1. Determine the number of subsets of a set with 4 elements.

Answer: Including all four elements, there are 24 = 16 subsets. 15 of those subsets are proper, 1 subset, namely {a,b,c,d}, is not.  
  
In general, if you have n elements in your set, then there are 2n subsets and 2n − 1 proper subsets.

1. Explain that intersection of two normal subgroups of G is again a normal subgroup of G.

Answer: Let H1 and H2 be any two subgroups of G.  
Then,

Since at least the identity element 'e' is common to both H1 and H2 . Since H1 and H2 are subgroups. Hence, H1 ∩ H2 is a subgroup of G and that is our theorem i.e. The intersection of two subgroups of a group is again a subgroup.

1. Calculate 7C3 (7 chose3).

Answer: By using the formula of combination . answer will be 35

1. Let G be a finite group and H be a subgroup of G. Then explain that order of H divides order of G. f G is a connected simple graph with n vertices with n>=3 such that G has Hamilton cycle then explain the complement of a disconnected graph is connected.

Answer : The statement that the order of a subgroup *H* of a finite group *G* divides the order of *G* is a fundamental result in group theory, and it directly follows from Lagrange's Theorem. To explain why this is true, let's first define some terms and then delve into the theorem itself.

**Definitions**

1. **Group (*G*)**: A set equipped with an operation that combines any two elements to form a third element, satisfying four conditions called the group axioms: closure, associativity, the existence of an identity element, and the existence of inverse elements.
2. **Subgroup (*H*)**: A subset of a group that is itself a group under the operation defined on the original group.
3. **Order of a Group**: The number of elements in the group.
4. **Cosets**: a subgroup H of a group G may be used to decompose the underlying set of G into disjoint, equal-size subsets called cosets.

**Lagrange's Theorem**

**Lagrange theorem**is one of the central theorems of abstract algebra. It states that in group theory, for any finite group say G, the order of subgroup H of group G divides the order of G. The order of the group represents the number of elements.

**Proof of Lagrange Statement:**

Let H be any subgroup of the order n of a finite group G of order m. Let us consider the cost breakdown of G related to H.

Now let us consider each coset of aH comprises n different elements.

Let H = {h1,h2,…,hn}, then ah1,ah2,…,ahn are the n distinct members of aH.

Suppose, ahi=ahj⇒hi=hj be the cancellation law of G.

Since G is a finite group, the number of discrete left cosets will also be finite, say p. So, the total number of elements of all cosets is np which is equal to the total number of elements of G. Hence, m=np

p = m/n

This shows that n, the order of H, is a divisor of m, the order of the finite group G. We also see that the index p is also a divisor of the order of the group.

Hence, proved, |G| = |H|

**Conclusion**

Lagrange's Theorem provides a fundamental insight into the structure of finite groups by relating the sizes of a group and its subgroups. It implies that the possible orders of subgroups of a given finite group are restricted to the divisors of the order of the group, which is a powerful tool in the study of group theory.

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5). What is the cardinality of the power set of an empty set?

Answer: As we know that an empty set is a set having zero or no element and the power set is the set of all the subsets of a set. Hence the power set with an empty set will be a null set and its cardinality will be zero.

6). Explain that every subgroup of a cyclic group is cyclic. iv) State and prove Lagrange's theorem on groups.

Answer: **Every Subgroup of a Cyclic Group is Cyclic**

In a cyclic group, every element is a power of a single generator. Let*G*=⟨*a*⟩ be a cyclic group generated by *a*. Any subgroup *H* of *G* is also generated by a power of *a*, making it cyclic.

**Lagrange's Theorem on Groups**

Lagrange's Theorem states that for any finite group *G* and any subgroup *H* of *G*, the order of *H* divides the order of *G*.

**Proof**:

1. **Coset Decomposition**: For any group *G* and subgroup *H*, the left cosets of *H* partition *G*. Each coset has the same number of elements as �*H*, and they are either identical or disjoint.
2. **Equivalence Classes**: Since *G* is the union of these disjoint cosets, the order of *G* is a multiple of the order of *H*.

Hence, ∣∣*H*∣ divides ∣*G*∣.

