PH1202 Experiment No. 05

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Contents

1 Aim

Magnetic Field Mapping using two identical coils

- Study of magnetic field due to one coil and calculation of its diameter.
- Study of *principle of superposition* of magnetic fields due to two coils by keeping the distance between the coils at a, where a is the radius of the coil, i.e., at *Helmholtz Configuration*.
- Study the magnetic field at a distance of *4 cm* on either side from the Helmholtz configuration.

2 Setup

1. Study of magnetic field due to one coil and calculation of its diameter.
2. Study of Principal of superposition of magnetic fields due to two coils by keeping the distance between the coils at a, where a is the radius of the coil, i.e., at Helmholtz configuration.
3. Study the magnetic field in the Helmholtz configuration and at a distance of 5 cm on either side from the Helmholtz configuration.

Figure 1: A photograph of the actual setup

A photograph of the entire apparatus is shown above. It consists of:

1. a pair of identical magnetic coils (having 500 number of turns at each coils), one of the coils is fixed and the other can move along its axis smoothly using a rail attached on it.
2. a Hall probe Gaussmeter head, mounted on another rail and can move along their common axis.
3. a constant current source and digital Gaussmeter are mounted on a common unit.

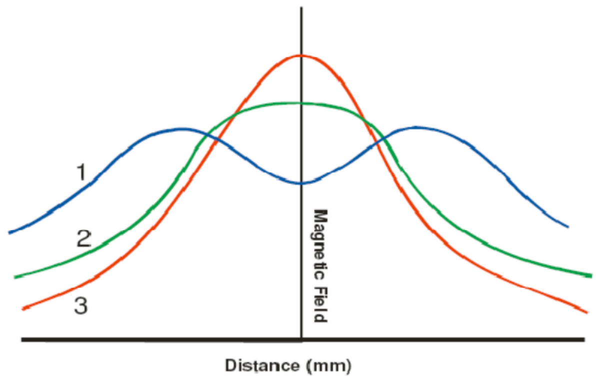

Figure 2: Demonstration of magnetic field profiles.
Line1: when the distance between the coils is > a;
Line 2: when the distance between the coils is = a;
and Line 3: when the distance between the coils is < a.

3 Description of the Apparartus

1. **Constant current source for coils**

   It is an IC regulated constant current source.

   Specification:

   (a) Current Range : $0 - 500\ mA$
   (b) Line Regulation : $\pm 0.2\%\ for\ \pm 10\%\ mains\ variation$
   (c) Load Regulation : $\pm 0.2\%\ for\ no\ load\ to\ full\ load$
   (d) Display : $3\frac{1}{2}\ digit\ 7\ segment\ LED\ display$
   (e) Power : $220 \pm 10\%\ Mains$
   (f) Protection : Protected against overload/short circuit

   The provision have been made to connect Coil 1 or Coil 2 separately or both the coils in Helmholtz Coil configuration.

2. **Digital Gaussmeter**

   A Hall Effect integrated circuits chip is used to increase the temperature stability and sensitivity. Laser trimmed thin film resistors on the chip provide high accuracy and temperature compensation to reduce null and gain shift over temperature.

   Specification:

   (a) Range : 0-200G
   (b) Resolution : 0.1G
   (c) Accuracy : $\pm 0.5\%$
   (d) Display : $3\frac{1}{2}$ digit 7 segment LED display
   (e) Transducer : Hall Effect IC sensor

3. **Current Carrying Coil Set-Up**

It has two coils, one is fixed. The other coil and magnetic field sensor can be moved smoothly along the axis of the coils with the help of lead-screw system, independently. The position of coil and magnetic sensor could be read on two separate scales.

Specification:

(a) Radius of the coils: Needs to be calculated

(b) Number of turn: 500

4 Theory

We know, the intensity of magnetic field at a point 'P', lying on the axis of a circular coil 'AB' of radius 'a' having 'n' turns at a distance 'x' from the centre 'O' of the coil in S.I. units, is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I a^2}{(a^2 + x^2)^{\frac{3}{2}}}$$

I is the current flowing through the coil, μ_0 is the permeability of the free space, which is equal to $4\pi \times 10^{-7}\ Hm^{-1}$

The units of B are $Tesla$ or Wbm^{-2}

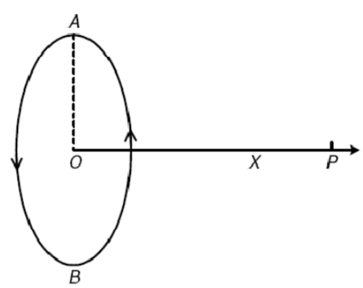

Figure 3:

The direction of the magnetic intensity at P is along OP produced if the current flows through the coil in the anticlockwise direction as seen from P. If the direction of the current is clockwise the field at P is along PO (Figure 3).

