QA (Quantative Analysis)

* **Tabulation** (Tabular form)

Ex. Out of a total number of 10,000 candidates who applied for job in a government department, 6854 were males, 3146 were graduate and other's non graduated. The number of male graduates was 2012. The num of candidates with experience was 2623 of whom 1860 were males. The number of graduates with experience was 1093 that include 323 females.

⇒ Sol^n

	Graduated			non-graduated			~~Experience~~ Tota		
	Experience	non-experien	Total	Ex.	Non-Ex	Total	Exp.	NonEx	Tota
male	770	1242	2012 ~~3146~~	1090	3752	4842	1860 ~~2623~~	4994	685
female	323	811	1134	440	1572	2012	763	2383	314
Total	1093	2053	3146	1530	5324	6854	2623 ~~2623~~	7377	10,0

Ex. Out of total 2000 students in a college 1400 were graduation and rest were Post graduation. Out of 1400, 100 were girls graduate. However, in all there were 600 Girls.

⇒ Sol^n

	Gradution	Post-Graduation	Total
Girls	100	500	600
Boys	1300	100	1400
Total	1400	600	2000

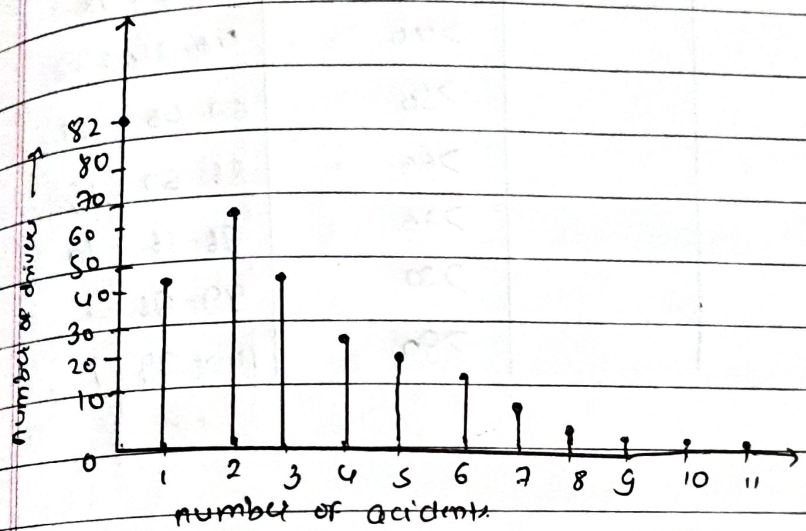
PAGE No.
DATE / /

* Line Diagram

Ex. following data shows the number of accidents sustained by 314 drivers over a five year's of time.

number of accidents : 0 1 2 3 4 5 6 7 8 9 10 11

number of drivers : 82 44 68 41 25 20 13 7 5 4 3 2 1

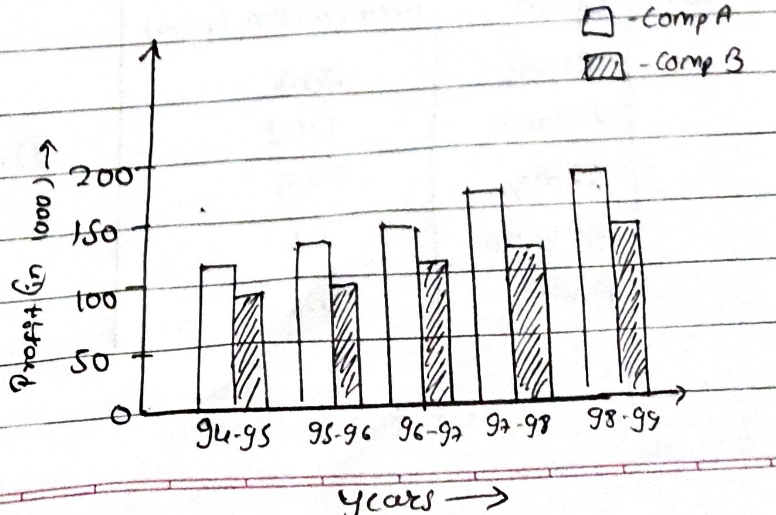


* Bar diagram

Ex. The data below give the yearly profit (in 1000 Rs) of two company

Year	Company A	Company B
1994-95	120	90
1995-96	135	95
1996-97	140	108
1997-98	160	120
1998-99	175	130

⇒ Soln



PAGE No.
DATE / /

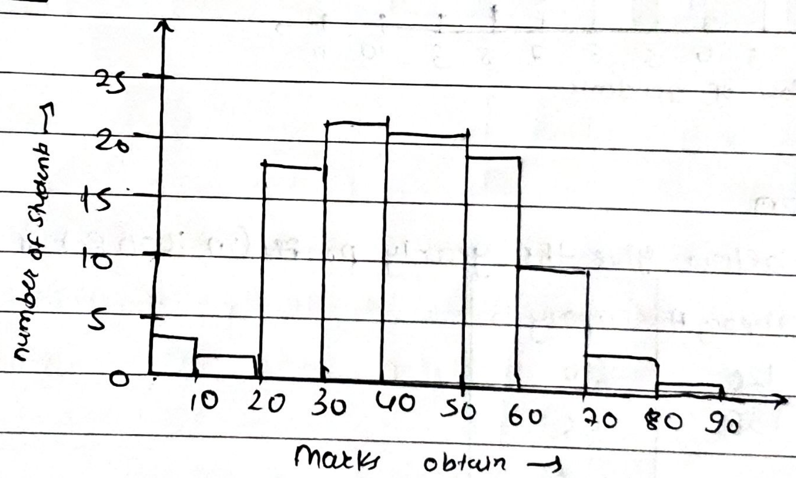
- convert cumulative frequency into frequency distribution

Histogram

Ex. Represent the adjoining distribution of marks in histogram.

marks obtain	number of students	⇒ Soln	marks obtain	no. of students
less than 10	4		>10	4
less than 20	6		>20	6 - 4 = 2
less than 30	24		>30	24 - 6 = 18
less than 40	46		>40	46 - 24 = 22
less than 50	67		>50	67 - 46 = 21
less than 60	86		>60	86 - 67 = 19
less than 70	96		>70	96 - 86 = 10
less than 80	99		>80	99 - 96 = 3
less than 90	100		>90	100 - 99 = 1

~~less than~~



Pie Diagram

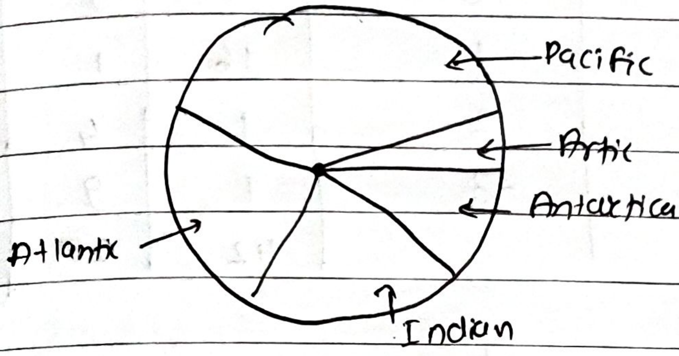
Ex.

