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PH1201 CLASS TEST - 03

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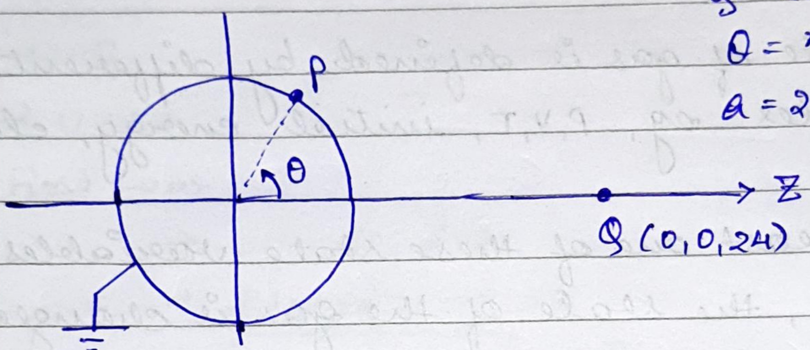
Q5. $R = 10cm$

$Q = 2\times10^{-6}\ C$

$\theta = \pi/3$

$a = 24$



$$\sigma(\theta) = \frac{q}{4\pi R}(R^2-a^2)(R^2+a^2-2Ra\cos\theta)^{-3/2}$$

$$= \frac{2\times10^{-6}}{4\pi\times10\times10^{-2}}(10^{-2}-(24\times10^{-2})^2)$$

$$(10^{-2}+(24\times10^{-2})^2-2\times10^{-1}\times24\times10^{-2}\times0.5)^{-3/2}$$

$$=(1.59\times10^{-6})\times(-0.0476)\times(109.84)$$

$$\simeq -8.3\times10^{-6}\ Cm^{-2}$$

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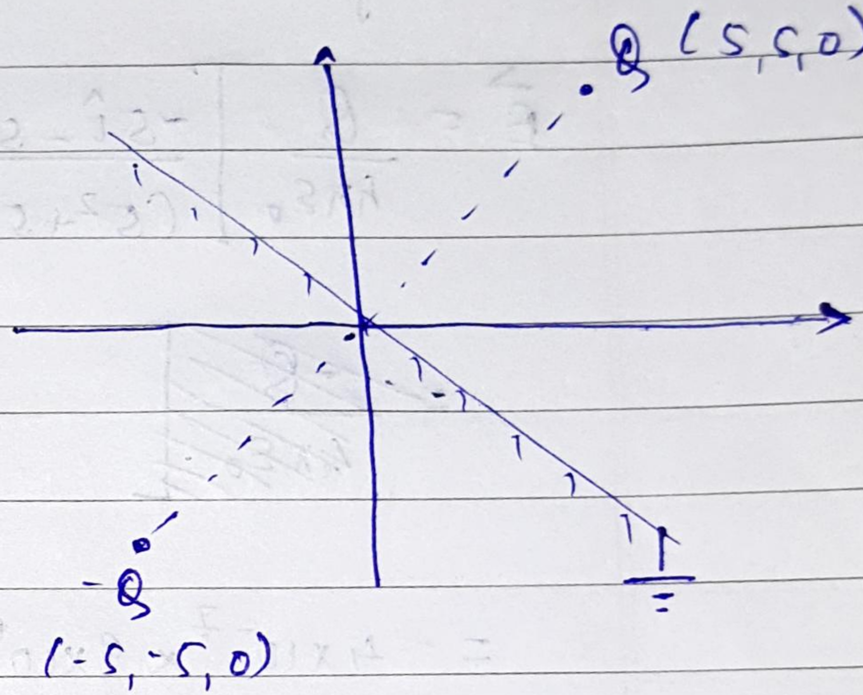
Q4. We can replace the conducting plate with a charge of $-Q$ placed at point $(-5,-5,0)$ m.



Let potential at pt. P (x,y,z) be V.

