assignement-1

1) $2x-3y+7z=5$, $3x+y-3z=13$, $2x+19y-47z=32$.

→

$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix}$ $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$

$= \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 0 & 22 & -54 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 13 \\ 27 \end{bmatrix}$

$R_2 \to R_2 - 3/2 R_1$

$= \begin{bmatrix} 2 & -3 & 7 \\ 0 & 11/2 & -27/2 \\ 0 & 22 & -54 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 11/2 \\ 27 \end{bmatrix}$

$= \begin{bmatrix} 2 & -3 & 7 \\ 0 & 11/2 & -27/2 \\ 0 & 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 11/2 \\ 5 \end{bmatrix}$

Rank of A = 2 rank of A:B = 3

$\rho(A) \neq \rho(A:B)$ Inconsistency.

ii) $2x-y+3z=8$, $-x+2y+z=4$, $3x+y-4z=0$

→ $A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$ $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$A = \begin{bmatrix} -1 & 2 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -38/5 \end{bmatrix} \qquad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$R_3 \to R_3 - \frac{7}{3} R_2$

$= -1 - \frac{35}{3}$

$-\frac{38}{3}$

$\rho(A) = \rho(A:B)$

Consistency

3) $4x - y = 12$, $-x + 5y - 2z = 0$ $-2x + 4z = -8$

$\to$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \qquad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -4 & 0 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 12 \\ 0 \\ -16 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -1 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -4 & 20 & -8 \\ 0 & -1 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

Page No. Date

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & -1 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 12 \\ 12 \\ -4 \end{bmatrix}$$

$\rho(A) = \rho(A:B)$
(consistency)

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 18 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 12 \\ 12 \\ 8 \end{bmatrix}$$

d) Find the soln. of the system of Equations
$x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=0$.

$\rightarrow$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$$\rho(A) = \rho(A:B)$$
$$= 2 \neq n$$
(Infinite Solution).

e) Find for what values of λ the given equations $3x+y-\lambda z=0$, $4x-2y-3z=0$, $2\lambda x+4y+\lambda z=0$, may posses non-trivial Solution and Solve them Completely in Each case.

Page No. | Date

$$\rightarrow A = \begin{bmatrix} 3 & +1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} = \begin{bmatrix} 3 & +1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda+3 & 4+1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 4 & -4\lambda \\ 12 & -6 & -9 \\ 2\lambda+3 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 4 & -4\lambda \\ 0 & -10 & -9+4\lambda \\ 2\lambda+3 & 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 4 & -4\lambda & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 5 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 12 & -6 & -9 & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 5 & 0 & 0 \end{bmatrix}$$

$$= 12x - 6y - 9z = 0$$

$$-10y + (-9+4\lambda)z = 0$$

$$(2\lambda+3)x + 5y = 0$$

$$\frac{-(2\lambda+3)x}{5} = y$$

$$12x - 6y - 9z = 0$$

$$12x + \frac{6(2\lambda+3)x}{5} - 9\left(\frac{10y}{(-9+4\lambda)}\right) = 0$$

$$12x + \frac{6(2\lambda+3)x}{5} + 9\left(\frac{\cancel{10}\,2}{(-9+4\lambda)}\left(\frac{2\lambda+3}{\cancel{5}}\right)x\right) = 0$$

$$12 + \frac{6(2\lambda+3)}{5} + \frac{18(2\lambda+3)}{(4\lambda-9)} = 0$$

Page No.
Date

$$12(4\lambda-9)5 + \underbrace{6(2\lambda+3)}_{(12\lambda+18)}(4\lambda-9) + 18(5)(2\lambda+3) = 0$$

$$240\lambda - 540 + 180\lambda + 270 + 48\lambda^2 - 108\lambda + 72\lambda - 162 = 0$$

$$48\lambda^2 + 384\lambda - 432 = 0$$

$$\lambda^2 + 8\lambda - 9 = 0$$

$$\lambda^2 + 9\lambda - \lambda - 9 = 0$$

$$\lambda(\lambda-1) + 9(\lambda-1) = 0$$

$$(\lambda+9)(\lambda-1) = 0 \quad \Rightarrow (\lambda = 1, \ \lambda = -9)$$

$\lambda = 1$

$$-x = y \qquad , z = -2y$$

$$12x - 6y - 9z = 0$$

$$12(-y) - 6y - 9(-2y) = 0$$

$$18y - 18y = 0$$

$$y = 0 \qquad \text{(trivial Solution)}$$

$\lambda_2 = -9$

$$y = \left(\frac{-(2\lambda+3)x}{5}\right) \qquad z = \frac{10y}{(-9+4\lambda)}$$

