ERROR ANALYSIS

I Introduction

Every measured physical quantity has an **uncertainty or error** associated with it. An experiment, in general, involves
(i) **Direct measurement** of various quantities (primary measurements) and
(ii) **Calculation** of the physical quantity of interest which is a function of the measured quantities. An uncertainty or error in the final result arises because of the errors in the primary measurements (assuming that there is no approximation involved in the calculation). Error analysis, therefore, consists of
(i) **Estimating** the errors in all primary measurements, and
(ii) **Propagating** the error at each step of the calculation. This analysis serves two purposes. First, the error in the final result is an indication of the precision of the measurement and, therefore, an important part of the result. Second, the analysis also tells us which primary measurement is causing more error than others and thus indicates the direction for further improvement of the experiment.

For example, in measuring 'g' with a simple pendulum, if the error analysis reveals that the errors in 'g' caused by measurements of l (length of the pendulum) and T (time period) are 0.5 cm/sec^2 and 3.5 cm/sec^2 respectively, then we know that there is no point in trying to devise a more accurate measurement of l. Rather, we should try to reduce the uncertainty in T by counting a larger number of periods or using a better device to measure time. Thus, error analysis **prior to the next improved version of the experiment** is an important aspect of planning.

Nomenclature
(i) **'Blunder'** denotes measured values that are totally wrong.
(ii) **'Systematic errors'** are errors which occur in every measurement in the same way-often in the same direction and of the same magnitude, e.g., length measurement with a faulty scale. These errors can, in principle, be eliminated or corrected for.
(iii) **'Random errors'** are errors which can cause the result of a measurement to deviate in either direction from its true value. We shall confine our attention to these errors, and discuss them under two heads: estimated and statistical errors.

II Estimated Errors

Estimating a primary error

An estimated error is an estimate of the maximum extent to which a measured quantity might deviate from its true value. For a primary measurement, the estimated error is often taken to be the "least count" of the measuring instrument. For example, if the length of a string is to be measured with a meter stick, the limiting factor of the accuracy in the least count, i.e. 0.1 cm. A note of caution is needed here.

What matters really is the **effective least count** and **not** the nominal least count. For example, in measuring electric current with an ammeter, if the smallest division corresponds to 0.1 amp, but the marks are far enough apart so that you can easily make out a quarter of a division, then the effective least count will be 0.025 amp. On the other hand, if you are reading a vernier scale where five successive marks on the vernier scale (say, 27th to 31st) look equally well in coincidence with the main scale, the effective least count is ~ 2 times the nominal one. Therefore, **make a judicious estimate of the least count.** The estimated error need not always be the **least count**.

Propagation of estimated errors

How to calculate the error associated with f, which is a function of the measured quantities a, b, and c?

Let $f = f(a, b, c).$ (1)

From differential calculus (Taylor's series in the 1st order)

$$df = \frac{\partial f}{\partial a} da + \frac{\partial f}{\partial b} db + \frac{\partial f}{\partial c} dc \ . \quad (2)$$

Eq. (2) relates the differential increment in f resulting from differential increments in a, b, c. Thus, if our errors in a, b, c (denoted as δa, δb, δc) are small compared to a, b, c, respectively, then we may say

$$\delta f = \left|\frac{\partial f}{\partial a}\right| \delta a + \left|\frac{\partial f}{\partial b}\right| \delta b + \left|\frac{\partial f}{\partial c}\right| \delta c \ . \quad (3)$$

where the modulus signs have been put because errors in a, b, and c are **independent** of each other and may be in the **positive or negative** direction. Therefore, the maximum possible error will be obtained only by adding absolute values of all the independent contributions. (All the δ's are considered positive by definition). Special care has to be taken when all errors are not independent of each other. This will become clear in special case (V) below.

Some simple cases

(i) For **addition or subtraction**, the absolute errors are added, e.g.,

$$\text{if } f = a + b - c \ , \ \text{then}$$
$$\delta f = \delta a + \delta b + \delta c \ . \quad (4)$$

(ii) For **multiplication and division,** the fractional (or percent) errors are added, e.g.,

$$\text{if } f = \frac{ab}{c} \ , \ \text{then}$$
$$\left|\frac{1}{f}\right| \delta f = \left|\frac{1}{a}\right| \delta a + \left|\frac{1}{b}\right| \delta b + \left|\frac{1}{c}\right| \delta c \ . \quad (5)$$