Ocean	Area (million sq. km)
Pacific	70.8
Atlantic	41.2
Indian	28.5
Antarctica	7.6
Artic	4.8

Draw pie chart

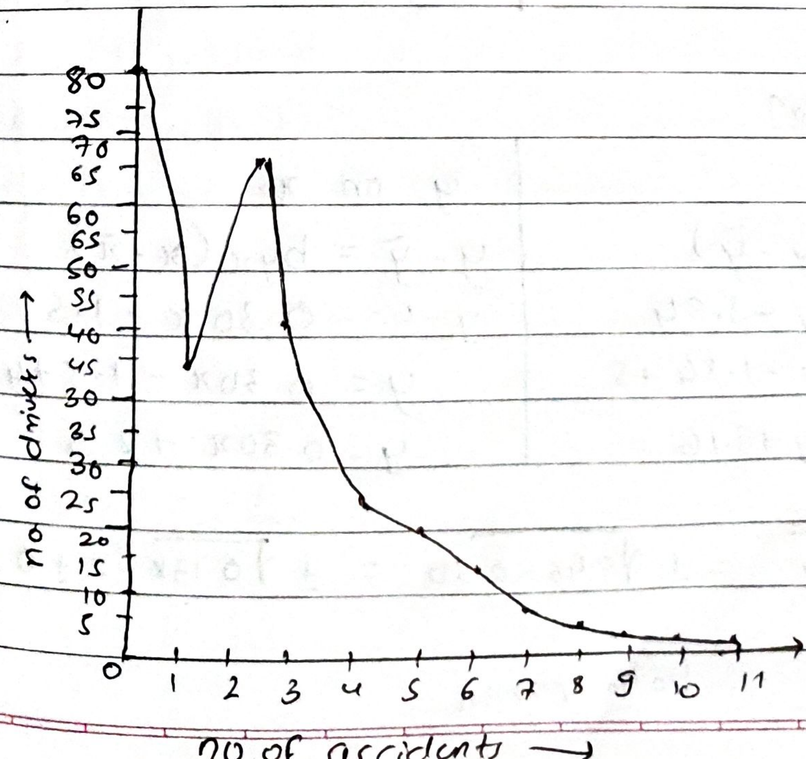
→ Sol^n

Ocean	Area (in million sq km)	Angle at centre
Pacific	70.8	$70.8/152.9 \times 360 = 166.7$
Atlantic	41.2	$41.2/152.9 \times 360 = 97.0°$
Indian	28.5	$28.5/152.9 \times 360 = 67.1$
Antarctica	7.6	$7.6/152.9 \times 360 = 17.9$
Artic	4.8	$4.8/152.9 \times 360 = 11.3$
Total	152.9	360°



* Frequency Polygon

no. of accidents	0	1	2	3	4	5	6	7	8	9	10	11
no. of drivers	80	44	68	41	25	20	13	7	5	4	3	2



PAGE No.	
DATE	/ /

Regression of equation with Arithmetic mean

x	1	3	5	7	9	4	6
y	2	4	3	7	5	6	1

⇒ Sol^n

	x	y	$x = x-\bar{x}$	$y = y-\bar{y}$	x^2	y^2	Σxy
	1	2	-4	-2	16	4	8
	3	4	-2	0	4	0	0
	5	3	0	-1	0	1	0
	7	7	2	3	4	9	6
	9	5	4	1	16	1	4
	4	6	-1	2	1	4	-2
	6	1	1	-3	1	9	-3
Σ	35	28			42	28	13

$\bar{x} = \frac{\Sigma x}{n} = \frac{35}{7} = 5$ | $\bar{y} = \frac{\Sigma y}{n} = \frac{28}{7} = 4$

① ~~Regression eq^n of x on~~ Regression Coefficients

$b_{xy} = \frac{\Sigma xy}{\Sigma y^2} = \frac{13}{28} = 0.46$ | $b_{yx} = \frac{\Sigma xy}{\Sigma x^2} = \frac{13}{42} = 0.30$

② Regression eq^n

x on y

$x - \bar{x} = b_{xy}(y-\bar{y})$

$x - 5 = 0.46y - 1.84$

$x = 0.46y - 1.84 + 5$

$x = 0.46y + 3.16$

y on x

$y - \bar{y} = b_{yx}(x-\bar{x})$

$y - 4 = 0.30x - 1.5$

$y = 0.30x - 1.5 + 4$

$y = 0.30x + 2.5$

$r = +\sqrt{b_{yx}\, b_{xy}} = \pm\sqrt{0.46 \times 0.30} = \pm\sqrt{0.138} = \pm 0.371$

$\therefore r = 0.371$ ... both positive

PAGE No
DATE / /

Ex. The regression lines of sample are $x + 6y = 6$ and $3x + 2y = 10$ find i) sample means $\bar{x}$ and $\bar{y}$, ii) the coefficient relation between x and y iii) find the value of y at $x = 12$

⇒ Soln

① regression lines pass through point $(\bar{x}, \bar{y})$
regression lines of sample are,
$\bar{x} + 6\bar{y} = 6$ ①, $3\bar{x} + 2\bar{y} = 10$ ②
After soloving ① and ②, we get
$\bar{x} = 3$, $\bar{y} = \frac{1}{2}$

② let, $x + 6y = 6$, coeffiect of ~~x on y~~ y on x
$\cancel{x = -\frac{6y}{6} + 6}$ $\cancel{x = -\frac{1}{6}y + 6}$ $y = -\frac{1}{6}x + 6$ — ①

let, $3x + 2y = 10$, coefficien of x on y
$x = -\frac{2}{3}y + \frac{10}{3}$ — ②

Comparing eqn ① and ② with general form, we get
$b_{xy} = -\frac{2}{3}$, $b_{yx} = -\frac{1}{6}$

Thus, $r = \pm\sqrt{b_{xy} b_{yx}} = \pm\sqrt{\left(-\frac{2}{3}\right)\left(-\frac{1}{6}\right)} = \frac{1}{3}$

Both b_{yx} and b_{xy} are negative. So, r is also negative.
the coefficient of co-relation is $-\frac{1}{3}$

③ value of y at $x = 12$
$y = -\frac{1}{6}x + 6$

$= -\frac{1}{6}(12) + 6$

$\boxed{\therefore y = -1}$

Ex. From the following, obtain two regression equations and estimate the yeid when rainfall is 29 cm. and the rainfall, when the yeild is 600 kg.

	yield (y)	Rainfall (x)
mean	508.4	26.7
S.D	36.8	4.6

$r = 0.52$

⇒ Soln

$\bar{x} = 26.7$, $\bar{y} = 508.4$, $\sigma x = 4.6$, $\sigma y = 36.8$, $r = 0.52$

$b_{yx} = r \dfrac{\sigma y}{\sigma x} = 0.52 \times \dfrac{36.8}{4.6} = 4.16$

$b_{xy} = r \dfrac{\sigma x}{\sigma y} = 0.52 \times \dfrac{4.6}{36.8} = 0.06$

regression line, y on x

$y - \bar{y} = b_{yx}(x - \bar{x})$

$y - 50.84 = 4.16x - 111.07$

$y = 4.16x + 397.3$

~~ret~~ x on y

$x - \bar{x} = b_{xy}(y - \bar{y})$

$x - 26.7 = 0.06y - 30.50$

$x = 0.06y - 3.8$

1. when rainfall is 29 cm

$y = 4.16(29) + 397.3$

$y = 517.8$ kg.

