Lecture 5
PREDICTIVE PARSER

SYNTAX ANALYSIS

CONTENT

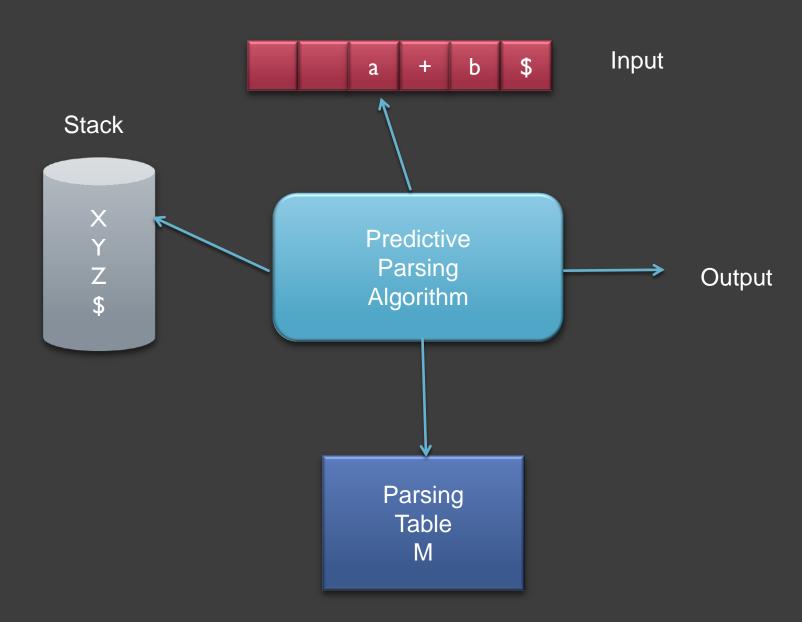
- LL (1) Parser
- Model of Non-Recursive Predictive Parser
- Construction of Predictive Parser Table
- Parsing a string

LL (I) GRAMMAR

LL (1) GRAMMAR

- Used to construct Predictive Parser
- Predictive Parser Recursive Descent Parser with no need of Backtracking
- First 'L' scanning input from Left to Right
- Second 'L' Leftmost Derivation
- "1" One input symbol of Look ahead at each step to make parsing action decisions
- Left Recursive and Ambiguous grammar is NOT LL(1)

Model Of Non-Recursive Predictive Parser



Construction Of Predictive Parsing Table

Input: Grammar G

Output: Parsing Table M

Table M has Non-Terminals as row and Terminals as columns

For Each Production A $\rightarrow \alpha$ of grammar do step 1 and 2

Step 1: For each terminal 'a' in FIRST (α)

Add A $\rightarrow \alpha$ to M [A , a]

Step 2:

Case 1:

If ε is in FIRST (α) then

for each terminal b in FOLLOW (A)

Add A \rightarrow α to M [A , b]

Case 2:

If ϵ is in FIRST (α) and \$ is in FOLLOW (A) then

for each terminal b in FOLLOW (A)

Add A \rightarrow α to M [A , b]

Step 3: Make each undefined entry of M be an error

$$E \rightarrow TE'$$

$$E' \to +TE' \mid \epsilon$$

$$T \rightarrow FT'$$

$$T' \to *FT' \mid \epsilon$$

$$\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$$

	id	+	*	()	\$
Е						
E'						
Т						
T'						
F						

Consider Production E → TE'

$$FIRST(TE') = FIRST(T) = \{(, id)\}$$

Add $E \rightarrow TE'$

to M [E , (] and M [E , id]

$$E \rightarrow TE'$$

$$E' \to +TE' \mid \epsilon$$

$$\mathsf{T} \to \mathsf{FT'}$$

$$T' \to *FT' \mid \epsilon$$

$$\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$$

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'						
Т						
T'						
F						

Consider Production E → TE'

Add $E \rightarrow TE'$

$$E \rightarrow TE'$$

$$E' \to +TE' \mid \epsilon$$

$$\mathsf{T} \to \mathsf{FT'}$$

$$T' \to *FT' \mid \epsilon$$

$$\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$$

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'						
Т						
T'						
F						

Consider Production E' → +TE'

Add
$$E' \rightarrow +TE'$$
 to M [E' , +]

$$E \rightarrow TE'$$

$$E' \to +TE' \mid \epsilon$$

$$\mathsf{T} \to \mathsf{FT'}$$

$$T' \to *FT' \mid \epsilon$$

$$\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$$

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$				
Т						
T'						
F						

Consider Production E' → +TE'

Add
$$E \rightarrow +TE'$$
 to M [E , +]

