ASSIGNMENT - 4
(PARTIAL DIFFERENTIATION)

① Given $z = f(x, y) = x^3 y - e^{xy}$, then evaluate following

[10marks]

ⓐ $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$

ⓑ $\dfrac{\partial^2 f}{\partial x \partial y}$

ⓒ $\dfrac{\partial^2 f}{\partial x^2}$

ⓓ $\dfrac{\partial^3 f}{\partial y^3}$

ⓔ $\dfrac{\partial^3 f}{\partial x^2 \partial y}$

②ⓐ find dy/dx where $y = \ln \sin 2x$, using partial differentiation

(5 marks)

ⓑ find dz/dt where $z = 2t^2 \sin t$, using partial differentiation

(5 marks)

③ⓐ Given $x + e^x = t$, find dx/dt and d^2x/dt^2 (5 marks)

ⓑ find the equation of the tangent line to the curve

(5 marks)

$x^3 - 3y^3 + xy + 21 = 0$ at the point $(1, 2)$

④ find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ where $z = xy$, $x = \sin(s+t)$, $y = s-t$.

(10 marks)

⑤ Let x, y be rectangular coordinates and r, θ be polar coordinates in a plane. Then equations relating them are

$$x = r \cos\theta, \quad y = r \sin\theta, \quad r = \sqrt{x^2 + y^2} \text{ and } \theta = \tan^{-1}(y/x).$$

Prove that $\dfrac{\partial x}{\partial \theta} \neq \dfrac{\partial \theta}{\partial x}$.

(10 marks)

⑥ A wire is bent to fit the curve $y = 1 - x^2$ (see figure below). A string is stretched from the origin to a point (x, y) on the curve. Find (x, y) to minimize the length of string. Use Lagrange multipliers method.
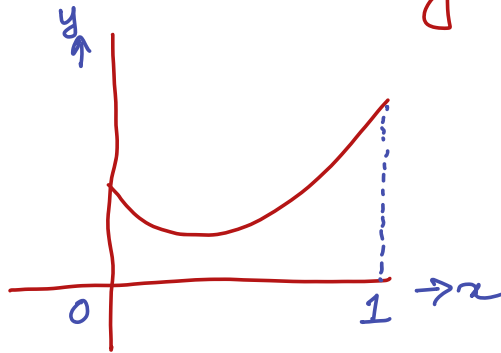
(15 marks)
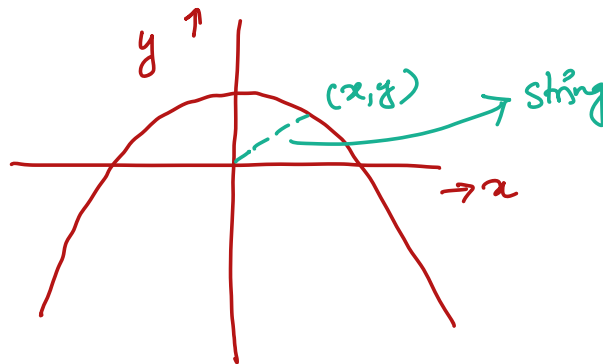
figure

⑦ Make the change of variables $r = x + \upsilon t, \ S = x - \upsilon t$ in the wave equation $\dfrac{\partial^2 F}{\partial x^2} - \dfrac{1}{\upsilon^2} \dfrac{\partial^2 F}{\partial t^2} = 0$ and solve the equation.
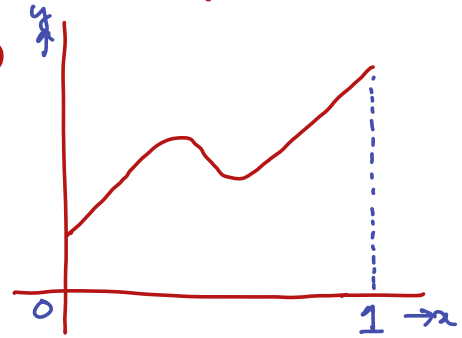
(10 marks)

⑧ Find $\dfrac{d I}{dx}$ where $I = \displaystyle\int_{\pi/4}^{x} \sin t \, dt$ and write a general form of defination of differentiation of integrals.

(10 marks)

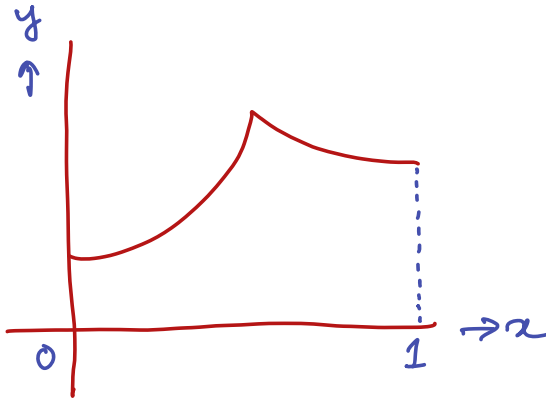**q)** Write down the boundary Points for the following figures:

a)

b)

c)

(15 marks)