UNIT III

Probability: Basic terminology, Mathematical probability, Statistical probability, Axiomatic approach to probability, Theorems onprobability.

Conditional Probability, Multiplication theorem of probability, Independent events, Pairwise/mutually independent events, Bayes' Theorem.

3.1 Probability

Q1. Define the term probability.

(OR)

What is probability?

Ans :

Introduction

An Italian mathematician, Galileo (1564 - 1642), attempted a quantitative measure of probability while dealing with some problems related to gambling. In the middle of 17th Century, two French mathematicians, Pascal and Fermat, laid down the first foundation of the mathematical theory of probability while solving the famous 'Problem of Points' posed by Chevalier-De-Mere. Other mathematicians from several countries also contributed in no small measure to the theory of probability. Outstanding of them were two Russian mathematicians, A. Kintchine and A. Kolmogoroff, who axiomised the calculus of probability.

If an experiment is repeated under similar and homogeneous conditions, we generally come across two types of situations.

(i) The net result, what is generally known as 'outcome' is unique or certain.

(ii) The net result is not unique but may be one of the several possible outcomes.

The situations covered by :

(i) are known as 'deterministic' or 'predectable' and situations covered by

(ii) are known as 'probabilistic' or 'unpredictable'.

'Deterministic' means the result can be predicted with certainty. For example, if r is the radius of the sphere then its volume is given by $V = \frac{4}{3}\pi r^3$ which gives uniquely the volume of the sphere.

There are some situations which do not lend themselves to the deterministic approach and they are known as 'Probabilistic'.

For example, by looking at the sky, one is not sure whether the rain comes or not.

In such cases we talk of chances or probability which can be taken as a quantitative measure of certainity.

Definitions

In a random experiment, let there be n mutually exclusive and equally likely elementary events. Let E be an event of the experiment. If m elementary events form event E (are favourable to E), then the probability of E (Probability of happening of E or chance of E), is defined as

$$P(E) = \frac{m}{n} = \frac{\text{Number of elementary events in E}}{\text{Total number of elementary events in the random experiment}}$$

If $\bar{E}$ denotes the event of non-occurrence of E, then the number of elementary events in $\bar{E}$ is n-m and hence the probability of $\bar{E}$ (non-occurrence of E) is

$$P(\bar{E}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(E) \Rightarrow P(E) + P(\bar{E}) = 1$$

Since m is a non-negative integer, n is a positive integer and $m \le n$, we have m

$$0 \le \frac{m}{n} \le 1.$$

Hence $0 \le P(E) \le 1$ and $0 \le P(\bar{E}) \le 1$.

1. **Statistical Probability**

Suppose an experiment is repeated 'w' times under essentially identical conditions.

Let an event A happens w-times then $\frac{m}{n}$ is defined as the relative frequency of A.

Statistical probability is also known as 'relative frequency' probability. The limit of this relative frequency as $n \to \infty$ is defined as the probability of A.

$$\boxed{\therefore\ P(A) = \lim_{n\to\infty} \frac{m}{n}} \quad 0 \le \frac{m}{n} \le 1$$

2. **Axiomatic Probability**

In axiomatic probability three axioms or postulates are explained on the basis of which probability is calculated. They are,

(i) Probability of an event ranges from, zero to one $\Rightarrow 0 \le P(A) \le 1$

(ii) Probability of entire sample space, $P(S) = 1$

(iii) If 'A' and 'B' are mutually exclusive events, then the probability of occurrence of either A or B is $P(A \cup B) = P(A) + P(B)$.

In general, there are three chances which can be expected for any events.

Q2. Explain the importance of probability.

Ans :

(i) The probability theory is very much helpful for making prediction. Estimates and predictions form an important part of research investigation. With the help of statistical methods, we make estimates for the further analysis. Thus, statistical methods are largely dependent on the theory of probability.

(ii) It has also immense importance in decision making.

(iii) It is concerned with the planning and controlling and with the occurrence of accidents of all kinds.

(iv) It is one of the inseparable tools for all types of formal studies that involve uncertainty.

(v) The concept of probability is not only applied in business and commercial lines, rather than it is also applied to all scientific investigation and everyday life.

(vi) Before knowing statistical decision procedures one must have to know about the theory of probability.

(vii) The characteristics of the Normal Probability Curve is based upon the theory of probability.

3.1.1 Basic Terminology

Q3. Explain the various terms used in probability theory.

Ans :

The following are the basic terms are used in probability theory :

(i) Random Experiment

If an 'experiment' is conducted, any number of times, under essentially identrfcal conditions, there is a set of all possible outcomes associated with it. If the result is not certain and is anyone of the several possible outcomes, the experiment is called a random trial or a random experiment. The outcomes are known as elementary events and a set of outcomes is an event. Thus an elementary event is also an event.

(ii) Outcome

The result of random experiment is usually referred as an outcome.

(iii) Event

An event is possible outcome of an experiment or a result of trial.

(a) **Simple Event:** In case of simple events we consider the probability of the happening or not happening of single events. For example, we might be interested in finding out the probability of drawing a red ball from a bag containing 10 white and 6 red balls.

(b) **Compound Events:** Compound events we consider the joint occurrence of two or more events. For example, if a bag contains 10 white and 6 red balls and if two successive draws of 3 balls are made, we shall be finding out the probability of getting 3 white balls in the first draw and 3 black balls in the second draw we are thus dealing with a compound event.

(iv) Mutually Exclusive Events

Two events are said to be mutually exclusive or incompatible when both cannot happen simultaneously in a single trial or, in other words, the occurrence of any one of them precludes the occurrence of the other.

For example, if a single coin is tossed either head can be up or tail can be up, both cannot be up at the same time. Similarly, a person may be either alive or dead at a point of time he cannot be both alive as well as dead at the same time.

To take another example, if we toss a dice and observe 3, we cannot expect 5 also in the same toss of dice. Symbolically, if A and B are mutually exclusive events, P (AB) = 0.

The following diagram will clearly illustrate the meaning of mutually exclusive events :

Disjoint Sets

It may be pointed out that mutually exclusive events can always be connected by the words "either ..... or". Events A, B, C are mutually exclusive only if either A or B or C can occur.

(v) Collectively Exhaustive Events

Events are said to be exhaustive when their totality includes all the possible outcomes of a random experiment. For example, while tossing a dice, the possible outcomes are 1, 2, 3, 4, 5 and 6 and hence the exhaustive number of cases is 6. If two dice are thrown once, the possible outcomes are :

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1,6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

The sample space of the experiment i e.. 36 ordered pairs (6^2). Similarly, for a throw of 3 dice exhaustive number of cases will be 216 (i.e. 6^3) and for n dice they will be 6^n.

Similarly, black and red cards are examples of collectively exhaustive events in a draw from a pack of cards.

(vi) Equally Likely Events

Events are said to be equally likely when one does not occur more often than the others. For example, if an unbiased coin or dice is thrown, each face may be expected to be observed approximately the same number of times in the long run. Similarly, the cards of a pack of playing cards are so closely alike that we expect each card to appear equally often when a large number of drawings are made with replacement. However, if the coin or the dice is biased we should not expect each face to appear exactly the same number of times.

