# PH1102

**Physics Laboratory I**

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# Experiment Number - 4

## Young's Modulus

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# Contents

# 1 Aim

To determine the Young's modulus of a metal bar by flexure method.

# 2 Prerequisites

You must thoroughly read and understand the content of Appendix-I: Supplementary Reading.

# 3 Apparatus

- Metal Bar
- Knife-edge Support
- Weight Hanger
- Different Weights
- Metre Scale
- Screw Gauge
- Vernier Callipers
- Travelling Microscope

# 4 Principle and Working Formula

## 4.1 Elasticity

The ability of a body to deform in response to applied forces and to recover its original shape when the forces are removed.

## 4.2 Stress

The restoring force per unit area of the material.

## 4.3 Strain

The amount of deformation experienced by the body in the direction of force applied, divided by the initial dimensions of the body.

## 4.4 The Stress-Strain Curve

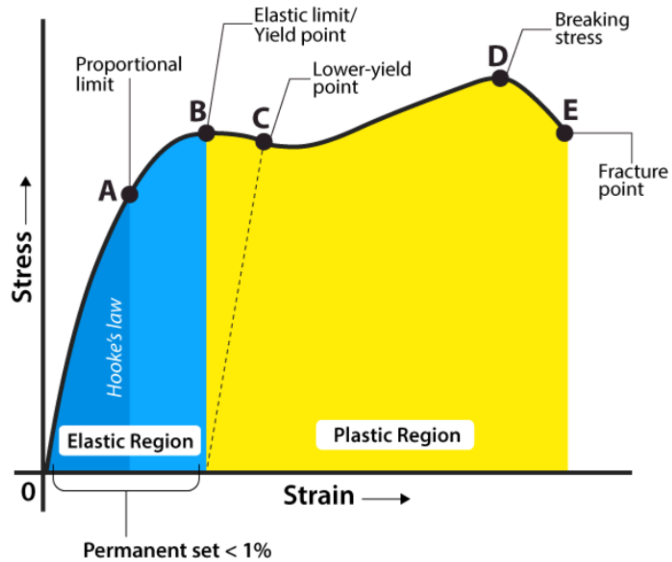


Figure 1: The Stress-Strain Curve depicting different sections of the graph

## 4.5 Explanation of the Stress-Strain Curve

(i) Proportional Limit: It is the region in the stress-strain curve that obeys Hooke's Law. In this limit, the stress-strain ratio gives us a proportionality constant known as Young's modulus. The point OA in the graph represents the proportional limit.

(ii) Elastic Limit: It is the point in the graph up to which the material returns to its original position when the load acting on it is completely removed. Beyond this limit, the material doesn't return to its original position, and a plastic deformation starts to appear in it.

(iii) Yield Point: The yield point is defined as the point at which the material starts to deform plastically. After the yield point is passed, permanent plastic deformation occurs. There are two yield points (i) upper yield point (ii) lower yield point.

(iv) Ultimate Stress Point: It is a point that represents the maximum stress that a material can endure before failure. Beyond this point, failure occurs.

(v) Fracture or Breaking Point: It is the point in the stress-strain curve at which the failure of the material takes place.

## 4.6 The different moduli to consider during the experiment

Y = Young's modulus, K = Bulk modulus,$\eta$ = Rigidity modulus, and $\sigma$ = Poisson's ratio.

