UNIT IV

Random variable: Definition of a random variable, discrete and continuous random variables, functions of random variables, probability density function and mathematical expectation of a random variable and properties of expectation.

Probability Distributions: Binomial, Poisson and Normal Distribution.

4.1 Random Variable

4.1.1 Definition of a Random Variable

Q1. Define Random Variable.

Ans :

Meaning

A variable that can many real values which are determined by the outcomes of a random experiment on a real line $(-\infty, +\infty)$. It is a chance or Stochastic Variable.

It is a function defined on the sample space, "S" of a random experiment. A Random variable takes different values as a result of the outcomes of a random experiment. A Random variable can be Discrete or Continuous.

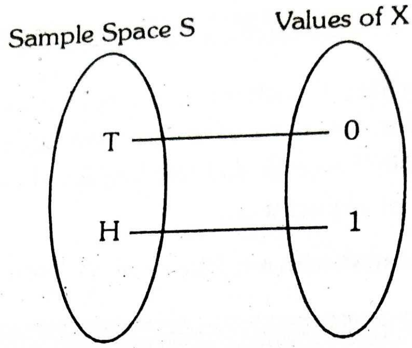
For example,

i) A random variable can also be used to describe the process of rolling a fair die and the possible outcomes.

The most obvious representation is to take the set {1, 2, 3, 4, 5, 6} as the sample space, defining the random variable X as the number rolled. In this case,

$$X = \begin{cases} 1, & \text{if a1 is rolled,} \\ 2, & \text{if a2 is rolled,} \\ 3, & \text{if a3 is rolled,} \\ 4, & \text{if a4 is rolled,} \\ 5, & \text{if a5 is rolled,} \\ 6, & \text{if a6 is rolled,} \end{cases} \qquad \rho x(x) = \begin{cases} \frac{1}{6}, & \text{if } x = 1,2,3,4,5,6 \\ 0, & \text{otherwise} \end{cases}$$

ii) Consider the experiment of tossing a single coin. Sample space S = {H, T}. Let X represent the number of heads. Thus X can assume the values 0 and 1. Let x be these values i.e. x = 0, 1. The relationship between the sample space and the values of X is shown in figure.



Association between S and x through X

Fig. : Outcomes on Tossing a Single Coin and Values of X

Example

Suppose the random experiment is the toss of a coin 3 times. The sample space consists of the 8 outcomes:

Ω = {(HHH) (HHT) (HTH) (HTT) (THH) (THT) (TTH) (TTT)}. Find the number of heads in these three tosses.

Sol :

Generally, rather than being interested in these outcomes, one is more concerned with the number of heads (or tails) in the 3 tosses. Thus one is interested in the number of heads being one, not whether the outcome was (HTT), (THT) or (TTH). That is, one is interested in the values of the random variable X = number of heads in the 3 tosses, where X can take on the values 0, i, 2, or 3; i.e., the set of outcomes of interest is the set {0, 1, 2, 3}.

A random variable is denoted by a capital letter (e.g., X, Y, W) and the values of the random variable by the corresponding lowercase letter (e.g., x, y, w). The set of all possible values of a random variable defines a new sample space. As before, an event is a subset of the sample space. For the random variable, the sample space is $\Omega_x = \{0, 1, 2, 3\}$. The event that the value of X is either 0 or 1 is the subset $E_1 = \{0, 1\}$. This is denoted by $X \in E_1$

4.1.2 Discrete and Continuous Random Variables

Q2. Explain different types of Random Variables.

Ans : **(Imp.)**

Random variables can be of two types as follows:

1. Discrete Random Variables

A discrete random variable is one which may take on only a countable number of distinct values such as 0, 1, 2, 3, 4, ... Discrete random variables are usually (but not necessarily) counts. If a random variable can take only a finite number of distinct values, then it must be discrete. Examples of discrete random variables include the number of children in a family, the Friday night attendance at a cinema, the number of patients in a doctor's surgery, the number of defective light bulbs in a box of ten.

The probability distribution of a discrete random variable is a list of probabilities associated with each of its possible values. It is also sometimes called the probability function or the probability mass function.

Suppose a random variable X may take k different values, with the probability that $X = x_i$ defined to be $P(X = x_i) = p_i$. The probabilities p_i must satisfy the following:

i) $0 \leq p_i \leq 1$ for each i,

ii) $p_1 + p_2 + ... + p_k = 1$.

Discrete random variables take on integer values, usually the result of counting.

Examples

i) Suppose that one flips a coin and count the number of heads. The number of heads results from a random process - flipping a coin. And the number of heads is represented by an integer value a number between 0 and plus infinity. Therefore, the number of heads is a discrete random variable.

ii) Suppose a variable X can take the values 1, 2, 3, or 4. The probabilities associated with each outcome are described by the following table:

Outcome	1	2	3	4
Probability	0.1	0.3	0.4	0.2

The probability that X is equal to 2 or 3 is the sum of the two probabilities: $P(X = 2 \text{ or } X = 3) = P(X = 2) + P(X = 3) = 0.3 + 0.4 = 0.7$. Similarly, the probability that X is greater than 1 is equal to $1 - P(X = 1) = 1 - 0.1 = 0.9$, by the complement rule.

2. **Continuous Random Variables**

A continuous random variable is one which takes an infinite number of possible values. Continuous random variables are usually measurements. For example height, weight, the amount of sugar in an orange, the time required to run a mile all can take infinite number of possible values.

A continuous random variable is not defined at specific values. Instead, it is defined over an interval of values, and is represented by the area under a curve (in advanced mathematics, this is known as an integral) The probability of observing any single value is equal to 0, since the number of values which may be assumed by the random variable is infinite.

Suppose a random variable X may take all values over an interval of real numbers. Then the probability that X is in the set of outcomes A, P (A) is defined to be the area above A and under a curve.

The curve, which represents a function p(x), must satisfy the following:

i) The curve has no negative values ($p(x) \geq 0$ for all x).

ii) The total area under the curve is equal to 1.

A curve meeting these requirements is known as a density curve.

4.2 Functions of Random Variables

4.2.1 Probability Mass Function and Probability Density Function

Q3. Define :

(a) Probability mass function

(b) Probability density function

Ans :

(a) Probability Mass Function

If X is a discrete random variable whose possible values are $x_1, x_2, \dots x_n$ with the corresponding probabilities as $P_1, P_2, \dots P_n$ then the probabilities of x_i is defined as,

$P_i = P(x_i) = P(X = x_i)$, where i = 1, 2 ... n.

This is called probability function or probability mass function (PMF) if it satisfies the conditions mentioned below,

(a) $P(x_i) \geq 0$ for all values. This means all values are non-negative.

(b) $\sum_{i=1}^{n} P(x_i) = 1$

This mean the total probability must always be unity.

(b) Probability Density Function

If X is a continuous random variable and F(x) is its continuous function then a non-negative function f(x) is known as a Probability Density Function of X if it satisfies the conditions mentioned below.

(a) $f(x) \geq 0$ for al real values.

(b) $\int_{-\infty}^{\infty} f(x)dx = 1$

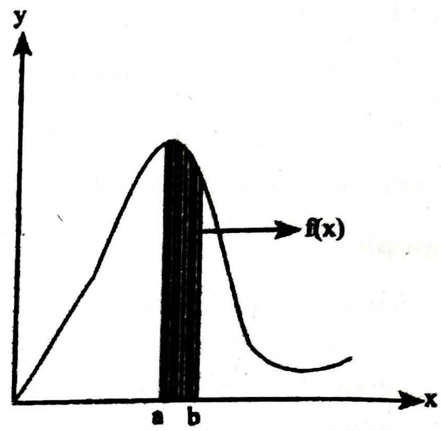
The function f(x) must be integrable on every interval of(a, b). This means the probability of an event in the interval (a, b) can be computed as,

$$P(a \leq X \leq b) = \int_{a}^{b} f(x)\,dx$$

$$= F(b) - F(a)$$

$\Rightarrow$ $P(a \leq X \leq b)$ = Area under f(x) which is enclosed with in a and b.

This can be diagramatically represented as follows,



4.3 Mathematical Expectation of a Random Variable and Properties of Expectation

Q4. Define Mathematical Expectation. What are the properties of a mathematical expectation?

Ans :

Definition

If x denotes a discrete random variable which can assume that values $x_1, x_2, \dots x_n$ with respective probabilities $p_1, p_2, \dots p_n$ then the mathematical expectation of x or simply expectation of x, denoted by E(x), is defined as

$$E(x) = x_1 p_1 + x_2 p_2 + \dots + x_n p_n$$

$$E(x) = \sum_{i=1}^{n} x_i p_i$$

where $p_i = 1$

Properties

(i) If 'a' is any constant then E(a) = a

Let x_i = a with probabilities p_i then

$$E(a) = \sum_{i=1}^{n} a\, p_i$$

$$E(a) = a\sum_{i=1}^{n} p_i$$

$$E(a) = a$$

$$\left(\because \sum_{i=1}^{n} p_i = 1\right)$$

(ii) **If x is a random variable and 'a' is constant then**

$$E(ax) = aE(x)$$

$$x = a x_i, \quad i = 1, 2, \dots n$$

$$ax = a x_i, \quad 1 = 1, 2, \dots n$$

$$E(ax) = \sum_{i=1}^{n} a x_i p_i$$

$$E(ax) = a\sum_{i=1}^{n} a x_i p_i$$

$$E(ax) = a E(x)$$

(iii) **If x is a random variable and 'a' and 'b' are constants**

Then

$$E(ax + b) = aE(x) + b$$

$$E(ax + b) = \sum_{i=1}^{n} (a x_i + b) p_i$$

$$E(ax + b) = \sum_{i=1}^{n} a x_i p_i + \sum_{i=1}^{n} b p_i$$

$$E(ax + b) = a\sum_{i=1}^{n} x_i p_i + b \sum_{i=1}^{n} p_i$$

$$E(ax + b) = aE(x) + b$$

$$\sum_{i=1}^{n} p_i = 1$$

Note :

If $a = 1$ and $b = -\bar{x}$ then

$$E(ax + b) = E(x - \bar{x}) = 0$$

$$E(x - \bar{x}) = E(x) - E(\bar{x})$$

$$E(x - \bar{x}) = EC(x) - \bar{x}$$

$$\because E(x) = \bar{x}$$

$$E(x - \bar{x}) = 0$$

Q5. Explain Raw and Central Moments using mathematical expectation.