$$= -\frac{(-18+3)x}{5} \qquad = \frac{10y}{-45} = \frac{2y}{-9}$$

$$= +3x$$

$$12x - 6y - 9z = 0$$

$$\cancel{12}^{4}\left(\frac{y}{\cancel{3}}\right) - 6y - \cancel{9}\left(\frac{2y}{-\cancel{9}}\right) = 0$$

$$4y - 6y + 2y = 0$$

$$y = 0 \qquad \text{trivial solution}$$

It has no trivial solution.

Page No.
Date

c) $x+y+z=1$ find values of λ
$x+2y+4z=\lambda$ of the
$x+4y+10z=\lambda^2$ given Equation
and solve the completely
in Each cases

$\rightarrow$ $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$

$R_2 = R_2 - R_1$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & (\lambda-1) \\ 1 & 4 & 10 & (\lambda^2-1) \end{bmatrix}$

$R_3 \to R_3 - 3R_2$

$= \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 3 & : & (\lambda-1) \\ 0 & 0 & 0 & : & (\lambda^2-1)-3(\lambda-1) \end{bmatrix}$

rank of $A = 2$ $n=3$
$(A:B) = 2$

1) (No unique solution)

2) Infinite Solution $\lambda^2-1-3\lambda+3=0$
$\lambda^2-3\lambda+2=0$
$\lambda^2-2\lambda-\lambda+2=0$
$\lambda(\lambda-2)-1(\lambda-2)=0$

$\lambda = 1, 2$

case $\lambda = 1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x+y+z=1$
$y+3z=0$

$z=K$
$y=-3K$
$x=1-2K$

case $\lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x+y+z=1$
$y+3z=1$

$z=K$
$y=1-3K$
$x=2K$

Q c) $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$

i) a unique Solution.
ii) no Solution
iii) an infinite number of Solution

$\rightarrow$ $$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$
$R_3 \rightarrow R_3 - R_1$

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & & 4 \\ 1 & 2 & \lambda-3 & & \mu-10 \end{bmatrix}$$

Case (i): $\lambda-3=0$ and $\mu-10\neq 0$
i.e $(\lambda=3, \mu\neq 0)$ $\rightarrow$ no Solution (consistent)

Case (ii): $\lambda-3=0$ and $\mu \in R$
i.e $\lambda \neq 3$
$\therefore$ the given system unique soln

Case (iii) $\lambda=3$ and $\mu=10$
(infinite soln).

Assignments

Q1) find set of vectors are LD or LID.

i) [1,0,0], [1,1,0], [1,1,1]

→ here no of unknowns = 3
$n = 3$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} \rho(A) \Rightarrow 3 \\ n = 3 \\ \rho(A) = n \end{array}$$

so it's linearly independent

2] [7 -3 11 -6] [-56 24 -88 48].

→ $n = 2$

$$A = \begin{bmatrix} 7 & -3 & 11 & -6 \\ -56 & 24 & -88 & 48 \end{bmatrix}$$

here max rank of A can be 2 $\quad \rho(A)_{max} \Rightarrow 2$
$\rho(A)_{max} = 2 \neq n$

so, it's linearly Dependent.

3) $\begin{bmatrix}-1 & 5 & 0\end{bmatrix}$, $\begin{bmatrix}16 & 8 & -3\end{bmatrix}$, $\begin{bmatrix}-64 & 56 & 9\end{bmatrix}$

$\rightarrow$ $n = 3$

$$A \Rightarrow \begin{bmatrix}-1 & 5 & 0 \\ 16 & 8 & -3 \\ -64 & 56 & 9\end{bmatrix} \qquad \begin{matrix}R_2 \rightarrow R_2 + 16R_1 \\ R_3 \rightarrow R_3 - 64R_1\end{matrix}$$

$$\Rightarrow \begin{bmatrix}-1 & 5 & 0 \\ 0 & 88 & -3 \\ 0 & -264 & 9\end{bmatrix} \qquad R_3 \rightarrow R_3 + 3R_2$$

$$\Rightarrow \begin{bmatrix}-1 & 5 & 0 \\ 0 & 88 & -3 \\ 0 & 0 & 0\end{bmatrix} \qquad \begin{matrix}\rho(A) = 2 \\ n = 3 \\ \rho(A) \neq n\end{matrix}$$

So, It's linearly Dependent.