(vii) Independent Event

Two or more events are said to be independent when the outcome of one does not affect, and is not affected by the other.

For example, if a coin is tossed twice, the result of the second throw would in no way be affected by the result of the first throw. Similarly, the results obtained by throwing a dice are independent of the results obtained by drawing an ace from a pack of cards.

To consider two events that are not independent, let A stand for a firm's spending a large amount of money on advertisement and B for its showing an increase in sales. Of course, advertising does not guarantee higher sales, but the probability that the firm will show an increase in sales will be higher if A has taken place.

(viii) **Dependent Event**

Dependent events are those in which the occurrence or non-occurrence of one event in any one trial affects the probability of other events in other trials. For example, if a card is drawn from a pack of playing cards and is not replaced, this will alter the probability that the second card drawn is, say an ace. Similarly, the probability of drawing a queen from a pack of 52 cards is $\frac{4}{52}$ or $\frac{1}{13}$. But if the card drawn (queen) is not replaced in the pack, the probability of drawing again a queen is 3/51 (\ the pack now contains only cards out of which there are 3 queens).

(xi) **Non-mutually Exclusive Events**

When two events can occur simultaneously in a single trial then such events are said to be non-mutually exclusive events.

Example, from a pack of cards, drawing a red card and drawing a queen are the two events. These two events can occur simultaneously while drawing a red queen.

Hence, these two events are said to be non-mutually exclusive events which can occur at the same time.

PROBLEMS

1. **What is the probability for a leap year to have 52 Mondays and 53 Sundays?**

Sol :

A leap year has 366 days i.e., 52 weeks and 2 days.

These two days can be any one of the following 7 ways :

(i) Mon & Tue

(ii) Tues & Wed

(iii) Wed & Thurs

(iv) Thurs & Fri

(v) Fri & Sat

(vi) Sat & Sun

(vii) Sun & Mon

Let E be the event of having 52 Mondays and 53 Sundays in the year.

Total number of possible cases is n = 7

Number of favourable cases to E is m = 1

(Sat & Sun is the only favourable case)

$$\therefore \quad P(E) = \frac{m}{n} = \frac{1}{7}$$

2. **Five digit numbers are formed with 0, 1,2, 3,4 (not allowing a digit being repeated in any number). Find the probability of getting 2 in the ten's place and 0 in the units place always.**

Sol :

Total number of 5 digit numbers using the digits 0, 1, 2, 3, 4 is

$= n = 4 \times 4 \times 3 \times 2 \times 1 = 96$ (or) $5! - 4! = 96$

Let E be the event of getting a number having 2 in 10's place and 0 in the units place.

So the number of numbers favourable to is = m = 3.2.1.1.1 = 6

$$\therefore \quad P(E) = \frac{m}{n} = \frac{6}{96} = \frac{1}{16}$$

3. In a class there are 10 boys and 5 girls. A committee of 4 students is to be selected from the class. Find the probability for the committee to contain at least 3 girls.

Sol :

A committee of 4 students out of 15 can be formed in $^{15}C_4$ ways i.e., n = $^{15}C_4$

Let E be the event of forming a committee with at least 3 girls.

Now the committee can have 1 boy, 3 girls or no boy, 4 girls. So the number of ways of forming the committee = The number of favourable ways to E

$$= {}^{10}C_1 \times {}^5C_3 + {}^{10}C_0 \times {}^5C_4 = 100 + 5 = 105$$

$$\therefore \quad P(E) = \frac{m}{n} = \frac{105}{^{15}C_4} = 0.0769$$

4. When two dice are thrown, find the probability that the sum of the numbers is either 10 or 11.

Sol :

When 2 dice are thrown sample spaces $6^2 = 36$

The no. of of possible outcomes

$$(4, 6)\ (5, 5)\ (5, 6)\ (6, 4)\ (6, 5) = \frac{5}{36}$$

Let "A" be the Event that number selected would be sum is 10.

'B' be the Even that number selected would be sum is 11.

3.1.2 Mathematical Probability

Q4. Define mathematical probability. Explain its merits and demerits.

Ans :

Definition

If a trial results in 'n' exhaustive, mutually exclusive and equally likely outcomes, and 'm' of them are favourable to the happening of an event E, then the probability 'p' of happening of E, denoted by P(E), is given by

$$p = P(E) = \frac{\text{Favourable number of outcomes}}{\text{Exhaustive number of outcomes}}$$

$$= \frac{m}{n}$$

The probability of non-happening of E is given by

$$q = P(\bar{E}) = \frac{\text{Unfavourable number of outcomes}}{\text{Exhaustive number of outcomes}}$$

$$= \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(E)$$

$\therefore$ $p + q = 1$ (or) $P(E) + P(\bar{E}) = 1$

$P(E) = 1 \Rightarrow$ E is called a certain event

$P(E) = 0 \Rightarrow$ E is called an impossible event

Merits

(i) It is the simpler and easiest way to compute probability.

(ii) It offers prior probabilities that can reflect ignorance which oftenly considered to be suitable before conducting an experiment.

(iii) It can be used in statistical mechanics.

Demerits

(i) If the various outcomes of a trial are not equally likely.

Ex.: The probability that a candidate will pass in a test is not 50%, since the two possible outcomes success and failure are not equally likely.

(ii) If the exhaustive number of outcomes of the random experiment is infinite (or) unknown.

3.1.3 Statistical Probability

Q5. Define statistical probability. Explain its merits and demerits.

Ans : **(Imp.)**

Definition

If a random experiment is repeated number of times under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the event happens to the number of trials when the number of trials become indefinitely large, is called statistical or empirical probability. It was given by Von Mises.

If an event E happens 'm' times out of 'n' trials, then

$$p = P(E) = \underset{n\to\infty}{\text{Lin}} \left(\frac{m}{n}\right)$$

The limit is finite and unique.

Merits

(i) This approach of probability can be used even in case of trials that are infinitely large.

(ii) It does not require all the elementary events to be equally likely.

Demerits

(i) It cannot determine the exact probability of an event.

(ii) It does not provide any guarantee for the existence of the limit of relative frequencies.

3.1.4 Axiomatic Approach to Probability

Q6. Explain the Axiomatic approach to probability.

(Imp.)

Ans :

The axiomatic approach to probability was introduced by the Russian mathematician A. N. Kolmogorov in the year 1933. Kolmogorov axiomised the theory of probability and his book Foundations of Probability, published in 1933, introduces probability as a set function and is considered as a classic. When this approach is followed, no precise definition of probability is given, rather we give certain axioms or postulates on which probability calculations are based. The whole field of probability theory for finite sample spaces is based upon the following three axioms :

1. The probability of an event ranges from zero to one. If the event cannot take place its probability shall be zero and if it is certain, i.e., bound to occur, its probability shall be one.

2. The probability of the entire sample space is 1, i.e., P(S) = 1.

3. If A and B are mutually exclusive (or disjoint) events then the probability of occurrence of either A or B denoted by $P(A \cup B)$ shall be given by :

$$P(A \cup B) = P(A) + P(B)$$

It may be pointed out that out of the four interpretations of the concept of probability, each has its own merits and one may use whichever approach is convenient and appropriate for the problem under consideration.