Ans :

The r^{th} moment of a random variable X about any point say A is called raw moment. It is denoted by μ'_r. From the expection of function of random variable we know that,

$$E[g(X)] = \begin{cases} \Sigma g(x).p(x); & \text{when X is discrete r.v} \\ \int_{-\infty}^{\infty} g(x)\cdot f(x); & \text{when X is continuous r.v} \end{cases}$$

Now consider that $g(x) = (x - A)^r$, where A denotes an arbitary constant and r is a non-negative integer. Then the r^{th} moment about 'A' (or raw moment) can be defined as,

$$E[(X - A)^r] = \mu'_r = \begin{cases} \sum_x (x-A)^r p(x) & ; \text{ when X is discrete r.v} \\ \int_{-\infty}^{\infty} (x-A)^r \cdot f(x)dx & ; \text{ when X is continuous r.v} \end{cases}$$

Note

If $A = 0$, then we have moment about origin given by,

$$\therefore \quad E[X^r] = \mu'_r = \begin{cases} \sum_x x^r \cdot p(x) ; & \text{when X is discrete r.v} \\ \int_{-\infty}^{\infty} x^n \cdot f(x)dx ; & \text{when X is discrete r.v} \end{cases}$$

$$\text{Put } r = 1 \Rightarrow E[X] = \mu = \left\{ \begin{matrix} \Sigma X(p(X) \;; & \text{when X is discrete r.v} \\ \int_{-\infty}^{\infty} x \cdot f(x)dx \;; & \text{when X is continuous r.v} \end{matrix} \right\} = \text{mean}$$

Put $r = 2 \Rightarrow E[X^2] = \mu'_2$

and $m_2 = \mu'_2 - (\mu'_1)^2$ = variance

$\Rightarrow$ Variance = $E(X^2) - [E(X)]^2$

Central Moment

The r^{th} moment of a random variable X about mean $\bar{X}$ is called central moment. It is denoted by μ_r. From the expectation of function of r.v. we know that,

$$E[g(X)] = \begin{cases} \Sigma g(x)p(x) \\ \int_{-\infty}^{\infty} g(x)\cdot f(x) \end{cases}$$

Now, consider that $g(x) = (x - \bar{X})^r$ where $\bar{X}$ denotes mean of X and r is a non-negative integer.

Then, the r^{th} moment about mean $\bar{X}$ (or central moment) can be defined as,

$$E\left[\left(X-\bar{X}\right)^r\right] = \mu_r = \begin{cases} \sum_x \left(x-\bar{X}\right)^r \cdot p(x) & ; \text{ when X is discrete r.v} \\ \int_{-\infty}^{\infty} \left(x-\bar{X}\right)^r \cdot f(x)dx & ; \text{ when X is continuous r.v} \end{cases}$$

Note

Put $r = 2 \Rightarrow E\left(X-\bar{X}\right)^2 = \mu_2 \rightarrow$ Variance

$r = 3 \Rightarrow E\left(X-\bar{X}\right)^3 = \mu_3 \rightarrow$ Third central moment

$r = 4 \Rightarrow E\left(X-\bar{X}\right)^4 = \mu_4 \rightarrow$ Fourth central moment.

Q6. Define variance. Explain the properties of variance.

Ans :

Definition

If X is a random variable, then variance of X is denoted by V(X) and is defined as

$$\begin{aligned} V(x) &= E\,[X - E(X)]^2 \\ &= E\,[X^2 + (E(X))^2 - 2\,XE(X)] \\ &= E(X^2) + E\,[E(X)]^2 - 2\,E(X)\,E(X) \\ &= E(X^2) + (E(X))^2 - 2(E(X))^2 \\ &= E(X^2) - (E(X))^2 \end{aligned}$$

$$\therefore \quad V(X) = E[X - E(X)]^2 = E(X^2) - (E(X))^2$$

Properties

(i) If C is a constant, then V(C) = 0.

Proof :

$$\begin{aligned} V(C) &= E[C-E(C)]^2 = E[C-C]^2 \quad (\because E(C)=C) \\ &= E(0)^2 = 0 \end{aligned}$$

$$\therefore \quad V(C) = 0$$

(ii) If X is a random variable and 'a' is a constant, then V(aX) = a^2 V(X).

Proof :

By definition,

$$V(X) = E[X - E(X)]^2$$

$$\begin{aligned} V(aX) &= E[aX - E(aX)]^2 = E[aX - aE(X)]^2 \\ &= a^2 E\,[X - E(X)]^2 = a^2\,V(X) \end{aligned}$$

(iii) If X is a random variable and 'a' and 'b' are constants, then V(aX+b) = a^2V(X).

Proof :

By Definition,

$$V(X) = E\,[X - E(x)]^2$$

$$V(aX+b) = E[aX+b-E(aX+b)]^2$$
$$= E[aX+b-E(aX)-E(b)]^2$$
$$= E[aX+b-aE(X)-b]^2$$
$$= E[aX-aE(X)]^2$$
$$= a^2 E[X-E(X)]^2 = a^2V(X)$$

(iv) If X and Y are two random variables and 'a' and 'b' are constants, then $V(aX+bY) = a^2 V(X)+b^2 V(Y)+2ab\ Cov(X, Y)$.

Proof.

By definition, $V(X) = E[X-E(X)]^2$

$$V(aX+bY) = E[aX+bY-E(aX+bY)]^2$$
$$= E[aX+bY-E(aX)-E(bY)]^2$$
$$= E[aX+bY-aE(X)-b\,E(Y)]^2$$
$$= E[a\{X-E(X)\}+b\{Y-E(Y)\}]^2$$
$$= E[(a\{X-E(X)\})^2+(b\{Y-E(Y)\})^2+2ab(X-E(X)(Y-E(Y))]$$
$$= E[a^2(X-E(X))^2+b^2(Y-E(Y))^2+2ab(X-E(X))(Y-E(Y))]$$
$$= a^2 E[X-E(X)]^2+b^2 E[Y-E(Y)]^2+2ab\,E[(X-E(X))(Y-E(Y))]$$
$$= a^2V(X)+b^2V(Y)+2ab\,cov(X, Y)$$
$$\therefore \quad V(aX+bY) = a^2V(X)+b^2V(Y)+2ab\,cov(X, Y)$$

(v) If X and Y are two random variables and 'a' and 'b' are constants, then $V(aX-bY) = a^2 v(X)+b^2 v(Y)-2ab\,cov(X, Y)$

Proof

By Definition :

$$V(X) = E(X-E(X))^2$$
$$V(aX-bY) = E[aX-bY-E(aX-bY)]^2$$
$$= E[aX-bY-E(aX)+E(bY)]^2$$
$$= E[aX-bY-aE(X)+bE(Y)]^2$$
$$= E[aX-aE(X)-bY+bE(Y)]^2$$
$$= E[a(X-E(X))-b(Y-E(Y))]^2$$
$$= E[a^2(X-E(X)^2+b^2(Y-E(Y))^2-2ab(X-E(X))(Y-E(Y))]^2$$
$$= a^2E[X-E(X)]^2+b^2E[Y-E(Y)]^2-2ab\,E[X-E(X)][Y-E(Y)]$$
$$= a^2v(X)+b^2v(Y)-2ab\,cov(X, Y)$$
$$\therefore \quad v(aX-bY) = a^2v(X)+b^2v(Y)-2ab\,cov(X, Y)$$

Q7. Define covariance. Explain the properties of covariance.

Ans :

Definition

If X and Y are two random variables then covariance between them is denoted by Cov(X, Y) and is defined as

$$\text{Cov}(X, Y) = E[(X-E(X))(Y-E(Y))]$$
$$= E[XY - XE(Y) - Y(E(X)) + E(X)E(Y)]$$
$$= E(XY) - E(X)E(Y) - E(Y)E(X) + E(X)E(Y)$$
$$= E(XY) - E(X)E(Y)$$
$$\therefore \text{Cov}(X, Y) = E[(X-E(X))(Y-E(Y))]$$
$$= E(XY) - E(X)E(Y)$$

If X and Y are independent then

$$E(XY) = E(X)E(Y)$$
$$\therefore \text{Cov}(X, Y) = E(XY) - E(X)E(Y)$$
$$= E(X)E(Y) - E(X)E(Y) = 0$$

Properties

(i) **Cov(aX, bY) = ab Cov(X, Y), 'a' & 'b' are constants.**

Proof :

$$\text{Cov}(X, Y) = E[(X-E(X))(Y-E(Y))]$$
$$\text{Cov}(aX, bY) = E[(aX-E(aX))(bY-E(bY))]$$
$$= E[a(X-E(X))\, b(Y-E(Y))]$$
$$= ab\, E[(X-E(X))(Y-E(Y))]$$
$$= ab\, \text{Cov}(X, Y)$$

(ii) **Cov (X+a, Y+b) = Cov(X, Y; 'a' and 'b' are constants.**

Proof :

$$\text{Cov}(X+a, Y+b) = E[\{X+a-E(X+a)\}\{Y+b-E(Y+b)\}]$$
$$= E[(X+a-E(X)-E(a))(Y+b-E(y)-E(b))]$$
$$= E[(X+a-E(X)-a)(Y+b-E(Y)-b)]$$
$$= E[(X-E(X))(Y-E(Y))] = \text{Cov}(X, Y)$$

(iii) **Cov(aX+b, CY+d) = ac Cov(X, Y), a, b, c, d are constant.**

Proof :

$$\text{Cov}(aX+b, cY+d) = E[(aX+b-E(aX+b))(cY+d-E(cY+d))]$$
$$= E[(aX+b-E(aX)-E(b))(cY+d-E(cY)-E(d))]$$
$$= E[(aX+b-aE(X)-b)(cY+d-cE(Y)-d)]$$
$$= E[(aX-aE(X))(cY-cE(Y))]$$
$$= ac\, E[(X-E(X))(Y-E(Y))]$$
$$= ac\, \text{Cov}(X, Y)$$

