Example 4.19 Determine the system function $H(z)$ of the lowest-order Chebyshev filter that meets the following specifications:

a. 3 dB ripple in the passband $0 \leqslant |\omega| \leqslant 0.3\pi$.

b. Atleast 20 dB attenuation in the stopband $0.6\pi \leqslant |\omega| \leqslant \pi$.

Use the bilinear transformation.

Solution

The specified magnitude frequency response is shown in Fig. Ex.4.19(a).

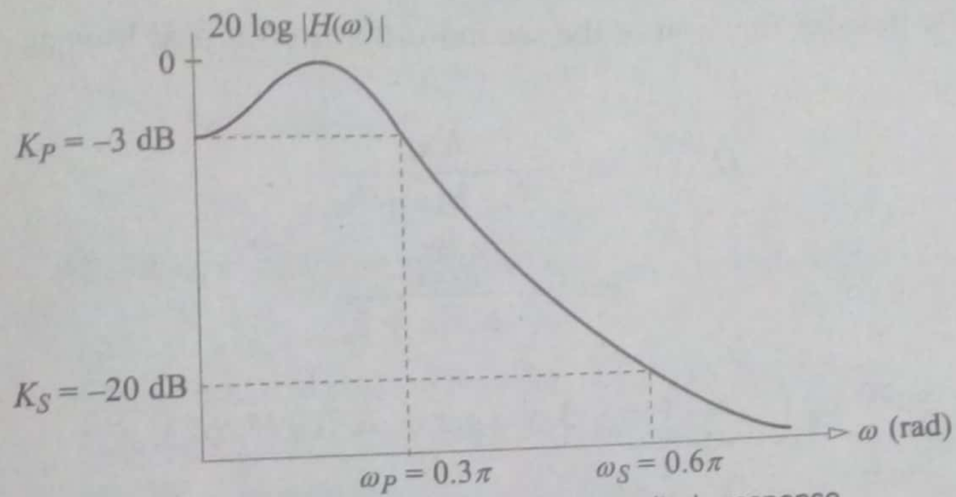

Fig. Ex.4.19(a) Specified lowpass magnitude response.

Step 1: Prewarping the band-edge frequencies ω_P and ω_S using $T = 1$ sec, we get

$$\begin{aligned}\Omega'_P &= \frac{2}{T}\tan\left(\frac{\omega_P}{2}\right)\\ &= \frac{2}{1}\tan\left(\frac{0.3\pi}{2}\right)\\ &= 1.019, \quad K_P = -3\text{ dB}\\ \Omega'_S &= \frac{2}{T}\tan\left(\frac{\omega_S}{2}\right)\\ &= \frac{2}{1}\tan\left(\frac{0.6\pi}{2}\right)\\ &= 2.75, \quad K_S = -20\text{ dB}\end{aligned}$$

Step 2: Let us design a prewarped analog lowpass Chebyshev I filter having a transfer function $H_a(s)$ to meet the specifications of step 1.

$$d = \sqrt{\frac{(1-\delta_P)^{-2}-1}{\delta_S^{-2}-1}} = 0.1$$

$$K = \frac{\Omega'_P}{\Omega'_S} = 0.3705$$

Minimum filter order, $$N = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)} = 1.8$$

Rounding off to the next larger integer, we get $N = 2$.

Referring to the normalized 3 dB ripple Chebyshev I filter tables (provided in Appendix II), we get for $N = 2$, the following filter coefficients.

$$b_0 = 0.7079478$$
$$b_1 = 0.6448996$$

Hence, the transfer function of the second-order normalized lowpass Chebyshev I filter is

$$H_2(s) = \frac{K_N}{s^2 + b_1 s + b_0} = \frac{\frac{b_0}{\sqrt{1+\epsilon^2}}}{s^2 + b_1 s + b_0}$$

Since, $K_P = 20\log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right) = -3$, we get $\epsilon^2 = 0.9952623$.

Hence, $$H_2(s) = \frac{\frac{0.7079478}{\sqrt{1+0.995263}}}{s^2 + 0.6448996s + 0.7079478} = \frac{0.50119}{s^2 + 0.6448996s + 0.7079478}$$

Since, we want the cutoff at Ω'_P, the required prewarped lowpass Chebyshev I filter $H_a(s)$ is obtained by applying lowpass-to-lowpass transformation to $H_2(s)$.

That is, $$H_a(s) = H_2(s)\Big|_{s \longrightarrow \frac{s}{\Omega'_P}} = \left.\frac{0.50119}{s^2 + 0.6448996s + 0.7079478}\right|_{s \longrightarrow \frac{s}{1.019}} = \frac{0.52}{s^2 + 0.6571526924s + 0.7351053856}$$

Step 3: Finally, the transfer function $H(z)$ of the digital filter is obtained by applying bilinear tranformation to $H_a(s)$ with $T = 1$ sec.

That is,
$$H(z) = H_a(s)\Big|_{s \longrightarrow \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$
$$= \frac{0.52}{4\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2 + 0.6571526924 \times 2\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 0.735105385}$$
$$= \frac{0.52\left(1+z^{-1}\right)^2}{6.0494 - 6.53z^{-1} + 3.420805z^{-2}}$$

Verification of the design

The frequency response of the digital filter is obtained by letting $z = e^{j\omega}$ in $H(z)$.

That is,
$$H\left(e^{j\omega}\right) = H(\omega) = \frac{0.52\left(1+e^{-j\omega}\right)^2}{6.0494 - 6.53e^{-j\omega} + 3.420805e^{-2j\omega}}$$
$$\Rightarrow \quad |H(\omega)| = \frac{0.52\left[(1+\cos\omega)^2 + \sin^2\omega\right]}{\sqrt{\begin{array}{l}(6.0494 - 6.53\cos\omega + 3.420805\cos 2\omega)^2 \\ +(6.53\sin\omega - 3.420805\sin 2\omega)^2\end{array}}}$$

Hence,
$$20\log|H(\omega)|_{\omega=0.3\pi} = -3 \text{ dB}$$
and
$$20\log|H(\omega)|_{\omega=0.6\pi} = -22.7 \text{ dB}$$