## 4.7 The relation between various moduli

$$\boxed{Y = 3K(1-2\sigma) = 2\eta(1+\sigma) = \frac{9K\eta}{3K+\eta}}$$

## 4.8 The actual working of the experiment

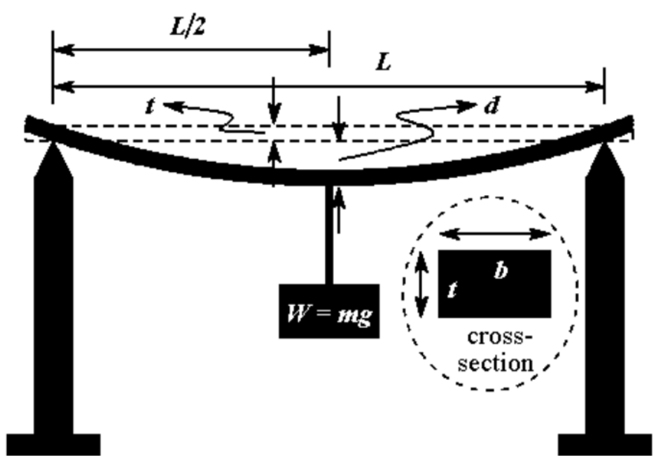


Figure 2: The schematic diagram of the experiment

A metal bar is kept horizontal with two knife-edge support at the two ends. When different suitable weight is hanged at the middle point of the bar it gets depressed (Fig. 2). The amount of depression $d$ is related to the Young's modulus $Y$ of the bar material. By experimentally measuring length, breadth and thickness of the bar and depression $d$ for different weights $W$ Young's modulus $Y$ can be determined. The bar in Fig. 2 can be regarded as two cantilevers, each of length $\frac{L}{2}$, fixed at the center (where weight is hanged) and loaded by $\frac{W}{2}$ at the two knife-edges. Each cantilever undergoes depression $d$ given by

$$d = \frac{WL^3}{4Ybt^3} = \frac{mgL^3}{4Ybt^3} \tag{1}$$

$$\Rightarrow d = \frac{mgL^3}{4bdt^3} \tag{2}$$

Actually, it is difficult to know the absolute value of depression, because, even without any additional weight, the bar will be depressed slightly due to its own weight. So, one measures relative depression as a function of added weights and plots a graph: $d$ vs. $m$, which is a straight line with a slope,

$$s = \frac{gL^3}{4Ybt^3} \tag{3}$$

From the slope $s$ of the $d$ vs. $m$ graph one can calculate Young's modulus as,

$$Y = \frac{gL^3}{4sbt^3} \tag{4}$$

Table 1: Symbols used in this experiment

| Symbol | Explanation | Unit |
|---|---|---|
| Y | Young's Modulus of the material of the beam | $\frac{N}{m^2}$ |
| M | Load Applied | kg |
| L | Distance between knife edges | m |
| g | Acceleration due to gravity | $\frac{m}{s^2}$ |
| b | Breadth of beam | m |
| t | Thickness of beam | m |
| s | Depression produced for 'M' kg load | m |

# 5 Procedure

1. Measure the breadth ($b$) of the bar using a Vernier caliper and tabulate the data.

2. Measure the thickness ($t$) of the bar using a screw gauge and tabulate the data.

3. Measure the length ($L$) of bar between two knife-edges i.e., separation between two knife-edges (Fig. 2) using a meter scale and tabulate the data.

4. Mark the point on the bar exactly at the center of the two knife-edges. Put the hanger of the weights exactly at that point. The hanger has pointer on top.

5. Focus the travelling microscope such that the pointer is clearly seen in it. Position the travelling microscope such that the pointer is touching the horizontal cross wire. Note down the reading of the vertical scale of the travelling microscope.

6. Add one weight block on the hanger. [Mass of all the weight blocks should be already written on them. If it is missing, weigh the block and write it down.] The bar will depress and the pointer will move in the microscope field of view. Adjust microscope vertical position to align the pointer back to the horizontal cross wire. Note the reading of the vertical scale of the travelling microscope.

7. Similarly, note microscope reading for addition of 4-5 more weight blocks one after another.

8. Then start reducing the weight one by one and each time adjust microscope to align the pointer at the horizontal cross wire and take microscope reading till the hanger is empty again.

