CS461 Computer Graphics Written Assignment 1

PRIYANSHU SINGH 170101049

OBJECTIVE: You are given 7 points that define two Bezier curves. Establish conditions between points to ensure the continuity of these Bezier curves.

A Bezier curve is a weighted sum of *n+1* control points, P_0, P_1, P_2...P_n where weights are Bernstein polynomials.

$$C(u) = \sum_{i=0}^{n} \mathrm{B}_{n,i}(\mathrm{u}) P_i$$

where $\mathbf{B_{n,i}(u)}$ is defined as follows:

$$\mathrm{B}_{n,i}(\mathrm{u}) = \frac{n!}{i!\,(n-i)!}\, u^i (1-u)^{n-i}$$

The Bezier curves of order *n+1* has *n+1* control points. These are the first three Bezier curve definitions:

$$\textbf{Linear}: C(u) = (1-u)P_0 + uP_1$$

$$\textbf{Quadratic}: C(u) = (1-u)^2 P_0 + 2(1-u)uP_1 + u^2 P_2$$

$$\textbf{Cubic}: C(u) = (1-u)^3 P_0 + 3(1-u)^2 uP_1 + 3(1-u)u^2 P_2 + u^3 P_3$$

So, we are given 7 points, let them be from P_1 - P_7. We have two Bezier curves w.r.t these points.

1. The 4^{Th} point (P_4) is the common point for both these curves and hence positional continuity ($\mathbf{C^0}$) is satisfied.

2. Since the control points are constants and independent of the variable *u*, computing the derivative curve **C**'(*u*) reduces to the computation of the derivatives of $\boldsymbol{B_{n,i}(u)}$'s

$$\mathrm{B}'_{n,i}(\mathrm{u}) = n(B_{n-1,i-1}(u) - B_{n-1,i}(u))$$

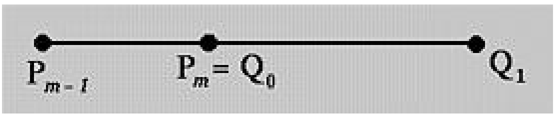
   Then computing the derivative of C(u) yields:

$$\mathrm{C}'(u) = \sum_{i=0}^{n-1} \mathrm{B}_{n-1,i}(\mathrm{u})\{\mathrm{n}(\mathrm{P}_{i+1} - \mathrm{P}_i)\}$$

To satisfy tangential continuity (**C^1**), we need to make sure tangent vector at *u=1* for the first curve (i.e. at the last control point of first curve) is equal to the tangent vector at *u=0* for the second curve (i.e. at the first control point of the curve).

The following image explains this concept clearly. Here P's are the point of 1st curve and Q's are the points of 2nd curve. Point (1) can be clearly seen from this picture. And now let us assume we denote 1st curve by C(u) and 2nd by D(u). So equating C' (1) to D' (0) we get:

$$C'(1) = m(P_m - P_{m-1}) = D'(0) = n(Q_1 - Q_0) \tag{1}$$



On solving for 4 control points i.e. *n=3 and m=3* we have from (1):

$$3(P_4 - P_3) = 3(P_5 - P_4)$$

$$\Rightarrow P_5 = 2(P_4 - P_3) \tag{2}$$

3. To satisfy curvature continuity (**C^2**), we need to make sure that the double derivative at *u=1* for the first curve is equal to the double derivate at *u=0* for the second curve. The double derivative is given by:

$$\mathrm{C}''(u) = \sum_{i=0}^{n-2} \mathrm{B}_{n-2,i}(u)\{\mathrm{n}(\mathrm{n}-1)(\mathrm{P}_{i+2} - 2\mathrm{P}_{i+1} + \mathrm{P}_i)\}$$

On solving for 4 control points i.e. *n=3* we have:

$$3 * 2 * (P_4 - 2P_3 + P_2) = 3 * 2 * (P_6 - 2P_5 + P_4)$$

Substituting the value of P_5 from (2), we have:

$$P_6 = P_2 + 4(P_4 - P_3) \tag{3}$$

From equations (2) and (3), we have the position of points P_5 and P_6 which ensures that the two Bezier curves satisfy **Positional**, **Tangential** and **Curvature** continuity.

APPENDIX:

Brief description of different types of continuity mentioned in the assignment.

1. *Positional Continuity (C^0)*: Two curves having a common endpoint, with no further condition, are positionally continuous. This is the less smooth type of joint, as the only condition to satisfy is the common endpoint, no matter what the flow of the two curves around the joint may be.

2. *Tangency Continuity (C^1)*: Two curves having a common point and tangent vectors lying along the same direction are said to have tangency continuity. The two curves may seem to have the same direction at the joint, but the way they change their direction may still be very different (the rate of this change of direction is called *curvature*).

3. *Curvature Continuity (C^2)*: Two curves having a common point, tangent vectors lying along the same direction, and having the same curvature (which is, the same rate of change of the direction) are said to have curvature continuity. The directions at the joint seem to change with the same "speed".