ASSIGNMENT - III

PH1101: Mechanics-I

Topic: Polar Coordinates

Time: 24 hour *(Last date: 3:00PM@ Jan 22, 2022)* Maximum Marks: 100

1. (a) Derive the base vectors in polar coordinates? (5M)
   (b) Sketch the cartesian and polar coordinate system and explain the axis?(5M)
   (c) How the cartesian and polar coordinates are related?(5M)
   (d) Derive $\frac{d\hat{r}}{dt}$ and $\frac{d\hat{\theta}}{dt}$ in Polar Coordinates?(5M) [20]

2. Derive and discuss
   (a) the velocity in Polar Coordinates? (10M)
   (b) the acceleration in Polar Coordinates? (10M) [20]

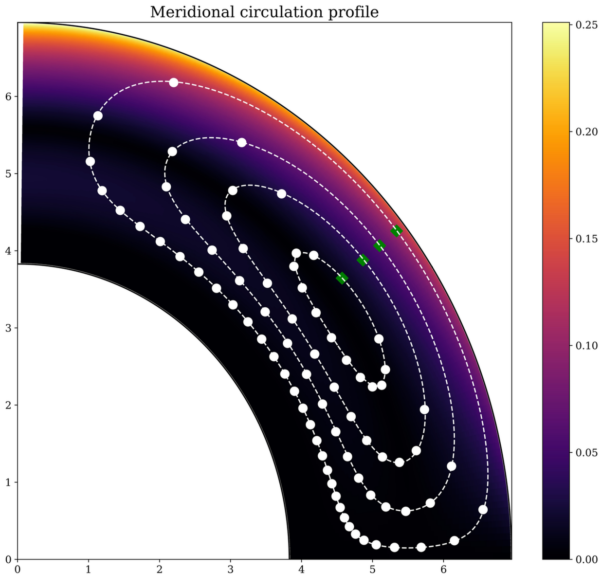
3. A bead moves along the spoke of a wheel at constant speed u meters per second. The wheel rotates with uniform angular velocity $\dot{\theta} = \omega$ radians per second about an axis fixed in space. At $t = 0$ the spoke is along the x axis, and the bead is at the origin. Find the bead's velocity at time t:
   (a) in polar coordinates; (5M)
   (b) in Cartesian coordinates.(5M)
   (c) Find the bead's acceleration in polar coordinates. (5M)
   (d) Sketch the trajectory of the bead. (5M) [20]

4. (a) A particle moves in a circle with $\theta = \omega t$ and $\vec{r} = rexp(\alpha t)$. Find the velocity as a function of time? (4M)
   (b) Find polar coordinates of the point $(2, -6)$ in a cartesian coordinates ?(2M)
   (c) Find cartesian coordinates of the point $(2, \frac{\pi}{4})$ in a polar coordinates?(2M)
   (d) Convert the equation $r = 4tan\theta sec\theta$ in terms of cartesian coordinates?(2M) [10]

5. Covert the given equations which are in terms of cartesian coordinates (x, y), in terms of polar coordinates (r, θ) (5M+5M)
   (a)
   $$\frac{4x}{3x^2 + 3y^2} = 6 - xy \tag{1}$$
   (b)
   $$x^2 = \frac{4x}{y} - 3y^2 + 2 \tag{2}$$
   (c) Show that the Polar curve $C : r = 2(cos\theta - sin\theta)$ represents a circle. Find its radius and cartesian coordinates of its centre? (5M) [15]

6. Our host star, the Sun, is known to harbor a gigantic circulation belt within its interior. The adjacent image shows a 2 dimensional annular cut-out of the Sun. The four green particles start flowing anti-clockwise along their respective velocity field streamlines (called the meridional circulation). The subsequent white dots denote their positions after a certain time interval along each of the four streamlines. The radial velocity and the angular velocity at any location can be approximately formulated as follows:
   $$v_r = \frac{1}{r^2}(2cos^2\theta - sin^2\theta); v_\theta = \frac{1}{r^3}(sin\theta cos\theta) \tag{3}$$

Student's name: Question 6 continues on the next page...



(a) Which system of coordinates would be the most suitable to study the motion of the particles here, explain (5M)?
(b) Write the equation of radial and tangential velocity (5M)?
(c) Assuming the initial position of a particle to be $(6, \frac{\pi}{3})$, calculate the its final radial position after 1 time unit (5M)?. [15]
