Step 1: Convert the above edge-band digital frequencies into analog frquencies using the form[...] $\Omega = \frac{\omega}{T}$ with $T = 1$ sec.

Hence,

$$\Omega_P = 0.2\pi, \quad \delta_P = 1 - 0.8 = 0.2$$

$$\Rightarrow \quad K_P = 20\log(1-\delta_P) = -1.94 \text{ dB}$$

$$\Omega_S = 0.6\,\pi, \quad \delta_S = 0.2$$

$$\Rightarrow \quad K_S = 20\log\delta_S = -14 \text{ dB}$$

Step 2: Design a chebyshev I lowpass analog prototype filter to meet the specifications listed [...] step 1.

$$d = \sqrt{\frac{(1-\delta_P)^{-2}-1}{\delta_S^{-2}-1}} = 0.153$$

$$K = \frac{\Omega_P}{\Omega_S} = 0.33$$

Filter order:

$$N \geqslant \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)}$$

$$\Rightarrow \quad N \geqslant 1.446$$

Hence, the minimum filter order is $N = 2$.

We know that

$$1 - \delta_P = \frac{1}{\sqrt{1+\epsilon^2}}$$

$$\Rightarrow \quad \epsilon = 0.75$$

$$a = \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{\frac{1}{N}} - \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{\frac{-1}{N}}$$

$$= 0.57735$$

$$b = \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{\frac{1}{N}} + \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{\frac{-1}{N}}$$

$$= 1.1547$$

$$\sigma_k = -a\sin\left[(2k-1)\frac{\pi}{2N}\right], \quad k = 1, 2, 3, 4$$

$$\Omega_k = b\cos\left[(2k-1)\frac{\pi}{2N}\right], \quad k = 1, 2, 3, 4$$

k	σ_k	Ω_k	s_k
1	−0.4082481	0.8164962	$-0.4082481 + j0.8164962$
2	−0.4082481	−0.8164962	$-0.4082481 - j0.8164962$
3	0.4082481	−0.8164962	$0.4082481 - j0.8164962$
4	0.4082481	0.8164962	$0.4082481 + j0.8164962$

Hence,
$$H_2(s) = \frac{K_N}{\prod\limits_{\substack{\text{LHP}\\\text{only}}} (s - s_k)} = \frac{K_N}{(s - s_1)(s - s_2)}$$

$$= \frac{K_N}{(s + 0.4082481 - j0.8164962)(s + 0.4082481 + j0.8164962)}$$

$$= \frac{K_N}{(s + 0.4082481)^2 + (0.8164962)^2}$$

$$= \frac{K_N}{s^2 + 0.8164962s + 0.833333}$$

where
$$K_N = \frac{b_0}{\sqrt{1+\epsilon^2}} = \frac{0.833333}{\sqrt{1 + (0.75)^2}} = 0.667$$

Thus,
$$H_2(s) = \frac{0.667}{s^2 + 0.8164962s + 0.833333}$$

Since, we want the cutoff at $\Omega_P = 0.2\pi$, let us apply lowpass-to-lowpass transfor on $H_2(s)$ and get $H_a(s)$.

That is,
$$H_a(s) = H_2(s)\Big|_{s \longrightarrow \frac{s}{0.2\pi}}$$

$$= \frac{0.667}{\left(\frac{s}{0.2\pi}\right)^2 + 0.8164962\left(\frac{s}{0.2\pi}\right) + 0.833333}$$

$$= \frac{0.263321}{s^2 + 0.51302s + 0.32899}$$

$$= \frac{0.263321}{(s + 0.25651)^2 + (0.51302)^2}$$

$$= \frac{0.263321}{0.51302} \times \frac{0.51302}{(s + 0.25651)^2 + (0.51302)^2}$$

$$= 0.513276 \times \frac{0.51302}{(s + 0.25651)^2 + (0.51302)^2}$$

Example 4.35 Design a digital Chebyshev I filter that satisfies the following constraints.

$$0.8 \leqslant |H(\omega)| \leqslant 1, \qquad 0 \leqslant \omega \leqslant 0.2\pi$$
$$|H(\omega)| \leqslant 0.2, \qquad 0.6\pi \leqslant \omega \leqslant \pi$$

Solution

We are given the following digital specifications:

Passband ripple: $\delta_P = 1 - 0.8 = 0.2$.