9. Record data for steps 5-8 in a table. Calculate average depression $d$ for a given amount of mass $m$ on the hanger.

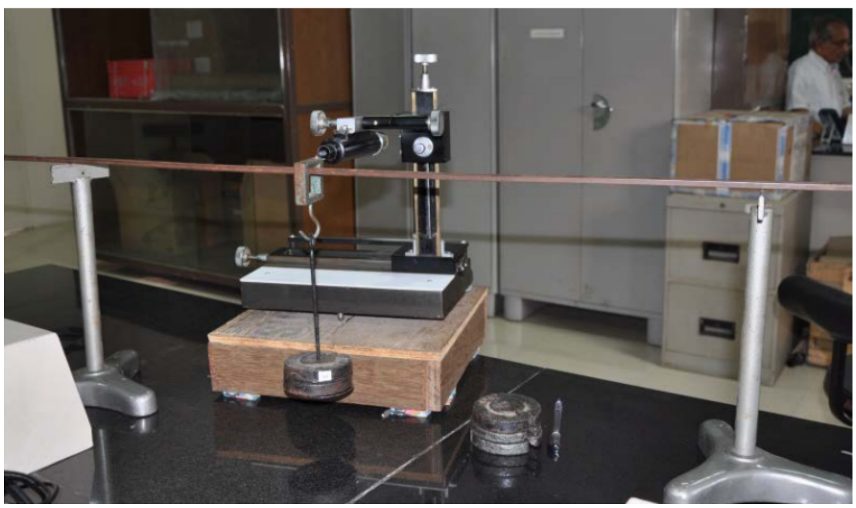

Figure 3: Photograph of experimental setup.

# 6 Observed Data

The length of the steel bar between two knife edges = 86 cm
Least count of the screw gauge = 0.001cm
Vernier constant = 0.002cm

Table 2: Table for detrmination of breadth of the bar using Vernier Callipers

| Sl. No. | MSR(cm) | VSR(cm) | VC(cm) | Breadth $[MSR + (VSR \times VC)]$(cm) | Average Breadth(cm) |
|---|---|---|---|---|---|
| 1 | 2.5 | 40 | 0.002 | 2.58 | |
| 2 | 2.5 | 16 | 0.002 | 2.532 | 2.55 |
| 3 | 2.5 | 25 | 0.002 | 2.55 | |

Table 3: Table for determination of thickness of the bar using screw gauge

| Sl. No. | LSR(mm) | CSR | LC(mm) | Thickness of the bar $[LSR + (CSR \times LC)]$(mm) | Average Thickness(mm) |
|---|---|---|---|---|---|
| 1 | 4.5 | 41 | 0.01 | 4.91 | |
| 2 | 4.5 | 36 | 0.01 | 4.86 | 4.88 |
| 3 | 4.5 | 38 | 0.01 | 4.88 | |

Table 4: Table for Measuring Depression for both Loading and Unloading mass

| Sl. No. | Mass on hanger(g) | Loading | | | Depression $(d_1)$(mm) | Unloading | | | Depression $(d_2)$(mm) | $d = \frac{d_1+d_2}{2}$(mm) |
|---|---|---|---|---|---|---|---|---|---|---|
| | | MSR (mm) | VSR | $x_1$ (mm) | | MSR (mm) | VSR | $x_2$ (mm) | | |
| 1 | 0 | 113 | 20 | 113.20 | 0 | 112.5 | 40 | 112.90 | 0 | 0 |
| 2 | 426 | 112 | 33 | 112.33 | 0.9 | 112 | 5 | 112.05 | 0.9 | 0.9 |
| 3 | 908 | 110.5 | 25 | 110.75 | 2.5 | 110.5 | 15 | 110.65 | 2.3 | 2.4 |
| 4 | 1393 | 109 | 10 | 109.10 | 4.1 | 109 | 23 | 109.23 | 3.7 | 3.9 |
| 5 | 1883 | 108 | 5 | 108.05 | 5.2 | 107.5 | 30 | 107.80 | 5.1 | 5.1 |
| 6 | 2843 | 105 | 38 | 105.38 | 7.8 | 105 | 30 | 105.30 | 7.6 | 7.7 |

