

Ques: You are given an array. You need to find the sum

1 → find range sum

2 - update at index

(index, value)

of given ranges.

You will get 'q' query to find the sum of that range

'q' query also includes a point update operation

For Ex: [5, 4, 12, 10, 2, 3]

q
1 - (2, 5)
2 - (3, 2)

.

q → queries n → elements arr

Time to build PrefixSum → O(n)

Time to get range sum → O(1)

Time to update element → O(n)

$0 < q < 10^4$

$0 < n < 10^9$

Fenwick Tree



Binary Index Tree (BIT)

Fenwick
Array



Every index has
the capacity
to store sum
of some pre-
calculated index
elements

Range?

(1, 1) (1, 2) (3, 3) (1, 4)
left right

Initialize
every index from
[1-n]
with '0'

Base-10
Index

Base-2
Binary

1
2
3
4
5
6
7

5	2	7	1	1	9	6	4
---	---	---	---	---	---	---	---

0 1 2 3 4 5 6 7

(1,1)	(1,2)	(3,3)	(1,4)	(5,5)	(5,6)	(7,7)
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0 1 2 3 4 5 6 7

Invert Right Most '1'

00

Base-10

0

Add 1

1

Index
finding

1. 2's Complement Index

2. & original index

3. + original index

Ex: - a

5	2	7	1	1	9	6	4
---	---	---	---	---	---	---	---

~~0 1 2 3 4 5 6 7~~

$(1,1) [12] (3,3) [1,4] (5,5) (5,6) (7,7)$

$$\begin{array}{r} 10 \\ 10 \\ \hline 100 \end{array}$$

0	5	7	10	15	9	15	4
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0

① $2^8 \rightarrow -a$

② $a \& -a$

③ $a + (a \& -a)$

Index \rightarrow Base 2 \rightarrow 2's Complement \rightarrow & ori index \rightarrow + ori index
1 01 11 01
2 10 10 10
4 100 100 100
3 11 01
5

$10 \rightarrow 100(4)$ Base 10

$100 \rightarrow 1000(8)$

$01 \rightarrow 100(4)$

01

$2^8 \rightarrow 10$

+ 1

$2^8 \rightarrow \frac{11}{11}$

$\cancel{8} 01$

$+ \frac{01}{10}$

```
void update(index, value){  
    while (index <= n) {  
        fen_arv[index] += value  
        index = index + (index & -index)  
    }  
}
```

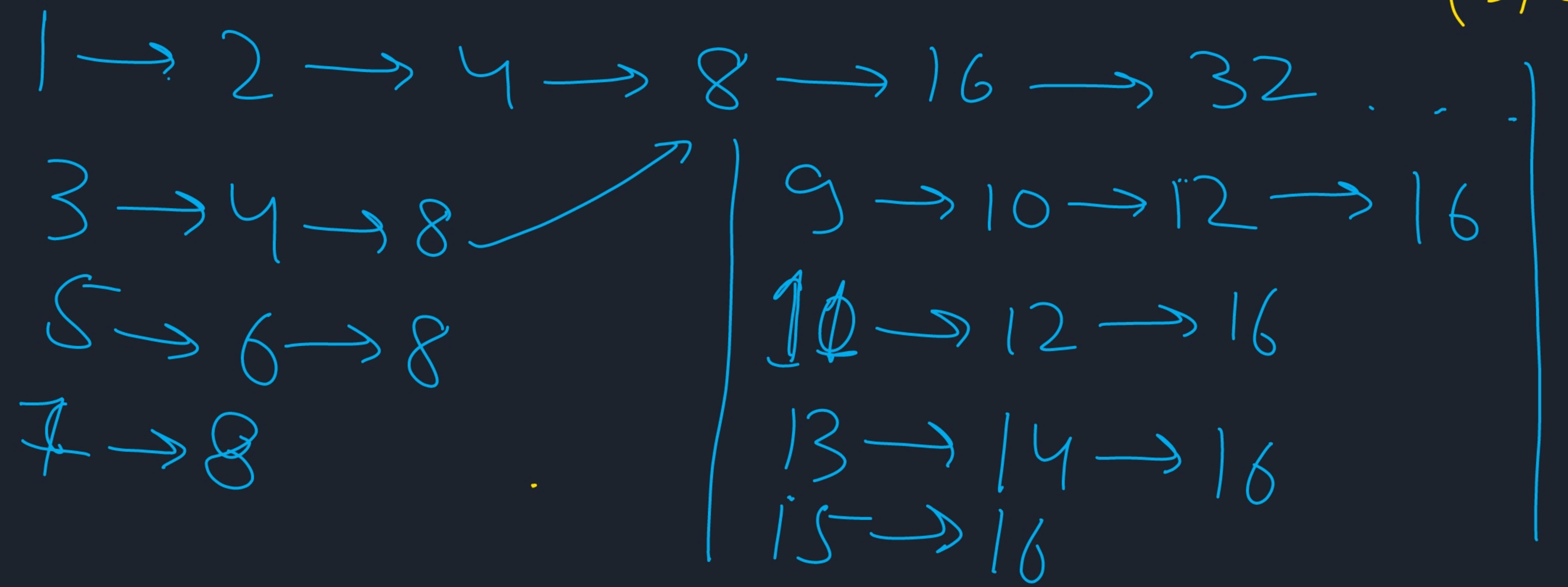
Diagram annotations:

- A blue bracket groups the two statements:

```
fen_arv[index] += value  
index = index + (index & -index)
```
- A blue arrow labeled "add" points from the first statement to the second.
- A red bracket labeled "Binary operation" encloses the expression $(index \& -index)$.
- A red bracket labeled "2's complement" encloses the expression $-index$.
- A red bracket labeled "Binary summation" encloses the entire addition operation $index + (index \& -index)$.

