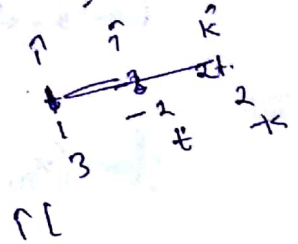


IISER Kolkata
Mid-Semester Examination
First Year Semester I; 2015
Time One Hour; Full Marks 20
Answer all questions

1. Calculate the area bounded by the pair of curves $y = 4x - x^2$, and $y = x$.
2. A curve is given by $x^2 - 4x + y^2 = 0$. Find the equation of the tangent and normal to the curve at $(0, 0)$.
3. Find $\frac{du}{dt}$ where $u = e^x \sin y + e^y \sin x$; $[x = \frac{1}{2}t; y = 2t]$.
4. Evaluate $\int \mathbf{A} \cdot \frac{d\mathbf{A}}{dt} dt$ between points P and Q where $\mathbf{A}(P) = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{A}(Q) = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.
5. Check whether the vector $\mathbf{F} = r\mathbf{r}$ is conservative or not. If it is, find a scalar function ϕ so that $\mathbf{F} = -\nabla\phi$.

$$f(n) = \frac{f^{(0)}(n)}{0!} (n-a)^0 + (-1)^1 \frac{f^{(1)}(n)}{1!} (n-a)^1 + (-1)^2 \frac{f^{(2)}(n)}{2!} (n-a)^2$$



IISER Kolkata
End-Semester Examination
First Year Semester I; 2015
MA1101: Mathematics-I

Time Three Hours; Full Marks 50

Answer all questions (Q1-5: 4 marks; Q6-10: 6 marks)

$$4xy^2 + 6xy^3 - 3xy^4$$

$$58xy^3 \cdot dy$$

$$2x \cdot \frac{40y}{80} + \frac{50x \cdot 0}{80 \cdot x}$$

1. Evaluate $\int_{(0,0)}^{(2,1)} \{(10x^4 - 2xy^3)dx - 3x^2y^2dy\}$ along the path $x^4 - 6xy^3 = 4y^2$.
2. Evaluate $\int_1^2 \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) dt$ where $\mathbf{A} = t\mathbf{i} - 3j + 2t\mathbf{k}$, $\mathbf{B} = \mathbf{i} - 2j + 2\mathbf{k}$, $\mathbf{C} = 3\mathbf{i} + t\mathbf{j} - \mathbf{k}$.
3. Evaluate $\int_S \mathbf{r} \cdot d\mathbf{S}$, either directly or by using any theorem that you know, where the surface S encloses a sphere of radius unity and \mathbf{r} is the position vector.

4. Show that $\nabla r^3 = 3r\mathbf{r}$.

5. The area of an ellipse is given as $\frac{1}{2} \int_C (xy - ydx)$. Find the area of the ellipse $x = a \cos \theta$, $y = b \sin \theta$.

6. A system of linear equations is given by

$$\begin{aligned} 3x + y + 2z &= 3, \\ 2x - 3y - z &= -2, \text{ and} \\ x + y + z &= 1. \end{aligned}$$

Write down the coefficient matrix. Find its inverse and hence find a solution for the system of equations.

7. A matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 4 \end{pmatrix}. \text{ Find the eigenvalues of the matrix. Find out the eigen vectors. Check that one}$$

can form an orthonormal basis with these vectors. Show by direct calculation that you can construct a matrix \mathbf{X} out of these eigenvectors which diagonalizes the matrix \mathbf{A} by a similarity transformation.

8(a). Show that $(\mathbf{AB})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1}$ where \mathbf{A} and \mathbf{B} are square matrices.

(b). \mathbf{A} and \mathbf{B} are Hermitian square matrices. If \mathbf{x} is an eigenvector of \mathbf{A} , show that it is also an eigenvector of \mathbf{B} .

9(a). With the help of a comparison test, show that the harmonic series

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

is a divergent series.

(b). With the help of a ratio test. check that

$$\frac{3}{4} + 2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right)^3 + \dots$$

is a convergent series.

10. Use Taylor's formula to verify that

$$(a). \ln(a+x) = \ln a + \frac{x}{a} - \frac{x^2}{2a^2} + \frac{x^3}{3a^3} + \dots$$

$$(b). \tan\left(\frac{\pi}{4} + x\right) = 1 + 2x + 2x^2 \text{ for small } x. \text{ Hence find the value of } \tan 46^\circ.$$