(Vernier Constant = 0.01mm)

# 7 Calculations

## 7.1 $d$ $vs.$ $m$ Graph

The graph for Depression (d) vs. Mass (m) is as follows:-

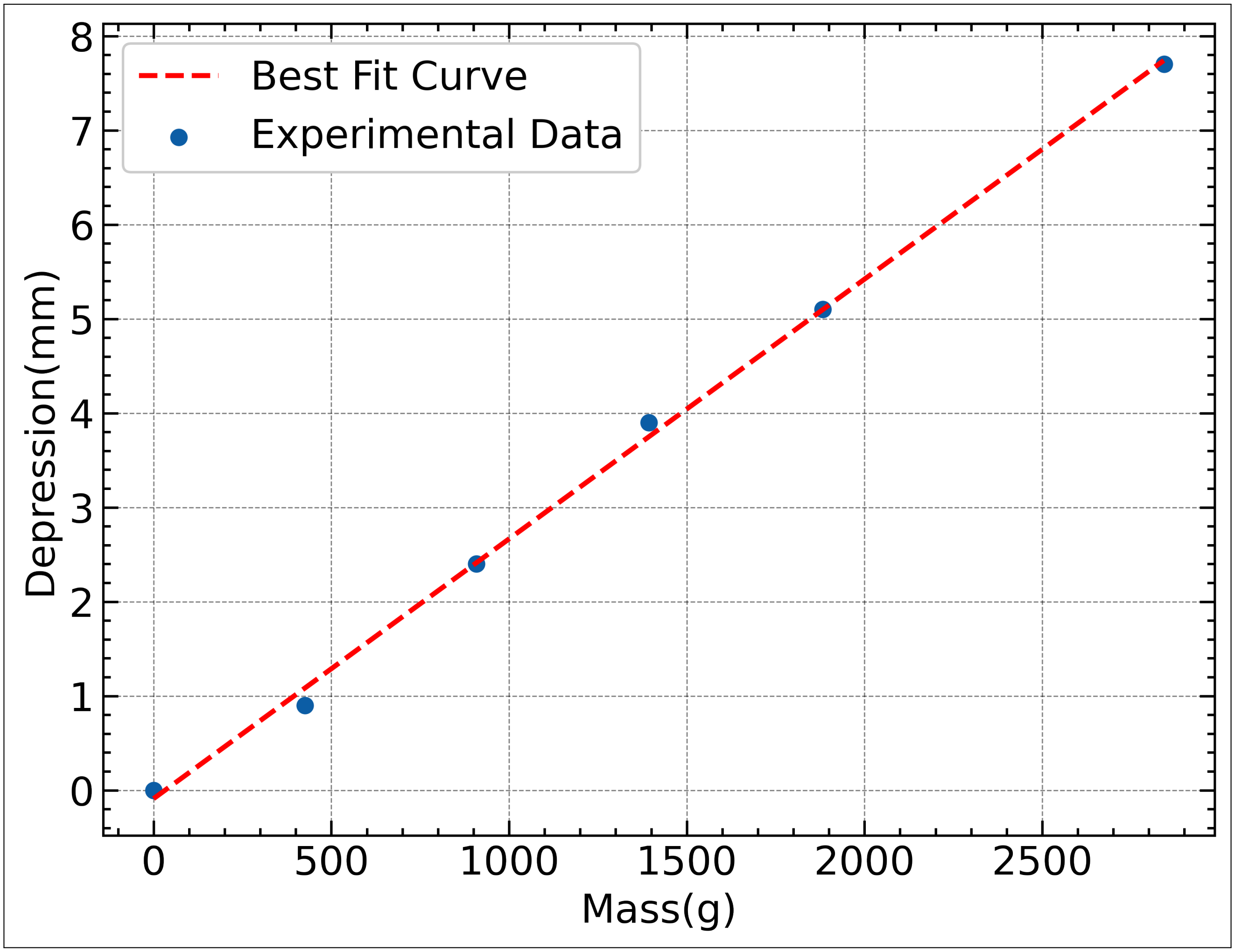


Figure 4: The graph for $d$ $vs.$ $m$

## 7.2 Curve-Fitting Algorithm/Code

The graph was drawn using the MatPlotLib, SciPy, and NumPy packages of Python. The curve fitting code (algorithm) used was :-

```
import matplotlib.pyplot as plt
plt.style.use(['science', 'notebook', 'grid'])
import numpy as np
from scipy.optimize import curve_fit
from matplotlib import pyplot

x = [0, 426, 908, 1393, 1883, 2843]
y = [113.05, 112.19, 110.70, 109.17, 107.93, 105.34]
plt.figure(dpi=500)
# define the true objective function
def objective(x, a, b):
   return a * x + b
# curve fit
popt, _ = curve_fit(objective, x, y)
# summarize the parameter values
a, b = popt
print('y = %.5f * x + %.5f' % (a, b))
# plot input vs output
pyplot.scatter(x, y, label='Experimental Data')
# define a sequence of inputs between the smallest and largest known inputs
x_line = np.arange(min(x), max(x), 1)
# calculate the output for the range
y_line = objective(x_line, a, b)
# create a line plot for the mapping function
pyplot.plot(x_line, y_line, '--', label='Best Fit Curve', color='red')
plt.xlabel('Mass(g)')
plt.ylabel('Depression(mm)')
plt.legend()
plt.savefig('Graphx.png')
pyplot.show()
```

This code resulted in a one degree polynomial as the best fit curve. The polynomial that it generated was:-

$$y = 0.00277x - 0.08847$$

So, it is visible from here that the slope of the line is,

$$|s| = 0.00277\ mmg^{-1}$$

.

### 7.3 Final Results

So, now we have:

- $L = 86cm = 0.86m$
- $g = 9.8ms^{-2}$
- $b = 2.55cm = 0.0255m$
- $t = 4.88mm = 0.00488mm$
- $s = 0.00277mmg^{-1} = 0.00277mkg^{-1}$

Therefore, on substituting the above values in the formula,

$$Y = \frac{gL^3}{4sbt^3}$$

one gets,

$$\Rightarrow Y = \frac{9.8 \times 0.86^3}{4 \times 0.00277 \times 0.0255 \times 0.00488^3} Nm^{-2}$$
$$\Rightarrow Y = 1.898375073 \times 10^{11} Nm^{-2} \approxeq 1.89 \times 10^{11} Nm^{-2}$$

# 8 Error Analysis

Literature Value of Young's Modulus of Steel $= 200 \times 10^9 Nm^{-2}$
(source: Click Here! )
Experimental Value of Young's Modulus of Steel $= 1.89 \times 10^{11} Nm^{-2} = 189 \times 10^9 Nm^{-2}$
(source: Check Here!)

Therefore, error is,

$$\Delta Y = 200 \times 10^9 Nm^{-2} - 189 \times 10^9 Nm^{-2}$$
$$\Rightarrow \Delta Y = 11 \times 10^9 Nm^{-2}$$

Percentage Error is,

$$\Delta Y\% = \frac{11 \times 10^9}{200 \times 10^9} \times 100\%$$
$$\Rightarrow \Delta Y = 5.5\%$$

which is within the accepted range.

# 9 Conclusion

In this experiment, we learned how to experimentally calculate the Young's modulus for a material, when it is shaped in the form of a long bar. Also, we learned the theoretical aspect of how the formula for Young's modulus for a material is derived. I enjoyed plotting the data points and performing the curve fitting.

# 10 Acknowledgements

- Creators of SciPy, NumPy, and MatPlotLib for providing the said packages.
- `https://byjus.com/physics/youngs-modulus-elastic-modulus/` for providing the literature value of Young's Modulus of Steel.
- Prof. Dhananjay Nandi for allowing us to view this experiment and create this lab report.