4) $\begin{bmatrix}1 & -1 & 1\end{bmatrix}$, $\begin{bmatrix}1 & 1 & -1\end{bmatrix}$, $\begin{bmatrix}-1 & 1 & 1\end{bmatrix}$, $\begin{bmatrix}0 & 1 & 0\end{bmatrix}$

$\rightarrow$ $n = 4$.

$$A = \begin{bmatrix}1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \\ 0 & 1 & 0\end{bmatrix} \qquad \begin{matrix}R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 + R_1\end{matrix}$$

$$A = \begin{bmatrix}1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 2 \\ 0 & 1 & 0\end{bmatrix} \qquad R_4 \rightarrow R_4 - \frac{1}{2}R_2$$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad R_4 \rightarrow R_4 - \tfrac{1}{2}R_3$$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \rho(A) = 3, \; n = 4$$

$\rho(A) \neq n = 3$

So, it's linearly independent

5] $[2\ -4]$ $[1\ 9]$ $[3\ 5]$

$\rightarrow$ $n = 3$

$$\begin{bmatrix} 2 & 1 & 3 \\ -4 & 9 & 5 \end{bmatrix}$$

$\rho(A)_{max} \Rightarrow 2$

So, it's linearly Dependent.

6) $[3\ -2\ 0\ 4]$ $[5\ 0\ 0\ 1]$ $[-6\ 1\ 0\ 1]$ $[2\ 0\ 0\ 3]$

$n = 4$

$$\Rightarrow \begin{bmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 5 & -6 & 2 \\ -2 & 0 & 1 & 0 \\ 4 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} R_3 \rightarrow R_3 - 4/3 R_1 \\ R_2 \rightarrow R_2 + 2/3 R_1 \end{array}$$

Page No.
Date

$$A = \begin{bmatrix} 3 & 5 & -6 & 2 \\ 0 & 16/3 & -3 & 4/3 \\ 0 & -17/3 & 9 & 1/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 5 & -6 & 2 \\ 0 & 10 & -9 & 4 \\ 0 & -17 & 27 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 + 17/16\, R_2$

$$A = \begin{bmatrix} 3 & 5 & -6 & 2 \\ 0 & 10 & 9 & 4 \\ 0 & 0 & 117/10 & 78/10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3$
$n = 4$

So, it's linearly Dependent.

7) $[3\ 4\ 7]$ $[2\ 0\ 3]$ $[8\ 2\ 3]$ $[5, 5, 6]$

$\to$ $n = 4$

$$A = \begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{bmatrix}$$

$\rho(A)_{max} \Rightarrow 3$
$n = 4$

So, its linearly Dependent.

"Assignment - 3"

(~ Eigen values & vectors)

1)

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$\rightarrow$ $A - \lambda I \Rightarrow \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix}$

$$-(2+\lambda)\left((1-\lambda)(-\lambda)-12\right) - 2(-2\lambda-6) - 3(-4+(1-\lambda)) = 0$$

$$-(\lambda+2)\left[-\lambda+\lambda^2-12\right] + 4\lambda + 12 + 12 - 3 + 3\lambda \Rightarrow 0$$

$$\lambda^2 - \lambda^3 + 12\lambda + 2\lambda - 2\lambda^2 + 24 + 4\lambda + 12 + 12 - 3 + 3\lambda = 0$$

$$-\lambda^3 - \lambda^2 + 21\lambda + 45 = 0$$

$$\lambda = -3, 5$$

for $\lambda = -3$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}$$

$$x + 2y - 3z = 0$$
$$2x + 4y - 6z = 0$$
$$-x - 2y + 3z = 0$$

2) $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$\rightarrow \quad A - \lambda I \Rightarrow \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$

$\rightarrow (4-\lambda)\left((1-\lambda)^2\right) + 1\left(+2(1-\lambda)\right) = 0$