**Conclusion**

Every subgroup of a cyclic group is cyclic, and Lagrange's Theorem provides a concise statement and proof about the relationship between the orders of groups and their subgroups in finite group theory.

7). If G is a connected simple graph with n vertices with n>=3 such that G has Hamilton cycle then explain the complement of a disconnected graph is connected.

1. Answer: **Connected Graph**: A graph is connected if there is a path between every pair of vertices in the graph.
2. **Simple Graph**: A graph with no loops (edges connecting a vertex to itself) and no multiple edges between the same pair of vertices.
3. **Hamiltonian Cycle**: A Hamiltonian cycle is a cycle in a graph that visits each vertex exactly once.
4. **Complement of a Graph**: The complement of a graph *G*, denoted as ‾*G*, is a graph that contains exactly the same set of vertices as *G* but has an edge between two vertices if and only if *G* does not have that edge.

**Proof**:

1. In *G*, every pair of vertices is connected by an edge since it has a Hamiltonian cycle, which visits each vertex exactly once.
2. The complement ‾*G* has an edge between two vertices if and only if they are not connected in *G*.
3. If ‾*G* were disconnected, it would imply that there exist at least two vertices in *G* that are not connected by an edge, contradicting the fact that *G* is connected.

Hence, the complement of a connected graph with a Hamiltonian cycle is connected.

8). The edge set F of the connected graph G is a cut set of G if and only if Explain (i) F includes at least one branch from every spanning tree of G (ii) if H ⊂F, then there is a spanning tree none of whose branches is in H b) Prove that there is one and only one path between every pair of vertices in a tree.

Answer: **Explanation:**

(i) F includes at least one branch from every spanning tree of G:

A cut set *F* of a connected graph *G* is a set of edges whose removal disconnects the graph. Now, if *F* includes at least one branch from every spanning tree of *G*, it implies that removing any edge from *F* will disconnect *G*. This is because a spanning tree is a subgraph of *G* that connects all vertices without forming any cycles. Thus, removing any edge from a spanning tree will disconnect the graph.

(ii) If *H*⊂*F*, then there is a spanning tree none of whose branches is in *H*:

If *H*⊂*F*, it means that removing the edges in *H* disconnects *G*. For each connected component formed by removing edges in *H*, there must exist a spanning tree (by definition of a spanning tree) that connects all vertices within that component. None of the edges in *H* are included in this spanning tree because removing any of them would disconnect the component.

**Proof:**

There is one and only one path between every pair of vertices in a tree:

1. **Existence of a Path**: Since a tree is a connected graph with no cycles, there exists a path between any pair of vertices. This path is unique because if there were two distinct paths between the same pair of vertices, it would form a cycle, contradicting the definition of a tree.
2. **Uniqueness of the Path**: Suppose there are two distinct paths *P*1​ and *P*2​ between vertices *u* and *v*. Since a tree is acyclic, *P*1​ and *P*2​ must intersect at some vertex other than *u* and *v*. However, this would create a cycle, which contradicts the assumption that the graph is a tree.

Hence, there is one and only one path between every pair of vertices in a tree.

9). Find the power set of a set with 3 elements.

Answer: The power set of a set is the set of all its subsets, including the empty set and the set itself.

Let's denote the set with 3 elements as *A*, and let's*A*={*a*,*b*,*c*}.

To find the power set of *A*, we need to list all possible combinations of its elements:

1. The empty set: {}
2. Singletons: {*a*},{*b*},{*c*}
3. Pairs: {*a*,*b*},{*a*,*c*},{*b*,*c*}
4. Triplets: {*a*,*b*,*c*}

Putting all of these together, we have the power set of *A*:P(*A*)={{},{*a*},{*b*},{*c*},{*a*,*b*},{*a*,*c*},{*b*,*c*},{*a*,*b*,*c*}}

So, the power set of a set with 3 elements contains 8 subsets.

10). Prove that the sum of two odd integers is always even.

Answer: To prove that the sum of two odd integers is always even, we can use the definition of odd integers.

An odd integer is of the form 2*k*+1, where *k* is an integer. Let's denote two odd integers as 2*k*1​+1 and 2*k*2​+1, where *k*1​ and *k*2​ are integers.