The probability of a event A, denoted by P(A) is so chosen as to satisfy the following three axioms.

i) $P(A) \geq 0 \Rightarrow$ This axiom states that the probability of occurrence of an event A in a random experiment may be zero or any positive number and it must not be negative number.

ii) $P(S) - 1 \Rightarrow$ This states that the sample space, S, itself is an event and since it is the event comprising all possible outcomes, it should have the highest possible probability, i.e., one.

iii) If $A \cap B = 0$, Then $P(A \cup B) = P(A) + P(B) \Rightarrow$ This axiom states that the probability of the event equal to the union of any number of mutually exclusive events is equal to the sum of the individual even probabilities.

PROBLEMS

5. A class consists of 6 girls and 10 boys. If a committee of 3 is chosen at random from the class, find the probability that (i) 3 boys are selected (ii) exactly 2 girls are selected.

Sol :

Total number of students = 16

n(S) = no. of ways of choosing 3 from 16 = ${}^{16}C_3$

i) Suppose 3 boys are selected. This can be done in ${}^{10}C_3$ ways.

Here $n(E) = {}^{10}C_7$

$\therefore \quad P(E) = \text{The probability that 3 boys are selected} = \dfrac{n(E)}{n(S)}$

$$= \frac{{}^{10}C_7}{{}^{16}C_7} = \frac{10 \times 9 \times 8}{16 \times 15 \times 14}$$

$$= \frac{3}{14} = 0.2143$$

(ii) Suppose exactly 2 girls are selected. Then

$n(E) = {}^{6}C_2 \times {}^{10}C_1$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{{}^{6}C_2 \times {}^{10}C_1}{{}^{16}C_3}$$

$$= \frac{15}{56} = 0.2678$$

6. A and B throw alternately with a pair of ordinary dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, show that his chance of winning is 30/61.

Sol : **(Imp.)**

When two dice are thrown , we have n(s) = 36

The probability of A throwing '6' $= \dfrac{5}{36}$ i.e., $P(A) = \dfrac{5}{36}$

The probability of A not throwing '6' is and is given by

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{5}{36} = \frac{31}{36}$$

The probability of B throwing '7' $= \dfrac{6}{36}$ i.e., $P(B) = \dfrac{6}{36} = \dfrac{1}{6}$

The probability of B not throwing 7 is $P(\bar{B}) = 1 - P(B) = 1 - \dfrac{6}{36} = \dfrac{5}{6}$

$\therefore$ Chances of winning of 'A' is

$$= P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A) + \ldots\ldots$$

$$= \left(\frac{5}{36}\right) + \left(\frac{31}{36} \times \frac{5}{6} \times \frac{5}{36}\right) + \left(\frac{31}{36} \times \frac{5}{6} \times \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36}\right) + \ldots\ldots$$

$$= \frac{5}{36}\left[1 + \left(\frac{31}{36} \times \frac{5}{6}\right) + \left(\frac{31}{36} \times \frac{5}{6}\right)^2 + \ldots\ldots\right] = \frac{5}{36}\left[\frac{1}{1 - \left(\frac{31}{36} \times \frac{5}{6}\right)}\right] = \frac{30}{61}$$

3.2 Theorems on Probability

3.2.1 Addition Theorem

Q7. Explain Addition theorem of probability.

(Imp.)

Ans :

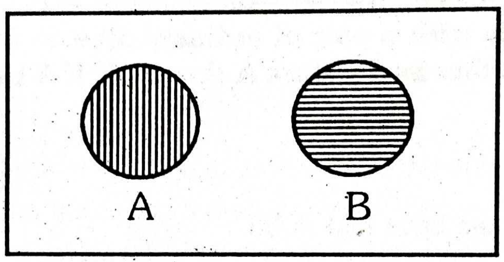
Adition theorem is different for mutually exclusive non-mutually exclusive events.

(i) For Mutually Exclusive Events

When 'A' and 'B' are two mutually exclusive events (i.e., both cannot occur at the same time) then the probability of occurrence of A or B is equal to the sum of their individual probabilities.

$$\boxed{\begin{aligned} &P(A \text{ or } B) = P(A) + P(B) \\ \Rightarrow\ &P(A \cup B) = P(A) + P(B) \end{aligned}}$$

Diagrametrically it can be represented as,



Mutually exclusive events

Figure: Mutually Exclusive Events

In case of 3 events A, B and C,

$$\boxed{P(A \text{ or } B \text{ or } C) = P(A \cup BB \cup C) = P(A)+P(B)+P(C)}$$

(ii) For Non-Mutually Exclusive Events

In case of non-mutually exclusive event (i.e., if the events occur together) these is a variation in the addition theorem.

When 'A' and 'B' are non-mutually exclusive events then the probability of occurrence of A or B is the sum of their individual probability which should be deducted from the probability of A and B occurring together.

$$\boxed{P(A \text{ or } B) \Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Diagrametically it can be represented as,

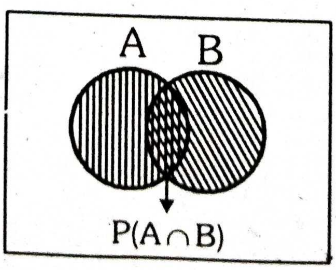


Fig.: Non-Mutually Exclusive Events

In case of three non-mutually exclusive events.

A, B and C the probability of occurence of A or B or C can be calculated by the following formula,

$$\boxed{P(A \text{ or } B \text{ or } C) = P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}$$

PROBLEMS

7. A card is drawn from a well shuffled pack of cards. What is the probability that is either a spade or an ace ?

Sol :

Let S is the sample space of all the simple events.

$\therefore \quad n(s) = 52$

Let A denote the event of getting a spade and B denote the event of getting an ace.

Then $A \cup B$ = The event of getting a spade or an ace

$A \cap B$ = The event of getting a spade and an ace

$$\therefore \quad P(A) = \frac{13}{52}, P(B) = \frac{4}{52}, P(A \cap B) = \frac{1}{52}$$

By Addition Theorem,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

8. Three students A, B, C are in running race. A and B have the same probability of winning and each is twice as likely to win as C. Find the probability that B or C wins.

Sol :

$A \cup B \cup C = S$ = Sample space of race

By data, $P(A) = P(B)$ and $P(A) = 2\,P(C)$ .....(1)

We have $P(A) + P(B) + P(C) = 1 \Rightarrow 2P(C) + 2P(C) + P(C) = 1$ [by (1)]

$$\Rightarrow P(C) = \frac{1}{5} P(A) = \frac{2}{5} \text{ and } P(B) = \frac{2}{5}$$

The probability that B or C wins $= P(B \cup C)$

$= P(B) + P(Q - P(B \cap C)$

$$= \frac{2}{3} + \frac{1}{5} - 0 = \frac{3}{5}$$

9. From a city 3 news papers A, B, C are being published. A is read by 20%, B is read by 16%, C is ready by 14% both A and B are read by 8%, both A and C are read by 5% both B and C are read by 4% and all three A, B, C are read by 2%. What is the percentage of the population that read at least one paper.

