Name: Priyanshu Mahato
Roll No.: pm21ms002.

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PH 1201 TUTORIAL - 1.

Q. $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$; $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$

Vector perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}(6+9) - \hat{j}(-2+12) + \hat{k}(6+24)$$
$$= 15\hat{i} - 10\hat{j} + 30\hat{k}$$

Now, unit vector of $\vec{A} \times \vec{B}$,

$$\Rightarrow \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{15\hat{i} - 10\hat{j} + 30\hat{k}}{\sqrt{15^2 + 10^2 + 30^2}} = \frac{1}{35}(15\hat{i} - 10\hat{j} + 30\hat{k})$$

$$= \frac{1}{7}(3\hat{i} - 2\hat{j} + 6\hat{k}) \quad \text{ans.}$$

Q2. $\vec{A} = 3\hat{i} - \hat{j} + 2\hat{k}$; $\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$; $\vec{C} = \hat{i} - 2\hat{j} + 2\hat{k}$

(a) $\vec{A} \cdot (\vec{B} \times \vec{C}) = (3\hat{i} - \hat{j} + 2\hat{k}) \cdot \{(2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - 2\hat{j} + 2\hat{k})\}$

$$= \begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & -1 \\ 1 & -2 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & -1 & 2 & 3 & -1 \\ 2 & 1 & -1 & 2 & 1 \\ 1 & -2 & 2 & 1 & -2 \end{vmatrix}$$

$2 \quad 6 \quad -4 \quad 6 \quad 1 \quad -8.$

$$= (6 + 1 - 8) - (2 + 6 - 4)$$
$$= (7 - 8) - (8 - 4)$$
$$= -1 - 4 = -5 \quad \text{ans.}$$

(b) $(\vec{A} \times \vec{B}) \cdot \vec{C}$

This is the same as STP (Scalar Triple Product) which was asked in the previous part of the question. So, the answer remains same.

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = \underline{-5}.$$

(c) $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

$$= (2\hat{i} + \hat{j} - \hat{k})\{(3\hat{i} - \hat{j} + 2\hat{k}) \cdot (\hat{i} - 2\hat{j} + 2\hat{k})\}$$

$$- (\hat{i} - 2\hat{j} + 2\hat{k})\{(3\hat{i} - \hat{j} + 2\hat{k}) \cdot (2\hat{i} + \hat{j} - \hat{k})\}$$

$$= (2\hat{i} + \hat{j} - \hat{k})(3 + 2 + 4) - (\hat{i} - 2\hat{j} + 2\hat{k})$$
$$(6 - 1 - 2)$$

$$= (2\hat{i} + \hat{j} - \hat{k})(9) - (\hat{i} - 2\hat{j} + 2\hat{k})(3)$$

$$= (18\hat{i} + 9\hat{j} - 9\hat{k}) - (3\hat{i} - 6\hat{j} + 6\hat{k})$$

$$= \underline{15\hat{i} + 15\hat{j} - 15\hat{k}} \quad \text{ans.}$$

(d) $(\vec{A}\times\vec{B})\times\vec{C}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}(1-2) - \hat{j}(-3-4) + \hat{k}(3+2)$$

$$= -\hat{i} + 7\hat{j} + 5\hat{k}$$

$$(-\hat{i}+7\hat{j}+5\hat{k})\times\vec{C} = (-\hat{i}+7\hat{j}+5\hat{k})\times(\hat{i}-2\hat{j}+2\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 7 & 5 \\ 1 & -2 & 2 \end{vmatrix}$$

$$= \hat{i}(14+10) - \hat{j}(-2-5) + \hat{k}(2-7)$$

$$= \underline{24\hat{i} + 7\hat{j} - 5\hat{k}} \quad \text{ans}$$

Q3. $\vec{a_1} = 3\hat{i} + \hat{j} - 2\hat{k}$; $\vec{a_2} = \hat{i} - 3\hat{j} + 4\hat{k}$

Area of $\parallel^{gm} = \frac{1}{2}(\vec{a_1} \times \vec{a_2})$

$= \frac{1}{2}\{(3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} - 3\hat{j} + 4\hat{k})\}.$