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow FT'$$

$$T' \to *FT' \mid \epsilon$$

$$\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$$

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$				
Т						
T'						
F						

Consider Production E' $\rightarrow \epsilon$

FIRST (
$$\epsilon$$
) = { ϵ }

Here find FOLLOW (
$$E'$$
) = {), \$}

Add E' $\rightarrow \epsilon$ to M [E' ,)] and M [E' , \$]

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow FT'$$

$$T' \to *FT' \mid \epsilon$$

$$\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$$

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т						
T'						
F						

Consider Production E' $\rightarrow \epsilon$

FIRST (
$$\epsilon$$
) = { ϵ }

Here find FOLLOW (
$$E'$$
) = {), \$}

Add E' $\rightarrow \epsilon$ to M [E,)] and M [E, \$]

$$E \rightarrow TE'$$

$$E' \to +TE' \mid \epsilon$$

$$T \rightarrow FT'$$

$$T' \to *FT' \mid \epsilon$$

$$\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$$

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т						
Τ'						
F						

Consider Production T → FT'

Add $T \rightarrow FT'$ to M [T , (] and M [T , id]

 $E \rightarrow TE'$

 $E' \to +TE' \mid \epsilon$

 $T \rightarrow FT'$

 $T' \to *FT' \mid \epsilon$

 $\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \to FT'$		
Τ'						
F						

Consider Production T → FT'

FIRST (FT') = FIRST (F) = { (, id }

Add $T \rightarrow FT'$ to M [T , (] and M [T , id]

 $E \rightarrow TE'$

 $E' \to +TE' \mid \epsilon$

 $\mathsf{T}\to\mathsf{FT'}$

 $T' \to *FT' \mid \epsilon$

 $\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
Τ'						
F						

Consider Production T' → *FT'

FIRST (*FT') = FIRST (*) = { * }

Add T' \rightarrow *FT' to M [T' , *]

 $E \rightarrow TE'$

 $E' \to +TE' \mid \epsilon$

 $\mathsf{T}\to\mathsf{FT'}$

 $T' \to *FT' \mid \epsilon$

 $\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \to FT'$		
T'			$T' \rightarrow *FT'$			
F						

Consider Production T' → *FT'

Add T' \rightarrow *FT' to M [T' , *]

 $E \rightarrow TE'$

 $E' \rightarrow +TE' \mid \epsilon$

 $T \rightarrow FT'$

 $T' \to *FT' \mid \epsilon$

 $\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \to FT'$		
T'			$T' \rightarrow *FT'$			
F						

Consider Production $T' \rightarrow \epsilon$

FIRST (ϵ) = { ϵ }

Here find FOLLOW (T') = { +,), \$ }

Add E' $\rightarrow \epsilon$ to M [T', +], M [T',)] and M [T', \$]

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow FT'$$

$$T' \to *FT' \mid \epsilon$$

$$\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$$

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F						

Consider Production $T' \rightarrow \epsilon$

FIRST (
$$\epsilon$$
) = { ϵ }

Here find FOLLOW (
$$T'$$
) = { +,), \$ }

Add E' $\rightarrow \epsilon$ to M [T', +], M [T',)] and M [T', \$]

$$E \rightarrow TE'$$

$$E' \to +TE' \mid \epsilon$$

$$\mathsf{T}\to\mathsf{FT'}$$

$$T' \to *FT' \mid \epsilon$$

$$\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$$

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \to FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F						

Consider Production $F \rightarrow (E)$

Add
$$F \rightarrow (E)$$
 to M [F, (]

$$E \rightarrow TE'$$

$$E' \to +TE' \mid \epsilon$$

$$\mathsf{T}\to\mathsf{FT'}$$

$$T' \to *FT' \mid \epsilon$$

$$\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$$

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \to FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F				$F \to (E)$		

Consider Production $F \rightarrow (E)$

Add
$$F \rightarrow (E)$$
 to M [F, (]

 $\mathsf{E}\to\mathsf{TE'}$

 $E' \to +TE' \mid \epsilon$

 $\mathsf{T}\to\mathsf{FT'}$

 $T' \to *FT' \mid \epsilon$

 $\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F				$F \to (E)$		

Consider Production $F \rightarrow id$

FIRST (id) = { id }

Add $F \rightarrow id$ to M [F, id]

$$\mathsf{E}\to\mathsf{TE'}$$

$$E' \to +TE' \mid \epsilon$$

$$\mathsf{T}\to\mathsf{FT'}$$

$$T' \to *FT' \mid \epsilon$$

$$\mathsf{F} \to (\mathsf{E}) \mid \mathsf{id}$$

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \to FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Consider Production $F \rightarrow id$

Add $F \rightarrow id$ to M [F, id]

	а	b	е	i	t	\$
S						
S'						
Е						

 $S \rightarrow iEtSS' \mid a$

 $S' \rightarrow eS \mid \epsilon$

 $\mathsf{E}\to\mathsf{b}$

Consider Production S → iEtSS'

