Q-1 

① $2A - B$

$\rightarrow 2.A = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ , $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $\therefore 2A - B = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$

② $||A||$ and angle of A relative to the X axis

$\rightarrow ||A|| = \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{1+4+9} = \sqrt{14}$

Angle of A relative to Positive X-axis:

$\cos\theta = \frac{A.X}{||A||.||X||} = \frac{1}{\sqrt{14}.1} = 1/\sqrt{14} = 0.27$ $\theta = \cos^{-1}(0.27)$

$\boxed{\theta = 74^\circ}$

③ $\hat{A}$, a unit Vector in direction of A

$A = (1, 2, 3)$ so, unit Vector $u = \frac{A}{||A||}$

$\therefore u = \frac{1}{||A||}.A = \frac{1}{\sqrt{14}}(1,2,3) = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

$= \left(\frac{\sqrt{14}}{14}, \frac{2\sqrt{14}}{14}, \frac{3\sqrt{14}}{14}\right)$

$= \left(\sqrt{14}/14, \sqrt{14}/7, 3\sqrt{14}/14\right)$

④ direction Cosine of A

$\cos\alpha = 1/\sqrt{14}$

$\cos\beta = 2/\sqrt{14}$

$\cos\gamma = 3/\sqrt{14}$

⑤ A.B & B.A are undefined as no. of column in A is not equal to no. of row in B

⑥ Angle between A and B.

~~$A.B = ||A||\,||B||.\cos\theta$~~

~~$4+10+18 \begin{bmatrix} 4 \\ 10 \\ 18 \end{bmatrix} = \sqrt{13}\,.\sqrt{77}.\cos\theta$~~

~~$\theta = \cos^{-1}\frac{(4,10,18)}{\sqrt{14}.77}$ $\therefore \theta = \cos^{-1}$~~

$A.B = ||A||\,||B||.\cos\theta$

$4+10+18 = \sqrt{14}\sqrt{77}.\cos\theta$

$\frac{32}{\sqrt{14}\sqrt{77}} = \cos\theta$

$\theta = \cos^{-1}\left(\frac{32}{\sqrt{14}.\sqrt{77}}\right)$

⑦ Vector Perpendicular to A : i.e B.

$$\cos 90^\circ = \frac{A.B}{||A|| \, ||B||}$$ , here $A.B = 0$ (from Property of perp. vector)

$(1,2,3).B = 0$

for $B = (-2,1,0)$ we will get $(A.B = 0)$

So, $B = (-2,1,0)$

⑧ $A \times B$ and $B \times A$.

no. of column in A is not equal to no. of row in B. So $A \times B$ is not possible. Same way no. of column in B is not equal to no. of row in A, so $B \times A$ is not possible.

⑨ a vector perpendicular to both A and B

→ $A = [1 \quad 2 \quad 3]$

$B = [4 \quad 5 \quad 6]$

$$A \times B = \begin{bmatrix} i & j & k \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= i\begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} - j\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} + k\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$$

$$= -3(i) + 6(j) - 3k$$

$A \times B = (-3, 6, -3)$

$|A \times B| = \sqrt{9+36+9} = \sqrt{54} = 3\sqrt{6}$.

Vector Perpendicular to both A & B :

$$\left( \frac{-3}{3\sqrt{6}}, \frac{6}{3\sqrt{6}}, \frac{-3}{3\sqrt{6}} \right)$$

$$= \left( -1/\sqrt{6}, 2/\sqrt{6}, -1/\sqrt{6} \right)$$

(10) linear dependancy between A, B, C.

$$\begin{vmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{vmatrix} \Rightarrow 0 = A\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + B\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + C\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{bmatrix}\begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

(11) $A^TB$ and $AB^T$.

① $A^TB$:

$$A^T = [1 \quad 2 \quad 3], \quad B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = [1.4 + 2.5 + 3.6]$$

$$= [4 + 10 + 18]$$

$$= [32]$$

② $AB^T$

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad B^T = [4 \quad 5 \quad 6]$$

$$A.B^T = \begin{bmatrix} 1.4 & 1.5 & 1.6 \\ 2.4 & 2.5 & 2.6 \\ 3.4 & 3.5 & 3.6 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$$

Q2

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 1 & 3 \end{bmatrix}.$$

1) $2A - B = \underbrace{\begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 6 \\ 0 & 10 & -2 \end{bmatrix}}_{2A} - \underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}}_{B}$

$$= \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$$

2) A.B & B.A.

$\to A.B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1+4+9 & 2+2-6 & 1-8+3 \\ 4-4+9 & 8-2-6 & 4+8+3 \\ 0+10+(-3) & 0+5+2 & 0+(-20)+ \end{bmatrix}$

$$= \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

$\to B.A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} = \begin{bmatrix} 1+8+0 & 2-4+5 & 3+6+(-1) \\ 2+4+0 & 4-2-20 & 6+3+4 \\ 3-6+0 & 6+4+5 & 9-6-1 \end{bmatrix}$

$$= \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

(3) ~~matrix $V^{-1}AV$, $V$ composed of eigenvector of A.~~

$(AB)^T$ and $B^TA^T$

$\rightarrow (AB)^T$ :

$$A.B = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

$$\rightarrow B^T.A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 4-4+9 & 0+10-3 \\ 2+2-6 & 8-2-6 & 0+5+2 \\ 1-8+3 & 4+8+3 & 0-20-1 \end{bmatrix} = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

(4) $|A|$ : $1[(-2.-1)-(5.3)] - 2[(-4-0)] + 3[20-0]$

$= 1[2-15] - 2[-4] + 60$

$= -13 + 8 + 60 = 55$ Ans

$|C|$ : $1[15-6] - 2[12+6] + 3[4+5]$

$= 9 - 36 + 27$

$= 0.$ Ans

(5) To from an orthogonal set,

$u.v = 0$

here, 3 will from an orthogonal set.

