SMALL-E : RSA Basics – Public Key Cryptography Challenge

Category: Cryptography

This challenge introduced the fundamentals of **RSA public key cryptography**, focusing on understanding how encryption works rather than performing brute-force attacks.

Objective

To analyze a provided RSA encryption script and determine how the encrypted message (ciphertext) could be decrypted by exploiting weak parameter choices.

Given Information

The challenge provided the following Python code snippet:

```
from Crypto.PublicKey import RSA
from Crypto.Util.number import bytes_to_long, long_to_bytes
import random

FLAG = "HQX{not_an_actual_flag}"
key = RSA.generate(2048, e = random.choice([3,5]))

msg = FLAG.encode()
m = bytes_to_long(msg)
c = pow(m, key.e, key.n)

print("e = {}".format(key.e))
print("n = {}".format(key.n))
print("ct = {}".format(c))
```

The outputs included:

- Public exponent `e`
- Modulus `n`
- Ciphertext `c`

Key Observations

- RSA key size was **2048 bits** (normally secure)
- Public exponent `e` was **very small** (`3` or `5`)
- The flag was encrypted **directly**, without padding (no OAEP)

These observations indicated a **textbook RSA vulnerability**.

Vulnerability Identified: Small Exponent Attack

In RSA, encryption is defined as:

```
c = m^e mod n
```

If:

- `e` is very small, and
- the message `m` is small enough that `m^e < n`

Then:

```
c = m^e   (no modulus applied)
```

This allows recovery of `m` by simply computing the **e-th root of the ciphertext**.

This is known as a **Small Public Exponent Attack**.

Exploitation Method

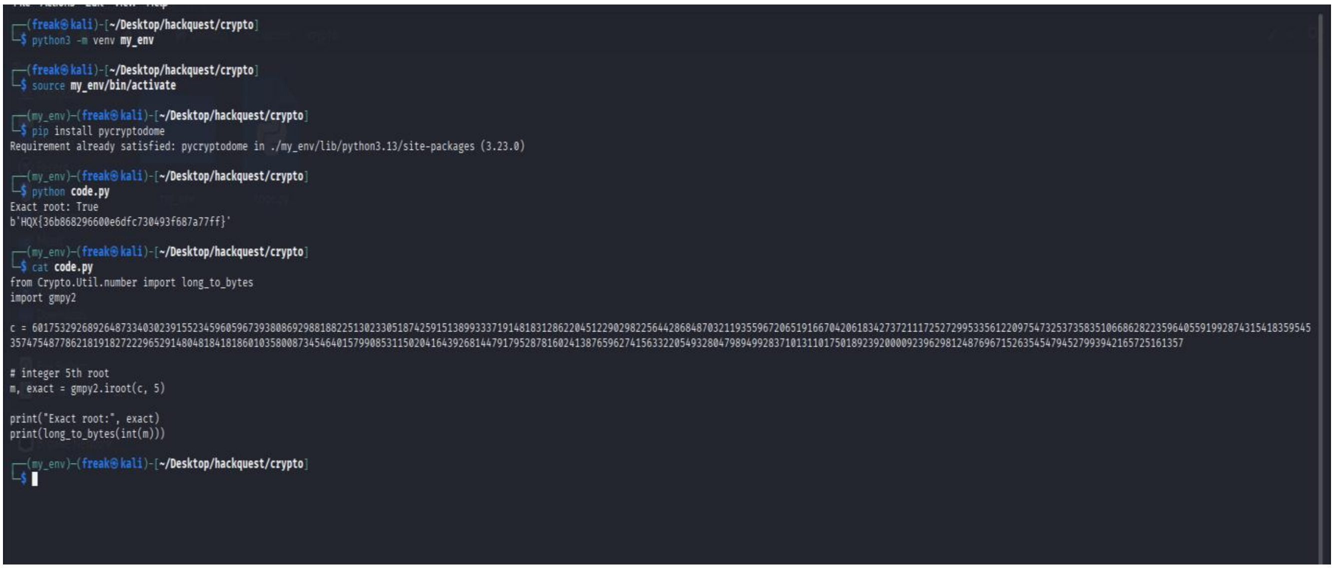
Step 1: Compute the integer e-th root

Using Python:

```
from Crypto.Util.number import long_to_bytes
import gmpy2

m = gmpy2.iroot(ct, e)[0]
print(long_to_bytes(m))
```

Since `m^e < n`, the root operation directly recovers the plaintext.



Result

The decrypted output revealed the original message:

```
HQX{36b868296600e6dfc730493f687a77ff}
```

The flag was successfully recovered without factoring `n`.

Flag Obtained

`HQX{36b868296600e6dfc730493f687a77ff}`

Conclusion

This challenge demonstrated how improper RSA implementation can completely break encryption security. By identifying the use of a small public exponent and lack of padding, the encrypted message was decrypted using a simple mathematical operation. The challenge reinforced the importance of secure parameter selection in public key cryptography.

Security Best Practice Note

In real-world applications:

- Use secure padding schemes like **RSA-OAEP**
- Avoid small public exponents without safeguards
- Never encrypt sensitive data directly with textbook RSA