Passband-edge frequency: $\omega_P = 0.2\pi$.

Stopband tolerance: $\delta_S = 0.2$.

Stopband-edge frequency: $\omega_S = 0.6\pi$.

4.37 Design a digital filter $H(z)$ that when used in an A/D-H(z)-D/A structure gives an equivalent analog filter with the following specifications:

Passband ripple : $\leqslant 3.01$ dB.

Passband edge : 500 Hz.

Stopband attenuation : $\geqslant 15$ dB.

Stopband edge : 750 Hz.

Sample rate : 2 KHz.

The filter is to be designed by performing a bilinear transformation on an analog system function, use Butterworth prototype. Also, plot the complete magnitude frequency response and obtain the difference equation realization.

□ **Solution**

The analog specifications are

$$\Omega_P = 2\pi \times 500 = \pi \times 10^3 \text{ rad/sec}, \qquad K_P = -3.01 \text{ dB}$$

$$\Omega_S = 2\pi \times 750 = 1.5\pi \times 10^3 \text{ rad/sec}, \qquad K_S = -15 \text{ dB}$$

Also, $T = \dfrac{1}{f_S} = \dfrac{1}{2000}$ secs.

Step 1: The corresponding digital specifications are obtained as follows.

$$\omega_P = \Omega_P T = \pi \times 10^3 \times \frac{1}{2000} = 0.5\pi \text{ rad}, \qquad K_P = -3.01 \text{ dB}$$

$$\omega_S = \Omega_S T = 1.5\pi \times 10^3 \times \frac{1}{2000} = 0.75\pi \text{ rad}, \qquad K_S = -15 \text{ dB}$$

Step 2: Prewarp the band-edge digital frequencies using $T = 1$ sec. Leave K_P and K_S unchanged.

$$\Omega'_P = \frac{2}{T}\tan\left(\frac{\omega_P}{2}\right) = 2\tan\left(\frac{0.5\pi}{2}\right) = 2, \qquad K_P = -3.01 \text{ dB}$$

$$\Omega'_S = \frac{2}{T}\tan\left(\frac{\omega_S}{2}\right) = 2\tan\left(\frac{0.75\pi}{2}\right) = 4.8282, \qquad K_S = -15 \text{ dB}$$

Step 3: Design an analog lowpass filter having the transfer function $H_a(s)$ to meet the prewarped specifications of Step 2.

$$N = \frac{\log\left[\left(10^{\frac{-K_P}{10}} - 1\right) \Big/ \left(10^{\frac{-K_S}{10}} - 1\right)\right]}{2\log\left(\frac{\Omega'_P}{\Omega'_S}\right)} = 1.944$$

Rounding off N to the next larger integer, we get $N = 2$.

The cutoff frequency Ω_C is found to satisfy the passband requirement exactly.

$$\Omega_C = \frac{\Omega'_P}{\left[10^{\frac{-K_P}{10}} - 1\right]^{\frac{1}{2N}}} = 2$$

Referring the normalized lowpass Butterworth filter tables, we get

$$H_2(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

Therefore, the required prewarped analog filter is obtained by applying lowpass-to-lowpass transformation to $H_2(s)$.

That is,

$$H_a(s) = H_2(s)|_{s \to \frac{s}{2}}$$

$$= \left.\frac{1}{s^2 + \sqrt{2}s + 1}\right|_{s \to \frac{s}{2}}$$

$$= \frac{4}{s^2 + 2\sqrt{2}s + 4}$$

Step 4: Apply bilinear transformation to $H_a(s)$ with $T = 1$ sec and get $H(z)$.

