RV College of Engineering®
Mysore Road, RV Vidyaniketan Post
Bengaluru - 560059, Karnataka, India

Department of Electronics and Communication Engineering

Date: 29.04.2025	Test - 1	Max. Marks:50+10
Semester:VI	UG	Duration: $1\frac{1}{2}$ Hrs+20mins
Course Title: Digital Signal Processing with ML		Course Code:EC364TA

SCHEME AND SOLUTIONS

1	Calculate discrimination and selectivity factor for the given low pass Butterworth filter to meet following specification. Fp=5 kHz, Fs= 9 kHz, δp= δs=0.1. Ans:Discrimination=0.0487 selectivity=0.55	02
2	In Bilinear transformation if Ω=1 maps to ω=π/2, Ω=2 maps to --------- value of ω. Ans: w=0.7 π or 2.21rad	01
3	Obtain the passband ripple factor for a Chebyshev filter type I that has -3dB pass band attenuation at a frequency of 100 rad/sec and 25dbdB stop band attenuation at frequency of 250 rad/sec Ans:0.292	01
4	What is Frequency Warping effect in Bilinear Transformation? How to overcome this effect?	01
5	If $s=\sigma+j\Omega$ and $z=re^{j\omega}$, then what is the condition on σ if r<1? Ans:$\sigma < 0$	01
6	What is the number of minima's present in the pass band of magnitude frequency response of a low pass filter having ripples in pass band of order 4? Ans: 2	01
7	The impulse response sequence of a linear phase filter are h(n) = (5, -8, 3, ,.....), complete the sequence h(n) assuming N = 7 for Type 3 **Ans:(5,-8,3,0,-3,8,-5)**	01
8	The phase of linear phase FIR filter of length N=13 is ____________. Ans: -6ω	01
9	What are the conditions to be satisfied for constant phase delay in linear phase FIR filter? Ans: The condition for constant phase delay are Phase delay, α = (N-1)/2 (i.e., phase delay is constant) Impulse response, h(n) = h(N-1-n) (i.e., impulse response is symmetric)	01
1	Determine the transfer function H(z) of the filter having ripples in pass band that will meet the following specifications: $\frac{1}{\sqrt{2}} \leq \lvert H(j\omega)\rvert \leq 1, \quad 0 \leq \lvert w\rvert \leq 0.3\pi$ $\lvert H(j\omega)\rvert \leq 0.1, \quad 0.6\pi \leq \lvert w\rvert \leq \pi$ Use the Bilinear Transformation. Assume T=1.	10

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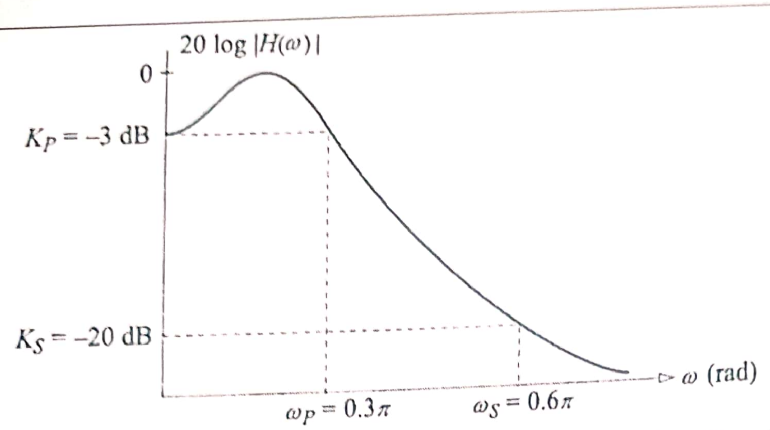


Prewarping the band-edge frequencies ω_P and ω_S using $T = 1$ sec, we get

$$\begin{aligned}
\Omega'_P &= \frac{2}{T}\tan\left(\frac{\omega_P}{2}\right) \\
&= \frac{2}{1}\tan\left(\frac{0.3\pi}{2}\right) \\
&= 1.019, \quad K_P = -3 \text{ dB} \\
\Omega'_S &= \frac{2}{T}\tan\left(\frac{\omega_S}{2}\right) \\
&= \frac{2}{1}\tan\left(\frac{0.6\pi}{2}\right) \\
&= 2.75, \quad K_S = -20 \text{ dB}
\end{aligned}$$