Now, let's find the sum of these two odd integers:(2*k*1​+1)+(2*k*2​+1)=2*k*1​+2*k*2​+1+1=2(*k*1​+*k*2​)+2

Notice that (*k*1​+*k*2​)+2 is an even integer because it is a multiple of 2. Specifically, it is twice some integer (*k*1​+*k*2​).

Therefore, the sum of two odd integers is always even.

This completes the proof.

11). Show using induction that i) any tree which has n nodes has n-1 edges ii) any rooted binary tree with depth d≥1 has atmost 2^d+1-1 nodes. iii) Assume that +,-, x and are left-associative, and that ^ is right-associative. The precedence order is ^, ×, +, −. Show the the postfix expression corresponding to the infix expression ""a + b × c − d ^ e ^.

Answer:

Sure, let's prove each statement step by step:

**(i) Proving that any tree with *n* nodes has *n*−1 edges using induction:**

**Base Case (n = 1)**: A tree with one node has no edges, and 1−1=01−1=0, which is correct.

**Inductive Step**: Assume the statement holds for a tree with *k* nodes, where *k*≥1. Now, consider adding a new node to this tree. To maintain the tree's connectivity (i.e., it remains a tree), we must also add an edge to connect this new node to an existing node. Thus, the number of edges increases by 1. Therefore, for *k*+1 nodes, the number of edges is (*k*+1)−1=*k*, which is consistent with the statement.

By induction, we have shown that any tree with *n* nodes has *n*−1 edges.

**(ii) Proving that any rooted binary tree with depth *d*≥1 has at most 2*d*+1−1 nodes:**

**Base Case (d = 1)**: For a binary tree with depth 1, it can have at most 21=2 nodes. And 21+1−1=221+1−1=2, which is correct.

**Inductive Step**: Assume the statement holds for a binary tree with depth *k*, where *k*≥1. Now, consider a binary tree with depth *k*+1. Each node in the *k*-level tree can have at most two children. Therefore, by adding one more level, each node in the *k*-level tree can have at most two new children. Thus, the total number of nodes in the *k*+1-level tree is at most twice the number of nodes in the *k*-level tree, plus one for the root. Therefore, the maximum number of nodes in a binary tree with depth *k*+1 is 2×(2*k*+1−1)+1=2*k*+1+1−1.

By induction, we have shown that any rooted binary tree with depth *d*≥1 has at most 2*d*+1−1 nodes.

**(iii) Finding the postfix expression corresponding to the infix expression "a + b × c - d ^ e ^ f":**

To convert an infix expression to postfix, we use the shunting yard algorithm, which uses a stack to keep track of operators and operands.

Given the precedence order and associativity, the postfix expression for the given infix expression is:

abc×+de^f^^−

12). Prove that a connected graph with n vertices and n1 edges is a tree.

Answer: Proof: We know that the minimum number of edges required to make a graph of n vertices connected is (n-1) edges. We can observe that removal of one edge from the graph G will make it disconnected. Thus a connected graph of n vertices and (n-1) edges cannot have a circuit. Hence a graph G is a tree.

13). Calculate the number of Eulerian paths in a complete graph with 8 vertices.

Answer: An Eulerian path in a graph is a path that traverses each edge of the graph exactly once. In a complete graph, every pair of vertices is connected by an edge, and thus, every vertex has the same degree.

For a complete graph with *n* vertices, each vertex has degree *n*−1 since it is connected to every other vertex in the graph. Therefore, for an Eulerian path to exist in a complete graph, the degree of every vertex must be even.

In a complete graph with 8 vertices, each vertex has degree *n*−1=8−1=7. Since 7 is an odd number, it's impossible for any vertex to have an even degree, and thus, it's impossible for a complete graph with an odd number of vertices to have an Eulerian path.

Therefore, there are 0 Eulerian paths in a complete graph with 8 vertices.

14). Show that if T is a full binary tree, then the number of leaves of T is one more than the number of internal vertices (non-leaves).

Answer: In a full binary tree, every internal node (non-leaf) has exactly two children. Therefore, for each internal node, we are adding two new leaves.

Let *L* be the number of leaves in the tree, and let �*I* be the number of internal vertices (non-leaves). Since each internal node adds two new leaves, the number of leaves in the tree increases by two for each internal node. Thus, *L*=*I*+2.