(iv) **Cov(aX+bY, cX+dY) = ac V(X) + bd V(Y) + (ad+bc) Cov(X, Y) a, b, c, d are constants.**

Proof :

$$\text{Cov}(aX+bY, cX+dY)$$
$$= E[(aX+bY-E(aX+bY))(cX+dY-E(cX+dY))]$$
$$= E[(aX+bY-E(aX)-E(bY))(cX+dY-E(cX)-E(dY))]$$

$= E[(aX+bY-aE(X)-bE(Y))\ (cX+dY-cE(X)-dE(Y))]$
$= E[\{a(X-E(X))+b(Y-E(Y))\}\ \{c(X-E(X))+d(Y-E(Y))\}]$
$= E[ac(X-E(X))^2+ad(X-E(X))(Y-E(Y))\ +bc(Y-E(Y))(X-E(X))+bd(Y-E(Y))^2\ ]$
$= ac\ E[X-E(X)]^2+ad\ E[(X-E(X))(Y-E(Y))]+bc\ E[(Y-E(Y))(X-E(X))]+bd\ E[Y-E(Y)]^2$
$= ac\ V(X)+ad\ cov(X, Y)+bc\ Cov(X, Y)+\ bd\ V(Y)$
$= ac\ V(X)\ +\ bd\ V(Y)\ +\ (ad+bc)\ Cov(X, Y)$

(v) $\mathbf{Cov\left(\frac{X-\bar{X}}{\sigma_x}, \frac{Y-\bar{Y}}{\sigma_y}\right) = \frac{1}{\sigma_x\sigma_y}\ Cov(X, Y)}$

Proof :

$$Cov\left(\frac{X-\bar{X}}{\sigma_x}, \frac{Y-\bar{Y}}{\sigma_y}\right)$$

$$= E\left[\left\{\frac{X-\bar{X}}{\sigma_X} - E\left(\frac{X-\bar{X}}{\sigma_X}\right)\right\}\left\{\frac{Y-\bar{Y}}{\sigma_Y} - E\left(\frac{Y-\bar{Y}}{\sigma_Y}\right)\right\}\right]$$

$$= E\left[\left\{\frac{X-\bar{X}}{\sigma_X} - \frac{1}{\sigma_X}E(X-\bar{X})\right\}\left\{\frac{Y-\bar{Y}}{\sigma_Y} - \frac{1}{\sigma_Y}E(Y-\bar{Y})\right\}\right]$$

$$= E\left[\left\{\frac{X-\bar{X}}{\sigma_X} - \frac{1}{\sigma_X}(E(X)-E(\bar{X}))\right\}\left\{\frac{Y-\bar{Y}}{\sigma_Y} - \frac{1}{\sigma_Y}(E(Y)-E(\bar{Y}))\right\}\right]$$

$$= E\left[\left(\frac{X-\bar{X}}{\sigma_X} - \frac{1}{\sigma_X}(\bar{X}-\bar{X})\right)\left(\frac{Y-\bar{Y}}{\sigma_Y} - \frac{1}{\sigma_Y}(\bar{Y}-\bar{Y})\right)\right]$$

$$= E\left[\left(\frac{X-\bar{X}}{\sigma_X}\right)\left(\frac{Y-\bar{Y}}{\sigma_Y}\right)\right]$$

$$= \frac{1}{\sigma_X\sigma_Y}\ E\left[(X-E(X))(Y-E(Y))\right]$$

$$= \frac{1}{\sigma_X\sigma_Y}\ Cov(X, Y) \quad (\because\ E(X) = \bar{X}\ \ E(Y) = \bar{Y})$$

PROBLEMS

1. Find expected number of heads in tossing two coins.

Sol :

If two coins are tossed, then

S = {HH, HT, TH, TT}

Let X = number of heads

Outcomes : HH HT TH TT

X :	2	1	1	0
P(x) :	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$\therefore$ $E(X) = \Sigma x\, P(x) = 2 \times \frac{1}{4} + 1 \times \frac{1}{4} + 1 \times \frac{1}{4} + 0 \times \frac{1}{4} = \frac{2}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$

2. If the probability distribution of X is

x :	– 3	6	9
P(x) :	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find E(X), E(X²), V(X) E(2X+1)²

Sol :

x :	– 3	6	9
P(x) :	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

$$E(X) = \Sigma x\, P(x)$$

$$= (-3)\left(\frac{1}{6}\right) + (6)\left(\frac{1}{2}\right) + (9)\left(\frac{1}{3}\right)$$

$$= -\frac{1}{2} + 3 + 3 = \frac{-1+12}{2} = \frac{11}{2}$$

$$E(X^2) = \Sigma x^2\, P(x) = (-3)^2\left(\frac{1}{6}\right) + (6)^2\left(\frac{1}{2}\right) + (9)^2\left(\frac{1}{3}\right)$$

$$= \frac{9}{6} + \frac{36}{2} + \frac{81}{3} = \frac{9+108+162}{6} = \frac{279}{6} = \frac{93}{2}$$

$$\therefore V(X) = E(X^2) - (E(X))^2 = \frac{93}{2} - \left(\frac{11}{2}\right)^2$$

$$= \frac{186-121}{4} = \frac{65}{4}$$

$$E(2X+1)^2 = E\,[4X^2+4X+1] = 4E(X^2) + 4E(X) + E(1)$$

$$= 4\left(\frac{93}{2}\right) + 4\left(\frac{11}{2}\right) + 1 = 186 + 22 + 1 = 209$$

3. If two dice are thrown, find the expected value of sum of the points on them.

Sol :

If two dice are thrown, then total number of points = 36

Let X = sum of the points on them.

Then X takes values : 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

$\therefore$ $P(X = 2) = (1, 1) = 1/36$

$P(X = 3) = (1, 2), (2, 1) = 2/36$

$P(X = 4) = (1, 3), (2, 2) (3, 1) = 3/36$

$P(X = 5) = (1, 4)(4, 1)(2, 3)(3, 2) = 4/36$

$$P(X = 6) = (1, 5)(5, 1)(2, 4)(4, 2)(3, 3) = \frac{5}{36}$$

$$P(X = 7) = (1, 6)(6, 1)(4, 3)(3, 4)(2, 5)(5, 2) = \frac{6}{36}$$

$$P(X = 8) = (2, 6)(6, 2)(4, 4)(5, 3)(3, 5) = \frac{5}{36}$$

$$P(X = 9) = (3, 6)(6, 3)(5, 4)(4, 5) = \frac{4}{36}$$

$$P(X = 10) = (5, 5)(6, 4)(4, 6) = \frac{3}{36}$$

$$P(X = 11) = (5, 6)(6, 5) = \frac{2}{36}$$

$$P(X = 12) = (6, 6) = \frac{1}{36}$$

$\therefore$

x :	2	3	4	5	6	7	8	9	10	11	12
P(x) :	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$\therefore \quad E(X) = \Sigma xP(x) = (2)\left(\frac{1}{36}\right)+(3)\left(\frac{2}{36}\right)+(4)\left(\frac{3}{36}\right)+(5)\left(\frac{4}{36}\right)+(6)\left(\frac{5}{36}\right)+7\left(\frac{6}{36}\right)+8\left(\frac{5}{36}\right)$$
$$+\,9\left(\frac{4}{36}\right)+10\left(\frac{3}{36}\right)+11\left(\frac{2}{36}\right)+12\left(\frac{1}{36}\right)$$
$$= \frac{2+6+12+20+30+42+40+36+30+22+12}{36} = \frac{252}{36} = 7$$

4. Let X be a random variable with the p.m.f.

x :	0	1	2	3
P(x) :	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{24}$	$\frac{1}{8}$

Find $E(X-1)^2$

Sol :

x :	0	1	2	3
P(x) :	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{24}$	$\frac{1}{8}$

$$E(X) = \sum_x xP(x) = 0\times\frac{1}{3}+1\times\frac{1}{2}+2\times\frac{1}{24}+3\times\frac{1}{8}$$

$$= 0+\frac{1}{2}+\frac{1}{2}+\frac{3}{8} = \frac{12+2+9}{24} = \frac{23}{24}$$

$$E(X^2) = \Sigma x^2 P(x) = (0)^2\left(\frac{1}{3}\right)+(1)^2\left(\frac{1}{2}\right)+(2)^2\left(\frac{1}{24}\right)+(3)^2\left(\frac{1}{8}\right)$$

$$= 0+\frac{1}{2}+\frac{4}{24}+\frac{9}{8} = \frac{12+4+27}{24} = \frac{43}{24}$$

$$\therefore\ E(X-1)^2 = E[X^2-2X+1] = E(X^2)-2E(X)+E(1)$$

$$= \frac{43}{24}-2\left(\frac{23}{24}\right)+1 = \frac{43-46+24}{24}$$

$$= \frac{21}{24} = \frac{7}{8}$$

5. A coin is tossed untial a head appears. What is the expectation of the number of tosses required.

Sol : **(Imp.)**

Let X = number of tosses required to get the first head.

Event	No. of Tosses X	P(x)
H	1	$\frac{1}{2}$
TH	2	$\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)=\frac{1}{4}$
TTH	3	$\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)=\frac{1}{8}$
TTTH	4	$\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)\cdot\left(\frac{1}{2}\right)=\frac{1}{16}$

$$E(X) = \sum_{x=1}^{\infty} xP(x) = (1)\frac{1}{2}+(2)\left(\frac{1}{4}\right)+(3)\left(\frac{1}{8}\right)+(4)\left(\frac{1}{16}\right)+....$$

$$\frac{E(X)}{2} = (1)\left(\frac{1}{4}\right)+(2)\left(\frac{1}{8}\right)+(3)\left(\frac{1}{16}\right)+(4)\left(\frac{1}{32}\right)+....$$

Consider

$$E(X)-\frac{E(X)}{2} = \left[(1)\left(\frac{1}{2}\right)+(2)\left(\frac{1}{4}\right)+(3)\left(\frac{1}{8}\right)+(4)\left(\frac{1}{16}\right)+...\right]$$

$$-\left[(1)\frac{1}{4}+(2)\frac{1}{8}+(3)\frac{1}{16}+(4)\left(\frac{1}{32}\right)+\ldots\right]$$

$$= 1\times\frac{1}{2}+1\times\frac{1}{4}+1\times\frac{1}{8}+1\times\frac{1}{16}+\ldots$$

$$\frac{E(X)}{2} = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots.$$

This is in geometric progression with $a = 1/2$ and $r = 1/2$

$$\therefore \quad \boxed{S_\infty = \frac{a}{1-r}}$$

$$\therefore \quad \frac{E(X)}{2} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$$\therefore \quad E(X) = 2$$

4.4 Probability Distributions

Q8. Define the term Probability Distribution Function.