------[2M]

$$d = \sqrt{\frac{(1-\delta_P)^{-2}-1}{\delta_S^{-2}-1}} = 0.1$$

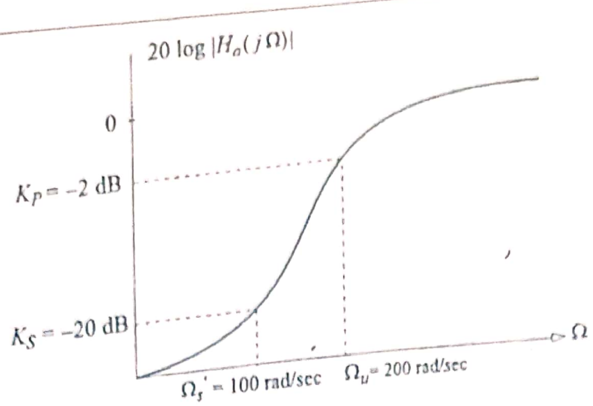
$$K = \frac{\Omega'_P}{\Omega'_S} = 0.3705$$

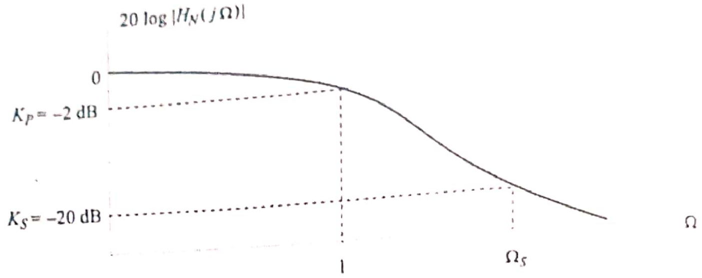
Minimum filter order, $$N = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)} = 1.8$$

Rounding off to the next larger integer, we get $N = 2$. ---------[2M]

$$\begin{aligned}
b_0 &= 0.7079478 \\
b_1 &= 0.6448996
\end{aligned}$$

	$$H_2(s) = \frac{K_N}{s^2 + b_1 s + b_0} = \frac{\frac{b_0}{\sqrt{1+\epsilon^2}}}{s^2 + b_1 s + b_0}$$ Since, $K_P = 20\log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right) = -3$, we get $\epsilon^2 = 0.9952623$. Hence, $$H_2(s) = \frac{\frac{0.7079478}{\sqrt{1+0.995263}}}{s^2 + 0.6448996s + 0.7079478} = \frac{0.50119}{s^2 + 0.6448996s + 0.7079478}$$ --------[1M] Since, we want the cutoff at Ω'_p, the required prewarped lowpass Chebyshev I filter $H_a(s)$ is obtained by applying lowpass-to-lowpass transformation to $H_2(s)$. That is, $$H_a(s) = H_2(s)\big\|_{s \longrightarrow \frac{s}{\Omega'_p}} = \frac{0.50119}{s^2 + 0.6448996s + 0.7079478}\bigg\|_{s \longrightarrow \frac{s}{1.019}} = \frac{0.52}{s^2 + 0.6571526924s + 0.7351053856}$$ That is, $$H(z) = H_a(s)\big\|_{s \longrightarrow \frac{2}{1}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]} = \frac{0.52}{4\left[\frac{1-z^{-1}}{1+z^{-1}}\right]^2 + 0.6571526924 \times 2\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 0.7351053856} = \frac{0.52\left(1+z^{-1}\right)^2}{6.0494 - 6.53z^{-1} + 3.420805z^{-2}}$$ ------[1M]	
2a	Design an analog high pass filter having monotonic pass band and stop band that will meet the following specifications: i. Maximum pass band attenuation =2dB ii. Pass band edge frequency= 200rad/sec iii. Minimum stop band attenuation= 20dB iv. Stop band edge frequency= 100rad/sec	07





Normalized lowpass magnitude frequency response.

--- [1M]

$$\Omega_S = \frac{\Omega_u}{\Omega'_S} = \frac{200}{100} = 2$$

---- [1M]

$$\Omega_P = 1, \qquad K_P = -2 \text{ dB}$$
$$\Omega_S = 2, \qquad K_S = -20 \text{ dB}$$

-------[2M]

we get, $N = 4$.

$$H_4(s) = \frac{1}{\left(s^2 + 0.76536s + 1\right)\left(s^2 + 1.84776s + 1\right)}$$

$$\begin{aligned}\Omega_C &= \frac{\Omega_P}{\left(10^{\frac{-K_P}{10}} - 1\right)^{\frac{1}{2N}}} \\ &= \frac{1}{\left(10^{0.2} - 1\right)^{\frac{1}{8}}} \\ &= 1.0693\end{aligned}$$

------[1M]

$$
\begin{aligned}
H_a(s) &= H_P(s)\Big|_{s\longrightarrow\frac{\Omega_R}{s}} \\
&= H_P(s)\Big|_{s\longrightarrow\frac{200}{s}} \\
&= H_4(s)\Big|_{s\longrightarrow\frac{200}{1.0693s}} \\
&= H_4(s)\Big|_{s\longrightarrow\frac{187.031}{s}} \\
&= \frac{s^4}{\left(s^2+143.1464s+34980.7521\right)\left(s^2+345.5892s+34980.7521\right)}
\end{aligned}
$$

------- [2M]

3

2b Compare and Contrast FIR and IIR Filters.

