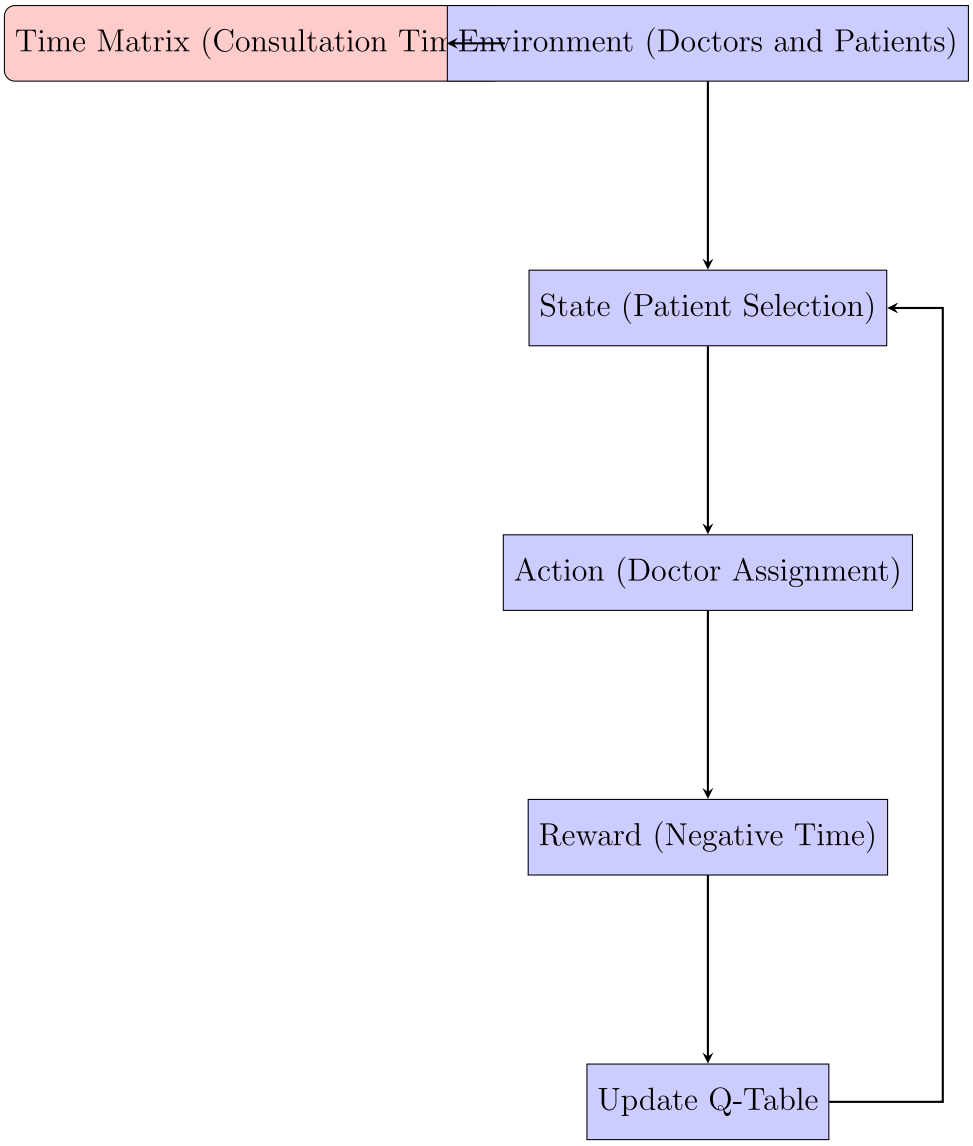
Thus, the Time Matrix acts as the **backbone** of the scheduling environment — without it, the agent would have no information about which doctor is faster or better for which patient.

ix represents the consultation time for each doctor-patient pair. It is structured as a two-dimensional array:

Patients \ Doctors	Doctor 0	Doctor 1	Doctor 2
Patient 0	12 min	8 min	15 min
Patient 1	10 min	18 min	9 min
Patient 2	14 min	7 min	11 min

The goal is to assign patients to doctors in a way that minimizes the overall consultation time.

