MA 1101 : Mathematics I

Problem 1.
Let A, B, C be sets. Prove that

(i) $A \cup B = B \cup A$, $A \cap B = B \cap A$.
(ii) $(A \cup B) \cup C = A \cup (B \cup C)$, $(A \cap B) \cap C = A \cap (B \cap C)$.
(iii) $A \subseteq B$ if and only if $A \cup B = B$.
(iv) $A \subseteq B$ if and only if $A \cap B = A$.
(v) $A \subseteq B$ if and only if $A \setminus B = \emptyset$.
(vi) $A \setminus (A \setminus B) = A \cap B$.
(vii) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.
(viii) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.
(ix) $A \Delta B = (A \cup B) \setminus (A \cap B)$.
(x) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.
(xi) $A \Delta (B \Delta C) = (A \Delta B) \Delta C$.
(xii) $A \Delta B = A \Delta C$ if and only if $B = C$.

Problem 2.
Let A, B, C, D be sets. Then,

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
(iii) $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$.
(iv) Is it true that $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$?
(v) Is it true that $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$?
(vi) Is it true that $(A \cup C) \times (B \cup D) = (A \times B) \cup (C \times D)$?

Problem 3.
Let $n \in \mathbb{N}$ and let X be a set of n elements. Calculate

(i) the number of subsets of X.
(ii) the number of non-empty subsets of X.
(iii) the number of ways one can choose two disjoint subsets of X.
(iv) the number of ways one can choose two non-empty disjoint subsets of X.