The value of the magnetic intensity is maximum at the centre O of the coil and is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n I}{a} = \frac{\mu_0 n I}{2a}$$

or,

$$B = \frac{4\pi \times 10^{-7} \times nI \times 10^4}{2a} Gauss$$

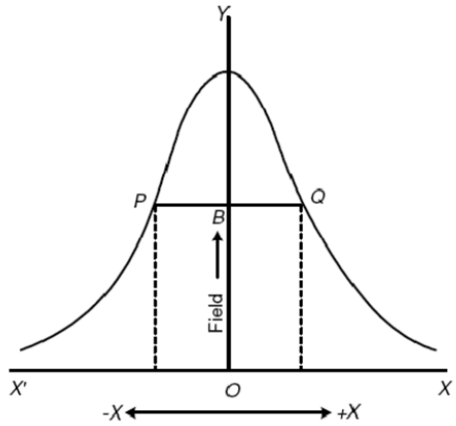

Figure 4:

If we move away from O towards the right or left, the intensity of the magnetic field decreases. A graph showing the relation between the intensity of the magnetic field B and the distance x is given in Figure 4. The curve is first concave towards O but the curvature becomes less and less, quickly changes sign at P and Q and afterwards becomes convex towards O. It can be shown that the points of inflexion P or Q where the curvature changes its sign lie at distances $\frac{a}{2}$ from the centre. Hence the distance between P and Q is equal to the radius of the coil.

5 Procedure

1. Switch 'ON' the main's power.
2. Set the current zero. Adjust magnetic field to zero.
3. Set a constant current that can be applicable for all configurations and the value should be greater than 350 mA.
4. Calculate the radius of the coil from a measured value of magnetic field.
5. Set the coils in Helmholtz configuration.
6. Study the superposition principle of magnetic field under Helmholtz configuration. Make an appropriate table. Graph required.
7. Study the magnetic field profile under Helmholtz configuration and at a distance of 5 cm on either side from the Helmholtz configuration. Make an appropriate table. Graph required.

6 Data Tabulation, Observation and Calculations

6.1 Determination of radius of coil

1. *Using the formula*

   From the following Table 1, we can see that when the Hall probe is at the centre of the loop ($x = 0$), the value of magnetic field is, $B = 11.1\ gauss = 11.1 \times 10^{-4}\ T$ The experiment is being conducted with a constant current, $I = 400\ mA = 0.4\ A$

   Therefore, we have,

$$
\begin{aligned}
B &= \frac{\mu_0 N I}{2a} \\
\Rightarrow a &= \frac{4\pi \times 10^{-7} \times 500 \times 0.4}{2 \times 11.1 \times 10^{-4}} \\
&= 0.1132\ m \\
\Rightarrow a &= 11.32\ cm
\end{aligned}
$$

2. *By measuring variation in magnetic field with position of probe, for constant current*

   For a constant current, **I = 400 mA**, the magnetic field along the axis of a single coil is measured at various positions and the following *Table 1* is made.

Sl. No.	**Position (cm)**	**Magnetic Field (Gauss)**
1	5.0	8.2
2	4.5	8.6
3	4.0	9.0
4	3.5	9.5
5	3.0	9.8
6	2.5	10.1
7	2.0	10.4
8	1.5	10.6
9	1.0	10.9
10	0.5	11.0
11	0	11.1
12	-0.5	11.1
13	-1.0	11.1
14	-1.5	11.0
15	-2.0	10.8
16	-2.5	10.6
17	-3.0	10.3
18	-3.5	10.0
19	-4.0	9.6
20	-4.5	9.3
21	-5.0	8.8