Sol :

Given, $P(A) = \frac{20}{100}$, $P(B) = \frac{16}{100}$, $P(C) = \frac{14}{100}$ and

$$P(A \cap B) = \frac{8}{100},\ P(A \cap C) = \frac{5}{100},\ P(B \cap C) = \frac{4}{100} \text{ and } P(A \cap B \cap C) = \frac{2}{100}$$

$$\therefore \quad P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{20}{100} + \frac{16}{100} + \frac{14}{100} + \frac{8}{100} - \frac{4}{100} - \frac{5}{100} + \frac{2}{100} = \frac{35}{100}$$

$\therefore$ Percentage of the population that read at least one paper $= \frac{35}{100} \times 100 = 35$

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10. One card is drawn at random from a pack of 52 cards. What is the probability that it is either a king or a queen?

Sol : **(Imp.)**

Probability of Drawing a King Card

Let P(A) denoted as probability of drawing a king card from a pack of cards.

Total number of king cards = 4

1 kind card is drawn from 4 king cards = $4c_1 = 4$.

Let total No. of Playing cards in a pack = 52.

1 card is drawn from 52 cards = $52c_1 = 52$

$$\therefore P(A) = \frac{4}{52}$$

Probability of Drawing a Queen Card

Let P(B) denoted as probability of drawing 1 Queen card from a pack of cards.

Total No. of Queen cards = 4

1 card is drawn from 4 Queen cards = $4c_1 = 4$.

Let total No. of playing cards in a pack = 52

1 card is drawn from 52 cards = $52c_1 = 52$.

$$\therefore P(B) = \frac{4}{52}$$

Probability of drawing a king or Queen is

$$P(A \cup B) = P(A) + P(B) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} \text{(or)} \frac{2}{13}.$$

11. **A bag contains 4 defective and 6 good Electronic Calculators. Two calculators are drawn at random one after the other without replacement. Find the probability that**

i) Two are good

ii) Two are defective and

iii) One is good and one is defective.

(Imp.)

Sol :

Total Number of calculators in a bag = 4 + 6 = 10

(i) **Probability of Drawing 2 good Calculators**

Let P(A) denoted as drawing 1 good calculator from total calculators.

$$\therefore P(A) = \frac{6}{10}$$

Let P(B) denoted as drawing 2^{nd} good calculator without replacing the 1^{st} calculator.

Total good calculators after first calculator is not replaced = 6 – 1 = 5.

Total Number of calculators after first calculator is drawn and not replaced = 10 – 1 = 9.

$$\therefore P(B) = \frac{5}{9}$$

∴ Probability of drawing 2 good calculators .

$$P(A) . P(B) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$$

(ii) **Probability of Getting Two Defectives**

Let P(A) denoted as drawing a defective calculator

Total Number of defective calculators = 4

1 Calculator is drawn from 4 = $4c_1$ = 4.

Total calculators in the bag = 4 + 6 = 10

One calculator is drawn from 10 = $10c_1$ = 10.

$$\therefore P(A) = \frac{4}{10}$$

Let P(B) denoted as drawing another defective calculator without replacing the first.

Total number of defective calculator after first calculator is drawn and not replaced = 4 – 1 = 3.

Total number of calculators in bag after first calculator is drawn and not replaced = 10 – 1 = 9.

$$\therefore P(B) = \frac{3}{9}$$

∴ Probability of drawing two are defective without replacing is P(A) . P(B).

$$= \frac{4}{10} \times \frac{3}{9} = \frac{12}{90} = \frac{2}{15}$$

(iii) One is good and one is defective

Let P(A) denoted as drawing 1 good calculator.

Number of good calculators = 6.

1 calculator is drawn from 6 = $6c_1$ = 6.

Total number 9 calculators in bag = 10

1 calculator is drawn from to = $10c_1$ = 10.

$$\therefore P(A) = \frac{6}{10}$$

Let P(B) denoted as drawing another calculator which is defective.

Total Number of defective calculators = 4.

1 calculator is drawn from 4 = $4c_1$ = 4.

Total number of calculators after first calculator is drawn and not replaced = 10 – 1 = 9.

$$\therefore P(B) = \frac{4}{9}$$

Probability of drawing 1 good and 1 bad is P(A) . P(B)

$$= \frac{6}{10} \times \frac{4}{9} = \frac{24}{90} = \frac{4}{15}.$$

3.2.2 Multiplication Theorem of Probability

Q8. Explain Multiplication theorem of probability.

Ans : **(Imp.)**

If 'A' and 'B' are two independent events then the probability of occurrence of both the events is equal to the product of their individual probabilities.

For independent events,

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

Similarly,

$$\boxed{P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)} \text{ and so on.}$$

If 'A' and 'B' are two dependent events, in such a case multiplication theorem is altered and is given as follows. For dependent events,

$$\boxed{\begin{aligned} P(A \cap B) &= P(A/B) \cdot p(B) \\ &= P(B/A) \cdot P(A) \end{aligned}}$$

Where, P(A/B) is a conditional probability of A given that B has occurred (Thq probability of occurrence of event A when event B has already occurred is the conditional probability of A given B).

PROBLEMS

12. Find the probability of drawing 2 red balls in succession from a bag containing 4 red and 5 black balls when the ball that is drawn first is not replaced (ii) replaced.

(Imp.)

Sol :

Let E_1 be the event of drawing a red ball in the first draw and £, be the event of drawing a red ball in second draw also.

(i) After the first draw the ball is not replaced. The first ball can be drawn in 9 ways and the second in 8 ways since the first ball is not replaced. Then both the balls can be drawn in 9x8 ways.

There are 4 ways in which £) can occur and 3 ways in which E_2 can occur, so that E_1 and E_2 can occur in 4×3 ways.

$$p\left(\frac{E_2}{E_1}\right) = P(E_2\text{, given the probability of } E_1)$$

$$= P(\text{2nd ball is red, given that first ball is red}) = \frac{3}{8}$$

$$\therefore \quad P(E_1 \cap E_2) = P(E_1) \times P\left(\frac{E_2}{E_1}\right) = \frac{4}{9} \times \frac{3}{8} = \frac{1}{6} \quad \left[\because P(E_1) = \frac{4}{9}\right]$$

(ii) Suppose the ball is replaced after the first draw. Then

$$P(E_1 \cap E_2) = \frac{4}{9} \cdot \frac{4}{9} = \frac{16}{81}$$

---

13. A class has 10 boys and 5 girls. Three students are selected at random one after another. Find the probability that (i) first two are boys and third is girl (ii) First and third are of same sex and the second is of opposite sex.

Sol :

Total no. of students = 10 + 5 = 15

(i) The probability that first two are boys and the third is girl is

$$P(E_1 \cap E_2 \cap E_3) = \frac{10}{15} \cdot \frac{9}{14} \cdot \frac{5}{13} = \frac{15}{91}$$

(ii) Suppose the first and third are boys and second is a girl

$$\text{Probability of the event} = P(E_1) = \frac{10}{15} \cdot \frac{5}{14} \cdot \frac{9}{13} = \frac{15}{91} = 0.1648$$

Suppose first and third are girls and second is boy.

$$\text{Then the probability of the event} = P(E_2) = \frac{5}{15} \cdot \frac{10}{14} \cdot \frac{4}{13} = \frac{20}{273}$$

$$\therefore \quad \text{Required probability} = P(E_1) + P(E_2)$$

$$= \frac{15}{91} + \frac{20}{273} = \frac{45 + 20}{273} = \frac{65}{273} = 0.238$$

14. Two marbles are drawn in succession from a box containing 10 red, 30 white 20 blue and 15 orange marbles, with replacement being made after each draw. Find the probability that

(i) Both are white

(ii) First is red and second is white.

