Feb 10, 2022 : **Power Series :**

$$\sum_{n=0}^{\infty} = a_n x^n$$

$$\boxed{S(x) = \sum_{n=0}^{\infty} = a_n x^n}$$

⇒ If we sum the two converging series we will get an another series which again converging

⇒ you can multiply the series with a constant with out changing its convergence.

⇒ $\underline{\sum_{n=0}^{\infty} a_n x^n}$ ⇒ Unique function that converges

The Taylor Series :

$f(x)$ ⇒ Expressing function $f(x)$ as a power series in x

$$f(x) = a_0 + a_1 x + a_2 x^2 + \dots = \boxed{\sum_{k=0}^{\infty} a_k x^k} \quad \text{— (A)}$$

where $a_0, a_1, a_2 \dots a_k$ are constants

Evaluate $f(x) \Rightarrow x=0 \Rightarrow \boxed{f(0) = a_0}$

$f(0) = a_0 + \underbrace{a_1(0) + a_2(0) + a_3(0) \ldots}_{\text{zeros}}$

$$\frac{df}{dx} = f'(x) = \frac{d}{dx}\left(a_0 + a_1 x + a_2 x^2 + \ldots\right)$$

$$f'(x) = a_1 + 2a_2 x + 3a_3 x^2 + \ldots$$

$\Rightarrow x=0$

$$\boxed{f'(0) = a_1} \Rightarrow a_1 = \frac{f'(0)}{1!}$$

Similarly $\boxed{f''(0) = 2a_2}$ ---- $\Rightarrow a_2 = \frac{f''(0)}{2!}$

differentiating $f(x)$ for K times, then

$$\boxed{f^{(K)}(0) = K!\, a_K} \Rightarrow a_K = \frac{1}{K!} f^{(K)}(0)$$

$$\boxed{K! = K \times (K-1) \times (K-2) \times \ldots \times 1}$$

Substitute all above in eqn (A)

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$

$$\boxed{f(x) = f(0) + f'(0)\, x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3 + \ldots}$$

This Expansion is Taylor Series

verification:

$$f'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3$$

$$f''(x) = 2a_2 + 3\times2\, a_3 x + 4\times3\, a_4 x^2$$

$$f'''(x) = 3\times2 a_3 + 4\times3\times2 a_4$$

$$\boxed{f(a+x) = f(a) + f'(a)x + \frac{f''(a)}{2!}x^2 + \frac{f'''(a)}{3!}x^3 + \dots}$$

This power Series Expansion is known as MacLaurin Series

Expansion of few Important functions

1. $\sin x$

$$\frac{d}{dx}\sin x = \underline{\cos x}$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\sin(0) = 0$$

$$\cos(0) = 1$$

$$\sin x = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

$$= 0 + 1\cdot x + \frac{(-0)}{2!}\cdot x^2 + \frac{(-1)}{3!}x^3$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots$$

Similarly $\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots$

The small angle approximation valid upto order of x^3

② Exponential function: e^x

$$\frac{d}{dx}e^x = e^x$$

$$e^0 = 1$$

$$e^x = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

$$= 1 + 1.x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n + \cdots$$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots \frac{1}{n!}x^n + \cdots$$

③ Algebraic functions:

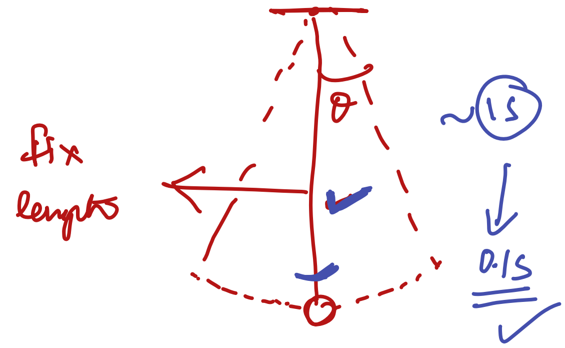
ⓐ $\frac{1}{1 \pm x}$

ⓑ $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

ⓒ $\frac{1}{\sqrt{1+x}}$

Important Problems :

$$T = 2\pi\sqrt{\frac{L}{g}}$$



L_0 L

initial final

$\Delta L \Rightarrow \Delta T$

Significant figures : [in physics units or constants]

what is the value of $\pi = 3.14$ [3]