2. when yeild is 600 kg

$x = 0.06(600) - 3.8$

$x = 32.2$ cm

PAGE NO
DATE / /

Ex. Following data is given the experience of machine operators and performance rating by number of good parts per 100 pieces.

Operators	1	2	3	4	5	6
Performance rating (x)	23	43	53	63	73	83
experience (y)	5	6	7	8	9	10

calculate the regression of line of performance rating on experience and also estimate the propable performance if operator has 11 year experience.

⇒ Soln

	x	y	x^2	y^2	$x \cdot y$
	23	5	529	25	115
	43	6	1849	36	258
	53	7	2809	49	371
	63	8	3969	64	504
	73	9	5329	81	657
	83	10	6889	100	830
Σ	338	45	21374	335	2735

$\bar{X} = \frac{\Sigma x}{n}$

$= 56.33$

$\bar{Y} = \frac{\Sigma y}{n}$

$= 7.5$

① regression coefficient

$$b_{yx} = \frac{(n \cdot \Sigma xy) - (\Sigma x \cdot \Sigma y)}{n \cdot \Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{(6 \cdot 2735) - (338 \cdot 45)}{(6 \cdot 21374) - (338)^2}$$

$b_{yx} = 0.085$

$$b_{xy} = \frac{(n \cdot \Sigma xy) - (\Sigma x \cdot \Sigma y)}{(n \cdot \Sigma y^2) - (\Sigma y)^2}$$

$$= \frac{(6 \cdot 2735) - (338 \cdot 45)}{(6 \cdot 335) - (45)^2}$$

$b_{xy} = 11.429$

② regression eqn

y on x

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

$y - 7.5 = 0.085x - 4.828$

$y = 0.085x + 2.67$

x on y

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

$x - 56.33 = 11.429y - 85.717$

$x = 11.429y - 29.387$

③ operator having 11 year experience

$x = 11.429(11) - 29.387$

$x = 96.33$

PAGE No
DATE / /

Ex. A departmental store gives in-service training, to its salesman which is followed by a test. It is considering whether it should terminate the service of any salesman who does not do well in the test. The following data give the test score and sales by nine salesman during a certain period of time.

Test score	14	19	24	21	26	22	15	20	19
Sales (in 1000)	31	36	48	37	50	45	33	41	39

Calculate the coefficient of correlation between the test score and the sales. Does it estimate the termination of service low cost. If the firm wants a minimum sale volume of Rs. 30,000, what is the minimum test score that will ensure continuation of service? Also estimate most probable sales volume of salesman making score

⇒ Soln

lets consider Test score as x and sales (in 1000) as y.

	x	y	$u = x - \bar{x}$	$v = y - \bar{y}$	u^2	v^2	$u.v$
	14	31	-6	-9	36	81	54
	19	36	-1	-4	1	16	4
	24	48	4	8	16	64	32
	21	37	1	-3	1	9	-3
	26	50	6	10	36	100	60
	22	45	2	5	4	25	10
	15	33	-5	-7	25	49	35
	20	41	0	1	0	1	0
	19	39	-1	-1	1	1	1
Σ	180	360	0	0	120	346	193

$\bar{X} = \frac{\Sigma x}{n} = \frac{180}{9}$

$\boxed{\bar{X} = 20}$

$\bar{Y} = \frac{360}{9} = \frac{\Sigma y}{n}$

$\boxed{\bar{Y} = 40}$

PAGE No.
DATE / /

Regression coefficients.

$b_{xy} = \frac{\Sigma xy}{\Sigma y^2} = \frac{193}{\cancel{360}\ 346}$

$b_{xy} = 0.5578$

$b_{yx} = \frac{\Sigma xy}{\Sigma x^2} = \frac{193}{120}$

$b_{yx} = 1.6083$

① Coefficient co relation

$r = \pm\sqrt{b_{yx}\, b_{xy}}$

$= \pm\sqrt{1.6083 \times 0.5578}$

$r = \pm 0.9471$ ... since b_{yx} and b_{xy} both are positive

$\boxed{r = 0.9471}$

② To obtain test score of 28, estimate sale

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

$y - 40 = 1.6083(\cancel{28}\,x - 20)$

$y = 1.6083x + \cancel{78}.\ 7.8340$

$\therefore$ Since $x_1 = 28$

$y = 1.6\cancel{0823}(28) + 7.8340$

$y = 52.866$

i.e., ~~$y = 52,866$~~ $y = 52,866$

③ To obtain sales of 30,000, estimate test score

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

$x - 20 = 0.5578y - 22.312$

$x = 0.5578y - 2.312$

$\therefore$ since, $y = 30$

$x = 0.5578(30) - 2.312$

$x = 14.42 \approx 14.$

$\therefore x = 14$

PAGE No.
DATE / /

$S_{xy} = \sigma_x \sqrt{1-r^2}$

$S_{yx} = \sigma_y \sqrt{1-r^2}$

Ex. Standard error rate

x	1	3	5	7	9	4	6
y	2	4	3	7	5	6	1

⇒ Sol^n

x	y	$u = x - \bar{x}$	$v = y - \bar{y}$	u^2	v^2	$u.v$
1	2	-4	-2	16	6	8
3	4	-1	0	1	0	0
5	3	0	-1	0	1	0
7	7	2	3	4	9	6
9	5	4	1	16	1	4
4	6	-1	2	1	4	-2
6	1	1	-3	1	9	-3
Σ 35	28	0	0	42	28	13

$\bar{x} = \frac{\Sigma x}{n} = \frac{35}{7}$

$\boxed{\bar{x} = 5}$

$\bar{y} = \frac{\Sigma y}{n} = \frac{28}{7}$

$\boxed{\bar{y} = 4}$

$\sigma_x = \sqrt{\frac{\Sigma u^2}{n}} = \sqrt{\frac{42}{7}}$

$\boxed{\sigma_x = 2.4}$

$\sigma_y = \sqrt{\frac{v^2}{n}} = \sqrt{\frac{28}{7}}$

$\boxed{\sigma_y = 2}$

$r = \frac{\Sigma u.v}{n.\sigma_x \sigma_y} = \frac{13}{7 \times 2.4 \times 2} = 0.39$ $\boxed{r = 0.39}$

$S_{xy} = 2.4 \sqrt{1-(0.39)^2}$

$\boxed{S_{xy} = 2.20}$

$S_{yx} = \sigma_y \sqrt{1-r^2} = 2\sqrt{1-(0.39)^2}$

$\boxed{S_{yx} = 1.84}$

PAGE No.	
DATE	/ /

— Book wala

Ex Standard error rate

→ Sol^n

x	y	$u=x-\bar{x}$	$v=y-\bar{y}$	u^2	v^2	$u.v$
1	2	−4	−2	16	4	8
3	4	−1	0	1	0	0
5	3	0	−1	0	1	0
7	7	2	3	4	9	6
9	5	4	1	16	1	4
4	6	−1	2	1	4	−2
6	1	1	−3	1	9	−3
35	28	0	0	42	28	13