Table 1: Magnetic Field in a single coil for steady current

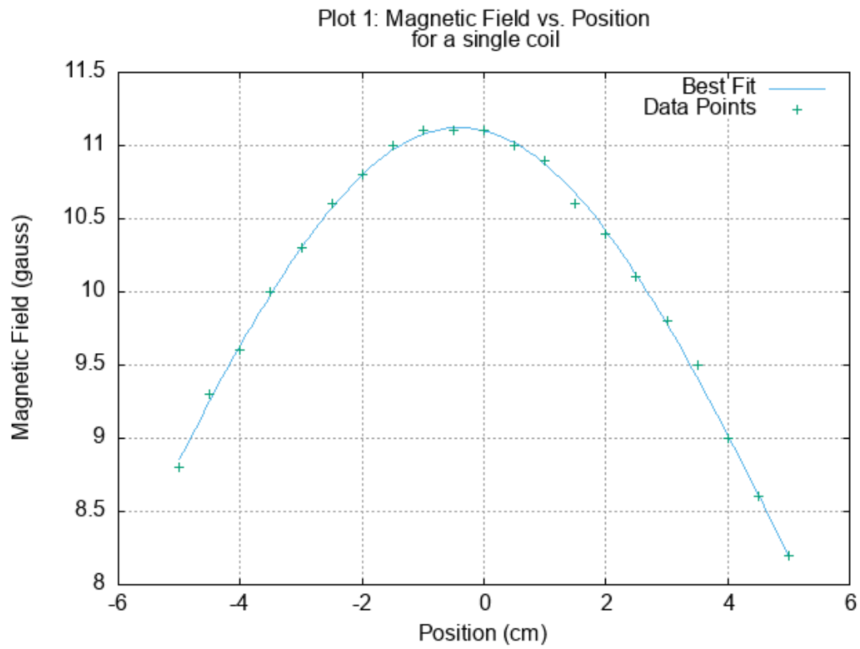

Figure 5: Magnetic Field *vs.* Position for a Single Coil

After fitting the data points obtained in Table 1, we obtain the graph as seen in Plot 1.

Using a suitable equation for fitting, we get the value of radius of the coil as, a = *11.36 cm* with a fitting error of **0.53%**.

3. *By measuring variation in magnetic field with current, for constant position of probe*

Sl. No.	Current (mA)	Magnetic Field (Gauss)
1	50	1.5
2	100	2.9
3	150	4.3
4	200	5.7
5	250	7.0
6	300	8.4
7	350	9.8
8	400	11.1
9	450	12.6
10	500	14.0

Table 2: Magnetic Field for varying current

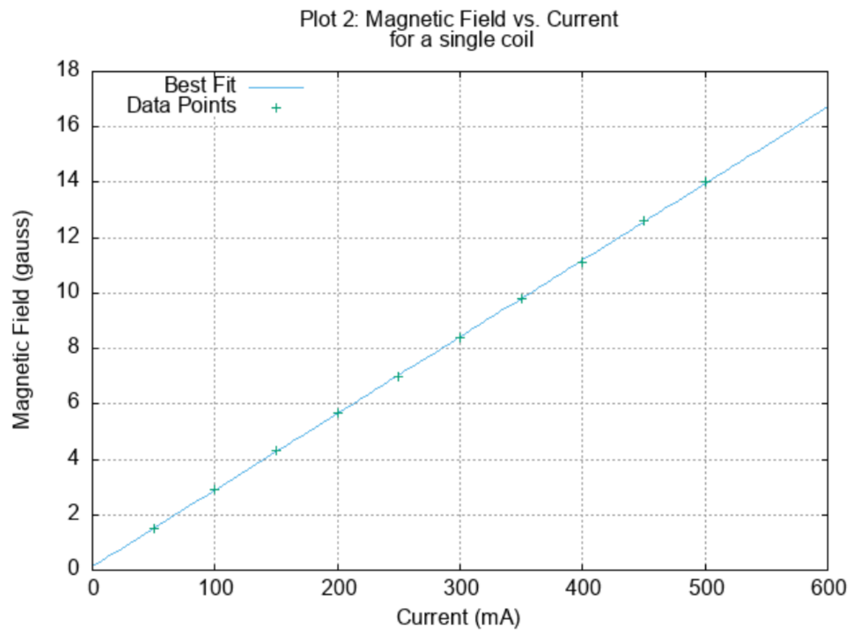

Figure 6: Magnetic Field *vs.* Current or a Single Coil

After fitting the data points obtained in Table 2, we obtain the graph as seen in Plot 2.

