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Following Paper ID and Roll No. to be filled in your Answer Book.

PAPER ID : 49908 Roll No. 1 2 3 2 0 8 0 2 2 4

B. Tech. Examination, 2024-25

(Odd Semester)

COMPLEX ANALYSIS AND INTEGRAL TRANSFORMS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION – A

1. Attempt all parts of the following : $8 \times 1 = 8$

   (a) Write C-R equations in Cartesian form.

   (b) Define harmonic function.

   (c) Write Cauchy integral formula for an analytic function.

   (d) Find the singularity for $F(z) = \frac{1}{z}$.

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(e) Find the Laplace transform of $f(t) = e^{-2t}$.

(f) Find $L^{-1}(1/s)$.

(g) Define Z-transform.

(h) Define fourier sine transform.

SECTION – B

2. Attempt any two parts of the following : $2 \times 6 = 12$

(a) Show that the function defined by $f(z) = \sqrt{|xy|}$ satisfy C-R equations at the origin but not analytic at that point.

(b) Evaluate :

$$\int_0^{1+i} \left(x - y + ix^2\right) dZ$$

(i) Along the straight line from $z = 0$ to $z = 1 + i$.

(ii) Along the imaginary axis from $z = 0$ to $z = i$ and then along a line parallel to real axis from $z = i$ to $z = 1 + i$.

(c) Find the inverse laplace transform of :

$$\log\left(1 + \frac{1}{s^2}\right)$$

(d) Find fourier transform of :

$$f(x) = \begin{cases} 1 & |x| \le a \\ 0 & |x| > a \end{cases}$$

hence evaluate :

$$\int_{-\infty}^{\infty} \frac{\sin \lambda a \cos \lambda x}{\lambda} d\lambda$$

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each questions. $8 \times 5 = 40$

3. (a) If $u = e^x (x \cos y - y \sin y)$ is a harmonic function find an analytic function $f(z) = u + i v$ such that $f(1) = e$.

(b) Find the bilinear transformation which maps that points $z = 1, i, -1$ in to the points $w = i, 0, -i$. Hence find the image of $|z| < 1$.

(c) Evaluate :

$$\oint_C \frac{e^z}{(z+1)^2} dz$$

where C is the circle $|z - 1| = 3$.

Use Cauchy Integral formula to evaluate

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$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}, \quad |z| = 3$$

4. (a) Evaluate the integral using Cauchy integral formula :

$$\int_C \frac{4-3z}{z\,(z-1)\,(z-2)}\,dz$$

where C is the circle $|z| = 3/2$.

(b) Find the Taylor's and Laurent's series which represent the function :

$$\frac{z^2-1}{(z+2)\,(z+3)}$$

when :

(i) $|z| < 2$

(ii) $2 < |z| < 3$

(c) Find two bilinear transformation where fixed points are 1 and 2.

Evaluate $\int \frac{12z-7}{(z-1)^2(2z+3)}\,dz \quad |z| = 2$

5. (a) Solve the simultaneous equations using Laplace transformation :

$$\frac{d^2x}{dt^2} + 5\frac{dy}{dt} - x = t, \quad 2\frac{dx}{dt} - \frac{d^2y}{dt^2} + 4y = 2$$

given that when :

$$t = 0,\ x = 0,\ y = 0,\ \frac{dx}{dt} = 0,\ \frac{dy}{dt} = 0$$

(b) Use convolution theorem to evaluate :

$$L^{-1}\left\{\frac{s}{\left(s^2+4\right)^2}\right\}$$

(c) Find L [f (t)] if :

$$F(t)=\begin{cases}(t-1)^2 & t>1\\ 0 & 0<t<1\end{cases}$$

6. (a) Solve by Z-transform the difference equation :

$$y_{k+2}+6\,y_{k+1}+9\,y_k=2^k;\ (y_0=y_1=0)$$

(b) Using Fourier sine integral, show that :

$$\int_0^\infty \left(\frac{1-\cos\pi\lambda}{\lambda}\right)\sin(x\lambda)\,d\lambda=\begin{cases}\pi/2 & 0<x<\pi\\ 0 & x>\pi\end{cases}$$

(c) Solve by Z transform :

$$y_{k+1}+y_k=1 \text{ if } y_0=0$$

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