$\bar{x} = 5$, $\bar{y} = 4$

$S_{xy} = \sqrt{\frac{\Sigma(x-x_c)^2}{n}} = \sqrt{\frac{35.93}{7}}$

$\boxed{S_{xy} = 2.26}$

$S_{yx} = \sqrt{\frac{\Sigma(y-y_c)^2}{n}} = \sqrt{\frac{23.95}{7}}$

$\boxed{S_{yx} = 1.84}$

Regression coefficients

$b_{yx} = \frac{\Sigma xy}{\Sigma u^2} = \frac{13}{42} = 0.309$

$b_{xy} = \frac{\Sigma uv}{\Sigma v^2} = \frac{13}{28} = 0.464$

Regression eq^n y on x

$(y-\bar{y}) = b_{yx}(x-\bar{x})$

$y-4 = 0.309x - 1.545$

$y = 0.309x + 2.455$

x on y

$(x-\bar{x}) = b_{xy}(y-\bar{y})$

$x-5 = 0.464y - 1.856$

$x = 0.464y + 3.144$

	x	y	x_c	y_c	$(x-x_c)^2$	$(y-y_c)^2$
	1	2	4.072	2.764	9.43	0.58
	3	4	5	3.382	4	0.38
	5	3	4.536	4	0.21	1
	7	7	6.392	4.618	0.36	5.67
	9	5	5.464	5.236	12.50	0.05
	4	6	5.928	3.691	3.71	5.33
	6	1	3.608	4.309	5.72	10.94
Σ	35	28			35.93	23.95

PAGE No.
DATE / /

* Partial correlation Coefficient

1) $r_{12.3} = \dfrac{r_{12} - r_{13} \cdot r_{23}}{\sqrt{(1-r_{13}^2)(1-r_{23}^2)}}$ x_1 and x_3 x_3 constant

2) $r_{13.2} = \dfrac{r_{13} - r_{12} \cdot r_{23}}{\sqrt{(1-r_{12}^2)(1-r_{23}^2)}}$ x_2 and x_3 x_2 constant

3) $r_{23.1} = \dfrac{r_{23} - r_{12} \cdot r_{13}}{\sqrt{(1-r_{12}^2)(1-r_{13}^2)}}$ x_2 and x_3 x_1 constant

Ex. $r_{12} = 0.70$, $r_{13} = 0.61$, $r_{23} = 0.40$ calculate $r_{12.3}$, $r_{13.2}$, $r_{23.1}$

⇒ Soln

$r_{12.3} = \dfrac{r_{12} - r_{13} r_{23}}{\sqrt{(1-r_{13}^2)(1-r_{23}^2)}} = \dfrac{0.70 - (0.61 \times 0.40)}{\sqrt{(1-(0.61)^2)(1-(0.40)^2)}} = 0.629$

$r_{13.2} = \dfrac{r_{13} - r_{12} r_{23}}{\sqrt{(1-r_{12}^2)(1-r_{23}^2)}} = \dfrac{0.61 - (0.70 \times 0.40)}{\sqrt{(1-(0.70)^2)(1-(0.40)^2)}} = 0.504$

$r_{23.1} = \dfrac{r_{23} - r_{12} r_{13}}{\sqrt{(1-r_{12}^2)(1-r_{13}^2)}} = \dfrac{0.40 - (0.61 \times 0.70)}{\sqrt{(1-(0.70)^2)(1-(0.61)^2)}} = -0.048$

PAGE No.
DATE / /

$R_{1.23} = R_{1.32}$

* Coefficient of Multiple Correlation

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - r_{12} r_{13} r_{23}}{1 - r_{23}^2}}$$

$$R_{2.13} = \sqrt{\frac{r_{12}^2 + r_{23}^2 - r_{12} r_{13} r_{23}}{1 - r_{13}^2}}$$

$$R_{3.12} = \sqrt{\frac{r_{13}^2 + r_{23}^2 - r_{12} r_{13} r_{23}}{1 - r_{12}^2}}$$

Ex. $r_{12} = 0.98$, $r_{13} = 0.44$, $r_{23} = 0.54$, calculate $R_{1.23}$, ~~$R_{13.2}$, $R_{23.1}$~~ $R_{2.13}$, $R_{3.12}$

⇒ Sol^n

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - r_{12} r_{13} r_{23}}{1 - r_{23}^2}} = \sqrt{\frac{0.98^2 + 0.44^2 - 0.98 \times 0.44 \times 0.54}{1 - 0.54^2}} = 0.986$$

$$R_{2.13} = \sqrt{\frac{r_{12}^2 + r_{23}^2 - r_{12} r_{13} r_{23}}{1 - r_{13}^2}} = \sqrt{\frac{(0.98)^2 + (0.54)^2 - 0.98 \times 0.44 \times 0.54}{1 - (0.44)^2}}$$

$$R_{3.12} \sqrt{\frac{r_{13}^2 + r_{23}^2 - r_{12} r_{13} r_{23}}{1 - r_{12}^2}} = \sqrt{\frac{(0.44^2) + (0.54)^2 - 0.98 \times 0.44 \times 0.54}{1 - (0.98)^2}}$$

PAGE NO.
DATE / /

* Multiple Linear Regression eq^n

1) x_1 on x_2 and x_3 $\rightarrow$ $X_1 = a_{1.23} + b_{12.3} X_2 + b_{13.2} X_3$

$\Sigma x_1 = N a_{1.23} + b_{12.3} \Sigma X_2 + b_{13.2} \Sigma X_3$

$\Sigma x_1 x_2 = a_{1.23} \Sigma x_2 + b_{12.3} \Sigma x_2^2 + b_{13.2} \Sigma x_2 x_3$

$\Sigma x_1 x_3 = a_{1.23} \Sigma x_3 + b_{13.2} \Sigma x_2 x_3 + b_{13.2} \Sigma x_3^2$

2) x_2 on x_1 and x_3 $\rightarrow$ $X_2 = a_{1.23} + b_{12.3} X_1 + b_{13.2} X_3$

$\Sigma x_2 = N a_{1.23} + b_{12.3} \Sigma x_1 + b_{13.2} \Sigma x_3$

$\Sigma x_1 x_2 = a_{1.23} \Sigma x_1 + b_{12.3} \Sigma x_1^2 + b_{13.2} \Sigma x_1 x_3$

$\Sigma x_2 x_3 = a_{1.23} \Sigma x_3 + b_{12.3} \Sigma x_1 x_3 + b_{13.2} \Sigma x_3^2$

3) x_3 on x_1 and x_2 $\rightarrow$ $x_3 = a_{1.23} + b_{12.3} x_1 + b_{13.2} x_2$

$\Sigma x_3 = N a_{1.23} + b_{12.3} \Sigma x_1 + b_{13.2} \Sigma x_2$

$\Sigma x_1 x_3 = a_{1.23} \Sigma x_1 + b_{12.3} \Sigma x_1^2 + b_{13.2} \Sigma x_1 x_2$

$\Sigma x_2 x_3 = a_{1.23} \Sigma x_2 + b_{12.3} \Sigma x_1 x_2 + b_{13.2} \Sigma x_2^2$

Ex.