Using a suitable equation for fitting, we get the slope of the fitted graph, $k = 0.02765\ gauss \cdot mA^{-1} = 0.002765\ TA^{-1}$ with a fitting error of 0.35%.

Now, we have,

$$\begin{aligned} a &= \frac{\mu_0 N}{2k} \\ &= \frac{4\pi \times 10^{-7} \times 500}{2 \times 0.002765} = 0.1136\ m \\ \Rightarrow a &= 11.36\ cm \end{aligned}$$

Therefore, the mean Helmholtz distance or the mean radius of the two coils,

$$= \frac{11.32 + 11.36 + 11.36}{3} = 11.35\ cm$$

6.2 Magnetic Field at Helmholtz Configuration

Sl. No.	**Position (cm)**	**Magnetic Field in Coil 1 B_1 (gauss)**	**Magnetic Field in Coil 2 B_2 (gauss)**	**Measured Superposed Magnetic Field B_1 & B_2 (gauss)**	**Expected Superposed Magnetic Field $B_1 + B_2$ (gauss)**
1	-13.5	6.6	1.9	8.0	8.5
2	-13.0	6.9	2.0	8.5	8.9
3	-12.5	7.3	2.1	9.0	9.4
4	-12.0	7.8	2.2	9.5	10.0
5	-11.5	8.2	2.3	10.1	10.5
6	-11.0	8.6	2.5	10.6	11.1

Sl. No.	Position (cm)	Magnetic Field in Coil 1 B_1 (gauss)	Magnetic Field in Coil 2 B_2 (gauss)	Measured Superposed Magnetic Field B_1 & B_2 (gauss)	Expected Superposed Magnetic Field $B_1 + B_2$ (gauss)
7	-10.5	9.0	2.6	11.2	11.6
8	-10.0	9.5	2.7	11.7	12.2
9	-9.5	9.9	2.9	12.3	12.8
10	-9.0	10.3	3.1	12.8	13.4
11	-8.5	10.6	3.2	13.3	13.8
12	-8.0	10.6	3.4	13.8	14.0
13	-7.5	11.1	3.6	14.2	14.7
14	-7.0	11.3	3.9	14.7	15.2
15	-6.5	11.5	4.1	15.0	15.6
16	-6.0	11.5	4.3	15.3	15.8
17	-5.5	11.5	4.6	15.6	16.1
18	-5.0	11.5	4.9	15.8	16.4
19	-4.5	11.4	5.2	16.0	16.6
20	-4.0	11.2	5.5	16.1	16.7
21	-3.5	10.9	5.8	16.2	16.7
22	-3.0	10.7	6.2	16.3	16.9
23	-2.5	10.4	6.6	16.4	17.0
24	-2.0	10.0	6.9	16.4	16.9
25	-1.5	9.6	7.4	16.4	17.0
26	-1.0	9.2	7.8	16.4	17.0
27	-0.5	8.8	8.2	16.4	17.0
28	0	8.4	8.6	16.4	17.0
29	0.5	8.0	9.1	16.4	17.1
30	1.0	7.6	9.5	16.4	17.1
31	1.5	7.2	9.9	16.4	17.1
32	2.0	6.8	10.3	16.4	17.1
33	2.5	6.5	10.6	16.4	17.1
34	3.0	6.1	10.9	16.4	17.0
35	3.5	5.7	11.2	16.3	16.9
36	4.0	5.4	11.4	16.2	16.8
37	4.5	5.1	11.6	16.0	16.7
38	5.0	4.8	11.7	15.8	16.5
39	5.5	4.6	11.7	15.6	16.3
40	6.0	4.4	11.7	15.4	16.1
41	6.5	4.1	11.6	15.0	15.7
42	7.0	3.9	11.4	14.6	15.3
43	7.5	3.7	11.2	14.2	14.9
44	8.0	3.5	11.0	13.8	14.5
45	8.5	3.3	10.6	13.3	13.9
46	9.0	3.2	10.3	12.8	13.5
47	9.5	3.0	10.0	12.3	13.0
48	10.0	2.9	9.5	11.7	12.4
49	10.5	2.8	9.2	11.2	12.0
50	11.0	2.7	8.7	10.7	11.4

Table 3: Study of magnetic field at Helmholtz position

The following graph is plotted by using the data obtained in **Table 3**.

The *green* and *purple* lines denote the magnetic field variation of each coil, individually. The probe is moved along the axis about the midpoint of the two coils (x = 0).