Add $S \rightarrow iEtSS'$ to M [S , i]

	а	b	е	i	t	\$
S				$S \rightarrow iEtSS'$		
S'						
Е						

$$S \rightarrow iEtSS' \mid a$$

$$S' \rightarrow eS \mid \epsilon$$

$$\mathsf{E}\to\mathsf{b}$$

Consider Production S → iEtSS'

Add
$$S \rightarrow iEtSS'$$
 to M [S , i]

	а	b	е	i	t	\$
S				$S \rightarrow iEtSS'$		
S'						
Е						

$$S \rightarrow iEtSS' \mid a$$

$$S' \to eS \mid \epsilon$$

$$\mathsf{E} \to \mathsf{b}$$

Consider Production $S \rightarrow a$

FIRST
$$(a) = \{a\}$$

Add
$$S \rightarrow a$$
 to M [S , a]

	а	b	е	i	t	\$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S'						
Е						

$$S \rightarrow iEtSS' \mid a$$

$$S' \to eS \mid \epsilon$$

$$\mathsf{E} \to \mathsf{b}$$

Consider Production $S \rightarrow a$

FIRST
$$(a) = \{a\}$$

Add
$$S \rightarrow a$$
 to M [S , a]

	а	b	е	i	t	\$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S'						
Е						

$$S \rightarrow iEtSS' \mid a$$

$$S' \to eS \mid \epsilon$$

$$\mathsf{E} \to \mathsf{b}$$

Consider Production $S' \rightarrow eS$

Add
$$S' \rightarrow eS$$
 to M [S' , e]

	а	b	е	i	t	\$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S'			$S' \to eS$			
E						

$$S \rightarrow iEtSS' \mid a$$

$$S' \to eS \mid \epsilon$$

$$\mathsf{E} \to \mathsf{b}$$

Consider Production $S' \rightarrow eS$

Add
$$S' \rightarrow eS$$
 to M [S' , e]

Example 2.	Exam	ple	2:
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 $S \rightarrow iEtSS' \mid a$

 $S' \rightarrow eS \mid \epsilon$

 $\mathsf{E}\to\mathsf{b}$

	а	b	е	i	t	\$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S'			$S' \rightarrow eS$			
Е						

Consider Production S' $\rightarrow \epsilon$

FIRST (ϵ) = { ϵ }

Here find FOLLOW (S') = { e, \$ }

Add S' $\rightarrow \epsilon$ to M [S', e] and M [S', \$]

 $S \rightarrow iEtSS' \mid a$

 $S' \rightarrow eS \mid \epsilon$

 $\mathsf{E}\to\mathsf{b}$

	а	b	е	i	t	\$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S'			$\begin{array}{c} S' \to eS \\ S' \to \epsilon \end{array}$			$S' \to \epsilon$
E						

Consider Production S' $\rightarrow \epsilon$

FIRST (ϵ) = { ϵ }

Here find FOLLOW (S') = { e, \$ }

Add S' $\rightarrow \epsilon$ to M [S', e] and M [S', \$]

Example	e 2:

 $S \to iEtSS' \mid a$

$$S' \rightarrow eS \mid \epsilon$$

 $\mathsf{E}\to\mathsf{b}$

	а	b	е	i	t	\$
S	$S \rightarrow a$			$S \rightarrow iEtSS'$		
S'			$\begin{array}{c} S' \to eS \\ S' \to \epsilon \end{array}$			$S' \to \epsilon$
E						

Consider Production $E \rightarrow b$

Here find FOLLOW (S') = { e, \$}

Add $E \rightarrow b$ to M [E , b]

Example 2:

$$S \to iEtSS' \mid a$$

$$S' \rightarrow eS \mid \epsilon$$

$$\mathsf{E}\to\mathsf{b}$$

	а	b	е	i	t	\$
S	$S \rightarrow a$			$S \to iEtSS'$		
S'			$\begin{array}{c} S' \to eS \\ S' \to \epsilon \end{array}$			$S' \to \epsilon$
E		$E \to b$				

Consider Production $E \rightarrow b$

Here find FOLLOW (
$$S'$$
) = { e, \$ }

Add
$$E \rightarrow b$$
 to M [E , b]

Construction Of Predictive Parsing Table

- For every LL grammar each parsing table entry uniquely identifies a production or signals an error
- For some grammars however M may have some entries that are multiply defined
- Such grammars are not LL(1) Grammar