x_1	4	6	7	9	13	15
x_2	15	12	8	6	4	3
x_3	30	24	20	14	10	4

$\Rightarrow$ Sol^n

	x_1	x_2	x_3	x_1x_2	x_1x_3	x_2x_3	x_1^2	x_2^2	x_3^2
	4	15	30	60	450	450	16	225	900
	6	12	24	72	288	288	36	144	576
	7	8	20	56	160	160	49	64	400
	9	6	14	54	84	840	81	36	196
	13	4	10	52	40	40	169	16	100
	15	3	4	45	12	12	225	9	16
Σ	54	48	102	339	720	1034	576	494	2188

x_1 on x_2 and x_3

$6a_{1.23} + 48b_{12.3} + 102b_{13.2} = 54$

$48a_{1.23} + 494b_{12.3} + 1034b_{13.2} = 339$

$102a_{1.23} + 1034b_{12.3} + 2188b_{13.2} = 720$

$a_{1.23} = 16.47$, $b_{12.3} = 0.38$, $b_{13.2} = -0.62$

$X_1 = 16.47 + 0.38X_2 - 0.62X_3$

$b_{12.3} = b_{21.3}$
$b_{13.2} = b_{31.2}$
$b_{23.1} = b_{32.1}$

Deviation of mean from actual mean

$\boxed{r_{1.23} = 0}$

1) x_1 on x_2 and x_3 → $x_1 = b_{12.3} x_2 + b_{13.2} x_3$

$b_{12.3} = \frac{\sigma_1}{\sigma_2} \times r_{12.3}$ | $b_{13.2} = \frac{\sigma_1}{\sigma_3} \times r_{13.2}$

2) x_2 on x_1 and x_3 → $x_2 = b_{12.3} x_1 + b_{23.1} x_3$

$b_{12.3} = \frac{\sigma_2}{\sigma_1} \times r_{12.3}$ | $b_{23.1} = \frac{\sigma_2}{\sigma_3} \times r_{23.1}$

3) x_3 on x_1 and x_2 $x_3 = b_{13.2} x_2 + b_{23.1} x_3$

$b_{13.2} = \frac{\sigma_3}{\sigma_1} \times r_{13.2}$ | $b_{23.1} = \frac{\sigma_3}{\sigma_2} \times r_{23.1}$

Trivariate distribution

$b_{12.3} = \frac{\sigma_1}{\sigma_2} \times r_{12.3}$

$b_{13.2} = \frac{\sigma_1}{\sigma_3} \times r_{13.2}$

$b_{23.1} = \frac{\sigma_2}{\sigma_3} \times r_{23.1}$

PAGE No.
DATE / /

least square method

1) x_1 on x_2 and x_3

$$(x_1 - \bar{x}_1) = \left(\frac{r_{12} - r_{13}r_{23}}{1 - r_{23}^2}\right)\left(\frac{s_1}{s_2}\right)(x_2 - \bar{x}_2) + \left(\frac{r_{13} - r_{12}r_{23}}{1 - r_{23}^2}\right)\left(\frac{s_1}{s_3}\right)(x_3 - \bar{x}_3)$$

2) x_2 on x_1 and x_3

$$(x_2 - \bar{x}_2) = \left(\frac{r_{12} - r_{13}r_{23}}{1 - r_{13}^2}\right)\left(\frac{s_2}{s_1}\right)(x_1 - \bar{x}_1) + \left(\frac{r_{23} - r_{12}r_{13}}{1 - r_{13}^2}\right)\left(\frac{s_2}{s_3}\right)(x_3 - \bar{x}_3)$$

3) x_3 on x_1 and x_2

$$(x_3 - \bar{x}_3) = \left(\frac{r_{13} - r_{13}r_{23}}{1 - r_{12}^2}\right)\left(\frac{s_3}{s_1}\right)(x_1 - \bar{x}_1) + \left(\frac{r_{23} - r_{12}r_{13}}{1 - r_{12}^2}\right)\left(\frac{s_3}{s_2}\right)(x_2 - \bar{x}_2)$$

PAGE No.
DATE / /

* F-test - test for significance of multiple correlation coefficient

$$F = \left[\frac{R^2_{1.(k)}}{1 - R^2_{1.(k)}}\right]\left(\frac{n-k-1}{2}\right) \quad \text{dof } (k, n-k-1)$$

Ex. Define multiple correlation and calculate $R_{1.23}$ from the following zero order coefficients $r_{12} = 0.9$, $r_{23} = 0.4$, $r_{13} = 0.5$
Also, obtain the coefficient of multiple determination and interpret it. If $n = 30$, how could you test the significance of $R_{1.23}$.

⇒ Soln

Given, $r_{12} = 0.9$, $r_{13} = 0.5$, $r_{23} = 0.4$, $n = 30$

① $$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - r_{12}r_{23}r_{13}}{1 - r_{23}^2}} = \sqrt{\frac{(0.9)^2 + (0.5)^2 + 0.9\times0.4\times0.5}{1-(0.4)^2}}$$

$$\boxed{R_{1.23} = 0.9129}$$

② Since $R_{1.23}$ is near to 1, the plane of regression x_1 on x_2 and x_3 is good fit

③ Significance of $R_{1.23}$

null hypothesis : $\rho_{1.23} = 0$

alternate " : $\rho_{1.23} \neq 0$

we have, $R_{1.23} = 0.9129$, $n = 30$, $k = 2$

$$\therefore F = \left[\frac{0.9129^2}{1-(0.9129)^2}\right]\left(\frac{30-2-1}{2}\right) = 67.48$$

Degree of freedom $(k, k-n-1) = (2, 27)$

Tabulated value $F(2,27)$ at 5% level of significan = 3.35

Since, $F >$ Tabulated value, H_0 is rejected

Hence, $R_{1.23}$ is significant

PAGE No
DATE / /

\# T-test

Ex. For given set of numerical data, foll. are $r_{12.3} = 0.65$, $n = 15$
Test the significance of partial correlation coefficient at 5% LOS.

⇒ Soln

null hypothesis

H_0 : $\rho_{1.23} = 0$ i.e, not significant.

Alternate hypothesis

H_1 : $\rho_{1.23} \neq 0$

$$t \cong \frac{r_{12.3}\sqrt{n-2-1}}{\sqrt{1-r_{12.3}^2}}$$

$$\therefore t = \frac{0.65\sqrt{15-2-1}}{\sqrt{1-(0.65)^2}}$$

$$\therefore \boxed{t = 2.9632}$$

Tabulated t at 12 d.f 5% level of significance is 2.179

Hence, we reject H_0

$\therefore r_{12.3} = 0.65$ is significant.

PAGE No.	
DATE	/ /

* Sampling with replacement and without replacement.

Ex. Assume that population contains of 5 students and marks obtained by them are in certain class are 20, 15, 12, 16, 18. Draw all possible random samples of 2 students when sampling perform with and without replacement. Calculate mean for each sample.