The *blue* line denotes the magnetic field variation of the combination of both the coils as determined from the experiment while the *red* line is the expected values of field variation.

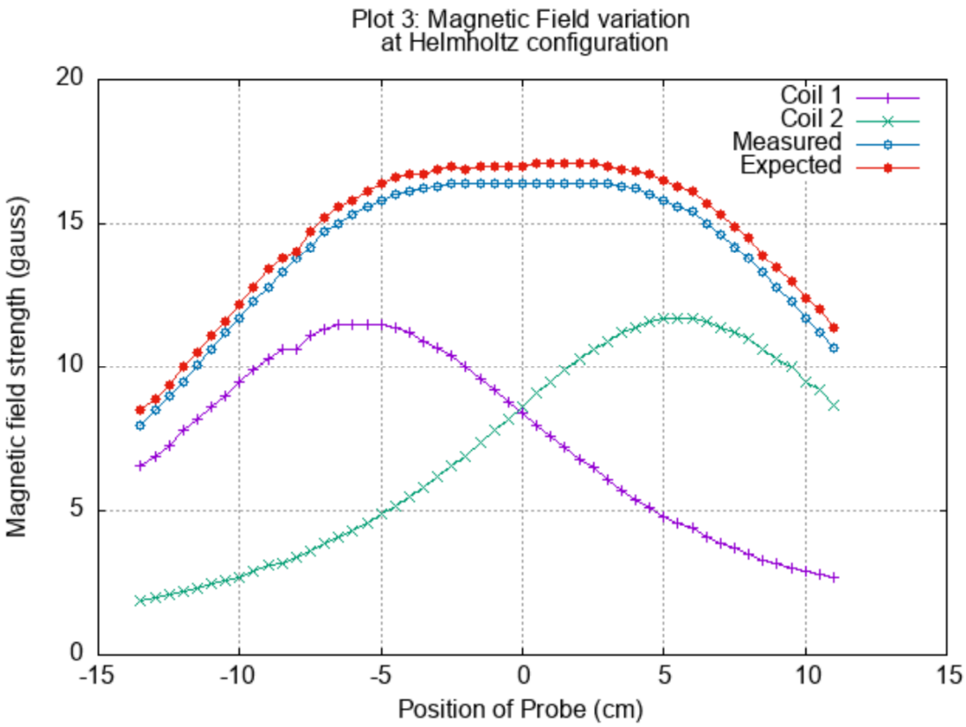

Figure 7: Magnetic Field Variation at Helmholtz Configuration

6.3 Variation of magnetic field about Helmholtz Configuration

Sl. No.	**Position (cm)**	**Magnetic Field in Coil 1 and 2 (gauss)**	
		dist. b/w coils > 4cm of Helmholtz posn.	**dist. b/w coils < 4cm of Helmholtz posn.**
1	-13.0	10.1	-
2	-12.5	10.5	-
3	-12.0	10.9	9.1
4	-11.5	11.3	9.6
5	-11.0	11.7	10.1
6	-10.5	12.0	10.7
7	-10.0	12.3	11.3
8	-9.5	12.6	11.7
9	-9.0	12.9	12.3
10	-8.5	13.2	12.9
11	-8.0	13.4	13.6
12	-7.5	13.5	13.7
13	-7.0	13.6	14.4
14	-6.5	13.7	15.1
15	-6.0	13.8	15.6
16	-5.5	13.8	16.2
17	-5.0	13.7	16.7

18	-4.5	13.6	17.2
19	-4.0	13.5	17.7
20	-3.5	13.4	18.0
21	-3.0	13.3	18.4
22	-2.5	13.1	18.6
23	-2.0	13.0	18.8
24	-1.5	12.9	19.0
25	-1.0	12.9	19.1
26	-0.5	12.8	19.1
27	0.0	12.8	19.3
28	0.5	12.7	19.3
29	1.0	12.8	19.2
30	1.5	12.8	19.0
31	2.0	12.9	18.8
32	2.5	13.0	18.5
33	3.0	13.1	18.2
34	3.5	13.2	17.8
35	4.0	13.4	17.4
36	4.5	13.5	16.9
37	5.0	13.6	16.4
38	5.5	13.7	15.8
39	6.0	13.7	15.2
40	6.5	13.8	14.6
41	7.0	13.7	13.9
42	7.5	13.6	13.3
43	8.0	13.5	12.7
44	8.5	13.4	12.1
45	9.0	13.1	11.4
46	9.5	12.1	11.0
47	10.0	11.7	10.4
48	10.5	11.3	10.0
49	11.0	10.9	9.5
50	11.5	10.4	-
51	12.0	10	-
52	12.5	9.6	-

Table 4:

The following graph is plotted by using the data obtained in **Table 3** and **Table 4**.

