# Expt. #07: Dynamical viscosity of a liquid and Stokes' law

**Aim:** To determine the coefficient of viscosity of a liquid using a falling ball viscometer (employing Stokes' law)

**Experimental apparatus:** The main experimental apparatus (Fig. 1) consists of a glass tube containing the experimental liquid. **P** and **Q** are two adjustable reference marks along the tube length. The entire glass system is supported by a wooden stand. Besides this you will need a few steel balls of different diameter, screw gauge, digital balance, vernier calliper, stop watch, and meter scale for this experiment.

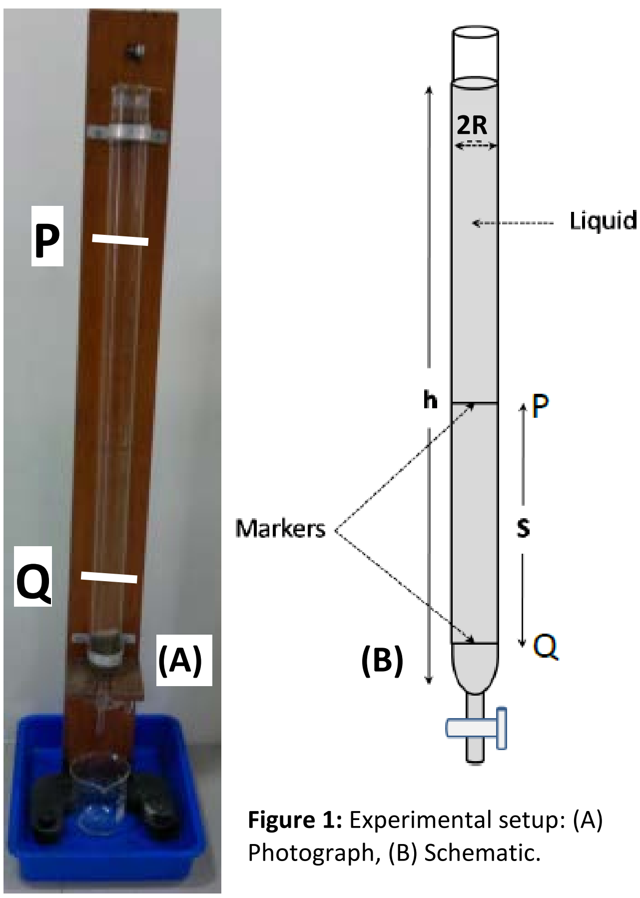


**Figure 1:** Experimental setup: (A) Photograph, (B) Schematic.

**Principle of the experiment:** A body moving in a fluid is acted on by a frictional force in the opposite direction of its velocity. The magnitude of this force depends on the geometry of the body, its velocity, and the internal friction of the fluid. A measure for the internal friction is given by the dynamic viscosity $\boldsymbol{\eta}$. For a spherical ball of radius $\boldsymbol{r}$ moving at velocity $\boldsymbol{v}$ in an infinitely extended fluid of dynamic viscosity $\boldsymbol{\eta}$, G. G. Stokes derived the viscous force to be: $\boldsymbol{F_v = 6\pi\eta r v}$ $(\mathbf{1})$. If the spherical ball (density $\boldsymbol{\rho}$) is dropped from rest at the upper surface of a vertical liquid (density $\boldsymbol{\sigma}$) column (as in Fig. 1), gravitational force $\boldsymbol{F_g = \frac{4}{3}\pi r^3 \rho g}$ ($\boldsymbol{g}$ = acceleration due to gravity) and buoyancy force $\boldsymbol{F_b = \frac{4}{3}\pi r^3 \sigma g}$ act on it besides viscous force $\boldsymbol{F_v = 6\pi\eta r v}$. Directions of forces are schematically shown in Fig. 2.

