Assignment 1

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Solution 1

(a) $P(\text{exactly one of } A \text{ or } B) = P(A) + P(B) - 2P(A \cap B)$
$= 0.3 + 0.4 - 2(0.2)$
$= 0.3$

(b) $P(\text{at least one of } A \text{ or } B) = P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$
$= 0.3 + 0.4 - 0.2$
$= 0.5$

(c) $P(\text{neither } A \text{ or } B) = 1 - 0.5$
$= 0.5$

Monty Hall Problem

Sol^n Let the event of choosing doors 1, 2 and 3 as D_1, D_2 and D_3 and C be the event in which contestant wins car.

~~Ist case~~ Let the car is behind door 1,

Ist case - when contestant decide to switch the door.

$P(C \mid D_1) = 0$ if contestant original choice was 1.

If 2 or 3, then, if contestant switches, then

$P(C \mid D_j) = 1, \quad j = 2, 3.$

and $P(D_j) = \frac{1}{3}, \quad j = 1, 2, 3$

By total Probability thm, $P(C) = \sum_{j=1}^{3} P(D_j) P(C \mid D_j)$

IInd case - when he didn't switch, $P(C \mid D_1) = 1$

and $P(C \mid D_j) = 0$, $j = 2, 3$, and $P(C) = \sum_{j=1}^{3} P(D_j) P(C \mid D_j$

$\Rightarrow P(C) = \frac{1}{3}$.

Therefor, Contestant should switch the door.

(3) Let A be that all balls in bag are red and B be the event that 3 red balls are drawn without replacement.

By Bayes theorem,

$$P(A \mid B) = \frac{P(A) \times P(B \mid A)}{P(B)} \quad -(1)$$

Now, $P(B) = P(B \mid A) \cdot P(A) + P(B \mid \sim A) \cdot P(\sim A)$

$$= 1 \cdot 1 + \frac{1}{4} \times 0$$

$$= 1$$

Substituting in (1), we get

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)} = 1$$

Soln 4 $\Rightarrow$

$$P_X(x) = \begin{cases} 0.1 & \text{for } x = 0.2 \\ 0.2 & \text{for } x = 0.4 \\ 0.2 & \text{for } x = 0.5 \\ 0.3 & \text{for } x = 0.8 \\ 0.2 & \text{for } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

- $P(X < 0.5) = \sum_{X<0.5} P(x)$

$= P(x=0.2) + P(x=0.4)$

$= 0.1 + 0.2 = \underline{0.3}$

- $P(0.25 < X < 0.75) = \sum_{0.25<X<0.75} P(x)$

$= P(x=0.4) + P(x=0.5)$

$= 0.2 + 0.2 = \underline{0.4}$

- $P(X = 0.2 \mid X < 0.6) = \dfrac{P(x=0.2)}{P(x=0.2) + P(x=0.4) + P(x=0}$

$= \dfrac{0.1}{0.1 + 0.2 + 0.2} = \underline{\dfrac{1}{5}}$

Soln 5 $\Rightarrow$

(1) Since $F(X)$ is continuous (Right).

~~Therefore~~ By continuity,

$\frac{2}{\cancel{3}} = \frac{7-6C}{\cancel{6}\,2}$ (at $x = 1$)

$\Rightarrow 6 = 7 - 6C$

$\Rightarrow 6C = 7 - 6$

$\Rightarrow \boxed{C = 1/6}$

Soln 5 (1) $\Rightarrow$ By Right Continuity Condition at $x=3$,

$$\frac{4c^2 - 9c + 6}{4} = 1$$

$$\Rightarrow 4c^2 - 9c + 2 = 0$$

$$\Rightarrow 4c^2 - 8c - c + 2 = 0$$

$$\Rightarrow 4c(c-2) - 1(c-2) = 0$$

$$\Rightarrow c = \frac{1}{4} \text{ or } c = 2$$

$c = 2$ is not possible as $F(x) > 0$.

(2) $$P(1 < X < 2) = F(2^-) - F(1)$$

$$= 0$$

$$P(2 \le X < 3) = F(3^-) - F(2^-)$$

$$= \frac{4c^2 - 9c + 6}{4} - \frac{7 - 6c}{6}$$

Putting value of c, $= \frac{1}{12}$

$$P(0 < X \le 1) = F(1) - F(0)$$

$$= \frac{22}{24} - \frac{2}{3}$$

$$= \frac{22 - 16}{24} = \frac{6}{24} = \frac{3}{12}$$

$$P(1 \le X \le 2) = F(2) - F(1^-)$$

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

$$P(X \ge 3) = F(3^-)$$

$$= 1$$

(6)

- $E[X] = \int_0^1 P(x) \cdot x \, dx = \frac{1}{2}$

~~$E[X^2]$~~

- $Var[X] = E[X^2] - (E[X])^2$

$$= \int_0^1 x^2 \cdot f(x) \, dx - \frac{1}{4}$$

$$= \frac{1}{3} - \frac{1}{4}$$

$$= \frac{1}{12}$$

- $E[Y]$

$E[X^2 + Y^2] = 1$

$\Rightarrow E[X^2] + E[Y^2] = 1$

$\Rightarrow E[Y^2] = \frac{2}{3}$

Now, $Var[Y] = E[Y^2] - (E[Y])^2$

$\Rightarrow \frac{5}{9} = \frac{2}{3} - (E[Y])^2$

$\Rightarrow E[Y] = \frac{1}{3}$

- $E[X+Y] = E[X] + E[Y]$

$$= \frac{1}{2} + \frac{1}{3}$$

$$= \frac{5}{6}$$