The **purple** lines denote the magnetic field variation of the combination of the two coils when kept at Helmholtz position. The **blue** line denotes the magnetic field variation of the combination of both the coils when the two coils were separated by a distance of $4cm$ more than the Helmholtz configuration, while the **red** line denotes the magnetic field variation of the combination of both the coils when the two coils were separated by a distance of $4cm$ less than the Helmholtz configuration.

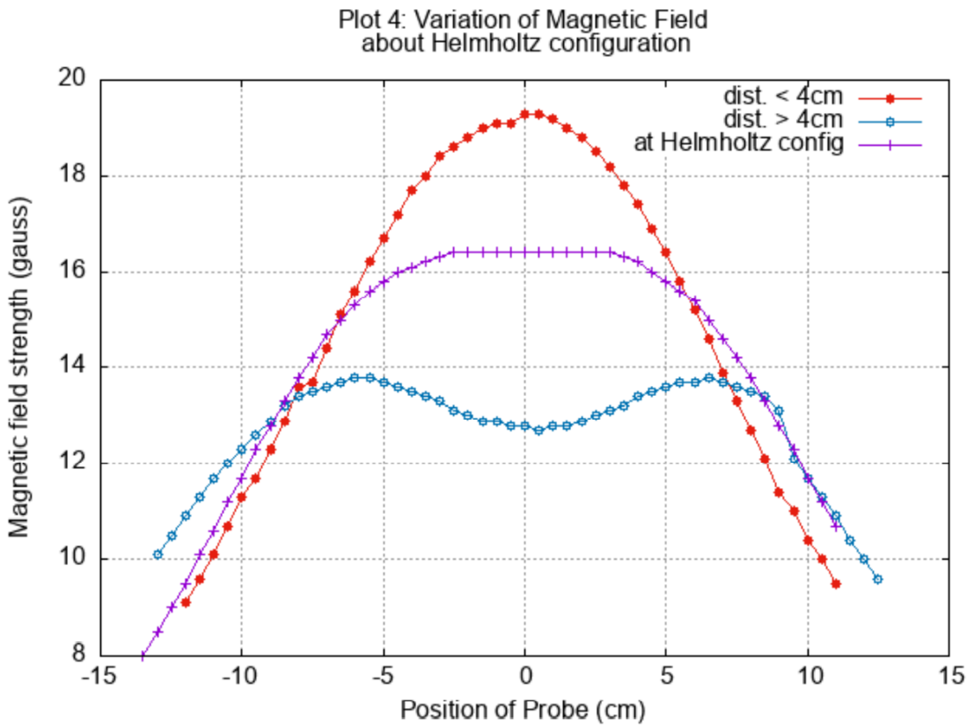

Figure 8: Variation of magnetic field about Helmholtz configuration

7 Error Analysis

Least count of the ammemeter, $\delta I = 1\ mA$
Least count of the gaussmeter, $\delta B = 0.1\ G$

Therefore, from the formula,

$$\begin{aligned} a &= \frac{4\pi\mu_0 \times N \times I}{2 \times B} \\ \Rightarrow \frac{\delta a}{a} &= \frac{\delta I}{I} + \frac{\delta B}{B} \\ &= \frac{1}{400} + \frac{0.1}{11.1} \\ \Rightarrow \delta a &= 0.012 \times 11.32 \\ \therefore \delta a &= 0.13\ cm \end{aligned}$$

So, using the formula, the error in calculating the radius of the coil comes out to be $\pm 0.13\ cm$.

Now, in **Plot 1** and **Plot 2**, the fitting errors come out to be 0.53% and 0.35%, respectively.

Therefore, the respective errors in calculating the radius of the coil from the two plots is $\pm 0.06\ cm$ and $\pm 0.04 cm$.

Hence, the net error in the whole experiment comes out to be $\pm 0.23/3 = \pm 0.08 cm$.