Sol :

Total no. of marbles in the box = 75

(i) Let E_1 be the event of the first drawn marble is white. Then

$$P(E_1) = \frac{30}{75}$$

Let E_2 be the event of second drawn marble is also white. Then

$$P(E_2) = \frac{30}{75}$$

The probability that both marbles are white (with replacement)

$$= P(E_1 \cap E_2) = P(E_1).\ P(E_2 \backslash E_1) = \frac{30}{75} \cdot \frac{30}{75} = \frac{4}{25}$$

(ii) Let E_1 be the event that the first drawn marble is red. Then

$$P(E_1) = \frac{10}{75} = \frac{2}{15}$$

Let E_2 be the event that the drawn marble is white. Then

$$P(E_2 \backslash E_1) = \frac{30}{75} = \frac{2}{5}$$

$\therefore$ The probability that the First marble is red and Second marble is white

$$= P(E_1 \cap E_2) = P(E_1).\ P(E_2 \backslash E_1)$$

$$= \frac{2}{15} \cdot \frac{2}{5} = \frac{4}{75}$$

15. Three boxes, practically indistinguishable in appearance have two drawers each. Box 1 contains a gold coin in first and silver coin in the other drawer, Box 2 contains a gold coin in each drawer and Box 3 contains a silver coin in each drawer. One box is chosen at random and one of its drawers is opened at random and a gold coin is found. What is the probability that the other drawer contains a coin of silver.

(Imp.)

Sol :

Let E denote the event that the box is choosen, i = 1,2,3.

$$P(E_i) = \frac{1}{3} \text{ for } i = 1, 2, 3.$$

Let A be the event that the gold coin is chosen. Then

$P(A|E_i)$ = Probability that a gold coin is choosen from the box i = 1, 2, 3.

$\therefore$ $P(A|E_1) = \frac{1}{2}$ ($\because$ The total no. of coins in box 1 is 2)

$P(A|E_2) = \frac{2}{2} = 1$ (There are two gold coins in box 2)

and $P(A|E_3) = \frac{0}{2} = 0$ (There is no gold coin in box 3)

(i) The probability that the drawn coin is gold

$$P(A) = P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + P(E_3)\,P(A|E_3)$$

$$= \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot 1 + 0 = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

(ii) The probability that the drawn coin is silver

$$P(B) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

---

16. Two digits are selected at random from the digits 1 through 9.

(i) If the sum is odd, what is the probability that 2 is one of the numbers selected ?

(ii) If 2 is one of the digits selected, what is the probability that the sum is odd ?

Sol :

The given set consists of five odd digits (1, 3, 5,7, 9) and four even digits (2, 4, 6, 8).

We know that

even + even = even

even + odd = odd

odd + even = odd

odd + odd = even

(i) Total number of events = $5 \times 4 = 20$

If '2' is one of the digits, then the other digit must be odd.

$\therefore$ Number of ways = 5

So, required probability $= \frac{5}{20} = \frac{1}{4}$

(ii) If 2 is selected, then the remaining number of digits = 8

$\therefore$ Total events = 8

If 2 is one of the digits selected, the probability that the sum is odd

$$= \frac{\text{Favourable cases for odd}}{\text{Total events}} = \frac{5}{8}$$

3.3 Independent Events, Pairwise/Mutually Independent Events

Q9. Explain briefly about Independent Events.

Ans : **(Imp.)**

Meaning

Independent events are those events whose occurrence is not dependent on any other event. For example, if we flip a coin in the air and get the outcome as Head, then again if we flip the coin but this time we get the outcome as Tail. In both cases, the occurrence of both events is independent of each other. It is one of the types of events in probability.

Let us learn here the complete definition of independent events along with its Venn diagram, examples and how it is different from mutually exclusive events.

In Probability, the set of outcomes of an experiment is called events. There are different types of events such as independent events, dependent events, mutually exclusive events, and so on.

If the probability of occurrence of an event A is not affected by the occurrence of another event B, then A and B are said to be independent events.

Consider an example of rolling a die. If A is the event 'the number appearing is odd' and B be the event 'the number appearing is a multiple of 3', then

$P(A) = 3/6 = 1/2$ and $P(B) = 2/6 = 1/3$

Also A and B is the event 'the number appearing is odd and a multiple of 3' so that

$P(A \cap B) = 1/6$

$P(A \mid B) = P(A \cap B) / P(B)$

$$= \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$$

$P(A) = P(A \mid B) = 1/2$, which implies that the occurrence of event B has not affected the probability of occurrence of the event A.

If A and B are independent events, then $P(A \mid B) = P(A)$

Using the Multiplication rule of probability, $P(A \cap B) = P(B) \cdot P(A \mid B)$

$P(A \cap B) = P(B) \cdot P(A)$

Note: A and B are two events associated with the same random experiment, then A and B are known as independent events if $P(A \cap B) = P(B) \cdot P(A)$

Q10. If events A and B are independent so are

(i) A and $\bar{B}$

(ii) B and $\bar{A}$

(iii) $\bar{A}$ and $\bar{B}$

Ans : **(Imp.)**

(i) Consider

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

Since A and B are independent

$$P(A \cap B) = P(A)\,P(B)$$

(ii) Consider

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$$= P(A) - P(A)\,P(B) = P(A)\,P(\bar{B})$$

Thus

$$P(A \cap \bar{B}) = P(A)\,P(\bar{B})$$

So, A and $\bar{B}$ are independent. Similarly we can prove (ii)

(iii) Consider

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$$

By DrMargan's Law

$$\begin{aligned} P(\overline{A \cup B}) &= 1 - P(A \cup B) \\ &= 1 - [P(A) + P(B) - P(A \cap B)] \\ &= 1 - [(A) + P(B) - P(A)\,P(B)] \end{aligned}$$

Since A and B are independent

Thus

$$\begin{aligned} P(\bar{A} \cap \bar{B}) &= [1 - P(A)]\,[1 - P(B)] \\ &= P(\bar{A})\;P(\bar{B}) \end{aligned}$$

So, $\bar{A}$ and $\bar{B}$ are independent.

3.4 Conditional Probability

Q11. Define Conditional Probability.

Ans :

Conditional Probability is the probability of occurrence of second event (B) given that the first event (A) has previously occurred.

