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PH1201 CLASS TEST

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Q1. (I) $\int_{C_I} \vec{v}\cdot d\vec{r} = \int_0^9 3x \cdot dy.$

$d\vec{r} = dy$ (as $x=$ const.)
$x = 3.$

$\Rightarrow \quad 3\times 3 \int_0^9 dy = 3\times 3 \times [y]_0^9 = 9\times 9 = \underline{81}$

(II) $d\vec{r} = dx$
$y = 9$

$\int_{C_{II}} \vec{v}\cdot d\vec{r} = \int_3^0 -2y \cdot dx = \int_0^3 2y\, dx = 18\,[x]_0^3$

$= \underline{54}$

(III) Eqn of line : $y = 3x+9$

For x-dir^n: $\int_0^{-3} -2y\,dx. = \int_{-3}^{0} 2y\,dx.$

$= \int_{-3}^{0} 2(3x+9)\,dx$

$= \int_{-3}^{0} (6x+18)\,dx$

$= \int_{-3}^{0} 6x\,dx + 18\int_{-3}^{0} dx.$

$= \cancel{6}^{3} \cdot \left[\frac{x^2}{\cancel{2}}\right]_{-3}^{0} + 18\,[x]_{-3}^{0}$

$= 3\cdot(-9) + 18\cdot(3)$

$= -27 + 54$

$= 27.$

For Y-dir^n: $\int_9^0 3x\,dy. = \int_9^0 \cancel{3}\left(\frac{y-9}{\cancel{3}}\right) dy$

$= \int_9^0 (y-9)\,dy = \int_9^0 y\,dy - \int_9^0 9\,dy$

$= \frac{1}{2}\left[y^2\right]_9^0 - 9\left[y\right]_9^0 = -\frac{81}{2} - 9(-9)$

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$$= -\frac{81}{2} + 81$$

$$= \frac{81}{2}$$

$$\therefore \int_{C_{III}} \vec{v} \cdot d\vec{r} = 27 + \frac{81}{2} = \frac{54+81}{2} = \frac{135}{2}$$

(IV) $y = x^2 - A^2$

For x-dirⁿ: $\int_{-3}^{3} -2y\,dx = -\int_{-3}^{3} 2(x^2 - A^2)\,dx$

$$= -\int_{-3}^{3} 2(x^2 - 9)\,dx$$

$$= \underline{\underline{72}}$$

For Y-dirⁿ: $\int 3x\,dy$.

$y = x^2 - A^2 \Rightarrow dy = 2x\,dx$.

$$= \int 3x \cdot 2x \cdot dx$$

$$= \int 6x^2\,dx = 6\int x^2\,dx = \cancel{6}^{2} \cdot \left[\frac{x^3}{\cancel{3}}\right]_{-3}^{3}$$

$$= 2 \cdot (27 + 27) = 108$$

$$\therefore \int_{C_{II}} \vec{v}\cdot d\vec{r} = \cancel{}\ 108 + 72 = 180.$$

$$\cancel{\text{(V)} \oint_C \vec{v}\cdot d\vec{r} \text{ (in opposite dir}^n\text{)} = (-81) + (-54) + \left(-\frac{135}{2}\right) + (-72) = -274.5 \text{ ans.}}$$

(V) $\oint_C \vec{v}\cdot d\vec{r}$ (opposite dir^n) $= (-81) + (-54) + \left(-\frac{135}{2}\right) + (-180)$

$= \underline{-382.5}$ ans

Q6. $\vec{B}$ and $(\vec{B}\cdot\vec{B})\vec{A} - (\vec{A}\cdot\vec{B})\vec{B}$

Taking dot product, we get,

$$= (\vec{B}\cdot\vec{B})(\vec{A}\cdot\vec{B}) - (\vec{A}\cdot\vec{B})(\vec{B}\cdot\vec{B})$$
$$= 0$$

Since, $\vec{B}$ and $(\vec{B}\cdot\vec{B})\vec{A} - (\vec{A}\cdot\vec{B})\vec{B}$ are both non-zero, $\cos\theta = 0 \Rightarrow \theta = 90^\circ$.

$\therefore$ Angle b/w vectors $= \underline{90.00^\circ}$

Q.

Q7. $\vec{A} = -2x^2y\,\hat{i} + 4xz\,\hat{j} - 4yz\,\hat{k}$

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{\nabla}\times\vec{A})$

Consider $\vec{\nabla}\times\vec{A} = \vec{B}$, then, the ~~go~~ expression reduces to,

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B})$

But we know that Div (curl) of a vector

is always zero.

$\therefore \vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0$

Q8. $\varphi = 2xz - y^2$

Maximum Derivative = Gradient.

$\vec{\nabla}\varphi = \vec{\nabla}(2xz - y^2)$.

$= \left\{ \frac{\partial}{\partial x}(2xz - y^2)\hat{i} + \frac{\partial}{\partial y}(2xz - y^2)\hat{j} \right.$

$\left. + \frac{\partial}{\partial z}(2xz - y^2)\hat{k} \right\}$.

$= 2z\hat{i} + (-2y)\hat{j} + (2x)\hat{k}$

$= 2z\hat{i} - 2y\hat{j} + 2x\hat{k}$

$= 2(z\hat{i} - y\hat{j} + x\hat{k})$

$\vec{\nabla}\varphi$ (at $(-1, 2, -2)$) $= 2(-2\hat{i} - 2\hat{j} - 1\hat{k})$

$= -4\hat{i} - 4\hat{j} - 2\hat{k}$

Magnitude $= \sqrt{4^2 + 4^2 + 2^2}$

$= \sqrt{16 + 16 + 4}$

$= \sqrt{36} = \pm 6 = \underline{\underline{6}}$ $\underline{\underline{\text{ans.}}}$