MA1201: Mathematics II End-semester Examination 2016

Attempt all questions **Duration: 3 hours** **Full Marks: 50**

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PART A: Multiple choice questions. Each question carries 3 Marks.

Mark the correct choices in OMR sheet. Show detailed calculations in the answer booklet.

OMRs will be collected after 2 hours

Q.1 The solution to the ODE $ty' + 2y = 4t^2$ given the initial condition $y(1) = 2$ is

A $y(t) = 2$
B $y(t) = \frac{1}{t^2} + t^2$
C $y(t) = t^2$
D $y(t) = \frac{1}{t^2}$

Q.2 The solution to the ODE $9y'' + 6y' + y = 0$ given the initial conditions $y(0) = 1, y'(0) = 5/3$ is

A $y(t) = \exp(-t/3)$
B $y(t) = 2t\exp(-t/3)$
C $y(t) = (1 + 2t)\exp(-t/3)$
D $y(t) = (3 + 2t)\exp(-t/3)$

Q.3 If we seek a power-series solution $y(x) = \sum_{n=0}^{\infty} a_n x^{n+s}$, the indical equation is given by

A $4s(s-1) + 2s = 0$
B $2(n+s+1)(2n+2s+1) = 0$
C $4s(s-1) + 2(s-1) = 0$
D $2(n+1)(2n+1) = 0$

Q.4 The general solution to the ODE $x^2y'' - 3xy' + 4y = 0$ can be written as

A $y(x) = c_1x^2 + c_2\log(|x|)$
B $y(x) = c_1x^2 + c_2$
C $y(x) = c_1x^2 + c_2x^2\log(|x|)$
D $y(x) = c_1\sin(x) + c_2\cos(x)$

Q.5 The stability analysis of the equation $x' = rx - x^3$ shows that there are

A 1 unstable, 2 stable fixed points if $r > 0$
B 1 unstable, 2 stable fixed points if $r < 0$
C 1 stable fixed point if $r > 0$
D 1 unstable fixed point if $r < 0$

Q.6 The general solution to the PDE $u_t + cu_x = 0$ for $-\infty < x < \infty$, $t > 0$ given the initial condition $u(x, 0) = f(x)$ is given by

A $u(x,t) =$ constant
B $u(x,t) = f(x - ct) + f(x + ct)$
C $u(x,t) = f(x + ct)$
D $u(x,t) = f(x - ct)$

Q.7 The solution to the 1-d heat equation $u_t = c^2u_{xx}$ satisfying the boundary conditions $u_x(0,t) = 0, u_x(L,t) = 0$ and the initial condition $u(x, 0) = f(x)$ is

A $u(x,t) = \sum_{n=0}^{\infty} A_n \exp[c^2n^2\pi^2t/L]\cos(n\pi x/L)$
B $u(x,t) = \sum_{n=0}^{\infty} A_n \exp[-c^2n^2\pi^2t/L]\cos(n\pi x/L)$
C $u(x,t) = \sum_{n=0}^{\infty} A_n \exp[-c^2n^2\pi^2t/L]\sin(n\pi x/L)$
D $u(x,t) = \sum_{n=0}^{\infty} A_n \exp[c^2n^2\pi^2t/L]\sin(n\pi x/L)$

Q.8 There are two boxes I and II, box I contains 1 red ball and 3 blue balls, box II contains 2 red balls and 1 blue ball. The value of the conditional probability that a ball is drawn from box I given that the color of the ball is blue, i.e., $P(\text{I}|\text{blue})$, is

A 5/11
B 5/8
C 1/8
D 3/11

Q.9 The variance of the number of successes after n trials of a Bernoulli process is n/4. The probability of success at each step of the Bernoulli trial is

A 1/4
B 3/4
C 1/2
D 2/3

Q.10 The mean outcome of the roll of a six sided fair die is 7/2. The variance of the outcomes is

A 7/2
B 35/12
C 35/72
D 1/6

P.T.O.

PART B: Each question carries 5 Marks.

Problem 1

5 Marks

Show that the Binomial distribution for large n and small success p, reduces to a Poisson distribution with mean, $\lambda = n\ p$.

Problem 2

5 Marks

Find the solution of the PDE

$$\begin{aligned} u_{xx} + u_{yy} &= 0 \\ \text{given that} \quad u(0,y) &= 0, \qquad u(L,y) = 0 \\ u(x,0) &= 0, \qquad u(x,H) = f(x) \end{aligned}$$

Problem 3

5 Marks

Find the particular solution, $y_p(t)$, to the ODE

$$t^2 y'' - 2y = 3t^2 - 1$$

given that $y_1(t) = t^2$ and $y_2(t) = 1/t$ are the solutions of the corresponding homogeneous equation.

Problem 4

5 Marks

Solve the PDE $\quad u_t - x\ u_x = 0$, satisfying the initial condition $u(x, t=0) = f(x) = \frac{1}{1+x^2}$.