

# Gradient Boosting Maths:

Exp	Degree	Salary	$\hat{y}$
2	BE	50K	60
3	PHD	60K	60
4	MASTER	70K	60

residual

regression.  $Loss = \sum_{i=1}^n \frac{1}{2} (y - \hat{y})^2$

we need to find  $\hat{y}$  value such that, this summation should go down, loss function should go down. (very minimum)

consider above example,

$$= \frac{1}{2} (50 - \hat{y})^2 + \frac{1}{2} (60 - \hat{y})^2 + \frac{1}{2} (70 - \hat{y})^2$$

first order derivative.  $\frac{d}{dx} x^n = nx^{n-1}$

$$= \frac{1}{2} (50 - \hat{y})(-1) + \frac{1}{2} (60 - \hat{y})(-1) + \frac{1}{2} (70 - \hat{y})(-1)$$

$$= -50 + \hat{y} - 60 + \hat{y} - 70 + \hat{y}$$

$$= -180 + 3\hat{y}$$

$$3\hat{y} = 180$$

$$\therefore y = 60$$

$= \hat{y}$  for base.   
 constant value

$$Loss = \frac{1}{2} (y - \hat{y})^2 = \frac{1}{2} (y - \hat{y})(-1)$$

$$\frac{\partial L}{\partial \hat{y}} = -y + \hat{y} = -(y - \hat{y}) \cdot \frac{\partial L}{\partial \hat{y}} = (y - \hat{y})$$

Instead of gamma, we have  $F_{m-1}(x_i) + \gamma$

$F_{m-1}(x_i)$  = previous model output.

To find  $\gamma(\bar{y})$  to minimize the loss. what is this loss,

$$\sum_{i=1}^n \frac{1}{2} (y_i - (60 + \gamma(\bar{y})))^2$$

find derivative.

(0.1) = learning rate

$$60 + (10) = 60 - 1.0 = 59.0$$



## Pseudo algorithm

① Input  $\{x_i, y_i\}$    
 Independent, dependent feature

② Loss function:  $d(y, F(x))$    
 Different for regression, classifn.   
 regression: Root mean square error, mean square error.

all the loss function should be differentiable. (derivative of loss funtn)   
 classifn: log loss, Hinge loss

③ How many decision we require,

① Initialize Model with constant value.

~~$F_0(x)$  Gamma~~

$$F_0(x) = \underset{\gamma}{\operatorname{argmin}} \left( \sum_{i=1}^n L(y_i, \gamma) \right) = \gamma(\text{Gamma})$$

$\gamma = \hat{y}$    
 Gamma = predicted value

② Iterate  $M = 1$  to  $M$    
  $M$  = no. of trees.   
 from 1 to  $M$ , I will compute pseudo residual, (pseudo error)

$$\text{error} = - \frac{\partial L(y, F(x_i))}{\partial F(x_i)}$$

based on salary -  $\hat{y}$

③ Create OT, error = dependent, feature   
 Fit a Base Learner  $h_m(x)$  = Independent   
  $\{x_i, \text{error}\}$    
 error = error value.

$$\gamma_m = \underset{\gamma}{\operatorname{argmin}} \sum_{i=1}^n L(y_i, F_m(x_i) + \gamma)$$

Compare with initial model

④ Updating Model =  $F_m(x) = F_{m-1}(x) + L(h(x))$