(iii) For raising to constant **powers**, including fractional powers, the fractional error is multiplied by the power, e.g.,

$$\text{if } f = a^{3.6} \ , \ \text{then}$$
$$\left|\frac{1}{f}\right| \delta f = \left|3.6 \times \frac{1}{a}\right| \delta a \ . \quad (6)$$

(iv) In **mixed calculations,** break up the calculation into simple parts, e.g.,

$$\text{if } f = \frac{a}{b} - c^{\frac{3}{2}} \ , \text{then}$$
$$\delta f = \left|\frac{1}{b}\right| \delta a + \left|\frac{a}{b^2}\right| \delta b + \left|\frac{3}{2} c^{\frac{1}{2}}\right| \delta c \ . \quad (7)$$

(v) *Consider* $f = \frac{ab}{c} - a^2 \ .$

The relation for error, **before** putting the modulus signs, is

$$\delta f = \left(\frac{b}{c}\right)\delta a + \left(\frac{a}{c}\right)\delta b - \left(\frac{ab}{c^2}\right)\delta c - 2a\delta a \quad .$$

Note that the δa factors in the first and fourth terms are **not** independent errors. Therefore, we must **not** add the absolute values of these two terms indiscriminately. Instead collect the coefficients of each independent errors **before** putting modulus signs, i.e.,

$$\delta f = \left|\frac{b}{c} - 2a\right|\delta a + \left|\frac{a}{c}\right|\delta b + \left|\frac{ab}{c^2}\right|\delta c \quad . \qquad (8)$$

III Statistical Errors

Statistical distribution and standard deviation

Statistical errors arise when making measurements on **random processes,** e.g., counting particles emitted by a radioactive source. Suppose we have a source giving off 10 particles/sec. on the average. In order to evaluate this count rate experimentally, we count the number of particles for, say, 20 seconds. Shall we get 200 counts? Not necessarily. In fact, we may get any number between zero and infinity. Therefore, in a measurement on a random process, **one cannot specify a maximum possible error**. A good measure of uncertainty in such a case is the standard deviation (s.d.) which specifies the range within which the result of any measurement is **"most likely"** to be. The exact definition of "most likely" depends on the distribution governing the random events. For all random processes whose probability of occurrence is small and constant, **Poisson distribution** is applicable, i.e.,

$$P_n = \frac{m^n}{n!}e^{-m} \quad , \qquad (9)$$

where P_n is the probability that you will observe a particular count n, when the expectation value is m.

It can be shown that if an infinite number of measurements are made, (i) their average would be **m** and (ii) their standard deviation (s.d.) would be $\sqrt{\mathbf{m}}$, for this distribution. Also, if m is not too small, then **68%** or nearly two-thirds of the measurements would yield numbers within one s.d. in the range $\mathbf{m \pm \sqrt{m}}$.

In radioactive decay and other nuclear processes, the Poisson distribution is generally valid. This means that we have a way of making certain conclusions without making an infinite number of measurements. Thus, if we measure the number of counts only once, say for 100 sec , and the number is, say 1608, then (i) our result for average count rate is 16.08/sec , and (ii) the standard deviation is $\sqrt{1608} = 40.1$ counts which corresponds to 0.401/sec. So our result for the count rate is **(16.1 ± 0.4) sec^{-1}.** The meaning of this statement must be remembered. **The actual count rate need not necessarily lie within this range, but there is 68% probability that it lies in that range.**

The experimental definition of s.d. for k measurements of a quantity x is

$$\sigma_x = \sqrt{\sum_{n=1}^{k}\left(\frac{\delta x_n^2}{k-1}\right)} \quad , \qquad (10)$$

where δx_n is the deviation of measurement x_n from the mean. However, since we know the distribution, we can ascribe the s.d. even to a single measurement.

Propagation of statistical errors

For a function f of independent measurements a, b, c, the statistical error σ_f is

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial a}\sigma_a\right)^2 + \left(\frac{\partial f}{\partial b}\sigma_b\right)^2 + \left(\frac{\partial f}{\partial c}\sigma_c\right)^2} \quad . \qquad (11)$$

A few simple cases are discussed below.