For statistically independent events, the conditional probability of event B given that event A has already occurred is simply the probability of event B. Symbolically, it can be represented as,

$$P(B/A) = P(B).$$

If E_1 and E_2 are two events in a sample space S and $P(E_1) \neq 0$, then the probability of E_2, after the event E_1 has occurred, is called the conditional probability of the event of E_2 given

E_1 and is denoted by $P\left(\frac{E_2}{E_1}\right)$ or $P(E_2 / E_1)$ and we define $P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)}$

Similarly we define $P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}$

We have

$$P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{n(E_1 \cap E_2)/n(S)}{n(E_1)/n(S)} = \frac{n(E_1 \cap E_2)}{n(E_1)}$$

$$= \frac{\text{Number of elements in } E_1 \cap E_2}{\text{Number of elements in } E_1}$$

Here

$E_1 \subset S$ and $P(E_1) > 0.$

We agreed to consider only those elements (sample points) of the event E_1 as the sample space and E_2 to be another subset of S. Since E_1 is the new sample space, the only elements of E_2 that concern us are also the elements of E_1 i.e., the elements of $E_2 \cap E_1$.

Then we define the probability of E_2 relative to the new sample space E_1

i.e., the conditional probability of E_2 given E_1.

Thus some authors define the conditional probability of E_2 given E_1

i.e., $P\left(\frac{E_2}{E_1}\right)$ as $\frac{n(E_2 \cap E_1)}{n(E_1)}$.

3.5 Bayes' Theorem

Q12. State and prove Baye's probability theorem.

Ans :

$E_1, E_2, \ldots, E_n$ are n mutually exclusive and exhaustive events such that P(E.) > 0 (z = 1, 2, , ri) in a sample space S and A is any other event in S intersecting with every E. (i.e., A can only occur in combination with any one of the events E_1 $E_2, \ldots, E_n$) such that $P(A) > 0$.

If E_1 is any of the events of E_1 E_2, ,E„ where $P(E_t)$, $P(E_2)$,....,P(EJ and $P(A/E_1)$, $P(A/E_2)$,....,$P(A/E_1)$ are known, then

$$P(E_K \backslash A) = \frac{P(E_K).P(A/E_K)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)+\ldots..+P(E_n).P(A/E_n)}$$

Proof :

$E_1, E_2, ..., E_n$ are n events of S such that $P(E_1) > 0$ and $E_i \cap E_j = \phi$ for $i \neq j$ where $i, j = 1, 2, ..., n$.

Also $E_1, E_2, ..., E_n$ are exhaustive events of S and A is any other event of S where $P(A) > 0$.

$$S = E_1 \cup E_2 \cup .... \cup E_N \text{ and}$$

$$A = A \cap S = A \cap (E_1 \cup E_2 \cup ..... \cup E_n)$$

$$= (A \cap E_1) \cup (A \cap E_2) \cup ..... \cup (A \cap E_n)$$

Here $A \cap E_1, A \cap E_2, ...$, are mutually exclusive events. Then

$$P(E_k/A) = \frac{P(E_K \cap A)}{P(A)} = \frac{P(E_K \cap A)}{P[(A \cap E_1) \cup (A \cap E_2) \cup ..... \cup (A \cap E_n)]}$$

$$= \frac{P(E_K \cap A)}{P(A \cap E_1) + P(A \cap E_2) + ... + P(A \cap E_n)}$$

$$= \frac{P(E_K).P(A/E_K)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + ... + P(E_n).P(A/E_n)}$$

Note :

Baye's theorem is also known as formula for the Probability of "Causes", i.e., probability of a particular (cause) E given that event A has happened (already).

$P(E_i)$ is 'a priori probability' known even before the experiment, $P(A/E_i)$ "Likelihoods and $P(E_i / A)$ 'Posteriori Probabilities' determined after the result of the experiment.

Q13. What are the applications of Baye's Theorem?

Ans :

Following points highlights the application of Baye's theorem,

1. In Baye's theorem, posterior probabilities can be known by rivising priori probabilities with the help of new information
2. The probability of occurrence of future events can be known by Baye's theorem.
3. Baye's theorem offers a powerful statistical tool.
4. Baye's theorem helps the business and management executives to take effective decisions in uncertain situation.

Baye's theorem is also known as 'Probability of Causes' as it helps in determining the probability which a particular effect has due to a specific cause.

PROBLEMS

17. In a certain college, 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body,

(a) What is the probability that mathematics is being studied ?

(b) If a student is selected at random and is found to be studying mathematics, find the probability that the student is a girl?

(c) Probability of maths student is a boy

Sol :

Given P (Boy) = $P(B) = \frac{40}{100} = \frac{2}{5}$

and P (Girl) = $P(G) = \frac{60}{100} = \frac{3}{5}$

Probability that mathematics is studied given that the student is a boy = $P\{M / B\} = \frac{25}{100} = \frac{1}{4}$

Probability that mathematics is studied given that the student is a girl = $P(M / G) = \frac{10}{100} = \frac{1}{10}$

(a) Probability that the student studied Mathematics = P(M)

$$= P(G)\,P(M/G) + P(B)\,P(M/B)$$

∴ By total probability theorem,

$$P(M) = \frac{3}{5}.\frac{1}{10} + \frac{2}{5}.\frac{1}{4}$$

$$= \frac{4}{25}$$

(b) By Baye's theorem, probability of mathematics student is a girl = P(G / M)

$$= \frac{P(G)P(M/G)}{P(M)} = \frac{\frac{3}{5}.\frac{1}{10}}{\frac{4}{25}} = \frac{3}{8}$$

(c) Probability of maths student is a boy = P(B / M)

$$= \frac{P(B)P(M/B)}{P(M)} = \frac{\frac{2}{5}.\frac{1}{4}}{\frac{4}{25}} = \frac{5}{8}$$

18. A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that the red ball drawn is from bag.

Sol :

Let A and B denote the events of selecting bag A and bag B respectively.

Then $P(A) = \frac{1}{2}$; $P(B) = \frac{1}{2}$

Let R denote the event of drawing a red ball.

Having selected bag A, the probability to draw a red ball from A = $P(R/A) = \frac{3}{5}$

Similarly $P(R/A) = \frac{5}{9}$

One of the bags is selected at random and from it a ball is drawn at random.

It is found to be red. Then the probability that the selected bag is B

$$= P(B).P(R/B) = \frac{P(B).P(R/B)}{P(A).P(R/A)+P(B)P(R/B)}$$

$$= \frac{\frac{1}{2}.\frac{5}{9}}{\frac{1}{2}.\frac{3}{5}+\frac{1}{2}.\frac{5}{9}} = \frac{25}{52}$$

19. A factory has two machines. Empirical evidence has established that machines I and II produce 30% and 70% of output respectively. It has also been established that 5% and 1% of the output produced by these machines respectively was defective. A defective item is drawn at random. What is the probability that the defective item was produced by either machine I or machine II ?

(Imp.)

Sol :

Let A_1 be the event of drawing of an item produced by machine1.

A_2 is the event of drawing an item produced by machine 2

$P(A_1)$ is the probability of getting an item produced by machine 1

$P(A_2)$ is the probability of getting an item produced by machine 2.

$P(B/A_1)$ is the probability of getting defective machine 1

$P(B/A_2)$ is the probability of getting defective machine 2.

