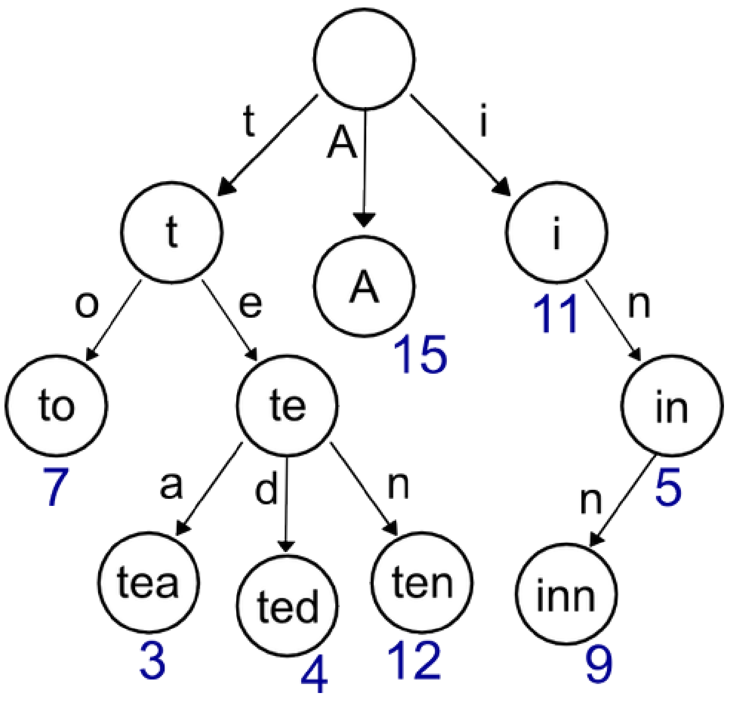
THREADS @ IIIT HYDERABAD

LALIT KUNDU

I will be writing in this post about Tries and the concept widely used in bit manipulation kind of problems. We'll see 2-3 problems which trie is helpful.

First we see what a trie is. Trie can store information about keys/numbers/strings compactly in a tree.
Tries consists of nodes, where each node stores a character/bit. We can insert new strings/numbers accordingly.
Here is an example trie of strings:

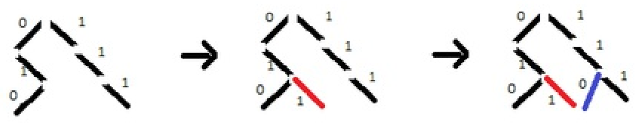


Source: Wikipedia.

But we will be dealing with numbers here, and particularly in binary bits. We'll see as we solve problems.

Problem1: Given an array of integers, we have to find two elements whose XOR is maximum.
Solution:
Suppose we have a data structure in which can satisfy two types of queries:
1. Insert a number X
2. Given a Y, find maximum XOR of Y with all numbers that have been inserted till now.

If we have this data structure, we'll insert integers as we go, and with query of 2nd type we'll find the maximum XOR.
Trie is the data structure we'll use. First, let's see how we insert elements in the trie.



Let's consider 3 bit numbers only. 010 and 111 are already into the trie here.

We are going to insert 011 into the trie.

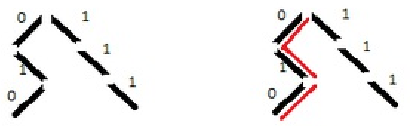
Since the first two bits, 01 are already there, we just have to add the 3rd bit 1 into the trie.

Now suppose we add 110 to the trie.

We add the third bit 0 to the trie.

So, we trace the path of the number we have to insert, we don't have to draw the existing path again.

Insertion of an N length key takes O(N) which is log2(MAX) where MAX is the maximum number to be inserted in trie, because there are at maximum log2(MAX) binary bits in a number.
This way we store all the data about all the numbers inserted into trie till now.
Now, for query of type 2:
Let's say our number Y is b1,b2...bn, where b1,b2.. are binary bits. We start from b1. Now for the XOR to be maximum, we'll try to make most significant bit 1 after taking XOR. So, if b1 is 0, we'll need a 1 and vice versa. In the trie, we go to the required bit side. If favorable option is not there, we'll go other side. Doing this all over for i=1 to n, we'll get the maximum XOR possible.



We have this already existing trie. Now, we want to maximise the xor with the number 100.

First bit is 1. So we'd like to have a 1 after XOR. So we go left side.
So our answer is of the form "1xx".
Next bit is 0. So we go right side. Our answer is now of form "11x".

For 3rd bit a 0 is there. So we'd like to go right. But we can't go to right.

Instead we'll continue with left only.

So, our answer is "110".

Query too is log2(MAX).

