Determination of Coefficient of Viscosity of a Liquid using Falling Ball Viscometer

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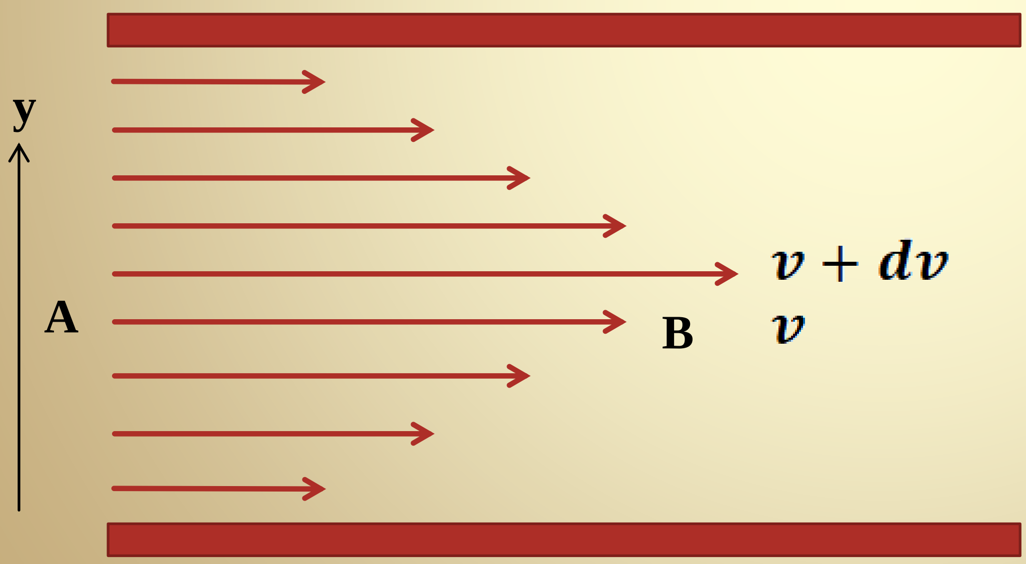
Viscosity

➢ What is Viscosity

Viscosity is a measure of a fluid's resistance to flow. It describes the internal friction of a moving fluid. The internal frictional force opposes the relative motion between the adjacent layers of fluid.

➢ Coefficient of Viscosity

Steady flow of liquid



Boundary (Stationary)

Shear stress (f) $\propto$ Velocity gradient

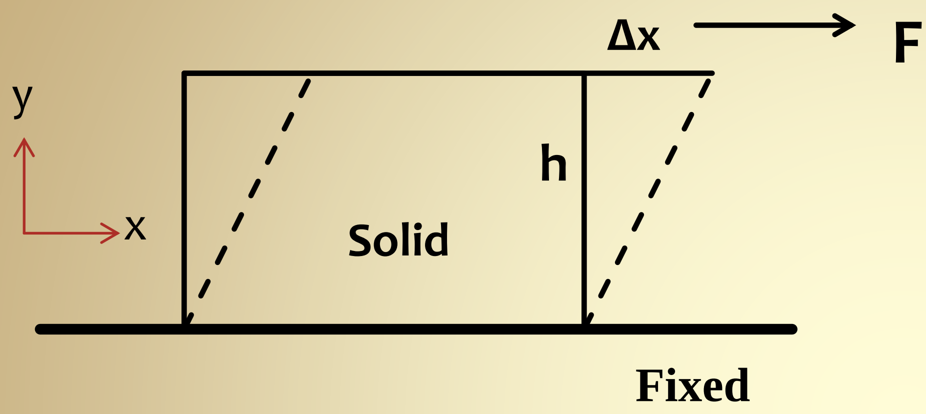
$$f \propto \frac{dv}{dy}$$

$$f = \eta \frac{dv}{dy}$$

η is the coefficient of viscosity

Viscosity

Solid Deformation:



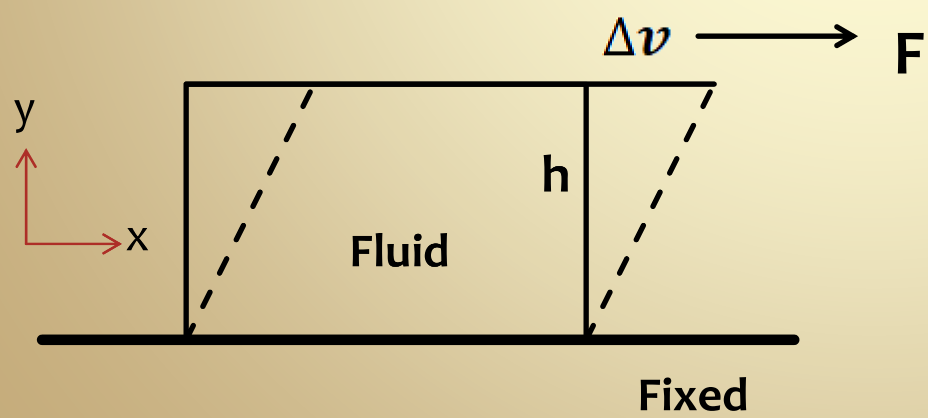
Shear stress ∝ Shear strain

$$\frac{F}{A} \propto \frac{\Delta x}{h}$$

Shear modulus
= Shear stress/Shear strain

$$= \frac{F/A}{\Delta x/h}$$

Fluid Deformation:



Shear stress ∝ Shear strain rate

$$\frac{F}{A} \propto \frac{\Delta v}{h}$$

Coefficient of viscosity (η)
= Shear stress/Shear strain rate

$$\eta = \frac{F/A}{\Delta v/h}$$

Dimension

- Dimension of coefficient of viscosity (η) : $M L^{-1} T^{-1}$

$$\eta = \frac{F/A}{\Delta v/h} = \frac{MLT^{-2}}{L^2} \cdot \frac{L}{LT^{-1}}$$

- SI unit Pa.S (Newton-second per square meter: N·s/m²)
- CGS: Poise (Dyne-second per square centimetre)
- Dynamic viscosity: η
- Kinematic viscosity: dynamic viscosity divided by the density

Determination of coefficient of Viscosity: Falling Ball Viscometer

Liquid: Castor Oil

Determination of coefficient of Viscosity

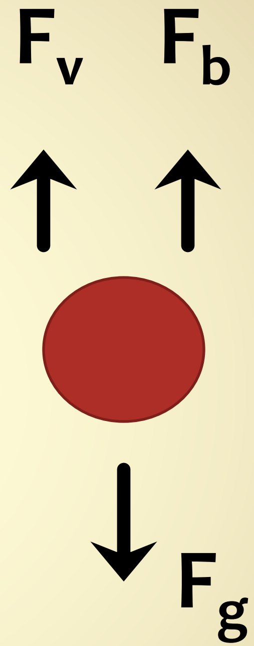
- **Gravitational force:**

$$F_g = mg = \frac{4}{3}\pi r^3 \rho g$$

- **Buoyant force:** $F_b = \frac{4}{3}\pi r^3 \sigma g$

- **Viscous force:** $F_V = 6\pi\eta r v$

($\boldsymbol{\rho}$: density of the ball; $\boldsymbol{\sigma}$: density of the liquid)



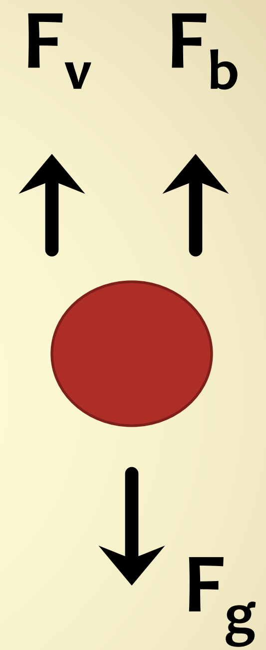
Falling Ball Viscometer

Determination of coefficient of Viscosity

➢ **Force Balance:**

$$F_g = F_b + F_v$$

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r V_t$$



➢ **Coefficient of Viscosity:**

$$\boxed{\eta = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{V_t}}$$

V_t : terminal velocity

Falling Ball Viscometer

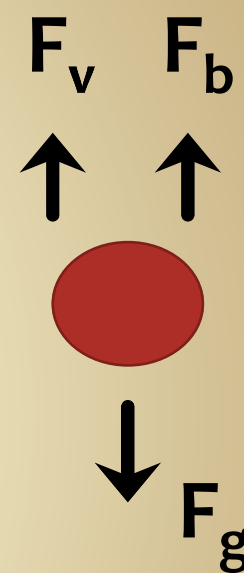
Equation of motion of a sphere falling through a viscous liquid