From the given data

$P(A_1) = 0.3$ (30%) $P(B/A_1) = \frac{5}{100} = 0.05$

$P(Q_2) = 0.7$ (70%) $P(B/A_2) = 1\% = \frac{1}{100} = 0.01$

Computation of posterior probabilities

Events	$P(A_1)$	$P(B/A_1)$	$P(A \cap B)$	$P(A_1/B)$
A_1	$P(A_1) = 0.3$	$P(B/A_1) = 0.05$	$0.3 \times 0.05 = 0.015$	$\frac{0.015}{0.022} = 0.682$
A_2	$P(A_2) = 0.7$	$P(B/A_2) = 0.01$	$0.7 \times 0.01 = 0.0007$	$\frac{0.007}{0.022} = 0.318$
	1.00	0.06	0.022	1.000

20. In a bolt factory, the Machines P,Q and R manufacture respectively 25%, 35% and 40% of the total of their outputs 5,4,2 percents respectively are defective bolts. A bolt is drawn at random from the product, and is known to be defective, What are the probabilities that it was manufactured by the machines P,Q and R.

Sol :

Let P(A), P(B), P(C) be the probabilities of the Events that the bolts are manufactured by the machines A, B & C respectively. Then

$$P(A) = \frac{25}{100} = 0.25, \quad P(B) = \frac{35}{100} = 0.35, \quad P(C) = \frac{40}{100} = 0.40$$

Let 'D' denotes that the bolts is defective. Then

$$P(D/A) = \frac{5}{100} = 0.05, \quad P(D/B) = \frac{4}{100}, \quad P(D/C) = \frac{2}{100}$$

(i) If bolt is defective, then the probability that it is from machine A.

By using Baye's Theorem

$$P(A/D) = \frac{P(D/A)P(A)}{P(D/A)P(A) + P(D/B)P(B) + P(D/C)P(C)}$$

$$= \frac{0.05(0.25)}{0.05(0.25) + 0.04(0.35) + 0.02(0.4)}$$

$$P(A/D) = \frac{0.0125}{0.0125 + 0.014 + 0.008} = \frac{0.0125}{0.0345} = 0.362$$

(ii) If bolt is defective, then the probability that it is from machine B.

$$P(B/D) = \frac{P(D/B)P(B)}{P(D/A)P(A) + P(D/B)P(B) + P(D/C)P(C)}$$

$$= \frac{0.04(0.35)}{0.05(0.25) + 0.04(0.35) + 0.02(0.40)}$$

$$P(B/D) = \frac{0.014}{0.0345} = 0.4057$$

(iii) It bolt is defective, then the probability that it is from machine C

$$P(C/D) = \frac{P(C/D)P(C)}{P(D/A)P(A) + P(D/B)P(B) + P(D/C)P(C)}$$

$$\frac{(0.02)(0.40)}{0.05(0.25) + 0.04(0.35) + 0.02(0.40)} = \frac{0.008}{0.0345}$$

$P(C/D) = 0.2318.$

21. A company has two plants for manufacturing scooters. Plant I manufacturers 80% of the Scooters and Plant II manufactures 20%. At the Plant I 85% Scooters are rated to be of standard quality and at plant II 65% Scooters are rated to be of standard quality. One Scooter was selected at random. What is the probability that

i) It is manufactured by Plant I

ii) It is manufactured by Plant II – which is of standard quality.

(Imp.)

Sol :

Let $P(E_1)$ denoted as manufacturing scooters from plant - I.

Probability of plant-I manufacturing scooters = 80% = 0.80.

$$\therefore P(A) = 0.8$$

Let rated $P\left(\frac{A}{E_1}\right)$ denotd as scooters are rated as standard quality.

Probability of scooters rated as standard quality in plant I = 85% = 0.85.

$$P\left(\frac{A}{E_1}\right) = 0.85$$

Let $P(E_2)$ denoted as manufacturing scooters from plant - II.

Probability of plant - II manufacturing scooters = 20% = 0.20.

$$\therefore P(E_2) = 0.20.$$

Let $P\left(\frac{A}{E_2}\right)$ denoted as scooters are rated standard quality in plant II.

Probability of scooters rated as standard quality in plant II is 65% = 0.65.

$$P\left(\frac{A}{E_2}\right) = 0.65$$

(i) **It is manufactured by plant I ;**

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\, P\left(\frac{A}{E_1}\right)}{P(E_1) . P\left(\frac{A}{E_1}\right) + P(E_2) . P\left(\frac{A}{E_2}\right)}$$

$$= \frac{0.80 \times 0.85}{(0.80 \times 0.85) + (0.2) \times (0.65)} = \frac{0.68}{0.68 + 0.13} = \frac{0.68}{0.81}$$

$$\therefore P\left(\frac{E_1}{A}\right) = 0.839.$$

(ii) **It is manufactured by Plant - II**

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2). P\left(\frac{P}{E_2}\right)}{P(E_1). P\left(\frac{A}{E_1}\right) + P(E_2). P\left(\frac{A}{E_2}\right)}$$

$$= \frac{(0.20) \times (0.65)}{(0.80 \times 0.85) + (0.2)(0.65)} = \frac{0.13}{0.68 + 0.13} = \frac{0.13}{0.81} = 0.1604.$$

Short Question and Answers

1. What is probability?

Ans :

An Italian mathematician, Galileo (1564 - 1642), attempted a quantitative measure of probability while dealing with some problems related to gambling. In the middle of 17th Century, two French mathematicians, Pascal and Fermat, laid down the first foundation of the mathematical theory of probability while solving the famous 'Problem of Points' posed by Chevalier-De-Mere. Other mathematicians from several countries also contributed in no small measure to the theory of probability. Outstanding of them were two Russian mathematicians, A. Kintchine and A. Kolmogoroff, who axiomised the calculus of probability.

2. Random Experiment.

Ans :

If an 'experiment' is conducted, any number of times, under essentially identrfcal conditions, there is a set of all possible outcomes associated with it. If the result is not certain and is anyone of the several possible outcomes, the experiment is called a random trial or a random experiment. The outcomes are known as elementary events and a set of outcomes is an event. Thus an elementary event is also an event.

3. Mutually Exclusive Events.

Ans :

Two events are said to be mutually exclusive or incompatible when both cannot happen simultaneously in a single trial or, in other words, the occurrence of any one of them precludes the occurrence of the other.

For example, if a single coin is tossed either head can be up or tail can be up, both cannot be up at the same time. Similarly, a person may be either alive or dead at a point of time he cannot be both alive as well as dead at the same time.

To take another example, if we toss a dice and observe 3, we cannot expect 5 also in the same toss of dice. Symbolically, if A and B are mutually exclusive events, P (AB) = 0.

4. Define mathematical probability.

Ans :

Definition

If a trial results in 'n' exhaustive, mutually exclusive and equally likely outcomes, and 'm' of them are favourable to the happening of an event E, then the probability 'p' of happening of E, denoted by P(E), is given by

$$p = P(E) = \frac{\text{Favourable number of outcomes}}{\text{Exhaustive number of outcomes}}$$

$$= \frac{m}{n}$$

The probability of non-happening of E is given by

$$q = P(\bar{E}) = \frac{\text{Unfavourable number of outcomes}}{\text{Exhaustive number of outcomes}}$$

$$= \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(E)$$

$\therefore$ $p + q = 1$ (or) $P(E) + P(\bar{E}) = 1$

$P(E) = 1 \Rightarrow$ E is called a certain event

$P(E) = 0 \Rightarrow$ E is called an impossible event

5. Define statistical probability.

Ans :

Definition

If a random experiment is repeated number of times under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the event happens to the number of trials when the number of trials become indefinitely large, is called statistical or empirical probability. It was given by Von Mises.