Initially, $\boldsymbol{F_v}$ is zero because ball started at rest. So, the ball accelerates downwards because there is a net downward force. Then $\boldsymbol{F_v}$ starts to increase. Eventually a force balance $\boldsymbol{F_v + F_b = F_g}$ is reached and the ball attains a steady terminal velocity $\boldsymbol{v_t}$. Using the force balance condition one can easily derive

$$\boldsymbol{\eta = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{v_t}} \qquad (\mathbf{2}).$$

Actually, Eq. (1) [derived under the assumption of infinitely extended liquid] should be corrected for the finite size of the liquid column. For the movement of the spherical ball along the axis of a liquid cylinder of radius $\boldsymbol{R}$ and length $\boldsymbol{h}$, the viscous force is $\boldsymbol{F_v = 6\pi\eta r v\left(1+2.4\frac{r}{R}\right)\left(1+3.3\frac{r}{h}\right)}$. For the experimental situation in our lab, $\boldsymbol{\frac{r}{R}\sim 0.1}$ and $\boldsymbol{\frac{r}{h}\sim 0.001}$. Thus, finite length correction $\boldsymbol{\left(1+3.3\frac{r}{h}\right)}$ may be ignored. Incorporating the correction due to finite radius of the liquid column, Eq. (2) becomes:

$$\boldsymbol{\eta = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{v_t\left(1+2.4\frac{r}{R}\right)}} \qquad (\mathbf{3}).$$

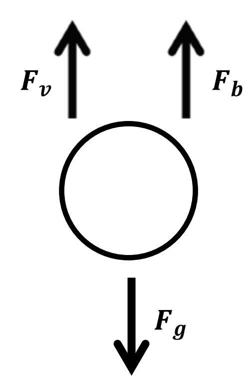


**Figure 2:** Force directions.

**Procedure:**

**A. Measure radius and density of steel balls**

1. Take steel balls of **3** different sizes, **5-7** balls of each **3** sizes. Measure diameters of all the balls and calculate the average radius $\boldsymbol{r}$ for balls of same sizes. Present data in tabulated form.
2. Use measured radius data to calculate total volume of the all the balls. Measure the total mass of all the balls together using a digital balance. Calculate density $\boldsymbol{\rho}$ of the balls using appropriate

formula. Present data in tabulated form. *** All balls are of same material; measuring mass of all balls together will reduce error. ***

**B. Measure terminal velocity**

1. Set the pointer **P** at about $\mathbf{10}$ cm below the top surface of the liquid column. (The distance of the pointer **P** should be chosen such that by the time a ball reach at **P**, it should attain terminal velocity. So, you should check if the balls are attaining terminal velocity before reaching at **P**.) Set the pointer **Q** about $\mathbf{80}$ cm below the pointer **P** (Fig. 1). Measure the actual distance $\boldsymbol{S}$ between **P** and **Q** using a meter scale and record data in tabular form.
2. Be prepared with a stop watch to measure the time a ball takes to cross the distance between **P** and **Q**.
3. Take **1** ball of the smallest size and drop it just above the top surface, near the axis of the liquid column. It is expected to attain terminal velocity before reaching to the pointer **P**.
4. Start the stop watch just when the ball crosses pointer **P** and stop the stop watch just when the ball crosses pointer **Q**. This gives the time $\boldsymbol{t}$ required to cross the distance $\boldsymbol{S}$ with a uniform (terminal) velocity $\boldsymbol{v_t}$.
5. Repeat steps **3** and **4** for all the balls and record data in tabular form. Calculate average terminal velocity for balls of same sizes.
6. Measure the inner diameter of the liquid containing glass cylinder by a vernier calliper and calculate its radius.
7. Ask your instructor for the value of the density $\boldsymbol{\sigma}$ of the liquid.
8. Use Eq. (3) to calculate coefficient of dynamic viscosity $\boldsymbol{\eta}$ for different size of balls. Calculate average $\boldsymbol{\eta}$.
9. Coefficient of dynamic viscosity $\boldsymbol{\eta}$ depends sensitively on the temperature. So, record the room temperature and mention it in the result.

**Error analysis and discussion:** Do it yourself. Consult the notes on error analysis available at the course webpage in WeLearn, if necessary.