Ans :

Probability distribution is a set of probabilities of all the possible outcomes of a random experiment.

Probability distribution is similar to frequency distribution. Probability distribution is based on the theoretical considerations, subjective assessment or on experience.

For example, 'X' is a random variable which can take the values x_1, x_2, x_3......

The probabilities associated with each of the possible values of 'X', $P(X = x_1) = P_i(i = 1, 2, 3, \ldots)$

Therefore, the collection of pairs (x_1, P_i), where $i = 1, 2, 3, \ldots$ is called probability distribution of random variable X.

The values of probability distribution are usually tabulated as given below.

$X = x_i$	$P(X = x_i)$
x_1	P_1
x_2	P_2
x_3	P_3
–	–
–	–
–	–
–	–
x_n	P_n

Probability distribution functions are usually represented as f(x), g(x), h(x) etc. Here f(x) is the function where $f(x) = P(X = x)$ which assigns probability to each possible outcome 'x', hence called as 'probability distribution'.

Q9. What are the different types of probability distribution function.

Ans :

There are two types of probability distribution function :

1. Discrete Probability Distribution Function
2. Continuous Probability Distribution Function.

1. **Discrete Probability Distribution Function:** The Discrete Probability Distributions are :
   i) Hyper – geometric distribution
   ii) Binomial distribution
   iii) Poisson distribution
   iv) Geometric distribution

2. **Continuous Probability Distribution Function :** The Continuous Probability distributions are :
   i) Uniform distribution
   ii) Normal distribution
   iii) Exponential distribution.
   iv) Beta distribution

4.4.1 Binomial Distribution

Q10. What is Binomial Distribution? State the assumptions of Binomial Distribution.

Ans : **(Imp.)**

Meaning

In probability theory and statistics, the binomial distribution is the discrete probability distribution of the number of successes in a sequence of n independent yes/no experiments, each of which yields success with probability p. Such a success/failure experiment is also called a Bernoulli experiment or Bernoulli trial.

In fact, when n = 1, the binomial distribution is a Bernoulli distribution. The binomial distribution is the basis for the popular binomial test of statistical significance.

This binomial distribution is also known as the Bernoulli distribution by the name of the Swiss Mathematician Jacob Bernoulli ho has derived it.

The binomial probability refers to the probability that a binomial experiment results in exactly x successes.

Definition

If an event E has probability p of occurring in each of n independent trials and that of failure in any trial is q (=1– p) then the probability that it will occur exactly r times in n trails is given by:

$$p(r) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$$

This probability distribution is called the binomial probability distribution. The binominal distribution is a discrete distribution with parameters n and p. If p, q are equal it is symmetrical, otherwise it is nonsymmetrical.

Where

p = probability of success in a single trial.

q = probability of failure a single trial.

$p+q = 1$:

n = Number of trails

r = Number of success in 'n' times.

Assumptions

1. Each trial has two mutually exclusive possible outcomes, i.e., success or failure.
2. Each trial is independent of other trials.
3. The probability of a success (say p) remains constant from trial to trial.
4. The probability of getting a head in a toss of coins is ½. This result must remain same in successive tosses.
5. The number of trials is fixed.

Q11. State the Properties of Binomial Distribution.

Ans :

The properties of binomial distribution are as follows,

1. It describes the distribution of probabilities when there are only two mutually exclusive outcomes for each trial of an experiment for example while tossing a coin, the two possible outcomes are head and tail.
2. The process is performed under identical conditions for 'n' number of times.
3. Each trial is independent of other trials.
4. The probability of success 'p' remains same for trial to trial throughout the experiment and similarly, the probability of failure ($q = 1 - p$) also remains constant overall the observations.
5. Binomial distribution is symmetrical when $p = 0.5$ [figure (i)] and it is skewed if $p \neq 0.5$, where 'n' can be any value.

   When $p > 0.5$ [figure (iii)], it is skewed to the right → negatively skewed.

   When $p < 0.5$ (ii),] it is skewed to the left g positively → skewed.

Hence, binomial distribution is 'Asymmetrical'

When $p > 0.5$ and $p < 0.5$

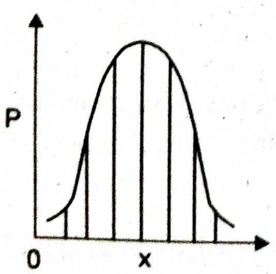


Figure (i)

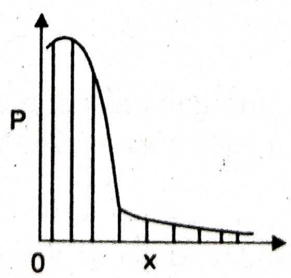


Figure (ii)

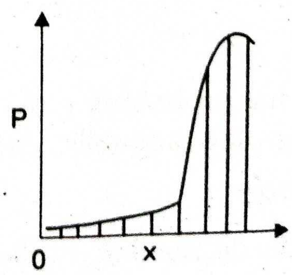


Figure (iii)

6. If 'n' is large and if neither 'p' nor 'q' is nearly zero, in such cases the binomial distribution is modified to normal distribution by standardizing the variable $\boxed{Z = \frac{X-np}{\sqrt{npq}}}$

Q12. State the Applications of Binomial Distribution.

Ans :

Binomial distribution is applicable in case of repeated trials such as,

1. Number of applications received for a junior assistant post during a period a particular period of time.
2. Number of births taking place in a hospital.
3. Number of candidates appearing for the screening test conducted by a company.

All the trials are statistical independent and each trials has two outcomes namely, success and failure.

PROBLEMS

6. **Fit a Binomial distribution to the following data.**

X	0	1	2	3	4
F(x)	122	60	15	2	1

Sol :

X	f	f(x)
0	122	0
1	60	60
2	15	30
3	2	6
4	1	4
	200	100

$$\text{Mean} = \frac{\varepsilon fX}{N} = \frac{100}{200} = 0.5$$

Mean = 0.5

np = 0.5

$$p = \frac{0.5}{4} = 0.13$$

p = 0.13

q = 1 – 0.13 = 0.87.

$p(r) = {}^nc_r\, p^r\, q^{n-r}$

n = 4 p = 0.13 r = 0, 1, 2, 3, 4 q = 0.87

$p(0) = 4c_0 \cdot (0.13)^0 (0.87)^{4-0}$

$= 1 \times 1 \times 0.5729 = 0.57.$

$p(1) = 4c_1 \cdot (0.13)^1 (0.87)^{4-1}$

$= 4 \times 0.13 \times 0.66 = 0.34.$

$p(2) = 4c_2 \cdot (0.13)^2 (0.87)^{4-2}$

$= 6 \times 0.017 \times 0.76 = 0.0775.$

$p(3) = 4c_3 \cdot (0.13)^3 (0.87)^{4-3}$

$= 4 \times 0.002 \times 0.87 = 0.0067$

$p(4) = 4c_4 \cdot (0.13)^4 (0.87)^0$

$= 1 \times 0.0003 \times 1 = 0.0003$

Fitting of Binomial Distribution

r	$p(r) = {}^nc_r\, p^r q^{n-r}$	$f(f) = N.(p(r))$
0	$p(0) = 4c_0\,(0.13)^0\,(0.87)^4 = 0.5729$	$200 \times 0.5729 = 114.58$
1	$p(1) = 4c_1\,(0.13)^1\,(0.87)^3 = 0.3432$	$200 \times 0.3432 = 68.64$
2	$p(2) = 4c_2\,(0.13)^2\,(0.87)^2 = 0.0775$	$200 \times 0.0775 = 15.5$
3	$p(3) = 4c_3\,(0.13)^3\,(0.87)^1 = 0.0067$	$200 \times 0.0067 = 1.34$
4	$p(4) = 4c_4\,(0.13)^4\,(0.87)^0 = 0.0003$	$200 \times 0.0003 = 0.06$
		200

7. Output of a production process is known to be 30% defective. What is the Probability that a sample of a 5 items would contain 0, 1, 2, 3, 4 and 5 defectives?

Sol :

If the appearance of a defective item is considered a success, then the probability of success in a trial, $p = 0.3$ thus $q = 1 - p = 1 - 0.3 = 0.7$. With $n = 5$, and $P(X = r) = {}^nc_r.\, q^{n-r}\, p^r$ we have

Number of successes (r)	Probability P(X = r)
0 ${}^5c_0(0.7)^5(0.3)^0$	0.1681
1 ${}^5c_1(0.7)^4(0.3)^1$	0.3602
2 ${}^5c_2(0.7)^3(0.3)^2$	0.3087
3 ${}^5c_3(0.7)^2(0.3)^3$	0.1323
4 ${}^5c_4(0.7)^1(0.3)^4$	0.0283
5 ${}^5c_5(0.7)^0(0.3)^5$	0.0024
Total	**1.0000**

Binomial Distribution has two parameters: n and p. It means that a Binomial Distribution can be specified completely by its n and p.

The mean of a binomial distribution is np and its Standard Deviation equals $\sqrt{npq}$.

8. **In an office, there are 150 employees. The pattern of their absence for duty in a particular month is recorded in the following table. Fit a Binomial Distribution to the data.**

Number of days absent	X	0	1	2	3	4
Number of Absentees	f(x)	28	62	46	10	4

Sol : **(Imp.)**

In the usual notations we have : n = 4; N = fx = 150 ... (1)

If p is the popular of the binomial distribution,

then np = Mean of the distribution = $\overline{X}$ ... (2)

Now $\bar{x} = \{\Sigma fx\}/\{\Sigma f\} = (0+62+92+30+16)/150$

$= 200 / 150 = 4/3$

Substituting in (2) we get

$4 \cdot p = 4/3$, or $p = 1/3$ and $q = 1 - p = 2/3$

The expected binomial probabilities are given by :

$$P(X = r) = {}^{n}c_{r}\, p^{r}\, q^{n-r}$$

$$= {}^{4}c_{r}\, (1/3)^{r}\, (2/3)^{4-r} \quad ...(3)$$

putting x = 0, 1, 2, 3, and 4 in (3) we get the expected binomial probabilities as given in the following table.