Predictive Parsing Algorithm

```
Let a be the first symbol of w
Let X be the top of the Stack symbol
while ( X != $)
   if (X == a)
      pop the stack and let 'a' be the next symbol of w
   else if (X is a terminal)
                                                // X != a and X is terminal
      Error ()
   else if ( M [ X , a ] is an error entry )
      Error ()
   else if ( M [ X , a ] = Y1 Y2 ... Yk )
      Output the production X \rightarrow Y1 Y2 ... Yk
      Pop the stack
      Push Yk Yk-1 ... Y1 onto the stack with Y1 on top
  Let X be the top stack symbol
```

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Predictive Parsing Algorithm

Matched Stack Input Action E \$ id + id * id \$

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Predictive Parsing Algorithm

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \to FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

	id	+	*	()	\$
E	$E \rightarrow TE'$			E → TE'		
E'		E' → +TE'			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \to FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Matched	Stack	Input	Action
	E \$	id + id * id \$	
	TE' \$	id + id * id \$	Output $E \rightarrow TE'$
	FT'E'\$	id + id * id \$	Output T→ FT'
	idT'E' \$	id + id * id \$	Output F → id

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Matched	Stack	Input	Action
	E \$	id + id * id \$	
	TE' \$	id + id * id \$	Output $E \rightarrow TE'$
	FT'E'\$	id + id * id \$	Output T→ FT'
	idT'E' \$	id + id * id \$	Output $F \rightarrow id$
id	T'E' \$	+ id * id \$	match id

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Matched	Stack	Input	Action
	E \$	id + id * id \$	
	TE' \$	id + id * id \$	Output $E \rightarrow TE'$
	FT'E'\$	id + id * id \$	Output T→ FT'
	idT'E' \$	id + id * id \$	Output $F \rightarrow id$
id	T'E' \$	+ id * id \$	match id
id	E' \$	+ id * id \$	Output T' $\rightarrow \epsilon$

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Matched	Stack	Input	Action
	E \$	id + id * id \$	
	TE' \$	id + id * id \$	Output $E \rightarrow TE'$
	FT'E'\$	id + id * id \$	Output T→ FT'
	idT'E' \$	id + id * id \$	Output $F \rightarrow id$
id	T'E' \$	+ id * id \$	match id
id	E' \$	+ id * id \$	Output $T' \to \epsilon$
id	+TE' \$	+ id * id \$	Output E' \rightarrow +TE'

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Matched	Stack	Input	Action
id +	TE' \$	id * id \$	match +

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	T' → *FT'		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Matched	Stack	Input	Action
id +	TE' \$	id * id \$	match +
id +	FT'E'\$	id * id \$	Output $T \rightarrow FT'$

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Matched	Stack	Input	Action
id +	TE' \$	id * id \$	match +
id +	FT'E'\$	id * id \$	Output $T \rightarrow FT'$
id +	idT'E' \$	id * id \$	Output $F \rightarrow id$

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Matched	Stack	Input	Action
id +	TE' \$	id * id \$	match +
id +	FT'E'\$	id * id \$	Output $T \rightarrow FT'$
id +	idT'E' \$	id * id \$	Output $F \rightarrow id$
id + id	T'E' \$	* id \$	match id

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Matched	Stack	Input	Action
id +	TE' \$	id * id \$	match +
id +	FT'E'\$	id * id \$	Output $T \rightarrow FT'$
id +	idT'E' \$	id * id \$	Output $F \rightarrow id$
id + id	T'E' \$	* id \$	match id
id + id	*FT'E' \$	* id \$	Output T'→ *FT'

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Matched	Stack	Input	Action
id +	TE' \$	id * id \$	match +
id +	FT'E'\$	id * id \$	Output $T \rightarrow FT'$
id +	idT'E' \$	id * id \$	Output $F \rightarrow id$
id + id	T'E' \$	* id \$	match id
id + id	*FT'E' \$	* id \$	Output T'→ *FT'
id + id *	FT'E' \$	id \$	match *

	id	+	*	()	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			$F \to (E)$		

Matched	Stack	Input	Action
id +	TE' \$	id * id \$	match +
id +	FT'E'\$	id * id \$	Output $T \rightarrow FT'$
id +	idT'E' \$	id * id \$	Output $F \rightarrow id$
id + id	T'E' \$	* id \$	match id
id + id	*FT'E' \$	* id \$	Output T'→ *FT'
id + id *	FT'E'\$	id \$	match *
id + id *	idT'E' \$	id \$	Output $F \rightarrow id$

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \to \epsilon$	$E' \to \epsilon$
Т	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \to \epsilon$	$T' \rightarrow *FT'$		$T' \to \epsilon$	$T' \to \epsilon$
F	$F \rightarrow id$			F → (E)		

Matched	Stack	Input	Action
id + id * id	T'E' \$	\$	match id
id + id * id	E' \$	\$	Output $T' \to \epsilon$
id + id * id	\$	\$	Output $E' \to \epsilon$