Adding one more internal node to the tree will add two more leaves. Therefore, for the tree with �+1*k*+1 nodes, the number of leaves will be *L*+2.

By the induction hypothesis, we know that *L*=*I*+2 for a full binary tree with *k* nodes. Substituting this into the expression for the tree with *k*+1 nodes, we get:

*L*+2=(*I*+2)+2=*I*+4=(*I*+1)+3

Therefore, for a full binary tree with *k*+1 nodes, the number of leaves is one more than the number of internal vertices.

This completes the proof by induction.

15). Calculate the number of surjective functions from a set with 3 elements to a set with 2 elements.

Answer: To calculate the number of surjective functions from a set with 3 elements to a set with 2 elements, we consider the number of ways each element in the codomain can be reached by elements in the domain.

Since the function must be surjective, each element in the codomain must be reached by at least one element in the domain.

There are two possible scenarios:

1. One element in the codomain is reached by two elements in the domain, and the other element is reached by one element in the domain.
2. The other element in the codomain is reached by two elements in the domain, and the first element is reached by one element in the domain.

In both scenarios, the element with two pre-images can be chosen in 22 ways, and the remaining element in the codomain will have only one pre-image.

So, the total number of surjective functions is 2×2=4.

Therefore, there are 44 surjective functions from a set with 33 elements to a set with 22 elements.

16). Prove that the sum of the eigenvalues of a square matrix is equal to the trace of the matrix.

Answer: To prove that the sum of the eigenvalues of a square matrix is equal to the trace of the matrix, let's denote the square matrix as *A*. We'll use the fact that the trace of a matrix is equal to the sum of its eigenvalues.

Let *λ*1​,*λ*2​,…,*λn*​ be the eigenvalues of *A*. We know that the trace of *A* is given by:

tr(*A*)=*i*=1∑*n*​*aii*​

where *aii*​ denotes the *i*-th diagonal element of *A*.

We also know that the eigenvalues *λ*1​,*λ*2​,…,*λn*​ of *A* satisfy the characteristic equation:

det⁡det(*A*−*λI*)=0

where *I* is the identity matrix of the same size as *A*.

Expanding the determinant of *A*−*λI*, we get:

det(*A*−*λI*)=(*λ*1​−*λ*)(*λ*2​−*λ*)⋯(*λn*​−*λ*)

Now, let's consider the term (*λ*1​−*λ*)(*λ*2​−*λ*)⋯(*λn*​−*λ*). When we expand this product, we get a sum of terms that include *λ* raised to different powers. The coefficient of *λn*−1 term in this expansion is exactly the negative of the trace of *A*.

Therefore, by Vieta's formulas, the sum of the eigenvalues *λ*1​,*λ*2​,…,*λn*​ is equal to the coefficient of *λn*−1, which is equal to the negative of the trace of *A*. Hence, the sum of the eigenvalues of a square matrix is equal to the trace of the matrix.

This completes the proof.

17). Show that i) every chain is a distributive complemented lattice ii) show that the following are equivalent a) a<=b b) a^b=0 c) aVb=1 d) b<=a.

Answer:

18). \* on R DEFINED BY X\*Y=X+Y+2XY for all x,y belongs to R . Identify i) (R, \*) is monoid or not. It is commutative ii) Which elements have inverse and what are they?

Answer:

19). Illustarte the total number of arrangements of the letters in the word "GALGOTIAS"". If the arrangement must start with a consonant and end with a vowel, Illustrate different arrangements are possible? Now, consider the word ""BANARAS"". Illustarte the arrangements are possible if the vowels must be together?

Answer: o find the total number of arrangements of the letters in the word "GALGOTIAS", we first count the number of letters in the word, which is 8. However, since the letter 'A' is repeated three times, 'G' is repeated twice, and the other letters are distinct, we have to divide by the factorials of the multiplicities of each letter to avoid overcounting.

The total number of arrangements is:

8!/3!×2!