Fitting of Binomial Distribution

X	P(x)	Frequency f(x) = N.P(x)
0	${}^{4}c_{0}\,(1/3)^{0}\,(2/3)^{4} = 16/81 = 0.1975$	29.63 = 30
1	${}^{4}c_{1}\,(1/3)\,(2/3)^{2} = 4(8/81) = 0.3951$	59.26 = 59
2	${}^{4}c_{2}\,(1/3)^{2}\,(2/3)^{2} = 4(3/21)(4/81) = 0.2963$	44.44 = 44
3	${}^{4}c_{3}\,(1/3)^{2}\,(2/3) = 4(2/81) = 0.0988$	14.81 = 15
4	${}^{4}c_{4}\,(1/3)^{4} = 1/81 = 0.0123$	1.85 = 2

Hence the fitted Binomial Distribution is :

X	0	1	2	3	4	Total
f(x)	30	59	44	15	2	150

4.4.2 Poisson Distribution

Q13. Define Poisson distribution. State the assumptions of Poisson distribution. **(Imp.)**

Ans :

Meaning

In probability theory and statistics, the Poisson distribution is a discrete probability distribution that expresses probability of a number of events occurring in a fixed period of time if these events occur with a known average rate and independently of the time since the last event. The Poisson distribution can also be used for the number of events in other specified intervals such as distance, area or volume.

Examples

1. The number of telephone calls received at a particular switch board per minute during a certain hour of the day.
2. The number of deaths per day in a district or town in a one year by disease (but not epidemic).
3. The number of cars passing a certain point per minute.
4. The number of persons bom deaf and dump per year in a city.
5. The number of typographical errors per page.
6. The number of printing errors per page
7. The number of defective blades in a pack of 100.

Mathematical Definition

It is defined by the probability function.

for x (no. of successeT) = 0, 1, 2, 3,

where m is fixed positive number.

e = 2.71828(app.)

Assumptions

1) The events must be statistically independent. This means that the events must not occur in clusters. For example, if a pupil is absent from school on Monday this has no effect on the probability that more absentees will occur in the same week. Or if a spelling error is found on one page, it does not mean there is a probability of finding other errors on the same page.
2) An event has a minimal probability of occurrence. The event, such as teacher transfer, has many opportunities to occur, but the probability that the event will occur at any opportunity is extremely small.
3) The probability of an event is proportional to the size of the area of probability. For example, the probability that a teacher will be transferred from a school is greater over a 10-year period than over a one - term period.

Q14. What are the Properties of Poisson distribution?

Ans :

Properties of Poisson Distribution are discussed in the following section.

i) Poisson Distribution is a Discrete Probability Distribution, since the random variable X can take only values 0. 1, 2, ... ∞.

ii) By putting r = 0,1,2,3,....., in (1), we obtain the probabilities of 0,1,2,3,.... successes respectively,

iii) Total probability is 1.

$$\Sigma P(r) = e^{-m} + me^{-m} + (m^2/2!)\, e^{-m} + (m^3/3!)\, e^{-m} + ...$$

$$= e^{-m}\,[1 + m + m^2/2! + m^3/3! + ...]$$

$$= e^{-m} \times e^{m} = e^{-m} \times m$$

$$= e^0 = 1$$

$$E(X) \doteq \text{Mean} = \sum_{r=0}^{\infty} rP(r) = \sum_{r=0}^{\infty} r(e^{-m})\,(m^r/r!)$$

$$= me^{-m} \times e^{m} = m$$

$$E(x^2) - [E(x)]^2 = \text{Variance} = \sum r^2 P(r) - \left[\sum rP(r)\right]^2$$

$$= \sum r^2 P(r) - (\text{mean})^2$$

$$= [me^{-m}][e^{m}(1+m)] - m^2$$

Note : One of the special properties associated with Poisson Distribution is

Mean = Variance = m

iv) If we know m, all the Probabilities of the Poisson Distribution can be obtained. Therefore, m is called as the parameter of the Poisson Distribution.

Q15. What are the Applications of Poisson distribution?

Ans :

Some practical situations where Poisson Distribution can be used.

i) Number of telephone calls arriving at a telephone switch board is a unit time (say, per minute)

ii) Number of customers arriving at the super market, say, per hour.

iii) The number of defects per unit of manufactured product (This is done for the construction of control chart for c in Statistical Quality Control)

iv) To count the number of radio-active element per unit of time (Physics)

v) The number of bacteria growing per unit time (Biology)

vi) The number of defective material say, pins, blades etc. in a package manufactured by a good concern.

vii) The number of suicides reported in a particular day.

viii) The number of casualties (persons dying) due to rare disease such as heart attack or cancer or snake bite in a year.

ix) Number of accidents taking place per day on a busy road.

x) Number of typographical errors per page in a typed material or the number of printing mistakes per page in a book.

Q16. What is meant by theoretical frequency distribution?

Ans :

It is defined as the distributions that are drawn from the expectations based on past (or) theoretical consideration. It is also known as expected frequency distribution (or) frequency distribution.

PROBLEMS

9. **It is known from past experience that in a certain industrial plant there are on the average 4 industrial accidents per month. Find the probability that in a given year there will be less than 4 accidents. Assume Poisson distribution ($e^{-4} = 0.0183$)**

Sol :

In the usual notations we are given m = 4. If the random variable X denotes the number of accidents in the industrial plant per month, then by Poisson Probability law,

$$P(X = r) = e^{-m}\, m^r / r! = e^{-4}\, 4^r / r! \quad \text{... (1)}$$

The required probability that there will be less than 4 accidents is given by

$$P(X < 4) = [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)]$$

$$P(X < 4) = e^{-4}\,[1 + 4 + 4^2/2! + 4^3/3!]$$

$$= e^{-4}\,[1 + 4 + 8 + 10.67]$$

$$P(X < 4) = e^{-4}\,[23.67] = [0.0183]\,[23.67]$$

$$= 0.4332.$$

10. **If 5% of the electric bulbs manufactured by a company are defective, using Poisson distribution find the probability than in a sample of 100 bulbs :**

(i) None is defective,

(ii) 5 bulbs are defective (Given : $e^{-5} = 0.07$).

Sol :

Here we are given n = 100,

P = Probability of a defective bulb = 5% = 0.05.

Since P is small and n is large we may approximate the given distribution by Poisson distribution. Hence the parameter m of the Poisson distribution is :

$$m = np = 100 \times 0.05 = 5$$

Let the random variable X denote the number of defective bulbs in a sample of 100. Then (by Poisson law)

$$P(X = r) = e^{-m}\, m^r / r! = e^{-5}\, 5^r / r!\ ;\ r = 0, 1, 2, \ldots\ldots \infty \quad \ldots(1)$$

i) The probability that none of the bulbs is defective is given by :

$$P(X = 0) = e^{-5} = 0.07 \text{ [from 1]}$$

ii) The probability of 5 defective bulbs is given by :

$$P(X = 5) = e^{-5} \times 5^5 / 5! = 0.07(3125/120)$$

$$= 0.007\,(625/24) = 4.375/24 = 0.1823.$$

11. In a Research Methodology Book, the following frequency mistakes per page were observed. Fit a Poisson distribution.

No. of Mistakes	0	1	2	3	4	5
No. of Pages	620	180	80	60	40	80

(Imp.)

Sol :

Steps in the fitting of Poisson distribution are as follows.

Step-1

Calculate the value of λ and probability of zero occurrence.

x	f	fx
0	620	0
1	180	180
2	80	160
3	60	180
4	40	160
5	80	400
	Σf = 1060	**Σfx = 1080**

Mean of Poisson distribution is given by 'λ'

$$\lambda = \frac{\Sigma fx}{\Sigma f} = \frac{1080}{1060} = 1.0188$$

$$\boxed{\therefore \lambda = 1.0188}$$

Probability in Poisson distribution is given by,

$$\boxed{P(X) = \frac{e^{-\lambda}.\lambda^{x}}{x!}}$$

$$\lambda = 1.0188$$

$$e^{-\lambda} = e^{-1.0188} = 0.3610$$

∴ Probability of zero occurrence,

$$P(0) = \frac{e^{-1.0188}.\lambda^{0}}{0!} = e^{-1.0188} = 0.3610$$

$$\boxed{\therefore P(0) = 0.3610}$$

Step-2

Calculate all the probabilities by using recurrence relation.

$$P(0) = 0.3610$$

$$P(1) = \frac{P(0).\lambda}{1} = 0.3610 \times 1.0188 = 0.3678$$

$$P(2) = \frac{P(1).\lambda}{2} = \frac{0.3678 \times 1.0188}{2} = 0.1874$$

$$P(3) = \frac{P(2).\lambda}{3} = \frac{0.1874 \times 1.0188}{3} = 0.0636$$

$$P(4) = \frac{P(3).\lambda}{4} = \frac{0.0636 \times 1.0188}{4} = 0.0162$$

$$P(5) = \frac{P(4).\lambda}{5} = \frac{0.0162 \times 1.0188}{5} = 0.0033$$

Step-3

Multiply each term of all probabilities with total frequency (Σf or N) to obtain expected frequencies.

x	**P(x)**	**f(x) = N P (x) = 1060 P(x)**
0	P(0) = 0.3610	f(0) = 1060 × 0.3610 = 382.66 ≅ 383
1	P(1) = 0.3678	f(1) = 1060 × 0.3678 = 389.868 ≅ 390
2	P(2) = 0.1874	f(2) = 1060 × 0.1874 = 198.644 ≅ 199
3	P(3) = 0.0636	f(3) = 1060 × 0.0636 = 67.416 ≅ 67
4	P(4) = 0.0162	f(4) = 1060 × 0.0162 = 17.172 ≅ 17
5	P(5) = 0.0033	f(5) = 1060 × 0.0033 = 3.50 ≅ 4
	ΣP(x) = 1	**Σf(x) = 1060**