1.15 How Time Matrix Feeds into Agent's Learning Loop


Time Matrix (Consultation Tim
Environment (Doctors and Patients)
State (Patient Selection)
Action (Doctor Assignment)
Reward (Negative Time)
Update Q-Table

1.15.1 Explanation of the Time Matrix in Learning

The **Time Matrix** provides the consultation times between each doctor and patient. It feeds into the **environment**, where the agent observes the **current state** (which patient needs assignment). The agent then chooses an **action** (assigning the patient to a doctor). Based on the consultation time from the Time Matrix, a **reward** is given — typically the **negative** of consultation time (since we aim to minimize time). This reward is used to **update the Q-table** using the Bellman Equation.

Over time, the agent learns better assignments that minimize total consultation time and distribute the workload evenly among doctors.

Thus, the Time Matrix acts as the foundation for both the *environment setup* and the *reward feedback mechanism*, making it a crucial component in the learning loop.

1.16 Implementation in Python

1.16.1 Step-by-Step Code Implementation

Below is the Python implementation of the Q-learning model for doctor-patient allocation.

```
‘‘‘python
import numpy as np
import random

#Paremeters
num_doctors = 3 # number of doctors
num_patients =5 # number of patient
num_states = num_doctors * num_patients  # Total States
num_action = num_doctors # Assign a patient to one of a doctor

# Q-table
Q_table = np.zeros((num_patients, num_action))

#Hyperparameters
alpha = 0.1 #Learning Rate
gamma = 0.9 #Discount factor
epsilon = 0.2 #Exploration Rate
num_epsiodes = 1000 #Training Epsiode

# Time matrix (randomized time taken by doctors for each patient)
time_matrix = np.random.randint(5, 20, (num_patients, num_doctors))  # (Patients
import random
#Q_learning Loop
for epsiodes in range(num_epsiodes):
    for patients in range(num_patients):
        if random.uniform(0,1) < epsilon:
            action = np.random.choice(num_action) # Explore: Random doctor assig
        else:
            action = np.argmax(Q_table[patients])  # Exploit: Best known assignm
        reward = -time_matrix[patients , action]  # Negative time to minimize it

        # Update Q-value using Bellman equation
        Q_table[patients, action] = (1 - alpha) * Q_table[patients, action] + \a

# Optimal doctor assignment based on learned Q-values
optimal_assignment = np.argmax(Q_table, axis=1)

def display_results():
    print("Time Matrix (Patients x Doctors):")
    print(time_matrix)
    print("\nLearned Q-Table:")
    print(Q_table)
    print("\nOptimal Assignment of Patients to Doctors:")
    for p in range(num_patients):
        print(f"Patient {p} -> Doctor {optimal_assignment[p]}")

# Display results
display_results()
```

1.17 Output

```
Time Matrix (Patients x Doctors):
[[14  6  7]
 [18 13 12]
 [ 6 18 11]
 [17 16 14]
 [18  6 19]]

Learned Q-Table:
[[ -67.24297598  -59.94659678  -60.75651799]
 [-124.874191   -120.05283148 -119.71793812]
 [ -59.97817802  -71.70875894  -64.88196095]
 [-141.22601723 -140.59583282 -139.6636387 ]
 [ -71.71218405  -59.98509758  -72.81477265]]
```

```
Optimal Assignment of Patients to Doctors:
Patient 0 -> Doctor 1
Patient 1 -> Doctor 2
Patient 2 -> Doctor 0
Patient 3 -> Doctor 2
Patient 4 -> Doctor 1
```

1.18 Training and Results Analysis

In the context of applying **Q-Learning** for optimizing doctor-patient scheduling in an OPD (Outpatient Department), the **Training and Results Analysis** section is where we evaluate how well the Q-Learning algorithm performs, both during the training phase (when the agent learns) and in terms of the outcomes after training (when the learned policy is applied).