To illustrate the different arrangements, let's denote the distinct letters as *G*1​,*G*2​,*A*1​,*A*2​,*A*3​,*L*,*O*,*T*,*I*. Then, the total number of arrangements is:

8!3!×2!=403206×2=33603!×2!8!​=6×240320​=3360

Now, if the arrangement must start with a consonant and end with a vowel, the possible arrangements are:

* Consonants: *G*1​,*G*2​,*L*,*T*,*S*
* Vowels: *A*1​,*A*2​,*A*3​,*O*,*I*

We have 5 choices for the first letter (consonant) and 5 choices for the last letter (vowel). For the remaining 6 positions, we have 6!/3!×2! arrangements.

So, the total number of arrangements satisfying the given condition is:

5×5×6!/3!×2!=5×5×60=1500

Now, considering the word "BANARAS", if the vowels must be together, we can treat the block of vowels as a single entity. So, we have *BNR*+(*AAA*)+*S*. The total number of arrangements in this case is:

4!/2!×3!/3!×2!=12

So, there are 12 possible arrangements if the vowels must be together.

20). A factory makes custom Bikes at an increasing rate. In the 1st Month only one Bike made, in the 2nd month two Bikes are made, and so on with n Bikes made in the nth month. (1) Set up the recurrence relation for the number of Bikes produced in the first n month by this factory. (2) How many Bikes are produced in the 1styear?

Answer: To set up the recurrence relation for the number of bikes produced in the first *n* months, we can observe that the number of bikes made in the *n*th month is simply *n*, as described in the problem. Therefore, the total number of bikes produced in the first *n* months is the sum of the bikes produced in each month from 1 to *n*.

Let *Bn*​ represent the number of bikes produced in the first *n* months. Then, we have:

*Bn*​=1+2+3+…+*n*

We can use the formula for the sum of the first *n* natural numbers to simplify this expression. The sum of the first *n* natural numbers is given by:

1+2+3+…+*n*=2*n*(*n*+1)​

Therefore, the recurrence relation for the number of bikes produced in the first *n* months is:

2*Bn*​=2*n*(*n*+1)​

Now, to find the number of bikes produced in the first year, we need to sum the number of bikes produced in each of the first 12 months:

Number of bikes produced in the 1st year=12Number of bikes produced in the 1st year=*B*12​

Using the recurrence relation we derived, we have:

*B*12​=212(12+1)​=212×13​=78

Therefore, 78 bikes are produced in the first year.

21). Examine a set of 20 different fruits - 8 apples, 6 oranges, and 6 bananas. Answer the following questions: a) In how many ways can you arrange all the fruits? b) If you select 5 fruits from the set, in how many ways can you arrange them? c) How many ways are there to distribute the 20 fruits among three children so that each child receives at least one fruit?"

Answer: Let's solve each question step by step:

a) In how many ways can you arrange all the fruits?

To find the total number of arrangements, we can use the concept of permutations. Since there are 20 fruits in total, we can arrange them in 20!20! ways. However, since there are repetitions of fruits (8 apples, 6 oranges, and 6 bananas), we need to divide by the factorial of the number of each type of fruit to correct for overcounting.

So, the number of arrangements is:

20!8!×6!×6!8!×6!×6!20!​

b) If you select 5 fruits from the set, in how many ways can you arrange them?

Similar to part a, we'll use permutations, but this time with only 5 fruits.

So, the number of arrangements of 5 fruits is:

5!�1!×�2!×�3!*n*1​!×*n*2​!×*n*3​!5!​

where �1*n*1​, �2*n*2​, and �3*n*3​ are the numbers of each type of fruit in the selection.

c) How many ways are there to distribute the 20 fruits among three children so that each child receives at least one fruit?

This can be solved using the concept of distributing indistinguishable objects into distinguishable groups, which is a problem of combinations with repetition.

We can approach this by first giving one fruit to each child. After this, we need to distribute the remaining 17 fruits among the three children. This can be done in (17+3−13−1)(3−117+3−1​) ways.

So, let's calculate each part:

a) Number of arrangements of all the fruits:

20!8!×6!×6!=20!(8!)28!×6!×6!20!​=(8!)220!​

b) Number of arrangements of 5 fruits:

5!3!×1!×1!=5!3!3!×1!×1!5!​=3!5!​

c) Number of ways to distribute 20 fruits among three children:

(17+3−13−1)=(192)(3−117+3−1​)=(219​)

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ChatGPT can make mistakes. Consider checking important information.

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