(i) For **addition or subtraction,** the **squares of errors** are added, e.g.

$$\begin{aligned} &\textit{if} \quad f = a + b - c \\ &\textit{then}, \quad \sigma_f^2 = \sigma_a^2 + \sigma_b^2 + \sigma_c^2 \quad . \end{aligned} \qquad (12)$$

(ii) For **multiplication or division**, the **squares of fractional errors** are added, e.g.

$$\begin{aligned} &\textit{if} \quad f = \frac{ab}{c}, \\ &\textit{then} \quad \left(\frac{\sigma_f}{f}\right)^2 = \left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2 + \left(\frac{\sigma_c}{c}\right)^2 \quad . \end{aligned} \qquad (13)$$

(iii) If a measurement is repeated n times, the error in the mean is a factor $\sqrt{\mathbf{n}}$ **less** than the error in a single measurement, i.e.,

$$\sigma_{\bar{f}} = \frac{\sigma_f}{\sqrt{n}} \quad . \qquad (14)$$

Note that Eqs. (11-14) apply to any statistical quantities a, b, etc, i.e., primary measurements as well as computed quantities whereas

$$\sigma_m = \sqrt{m} \qquad (15)$$

applies only to a directly measured number, say, number of α-particle counts **but not to computed quantities like** count rate.

IV Miscellaneous

Repeated measurements

Suppose a quantity f, whether statistical in nature or **otherwise**, is measured n times. The best estimate for the actual value of f is the $\bar{f}$ average of all measurements. It can be shown that this is the value with respect to which the sum of squares of all deviations is a minimum. Further, **if** errors are assumed to be randomly distributed, the error in the **mean value** is given by

$$\delta_{\bar{f}} = \frac{\delta_f}{\sqrt{n}} \quad , \qquad (16)$$

where δ_f is the error in one measurement. Hence one way of **minimizing random errors** is to **repeat the measurement** many times.

Significant figures

A result statement such as **f = 123.4678 ± 1.2331 cm** contains many superfluous digits. Firstly, the digits 678 in quantity f do not mean anything because they represent something much smaller than the uncertainty δf. Secondly δf is an approximate estimate for error and should not need more than two significant figures. The correct statement would be **f = 123.5 ± 1.2 cm**.

V Instructions

1. Calculate the **estimated/statistical error** for the final result. In any graph you plot, show **typical error bars.**
2. If the same quantity is measured/calculated many times, you need not determine the errors each time, similarly one **typical error bar** on the graph will be enough.
3. Where the final result is a known quantity (for example, e/m), show the **deviation** of your result from the **standard value**. If this is much greater or even less than the **estimated error**, this is abnormal and **requires explanation.**
4. Where a quantity is determined many times, the standard deviation should be calculated from Eq.(10). Normally, **the s.d. should not be much different from the estimated error.**

VI Mean and Standard Deviation

If we make a measurement x_1 of a quantity x, we expect our observation to be close to the quantity but not exact. If we make another measurement we expect a difference in the observed value due to random errors. As we make more and more measurements we expect them to be distributed around the correct value, assuming that we can neglect or **correct for systematic errors and avoid blunders.** If we make a very large number of measurements we can determine how the data points are distributed in the so-called parent **distribution**. In any practical case, one makes a finite number of measurements and one tries to describe the parent distribution as best as possible.
Consider N measurements of quantity x, yielding values $x_1, x_2 \ldots x_N$. One defines

$$Mean \;:\; \bar{x} \;=\; \lim_{N\to\infty}\left[\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)\right] \;, \qquad (17)$$

which is equivalent to the centroid or average value of the quantity x.

Deviations: The deviation d_i of any measurement x_i from the mean $\bar{x}$ of the parent distribution is defined as $d_i = x_i - \bar{x}$ Note that if $\bar{x}$ is the true value of the quantity being measured, d_i is also the true error in x_i .

The arithmetic average of the deviations for an infinite number of observations must vanish, by definition of (Eq.(18). There are several indices one can use to indicate the spread (dispersion) of the measurements about the central value, i.e., the mean value. The dispersion is a measure of precision. One can define average deviation d as the average of the **magnitudes** of the deviations (absolute values of the deviations)

$$d \;=\; \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\left(\left|x_i - \bar{x}\right|\right) \;.$$

This can be used as a measure of the dispersion of the expected observation about the mean. However, a more appropriate measure of the dispersion is found in the parameter called standard deviation σ, defined as

$$\sigma^2 \;=\; \lim_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2\right] \;=\; \lim_{N\to\infty}\left(\frac{1}{N}\sum x_i^2\right) - \left(\bar{x}\right)^2 . \qquad (18)$$

σ^2 is known as **VARIANCE and the STANDARD DEVIATION** σ is the square root of the variance. In other words it is the root mean square (rms) of the deviations given by

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} d_i^2}{(N-1)}} \qquad (19)$$

where the denominator is N-1 ~ N for large N.