Thus, the theoretical fitted poisson distribution is as follows,

Number of Mistakes Per Page	0	1	2	3	4	5
Number of pages	383	390	199	67	17	4

12. A book containing 1000 pages has 0, 1, 2, 3 or 4 misprints per page as shown below :

Number of misprints	**0**	**1**	**2**	**3**	**4**
Number of pages	**500**	**340**	**120**	**30**	**10**

Fit Poisson distribution to the data and compare the theoretical frequencies into those given in the question

Sol :

Steps in the Fitting of Poisson Distribution

Step 1

Calculate the variation of 'λ' and probability of zero occurrence,

x	f	fx
0	500	0
1	340	340
2	120	240
3	30	90
4	10	40
Total	**Σf = 1000**	**Σfx = 710**

Mean, $$\lambda = \frac{\Sigma fx}{\Sigma f} = \frac{710}{1000} = 0.71$$

$$\boxed{\therefore\ \lambda = 0.71}$$

Probability of 'X' is Poisson distribution is given by,

$$\boxed{P(X) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}}$$

∴ Probability of zero occurrence is calculated as follows.

$$P(0) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= e^{-\lambda} \quad [\because\ \lambda^0 = 1 \text{ and } 0! = 1\ ]$$

$$= e^{-0.71}$$

$$= 0.492$$

$$\boxed{\therefore\ P(0) = 0.492}$$

Step 2

Calculate each term of probability by using recurrence relation

$$P(0) = 0.492 \qquad [\text{ From step 1 }]$$

$$\therefore\ P(1) = \frac{P(0) \times \lambda}{1} = 0.492 \times 0.71$$

$$\therefore \quad P(2) = \frac{P(1) \times \lambda}{2}$$

$$= \frac{0.349 \times 0.71}{2}$$

$$\therefore \quad P(2) = 0.123895 \cong 0.124$$

$$P(3) = \frac{P(2) \times \lambda}{3} = \frac{0.124 \times 0.71}{3}$$

$$\therefore \quad P(3) = 0.0293$$

$$P(4) = \frac{P(3) \times \lambda}{4} = \frac{0.0293 \times 0.71}{4} = 0.0052.$$

Step 3

Multiply each term of probability with total frequency (Σf) to obtain the expected frequencies is shown in table.

X	P(X)	f(x) = N.P(X) = 1000.P(X)
0	p(0) = 0.492	f(0) = 0.492 × 1000 = 492
1	p(1) = 0.349	f(1) = 0.349 × 1000 = 349
2	p(2) = 0.124	f(2) = 0.124 × 1000 = 124
3	P(3) = 0.0293	f(3) = 0.0293 × 1000 = 29.3
4	P(4) = 0.0052	f(4) = 0.0052 × 1000 = 5.2

Table: Computation of Expected Frequencies

The comparison of theoretical frequencies (that are fitted by Poisson distribution) with given frequencies is given below.

Number of misprints	0	1	2	3	4
Number of page (given)	500	340	120	30	10
Theoretical frequencies	492	349	124	29.3	5.2

4.4.3 Normal Distribution

Q17. Describe briefly about Normal Distribution.

(OR)

What is normal distribution.

(OR)

Explain normal probability distribution.

(Imp.)

Ans :

Introduction

The Normal Distribution was discovered by De Moivre as the limiting case of Binomial model in 1733. It was also known to Laplace no later than 1774, but through a historical error it has been credited to Gauss who first made reference to it in 1809. Throughout the 18th and 19th centuries, various efforts were made to establish the Normal model as the underlying law ruling all continuous random variables – thus the name Normal. The Normal model has, nevertheless, become the most important probability model is statistical analysis.

The normal Distribution in approximation to Binomial Distribution, whether or not p is equal to q, the Binomial Distribution tends to the form of the continuous cure when n becomes large at least for the material part of the range. As a matter of fact, the correspondence between Binomial and the Normal curve is surprisingly close even for low values of n provide dp and q are fairly near to equality. The limiting frequency curve, obtained as n, becomes large and is called the Normal frequency curve or simply the Normal curve.

Probability Density Function

A random variable X is said to have a Normal Distribution with parameters m (mean) and s^2 (Variance), if the density function is given by :

$$y = f(x) = \frac{1}{\sigma\sqrt{2x}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where

X = Value of the continous random variable

μ = Mean of random variable

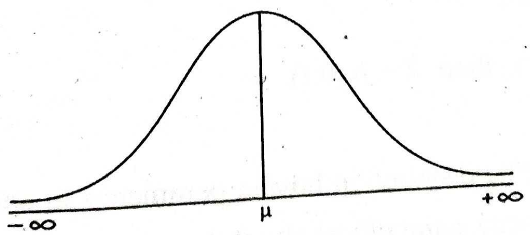
e = Mathematical constant (2.7783)

T_1 = Mathematical constant (3.14)

σ = Standard deviation.

Normal Distribution Graph

If we draw the graph of Normal Distribution, the curve obtained will be known as Normal curve and is given below :



The Graph of y = f(x) is a famous 'bell shaped' curve. The top of the bell is directly above the mean m. For large values of m, the curve tends to flatten out and for small values of s^2, it has a sharp peak.

When we say that curve has unit area we mean that the total area under the Normal Distribution between ($-\infty$ to ∞) is equal to 1.

Q18. Explain the properties of normal distribution.

Ans :

Following are the properties of normal, distribution.

1. The normal curve is 'bell-shaped' and symmetrical about the mean (skewness = 0). If the curve is folded along its central vertical axis the curves either side of the axis would coincide.
2. The height of the normal curve is maximum at its mean. Hence, the mean and mode coincide. Thus in normal distribution, mean, mode and median are equal.
3. The height of the curve is maximum at its mean but reduces as it goes towards either of the direction but never touches the base. Hence, the curve is known as ASYMPTOTIC. The range in unlimited or infinite in both the directions.
4. As there is only one maximum point, the normal curve has only one mode and it known as 'unimodal'.
5. The points of inflection i.e., the points where the change in curvature occurs are $\bar{x} \pm \sigma(\text{or})\mu \pm \sigma$.
6. The variables used in Binomial and Poisson are discrete variables whereas nonnal distribution has continuous random variable.

4.4.3.1 Standard Normal Distribution Properties

Q19. What is Standard Normal Distribution. State the properties of Standard Normal Distribution.

Ans :

A Random Variable with any mean and standard deviation can be transformed to a Standard Normal Variate (SNV) by subtracting the mean and dividing by the standard deviation. For a Normal Distribution with mean m and standard deviation s, the SNV 'Z' is obtained as

$$Z = \frac{x-\mu}{\sigma}$$

Where Z = the distance, expressed as a multiple of the standard deviation, that the value X lies away from the mean. The SNV Z has mean zero and variance '1'.

x= Random variable,

μ = Mean

σ = standard deviation.

In symbols, if $X \sim N(\mu, \sigma^2)$, then $Z \sim N(0,1)$.

Properties

1. It is a continuous probability distribution having parameters m and σ.
2. The normal curve is perfectly symmetrical about the mean (m) and is bell shaned. It means that if we fold the curve along the vertical line at the centre, the two halves of the curve work incide. The two tails of the curve on either side of x = m extends to infinity.
3. Mean = Median = Mode, Skewness = 0.
4. It has only one mode.

5. Q_1 and Q_3 are equidistant from median (or mean)

$Q_1 = m - 0.67\,\sigma$ $Q_3 = m + 0.67\,\sigma$ (app.)

Quartile Deviation = 0.67 σ, mean deviation (about mean) = 0.80 σ = 4/5 σ.

6. The maximum ordinate is at x = m its value is $\frac{1}{\sigma\sqrt{2\lambda}}$.

7. The area under the curve means the area lying between the curve and the horizontal axis and is equal to the number of frequencies. The s.d. (σ) distributes this area as follows:

i) 68.27% of the area lies between $x = -\sigma$ to $x = +\sigma$, 34.134% area will lie on either side of the mean.

ii) 95.45% of the area lies between $x = -2\sigma$ to $x = +2\sigma$ i.e. 47.725% area will lie on either side of the mean.

iii) 99.73% of the area lies between $x = -3\sigma$ to $x = +3\sigma$, 49.865% area will lie on either side of the mean.

4.4.3.2 Applications

Q20. What are the applications of normal distribution?

Ans : (Imp.)

1. **Production/Operations**

(i) A workshop produces a known quantity of units per day. The average weight of unit and standard deviation are given. Assuming normal distribution, we can find how many units are expected to weigh less than greater than some given weight.

(ii) A company manufacturers Electric bulbs and find that lifetime of the bulbs is normally distributed with some average life in hours and standard deviation in hours. On the basis of the information it can be estimated that the number of bulbs that is expected to burn for more than specified hours and less than specified hours.

2. **Finance / Accounting / Receivables**

In a business, the amount of daily collection is given for a particular period, we can estimate the average daily collection and Standard Deviation of this business. Assuming the daily collection follows a Normal Distribution.

3. **Health Care and Insurance Services**

(i) In patient's medical sample information, many parameters like cholesterol, urea in blood, hemoglobin, sugar, lipid profile, blood pressure etc., are used for diagnostic testing purpose. If each parameter follows Normal Distribution, we can calculate number of patients having abnormal and normal levels to decide upon the treatment to be followed.

(ii) In insurance industry, if insurance Premium follows normal distribution, we can calculate how many persons fall above or below a certain insured amount to plan marketing strategies by the management.

4. **Personnel**

(i) Given with average and variance of a wage distribution of a group of workers one can estimate the number of workers in different wage ranges.

ii) Given a distribution of training hours of a category of employees, it can be planned for the number of hours required for training an employee to suit a particular work requirement.

4.3.3.3 Importance of Normal Distribution

Q21. Explain the Importance of Normal Distribution.

(OR)

Explain the significance of normal distribution.