$\Rightarrow (4-\lambda)(1-\lambda)^2 + 2(1-\lambda) = 0$

$(1-\lambda)\left[(4-\lambda)(1-\lambda) + 2\right] = 0$

$(1-\lambda)\left[6 - 5\lambda + \lambda^2\right] = 0$

$(\lambda-1)(\lambda-2)(\lambda-3) = 0$

Eigen Values $\lambda = 1, 2, 3$

for $\lambda = 1$

$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$

$-2x \Rightarrow 0 \quad . \; x = 0, \quad z = 0$

$3x + z = 0 \qquad y = K$

$\therefore$ Eigen Vector $= K \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Page No.
Date

for $\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix}$$

$-2x - y = 0$

$2x = -y$

$-2x - z = 0$

$z = -2x$

$x = K, \quad y = -2K, \quad z = -2K.$

Eigen Vector $= K \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$

for $\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix}$$

$-2x - 2z = 0$

$x = -z$

$-2x - 2y = 0$

$x = -y$

$(x = K, \; y = -K, \; z = -K)$

$\therefore$ Eigen vectors $K \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

3) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$\rightarrow$

$A - \lambda I \Rightarrow \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$

Page No.
Date

$$(5-\lambda)\left((-\lambda)(3-\lambda)\right) = 0$$

$$(\lambda-5)(\lambda)(\lambda-3) = 0$$

Eigen vectors : $\boxed{\lambda = 5, 0, 3}$

- for $\lambda = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix}$$

$x = -2K, y = 0, z = K$

$-x - 2z = 0$

$(x = -2z)$

$-5y = 0$

$(y = 0)$

Eigen Vectors $= K \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

for $\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$x = 0$

$-x + 3z = 0$

$z = 0$

$x = 0, y = K, z = 0$

Eigen Vectors $= K \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Page No.
Date

· for $\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$y = 0$
$x = 0$
$z = K$

Eigen Vectors $= K \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

4) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & +4 \\ 0 & 0 & -2 \end{bmatrix}$

$\rightarrow A - \lambda I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}$

$\Rightarrow -1 \left((3-\lambda)(-2-\lambda) \right) = 0$

$(3-\lambda)(\lambda+2) = 0$

Eigen Values $\lambda = 3, -2$

· for $\lambda = +3$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix}$$

$x = 0$
$y = 0$
$z = K$

Eigen vectors $= K \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

∴ for $\lambda = -2$

$x = 0$, $y = -4/5K$
$z = K$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$5y + 4z = 0$

$y = -4/5z$

$$\text{Eigen Vectors} = k \begin{bmatrix} 0 \\ -4/5 \\ 1 \end{bmatrix}$$

Priyanshu Singh (086)

Page No.

Date

ussignment NO - 4

① find the rank of the matrix A by reducing in row reduced Echelon form

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

→

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_4 \leftarrow R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -5 & 5 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -5 & 5 \end{bmatrix}$$

Priyanshu Singh (086)

Page No. | Date

$R_4 \leftarrow R_4 + R_2$ $\qquad$ $R_3 \leftarrow -1/3 R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -13 & 8 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 13 & 8 \end{bmatrix}$$

$R_2 \leftarrow R_2 + 4R_3$
$R_1 \leftarrow R_1 + 2R_3$
$R_2 \leftarrow R_2 + 5/3 R_3$
$R_2 \leftarrow -1/4 R_2$
$R_1 \leftarrow R_1 - 2R_3$
$R_2 \leftarrow R_2 - 2R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -13 & 8 \end{bmatrix}$$

$\therefore$ rank of Matrix is $A = 4$.

Q2) Let W be the vector space of all symmetric 2×2 matrices and let $T: W \to P_2$ be the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$.

Find the rank and nullity of T.

$\rightarrow$

$$T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$$

let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ then

$T(A) = (a-b) + (b-c)x + (c-a)x^2 = a - bx + c(x^2 - x + 1)$

$\therefore$ the image of T is the set of all polynomials of degree at most 2, denoted as P_2.

Priyanshu Singh

Page No.
Date

Rank of T :

the rank of T' is the dimension of its image . since P_2 has a dimension of 3 (coefficients for x^0, x^1, and x^2), the rank of T is 3.

· The Null Space of Symmetric matrices

$$T(A) = 0$$

this leads to the system of Equations

$a - b = 0$ $\quad$ $b - c = 0$ $\quad$ $c - a = 0$

$\therefore a = b = c$

i.e. T is the set of Symmetric matrices of the form

$\begin{bmatrix} t & t \\ t & t \end{bmatrix}$, where t is any scaler.