$= \frac{1}{2}\left\{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}\right\}.$

$= \frac{1}{2}\{\hat{i}(4-6) - \hat{j}(12+2) + \hat{k}(-9-1)\}.$

$= \frac{1}{2}\{-2\hat{i} - 14\hat{j} - 10\hat{k}\}.$

$= -\hat{i} - 7\hat{j} - 5\hat{k}$

Q4. Projection vector of $\vec{A}$ ∥ to $\vec{B}$ $= \frac{(\vec{A}\cdot\vec{B})}{|B|^2}\vec{B}$

$= \frac{(8+9+6)}{\sqrt{26}^2}(4\hat{i}-3\hat{j}+\hat{k})$

$= \frac{23}{\sqrt{26}^2}(4\hat{i}-3\hat{j}+\hat{k}) = \frac{23}{26}(4\hat{i}-3\hat{j}+\hat{k})$

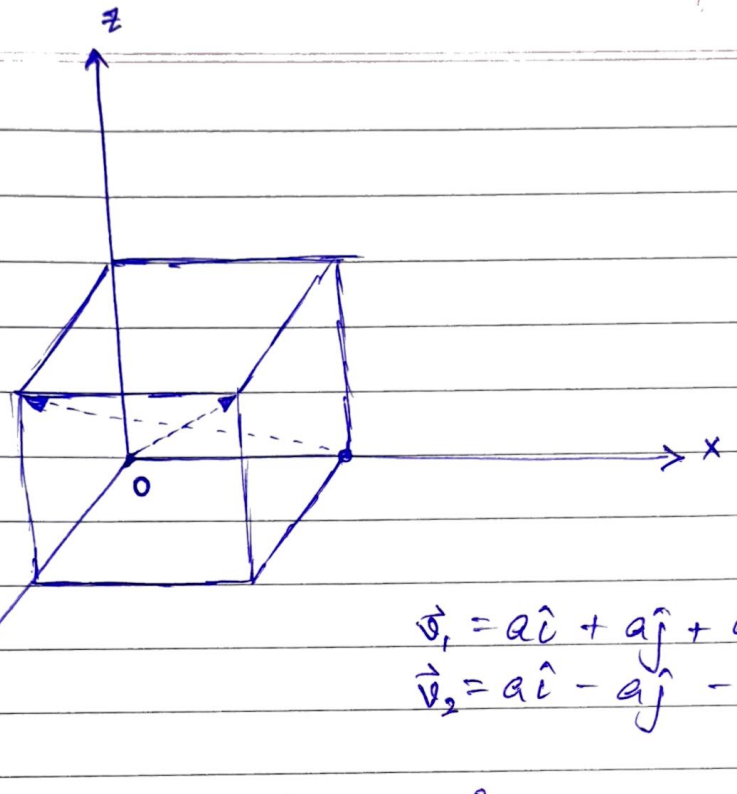
Projection vector of $\vec{A}$ $\perp^r$ to $\vec{B}$ $= \frac{|\vec{A}\times\vec{B}|}{|B|^2}\vec{B}$

$= \frac{|(2\hat{i}-3\hat{j}+6\hat{k})\times(4\hat{i}-3\hat{j}+\hat{k})|}{(\sqrt{26})^2}(4\hat{i}-3\hat{j}+\hat{k})$

$= \frac{|15^2+22^2+6^2|}{26}(4\hat{i}-3\hat{j}+\hat{k})$

$= \frac{\sqrt{745}}{26}(4\hat{i}-3\hat{j}+\hat{k})$

Qs.



$\vec{v}_1 = a\hat{i} + a\hat{j} + a\hat{k}$

$\vec{v}_2 = a\hat{i} - a\hat{j} - a\hat{k}$

Angle b/w body diagonal $= \cos^{-1}\left(\frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1||\vec{v}_2|}\right)$

$= \cos^{-1}\left|\left(\frac{a^2 - a^2 - a^2}{\sqrt{3}a\sqrt{3}a}\right)\right|$

$= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right).$