$$m\frac{dv}{dt} = F_g - F_b - F_v$$

$$= mg - \frac{4}{3}\pi r^3 \sigma g - 6\pi\eta r v$$

$$= V p g - V\sigma g - 6\pi\eta r v$$

$$= m_0 g - \gamma v \quad ; \; m_0 = V(p-\sigma); \gamma = 6\pi\eta r$$



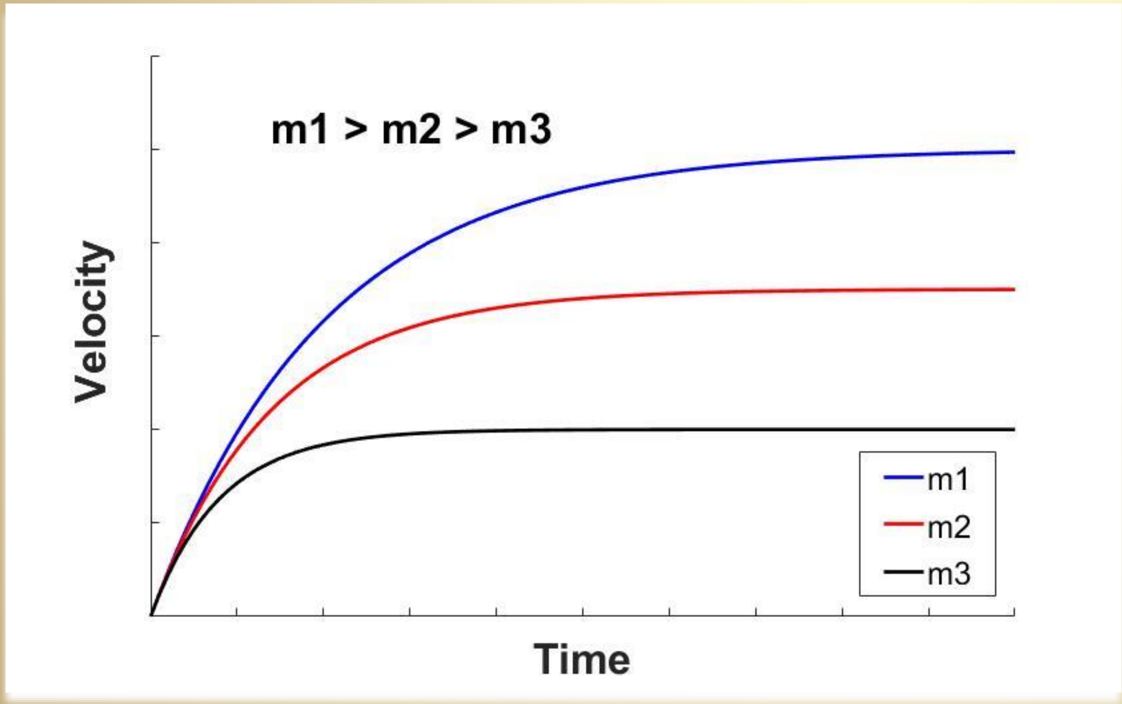
$$\Rightarrow \quad \frac{dv}{dt} = -bv + c \; ; \quad \left(b = \frac{\gamma}{m};\; c = \frac{m_0 g}{m}\right)$$

$$v(t) = \frac{c}{b}\left(1 - e^{-bt}\right) = \frac{m_0 g}{\gamma}\left(1 - e^{-\frac{\gamma}{m}t}\right)$$

Terminal Velocity

- **Velocity of a spherical ball falling thorugh a liquid:**

$$v(t) = \frac{c}{b}\left(1 - e^{-bt}\right) = \frac{m_0 g}{\gamma}\left(1 - e^{-\frac{\gamma}{m}t}\right)$$



Terminal velocity:

$$V_t = \frac{m_0 g}{\gamma}$$

$$\boxed{V_t = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}}$$

$$\boxed{V_t \propto r^2}$$

Viscous Force: Correction

- **Finite radius (R) and length (h) of the liquid cylinder**

$$F_V = 6\pi\eta r v \left(1 + 2.4\frac{r}{R}\right)\left(1 + 3.3\frac{r}{h}\right)$$

- **In our experimental set up:** $\frac{r}{R} \sim 0.1; \quad \frac{r}{h} \sim 0.002$

- **Coefficient of Viscosity:** $\boxed{\eta = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{V_t\left(1 + 2.4\frac{r}{R}\right)}}$

Experimental demonstration to determine Coefficient of Viscosity of a Liquid (Castor Oil)

Result & Analysis of Experimental Data

- **Determination of radius and mass of the spherical balls (for three different radii)**

S. No.	Linear Scale reading (in cm)	Circular Scale Reading	Diameter (in cm)	Mean Diameter (in cm)	Mean radius (in cm)	Mass of 10 balls (in gm)	Mass per ball (in gm)	Volume per ball in cm^3	Density per ball (gm/cc)

- **Coefficient of Viscosity:** $$\eta = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{V_t\left(1 + 2.4\frac{r}{R}\right)}$$

Result & Analysis of Experimental Data

- **Measurement of terminal velocity**

Sl. No.	Time for ball 1, small (in s)	Mean time (in s)	Time for ball 2, medium (in s)	Mean Time (in s)	Time for ball 3, large (in s)	Mean Time (in s)	Velocity for ball 1 (cm/s)	Velocity for ball 2 (cm/s)	Velocity for ball 3 (cm/s)

- **Determination of inner diameter of the glass cylinder**

- **Plot of terminal velocity Vs $(\text{radius})^2$ of the spherical ball**

- **Calculate coefficient of viscosity**

Writing a Report on Experiment

- **A brief write up on Aim, Theory, Working formula, Diagram**

- **Tabulate data on lab notebook /Soft copies (from given data)**

- **Results: analysis of data, plot (graph paper/plotting software)**

- **Estimation of error: Error analysis**
(viscosity of Castor oil is 0.650 Pascal.S at 25^0 C**)**

- **Discussions and probable sources of error**

- **Submit a PDF copy of the report on "Welearn"**

Questions??