Problem2: Given an array of integers, find the subarray with maximum XOR.
Solution:
Let's say F(L,R) is XOR of subarray from L to R.
Here we use the property that F(L,R)=F(1,R) XOR F(1,L-1). How?
Let's say our subarray with maximum XOR ended at position i. Now, we need to maximise F(L,i) ie. F(1,i) XOR F(1,L-1) where L<=i. Suppose, we inserted F(1,L-1) in our trie for all L<=i, then it's just problem1.

```
ans=0
pre=0
Trie.insert(0)
for i=1 to N:
    pre = pre XOR a[i]
    Trie.insert(pre)
    ans=max(ans, Trie.query(pre))
print ans
```

You can try this problem here: ACM-ICPC Live Archive

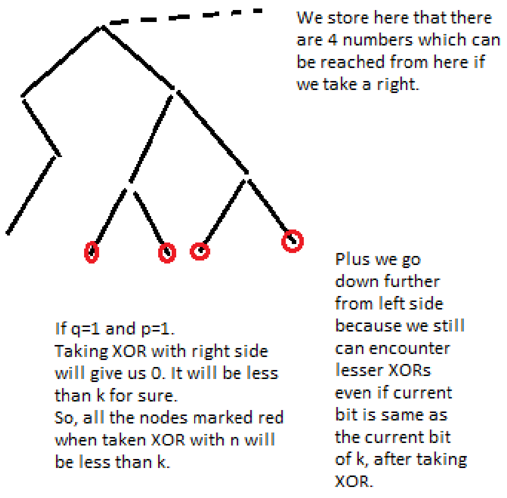
Problem3: Given an array of positive integers you have to print the number of subarrays whose XOR is less than K.

Solution:
This again uses the concepts we have seen till now. We'll just go like previous question.
For each index i=1 to N, we can count how many subarrays ending at ith position satisfy the given condition.

```
ans=0
p=0
for i=1 to N:
    q=p XOR A[i]
    ans += Trie.query(q,k)
    Trie.insert(q)
    p=q
```

query(q,k) returns how many integers already exist into structure which when taken xor with q return an integer less than k.
We compare the corresponding bits of q and k, beginning from most significant bits. Suppose p and q are the respective bits we are considering now.

If q is 1, and p is 0, then we do this:


We store here that there are 4 numbers which can be reached from here if we take a right.
If q=1 and p=1. Taking XOR with right side will give us 0. It will be less than k for sure. So, all the nodes marked red when taken XOR with n will be less than k.
Plus we go down further from left side because we still can encounter lesser XORs even if current bit is same as the current bit of k, after taking XOR.

Similarly we can very easily work out other 3 cases ie. (q=1,p=1), (q=0,p=1) and (q=0,p=1).

So, we need to alter our structure here, we also keep the number of leaf nodes reachable from current node if I go to the left side and similarly for right side. Because, otherwise, the complexity will increase, if we traverse the whole tree again and again. We can do this while inserting numbers into the tree very easily.

This problem was set in CodeCraft'14. You can pratice here: SPOJ.com - Problem SUBXOR

Now, let's talk about implementations.
For implementation of a trie in C/CPP we can keep nodes and left and right pointers. We can write recursive functions.

```
insert(root, num, level):
    if level==-1: return root
    x=level'th bit of num
    if x==1:
        if root->right is NULL: create root->right
        else: insert(root->right,num,level-1)
    else:
        if root->left is NULL: create root->left
        else: insert(root->left,num,level-1)
```

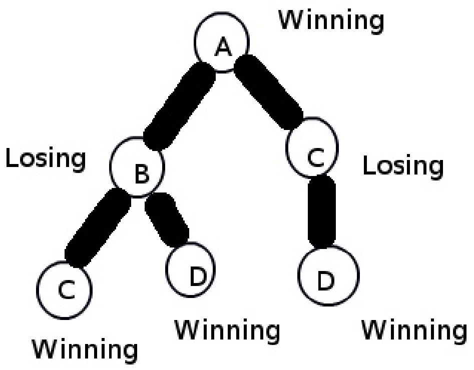
For queries also, we recursively traverse the tree.

Update:
Another problem using Trie(yay! :P).
Problem - B - Codeforces

Sub-Problem: Given a group of *n* non-empty strings. During the game two players build the word together, initially the word is empty. The players move in turns. On his step player must add a single letter in the end of the word, the resulting word must be prefix of at least one string from the group. A player loses if he cannot move.

We need to find which player(first or second) has the winning strategy.

So, the idea here is again to build a trie of all strings. Why? Because a trie stores information about all prefixes.
Now, we will try to evaluate for each node if first player has a winning strategy or not. We can do this recursively. For a node v, for each node u such that u is a immediate child of v, if first player has a losing strategy for u, then for node v first player has a winning strategy.
For example, say we have "abc", "abd", "acd".
Our trie would look like:



All leaf nodes have winning strategy.