Ans :

The Normal Distribution has great significance in statistical data analysis, because of the following reasons :

(i) The Normal Distribution has a remarkable property stated in the central limit theorem, which asserts that if $X_1, X_2, \ldots\ldots X_n$ are n independent and identically random samples from a Normal Distribution which mean (m) and standard deviation (s), then the sample mean $(\bar{x})$ is also a Normal Distribution with mean (m) and standard error $\left(\frac{\sigma}{\sqrt{n}}\right)$. This result is true even if the population from which the samples are drawn is not a Normal Distribution subject to condition that n, the sample size is sufficiently large (n>30).

(ii) Even if a variable is not Normally distributed, it can sometimes be brought to Normal form by simple transformation of variable. For example, if Distribution of X is skewed, the Distribution of $\sqrt{X}$ might come out to be Normal.

(iii) Many of the sampling Distributions like Student's t, Snedecor's F, etc. also tend to Normal Distribution.

(iv) The sampling theory and tests of significance are based upon the assumption that samples have been drawn from a Normal population with mean m and variance s^2.

(v) Normal Distribution find large applications in Statistical Quality Control.

(vi) An n becomes large, the Normal Distribution serves as a good approximation for many discrete Distributions (such as Binomial, Poisson,. etc.).

(vii) In theoretical statistics many problems can be solved only under the assumption of a Normal population. In applied work we often find that methods developed under the normal probability law yield "satisfactory" results, even when the assumption of a normal population is not fully met, despite the fact that the problem can have a normal solution only if such a premise is hypothesized.

PROBLEMS

13. If the salary of workers in a factory is assumed to follow a Normal Distribution with a mean of Rs. 500 and a S.D. of Rs. 100, find Number of workers whose salary vary between Rs. 400 and Rs. 650, give the number of workers in the factory as 15,000 ?

Sol :

The required area will be calculated only after finding the corresponding Z values as shown below.

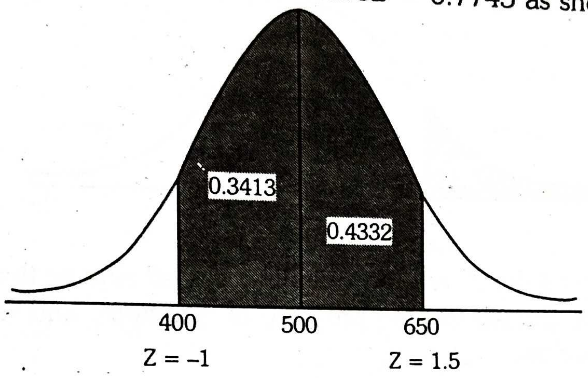
$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{(400-500)}{100} = -1$$

(left of the mean)

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{(650 - 500)}{100} = +1.5$$

(right of the mean)

Now we read the area between Z = 0 to Z = 1 from table as 0.3413. Because of symmetry the area between Z = –1 to Z = 0 is same as that of the area between Z = 0 to Z = 1. Again the area between Z = 0 to Z = + 1.5 is read from table as 0.4332. Thus the desired area between is x_1 = 400 and x_2 = 650 (i.e., Z = –1 to Z = + 1.5) is (0.3413 + 0.4332 = 0.7745 as shown in the figure.



Hence, the number of workers whose salary will be between 400 and 650 is given by 0.7745 5 15,000 = 11,618.

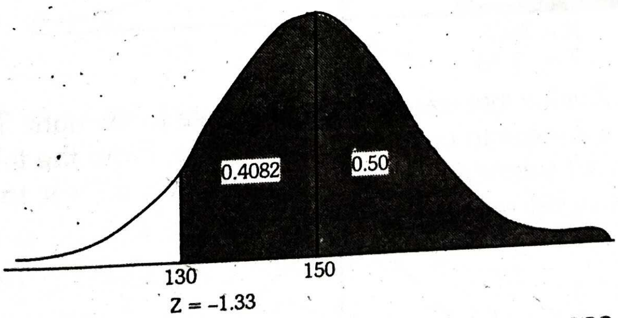
Hence, the number of workers whose salary will be between 400 and 650 is given by 0.7745 5 15,000 = 11,618.

14. A large flashlight is powered by 5 batteries. Suppose that the life of a battery is normally distributed with m = 150 hours and s = 15 hours. The flashlight will cease functioning if one or more of its batteries go dead. Assuming the lives of batteries are independent, what is the probability that flashlight will operate more than 130 hours ?

Sol :

The required area will be calculated only after finding the corresponding Z value as shown below.

$$z = \frac{x - \mu}{\sigma} = \frac{(130 - 150)}{15} = -1.33$$



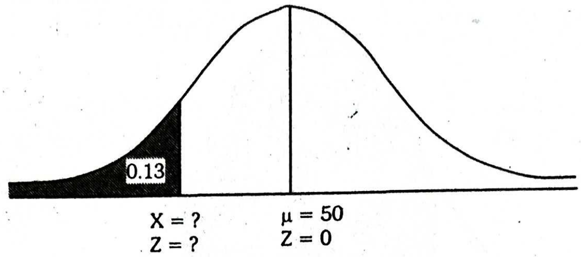
From the table we read the area from Z = 0 to Z = 1.33 as 0.4082. Due to symmetry, the area from Z = –1.33 to Z = 0 is same as 0.4082. The area to the right of mean (i.e., beyond Z = 0) is 0.5 The required area (≥ 130 hours of operating time of flash light) is {0.4082 + 0.5} = 0.9082. Hence the probability that the flashlight will operate for more than 130 hours is 0.9082 and is shown in the figure below.

15. Given a normal distribution with m = 50 and s = 10, find the value of X that has (i) 13% of the area to its left and (ii) 14% of the area to its right.

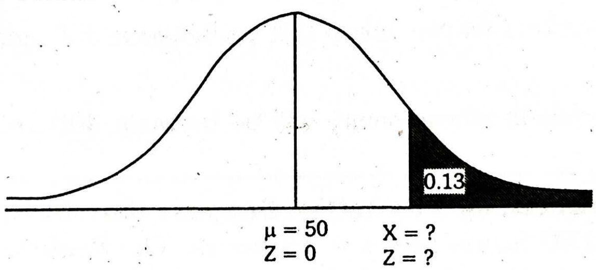
Sol :

We solved fist going from a value of X to a Z value and then computing desired area. In this example, it just reverse that we begin with a known area of probability, read Z value and then determine X by rearranging the formula given below $z = \frac{x - \mu}{\sigma}$ to give X = sZ + m

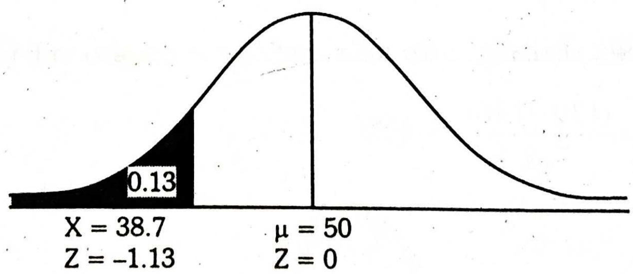
i) An area of 0.13 to the left of the desired X value is shaded in the following figure. We require a Z value that leaves an area of 0.13 to the left i.e., $P(Z < z_0) = 0.13$ as shown below.



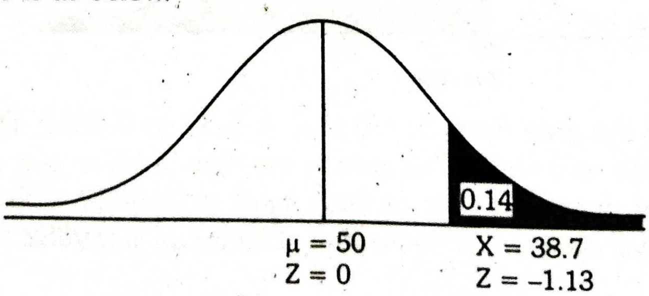
But it is not possible to read the Z value (or area) on the left side as the tables are not available for this purpose in this book. So, we use the property of symmetry and read the corresponding Z on the right side as shown below.



The table value for an area of 0.13 is 1.13. Because of symmetry this will be (–1.13) on the left side. By substituting Z = –1.13 into X = aZ + m we have X = 10(–1, 13) + 50 = 38.7. Finally, values are as given below.



ii) In this case we require a Z value that leaves 0.14 of the area to the right. This means an area of 0.46 lies between Z = 0 and a Z value to be read from the table. From the tables, w can read this value as Z = 1.08. Once again we substitute the value of Z into $X = \sigma Z + \mu$ to get X = 10(1.08) + 50 = 60.8. The final picture is as below.



Short Question and Answers

1. Define Random Variable.

Ans :

Meaning

A variable that can many real values which are determined by the outcomes of a random experiment on a real line $(-\infty, +\infty)$. It is a chance or Stochastic Variable.

It is a function defined on the sample space, "S" of a random experiment. A Random variable takes different values as a result of the outcomes of a random experiment. A Random variable can be Discrete or Continuous.

2. Discrete Random Variables

Ans :

A discrete random variable is one which may take on only a countable number of distinct values such as 0, 1, 2, 3, 4, ... Discrete random variables are usually (but not necessarily) counts. If a random variable can take only a finite number of distinct values, then it must be discrete. Examples of discrete random variables include the number of children in a family, the Friday night attendance at a cinema, the number of patients in a doctor's surgery, the number of defective light bulbs in a box of ten.

The probability distribution of a discrete random variable is a list of probabilities associated with each of its possible values. It is also sometimes called the probability function or the probability mass function.

3. Continuous Random Variables

Ans :

A continuous random variable is one which takes an infinite number of possible values. Continuous random variables are usually measurements. For example height, weight, the amount of sugar in an orange, the time required to run a mile all can take infinite number of possible values.

A continuous random variable is not defined at specific values. Instead, it is defined over an interval of values, and is represented by the area under a curve (in advanced mathematics, this is known as an integral) The probability of observing any single value is equal to 0, since the number of values which may be assumed by the random variable is infinite.

4. Probability Mass Function

Ans :

If X is a discrete random variable whose possible values are $x_1, x_2, \ldots x_n$ with the corresponding probabilities as $P_1, P_2, \ldots. P_n$ then the probabilities of x_i is defined as,

$$P_i = P(x_1) = P(X = x_i), \text{ where } i = 1, 2 \ldots n.$$

This is called probability function or probability mass function (PMF) if it satisfies the conditions mentioned below,

(a) $P(x_i) \geq 0$ for all values. This means all values are non-negative.

