Grokking as a phase transition in optimisation problems

Seasons of code

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1. The k-XORSAT problem
2. Generalisation of Neural Networks on small algorithmically generated datasets
3. Grokking visualised in learning simple mathematical relations

The k-XORSAT problem

The Problem

- It is a boolean system of XOR equations having k variables each.
- Determining whether such a system is SAT or not can be done through gauss elimination, but the optimisation version of it, that asks "what is the maximum number $M_s(F)$ of equations that are SAT simultaneously among a system F of UNSAT equations" is pretty tough.
- No approximation algorithm exists for XORSAT with r $> \frac{1}{2}$ that is guaranteed to satisfy at least r$M_s(F)$ equations for any F. Interestingly, r $= \frac{1}{2}$ can be achieved on average by random guessing. T

Modelling random k XORSAT systems

Fixed Size Ensemble

- To make a random system of M equations of k variables each using n variables under the FSE, we do the following:
  - Pick up random k indices $i_1, i_2, ..., i_k from 1, 2, ...n$
  - Randomly sample v from 0, 1
  - Construct the equation where '+' denotes XOR:

$$x_{i_1} + x_{i_2} + ... + x_{i_k} = v \tag{1}$$

Fixed Probability Ensemble

- To make a random system of some equations of k variables each using n variables under the FPE, we do the following:
  - Scan the set of all 2*$\binom{n}{k}$ XOR equations and select each with a fixed probability 'p'
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$$E[numberofequations] = p \cdot 2 \cdot \binom{n}{k} \tag{2}$$

    To get an average of M equations, we keep

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