$\therefore \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r}_1 = 5\hat{i} + 5\hat{j}$

$\vec{r}_2 = -5\hat{i} - 5\hat{j}$

$\vec{R}_1 = \vec{r} - \vec{r}_1 = (x-5)\hat{i} + (y-5)\hat{j} + (z\hat{k})$

$\vec{R}_2 = \vec{r} - \vec{r}_2 = (x+5)\hat{i} + (y-5)\hat{j} + (z\hat{k})$

$$V(\vec{r}) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{|\vec{R}_1|} - \frac{1}{|\vec{R}_2|}\right]$$

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$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{\vec{R_1}}{R_1^3} - \frac{\vec{R_2}}{R_2^3}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{(x-5)\hat{i} + (y-5)\hat{j} + z\hat{k}}{\{(x-5)^2 + (y-5)^2 + z^2\}^{3/2}} - \frac{(x+5)\hat{i} + (y+5)\hat{j} + z\hat{k}}{\{(x+5)^2 + (y+5)^2 + z^2\}^{3/2}}\right]$$

Now, for P $\equiv (0,0,0)$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{-5\hat{i} - 5\hat{j}}{(5^2+5^2)^{3/2}} - \frac{5\hat{i} + 5\hat{j}}{(5^2+5^2)^{3/2}}\right]$$

$$\cancel{= \frac{-Q}{4\pi\varepsilon_0}\Big[}$$

$$= 4\times10^{-7} \times 9\times10^{+9}\left[\frac{-5\hat{i} - 5\hat{j} - 5\hat{i} - 5\hat{j}}{(50)^{3/2}}\right]$$

$$\cancel{= 3.6\times10^{-15} \times \left[\frac{-10\hat{i} - 10\hat{j}}{353.55}\right]}$$

$$\cancel{= -1.018\times10^{-17}(10\hat{i} + 10\hat{j})}$$

$$= -1.018\times10^{-16}(\hat{i} + \hat{j})$$

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$$= 3600 \left[\frac{-10\hat{i} - 10\hat{j}}{(50)^{3/2}} \right]$$

$$= -36000 \left[\frac{\hat{i} + \hat{j}}{(50)^{3/2}} \right]$$

$$= -101.82 \left[\hat{i} + \hat{j} \right]$$

$$\Rightarrow |\vec{E}| = -101.82 \times \sqrt{2}$$

$$= -144$$

Now, for plate,

$$E_{above} + E_{below} = \sigma / \varepsilon_0$$

$$\Rightarrow -144 = \sigma / \varepsilon_0$$

$$\Rightarrow \sigma = -144 \times 8.85 \times 10^{-12} \ Cm^{-2}$$

$$\Rightarrow \boxed{\sigma = -1.27 \times 10^{-9} \ Cm^{-2}}$$

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Q2. $V(x,y) = \dfrac{q}{4\pi\varepsilon_0}\left[\dfrac{1}{\sqrt{(x-a)^2+(y-b)^2+z^2}} + \dfrac{1}{\sqrt{(x+a)^2+(y+b)^2+z^2}}\right.$

$$\left. - \frac{1}{\sqrt{(x+a)^2+(y-b)^2+z^2}} - \frac{1}{\sqrt{(x-a)^2+(y+b)^2+z^2}}\right]$$

where $a=b$;

$$\frac{\partial V}{\partial x} = \frac{q \times (-1)}{4\pi\varepsilon_0 (2)}\left[\frac{2(x-a)}{\{(x-a)^2+(y-a)^2+z^2\}^{3/2}}\right.$$

$$+ \frac{2(x+a)}{\{(x+a)^2+(y+b)^2+z^2\}^{3/2}} - \frac{2(x+a)}{\{(x+a)^2+(y-b)^2+z^2\}^{3/2}}$$

$$\left. - \frac{2(x-a)}{\{(x-a)^2+(y+b)^2+z^2\}^{3/2}}\right].$$

$$\Rightarrow \sigma = -\varepsilon_0 \left(\frac{\partial V}{\partial x}\right)_{(0,9,0)}$$

$$\Rightarrow \sigma = -\varepsilon_0 \left(\frac{\partial V}{\partial x}\right)_{(0,9,0)} = \frac{Q}{8\pi}\left[\frac{-4\times 9}{(81)^{3/2}} + \frac{4\times 9}{(5\times 81)^{3/2}}\right]$$

$$= (0.0795)\left[\frac{-4\times 9}{729} + \frac{4\times 9}{8150.47}\right]$$

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$$\sigma = (0.0795)(-0.0494 + 0.00442)$$

$$\Rightarrow \boxed{\sigma = -3.57 \times 10^{-10}\ cm^{-2}}$$ ans.

Q3. $$\frac{\sigma_A}{\sigma_B} = -\left(\frac{R_1 + d}{R_2 + d}\right)^2$$

$$= -\left(\frac{10+1}{5+1}\right)^2$$

$$= -\frac{121}{6}$$

$$\Rightarrow \frac{\sigma_A}{\sigma_B} = -20.17.$$