
MA 1101 : Mathematics I

Problem 1.

Let $X, Y, Z \neq \emptyset$, let $f : X \rightarrow Y$ and let $g : Y \rightarrow Z$. Prove that

- ~~(i)~~ $g \circ f$ is one-one if f, g are one-one.
- ~~(ii)~~ $g \circ f$ is onto if f, g are onto.
- ~~(iii)~~ $g \circ f$ is bijective if f, g are bijective.
- ~~(iv)~~ g is onto if $g \circ f$ is onto. Is f onto if $g \circ f$ is onto?
- ~~(v)~~ f is one-one if $g \circ f$ is one-one. Is g one-one if $g \circ f$ is one-one?
- ~~(vi)~~ g is one-one if $g \circ f$ is one-one and f is onto.

Problem 2.

Let $W, X, Y, Z \neq \emptyset$ and let $f : W \rightarrow X, g : X \rightarrow Y, h : Y \rightarrow Z$. Show that

$$(h \circ g) \circ f = h \circ (g \circ f).$$

Problem 3.

Check whether the following functions are one-one and/or onto.

- ~~(i)~~ $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) := x^2 + x$.
- ~~(ii)~~ $f : \mathbb{N} \rightarrow \mathbb{N}, f(x) := \left\lceil \frac{n+1}{2} \right\rceil$, where $\lceil \cdot \rceil$ denotes the greatest integer function.
- (iii)** $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) := x + [x]$.
- ~~(iv)~~ $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) := x - [x]$.
- ~~(v)~~ $f : \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R}, f(x) := \frac{x+1}{x-1}$.
- (vi)** $f : (-1, 1) \rightarrow \mathbb{R}, f(x) := \frac{x}{1-|x|}$.