Physics Laboratory PH1102

Lab Report

Experiment No.: 03

- STOKES LAW AND VISCOSITY -

By : Priyanshu Mahato
Roll No. : pm21ms002
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Contents

1 Aim

To determine the coefficient of viscosity of a liquid using a falling ball viscometer (employing Stokes' law).

2 Experimental apparatus

The main experimental apparatus consists of a glass tube containing the experimental liquid. **P** and **Q** are two adjustable reference marks along the tube length. The entire glass system is supported by a wooden stand. Besides this you will need a few steel balls of different diameter, screw gauge, digital balance, vernier calliper, stop watch, and meter scale for this experiment.

3 Principle of the Experiment

A body moving in a fluid is acted on by a frictional force in the opposite direction of its velocity. The magnitude of this force depends on the geometry of the body, its velocity, and the internal friction of the fluid. A measure for the internal friction is given by the dynamic viscosity η. For a spherical ball of radius r moving at velocity v in an infinitely extended fluid of dynamic viscosity η, G. G. Stokes derived the viscous force to be:

$$F_v = 6\pi\eta r v \tag{1}$$

If the spherical ball (density ρ) is dropped from rest at the upper surface of a vertical liquid (density σ) column , gravitational force $F_g = \frac{4}{3}\pi r^3 \rho g$ (g = acceleration due to gravity) and buoyancy force $F_b = \frac{4}{3}\pi r^3 \sigma g$ act on it besides viscous force $F_v = 6\pi\eta r v$.

Initially, F_v is zero because ball started at rest. So, the ball accelerates downwards because there is a net downward force. Then F_v starts to increase. Eventually a force balance $F_v + F_b = F_g$ is reached and the ball attains a steady terminal velocity v_t. Using the force balance condition one can easily derive,

$$\eta = \frac{2}{9}\frac{r(\rho - \sigma)g}{v_t} \tag{2}$$

Actually, Eq. (1) [derived under the assumption of infinitely extended liquid] should be corrected for the finite size of the liquid column. For the movement of the spherical ball along the axis of a liquid cylinder of radius R and length h, the viscous force is,

$$F_v = 6\pi\eta r v \left(1 + 3.3\frac{r}{h}\right)\left(1 + 2.4\frac{r}{R}\right)$$

For the experimental situation in our lab, $\frac{r}{R} \approx 0.1$ and $\frac{r}{h} \approx 0.001$. Thus, finite length correction $\left(1 + 3.3\frac{r}{h}\right)$ may be ignored. Incorporating the correction due to finite radius of the liquid column, Eq. (2) becomes:

$$\eta = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{v_t\left(1 + 2.4\frac{r}{R}\right)} \tag{3}$$

4 Diagram

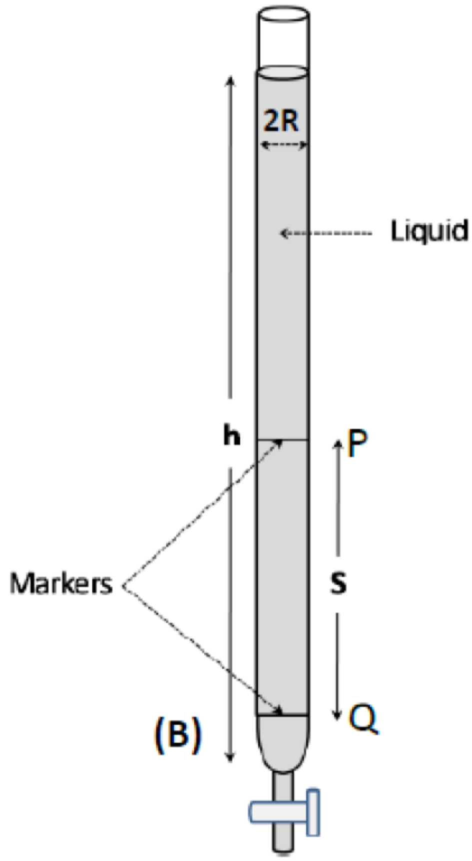


Figure 1: Schematic Diagram

5 Results and Observations

Tables:

1. Determination of radius and mass of the spherical balls (for three different radii)
   Density of Castor oil = 0.961 $g \cdot cc^{-1}$
   Today's temperature = 23°C
   Least count of the Screw Gauge = 0.001 cm
   Least count of Digital Balance = 0.01 g
   Least count of stopwatch = 0.01 s

   (a) Ball 1 :