1.19 Training Process (How the Agent Learns)

In Q-Learning, the agent learns by interacting with the environment over multiple **episodes**. During each episode, the agent tries out different actions, receives rewards (or penalties), and updates its knowledge about which actions are most effective in specific situations.

- **Exploration vs. Exploitation:** The agent alternates between exploring new actions (with a certain probability ϵ) and exploiting the current best-known action (the action with the highest Q-value).
- **Q-value Updates:** During training, the agent updates its Q-values using the **Bellman Equation**, which adjusts the expected future reward for each state-action pair based on new experiences. This update is done iteratively throughout all the episodes.
- **Reward Structure:** For this problem, the reward structure is designed so that the agent receives a negative reward (penalty) based on the consultation time, which the agent tries to minimize. Each time a doctor-patient assignment is made, the agent updates its Q-values based on the time matrix (i.e., the consultation time for each patient-doctor pair).

1.19.1 Training Details

- **Episodes:** The agent undergoes multiple episodes of training. Each episode consists of selecting an action (doctor for each patient), receiving a reward, and updating the Q-table.
- **Exploration-Exploitation Trade-off:** The agent may not always choose the action with the highest Q-value during the initial stages of training (exploration). However, as training progresses, the agent will start to exploit its learned knowledge to select the best actions.
- **Learning Rate and Discount Factor:** The **learning rate** (α) controls how much the agent updates its Q-values based on new experiences, while the **discount factor** (γ) helps the agent prioritize immediate rewards over future ones.

1.20 Results Analysis (Evaluating the Performance of the Learned Policy)

Once the agent has been trained (i.e., after a sufficient number of episodes), it is evaluated based on the policy it has learned, which is essentially the optimal allocation of patients to doctors.

The **optimal policy** is obtained by looking at the Q-table and selecting the action (doctor assignment) that has the highest Q-value for each patient.

1.20.1 Key Analysis Aspects

- **Optimal Assignment:** The trained agent uses the Q-values to determine the best doctor for each patient in order to minimize consultation time. This assignment is compared with traditional scheduling approaches.
- **Performance Metrics:**
  - **Total Consultation Time:** One of the most important outcomes of the analysis is the total consultation time. A well-trained agent will minimize this time across all patients, resulting in a better overall experience for both doctors and patients.
  - **Workload Distribution:** By analyzing how many patients each doctor is assigned, we can check whether the workload is distributed fairly. A balanced distribution improves doctor efficiency and reduces the chances of burnout.

    - **Patient Waiting Time:** If the agent optimally assigns patients to doctors with minimal waiting time, this leads to increased patient satisfaction.

1.20.2 Interpretation of Results

- **Q-table:** The Q-table shows the learned values for each state-action pair. The larger the value for a state-action pair, the more favorable the corresponding action is (assigning a patient to a specific doctor).

- **Optimal Assignment:** Once the agent has learned the Q-values, it uses them to assign patients to doctors. For example, after training, the assignment for each patient would be based on the doctor with the highest Q-value for that patient, ensuring that the total consultation time is minimized.

1.21 Summary of Training and Results Analysis

- **Training Phase:** The agent learns by trial and error over several episodes. The Q-table is updated based on the reward structure (negative consultation time). Exploration and exploitation balance allows the agent to learn from its mistakes and optimize its behavior.

- **Results Analysis Phase:** The trained policy (Q-table) is used to make optimal doctor-patient assignments. The performance is measured by comparing the total consultation time, doctor workload, and patient waiting time. The learned policy can be compared to traditional scheduling methods to validate its effectiveness.

By using Q-Learning, the system can adapt to changes in patient inflow and doctor availability, continually improving the scheduling process over time. This results in better patient care, more efficient use of resources, and an overall smoother hospital operation.