FIR filter	IIR filter
These filters can be easily designed to have perfectly linear phase.	These filters do not have linear phase.
FIR filters can be realized recursively and non-recursively.	IIR filters can be realized recursively.
Greater flexibility to control the shape of their magnitude response.	Less flexibility,usually limited to kind of filters.
Errors due to roundoff noise are less severe in FIR filters, mainly because feedback is not used.	The roundoff noise in IIR filters are more.

06

3a A digital low pass filter is required to meet the following specifications:

i. Pass band ripple $\leq 1dB$
ii. Passband edge frequency: 4 KHz
iii. Stopband attenuation: ≥40dB
iv. Stopband edge frequency: 6KHz
v. Sample rate: 24KHz

Apply bilinear transformation on an analog system function. Determine the order of Butterworth and Chebyshev 1 that must be used to meet the specification in the digital implementation. Comment on the result obtained.

$$
\begin{aligned}
K_P &= -1\text{ dB}, \quad \Omega_P = 2\pi \times 4\times 10^3\text{ rad/sec} \\
K_S &= -40\text{ dB}, \quad \Omega_S = 2\pi \times 6\times 10^3\text{ rad/sec} \\
T &= \frac{1}{fs} = \frac{1}{24\times 10^3}\text{ sec}
\end{aligned}
$$

$$
\begin{aligned}
\omega_P &= \Omega_P T = 2\pi \times 4\times 10^3 \times \frac{1}{24\times 10^3} = \frac{\pi}{3}\text{ rad}, \\
\omega_S &= \Omega_S T = 2\pi \times 6\times 10^3 \times \frac{1}{24\times 10^3} = \frac{\pi}{2}\text{ rad},
\end{aligned}
$$

------[2M]

	$$\Omega'_P = \frac{2}{1}\tan\left(\frac{\omega_P}{2}\right) = 1.155, \quad K_P = -1\text{ dB}$$ $$\Omega'_S = \frac{2}{1}\tan\left(\frac{\omega_S}{2}\right) = 2, \quad K_S = -40\text{ dB}$$ --------[1M] **Butterworth filter:** $$N = \frac{\log\left[\left(10^{\frac{-K_P}{10}} - 1\right)/\left(10^{\frac{-K_S}{10}} - 1\right)\right]}{2\log\left(\frac{\Omega'_P}{\Omega'_S}\right)} = 9.618$$ Rounding off to next larger integer, we get $N = 10$. --------[2M] **Chebyshev I filter:** $$K_P = 20\log(1-\delta_P) = -1$$ $$\Rightarrow \delta_P = 0.11$$ $$K_S = 20\log\delta_S = -40$$ $$\Rightarrow \delta_S = 0.01$$ $$d = \sqrt{\frac{(1-\delta_P)^{-2}-1}{\delta_S^{-2}-1}} = 5.1234\times 10^{-3}$$ $$K = \frac{\Omega'_P}{\Omega'_S} = 0.5775$$ $$N \geqslant \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)}$$ $$\Rightarrow N \geqslant 5.21$$ Hence, minimum filter order $N = 6$. -----[1M]	
3b	Compare Elliptic and Bessel filters in terms of frequency response and phase characteristics.	03

	Aspect	Elliptic Filter	Bessel Filter	
	Passband ripple	Ripple is allowed in the passband	Maximally flat no ripple	
	Stopband attenuation	Very sharp roll off best selectivity per order	Gradual roll off poor selectivity	
	Transition bandwidth	Very narrow sharpest among classic filters	Very wide slow transition	
	Order efficiency	Requires lower order for a given spec	Needs much higher order to match attenuation	
	Aspect	Elliptic Filter	Bessel Filter	
	Phase linearity	Poor non linear phase response	Excellent nearly linear phase in passband	
4a	Consider a fifth-order lowpass Chebyshev -1 filter with the passband of 1 KHz and passband ripple of 1dB. What is the attenuation of this filter in dB at f=1KHz and f=2KHz.			05