$u = (1,2,1)$, $v = (2,1,-4) \rightarrow u.v = 2+2-4 = 0$

$u = (2,1,-4)$, $v = (3,-2,1) \Rightarrow u.v = 6-2-4 = 0$

$u = (1,2,1)$, $v = (3,-2,1) \Rightarrow u.v = 3-4+1 = 0$

(6) $A^{-1}$ :

$$A: \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix}$$

$\det(A) = 55$

$$A^T = \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

det of each 2x2 matrices.

$A_{11}: -13$ $\quad A_{21}: -4$ $\quad A_{31}: 20$

$A_{12}: -17$ $\quad A_{22}: -1$ $\quad A_{32}: 5$

$A_{13}: 12$ $\quad A_{23}: -9$ $\quad A_{33}: -10.$

$$A^{-1} = \frac{1}{\det(A)}\left(\text{adj}(A)\right)$$

$$= \frac{1}{55}\begin{bmatrix} -13 & -4 & 20 \\ -17 & -1 & 5 \\ 12 & -9 & -10 \end{bmatrix} = \begin{bmatrix} -13/55 & -4/55 & 20/55 \\ -17/55 & -1/55 & 1/11 \\ 12/55 & -9/55 & -10/55 \end{bmatrix}$$

Ans

$B^{-1}$ :

$$B: \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix}$$

$$\det(B) = 1(1+8) - 2(2+12) + 1(-4-3)$$
$$= 1(9) - 2(14) + 1(-7)$$
$$= 9 - 28 - 7 = 9 - 35 = -26$$

det of each 2x2 matrices:

$B_{11}: 9$ $\quad B_{21}: 14$ $\quad B_{31}: -7$

$B_{12}: 4$ $\quad B_{22}: -2$ $\quad B_{32}: -6$

$B_{13}: -9$ $\quad B_{23}: -6$ $\quad B_{33}: -3$

$$B^{-1} = \frac{1}{\det(B)}\left(\text{adj}(B)\right)$$

$$= \frac{1}{-26}\begin{bmatrix} 9 & 14 & -7 \\ 4 & -2 & -6 \\ -9 & -6 & -3 \end{bmatrix} = \begin{bmatrix} -9/26 & -14/26 & 7/26 \\ -4/26 & 1/13 & 6/26 \\ 9/26 & 6/26 & 3/26 \end{bmatrix}$$

Ans

Q-3:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$$

① $A - \lambda I = 0$

$$\rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\rightarrow \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} = 0$$

$\rightarrow (1-\lambda)(2-\lambda) - 6 = 0$

$\rightarrow 2 + \lambda^2 - 3\lambda - 6 = 0$

$\lambda^2 - 3\lambda - 4 = 0$

$(\lambda - 4)(\lambda + 1) = 0$

$\boxed{\lambda = 4 \quad \text{or} \quad \lambda = -1}$ eigen values:

For $\lambda = 4$

eigen vector:

$$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix}$$

For $\lambda = -1$

eigen vector

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$$

Ans

② $V^{-1}.AV$ where $V$ is composed of eigen vectors of A

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \quad V = \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix}_{\text{for } \lambda = 4}, \quad V = \begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix}_{\lambda = -1}$$

$V^{-1} = \dfrac{\text{adj}(V)}{|V|} = 1/\tfrac{5}{2} \begin{bmatrix} 1 & 1 \\ -3/2 & 1 \end{bmatrix}$

~~$V^{-1} = 2 / 0$ for both~~ $= \begin{bmatrix} 2/5 & 2/5 \\ -3/5 & 2/5 \end{bmatrix}$

hence,

$V^{-1}.AV = 0.$

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(3) Dot Product between eigenvector of A.

$$= \begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$$

$$\Rightarrow \quad = \begin{bmatrix} 1/2 \\ 3/2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= -1 + 3/2 = \underline{1/2}$$ Ans

(4) $B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$

$$B - \lambda I = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{bmatrix}$$

$$= \lambda^2 - 7\lambda + 6 = (\lambda - 6)(\lambda - 1)$$

$$\lambda = 6 \quad \text{or} \quad \lambda = 1$$

for $\lambda = 6$,

$$\begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-4x_1 - 2x_2 = 0$$

$$4x_1 = -2x_2 \qquad x_1 = \underline{-x_2/2}$$

for $\lambda = 1$

$$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

dot Product: $\begin{bmatrix} -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = -2 + 2 = 0$

(5) Properties

- matrix is not singular as it determinant is non-zero
- dot product of eigenvectors is 0, so they both are perpendicular to each other

Q4 D) $f(x) = x^2+3$ , $g(x,y) = x^2+y^2$

① → $f'(x) = d/dx\,(x^2+3)$

$= 2x$

→ $f''(x) = d/dx\,(f'(x))$

$= d/dx\,(2x) = \underline{2}$

② $\dfrac{\partial g}{\partial x} = \dfrac{\partial}{\partial x}(x^2+y^2)$

$= 2x$

$\dfrac{\partial g}{\partial y} = \dfrac{\partial}{\partial y}(x^2+y^2)$

$= 2y$

③ gradient vector

$\nabla g(x,y) = \dfrac{d(g)}{dx} + \dfrac{d(g)}{dy}$

$= (2x + 2y)$

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Homework 0.