Significance: $\bar{x}$, the mean, is a parameter which characterizes the information we are seeking when we perform an experiment. The mean is, of course, not the most probable value if the parent distribution is not symmetrical but nevertheless it is a reasonable parameter to characterize the distribution. In place of mean, one can characterize the distribution in terms of **median** or **most probable value (mode)**. It can be proved that if we use the average **(mean)** of the measured values for calculating the deviations, the sum of **the square of the deviations is a minimum**. The standard deviation is ***simply*** related to this minimum value of the square of the deviations and is used for specifying error quantitatively.

The standard deviation characterizes the uncertainties associated with our experimental attempts to determine the ***"true"*** value, namely the mean value for all practical purposes. σ, for a given finite number of observations, is the uncertainty in determining the mean of the parent distribution. Thus it is an appropriate measure of the uncertainty in the observations.

VII Method of Least Squares (Linear)

Our data consist of pairs of measurements (x_i, y_i) of an independent variable x and a dependent variable y. We wish to fit the data to an equation of the form

$$y = a + bx \qquad (20)$$

by determining the values of the coefficients a and b such that the discrepancy is minimized between the values of our measurements y_i and the corresponding values $y = f(x_i)$ given by Eq. (24). We **cannot determine the coefficients exactly** with only a **finite number of observations**, but we do want to extract from these data the most probable estimates for the coefficients.

The problem is to establish **criteria for minimizing the discrepancy** and optimizing the estimates of the coefficients. For any arbitrary values of a and b, we can calculate the deviations δy_i between each of the observed values y_i and the corresponding calculated values

$$\delta y_i = y_i - a - bx_i \quad . \qquad (21)$$

If the coefficients are well chosen, these deviations should be relatively small. The sum of these deviations is **not a good measure** of how well we have approximated the data with our calculated straight line because large positive deviations can be balanced by large negative ones to yield a small sum even when the fit is bad. We might however consider **summing up the absolute values** of the deviations, but this leads to difficulties in obtaining an analytical solution. We consider instead the sum of the squares of deviations.

There is no unique correct method for optimizing the coefficients which is valid for all cases. There exists, however, a method which can be **fairly well justified**,

which is **simple and straightforward**, and is **accepted by convention**. This is the method of least squares which uses, in turn, the method of maximum likelihood. This is explained in greater details in the longer version of this note which I followed in my initial 3-hr lecture in early August, 2012.

Examples from UG Lab Set-up

EXAMPLE: Not specific to any experiment

a) Where the final result is a known quantity, show the **deviation** of your result from the **standard value**. If this is much greater or even less than the **estimated error**, this is abnormal and **requires explanation.**
b) Where a quantity is determined many times, say 10, the standard deviation should be calculated from Eq. (10). Normally, **the s.d. should not be much different from the estimated error.**
c) Where a quantity is determined, say, only 3 times, the **spread** about the mean **should not be much different from the estimated error.**

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d) Given data: Radius of an object = 2.4 or 2.42 or 2.425. What % error will you assign ?
e) Find the radius of, say, a hazy (not sharp) diffraction ring from a circular aperture and quote its error. How will you do this ?

Experiment #1: Air Track:Verification of Newton's law

Dependence of acceleration (a) on the applied force (F) for a fixed mass: The error in the slope (i.e., mass) of the ***F*** vs ***a*** plot, as given by the linear fit programme, is an indicator of the accuracy of the data as well as the degree of validity of Newton's second law. Compare with the directly measured mass. The difference should be ~ error in slope (i.e., mass).

Dependence of acceleration (a) on the mass (m) of the slider for a fixed applied Force (F): Thinking along the same line, here also one can find the error in the slope (i.e., force applied) from the ***m*** vs **1/*a*** graph and check whether the error is consistent with the difference between the directly measured value of the force applied and the above slope.

Experiment #2: Study of projectile motion

Measurements and errors:

Measure angle of projection θ, initial velocity v, and range R for 2 plunger positions and 3 values of θ.
For each combination, calculate range from $R = (v^2 \sin 2\theta)/g$ and compare with the average value of R found above.
Is the difference lying within the s.d of R ?

For a given combination plunger position and θ, find (x,y) of at least 6 points. Fit a parabola (using path equation) and use its χ^2 as a measure of the goodness of the fit (i.e.,

how well does a parabola describe your data) as well as the accuracy of y. You can check the latter by measuring y, say 5 times, for any given value of x.