That is,

$$H(z) = H_a(s)|_{s \to \frac{2}{1}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{4}{\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 2\sqrt{2}\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right] + 4}$$

$$= \frac{1 + 2z^{-1} + z^{-2}}{3.4142 + 0.5858z^{-2}}$$

$$= \frac{(1 + z^{-1})^2}{3.4142 + 0.5858z^{-2}}$$

Difference equation realization

Let

$$H(z) = \frac{Y(z)}{X(z)}$$

Then,

$$\frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1} + z^{-2}}{3.4142 + 0.5858z^{-2}}$$

Cross multiplying and taking inverse $\mathcal{Z}$-transform yield

$$3.4142y(n) + 0.5858y(n-2) = x(n) + 2x(n-1) + x(n-2)$$

$$\Rightarrow \quad y(n) = -0.1715y(n-2) + 0.2928x(n) + 0.5857x(n-1) + 0.2928x(n-2)$$

Verification of the design

Letting $z = e^{j\omega}$ in $H(z)$, we get

$$H\left(e^{j\omega}\right) = H(\omega) = \frac{(1 + e^{-j\omega})^2}{3.4142 + 0.5858e^{-j2\omega}}$$

$$= \frac{[(1 + \cos\omega) - j\sin\omega]^2}{(3.4142 + 0.5858\cos 2\omega) - j\,0.5858\sin 2\omega}$$

$$\Rightarrow \quad |H(\omega)| = \frac{[(1 + \cos\omega)^2 + \sin^2\omega]}{\sqrt{(3.4142 + 0.5858\cos 2\omega)^2 + (0.5858\sin 2\omega)^2}}$$

Therefore,

$$20\log|H(\omega)|_{\omega=0.5\pi} = -3.01 \text{ dB}$$

$$20\log|H(\omega)|_{\omega=0.75\pi} = -15.44 \text{ dB}$$

The complete magnitude frequen...

xample 4.38 Design an IIR digital filter that when used in the prefilter A/D-H(z)-D/A structur will satisfy the following specifications (use Chebyshev protype):

a. lowpass filter with −2 dB cutoff at 100 Hz,

b. stopband attenuation of 20 dB or greater at 500 Hz, and

c. sampling rate of 4000 samples/sec.

Verify the design.

Solution

are given the following analog requirements:

$$\Omega_P = 2\pi \times 100 = 200\pi \text{ rad/sec}, \qquad K_P = -2 \text{ dB}$$
$$\Omega_S = 2\pi \times 500 = 1000\pi \text{ rad/sec}, \qquad K_S = -20 \text{ dB}$$

Also, $$T = \frac{1}{4000} \text{ secs}$$

In the present problem, the value of T is provided. In practice, the value of T is founc pass sampling theorem: $T \leqslant \frac{1}{2f_x}$, where f_x is the highest frequency present in the input).

p 1: Convert the above analog frequencies into equivalent digital frequencies using the r $\omega = \Omega T$ with $T = \frac{1}{4000}$ secs. The values of K_P and K_S remain unchanged.

$$\omega_P = \Omega_P T = 0.05\pi, \qquad K_P = -2 \text{ dB}$$
$$\omega_S = \Omega_S T = 0.25\pi, \qquad K_S = -20 \text{ dB}$$

Step 2: Prewarp the band-edge frequencies ω_P and ω_S using $T = 1$ sec.

$$
\begin{aligned}
\Omega'_P &= \frac{2}{T}\tan\left(\frac{\omega_P}{2}\right) \\
&= \frac{2}{1}\tan\left(\frac{0.05\pi}{2}\right) \\
&= 0.1574, \qquad K_P = -2 \text{ dB} \\
\Omega'_S &= \frac{2}{T}\tan\left(\frac{\omega_S}{2}\right) \\
&= \frac{2}{1}\tan\left(\frac{0.25\pi}{2}\right) \\
&= 0.8284, \qquad K_S = -20 \text{ dB}
\end{aligned}
$$

Step 3: Design the prewarped analog Chebyshev I filter having the transfer function $H_a(s)$ to meet the specifications of Step 2.

