# MA 1101 : Mathematics I

**Problem 1.**
Define a relation $\sim$ on $\mathbb{R} \times \mathbb{R}$ as

$$(x_1, x_2) \sim (y_1, y_2) \text{ if } x_1 = y_1.$$

(i) Check that $\sim$ is an equivalence relation.

(ii) Identify and draw the equivalence classes.

**Problem 2.**
Define a relation $\sim$ on $\mathbb{R} \times \mathbb{R}$ as

$$(x_1, x_2) \sim (y_1, y_2) \text{ if } x_1^2 + x_2^2 = y_1^2 + y_2^2.$$

(i) Check that $\sim$ is an equivalence relation.

(ii) Identify and draw the equivalence classes.

**Problem 3.**
Define a relation $\sim$ on $\mathbb{N} \times \mathbb{N}$ as

$$(m, n) \sim (p, q) \text{ if } m + q = n + p.$$

(i) Check that $\sim$ is an equivalence relation.

(ii) Identify and draw the equivalence classes.

**Problem 4.**
Define a relation $\sim$ on $\mathbb{R} \times \mathbb{R} \setminus \{(0, 0)\}$ as

$$(x_1, x_2) \sim (y_1, y_2) \text{ if } (y_1, y_2) = \alpha(x_1, x_2), \text{ for some } \alpha \neq 0.$$

(i) Check that $\sim$ is an equivalence relation.

(ii) Identify and draw the equivalence classes.

**Problem 5.**
Let $n \in \mathbb{N}$ and let $X$ be a set of $n$ elements. Calculate the number of

(i) relations on $X$.

(ii) reflexive relations on $X$.

(iii) symmetric relations on $X$.

(iv) reflexive and symmetric relations on $X$.