Experiment #4: Study of reversible pendulum

While calculating the error in g given by $\boldsymbol{g = 4\pi^2(a+b)^2/T^2}$, you will find ~ 0.1% error in a+b, much more will come from T. So T needs most attention. Just keep in my mind the note given below which is applicable to all experiments where T is to be measured.

NOTE: For any experiment where you have to measure time-period T, take as many oscillations as possible. This will minimize % error in the measured total time, t (What is the error in t? ~ 0.4 s. Why ???). However, on the other hand, damping will show up if you take too many oscillations. You have to strike a compromise.

Experiment #5: Study of coupled pendula

Measurements and errors

Measure M_{bob}, M_{rod}, L, and h (Fig. 2) and l (Fig. 3): their errors are trivial to calculate.

For each l = 25, 35, and 45 cm do:
In ω_+ configuration (Page 4), measure time for at least 20 oscillations and repeat at least 4 times, calculate ω_+ and find its error.
Measure ω_{beat} as suggested (Page 5) and estimate its error. Calculate ω_- and its error.
Finally find κ_{exp} from Eq. (11), its error as well as the spread for the 3 values of l.
Is the **spread** compatible with the **estimated** error ?
Estimate theoretically κ_{estim} from Eq. (18) by measuring m, L_S and d. The difference between κ_{exp} and κ_{estim} should be within the **error bar/spread** of κ_{exp}.

Experiment #7: Determination of Young's modulus

Measurements and errors:

Measure breadth (b), thickness (t), length (L) of the bar.
Measure average depression d as a function of mass, m added (increasing and decreasing).
Plot d vs. m and find the slope of the best-fitted straight line along with its s.d. The error in the slope is s.d/√N, where N = no. of data points (no. of values of m).
Find $Y = m\ g\ L^3/4\ b\ d\ t^3$ and its **estimated** error.
Again, be careful about measurements of L and t. Why ???
Make sure that the **deviation** from the standard value of Y is consistent with the **estimated error.**

Experiment #8: Determination of modulus of rigidity

Measurements and errors:

Measure length (l) of the wire, its diameter and hence the radius (r), diameter of the mass block and hence its radius R. Mass, M of the mass block will be supplied to you.
Rotate the mass block by < 10° and release it to oscillate. Measure time, t taken for 20 oscillations and repeat it 10 times. Calculate T.

Find errors in l, r, R, M and T & hence the estimated error in $\eta = 4\pi M l R^2 / T^2 R^4$ after error propagation.

Which of these, l, r, R, M, and T need more attention than others?

NOTE: For any experiment where you have to measure time-period T, take as many oscillations as possible. This will minimize % error in the measured total time, t (What is the error in t? ~ 0.4 s. Why???). However, on the other hand, damping will show up if you take too many oscillations. You have to strike a compromise.

Experiment #9: Surface tension by direct force measurement

Measurements and errors:

Measure the diameter of the metal ring (a few times), the effective L.C of the caliper will give you the % error.
The tensile force F is to be measured (at least 10 times) for each of the 2 rings and 2 liquids. The error in the mean of F could be taken as the s.d of the 10 values divided by $\sqrt{10}$.
The rest is trivial for estimating the error in γ.
The spread in the 2 values of γ for each ring and their **deviations** from the standard values should be consistent with the **estimated error**.

Experiment #10: Determination of the coefficient of viscosity using Stokes' law

Principle of the experiment: Steel balls of radius r (a few mm) are released from rest and they fall through a viscous liquid in a cylindrical glass container of radius R (a few cm). The radii are measured with a screw gauge and a slide caliper (I hope you know what these instruments are and how to use them. Also, Vernier Constant, etc.).The balls, soon after they are released, attain terminal velocity which is measured by timing (stop-watch) the fall through a distance D, typically of ~ 50 cm.
Measurements and errors: Errors in r and R are less/much less than 1 %. Why?
If error in r is x %, what is the % error in the density?
Error in D ? 0.1 %. Why?
Typical time of fall is 3-10 s and then what is the % error of time measurement?
Answer: >10 % if time is 3 s.
Hint: LC of stop-watch or your reaction time, which is the effective LC?

Experiment #11: Determination of Joule's constant (Mechanical equivalent of heat)

Measurements and errors

Measure mass (m) and diameter and hence the radius(R) of the aluminum cylinder, mass (m_b) of the empty bucket (along with the tied nylon rope).
For 3 different masses put in the bucket (say, 1.5, 2.5, and 3.5 kg), measure room temperature (T_R), starting (T_i) and stopping temperature (T_f) for N full turns of the crank.
Error estimations for m, R, m_b, T_R , and N are simple(1/2 of effective LC) but those of T_i and T_i are quite tricky. Find the best way of estimating them. Unless you do it properly, the **deviation** from the standard value of J as well as the spread of your 3 values will be much more than the **estimated error.**

Experiment #12: Determination of linear coefficient of thermal expansion

Measurements and errors:

Materials: copper, brass, and aluminum, measure each 3 times (9 sets).