Given

$$K_P = -2 = 20\log(1-\delta_P)$$

$$\Rightarrow \quad \delta_P = 0.20567$$

and

$$K_S = -20 = 20\log\delta_S$$

$$\Rightarrow \quad \delta_S = 0.1$$

$$d = \sqrt{\frac{(1-\delta_P)^{-2}-1}{\delta_S^{-2}-1}} = 0.077$$

$$K = \frac{\Omega'_P}{\Omega'_S} = 0.19$$

$$N \geqslant \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)} = 1.39$$

$$\Rightarrow \quad N \geqslant 1.39$$

Minimum filter order, $N = 2$.

To find $H_2(s)$:

$$K_P = 20\log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right) = -2$$

$$\Rightarrow \quad \epsilon = 0.76478$$

$$
\begin{aligned}
a &= \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{1}{N}} - \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{-1}{N}} \\
&= 0.56839 \\
b &= \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{1}{N}} + \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{-1}{N}} \\
&= 1.15024
\end{aligned}
$$

We know that

$$\sigma_k = -a\sin\left[(2k-1)\frac{\pi}{2N}\right], \qquad k = 1, 2, \cdots 2N$$

$$\Rightarrow \quad \sigma_k = -0.56839\sin\left[(2k-1)\frac{\pi}{4}\right], \quad k = 1, 2, 3, 4$$

and

$$\Omega_k = b\cos\left[(2k-1)\frac{\pi}{2N}\right], \qquad k = 1, 2, \cdots 2N$$

$$\Rightarrow \quad \Omega_k = 1.15024\cos\left[(2k-1)\frac{\pi}{4}\right], \quad k = 1, 2, 3, 4$$

k	σ_k	Ω_k	$s_k = \sigma_k + j\Omega_k$
1	−0.40191	0.81334	$-0.40191 + j0.81334$
2	−0.40191	−0.81334	$-0.40191 - j0.81334$

The values of $k = 3$ and 4 give the poles of $H_2(s)H_2(-s)$ on the right-half of the s-plane and hence are not considered. In fact, $k = 3$ and 4 give the poles of $H_2(-s)$.

Thus, $$H_2(s) = \frac{K_N}{\prod\limits_{\substack{\text{LHP}\\\text{only}}}(s - s_k)} = \frac{K_N}{(s-s_1)(s-s_2)}$$

$$= \frac{K_N}{(s + 0.40191 - j0.81334)(s + 0.40191 + j0.81334)}$$

$$= \frac{K_N}{(s+0.40191)^2 + (0.81334)^2}$$

$$= \frac{K_N}{s^2 + \underbrace{0.80382}_{b_1}s + \underbrace{0.82305}_{b_0}}$$

Since N is even, the normalizing factor $K_N = \frac{b_0}{\sqrt{1+\epsilon^2}}$.

Hence, $$K_N = \frac{0.82305}{\sqrt{1+(0.76478)^2}} = 0.65377$$

Therefore, $$H_2(s) = \frac{0.65377}{s^2 + 0.80382s + 0.82305}$$

Since, we want the cutoff at $\Omega'_p = 0.1574$, we apply lowpass-to-lowpss transformation to $H_2(s)$ and get the required lowpass analog filter $H_a(s)$.

That is, $$H_a(s) = H_2(s)\Big|_{s \longrightarrow \frac{s}{0.1574}}$$

$$= \frac{0.65377}{\left(\frac{s}{0.1574}\right)^2 + 0.80382\left(\frac{s}{0.1574}\right) + 0.82305}$$

$$= \frac{0.0162}{s^2 + 0.12652s + 0.02039}$$