Sl. No.	Linear Scale Reading (in cm.)	Circular Scale Reading	Diameter (in cm)	Mean Diameter (in cm)	Mean Radius (in cm)	Mass of 10 balls (in gm)	Mass per ball (in gm)	Volume per ball (in cm^3)	Density (in gm/cm^3)
1	0.3	15	0.315	0.3166	0.1583	1.31	0.131	0.01662	7.884
2	0.3	18	0.318						
3	0.3	17	0.317						
4	0.3	15	0.315						
5	0.3	16	0.316						
6	0.3	19	0.319						
7	0.3	16	0.316						

   (b) Ball 2 :

Sl. No.	Linear Scale Reading (in cm.)	Circular Scale Reading	Diameter (in cm)	Mean Diameter (in cm)	Mean Radius (in cm)	Mass of 10 balls (in gm)	Mass per ball (in gm)	Volume per ball (in cm^3)	Density (in gm/cm^3)
1	0.3	43	0.343	0.3446	0.1723	2.56	0.256	0.0214	11.95
2	0.3	43	0.343						
3	0.3	44	0.344						
4	0.3	46	0.346						
5	0.3	45	0.345						
6	0.3	46	0.346						
7	0.3	45	0.345						

(c) Ball 3 :

Sl. No.	Linear Scale Reading (in cm.)	Circular Scale Reading	Diameter (in cm)	Mean Diameter (in cm)	Mean Radius (in cm)	Mass of 10 balls (in gm)	Mass per ball (in gm)	Volume per ball (in cm^3)	Density (in gm/cm^3)
1	0.4	24	0.424	0.426	0.213	4.47	0.447	0.0405	11.037
2	0.4	26	0.426						
3	0.4	28	0.428						
4	0.4	24	0.424						
5	0.4	28	0.428						
6	0.4	26	0.426						
7	0.4	24	0.424						

2. Measurement of terminal velocity
   Distance travelled by Balls is 80 cm = 0.8 m

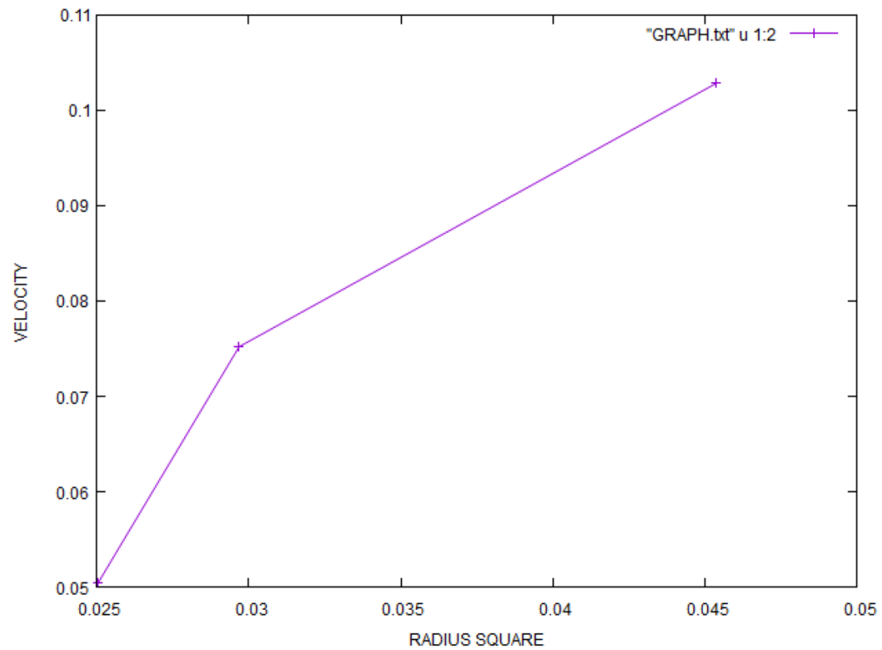
Sl. No.	Time for Ball 1 (in s)	Mean Time (in s)	Time for Ball 2 (in s)	Mean Time (in s)	Time for Ball 3 (in s)	Mean Time (in s)	Velocity for Ball 1 (in m/s)	Velocity for Ball 2 (in m/s)	Velocity for Ball 3 (in m/s)
1	15.72	15.85	10.66	10.64	7.68	7.78	0.0505	0.0752	0.1028
2	15.93		10.65		7.81				
3	15.88		10.69		7.72				
4	15.85		10.57		7.81				
5	15.75		10.66		7.82				
6	16.03		10.63		7.69				
7	15.78		10.59		7.90				

3. Determination of the Inner Diameter of Glass Cylinder
   Vernier Constant of Vernier Calliper = 0.002 cm

Sl. No.	Main Scale Reading (in cm)	Vernier Reading	Diameter (in cm)	Average Diameter (in cm)
1	4.6	5	4.61	4.609
2	4.6	5	4.61	
3	4.6	4	4.608	