⇒ Soln

① with replacement

Suppose 5 students be identified as A, B, C, D, E, number of possible samples of $n = 2$ students which can be selected from population of $N = 5$ units with replacement is

$$(N)^n = (5)^2 = 25$$

So, the total number of samples will be 25

let $X1$ be marks of student selected first

let $X2$ be marks of —"— second

No.	sample Students	Sample marks $(X1, X2)$	Sample mean
1	(A, A)	(20, 20)	20
2	(A, B)	(20, 15)	17.5
3	(A, C)	(20, 12)	16
4	(A, D)	(20, 16)	18
5	(A, E)	(20, 18)	19
6	(B, A)	(15, 20)	17.5
7	(B, B)	(15, 15)	15
8	(B, C)	(15, 12)	13.5
9	(B, D)	(15, 16)	15.5
10	(B, E)	(15, 18)	16.5
11	(B, A)	(12, 20)	16
12	(C, B)	(12, 15)	13.5
13	(C, C)	(12, 12)	12
14	(C, D)	(12, 16)	14
15	(C, E)	(12, 18)	15
	(D, A)	(16, 20)	18
	(D, B)	(16, 15)	13.5

18	(D, C)	(16,12)	14
19	(D,D)	(16,16)	16
20	(D,E)	(16,18)	17
21	(E,A)	(18,20)	19
22	(E,B)	(18,15)	16.5
23	(E,C)	(18,12)	15
24	(E,D)	(18,16)	17
25	(E,E)	(18,18)	18

② without replacement

The number of samples that can be drawn without replacement

$\binom{5}{2} = 10$ samples

No.	Sample students	Sample marks (x_1, x_2)	Sample mean
1	(A,B)	(20,15)	17.5
2	(A,C)	(20,12)	16
3	(A,E)	(20,16)	18
4	(A,D)	(20,18)	19
5	(B,C)	(15,12)	13.5
6	(B,E)	(15,16)	15.5
7	(B,D)	(15,18)	16.5
8	(C,E)	(12,16)	14
9	(C,D)	(12,18)	15
10	(D,E)	(16,18)	17

σ = standard deviation $\quad z = 2.58 \quad 1\%$

n = sample size , z = value at LOC (level of confidence) $\quad z = 1.96 \quad 5\%$

d = difference between population mean and sample mean

Sampling Solved Sums

all sums we need to cal n

Q. Determine sample size, if $\sigma = 6$, population mean = 25, sample mean = 23 and the desired degree of precision is 99 %.

⇒ Soln

Given,

$\sigma = 6$, $d = 25 - 23 = 2$, $z = 2.58$ (at 1% level)

formula ① $\quad n = \left(\frac{z\sigma}{d}\right)^2 = \left(\frac{2.58 \times 6}{2}\right)^2$

$= (7.728)^2$

$= 59.72 \approx 60$

$\therefore$ The sample size is 60

formula ② $\quad$ If $\sigma = 10$ and $\sigma_x = 0.25$, standard error of mean $\boxed{\sigma_x = \frac{\sigma}{\sqrt{n}}}$

Here,

$n = \left(\frac{\sigma}{\sigma_x}\right)^2 = \left(\frac{10}{0.25}\right)^2$

$= 19.71$

formula ③ $\quad$ Standard error of proportion, $p = 0.5$, $q = 0.5$, $\sigma_p = 0.005$

⇒

$n = \frac{pq}{\sigma_p^2} = \frac{(0.5 \times 0.5)}{(0.005)^2}$

$= 10{,}000$

PAGE No.
DATE / /

Statistic Sums

* Test for number of success

n = size of sample
p = probability of success
$q = (1-p)$ probability --- failure

Q. In 324 throws 6 faced dice, odd point appears 180 times. Can we say that dice is 5% level of significance?

⇒ Sol^n

Given, $n = 324$, $p = q = \frac{1}{2}$, odd occurence = 180
even occurence = 162

$$\text{Standard error} = \sqrt{npq}$$
$$= \sqrt{324 \times \frac{1}{2} \times \frac{1}{2}} = 9$$

$$\frac{\text{Difference}}{\text{S.E}} = \frac{180-162}{9} = 2$$

Since, difference is more than 1.96 at 5% level of sign the hypothesis is rejected.

* Test for Proportion of Success

Q. 500 apples are taken at random from large basket and 50 are found to be bad. Estimate the proportion of bad apples and assign limits with %.

⇒ Sol^n

Proportion of bad apples $= \frac{50}{500} = 0.1$, $n = 500$

Hence, $p = 0.1$, $q = 0.9$

$$SE = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.1 \times 0.9}{500}} = 0.013$$

$$= \left| p \pm 3\sqrt{\frac{pq}{n}} \right| \times 100 = |0.1 \pm (0.039)| \times 100$$

$= |0.61 \text{ and } 0.139| \times 100$

$= 6.1$ and 13.9

* Test for Difference Between Proportion.

Q. In a simple Random sample of 600 men taken from big city, 400 are found to be smokers. In another simple random sample 900 men taken from another city 450 were smokers. Do the data indicate that there is significant different in habit of smoking between two cities

⇒ Soln

Given,

$$P_1 = \frac{x_1}{n_1} = \frac{400}{600} = 0.667, \quad P_2 = \frac{x_2}{n_2} = \frac{450}{900} = 0.5$$

$$S.E\, z(p_1 - p_2) = \sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$p = \frac{x_1 + x_2}{n_1 + n_2} = \frac{400 + 450}{600 + 900} = \frac{17}{30}$$

$$q = 1 - p = 1 - \frac{17}{30} = \frac{13}{30}$$

$$S.E = \sqrt{\frac{17}{30} \times \frac{13}{30}\left(\frac{1}{600} + \frac{1}{900}\right)}$$

$$= 0.026$$

$$\frac{\text{Difference}}{S.E} = \frac{0.667 - 0.5}{0.026} = 6.42$$

So, difference is more than 2.58 1% LOS, the hypothesis is rejected.

for two mean difference

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S} \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

PAGE No.
DATE / /

$$S = \sqrt{\frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}}$$

Q. Test the significance of the mean of Random sample
Sample : 24, 26, 30, 20, 20, 18 , $n = 6$, $\mu = 25$

⇒ Sol^n

x	$(x - \bar{x})$	$(x - \bar{x})^2$
24	1	1
26	3	~~27~~ 9
30	7	49
20	-3	9
20	-3	9
18	5	25
$\Sigma = 138$		$\Sigma = 102$

$$\bar{x} = \frac{138}{6} = 23$$

$$S = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n-1}} = \sqrt{\frac{102}{5}} = 4.517$$

~~t =~~

$$t = \frac{(\bar{x} - \mu)}{S}\sqrt{n}$$

$$= \frac{(23 - 25)}{4.517}\sqrt{6} = 1.086$$

$v = n - 1$

$= 6 - 1 = 5$

$v = 5$. $t_{0.01} = 4.032$

less than table value , hypothesis accepted

* Biased, unbiased and consistent

1) $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$ ... unbiased estimator and consiten

$E(\hat{\theta}) = \theta$ ... unbiased estimator

2) $\hat{p} = \frac{x}{n}$ ... unbiased and consistent

$E(\hat{\theta}) \neq \theta$ ... biased estimator

3) $S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$ ... unbiased and consistent

$E(\hat{\theta}) > \theta$ ... +ve biased

4) $S^2 = \frac{\sum (x_i - \bar{x})^2}{n}$ ... biased ~~con~~ and consistent

$E(\hat{\theta}) < \theta$ ... -ve biased

Regression Hypothesis

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

* Regression eqⁿ : $Y = \beta_0 + \beta_1 X$

$H_0 : \beta_1 = 0$ (no relation exists) .. null
$H_1 : \beta_1 \neq 0$ (relation exists) .. alternate

Sum of square = Sum of squares + Sum of square errors
(SST) (SSR) (SSE)

$$\Sigma (Y - \bar{Y})^2 = \Sigma (\hat{Y} - \bar{Y})^2 + \Sigma (Y - \hat{Y})^2$$

Y = observed value of Y $\bar{Y}$ = mean value of Y
$\hat{Y}$ = Predicted value of Y (regression line)