Ans

$$|H(j\Omega)| = \frac{1}{\left[1+\epsilon^2 T_N^2\left(\frac{\Omega}{\Omega_P}\right)\right]^{\frac{1}{2}}}$$

With $N = 5$, we get

$$|H(j\Omega)| = \frac{1}{\left[1+\epsilon^2 T_5^2\left(\frac{\Omega}{2\pi\times10^3}\right)\right]^{\frac{1}{2}}}$$

Given $K_P = -1$.
Since, $K_P = 20\log\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$, we get

$$20\log\left[\frac{1}{\sqrt{1+\epsilon^2}}\right] = -1$$

$$\Rightarrow \quad \epsilon = 0.50885$$

that.
$$T_5(x) = 16x^5 - 20x^3 + 5x$$

$$\left(\frac{\Omega}{2\pi\times10^3}\right) = 16\left(\frac{\Omega}{2\pi\times10^3}\right)^5 - 20\left(\frac{\Omega}{2\pi\times10^3}\right)^3 + 5\left(\frac{\Omega}{2\pi\times10^3}\right)$$

$$|H(j\Omega)| = \frac{1}{\left[1+(0.50885)^2\left(16\left(\frac{\Omega}{2\pi\times10^3}\right)^5 - 20\left(\frac{\Omega}{2\pi\times10^3}\right)^3 + 5\left(\frac{\Omega}{2\pi\times10^3}\right)\right)^2\right]^{\frac{1}{2}}}$$

$$A_1 = -20\log|H(j\Omega)|_{\Omega=2\pi\times10^3} = 1\text{ dB}$$

$$A_2 = -20\log|H(j\Omega)|_{\Omega=2\pi\times2\times10^3} = 45.31\text{ dB}$$

4b	Derive the mathematical relationship for the bilinear transformation which used to map the analog s-domain to the digital z-domain in digital filter design.	05
5a	Transform the analog filter $$H_a(s) = \frac{s+1}{s^2+5s+6}$$ Into H(z) using Impulse Invariant transformation. Take T=0.1 sec.	06

$$= \frac{C_1}{s+2} + \frac{C_2}{s+3}$$

$$C_1 = \left.\frac{s+1}{s+3}\right|_{s=-2} = -1$$

$$C_2 = \left.\frac{s+1}{s+2}\right|_{s=-3} = 2$$

$H_a(s)$ are $s_1 = -2$ and $s_2 = -3$.

$$H(z) = \sum_{i=1}^{N} \frac{C_i}{1 - e^{s_i T} z^{-1}}$$

$$\Rightarrow \quad H(z) = \sum_{i=1}^{2} \frac{C_i}{1 - e^{s_i T} z^{-1}}$$

$$= \sum_{i=1}^{2} \frac{C_i\, z}{z - e^{s_i T}}$$

$$H(z) = \frac{C_1 z}{z - e^{s_1 T}} + \frac{C_2 z}{z - e^{s_2 T}}$$

$$= \frac{-z}{z - e^{-0.2}} + \frac{2z}{z - e^{-0.3}}$$

$$= \frac{-z}{z - 0.8186} + \frac{2z}{z - 0.7408}$$

$$= \frac{z^2 - 0.8964z}{z^2 - 1.559z + 0.6065}$$

$$= \frac{1 - 0.8964\, z^{-1}}{1 - 1.559\, z^{-1} + 0.6065\, z^{-2}}$$

5b	The impulse response of linear phase FIR filter starts at the values h(0)=1 , h(1)=3, h(2)= -2. For N being odd and even find the conditions of the smallest order FIR filter that satisfies the linear phase condition.	04
Ans	*Case* (i): N odd and $h(n) = h(N-1-n)$. With $N = 5$, we have $h(n) = \{1, 3, -2, 3, 1\}$ $\quad$ $h(n) = \{2, -5, -2, -5, 2\}$	

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(ii): N even and $h(n) = h(N-1-n)$.
With $N = 6$, we have

$$h(n) = (1, 3, -2, -2, 3, 1)$$

$h(n) = \{2, -5, -2, -2, -5, 2\}$

(iii): N odd and $h(n) = -h(N-1-n)$.
With $N = 7$, we have

$$h(n) = (1, 3, -2, 0, 2, -3, -1)$$

$h(n) = \{2, -5, -2, 0, 2, 5, 2\}$

(iv): N even and $h(n) = -h(N-1-n)$.
With $N = 6$, we have

$$h(n) = (1, 3, -2, 2, -3, -1)$$

$h(n) = \{2, -5, -2, 2, 5, -2\}$