If an event E happens 'm' times out of 'n' trials, then

$$\therefore \quad p = P(E) = \underset{n\to\infty}{\text{Lin}} \left(\frac{m}{n}\right)$$

The limit is finite and unique.

6. Axiomatic approach to probability.

Ans :

The axiomatic approach to probability was introduced by the Russian mathematician A. N. Kolmogorov in the year 1933. Kolmogorov axiomised the theory of probability and his book Foundations of Probability, published in 1933, introduces probability as a set function and is considered as a classic. When this approach is followed, no precise definition of probability is given, rather we give certain axioms or postulates on which probability calculations are based. The whole field of probability theory for finite sample spaces is based upon the following three axioms :

1. The probability of an event ranges from zero to one. If the event cannot take place its probability shall be zero and if it is certain, i.e., bound to occur, its probability shall be one.

2. The probability of the entire sample space is 1, i.e., $P(S) = 1$.

3. If A and B are mutually exclusive (or disjoint) events then the probability of occurrence of either A or B denoted by $P(A \cup B)$ shall be given by :

$$P(A \cup B) = P(A) + P(B)$$

It may be pointed out that out of the four interpretations of the concept of probability, each has its own merits and one may use whichever approach is convenient and appropriate for the problem under consideration.

7. Explain Multiplication theorem of probability.

Ans :

If 'A' and 'B' are two independent events then the probability of occurrence of both the events is equal to the product of their individual probabilities.

For independent events,

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

Similarly,

$$\boxed{P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)} \text{ and so on.}$$

If 'A' and 'B' are two dependent events, in such a case multiplication theorem is altered and is given as follows. For dependent events,

$$\boxed{\begin{aligned} P(A \cap B) &= P(A/B) \cdot p(B) \\ &= P(B/A) \cdot P(A) \end{aligned}}$$

Where, P(A/B) is a conditional probability of A given that B has occurred (Thq probability of occurrence of event A when event B has already occurred is the conditional probability of A given B).

8. Independent Events.

Ans :

Meaning

Independent events are those events whose occurrence is not dependent on any other event. For example, if we flip a coin in the air and get the outcome as Head, then again if we flip the coin but this time we get the outcome as Tail. In both cases, the occurrence of both events is independent of each other. It is one of the types of events in probability.

Let us learn here the complete definition of independent events along with its Venn diagram, examples and how it is different from mutually exclusive events.

In Probability, the set of outcomes of an experiment is called events. There are different types of events such as independent events, dependent events, mutually exclusive events, and so on.

9. Define Conditional Probability.

Ans :

Conditional Probability is the probability of occurrence of second event (B) given that the first event (A) has previously occurred.

For statistically independent events, the conditional probability of event B given that event A has already occurred is simply the probability of event B. Symbolically, it can be represented as,

$$P(B/A) = P(B).$$

10. Applications of Baye's Theorem.

Ans :

Following points highlights the application of Baye's theorem,

1. In Baye's theorem, posterior probabilities can be known by rivising priori probabilities with the help of new information
2. The probability of occurrence of future events can be known by Baye's theorem.
3. Baye's theorem offers a powerful statistical tool.
4. Baye's theorem helps the business and management executives to take effective decisions in uncertain situation.

Exercise Problems

1. A class has 10 boys and 5 girls. Three students are selected at random one after another. Find the probability that

   (i) first two are boys and third is girl

   (ii) First and third are of same sex and the second is of opposite sex.

   [Ans : $\frac{15}{91}$, $\frac{20}{273}$]

2. Two aeroplanes bomb a target in succession. The probability of each correctly scoring a hit is 0.3 and 0.2 respectively. The second will bomb only if the first misses the target. Find the probability that

   (i) target is hit

   (ii) both fails to score hits.

   [Ans : 0.44]

3. Box A contains 5 red and 3 white marbles and box B contains 2 red and 6 white marbles. If a marble is drawn from each box, what is the probability that they are both of same colour.

   [Ans : 0.109]

4. Two marbles are drawn in succession from a box containing 10 red, 30 white 20 blue and 15 orange marbles, with replacement being made after each draw. Find the probability that

   (i) Both are white

   (ii) First is red and second is white.

   [Ans : $\frac{4}{25}$, $\frac{4}{75}$]

Choose the Correct Answers

1. An event in the probability that will never be happened is called as ______. [d]

(a) Unsure event (b) Sure event

(c) Possible event (d) Impossible event

2. What will be the value of P(not E) if P(E) = 0.07? [c]

(a) 90 (b) 007

(c) 93 (d) 72

3. What will be the probability of getting odd numbers if a dice is thrown? [a]

(a) 1/2 (b) 2

(c) 4/2 (d) 5/2

4. What is the probability of getting a sum as 3 if a dice is thrown? [b]

(a) 2/18 (b) 1/18

(c) 4 (d) 1/36

5. What is the probability of getting an even number when a dice is thrown? [b]

(a) 1/6 (b) 1/2

(c) 1/3 (d) 1/4

6. The probability of getting two tails when two coins are tossed is ______. [d]

(a) 1/6 (b) 1/2

(c) 1/3 (d) 1/4

7. What is the probability of getting the sum as a prime number if two dice are thrown? [b]

(a) 5/24 (b) 5/12

(c) 5/30 (d) 1/4

8. What is the probability of getting atleast one head if three unbiased coins are tossed? [a]

(a) 7/8 (b) 1/2

(c) 5/8 (d) 8/9

9. What is the probability of getting 1 and 5 if a dice is thrown once? [b]

(a) 1/6 (b) 1/3

(c) 2/3 (d) 8/9

10. What will be the probability of losing a game if the winning probability is 0.3? [c]

(a) 0.5 (b) 0.6

(c) 0.7 (d) 0.8

Fill in the Blanks

1. Bayes theorem is also called as ________ as it helps in determining cause of the probability of a particular effect.
2. A ________ probability is the probability of occurrence of two or more simple events.
3. The outcomes of random experiment are ________.
4. ________ approach of probability assumes that all the possible outcomes of an experiment are mutually exclusive and equally likely.
5. ________ theorem states that if two events A and B are independent, the probability that they both will occur is equal to the product of their individual probabilities.
6. ________ theorem helps in calculating or revising the probabilities in the light of additional information.
7. When two events can occur simultaneously in a single trial then such events are said to be ________
8. The axiomatic approach to probability was introduced by the Russian mathematician in the year 1933.
9. $P(A \cup B) =$ ________.
10. ________ Probability is the probability of occurrence of second event (B) given that the first event (A) has previously occurred.

Answers

1. 'Probability of Causes'
2. Joint
3. Events
4. Classical approach or A prior approach
5. Multiplication theorem of probability
6. Baye's
7. Non-mutually exclusive events.
8. A. N. Kolmogorov
9. P (A) + P (B)
10. Conditional