* Coefficient of determination : r^2 $0 \leq r^2 \leq 1$

$$r^2 = \frac{SSR}{SST} = \frac{\Sigma (\hat{Y} - \bar{Y})^2}{\Sigma (Y - \bar{Y})^2} \quad / \quad r = 1 - \frac{SSE}{SST}$$

* Mean square error (MSE) : σ_ε^2

$$MSE = \frac{SSE}{d.f} = \frac{\Sigma (Y - \hat{Y})^2}{n - k} \qquad MSR = \frac{SSR}{K-1}$$

* Standard error of estimate : S_{YX}

$$S_{YX} = \sqrt{MSE}$$

n = sample size
K = 2 (ie parameters of estimate)

* Mean Absolute Error (MAE)

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |\hat{y}_i - y_i|$$

PAGE No.		
DATE	/ /	

Ex.

x	1	2	3	4	5
y	1	1	2	2	4

calculat ~~MSR~~ SSR then MSR

⇒ Sol^n

x	y	x^2	xy	$\hat{y} = b_0 + b_1 x$	$(\hat{y} - \bar{y})$
1	1	1	1	0.60	1.96
2	1	4	2	1.30	0.49
3	2	9	6	2.00	0
4	2	16	8	2.7	0.49
5	4	25	20	3.4	1.96
15	10	55	37		4.9

$$b_1 = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{5(37) - (15 \times 10)}{5(55) - (15)^2}$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{10}{5} = 2$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{15}{5} = 3$$

$b_1 = 0.7$

$b_0 = \bar{y} - b_1 \bar{x}$

$= 2 - 0.7(3)$

$\boxed{b_0 = -0.10}$

$SSR = \Sigma(\hat{y} - \bar{y})^2 = 4.9$

$$MSR = \frac{SSR}{K-1} = \frac{4.9}{2-1} = 4.9$$

PAGE No.
DATE / /

Ex. calculate coefficient of determination, r^2, SSE, error of estimate

⇒ Sol^n

	x	y	$(y-\bar{y})^2$	$(y-\hat{y})^2$
	1	1	1	0.16
	2	1	1	0.09
	3	2	0	0
	4	2	0	0.49
	5	4	4	0.36
Σ	15	10	6	1.10

$$r^2 = \frac{SSR}{SST} = \frac{4.9}{6} = 0.82$$

$$SST = \Sigma(y-\bar{y})^2 = 6$$

$$\boxed{r^2 = 0.82}$$

$$SSE = \Sigma(y-\hat{y})^2 = 1.10$$

$$MSE = \frac{SSE}{n-k} = \frac{1.10}{5-2} = 0.3667$$

Standard error of estimate:

$$S_{yx} = \sqrt{MSE} = \sqrt{0.3667}$$

$$S_{yx} = 0.6056$$

* F test

$$F = \frac{MSR}{MSE} \qquad F(k-1, n-k)$$

Reject Ho if $F_{calc} > F_{cv}$

$$F = \frac{4.9}{0.6056} = 13.36$$

At, $\alpha = 0.05$, $F_{cv}(1,3) = 10.13$

Hence, reject Ho

* T- test

$$t_{n-k} = \frac{b_1}{\text{Std error of } b_1}$$

Reject Ho, if $t_{calc} > t_{cv}$

$$\text{standard error } S(b_1) = \frac{\sqrt{MSE}}{\sqrt{\Sigma(x-\bar{x})^2}}$$

PAGE No.

DATE / /

* Multiple Regression

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots \beta_k x_k + \varepsilon$$

Y is dependent

x_i is independent

ε is random error

~~$\beta = 0$~~ Same calculate of multiple regression

$$\cancel{X_1}_2 = a_{1.23} + b_{12.3} X_1 + b_{13.2} X_3$$

* F test

$H_0 : \beta_1 = \beta_2 = 0$ ... no relation

H_1 : ~~At~~ least one of β_i is not equal to 0

$$F_{(k-1, n-k)} = \frac{MSR}{MSE}$$

$MSR = \frac{SSR}{k-1}$, $\cancel{MSE}$ MSE $= \frac{SSE}{n-k}$

$$F(k-1, n-k) = \frac{r^2}{(k-1)} \Big/ \frac{1-r^2}{n-k}$$

, $k = 3$

r^2 = coefficient of determination

* t test

$H_0 : \beta_1 = \beta_2 = 0$ --- does not contribute y

$H_1 : \beta_1 \neq 0, \beta_2 \neq 0$

$t = \frac{b_1}{\text{std. erro of } b_1}$, $t = \frac{b_2}{\text{std. err. of } b_2}$

$$S(b_1) = \frac{\sqrt{MSE}}{\sqrt{\Sigma(x-\bar{x})^2}}$$

PAGE No.
DATE / /

Ex. A random sample of $n = 6$ has elements 6, 10, 13, 14, 18, 20. Compute a point of estimate of
1) population mean, 2) the population std. deviation 3) Standard error of mean

⇒ Sol^n

	X	X^2
	6	36
	10	100
	13	169
	14	196
	18	324
	20	400
Σ	81	1225

$$\bar{X} = \frac{\Sigma x}{n} = \frac{81}{6} = 13.6$$

① point of estimate of population mean $(\mu) = 13.5$

② Sample standard deviation

$$S = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{1225}{6} - \left(\frac{81}{6}\right)^2}$$

$\boxed{S = 4.68}$ $\therefore \sigma = 4.68$

$$S_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

③ Sample standard error

$$S_{\bar{x}} = \frac{S}{\sqrt{n}} = \frac{4.68}{\sqrt{6}}$$

$\boxed{S_{\bar{x}} = 1.91}$

$SE_r = \frac{1-r^2}{\sqrt{n}}$ ... coefficient correlation $\quad S.E_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

PAGE No.

DATE / /

Ex. Calculate standard error of mean

Amount (m)	39	49	59	69	79	89	99
firms (no) (f)	2	3	11	20	32	25	7

⇒ Sol^n

m	f	d = (m-69)/10	fd	$f \times d^2$
39	2	-3	-6	18
49	3	-2	-6	12
59	11	~~0~~ -1	-11	11
69	20	0	0	0
79	32	1	32	32
89	25	2	50	100
99	7	3	21	63
Σ	N = 100		80	236

$$\sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2} = \sqrt{\frac{236}{100} - \left(\frac{80}{100}\right)^2}$$

$\boxed{\sigma = 13.11}$

$$S.E_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{13.11}{\sqrt{100}}$$

$\boxed{SE_{\bar{x}} = 1.311}$

PAGE No.	
DATE	/ /

* Mean deviation from mean

Individual series : $MD_{\bar{x}} = \frac{\Sigma|x-\bar{x}|}{n}$

MD from mean

~~Continuous~~ Discrete series : $MD_{\bar{x}} = \frac{\Sigma f|x-\bar{x}|}{\Sigma f}$

$\frac{MD_{\bar{x}}}{mean}$

Continuous series : $MD_{\bar{x}} = \frac{\Sigma f|m-\bar{x}|}{\Sigma f}$

Ex. Calculate mean deviation from mean
7, 4, 10, 9, 15, 12, 7, 9, 7

⇒ Soln

	x	$x-\bar{x}$
	7	1.89
	4	4.89
	10	1.11
	9	0.11
	15	6.11
	12	3.11
	7	1.89
	9	0.11
	7	1.89
Σ	80	21.11