	5	7	4	2	8	9	6	5	10	12	-2	-8	1	6	0	-1	5
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
(1,1)	(1,2)	(3,3)	(4,4)	(5,5)	(5,6)	(7,7)	(1,8)	(9,9)	(9,10)	(11,11)	(11,12)	(13,13)	(13,14)	(15,15)	(1,16)	(17,17)	



	5	7	4	2	8	9	6	5	10	12	-2	-8	1	6	0	-1	5
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

46

~~68~~
~~56~~
~~46~~
~~58~~
~~58~~
~~59~~
~~65~~
~~64~~

	5	12	4	¹⁸ 16	8	17	6	⁴⁶ 41	10	²² 10	-2	¹² 25	²¹ 10	1	7	0	⁴¹ 38	5
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	

$(1,1)$ $(1,2)$ $(3,3)$ $(4,4)$ $(5,5)$ $(5,6)$ $(7,7)$ $(1,8)$ $(9,9)$ $(9,10)$ $(11,11)$ $\cancel{(11,12)}$ $(13,13)$ $(13,14)$ $(15,15)$ $(1,16)$ $(17,17)$
 $\underbrace{\hspace{1cm}}$ $\underbrace{\hspace{1cm}}$

	5	12	4	18	8	17	6	46	10	22	-2	12	1	7	0	64	5
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

	(1,1)	(1,2)	(3,3)	(1,4)	(5,5)	(5,6)	(7,7)	(1,8)	(9,9)	(3,10)	(1,11)	(5,12)	(13,13)	(13,14)	(15,15)	(1,16)	(17,17)
5	12	4	18	8	17	6	46	10	22	-2	12	1	7	0	64	5	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Sum

$$(1, 16) \rightarrow 64$$

$$(1, 16) \rightarrow 16 \rightarrow \begin{array}{r} 2^4 \\ \times \\ 1 \end{array} + \begin{array}{r} 2^3 \\ \times \\ 0 \end{array} + \begin{array}{r} 2^2 \\ \times \\ 0 \end{array} + \begin{array}{r} 2^1 \\ \times \\ 0 \end{array} + \begin{array}{r} 2^0 \\ \times \\ 0 \end{array}$$

$$(1, 12) \rightarrow 46 + 12 \rightarrow 58$$

$$(1, 6) \rightarrow 18 + 17 \rightarrow 35$$

$$(1, 5) \rightarrow 18 + 8 \rightarrow 26$$

$$(1, 17) \rightarrow 64 + 5 \rightarrow 69$$

$$2^4 = \boxed{16} \text{ index}$$

	(1,1)	(1,2)	(3,3)	(1,4)	(5,5)	(5,6)	(7,7)	(1,8)	(9,9)	(9,10)	(1,11)	(5,12)	(13,13)	(13,14)	(15,15)	(11,16)	(17,17)
↓	5	12	4	18	8	17	6	46	10	22	-2	12	1	7	0	64	5
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Sum left right
 $(1, 16) \rightarrow 64$ $(1, 16) \rightarrow 16 \rightarrow 2^4 = \boxed{16}$ index

$(1, 12) \rightarrow 46 + 12 \rightarrow 58 \rightarrow 12 \rightarrow 2^3 + 2^2$

$(1, 6) \rightarrow 18 + 17 \rightarrow 35$

$(1, 5) \rightarrow 18 + 8 \rightarrow 26$

$(1, 17) \rightarrow 64 + 5 \rightarrow 69$

1. \rightarrow 2's complement of index
 2. & with orig. Index
 3. — with original Index

(1,1)	(1,2)	(3,3)	(1,4)	(5,5)	(5,6)	(7,7)	(1,8)	(9,9)	(3,10)	(1,11)	(9,12)	(13,13)	(13,14)	(15,15)	(11,16)	(17,17)	
5	12	4	18	8	17	6	46	10	22	-2	12	1	7	0	64	5	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Sum

$$(6, 13) \rightarrow \text{sum}(13) - \text{sum}(6-1) = \underline{\underline{59 - 26 = 33}}$$

left right

$\rightarrow \text{sum}(\text{right}) - \text{sum}(\text{left}-1)$

int range-sum(left, right){

 return sum(right) - sum(left-1);

}

Time of Construction of fenwick tree : $O(n \log n)$

Time of Calculation of Sum/RangeSum : $O(\log n)$

Time of Updating a point value in fen_wav : $O(\log n)$