3.14176 [6]

3.1415926 [8]

3.1415 [5]

0.00021 (2)

0.000210 (3)

0.02001 (4)   4   1 2

Experimental Uncertainity :-

4 ← 72.53 ± 0.20

2

(72.33 to 72.73)

0.02001 ± 0.01 ×

4   1

1234   1

± 10.9

72.53

0.02001

0.0

72.53000 72.53 ± 0.02001

72.527999 to 72.532001

$$\begin{cases} 7200 & (4) \\ 7200.00 & (6) \end{cases}$$

4

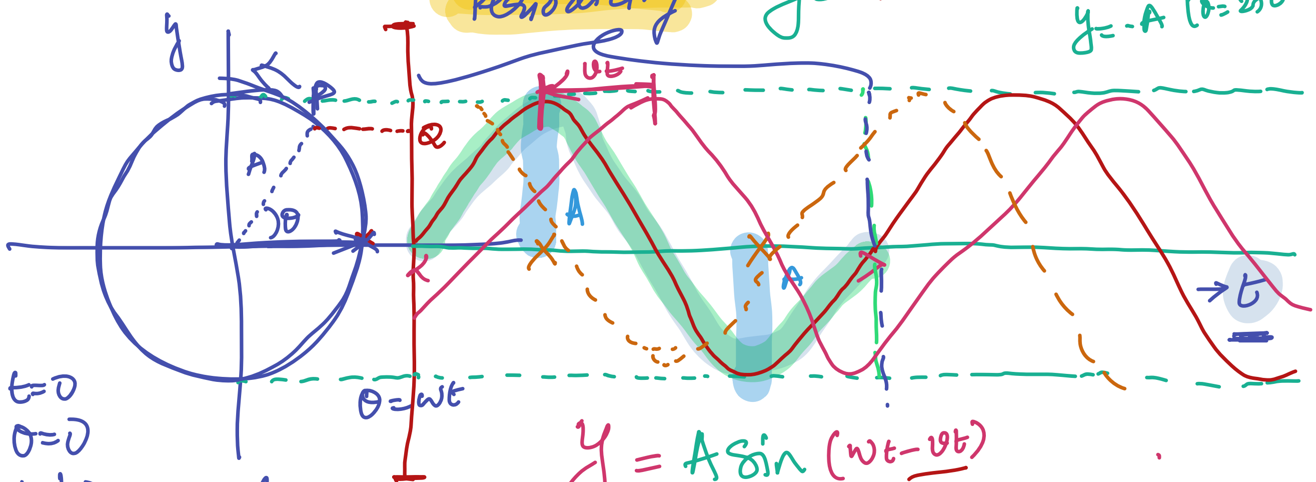
General Constants like g, G, c ⑥

Fourier Series and Transforms:

Periodicity

$y = A\sin\omega t$ $\quad y = A\ (\theta = \pi/2)$

$y = -A\ (\theta = 270)$



$t = 0$

$\theta = 0$

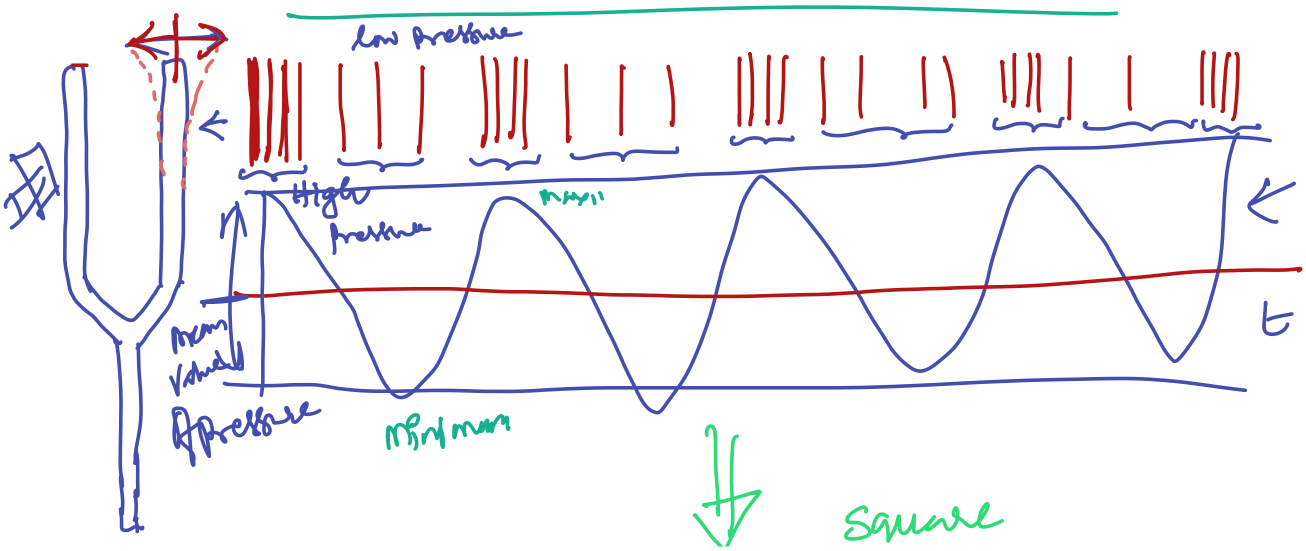
$t \neq 0$ circular motion

$y = A\sin(\omega t - vt)$

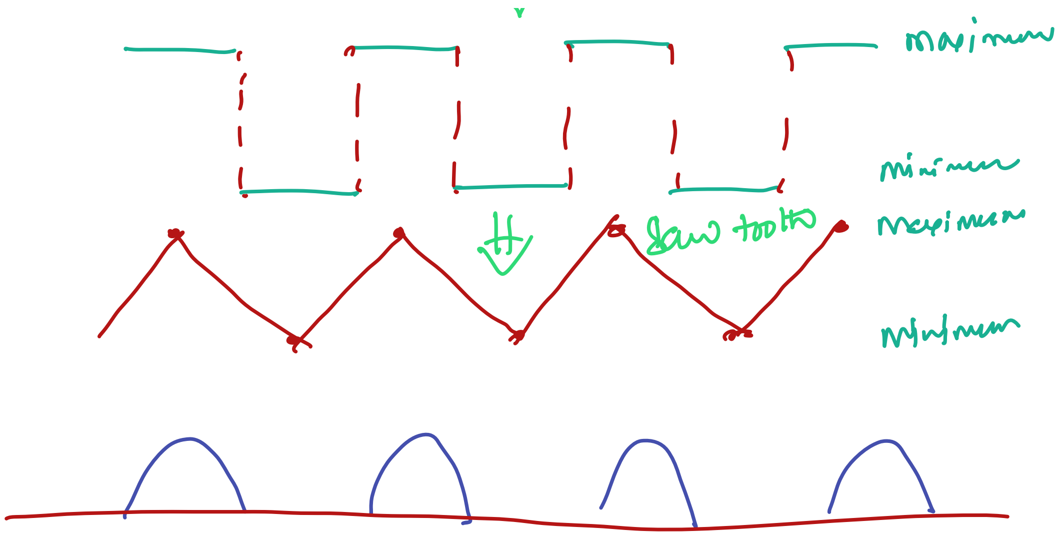
$\theta = \omega t \Rightarrow$ $\quad 2\pi = \omega t$

$\Rightarrow \boxed{t = \frac{2\pi}{\omega} = T}$

"Simple Harmonic motion"



Square



Rectified half-wave