$\bar{x} = \frac{\Sigma x}{n} = \frac{80}{9}$

$\boxed{\bar{x} = 8.89}$

$MD_x = \frac{\Sigma|x-\bar{x}|}{n} = \frac{21.11}{9} = 2.35$

PAGE No.
DATE / /

Ex. Calculate mean deviation from mean

x	4	6	8	10	12	14	16
f	2	1	3	6	4	3	1

$\Rightarrow$ Solⁿ

x	f	fx	$x-\bar{x}$	$f\|x-\bar{x}\|$
4	2	8	6.2	
6	1	6	4.2	
8	3	24	2.2	
10	6	60	0.2	
12	4	60	1.8	
14	3	$\cancel{8}$42	3.8	
16	1	16	5.8	
	20	204		

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{204}{20}$

$\bar{x} = 10.2$

$MD_{\bar{x}} = \frac{\Sigma f|x-\bar{x}|}{\Sigma f} = \frac{48.8}{20} = 2.4$

Ex Calculate mean deviation from mean.

marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90
students	4	6	8	20	25	15	10	9	3

⇒ Soln

x	f	m	$\lvert m-\bar{x}\rvert$	$f\lvert m-\bar{x}\rvert$	fm
10-20	4	15	40.4	161.6	60
20-30	6	25	30.4	182.4	150
30-40	8	35	20.4	163.2	280
40-50	20	45	10.4	208	900
50-60	25	55	0.4	10	1375
60-70	15	65	10.4	166.0	975
70-80	10	75	20.4	204	750
80-90	9	85	30.4	273	765
90-100	3	95	40.4	121.2	285
Σ	100			1480	5540

$$\bar{x} = \frac{\Sigma fm}{\Sigma f} = \frac{5540}{100} = 55.40$$

$$MD\bar{x} = \frac{\Sigma f\lvert m-\bar{x}\rvert}{\Sigma f} = \frac{1480}{100} = 14.8$$

Coefficient of ~~mean~~ $MD\bar{x} = \frac{MD\bar{x}}{\bar{x}} = \frac{14.8}{55.40} = 0.267$

PAGE No.

DATE / /

* Mean deviation from mode

1) Individual series : $MD_z = \frac{\Sigma|x-z|}{n}$

2) Discrete series : $MD_z = \frac{\Sigma f|x-z|}{\Sigma f}$

3) Continuous series : $MD_z = \frac{\Sigma f|m-z|}{\Sigma f}$ Coefficient $= \frac{MD_z}{mode}$

Ex. 7, 4, 10, 9, 15, 12, 7, 9, 7

⇒ Soln

arranging in ascending order

4, 7, 7, 7, 9, 9, 10, 12, 15 $\boxed{z = 7}$

x	$\|x-z\|$
7	0
4	3
10	3
9	2
15	8
12	5
7	6
9	4
7	0
Σ	23

$MD_z = \frac{\Sigma|x-z|}{n} = \frac{23}{7}$

$\boxed{MD_z = 2.56}$

PAGE No.
DATE / /

Ex: Calculate mean deviation from mode

x	4	6	8	10	12	14	16
f	2	1	3	6	4	3	1

⇒ Solⁿ.

x	f	|x − z|	f|x − z|
4	2	6	12
6	1	4	4
8	3	2	6
$\boxed{10}$	$\boxed{6}$	0	0
12	4	2	8
14	3	4	12
16	1	6	6
	20		48

$z = 10$

$$MD_z = \frac{\Sigma f|x-z|}{\Sigma f} = \frac{48}{20}$$

$$= 2.4$$

PAGE No.

DATE / /

Ex. Calculate mean deviation from mode

Scale	140-150	150-160	160-170	170-180	180-190	190-200
Freq	4	6	10	18	~~9~~ 7	3

⇒ Soln

x	f	m	|m - z|	f|m - z|
140-150	4	145	29	116
150-160	6	155	19	114
160-170	10	165	9	90
170-180	18	175	1	18
180-190	~~9~~ 7	185	11	77
190-200	3	195	21	63
	50			478

$$\text{mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i \quad , L = 170, f_1 = 18, f_0 = 10, f_2 = 7$$

$i = 10$

$$= 170 + \frac{18 - 10}{2(18) - 10 - 7} \times 10$$

$$\text{mode}(z) = 174$$

$$MD_z = \frac{\Sigma f|m - z|}{\Sigma f} = \frac{478}{50}$$

$$= 9.56$$

PAGE No.
DATE / /

* Mean deviation from median

1) Individual series : $MD_m = \frac{\Sigma |x-m|}{n}$

2) Discrete series : $MD_m = \frac{\Sigma f|x-m|}{\Sigma f}$

3) Continous series : $MD_m = \frac{\Sigma f|m-M|}{\Sigma f}$

Coefficient = $\frac{MD_m}{median}$

Ex. Calculate mean deviation from median
7, 4, 10, 9, 15, 12, 7, 9, 7

⇒ Soln
Arranging in ascending
4, 7, 7, 7, 9, 9, 10, 12, 15

x	$\|x-M\|$
~~4~~ 7	2
~~7~~ 4	5
~~7~~ 10	1
~~7~~ 9	0
15	6
12	3
7	2
9	0
7	2
Σ	21

median = $\left(\frac{n+1}{2}\right)^{th} = \left(\frac{9+1}{2}\right) = 5$

$\boxed{\text{median} = 9}$

$MD_m = \frac{\Sigma |x-m|}{n} = \frac{21}{9} = 2.33$

PAGE No.
DATE / /

Ex. Calculate mean deviation from median

x	4	6	8	10	12	14	16
F	2	1	3	6	4	3	1

⇒ Sol^n

x	f	CF	|x - M|	f|x - m|
4	2	2	6	12
6	1	3	4	4
8	3	6	2	6
10	6	12	0	0
12	4	16	2	8
14	3	19	4	12
16	1	20	6	6
	20			48

$$median = \left(\frac{N+1}{2}\right) = \left(\frac{20+1}{2}\right) = 10.5$$

$median (M) = 10$

$$MDm = \frac{\Sigma f|x-m|}{\Sigma f} = \frac{48}{20} = 2.4$$

Q. Calculate mean deviation from median

x	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
F	4	6	8	20	25	15	10	9	3

=> Sol^n

x	F	CF	m	|m - M|	F|m - M|
10-20	4	4	15	39.8	159.2
20-30	6	10	25	29.8	178.8
30-40	8	18	35	19.8	158.4
40-50	20	38	45	9.8	196.0
50-60	25	63	55	0	5
60-70	15	78	65	10.28	153.
70-80	10	88	75	20.2	202
80-90	9	97	85	30.2	~~120.~~ 271.6
90-100	3	100	95	40.2	120.6
	100				1444.8

$$\text{median} = L + \left(\frac{N/2 - cf}{f}\right) \times i \qquad L = 50,\ cf = 38,\ f = 25,\ i = 1$$

$$= 50 + \left(\frac{50 - 38}{25}\right) \times 10 = 54.8$$

$$\left(\frac{N}{2}\right) = \frac{100}{2} = 50$$

$$MD_M = \frac{\sum f|m - M|}{\sum f} = \frac{1444.8}{100}$$

$$= 14.448$$