(b) $\sum_{i=1}^{n} P(x_i) = 1$

This mean the total probability must always be unity.

5. Probability Density Function

Ans :

If X is a continuous random variable and F(x) is its continuous function then a non-negative function f(x) is known as a Probability Density Function of X if it satisfies the conditions mentioned below.

(a) $f(x) \geq 0$ for al real values.

(b) $\int_{-\infty}^{\infty} f(x)dx = 1$

The function f(x) must be integrable on every interval of(a, b). This means the probability of an event in the interval (a, b) can be computed as,

$$P(a \le X \le b) = \int_a^b f(x)\,dx$$

$$= F(b) - F(a)$$

$\Rightarrow$ $P(a \le X \le b)$ = Area under f(x) which is enclosed with in a and b.

6. Probability Distribution.

Ans :

Probability distribution is a set of probabilities of all the possible outcomes of a random experiment.

Probability distribution is similar to frequency distribution. Probability distribution is based on the theoretical considerations, subjective assessment or on experience.

For example, 'X' is a random variable which can take the values x_1, x_2, x_3......

The probabilities associated with each of the possible values of 'X', $P(X = x_1) = P_i(i = 1, 2, 3,....)$

Therefore, the collection of pairs (x_1, P_i), where i = 1, 2, 3,..... is called probability distribution of random variable X.

7. What is Binomial Distribution.

Ans :

Meaning

In probability theory and statistics, the binomial distribution is the discrete probability distribution of the number of successes in a sequence of n independent yes/no experiments, each of which yields success with probability p. Such a success/failure experiment is also called a Bernoulli experiment or Bernoulli trial.

In fact, when n = 1, the binomial distribution is a Bernoulli distribution. The binomial distribution is the basis for the popular binomial test of statistical significance.

This binomial distribution is also known as the Bernoulli distribution by the name of the Swiss Mathematician Jacob Bernoulli ho has derived it.

8. Properties of Binomial Distribution.

Ans :

The properties of binomial distribution are as follows,

1. It describes the distribution of probabilities when there are only two mutually exclusive outcomes for each trial of an experiment for example while tossing a coin, the two possible outcomes are head and tail.
2. The process is performed under identical conditions for 'n' number of times.
3. Each trial is independent of other trials.
4. The probability of success 'p' remains same for trial to trial throughout the experiment and similarly, the probability of failure (q = 1 – p) also remains constant overall the observations.
5. Binomial distribution is symmetrical when p = 0.5 [figure (i)] and it is skewed if p ≠ 0.5, where 'n' can be any value.

9. Applications of Binomial Distribution.

Ans :

Binomial distribution is applicable in case of repeated trials such as,

1. Number of applications received for a junior assistant post during a period a particular period of time.
2. Number of births taking place in a hospital.
3. Number of candidates appearing for the screening test conducted by a company.

10. Define Poisson distribution.

Ans :

Meaning

In probability theory and statistics, the Poisson distribution is a discrete probability distribution that expresses probability of a number of events occurring in a fixed period of time if these events occur with a known average rate and independently of the time since the last event. The Poisson distribution can also be used for the number of events in other specified intervals such as distance, area or volume.

11. Applications of Poisson distribution

Ans :

Some practical situations where Poisson Distribution can be used.

i) Number of telephone calls arriving at a telephone switch board is a unit time (say, per minute)

ii) Number of customers arriving at the super market, say, per hour.

iii) The number of defects per unit of manufactured product (This is done for the construction of control chart for c in Statistical Quality Control)

iv) To count the number of radio-active element per unit of time (Physics)

v) The number of bacteria growing per unit time (Biology)

vi) The number of defective material say, pins, blades etc. in a package manufactured by a good concern.

12. Applications of normal distribution?

Ans :

1. **Production/Operations**

(i) A workshop produces a known quantity of units per day. The average weight of unit and standard deviation are given. Assuming normal distribution, we can find how many units are expected to weigh less than greater than some given weight.

(ii) A company manufacturers Electric bulbs and find that lifetime of the bulbs is normally distributed with some average life in hours and standard deviation in hours. On the basis of the information it can be estimated that the number of bulbs that is expected to burn for more than specified hours and less than specified hours.

2. **Finance / Accounting / Receivables**

In a business, the amount of daily collection is given for a particular period, we can estimate the average daily collection and Standard Deviation of this business. Assuming the daily collection follows a Normal Distribution.

3. **Health Care and Insurance Services**

(i) In patient's medical sample information, many parameters like cholesterol, urea in blood, hemoglobin, sugar, lipid profile, blood pressure etc., are used for diagnostic testing purpose. If each parameter follows Normal Distribution, we can calculate number of patients having abnormal and normal levels to decide upon the treatment to be followed.

(ii) In insurance industry, if insurance Premium follows normal distribution, we can calculate how many persons fall above or below a certain insured amount to plan marketing strategies by the management.

13. Significance of normal distribution.

Ans :

The Normal Distribution has great significance in statistical data analysis, because of the following reasons :

(i) The Normal Distribution has a remarkable property stated in the central limit theorem, which asserts that if $X_1, X_2, \ldots X_n$ are n independent and identically random samples from a Normal Distribution which mean (m) and standard deviation (s), then the sample mean $(\bar{x})$ is also a Normal Distribution with mean (m) and standard error $\left(\frac{\sigma}{\sqrt{n}}\right)$. This result is true even if the population from which the samples are drawn is not a Normal Distribution subject to condition that n, the sample size is sufficiently large (n>30).

(ii) Even if a variable is not Normally distributed, it can sometimes be brought to Normal form by simple transformation of variable. For example, if Distribution of X is skewed, the Distribution of $\sqrt{X}$ might come out to be Normal.

(iii) Many of the sampling Distributions like Student's t, Snedecor's F, etc. also tend to Normal Distribution.

Exercise Problems

1. The probability density function of a variate X is as follows:

X = x	0	1	2	3	4	5	6
P(x)	K	3K	5K	7K	9K	11K	13K

(i) Find $P(X<4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$

(ii) What will be the minimum value of K so that $P(X \leq 2) > 0.3$.

[Ans : Set No. 4]

2. A random variable X has the following probability function :

X	0	1	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$7K^2 + K$

(i) Find the value of K

(ii) Evaluate $P(X<6)$, $P(X \geq 6)$

(iii) Evaluate $P(0<X<5)$

[Ans : Set No. 3]

3. A discrete random variable X has the following probability distribution

Value of X	0	1	2	3	4	5	6	7	8
P(X = x)	k	3k	5k	7k	9k	11k	13k	15k	17k

(i) Find the value of 'k'

(ii) Find $P(X \leq 3)$, $P(0<X<3)$, $P(X \geq 3)$

[Ans : Set No. 1]

4. A discrete random variable X has the following probability distribution

Value of X	1	2	3	4	5	6	7	8
P(X = x)	2k	4k	6k	8k	10k	12k	14k	4k

(i) Find the value of 'k'

(ii) Find $P(X < 3)$ and $P(X \geq 5)$

(iii) Find the distribution function of X.

[Ans : Set No. 3]

Choose the Correct Answer

1. If the random variable X assumes only a finite or count ably infinite set of values, it is known as ______ variable. [b]
   (a) Continues random variable (b) Discrete random variable
   (c) Both a and b (d) None of the above
2. Which of the following can be taken as examples of discrete probability distribution. [d]
   (a) Binomial distribution (b) Normal distribution
   (c) Poisson distribution (d) (a) and (c).
3. The Poisson distribution is given by which formula. [a]
   (a) $P(r) = \frac{e^{-m}m^r}{r!}$ (b) ${}^nP_r \cdot p^r \cdot q^{n-r}$
   (c) $\frac{n!}{r!(n-r)!}$ (d) $\frac{X-\mu}{\sigma}$
4. The standard deviation of a Binomial distribution is given by, [c]
   (a) $\sigma = \sqrt{npq}$ (b) $\sigma = \sqrt{\pi}$
   (c) a or b (d) $\sigma = n \times p \times q$
5. If probability of success (P) = 0.2, what would be the probability of failure (q) [c]
   (a) q = 0.5 (b) q = 0.2
   (c) q = 0.8 (d) q = 0.4.
6. Who developed Poisson distribution ? [a]
   (a) S. Poisson (b) Pearson and pecar
   (c) James Poisson (d) Bernouli.
7. When we use Poisson distribution. [c]
   (a) When probability of success is constant
   (b) When probability of success is not constant
   (c) When probability of any individual event being a failure is small.
8. ______ probability distributions are skewed to right. [b]
   (a) Binomial (b) Poisson
   (c) Normal (d) Both a and c.
9. Poisson distribution is given by the formula $P(r) = \frac{e^{-m}m^r}{r!}$. In this formula m stands for ______ [a]
   (a) Mean of Poisson distribution (b) Median of Poisson distribution
   (c) Mode of Poisson distribution (d) None of the above.
10. ${}^nC_r = ?$ [d]
   (a) ${}^nC_r = \frac{n!}{n!(n-r)!}$ (b) ${}^nC_r = \frac{r!}{n!(n-r)!}$
   (c) ${}^nC_r = \frac{n!}{r \times n!}$ (d) ${}^nC_r = \frac{n!}{r!(n-r)!}$

Fill in the blanks

1. A ________ can also be used to describe the process of rolling a fair die and the possible outcomes.

   A ________ is not defined at specific values.

3. If X and Y are two random variables then covariance between them is denoted by ________

4. Probability distribution is similar to ________

5. The Normal Distribution was discovered by ________ De Moivre

6. SNV Stands for ________

7. The r^{th} moment of a random variable X about mean $\bar{X}$ is called ________ .

8. If two dice are thrown, then total number of points = ________

9. There are ________ types of probability distribution function :

10. The ________ can also be used for the number of events in other specified intervals such as distance, area or volume.

Answers

1. Random Variable
2. Continuous Random Variable
3. Cov(X, Y)
4. Frequency Distribution
5. De Moivre
6. Standard Normal Variate
7. Central moment
8. 36
9. Two
10. Poisson distribution