MATRIX

A matrix is an arrangement of mXn scalars from a given field F.

$$X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}_{2X2} \longrightarrow \text{Rows}$$

Field is a set on which addition and multiplication are defined and behave as operations on real and rational numbers.

$$Y = \begin{pmatrix} p & q & r \\ u & v & w \end{pmatrix}_{2X3}$$

$\downarrow$

Columns

$$A \equiv \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{mXn} \qquad a_{ij} \in F$$

$x = (a \quad b)$ Row vector

Matrix **dimension/order/size**: mXn

$y = \begin{pmatrix} q \\ v \end{pmatrix}$ Column vector

$$A \equiv \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{mXn}$$

Zero matrix : $a_{ij} = 0 \quad \forall\ i, j$

Square matrix: $m{=}n$

Diagonal matrix: $a_{ij} = 0 \quad \forall\ i \neq j$ and $a_{ii} = d_i$

Identity matrix: $d_i = 1 \quad \forall \ \ i$

Tridiagonal matrix:

$$A \equiv \begin{pmatrix} a_{11} & a_{12} & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & a_{23} & 0 & \cdots & 0 \\ 0 & a_{32} & a_{33} & a_{34} & \cdots & 0 \\ 0 & & \ddots & \ddots & \ddots & \end{pmatrix}_{nXn}$$

Band-diagonal matrix:

Upper / Lower triangular matrices:

Symmetric matrix

$$a_{ij} = a_{ji} \quad \forall \quad i, j \quad \text{and} \quad a_{ij} \in \mathbb{R}$$

$$A = \begin{pmatrix} a & d & e \\ d & b & f \\ e & f & c \end{pmatrix}_{3X3}$$

Hermitian matrix

$$\underbrace{a_{ij} = \bar{a}_{ji}}_{\text{Complex conjugate}} \quad \forall \quad i, j \quad \text{and} \quad a_{ij} \in \mathbb{C}$$

$$B = \begin{pmatrix} a & x + iy \\ x - iy & c \end{pmatrix}_{2X2}$$

The set of all mXn matrices is denoted by $M_{m,n}(F)$ where $F \equiv \mathbb{R}$ or $\mathbb{C}$

Let $A, B, C, D \in M_{m,n}(F)$

Equality: two matrices A and B are equal iff they have same dimension and $a_{ij} = b_{ij} \quad \forall i, j$

Scalar Multiplication: $B = \alpha A \quad \alpha \in F$ such that $b_{ij} = \alpha\, a_{ij} \quad \forall \quad i, j$

Addition: $C = A + B$ such that $c_{ij} = a_{ij} + b_{ij} \quad \forall \quad i, j$

Difference: $D = A - B = A + (-1)B$

Matrix Addition

Let $A, B, C, D \in M_{m,n}(F)$

Commutivity: A + B = B + A

Associativity: A + (B + C) = (A + B) + C

Distributivity: x (A + B) = x A + x B

Zero matrix: A + **O** = A

(x + y) A = x A + y A

x (y A) = x y A

o A = **O**

x **O** = **O**