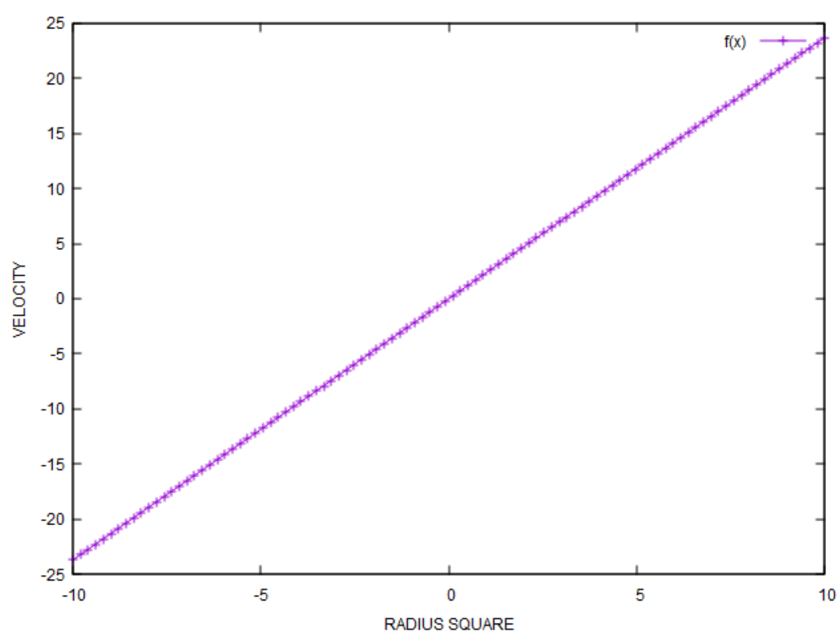
6 Plot

Plot of Velocity (v) in ms^{-1} vs. Square of Radius (r^2) in cm^2:



7 Inference

From the above graph, we see that with the given set of data, the resultant line is not a straight line. To remove it, we need to find the best possible fit of the line. If we fit the line within the given data-points in the best possible manner, the slope of the line comes out to be 243.615 $cm^{-1}s^{-1}$.
The fitted graph is:



This value is the value of the expression: $\frac{2}{9}\frac{(\rho-\sigma)g}{\eta\left(1+2.4\frac{r}{R}\right)}$

According to the tables, the average value of r is : 0.1812 cm. The value of R is 2.305 cm.
Hence, substituting the values (taking the density of Castor oil as 0.961 $g \cdot cc^{-1}$ and density of the material as the average value 10.29 $g \cdot cc^{-1}$), we get the value of η as : 6.35 $poise$ = 0.635 $Pa \cdot s$
The given value of co-efficient of viscosity of Castor oil is 0.650 $Pa \cdot s$

8 Error Analysis

The error in the calculation of η is :

$$\begin{aligned} Absolute\ Error &= |0.650 - 0.635| \\ &= 0.015\ Pa \cdot s \\ Percentage\ Error &= \frac{0.015}{0.650} \times 100\% \\ &= 2.3\% \end{aligned}$$

Systematic/Instrumentation Error
We used Equation 3 to find the value of η. In the process, errors of the measured quantities get accumulated and propagated to give a *systematic error* in the value of η. Using Partial Derivatives in the Equation 3, we get,

$$\frac{\delta\eta}{\eta} = \left|\frac{2}{r} - \frac{2.4}{R+2.4r} - \frac{9m}{4\pi r^4(\rho-\sigma)}\right|\delta r + \left|\frac{1}{x}\right|\delta x + \left|\frac{1}{t}\right|\delta t + \left|\frac{2.4r}{R(R+2.4r)}\right|\delta R + \left|\frac{3}{4\pi r^3(\rho-\sigma)}\right|\delta m$$

Now, substituting different parameters and least counts of the instruments, as individual errors in the above formula, we obtain,

1. **Ball 1:**
   For Ball 1, the systematic error comes out to be about 9.8%
2. **Ball 2:**
   For Ball 2 the systematic error comes out to be about 6.6%
3. **Ball 3:**
   For Ball 3 the systematic error comes out to be about 3.4%

9 Sources of Error

1. The point where the glass bead attains the terminal velocity is an eye-estimated length and has 100% chance for errors.
2. The inaccuracy in screw-gauge, vernier-calliper and digital balance can contribute to the errors of the constituting variables.
3. Inaccuracy in working with the stopwatch as the spherical watch passes the two reference marks (namely, **P** and **Q**).

10 Conclusion

The Experimental value of the co-efficient of viscosity of Castor Oil differs from the actual one by 2.3 %.