IISER Kolkata
Mid-Semester Examination
First Year Semester I; 2015
Time One Hour; Full Marks 20
Answer all questions

1. Calculate the area bounded by the pair of curves $y = 4x - x^2$, and $y = x$.

2. A curve is given by $x^2 - 4x + y^2 = 0$. Find the equation of the tangent and normal to the curve at $(0, 0)$.

3. Find $\frac{du}{dt}$ where $u = e^x \sin y + e^y \sin x$; $[x = \frac{1}{2t}; y = 2t]$.

4. Evaluate $\int \mathbf{A}.\frac{d\mathbf{A}}{dt} dt$ between points P and Q where $\mathbf{A}(P) = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{A}(Q) = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.

5. Check whether the vector $\mathbf{F} = r\mathbf{r}$ is conservative or not. If it is, find a scalar function ϕ so that $\mathbf{F} = -\nabla\phi$.

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$f(x) = \frac{f^0(x)}{0!}(x-a)^0 + (-1)^1 \frac{f'(x)}{1!}(x-a)^1 + (-1)^2 \frac{f^2(x)}{2!}(x-a)^2$

IISER Kolkata
End-Semester Examination
First Year Semester I; 2015
MA1101: Mathematics-I
Time Three Hours; Full Marks 50
Answer all questions (Q1-5: 4 marks; Q6-10: 6 marks)

$4axy^2 + 6xy^3 - 3xy^3$

$58xy^3 \cdot dy$

1. Evaluate $\int_{(0,0)}^{(2,1)}\{(10x^4 - 2xy^3)dx - 3x^2y^2dy\}$ along the path $x^4 - 6xy^3 = 4y^2$.

2. Evaluate $\int_1^2 \mathbf{A}.(\mathbf{B}\times\mathbf{C})dt$ where $\mathbf{A} = t\mathbf{i} - 3\mathbf{j} + 2t\mathbf{k}$, $\mathbf{B} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{C} = 3\mathbf{i} + t\mathbf{j} - \mathbf{k}$.

3. Evaluate $\int\int_S \mathbf{r}.d\mathbf{S}$, either directly or by using any theorem that you know, where the surface S encloses a sphere of radius unity and $\mathbf{r}$ is the position vector.

4. Show that $\nabla r^3 = 3r\mathbf{r}$.

5. The area of an ellipse is given as $\frac{1}{2}\int_C (\cancel{xy}\ xdy - ydx)$. Find the area of the ellipse $x = a\cos\theta, y = b\sin\theta$.

6. A system of linear equations is given by
$3x + y + 2z = 3,$
$2x - 3y - z = -2,$ and
$x + y + z = 1.$
Write down the coefficient matrix. Find its inverse and hence find a solution for the system of equations.

7. A matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 4 \end{pmatrix}.$$

Find the eigenvalues of the matrix. Find out the eigen vectors. Check that one can form an orthonormal basis with these vectors. Show by direct calculation that you can construct a matrix **X** out of these eigenvectors which diagonalizes the matrix **A** by a similarity transformation.

8 (a). Show that $(\mathbf{AB})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1}$ where **A** and **B** are two Hermitian commuting square matrices.

(b). **A** and **B** are Hermitian square matrices. If **x** is an eigenvector of **A**, shoow that it is also an eigenvector of **B**.

9(a). With the help of a comparison test, show that the harmonic series
$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ...$
is a divergent series.

(b). With the help of a ratio test. check that
$\frac{3}{4} + 2(\frac{3}{4})^2 + 3(\frac{3}{4})^3 + ...$
is a convergent series.

10. Use Taylor's formula to verify that

(a). $\ln(a+x) = \ln a + \frac{x}{a} - \frac{x^2}{2a^2} + \frac{x^3}{3a^3} + \cdots$

(b). $\tan(\frac{\pi}{4} + x) = 1 + 2x + 2x^2$ for small x. Hence find the value of $\tan 46^o$.