$\therefore$ Dimension = 1 (using only t)

$\therefore$ rank T is 3

the nullity of T is 1.

Priyanshu Singh (088)

Page No.

Date

Q3) let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, find the Eigenvalues and Eigenvectors of A^{-1} and $A + 4I$.

$\rightarrow$ $(A - \lambda I) = 0$ $\qquad A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

λ is the Eigenvalue

$$\det\left(\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix}\right) = 0$$

$$(2-\lambda)^2 - (-1)(-1) = 0$$
$$(2-\lambda)^2 - 1 = 0$$
$$(\lambda - 1)(\lambda - 3) = 0$$

So, the Eigenvalues λ is 1 and 3.

for $\lambda = 1$

$$(A - I)x = 0$$
$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore$ leads $x_1 - x_2 = 0$, which means the Eigenvector is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for $\lambda = 3$

$$(A - 3I)x = 0$$
$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore$ $-x_1 - x_2 = 0$

which means the Eigenvector is $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Priyanshu Singh (008)

Page No.

Date

* A^{-1}

for A, the Eigenvalues are 1 and 3.

$\therefore$ the Eigenvalues of A^{-1} are $\frac{1}{1}$ and $\frac{1}{3}$

which are 1 and $\frac{1}{3}$

$A+4I$:

Eigenvalues of $A+4I$ is $1+4=5$ and $3+4=7$.

are 1 and 3

Q4) Solve by gauss-seidal Method (take three iteration)

$$3x-0.1y-0.2z=7.85$$
$$0.1x+7y-0.3z=-19.3$$
$$0.3x-0.2y+10z=71.4$$

with initial Values $x(0)=0$, $y(0)=0$, $z(0)=0$

→ Iteration 1:

$$x^{(1)}=\frac{7.85-0.1(0)-0.2(0)}{3}\approx 2.6167$$

$$y^{(1)}=\frac{19.3-0.1(2.6167)-0.3(0)}{7}\approx -2.7677$$

$$z^{(1)}=\frac{71.4-0.3(2.6167)-0.2(2.76774)}{10}$$

$$\approx 7.1408$$

Priyanshu Singh(0881).

Page No.
Date

Iteration 2:

$$x^{(2)} = \frac{7.85 - 0.1(2.6167) - 0.2(7.1408)}{3} \approx 2.9255$$

$$y^{(2)} = \frac{19.3 - 0.1(2.9255) - 0.3(7.1408)}{7} \approx 3.0123$$

$$z^{(2)} = \frac{71.4 - 0.3(2.9255) - 0.2(3.0123)}{10} \approx 7.0132$$

Iteration 3:

$$x^{(3)} = \frac{7.85 - 0.1(2.9255) - 0.2(7.0132)}{3} \approx 3.0032$$

$$y^{(3)} = \frac{19.3 - 0.1(3.0032) - 0.3(7.0132)}{7} \approx 3.001$$

$$z^{(3)} = \frac{71.4 - 0.3(3.0032) - 0.2(3.0001)}{10} \approx 7.00$$

$x = 3.0032$, $y \approx 3.0001$, $z \approx 7.000$.

Q-5) Define Consistent and inconsistent System of Equations. Hence Solve the following System of Equation if Consistent

$x + 3y + 2z = 0$, $2x - y + 3z = 0$, $3x - 5y + 4z = 0$,
$x + 17y + 4z = 0$.

→ we Expressed it in matrix form $Ax = 0$. where A is the coefficient matrix:

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$$

Priyanshu Singh (06

after performing rows reduction, we obtained Echelon form

$$\begin{bmatrix} 1 & 3 & 2 & : & 0 \\ 0 & -7 & 1 & : & 0 \\ 0 & 0 & 0 & : & 0 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

This Corresponds to the System:

$$x + 3y + 2z = 0$$
$$-7y + z = 0$$

Now let's Express the Variables in terms of parameters. let $y = t$.

$$x = -3t$$
$$z = 7t$$

So, the System has infinitely many Solutions given by

$$x = -3t$$
$$y = t$$
$$z = 7t$$

$\therefore$ the System is Consistent and dependent.

Priyanshu Singh (088).

Page No.

