classmate

Date
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HW-12

1

$\omega_1 : \{ (2,3), (3,2) (3,5) \}$
$\omega_2 : \{ (0,0), (1,2), (2,0) \}$

Initial seperator is $\{1,1,1\}$

$$\Rightarrow \omega^0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & 1 \\ 3 & 5 & 1 \\ 0 & 0 & 1 \\ 1 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x_1^T \\ x_2^T \\ x_3^T \\ x_4^T \\ x_5^T \\ x_6^T \end{bmatrix}$$

class A $\rightarrow +1$
class B $\rightarrow -1$

$O_1 = f(\omega^T x_1) = f(6) = 1$
$O_2 = f(6) = 1$
$O_3 = f(9) = 1$
$O_4 = f(1) = 1$
$O_5 = f(4) = 1$
$O_6 = f(3) = 1$

$t_1 = t_2 = t_3 = 1$ and
$t_4 = t_5 = t_6 = -1$

$\omega' = \omega^0 + \eta \sum_i (t_i - O_i) x_i$
$\omega = \omega^0 - \eta 2 (x_4 + x_5 + x_6)$

For $\eta = \frac{1}{2}$ $\qquad \omega' = \omega^0 - x_4 - x_5 - x_6$

$$\boxed{w^1 = \begin{bmatrix} -2 \\ -1 \\ -2 \end{bmatrix}}$$

$$O_1 = f(-4-3-3) = f(-10) = -1$$

Illy,

$$O_2 = O_3 = O_4 = O_5 = O_6 = -1$$

$$t_1 = t_2 = t_3 = 1$$

$$t_4 = t_5 = t_6 = -1$$

$$w^2 = w^1 + \eta \sum (t_i - O_i) x_i$$

$$= w^1 + \frac{1}{2}[2x_1 + 2x_2 + 2x_3]$$

$$w^2 = \begin{bmatrix} -2 \\ -1 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ 1 \end{bmatrix}$$

$$\boxed{w^2 = \begin{bmatrix} 6 \\ 9 \\ 1 \end{bmatrix}}$$

2) a)

$$x_1 = 1$$

$$w^0 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & 1 \\ 2 & 2 & 1 \\ -2 & -2 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$t_1 = t_3 = t_5 = t_6 = 1$$

$$t_2 = t_4 = -1$$

class 1 if $w^T x > 0$

class 2 if $w^T x \leq 0$

$$O_1 = [1\,0\,-1]\begin{bmatrix}1\\1\\1\end{bmatrix} = f(0) = -1$$

Ily,

$O_2 = -1$, $O_3 = 1$, $O_4 = 1$, $O_5 = -1$, $O_6 = -1$

$$w^1 = w^0 + \eta \sum_i (t_i - o_i) x_i$$
$$= w^0 + \eta 2 (x_1 + x_5 + x_6)$$

for $\eta = 1/2$

$$w^1 = w^0 + x_1 + x_5 + x_6$$

$$\boxed{w^1 = \begin{bmatrix}2\\1\\2\end{bmatrix}}$$

$O_1 = f(5) = 1$

Ily $O_2 = O_4 = -1$

$O_3 = O_5 = O_6 = 1$

For all $i = 1$ to $i = n$, $t_i = O_i$

$$\therefore w^2 = w^1 + \eta \sum_i (t_i - O_i) x_i$$

$$\boxed{w^2 = w^1}$$

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2(b) First sample – $(1,1,-1)$

$\Rightarrow t_1 = t_2 = t_4 = -1$

$t_3 = t_5 = t_6 = 1$

x_1 didn't change

$\Rightarrow O_1 = O_2 = O_4 = O_5 = O_6 = -1 \; ; O_3 = 1$

$$w^1 = w^0 + \eta \Sigma (t_i - O_i) x_i$$

$(\eta = 1/2)$ $$= w^0 + \frac{1}{2} 2 [x_5 + x_6]$$

$$= w^0 + x_5 + x_6$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

For second iteration =

$O_1 = O_3 = O_6 = 1$

$O_2 = O_4 = O_5 = -1$

$$w^2 = w^1 + \eta \Sigma (t_i - O_i)$$

$$= w^1 + (-x_1) + x_5$$

$$w^2 = [-1 \ 0 \ 1]^T$$

For third iteration

$O_1 = O_3 = O_6 = -1 \quad ; \quad O_2 = O_4 = O_5 = 1$

$$w^3 = w^2 - x_2 + x_3 - x_4 + x_6$$

$$= \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$$

lly

$$w^4 = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \quad w^5 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \quad w^6 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$w^7 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ —— scaled version of w^1

Hence oscillation of values.

③ **Gradient Descent** | **Newton's Method**

Gradient Descent	Newton's Method
→ first order	→ secon order
→ less computatⁿ required in each iteratⁿ	→ more computⁿ in each iteration
→ more iterations (seen in picture attached)	→ less iteratⁿ

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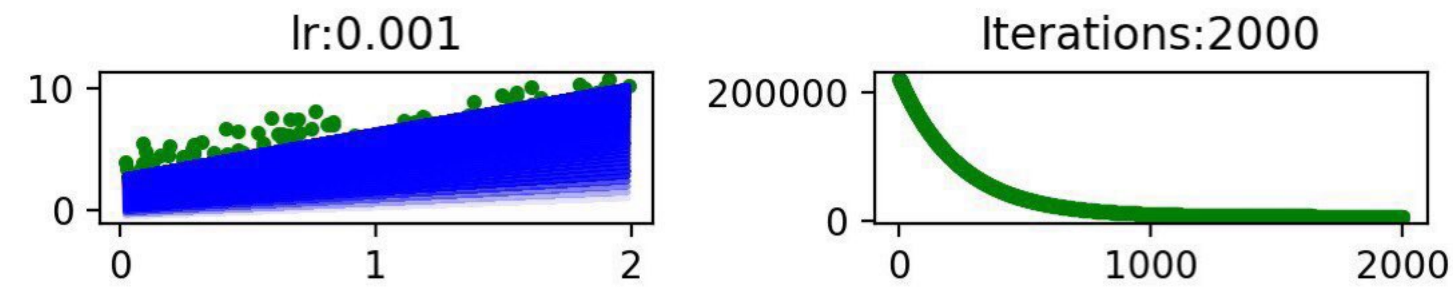

<u>shows how different learning rates and iterations useful for GD</u>