Also, from the fitting equation of **Plot 1**, we find that there is a zero error of 0.3971 cm in the instrument which has been used to measure the position of the probe.

8 Results

From this experiment, the Helmholtz distance comes out to be $11.35 \pm 0.08 cm$.

We have successfully measured the magnetic field variation at the Helmholtz configuration and at a distance of 4 cm on either side from the Helmholtz configuration.

9 Discussion

From the graphs, we can see that the superposition principle holds true. The experimental values obtained are close to the expected value of resultant magnetic field, which we are supposed to get at Helmholtz configuration.

We have also seen that when the coils are moved closer to each other in comparison to the Helmholtz configuration, the magnetic field in between the coils increases as compared to the region on either side of the two coils, and when the coils are moved farther in comparison to the Helmholtz configuration, the magnetic field in between the two coils decreases as compared to the region on either side of the two coils and also we get maximal in Magnetic Field at the center of the coils in this particular configuration.

There might be many factors affecting the experiment, while it is being conducted, which can lead to a deviation from the expected values. The presence of unmonitored residual magnetic fields may lead to the erroneous calculation of the radius of the coils. There might also be fluctuations in the current, which may lead to the formation of varying magnetic field at a particular point.

The instruments used might also be faulty, or there might be an error because of the inability of the person conducting the experiment.

Taking everything into account, we can say that the desired results have been obtained.

10 Appendix

10.1 Fitting for Plot 1

```
FIT:    data read from "data1a.dat" u 1:2
        format = x:z
        #datapoints = 21
        residuals are weighted equally (unit weight)

function used for fitting: B(x)
        B(x) = k/(a**2 + (x+e)**2)**1.5
fitted parameters initialized with current variable values

iter      chisq       delta/lim  lambda   k             a             e
   0 2.0693071017e+03   0.00e+00  5.63e-01    1.000000e+00   1.000000e+00   1.000000e+00
  65 2.4853283649e-02  -8.86e-05  5.63e-09    1.630151e+04   1.135890e+01   3.971093e-01

After 65 iterations the fit converged.
final sum of squares of residuals : 0.0248533
rel. change during last iteration : -8.85807e-10

degrees of freedom    (FIT_NDF)                        : 18
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.0371583
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.00138074

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
k               = 16301.5          +/- 244.4        (1.499%)
a               = 11.3589          +/- 0.06002      (0.5284%)
e               = 0.397109         +/- 0.01406      (3.542%)

correlation matrix of the fit parameters:
                k      a      e
k               1.000
a               0.999  1.000
e               0.211  0.206  1.000
```

Figure 9:

The fitting equation which was used,

$$B = \frac{k}{(a^2 + (x+e)^2)^{\frac{3}{2}}}$$

Here, **B** is the magnetic field and **a** is the radius of the coil.
e represents the zero error of the instrument which was used to measure the position of the probe.

From the fit, we come to know that, radius of the coil is 11.36 cm with a fitting error of 0.53%, and the zero error, $e = 0.3971$ cm which needs to be subtracted from the data.

10.2 Fitting for Plot 2

```
FIT:    data read from "data1b.dat" u 1:2
        format = x:z
        x range restricted to [0.00000 : 600.000]
        #datapoints = 10
        residuals are weighted equally (unit weight)

function used for fitting: B(x)
        B(x) = k*x + c
fitted parameters initialized with current variable values

iter      chisq       delta/lim  lambda   k             c
   0 9.1469061000e+05   0.00e+00  2.19e+02    1.000000e+00   1.000000e+00
   6 1.5515151515e-02  -1.21e-04  2.19e-04    2.764848e-02   1.266667e-01

After 6 iterations the fit converged.
final sum of squares of residuals : 0.0155152
rel. change during last iteration : -1.20806e-09

degrees of freedom    (FIT_NDF)                        : 8
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.0440386
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.00193939

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
k               = 0.0276485        +/- 9.697e-05    (0.3507%)
c               = 0.126667         +/- 0.03008      (23.75%)

correlation matrix of the fit parameters:
                k      c
k               1.000
c              -0.886  1.000
```

Figure 10:

The fitting equation which was used,

$$B = kI + c$$

Here, **B** is the magnetic field and **I** is the current flowing through the coil.
From the fit, we come to know that, $k = 0.02765\ G\ mA^{-1}$ with a fitting error of 0.35%.