Date

Q6) Determine whether the function $T: P_2 \to P_2$ is linear transformation or not: where $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

$\to$

$T(a+bx+c) = (a+1)+(b+1)x+(c+1)x^2$ is a linear transformation, we need to check two properties

1> Additivity: $T(U+V) = T(U)+T(V)$

2> homogeneity of degree 1:

$T(Ku) = KT(u)$ for all u in the domain of T and all scalars K.

1) $T(U+V) = T((a_1+b_1x+c_1)+(a_2+b_2x+c_2))$

$= T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2))$

$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$

$= (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)+(b_2+1)x+(c_2+1)x^2$

$= T(a_1+b_1x+c_1)+T(a_2+b_2x+c_2)$.

So function is additive

Homogeneity of Degree 1:

$T(KU) = T(K(a+bx+c))$

$= T(Ka+Kbx+Kc)$

$= (Ka+1)+(Kb+1)x+(Kc+1)x^2$

$= K(a+1)+K(b+1)x+K(c+1)x^2$

$= KT(a+bx+c)$

So, the function is homogeneous of degree 1.

it indeed linear transformation

Priyanshu Singh (088)

Page No.

Date

Q7) Determine the whether the Set $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In cases S is not a basis determine the dimension and the basis of the sub space spanned by S

→ $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ can be arranged as a matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

Now, let's perform row reduction to obtained the Echelon form:

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2R_1$ and $R_3 \leftarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \quad R_3 \leftrightarrow R_3 + \frac{9}{5} R_2$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

third row of zeros indicates that the vectors in S are linearly dependent

Priyanshu Singh (CA)

Page No
Date

for basis of the Subspace Spanned by S.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \end{bmatrix}$$

$\therefore$ $(1,3,-2)$ and $(0,-5,5)$. these vectors form a basis for the subspace spanned by S.

$\therefore$ Dimension of Subspace Spanned by S = 2.

$\therefore$ set S is not a basis of R^3 because the row-reduced form has a row of zeros.

$\therefore$ the basis for the subspaces Spanned by S is $\{(1,3,-2), (0,-5,5)\}$.

$\therefore$ The dimension of the Subspace is 2.

Q8) Using Jacobi's method (perform 3 iterations), solve $3x-6y+2z=23$, $-4x+y-z=-15$, $x-3y+7z=16$, with initial values $x_0=1$, $y_0=1$, $z_0=1$

$\rightarrow$

$$3x-6y+2z=23$$
$$-4x+y-z=-15$$
$$x-3y+7z=16$$

with initial values $x^{(0)}=1$, $y^{(0)}=1$, $z^{(0)}=1$.

Iteration 1 :

$$x^{(1)} = \frac{23+6y^{(0)}-2z^{(0)}}{3} \approx 8.0$$

$$y^{(1)} = \frac{-15+4x^{(0)}+z^{(0)}}{1} \approx -9.0$$

$$z^{(1)} = \frac{16-x^{(0)}+3y^{(0)}}{7} \approx 2.0$$

(Priyanshu 088)

Page No.
Date

Iteration 2:

$$x^{(2)} = \frac{23+6y^{(1)}-2z^{(1)}}{3} \approx 5.0$$

$$y^{(2)} = \frac{-15+4x^{(1)}+z^{(1)}}{1} \approx -5.0$$

$$z^{(2)} = \frac{16-x^{(1)}+3y^{(1)}}{7} \approx -3.0$$

Iteration 3:

$$x^{(3)} = \frac{23+6y^{(2)}-2z^{(2)}}{3} \approx 6.0$$

$$y^{(3)} = \frac{-15+4x^{(2)}+z^{(2)}}{1} \approx -6.0$$

$$z^{(3)} = \frac{16-x^{(2)}+3y^{(2)}}{7} \approx 2.0$$

Priyanshu Singh (088).

Page No. | Date

Q9) one common application of matrix operations in image processing is in the field of image filtering, particularly with convolution operations. Convolution involves applying a filter (also known as a kernel or masks) to an image to enhance or modify certain features. The filter is a typically a small matrix and it convolved with the image matrix through a series of element-wise multiplication and summations.

Q10) In Computer Vision, linear transformation are
-> often used to manipulate and transform 2D images one common linear transformation is rotation..

linear transformation: rotation in 2D.

1. Rotation matrix
2. Image Representation.
3. Transformation Operation (Each pixel's coordinates are transform using the rotation matrix)
4. Interpolation.