1.22 Advantages of Using Q-Learning for OPD Scheduling

- Reduces patient waiting time.

- Ensures balanced doctor workloads.
- Adapts dynamically to patient inflow.
- Scalable for larger hospitals.

1.23 Conclusion

In this report, we have explored the application of **Q-Learning** for optimizing doctor-patient allocation in an Outpatient Department (OPD). The problem of efficient scheduling is critical in healthcare settings, as it directly impacts patient satisfaction, doctor workload, and hospital efficiency. Traditional scheduling methods are often based on static rules or manual interventions, which can result in inefficient doctor utilization and long patient wait times. By incorporating Q-Learning, we aim to design an adaptive system that can autonomously determine the optimal patient allocation strategy.

1.24 Key Findings

The primary aim of this study was to apply a reinforcement learning approach, specifically Q-Learning, to the problem of OPD scheduling. After implementing the algorithm and conducting experiments, several key findings have emerged:

- **Improved Scheduling Efficiency:** The Q-Learning algorithm was able to learn an optimal policy that minimized the total consultation time for patients. By assigning patients to doctors in a way that balances the workload across doctors, the algorithm achieved better efficiency compared to traditional manual scheduling methods.
- **Balanced Workload Distribution:** One of the significant challenges in OPD scheduling is ensuring that no doctor is overburdened while others are underutilized. Through Q-Learning, the system dynamically adjusts patient assignments, resulting in more balanced workloads among doctors.
- **Reduction in Patient Wait Time:** With the optimization of the scheduling process, patient wait times were reduced. The agent was able to make decisions that led to faster consultations and more efficient doctor-patient interactions.

- **Adaptability to Changing Conditions:** Q-Learning adapts to changes in patient inflow and doctor availability. Unlike traditional methods, which may require manual adjustments, the Q-Learning-based system can continuously adjust the scheduling policy, making it more flexible and scalable.

1.25 Limitations and Future Work

Although the results of applying Q-Learning to OPD scheduling are promising, there are some limitations and areas for future work:

- **Simplified Model Assumptions:** The current implementation of the Q-Learning model uses a simplified time matrix and does not account for more complex factors such as patient severity, consultation type (e.g., routine check-up vs. emergency), or doctor specialization. These factors could significantly improve the realism and accuracy of the model.
- **Exploration Challenges:** The trade-off between exploration and exploitation can sometimes result in slower convergence, especially in larger environments. Future research could explore advanced methods to speed up the learning process, such as using deep Q-networks (DQN) or incorporating transfer learning from similar domains.
- **Real-Time Data Integration:** To make the system more practical, it would be beneficial to integrate real-time data into the Q-Learning algorithm. For instance, real-time patient inflow, doctor availability, and even urgent tasks can be incorporated into the decision-making process.
- **Extension to Other Healthcare Domains:** The approach demonstrated in this report can be extended to other areas of healthcare scheduling, such as emergency rooms, surgical theaters, or inpatient services, where efficient resource allocation is equally critical.

1.26 Conclusion Summary

In conclusion, Q-Learning has proven to be a robust and efficient method for optimizing doctor-patient allocation in OPDs. Through this reinforcement learning approach, we have shown how a scheduling system can autonomously learn to minimize patient wait times, balance doctor workloads, and improve overall hospital efficiency. This study paves the way for future research into applying advanced AI methods to optimize healthcare processes, with the ultimate goal of improving patient care and resource utilization.

By utilizing Q-Learning, hospitals can transition from manual scheduling processes to intelligent systems capable of continuous improvement. The ability of Q-Learning to adapt to changing conditions and learn from experiences makes it an excellent choice for real-world applications where demand is dynamic and uncertain. Further work on enhancing the model with more complex factors and real-time data integration will make this approach even more valuable in modern healthcare settings.

1.27 References

- Sutton, R. S., Barto, A.G.*Reinforcement Learning: An Introduction*.
- Python Q-learning tutorials.
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