For each set, measure rotational change ($\Delta\theta$) of Rotary Motion Sensor, temperature change (ΔT), Radius (*r*) of the Rotary Sensor pin, and length L of the metal tube.

Errors in ΔT and $\Delta\theta$ to be found from their stability initially and again after becoming almost constant. Errors in r and L from half of the respective effective least count (LC).

NOTE: For each metal the **spread** in the 3 values of α as well as the **deviation** from the standard value must be consistent with the **estimated error.**

Experiment #13: Measurement of specific heat capacity

Principle of the experiment: Known mass (m_1) of a solid is heated to a known temperature (T_1). It is then put into a known mass (m_w) of water kept in a calorimeter at a known temperature (T_w). Heat exchange takes place between the water and calorimeter and the solid and they together come to an equilibrium temperature (T_e) which is measured. Using the condition that heat lost by solid is equal to the heat absorbed by the water and calorimeter, the specific heat capacity of the solid is

$$c = [(C + c_w m_w).(T_e - T_w)] / [m_1.(T_1 - T_e)],$$

where C = heat capacity of the calorimeter (supplied to be 80 J/°C) and c_w = specific heat capacity of water, known to be 4.19 J/g°C.
Measurements and errors: Measured quantities are m_1, m_w , T_1, T_w, and T_e.
Which ones have to be measured more accurately than the others?
m_1, and m_w are typically a few hundred g and if the accuracy of the balance is 0.1 g, their accuracy is a lot better than 0.1 %. Typical T_1 is ~ 100 °C, T_w ~ 30 °C, and T_e ~ 40 °C. Their accuracies are ~ 0.25 °C (1/2 of LC of 1/2 °C). If you use a magnifying glass, the

error could still be below 0.25 °C. Typical T_e - T_w ~ 10 °C and hence has 5 % error while T_1 - T_e (~ 60 °C) has < 1 % error. So T_e - T_w needs most attention than T_1 - T_e.

Experiment #15: Study of single slit and double slit diffraction pattern

To determine the slit width, d for the **single slit** diffraction, measure the linear position, *y* of the intensity minima, the distance between the slit and the detector, *D* and use the known value of the wavelength of light used, λ.
Error in D is trivial to find.
Measure y several times for each minima on both sides. Their spread could be used as a measure of its error.
The estimated error in d should be comparable with the difference between the experimental value and the value supplied to you.

For the **double slit** diffraction, perform similar measurements as for the single slit and determine the slit width (a) and the slit separation (d). Proceed similarly to estimate their errors, etc.

Experiment #16: Study of Malus' law

As suggested in the manual, comment on the accuracies in angle and intensity measurements and compare these with parameters *c* and *a*, respectively. Are they comparable? If not, why not?

The deviation between the experimental and the analytical values of $\cos^2 \varphi$ vs. φ could be used as an indicator of the overall experimental accuracy.

Experiment #18: Equipotential lines and electrostatic field mapping

This experiment is very interesting and educative and needs experimental skill. Good plots are signs of accurate measurements. It is difficult to quantify errors here (may not be necessary as well). However, you should suggest how to take good/better data and discuss in brief the possible sources of error.

Experiment #19: Magnetic field mapping

Measurements and errors

Make sure that B = 0 when I = 0.
Keep the current through the coil constant (~ 400 mA) throughout the experiment.
Measure B, say at x = 0. What is the error in B? (Field stability for a constant current is a possible indicator). Compute the error in the calculated B from Eq. (1).
Find the coil radius 'a' from Eq. (1) using the above B, I, and N = 500 and compare with the value measured with a scale. Is the difference compatible with the estimated error ?

Superposition principle (SP):

For all values of x, measure B(x) with currents in Coil 1 alone (Graph 1), then Coil 2 alone (Graph 2), then both the coils together (Graph 3). Plot the 3 graphs.

On the same plot, assuming the superposition principle for magnetic fields, show the sum of the first 2 curves (Graph 4) and compare with Graph 3.

The difference could be used as a measure of the accuracy of the experiment as well as the validity of the SP. It will be instructive to plot the difference vs. x.

Discuss your results.