

ustering: divides data into a fixed number of clusters by optimizing a criterion, like k-means minimizing within-cluster variance. K-means is a popular partitioning method that assigns points to the nearest cluster centroid iteratively. Its time complexity is $O(n \times k \times i \times d)$, where n = number of points, k = clusters, i = iterations, and d = dimensions. Agglomerative (Bottom-up): Starts with each data point as its own cluster and merges them based on distance until all points belong to one cluster. Divisive (Top-down): Starts with all data points in one cluster and recursively splits clusters into smaller ones until each point is separate. Cluster validity measures assess the quality of clustering results and are classified into three types: internal, external, and relative. Evaluate clusters based on the data, focusing on compactness and separation. An example is the Silhouette Coefficient (S), which for each point uses: a = average distance to points in the same cluster b = minimum average distance to points in other clusters. The silhouette score ranges from -1 to 1, where values close to 1 indicate well-separated clusters. Compare clustering results on known labels using metrics like Precision, Recall, and F1 Score.

PCA (Principal Component Analysis) is a dimensionality reduction technique that transforms correlated variables into a smaller set of uncorrelated variables called principal components. It is often used for data visualization, feature selection, and as a pre-processing step for machine learning models. A first matrix, the covariance matrix, measures how variables vary together – positive covariance means variables increase or decrease together, while negative covariance means they move inversely. This matrix is then converted into eigenvectors and eigenvalues, representing the direction of maximum variance. The eigenvalues represent the amount of variance explained by each principal component. The principal component with the highest eigenvalue represents the direction with the most variance in the data. The next principal component is orthogonal to the first, capturing the second-most variance, and so on. Principal components are often used to reduce data dimensionality while preserving essential information.